1. (Asymmetric Random Walk). Let $S_n$ be an asymmetric random walk with $p > \frac{1}{2}$. That is, $S_n = X_1 + X_2 + \ldots + X_n$ where $P(X_i = 1) = p$ and $P(X_i = -1) = 1 - p$. Let $\sigma^2 = 1 - (p - q)^2$. Show that, $Z_n$, is a martingale where

$$Z_n = (S_n - (p - q)n)^2 - \sigma^2 n.$$

Suppose $p > 1/2$ and let $\phi(x) = (q/p)^x$. Show that $\phi(S_n)$ is a martingale.

Let $T_x = \inf\{n : S_n = x\}$. Then for $a < 0 < b$ show that

$$P(T_a < T_b) = \frac{\phi(b) - \phi(0)}{\phi(b) - \phi(a)}.$$

Also, show that

$$E(T_b) = \frac{b}{2p - 1}.$$

2. (Brownian Motion). Let $B_t$ be a standard Brownian motion.

- Which of the following are also a Brownian motion: $-B_t$, $\sqrt{t}B_1$ or $B_{2t} - B_t$?
- Find the mean function and variance for the following: $X_t = |B_t|$ and $Y_t = e^{B_t}$.

3. (AI and Deep Learning).

Mark Cuban a famous tech entrepreneur recently said in an interview to CNBC (May, 2017) that “study AI or you’ll be a dinosaur in 3 years”.

Click on the hyperlinks below and read the two articles. Pick one of the articles and discuss its implications for business applications.

(a) Companies already using AI

(b) Digital Marketing

In particular, your answer should include a discussion of the appropriate machine learning tools that you’ve learned in class. Provide a description of the problem that AI is designed to solve and a discussion of input/output variables used.