1. **Fiscal theory**

- A very simple version

1. One period, \( t \).

2. Government debt \( B_{t-1} \) outstanding, comes due at \( t \). (May include \( M_{t-1} \))

3. At the end of \( t \), you must pay net taxes \( p_t s_t \) using money.

4. Equilibrium: \( p_t \) adjusts until

\[
\frac{B_{t-1}}{p_t} = p_t s_t = s_t.
\]

\( B_{t-1}/p_t = s_t \)

---

**House** --- **Print m, burn B** --- **Transactions** --- **Burn m** --- **Basic**
What have we done? Determine the price level with

1. No frictions, no transactions costs, no inventory demand, no OG (time zero markets ok).

2. Fiat money: not commodity money, commodity standard, or explicit redemption promise (peg, currency board).

3. Inflation feels like “aggregate demand,” “too much money chasing too few goods.” Wouldn’t notice!

(a) $p_t$ too low? People have more $M_t = B_{t-1}$ than needed to pay $p_t s_t$. Try to buy goods $\Rightarrow p_t$ rises

(b) $p_t$ too high? People try too sell goods $\Rightarrow p_t$ falls

4. Survives obvious observations: hyperinflations, currency reforms.

5. Argentina, Russia, East Asia. Would any open market operation have worked?

6. No limits on private money – banknotes, checks, etc. No limits on money supply. Only total, government, debt matters.
7. No limit on transactions innovation – credit cards, electronic payment systems, etc.

8. Works in a cashless economy – no money at all.

\[
\text{house } \quad B_{t-1} \quad \rightarrow \quad \text{(Transactions ?)} \quad p_t \quad \rightarrow \quad B_{t-1}/p_t = s_t \quad \text{Cashless}
\]

9. Composition of government debt does not matter.


- Even if it’s not true, (yet, in the US) it’s interesting!

1. Throws out Hume, Fisher, Friedman, MV=PY.

2. Frictionless models as a baseline, first approximation. Later, add frictions (transactions/liquidity spreads, sticky prices, traded/nontraded goods, adjustment costs, ...).
3. Like the term structure. Why is the 0-1 day spread so different from the 29.5-30 spread?

4. Add frictions to study (say) what central bank can do to smooth inevitable exchange rate collapse.

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    title={Price level, exchange rate},
    xlabel={Time},
    ylabel={},
]
\addplot[smooth,mark=none] coordinates {
    (0,0)
    (1,1)
    (2,2)
    (3,3)
};
\end{axis}
\end{tikzpicture}
\end{center}

- Antecedents:

  1. Tax theory of money. (Starr, Chase, others?)

  2. Implicit redemption promise; backing theories of money
3. Wealth effects of government bonds. (Patinkin,..)

4. Asset demand for money (Tobin).


- Paper: studies infinite period version,

1. Standard CIA model, with “bank” only open AM, so must hold M from transactions overnight. $Mv = Py$ is an equilibrium condition.

2. Open bank PM (as above), $v = \infty$. M holdings drop to zero.

3. Still, $\frac{B_{t-1}}{p_t} = Et \sum_{j=0}^{\infty} m_{t,t+j}s_{t+j}$ determines price level.

4. Proper model: Government choice $\{s_t, B_t\}$; consumer’s problem, definition of equilibrium, etc.
Money as stock analogy.

1. Microsoft stock becomes numeraire, unit of account, and medium of exchange. 

   \[
   \frac{\text{number of shares}}{\text{price level}} = \text{Expected present value of future dividends or earnings.}
   \]

   (price level= shares per good, not goods/share.) This economy could work. 
   Note: no frictions.

2. The fiscal theory: nominal debt, is a residual claim to government primary surpluses. No surpluses? Inflate away debt. Therefore, 

   \[
   \frac{\text{nominal government debt}}{\text{price level}} = \text{Expected present value of future primary surpluses.}
   \]

   (Rolled over rather than long term, but this does not make any difference.)

3. Bad news. Explaining stock price movements by subsequent dividends is not a great empirical success!
2 Theoretical controversies

• “Ricardian and non Ricardian Regimes”

• One period

\[
\frac{B_{t-1}}{p_t} = s_t
\]

What if \(s(p)\)? Can be no equilibrium or infinite number of equilibria. For example, if nominal rather than real taxes are fixed, \(s(p_t) = S\) (or \(s(t)(p_t) = S/p_t\))

\[
B_{t-1} = S,
\]

\(p\) can’t help. (In fact real taxes are set: \(p_t s_t = \tau(p_t y_t)\).)

• Infinite period. Policy \(s_t(\{p_t\})\) s.t.

\[
\frac{B_{t-1}}{p_t} = E_t \sum m_{t+j} s_{t+j}
\]

holds for any \(\{p_t\}\). Easier: any positive reaction of surplus to outstanging debt

\[
s_t = s_0 + \alpha \left( \frac{B_{t-1}}{p_t} \right), \alpha > 0
\]

will work
Isn’t this the “government budget constraint?”

1. *Must* the gov’t set \( s \) (ex-post) to \( B_{t-1}/p_t \)?

2. What about Porsches? \( B = 10 \) in pocket, \( s \) =Porsche purchase policy, \( B/p = s \)?

3. Mustn’t we write “The government chooses \( \{B_{t-1}, s_t\} \) subject to \( \frac{B_{t-1}}{p_t} = s_t \)?”

4. Does the government “threaten to violate its budget constraint at off equilibrium prices?”

5. If so, a special assumption, interesting new economics or a mistake.

- Kocherlakota and Phelan (1999)

The key force behind the fiscal theory is that a government is fundamentally different from households. Households need to satisfy their budget constraint for all prices, regardless of whether or not those prices are equilibria. A government does not.
The recently revived ‘fiscal theory of the price level’ is an example of a research programme that it fatally flawed, conceptually and logically. The source of the fallacy is an economic misspecification. The proponents of the fiscal theory do not accept the fundamental proposition that the government’s intertemporal budget constraint is a constraint on the government’s instruments that must be satisfied for all admissible values of the economy-wide endogenous variables [prices]. Instead they require it to be satisfied only *in equilibrium* (p.1).

Ljungqvist and Sargent (2000, p.507), Bohn (1999): The fiscal theory assumes that the government violates a budget constraint at off-equilibrium prices, and that the government is special in its ability to do so.

Marimon (2001): “a theory that does not respect Walras’ law.”
• Woodford (2001): government violates budget constraint

Even if we wish to analyze the behavior of an optimizing government, the government should not optimize subject to given market prices and a given budget constraint, as private agents are assumed to in the theory of competitive equilibrium. For the government is a large agent, whose actions can certainly change equilibrium prices, and an optimizing government surely should take account of this in choosing its actions.

• Cochrane (1999) too!

• Answer: **No.** $B_{t-1}/p_t$ is a valuation equation, a market-clearing condition, not a budget constraint. The government may choose $\{B_{t-1}, s_t\}$ and let $\{p_t\}$ adjust. *Nothing special about the government.* It may also choose to be Ricardian, in which case $p$ must be determined elsewhere.

• One period model: If $p$ is too low, households left with $B_{t-1}$. That’s ok. If $p$ is too high, agents run out of money before taxes are paid. That’s ok. Markets don’t clear when a market clearing condition is violated. No budget constraint is violated.
• What if agents discover a wallpaper demand for money at the end of the period? Then \( B_{t-1} \) is outstanding in equilibrium. A change in preferences can’t affect a budget constraint.

• If \( p_t \) is too low, must the government raise \( s_t \) so \( B_{t-1}/p_t = s_t \) again?

1. If \( p \) is too high, there is no budget constraint that forces Microsoft to increase earnings so

\[
p_t \times \text{shares} = E \sum m_{t,t+j} e_{t+j}
\]

It can honor promises of equity at any \( p_t \)

2. IOU for beer after seminar.

3. Similarly, the government can honor the promises of nominal debt (exchange for \( M \), accept \( M \) for tax payments), for any off-equilibrium \( p \)

\[
\frac{B_{t-1}}{p_t} = E_t \sum m_{t,t+j} \tilde{s}_{t+j}
\]

is not a budget constraint.
- Can the government change $B_{t-1}$, hold $s_t$ fixed, count on $p_t$ to change?

1. No, if “choose $\{B, s\}$ given $p$ subject to $B_{t-1}/p_t = s_t$”


4. If a currency reform/split is possible, then $B_{t-1}/p_t = s_t$ is not a budget constraint!

5. Note careful marketing difference between reform (no change in $s_{t+j}$, please change $p_t$) and treasury issue (we will increase $s_{t+j}$, please no change in $p_t$). Note careful marketing difference between IPO and split.

- Equity: $\{\text{earnings, shares}\}$ is a definition of securities. It comes before any prices are announced, and cannot be contingent on prices, or you issue infinite amounts.

- $\{B_{t-1}, s_t\}$ is the same. Nominal government debt is equity. The quantity issued is a definition of units, a definition of a security, like share issues; not a decision of how many units of a defined good or security to buy or sell.
• At some point, \( s \) is limited. Every government must be non Ricardian at some point. Hence, it must be possible to be Non-Ricardian.

• The government does face budget constraints in other transactions, buying and selling goods and securities whose values have been defined, just as private agents do.

1. Real debt – indexed or foreign-denominated; gold or foreign currency = numeraire. Then the security is determined.

\[
\frac{B_{t-1}}{p_t} = E_t \left( m_{t,t+1} \frac{B_t}{p_{t+1}} \right) + s_t
\]

\[
\frac{B_{t-1}}{p_t} = E_t \sum m_{t,t+j} s_{t+j}
\]

really are constraints.

(a) Price of gold or foreign currency rises? Must raise \( s \) ex-post

(b) Issue more debt without increasing \( s \)? Can’t do it. \( p \) will not change.
2. Same for private agents: If you issue state-noncontingent, completely non-defaultable debt, you have to raise the real resources (ex-post) to pay it off one way or another.

- Real debt = debt. Nominal government debt = Equity.

- Alternative, equity-like securities? State-contingent default without inflation? Or is it a good idea to have a lot of voters mad when the government “defaults,” not just a few bondholders?

- Summary: Ricardian regime is a possible choice by governments, to give price power to its Fed, within range of taxing power. It is not a requirement, fiscal regimes or mixed regimes are possible.
The same arguments hold for the flow constraint.

1. Is the fiscal theory limited to picking $p_0$, “solving initial indeterminacies,” or can the government freely pick $B_{t-1}$ to set $E_{t-1}(p_t)$ (or $p_t$ in perfect foresight models) at each date?

2. 

$$B_{t-1} = p_t s_t + Q_t B_t$$

$$B_{t-1} = p_t s_t + E_t \left( m_{t+1} \frac{p_t}{p_{t+1}} \right) B_t$$

$$\frac{B_{t-1}}{p_t} = s_t + E_t \left( m_{t+1} \frac{B_t}{p_{t+1}} \right)$$

Given $\{s_t, B_{t-1}\}$, the choice of $B_t$ follows from this “flow constraint,”

$$B_t = \frac{B_{t-1}}{p_t} - s_t$$

$$= E_t \left( m_{t+1} \frac{1}{p_{t+1}} \right)$$

3. No, by exactly the same logic. Currency reform (change $B_{t-1}$) is possible.
2.1 Explosive debt

- Is the equilibrium maintained by an incredible threat by the government to let debt explode at off-equilibrium prices?

- Doesn’t Maastricht and the US experience of the 1990s tell us that governments will follow Ricardian policies, even if they don’t have to?

- Example: perfect-foresight, constant interest rate $r$. What if $\{p_t\}$ satisfy

$$
\frac{B_{t-1}}{p_t} = s_t + \frac{1}{r} \frac{B_t}{p_{t+1}}.
$$

but the overall level is too low

$$
\frac{B_{t-1}}{p_t} > \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}
$$

– “off the equilibrium path?”

- Answer: subtract $\sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}$; multiply by $r$,

$$
\left( \frac{B_t}{p_{t+1}} - \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+1+j} \right) = r \left( \frac{B_{t-1}}{p_t} - \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j} \right)
$$
The real value of debt grows explosively, violating the transversality condition. The government does “threaten explosive debt at off-equilibrium prices”

1. *So does Microsoft.* If Microsoft stock is overpriced, but grows at the proper rate of return, the value of Microsoft stock grows explosively.

2. This fact violates no budget constraint. Microsoft and the government *can* ignore exploding or imploding secondary-market prices.


4. One period debt: Extra revenue from selling new bonds at “too low” $p_{t+1}$ pays off extra value of old bonds at “too low” $p_t$.

5. *It’s worse than you think.* In complete markets, off equilibrium prices are *arbitrage opportunities*; agents can make immediate, unbounded profits. Sell claim, buy dividend stream, go to lunch.
• Exploding off-equilibrium debt is possible, but is it credible?

1. Didn’t the US raise taxes in response to 80s debt increase? Doesn’t Maastricht require taxes to increase if debt increases?

2. Increases in nominal debt at constant price levels are not the same as an (off-equilibrium) deflation with constant nominal debt. 80’s, Maastricht, WWII are not relevant. (Abrogation of gold clauses?)

3. \[
\frac{B_{t-1}}{p_t} = E_t \sum m_{t,t+j} \tilde{s}_{t+j}.
\]

If the government issues more \( B_{t-1} \) with no change in \( \tilde{s}_{t+j} \), \( p_t \) changes 1 for 1, the real value of the debt does not change, and the government raises no revenue. To raise revenue by increases in nominal debt, the government must (implicitly) promise more \( s_{t+j} \). Thus, “rules” to increase taxes when real debt rises because nominal debt increased at constant \( p_t \) just mean “keep your commitments.”

• No test, no history. Alas, we never observe off-equilibrium behavior. That’s the
point!

\[ \frac{B_{t-1}}{p_t} = E_t \sum m_{t,t+j} \tilde{s}_{t+j} \]

holds in both Ricardian and NonRicardian regimes, in equilibrium. Granger tests whether \( s \) causes \( B/p \) or vice versa are pointless. War, recession of a government that wants smooth prices looks the same in either regime.
3  Fiscal and quantity theories

- Special cases of
  \[ M_tv = pty \] \hspace{1cm} (3)
  \[
  \frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j} \] \hspace{1cm} (4)

1. Government chooses \( \{B_t, M_t, \tilde{s}_t\} \).

2. Two equations in one unknown \( \Rightarrow \) policies must be coordinated (Sargent)

3. \( \Leftrightarrow \) Equilibria only exist for a restricted set of \( \{B_t, M_t, \tilde{s}_t\} \)

4. “Regimes,” “Game of chicken” stories to achieve coordination. No unusual economics in the fiscal regime; no clear line between “regimes.”
\[
\begin{align*}
M_{t+1}p_t & = p_t y \\
\frac{B_{t-1}}{p_t} & = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\end{align*}
\]

- **Monetarist.**

1. Fed fixes \( \{M_t\} \Rightarrow \{p_t\} \) with (3)

2. Treasury adjusts \( \{\tilde{s}_t\} \) so (4); “Ricardian” fiscal policy

3. Example: Fed deflates, Treasury must raise taxes to pay gift to bondholders.

4. Treasury need not feel passive.

5. CIA models: seigniorage rebated by lump-sum transfer.
\[
\begin{align*}
  M_t v &= pty \\
  \frac{B_{t-1}}{p_t} &= E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\end{align*}
\]

- **Fiscal.**

1. Treasury sets \( \{ \tilde{s}_t \} , \{ B_t \} \). \( \Rightarrow \) \( \{ p_t \} \) with

\[
\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\]

2. Fed “passively” sets

\[ M_t = pty/v. \]

3. Passive money – real bills; “meet needs of trade” – need not lead to undetermined price level.

4. Example: If \( \{ s_{t+j} \} \) falls, Fed must raise \( M_t \).

5. Fed need not feel passive.
\[
\begin{align*}
M_{tv} &= p_{ty} \\
\frac{B_{t-1}}{p_t} &= E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\end{align*}
\]

- Both equations hold in equilibrium. **You cannot test for the Money vs. Fiscal regimes from time series data. You cannot test off-equilibrium threats from data on an equilibrium.**

- “Money as stock”.

1. Can sensibly delete \( M_{tv} = p_{ty} \); or \( v = \infty \) in a fiscal regime.

2. Treasury sets \( \{\tilde{s}_t\}, \{B_t\} \), then

\[
\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\]

determines \( \{p_t\} \). Period.
1. Break surplus into fiscal and seigniorage components:

\[ \tilde{s}_t = s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_{t+j}} \]

\[
\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[ s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_{t+j}} \right]
\]

2. Treasury sets \( \{B_t\} ; \{s_t\} \), not \( \{\tilde{s}_t\} \)

3. Fed sets \( \{M_t\} \).

4. Many paths of \( \{M_t\} \) produce equilibria.

5. Low \( M_t \) now \( \Rightarrow \) low \( p_t \) now \( \Rightarrow \) high \( B_{t-1}/p_t \) \( \Rightarrow \) must have higher future \( \Delta M_{t+j} \), \( \Delta p_{t+j} \). Future inflation tax pays for lower \( \{s_{t+j}\} \).

6. High \( M_t \) now \( \Rightarrow \) high \( p_t \) now \( \Rightarrow \) lower \( B_{t-1}/p_t \) \( \Rightarrow \) lower future \( \Delta M_{t+j} \), \( \Delta p_{t+j} \). Implicit default on outstanding bonds pays for lower \( \{s_{t+j}\} \).

7. Real vs. nominal debt. *Nominal debt gives a direct, non seigniorage channel.*
• *Fiscal rehabilitation of interest rate targets.*

1. Deterministic:

(a) Fed sets \( \{M_t\} \) to hit \( p_{t+1}/p_t = \pi \)

(b) If Treasury is Ricardian, this determines inflation rate but not initial price level \( p_0 \). "Price level indeterminacy"

(c) Make Treasury Non-Ricardian; determine \( p_0 \) with

\[
\frac{B^{-1}}{p_0} = \sum_{t=0}^{\infty} \beta^t s_t
\]
2. Interest rate targets, Stochastic case:

(a) Fed sets \( \{M_t\} \) to target \( E_t(p_{t+1}/p_t) = \pi \).

(b) If the Treasury is Ricardian,

\[
\frac{p_{t+1}}{p_t} = \pi + \varepsilon_{t+1}; \quad \frac{M_{t+1}}{M_t} = \pi + \varepsilon_{t+1};
\]

But any \( \varepsilon_{t+1} \) is possible; “indeterminacy” each day.

(c) If the Treasury is NonRicardian we can determine \( p_t \) each day with

\[
\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j}
\]
Hyperinflation dynamics and indeterminacies

1. Ingredients: Perfect foresight, Fed sets the money supply; e.g. \( M_t = M \), Interest elastic money demand.

\[
M \bar{v} \left( \frac{p_{t+1}}{p_t} \right)^b = p y 
\]

\[
\ln p_{t+1} - \ln \left( \frac{M \bar{v}}{y} \right) = \frac{1 + b}{b} \left[ \ln p_t - \ln \left( \frac{M \bar{v}}{y} \right) \right]
\]

2. Treasury “passive” solutions. \( p_0 \) is not determined. “indeterminacy.”
3. Determine $p_0$:

(a) Standard selection: arbitrarily pick $p_0 = M\bar{v}/y$ “Minimum state variable” “Restrict to local equilibria” or other philosophical criterion.

(b) Woodford 1995: Add Ricardian fiscal policy; determine $p_0 = B_{-1}/\sum \frac{1}{r_t} s_t$. If $\{p_t\}$ is explosive, so what?

(c) McCallum: Doesn’t work if $B_{-1}/\sum \frac{1}{r_t} s_t < M\bar{v}/y$. then $p_t \rightarrow 0$; $M/p_t \rightarrow \infty$ violating the consumer’s transversality condition.

(d) Answer: Yes. A “coordination” restriction on $\{B_t, M_t, s_t\}$ that can give an equilibrium, in one particular specification. Not a criticism of “fiscal theory” in general.
• Important. What determines the price level in the US today?

1. $Mv = py$, control of $M$? No.


3. Interest rate target, Taylor rule determinacy? Claim

   \[ i_t = \bar{i} + \phi_{\pi} \pi_t + \phi_y y_t \]

   (or $\pi_{t-1}$, or $E_t \pi_{t+1}$, etc.)

   $\phi_{\pi} > 1 \iff$ price level determined. *The only potential alternative to a fiscal regime.*

• What do we need for Taylor rule to work? Must you believe in sticky price-Neo Keynesian models? Or adaptive expectations irrational models?

• Wall Street Journal, Econ 101: $\pi_t \uparrow \rightarrow i \uparrow \rightarrow r_t \uparrow \rightarrow y_{t+1} \downarrow \rightarrow \pi_{t+1} \downarrow$. Result: stable dynamics for $\pi_t$.

   \[ \pi_t = (1 - \alpha)\bar{\pi} + \alpha \pi_{t-1} + \varepsilon_t. \]
Formal model?

- Taylor 1999 JME

\[
y_t = -\beta(i_t - \pi_t - r) + u_t \quad \text{“IS”}
\]

\[
\pi_{t+1} = \pi_t + \alpha y_t + e_{t+1} \quad \text{“Phillips”}
\]

\[
i_t = g_0 + g_{\pi} \pi_t + g_y y_t \quad \text{“Taylor rule”}
\]

Intuition: \( \pi_t \uparrow \), then \( i_t \) up more in Taylor rule. This sends \( i_t - \pi_t \) up and \( y_t \) down in IS, and then \( \pi_{t+1} \) down in Phillips: stable \( \pi \) dynamics. Formally, (simple version, \( g_y = 0 \))

\[
\pi_{t+1} = \pi_t + \alpha \left[ -\beta(i_t - \pi_t - r) + u_t \right] + e_{t+1}
\]

\[
\pi_{t+1} = \pi_t - \alpha \beta \left( g_0 + g_{\pi} \pi_t \right) + \alpha \beta \pi_t + \alpha \beta r + \alpha u_t + e_{t+1}
\]

\[
\pi_{t+1} = \left[ 1 + \alpha \beta (1 - g_{\pi}) \right] \pi_t - \alpha \beta g_0 + \alpha \beta r + \alpha u_t + e_{t+1}
\]

\( g_{\pi} > 1 \) now means \( \left[ 1 + \alpha \beta (1 - g_{\pi}) \right] < 1 \), stable dynamics, and deterministic.

- “Old fashioned:” IS has only current interest rate. Real rate is \( i \) minus past inflation. Mechanical accelerationist Phillips curve. No \( E_t \) anywhere.
• Woodford (*Interest and Prices*). Completely written down model, beautifully solved (of course). Money in U. *Explicit Ricardian regime.* (Also Benhabib, Schmitt-Grohé and Uribe in continuous time.)

1. Central equilibrium conditions:

\[ i_t = \bar{i}_t + \phi \pi_t \pi_t \]  
\[ i_t = r_t + E_t \pi_{t+1} \]  

(2.18) Rule  
(1.30) Fisher

\[
\bar{i}_t + \phi \pi_t = r_t + E_t \pi_{t+1} \\
\phi \pi_t = E_t \pi_{t+1} + r_t - \bar{i}_t 
\]

(2.19)

2. Woodford: Examine only “unique bounded solution.” With \( \phi > 1 \),

\[
\pi_t = \sum_{j=0}^{\infty} \phi^{-(j+1)} E_t (r_{t+j} - \bar{i}_{t+j}) 
\]

(2.20)

Woodford’s point: Taylor isn’t enough for optimal policy. *Taylor intercept \( \bar{i} \) must move to match real rate changes \( r \).* “Neo-Wicksellian”

3. (Note: \( i_t = \phi \pi_t + \bar{i}_t \); \( \bar{i}_t \) optimally set will ruin estimates!)
4. Determinacy:

\[ E_t \pi_{t+1} = \phi_\pi \pi_t - (r - \bar{\gamma}) \] (2.19).

Solutions

\[ \pi_t = \bar{\pi} + \omega_t \]
\[ \omega_{t+1} = \phi_\pi \omega_t + \varepsilon_{t+1} \]

(a) \( \phi_\pi < 1 \): Multiple solutions. “Indeterminacy.”

(b) \( \phi_\pi > 1 \): Multiple solutions. But all except \( \omega = 0 \) are explosive. \( \pi_t = \bar{\pi} \) is the “unique locally bounded equilibrium.”

5. Who said price level must be bounded? No equilibrium condition (including transversality) is violated by nominal explosions? Where is Woodford (1995)?

6. Rule: *Not* WSJ dynamics. No Phillips curve here. *Fed threatens hyperinflation if actual inflation is 0.1 too big.* This “coordinates expectations.”

- JC conclusion. Taylor rule does not give price level determinacy in frictionless models. Bring back the equation, a non-Ricardian regime!
Neo Keynesian models: Clarida, Gali, Gertler, King, Woodford, etc.

- Model

\[
\text{IS} : \quad y_t = E_t y_{t+1} - \sigma [i_t - E_t \pi_{t+1}] + x_{dt}
\]
\[
\text{Phillips} : \quad \pi_t = E_t \pi_{t+1} + \kappa (y_t - \bar{y}) + x_{\pi t}
\]
\[
\text{Rule} : \quad i_t = \bar{i}_t + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (y_t - \bar{y})
\]

- Compare to Taylor

\[
\text{IS} : \quad y_t = -\beta (i_t - \pi_t - r) + u_t
\]
\[
\text{Phillips} : \quad \pi_{t+1} = \pi_t + \alpha y_t + e_{t+1}
\]

New Features.

1. IS has $E_t y_{t+1}$ with unit coefficient. From $1 = E(mR)$. Thus all future $r$ matter!

2. Phillips has expected future not past inflation.
• Result: Woodford dynamics, not Taylor dynamics. ($\phi_y = 0$ case)

$$
\pi_t = -\frac{\kappa \sigma}{(1 + \kappa \sigma \phi_\pi)} l + \frac{1 + \kappa \sigma}{1 + \kappa \sigma \phi_\pi} E_t \pi_{t+1}
$$

• JC conclusion: *Neo Keynesian models do not deliver price determinacy from Taylor rules either. Choice: Mechanistic Keynesian models, or NonRicardian Regime.*
4 Long term debt

- \( \frac{\text{nominal debt}}{\text{price level}} = \text{present value of real surpluses.} \) (8)
  
  If surpluses fall, maybe long term bond prices fall instead of price rising \(\iff\) future not current inflation.

- What is the effect of surplus news? Example: Perpetuity pays coupon \(c_t\). Then, \(\frac{c_t}{p_t} = s_t\)

  just like the one period model! News of \(s_{t+j}\) has no effect at \(t\). (8) is still there; \(s_{t+j}\) news changes \(E_t p_{t+j}\) and hence lowers long-term bond price. The maturity structure of the debt has an important effect on the time-series relation between surpluses and inflation.
• What happens if the government sells more debt $B_t$ at $t$, holding $\{s_t\}$ constant?

1. One period debt:

$$\frac{B_{t-1}}{p_t} = E_t \sum \beta^j s_{t+j}$$

$$\frac{B_t}{p_{t+1}} = E_{t+1} \sum \beta^j s_{t+1+j}$$

$$Q_t = \beta E_t \left( \frac{p_t}{p_{t+1}} \right)$$

No effect on $p_t$. $p_{t+1}$ rises. Bond price $Q_t$ falls. $Q_tB_t =$ Revenue from debt sale = constant. “Unit elastic demand curve”
2. Debt sale $B_t$ with long term debt. 2 period example. $B_{\text{when issued}}$ (when due)

\[
\frac{B_1(2)}{p_2} = s_2
\]

\[
\frac{B_0(1)}{p_1} + \beta E_1 \left( \frac{1}{p_2} \right) B_0(2) = s_1 + \beta E_1(s_2)
\]

\[
\frac{B_0(1)}{p_1} + \beta E_1 \left( \frac{s_2}{B_1(2)} \right) B_0(2) = s_1 + \beta E_1(s_2)
\]

If $B_0(2) > 0$, debt sales $B_1(2) - B_0(2)$ now lower $p_1$! (It still raises $p_2$)

Selling more 2 debt at 1 devalues outstanding 2 claims – lowers their value. “State contingent default” of long term debt, not debt due today. The Fed can affect the time-path of inflation.

3. Before, if $s_t$ is low, $p_t$ will be high (as in the last period). Now, you can meet $s$ shortage with future rather than current inflation.

- *Dynamics depend sensitively on the maturity structure of government debt.*