In this article and as our title suggests, we demonstrate a method for understanding the intuition behind the Black-Litterman asset allocation model. To do this, we use examples to show the difference between the traditional mean-variance optimization process and the Black-Litterman process. We show that the mean-variance optimization process, while academically sound, can produce results that are extreme and not particularly intuitive. In contrast, we show that the optimal portfolios generated by the Black-Litterman process have a simple, intuitive property:

- The unconstrained optimal portfolio is the market equilibrium portfolio plus a weighted sum of portfolios representing an investor’s views.
- The weight on a portfolio representing a view is positive when the view is more bullish than the one implied by the equilibrium and other views.
- The weight increases as the investor becomes more bullish on the view as well as when the investor becomes more confident about the view.

December 1999
Appendix C

1. Given the expected returns $\mu$ and the covariance matrix $\Sigma$, the unconstrained maximization problem $\max w' \mu - \frac{1}{2} \delta w' \Sigma w$ has a solution of $w^* = \delta \mu \Sigma^{-1}$.

2. Given the covariance matrix $\Sigma$, the minimum variance portfolio is $w_{min} = \delta \Sigma^{-1} \iota$, where $\iota$ is a vector with all elements being one.

3. The solution to the risk constrained optimization problem $\max w' \mu$, subject to $w' \Sigma w = \sigma^2$, can be expressed as $w^* = \delta \mu \Sigma^{-1} \Sigma^{-1/2} \iota$, where $w^*$ is the solution of the unconstrained problem.

4. The risk and budget constrained optimization problem can be formulated as $\max w' \mu$, subject to $w' \Sigma w = \sigma^2$ and $w' \iota = \iota$.

5. The risk-, budget-, and beta-constrained optimization problem can be formulated as $\max w' \mu$, subject to $w' \Sigma w = \sigma^2$, $w' \iota = \iota$, and $w' \Sigma w = \iota$.

References


executive Summary

be the expected returns by using

\[ \sum (\mu \tau \tau = \Lambda) \]

There are 11 assets in the market. The market portfolio (equilibrium portfolio) is

\[ \Omega \]

is normally distributed with mean zero and covariance

\[ \Sigma \]

The mean of the expected returns is

\[ \mu \]

additional view will have a zero weight if

\[ \lambda \]

is given by the following formula

\[ \Pi \Sigma = \delta \]

Let

\[ \mu \]

has one additional view, represented by

\[ \lambda \]

is normally distributed with mean zero and covariance

\[ \Pi \]

is the neutral reference point of the Black-Litterman model. Presumably it is because of the way the views are being translated into

the view into the expected returns. Can she possibly do better?


diagram
Sachs Asset Management’s prior written consent.

in light of their experience, circumstances and financial resources.

assess the exposure to risk. Investors should carefully consider whether such investments are suitable for them.

originally invested and may possibly result in unquantifiable further loss exceeding the amount invested.

exchange, may involve contingent liability resulting in a need for the investor to pay more than the amount.

a relatively small movement in price of the underlying security or benchmark may result in a disproportionately

restricted or illiquid, there may be no readily available market and there may be difficulty in obtaining reliable

Past performance is not a guide to future performance and the value of investments and the income derived from

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inform themselves and take appropriate advice as to any applicable legal requirements and any applicable

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Appendix A

Table 1: Annualized volatilities, market-capitalization weights, and equilibrium expected returns for the equity markets in

Table 2: Correlations among the equity index returns.

Goldman Sachs Investment Management

Investment Management Research The Intuition Behind Black-Litterman

Second, investment managers tend to think in terms of weights in a

portfolio rather than balancing expected returns against the contribution to

portfolio risk—the relevant margin in the Markowitz framework. When

managers try to optimize using the Markowitz approach, they usually find

generate very extreme portfolios

What this investor finds is that

What this investor finds is that

The weight for France now is -94.8 percent!

The weight for France now is -94.8 percent!

The weight for France now is -94.8 percent!

The weight for France now is -94.8 percent!

The weight for France now is -94.8 percent!

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The weight for France now is -94.8 percent!

The weight for France now is -94.8 percent!
or Expected Returns

Investment Management Research
The Intuition Behind Black-Litterman

Members of Goldman Sachs Investment Management

General

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There are many different ways to translate the views or expected returns. For example, the investor could simply add the expected return for Germany to the expected return for the rest of Europe to achieve the desired return. However, this approach may suggest that Germany will outperform the rest of Europe, which already implies that Germany will outperform the rest of Europe. To be precise in expressing her view, she sets the expected return for Germany 5% higher than the (market capitalization) weighted average of the expected returns of France and the United Kingdom. She sets the sum of market capitalization-weighted expected returns for the rest of Europe as the risk aversion parameter representing the world average risk tolerance.

Expected Returns Shifted for European Countries

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Table 1: Annualized volatilities, market-capitalization weights, and equilibrium expected returns for the equity markets in

<table>
<thead>
<tr>
<th>Country</th>
<th>Volatility (%)</th>
<th>Weight (%)</th>
<th>Expected Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>16.0</td>
<td>1.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Germany</td>
<td>27.1</td>
<td>5.5</td>
<td>9.0</td>
</tr>
<tr>
<td>Japan</td>
<td>21.0</td>
<td>11.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Canada</td>
<td>0.488</td>
<td>0.664</td>
<td>0.515</td>
</tr>
<tr>
<td>France</td>
<td>0.478</td>
<td>0.655</td>
<td>0.861</td>
</tr>
<tr>
<td>UK</td>
<td>0.512</td>
<td>0.608</td>
<td>0.783</td>
</tr>
</tbody>
</table>

Goldman Sachs Investment Management

Specifying a Starting Point

This chart is to be used for illustrative purposes only.

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Chart 1A. Optimal Weights, Traditional Mean-Variance Approach

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Executive Summary

This corresponds to the case where the new view is $qp = \delta - \Sigma$. The vector $\delta - \Sigma$ (EX ᴞ). The investor has the world average risk tolerance. The objective of the investor is to maximize the utility implied by the old expected returns already. In this case, the new view has no impact at all. For a particular view the new view on the portfolio views. The weights for these portfolios are given by the elements of the vector $ww_{\text{w}}$. Since $\lambda$ is normally distributed with mean zero and covariance $\mu$, its weight $\omega$ to understand.

Conclusion

The Black-Litterman model starts with equilibrium expected returns (as derived via the Capital Asset Pricing Model). This set of equilibrium weights $\tau$ provides both a reference point for expected return assumptions as well as a framework has had surprisingly little impact. Why is that the case? We cite practical issues in using the Markowitz framework by allowing the portfolio manager to express views about portfolios, rather than a complete vector of expected returns. Having formed the foundation of the view into the expected returns. Can she possibly do better?

The Black-Litterman asset allocation model addresses those practical issues simply $L$. The Litterman model provides the appropriate weights on the portfolios, based each of the portfolios about which a view is expressed against its

Appendix B

1. There are $n$ funds to consider. The market portfolio is given by $\mu_{w}$. The vector of stock returns is $\mu$. The portfolio is expressed as a $w = \mu_{w} / \sqrt{\mu_{w}^T \Sigma^{-1} \mu_{w}}$ solution of the Markowitz optimization problem.
2. The market equilibrium is expressed as $\mu_{w} = \mu_{0} + \Omega \mu_{1}$, where $\Omega$ is a matrix of expected returns of assets as well as the optimal portfolio weights. The model combines the views with equilibrium, producing both the set of expected returns of assets as well as the optimal portfolio weights.
In the case when the investor does have views about the market, the Black-Litterman model is used to incorporate these views into the equilibrium expected returns. The model takes these views and constructs a set of expected returns on each asset. Although we manage many portfolios using different benchmarks, different targeted risk levels, and different constraints on the portfolios, the same set of expected returns from the Black-Litterman model is used for all of the expected returns away from their starting values in a manner consistent with the same set of views, and all will have exposures to the same set of historically profitable return-generating factors.

The real power of the Black-Litterman model arises when there is a real demand for these assets because the investors hold the same belief, the demand for these assets will exactly equal the expected supply. This set of expected returns is the optimal allocation point of the Black-Litterman model. The investor then can express her views in this manner.

In the Black-Litterman model, a view is a general statement about the expected return for any portfolio. Views are not combined with the market equilibrium expected returns as in the traditional approach. Instead, the views are combined with the market equilibrium expected returns to arrive at a set of expected returns. The views can be expressed using a variety of linear and nonlinear constraints. This property is quite intuitive, since it is reasonable to expect that a view on the market will be represented in the optimal portfolio.

The Black-Litterman model starts with equilibrium expected returns according to the Capital Asset Pricing Model (CAPM), which are then adjusted to reflect the investor’s views. The views can be either positive or negative, depending on whether the investor believes that asset prices are too high or too low. In contrast to the traditional approach, the Black-Litterman model adjusts all of the expected returns away from their starting values in a manner consistent with the same set of views, and all will have exposures to the same set of historically profitable return-generating factors.

Example: consider the case when the investor has one or more views about the market, the Black-Litterman model adjusts all of the expected returns away from their starting values in a manner consistent with the same set of views, and all will have exposures to the same set of historically profitable return-generating factors.
The solutions to the unconstrained optimization problem as well as to several special constrained optimization problems are given in Appendix C.

Chart 5. Black-Litterman Model with Two Views, Risk Constrained

With this constraint, there is a special minimum-variance portfolio of the market equilibrium portfolio (Chart 5).

From the graph of the expected returns in Chart 2A, it seems counter-intuitive to see that the expected returns for both France and the United Kingdom are raised. On the contrary, the view expressed does not say anything about the risk level of these two countries. The deviation of the optimal portfolio from the equilibrium weights is positively correlated with the view portfolio and the view raises the expected returns for both France and the United Kingdom. The same intuition can be derived from the graph of the optimal portfolio weights in Chart 2B. Compared to the equilibrium weights, the optimal portfolio increases the weight in Germany and decreases the weights in both France and the United Kingdom. One can see that the deviations from the equilibrium weights are proportional to the portfolio of long Germany and short France and the United Kingdom—exactly the portfolio representing the investor's view. This result is very intuitive. Since the investor has no view about the portfolio of the market equilibrium, the optimal portfolio weights are the equilibrium weights.

The optimal portfolio weights in Chart 2B are computed by solving the unconstrained problem. Compared to the equilibrium weights, the optimal portfolio increases the weight in Germany and decreases the weights in both France and the United Kingdom. The deviations from the equilibrium weights are proportional to the portfolio of long Germany and short France and the United Kingdom—exactly the portfolio representing the investor's view. This result is very intuitive. Since the investor has no view about the portfolio of the market equilibrium, the optimal portfolio weights are the equilibrium weights.

<table>
<thead>
<tr>
<th>Asset</th>
<th>France</th>
<th>Germany</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>-0.05</td>
<td>0.02</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

This chart is to be used for illustrative purposes only.

In many cases, in addition to the risk constraint, the investor faces a budget constraint which forces the sum of the total portfolio weights to be one. With this constraint, there is a special minimum-variance portfolio of the market equilibrium portfolio (Chart 2C). The optimal portfolio is a linear combination of the unconstrained optimal portfolio and the equilibrium weights. The deviations from the equilibrium portfolio are positively correlated with the view portfolio and the view raises the expected returns for both France and the United Kingdom. The same intuition can be derived from the graph of the optimal portfolio weights in Chart 2D. Compared to the equilibrium weights, the optimal portfolio increases the weight in Germany and decreases the weights in both France and the United Kingdom. One can see that the deviations from the equilibrium weights are proportional to the portfolio of long Germany and short France and the United Kingdom—exactly the portfolio representing the investor's view. This result is very intuitive. Since the investor has no view about the portfolio of the market equilibrium, the optimal portfolio weights are the equilibrium weights.

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Appendix C.

In general, when there are constraints, the easiest way to find the optimal portfolio is to use the Black-Litterman model to generate unconstrained optimal weights, multiplying it by the ratio of the targeted portfolio to the market equilibrium portfolio (Chart 5). However, because of the scaling, the deviation from the equilibrium portfolio is no longer a weighted sum of the Germany/Europe portfolio and the global minimum-variance portfolio. The parameters are adjusted in a way that the combination satisfies both the risk constraint and the budget constraint (Chart 6).

All these examples display a very important property of the Black-Litterman model. In Chart 2C, one can see that the deviations of the optimal portfolio from the equilibrium weights are proportional to the portfolio of long Germany and short France and the United Kingdom. One can see that the deviations from the equilibrium weights are exactly the portfolio representing the investor's view. This result is very intuitive. Since the investor has a view about this portfolio, she simply invests in this portfolio, whereas the unconstrained optimal portfolio is a weighted sum of the Germany/Europe portfolio and the global minimum variance portfolio. The parameters are adjusted in a way that the combination satisfies both the risk constraint and the budget constraint (Chart 7).

For the graph of the expected returns in Chart 2A, it seems counterintuitive that the view expressed does not say whether any of the other countries will go up or down, yet both France and the United Kingdom increase as well. The same intuition applies to the other countries in Chart 2A.
These views are combined with the vector of the expected returns in the market equilibrium:

$$\mathbf{q} = \sum \lambda \mathbf{a}_i$$

In the Quantitative Strategies group at Goldman Sachs Asset Management, the Black-Litterman Model is the central framework for our modeling process. Our process starts with finding a set of views that are profitable. Although we manage many portfolios for many clients, using different benchmarks, and construct a set of expected returns on each asset. We forecast the expected returns on portfolios which incorporate these factors and portfolios based on momentum factors are consistently profitable. We then solve for the weights of the portfolios in question, subject to the constraints of the problem and to the optimization problem presented in the Black-Litterman model. The views, which are expressed in the form of expected returns, are combined with the market equilibrium (capitalization weights) portfolio. In the unconstrained case, if all investors hold the same belief, the demand for these assets will exactly balance the supply, and the expected return for any portfolio will be the same. This implies that the market equilibrium is the unique portfolio with the highest expected return, and the weights of the assets in this portfolio are the capitalization weights. In the constrained case, the weights are adjusted to account for the constraints, and the expected return is still priced in the market. The real power of the Black-Litterman model arises when there is a degree of uncertainty associated with it, and thus the Black-Litterman model is constructed to provide a framework for incorporating and expressing these views for a robust optimization framework.

The view that German equity will outperform the rest of Europe is now precisely expressed as an expected return of 5% for the portfolio of a long position in German equity and short positions in the rest of the European markets. The Black-Litterman model allows investors to express views about the markets, which are then combined with the market equilibrium weights to construct an optimal portfolio. The expected return of the portfolio is higher in the presence of these views. One of the features of the Black-Litterman model is that the investor can express different degrees of confidence about the views. This allows the investor to capture additional expected return. In Chart 3A we see how the expected return of the portfolio changes as the degree of confidence in the view changes. For example, the expected return of the portfolio increases from 3% to 4% per annum as the degree of confidence increases from 50% to 100%. These effects are illustrated in Chart 3B.

In Chart 1A, in addition to the original view that German equity will outperform the rest of Europe, we add a new view that the Canadian equity market will outperform the US equity market by 25 Basis Points (BPs) per annum. The deviations from the long position in the view are 3% per annum for the portfolio of Canada versus the United States and 3.25% per annum for the portfolio of Canada versus the United States and 3.25% per annum for the portfolio of Germany versus the rest of Europe. The table shows the weights of the portfolios in the views and the optimal deviations from the market equilibrium weights. The 100% confidence level on the view that German equity will outperform the rest of Europe results in an overweight in Germany and underweight in the rest of Europe, which is the direct result of the second view. The weights are shown in Chart 1B.

One of the features of the Black-Litterman model is that the investor can express different degrees of confidence about the views. This allows the investor to incorporate additional expected return. In Chart 1B, we see how the expected return of the portfolio changes as the degree of confidence in the views changes. For example, the expected return of the portfolio increases from 3% to 4% per annum as the degree of confidence increases from 50% to 100%. These effects are illustrated in Chart 1B.

The next question is how does the weight on a portfolio change when the view changes? For example, in Chart 1A we see that the expected return of the portfolio of a long position in German equity and short positions in the rest of the European markets is now 5% per annum. This can be interpreted as the investor becoming more confident in the view that German equity will outperform the rest of Europe. In the unconstrained case, if all investors hold the same belief, the demand for these assets will exactly balance the supply, and the expected return for any portfolio is identical to the expected return of the market equilibrium portfolio. In the constrained case, the weights are adjusted to account for the constraints, and the expected return is still priced in the market. Therefore, in general, the expected return of a portfolio is higher if the investor is more confident in the view. The table shows the weights of the portfolios in the views and the optimal deviations from the market equilibrium weights. The 100% confidence level on the view that German equity will outperform the rest of Europe results in an overweight in Germany and underweight in the rest of Europe, which is the direct result of the second view. The weights are shown in Chart 2B.

The most significant feature of the Black-Litterman model is that the investor can express different degrees of confidence about the views. This allows the investor to incorporate additional expected return. In Chart 2B, we see how the expected return of the portfolio changes as the degree of confidence in the views changes. For example, the expected return of the portfolio increases from 3% to 4% per annum as the degree of confidence increases from 50% to 100%. These effects are illustrated in Chart 2B.
**The Practical Application of the Black-Litterman Model**

In the Quantitative Strategies group at Goldman Sachs Asset Management, we develop quantitative models and use these models to manage portfolios. The Black-Litterman model is the central framework for our modeling and portfolio construction. Model Portfolios have exposures to the same set of historically profitable return-generating factors, and this allows us to manage many portfolios for many clients, using different benchmarks, and constructs a set of expected returns on each asset. Although we use the Black-Litterman model without this particular view.

The Black-Litterman approach combines the information from the equilibrium and investor views. In the case when the investor has one or more views about the market, the Black-Litterman model, which is the market equilibrium (capitalization weights) portfolio. In the unconstrained case, optimal weights are no longer obvious or intuitive. Nonetheless, the investor’s intended views can be integrated into the Black-Litterman expected returns formula. The uncertainty of the view is represented by the presence of a view, and this is quite natural because the view includes a market price of risk. The confidence level of the view is expressed in the view. This is quite natural because the view includes a market price of risk.

When will the weight on a portfolio be positive, negative, or zero? It turns out that the weight on a portfolio can be positive, negative, or zero. If the weight on a portfolio is positive, then it expresses the investor’s view about the markets. If the weight on a portfolio is negative, then it expresses the investor’s view against the markets. If the weight on a portfolio is zero, then the investor has no view about the markets.

We can deduce that the weight on the portfolio is positive, if and only if the expected return on the same portfolio generated by the Black-Litterman model is higher.

The Black-Litterman model without this particular view.

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We can deduce that the weight on the portfolio is positive, if and only if the expected return on the same portfolio generated by the Black-Litterman model is higher.
The Constrained Optimal Portfolio

Arriving at the optimal portfolio is sometimes more complex in the presence of constraints. In general, when there are constraints, the easiest way to find the optimal portfolio is to solve the unconstrained optimization problem first. From there, you could remove the constraints and find the optimal portfolio in a unconstrained case.

Alternatively, you could see a very important property of the Black-Litterman model. In the Black-Litterman model, the optimal portfolio is a linear combination of the equilibrium weights and the investor's view. The equation is as follows:

\[ w_{opt} = \frac{1}{1 + \frac{1}{\lambda}} \left( \frac{1}{\lambda} w_{equilibrium} + \Delta w \right) \]

where \( w_{opt} \) is the optimal portfolio, \( w_{equilibrium} \) is the equilibrium portfolio, \( \Delta w \) is the investor's view, and \( \lambda \) is the risk aversion parameter.

In the case of having a risk constraint, the investor can calculate the constrained optimal portfolio weights by taking the unconstrained optimal weights, multiplying it by the ratio of the risk-free rate to the variance of the portfolio, and then adding the investor's view. This method is known as the constant expected return portfolio method.

In the case of having a budget constraint, the investor can calculate the constrained optimal portfolio weights by taking the unconstrained optimal weights, multiplying it by the ratio of the total portfolio value to the variance of the portfolio, and then adding the investor's view. This method is known as the constant expected return portfolio method.

Conclusion

All these examples display a very important property of the Black-Litterman model: its ability to handle constraints. The Black-Litterman model is very flexible and can be used in a wide range of situations. The Black-Litterman model is a powerful tool that can be used to construct optimal portfolios under various constraints.
Appendix C.

With this constraint, there is a special global minimum-variance portfolio and a corresponding set of optimal portfolio weights. The optimal portfolio is a linear combination of the unconstrained optimal portfolio weights and the balanced capitalization-weighted portfolio. The combination is chosen in such a way that the combination satisfies both the risk constraint and the budget constraint.

In many cases, in addition to the risk constraint, the investor faces a budget constraint which forces the sum of the total portfolio weights to be one.

The optimal portfolio weights are computed by solving the constrained problem. Compared to the unconstrained weights, the optimal portfolio requires the weight on France and decreases the weights in both France and the United Kingdom.

The investor has a view about this portfolio. She simply invests in this portfolio, and up to her wanted weight, 80% in that country.

In the last example, a beta constraint is added to the budget and risk constraints. A beta constraint forces the beta of the portfolio with respect to the market portfolio to be 1. This is done to ensure that the optimal portfolio is positively correlated with the market portfolio and the view portfolio. If the market portfolio and the market capitalization-weighted portfolio are positively correlated, the expected return of the optimal portfolio raises the expected return of the market capitalization-weighted portfolio.

Since the investor has a view about the portfolio, the optimal weights in the portfolio, up to her wanted weight, are the equilibrium weights.

In many cases, in addition to the risk constraint, the investor faces a budget constraint which forces the sum of the total portfolio weights to be one. With this constraint, there is a special global minimum-variance portfolio and a corresponding set of optimal portfolio weights. The optimal portfolio is a linear combination of the unconstrained optimal portfolio weights and the balanced capitalization-weighted portfolio. The combination is chosen in such a way that the combination satisfies the risk constraint and the budget constraint.
In the Quantitative Strategies group at Goldman Sachs Asset Management, we develop quantitative models and use these models to manage portfolios. The Black-Litterman model is the central framework for our modeling process. Our process starts with finding a set of views that are profitable. For example, it is well known that portfolios based on certain value factors and portfolios based on momentum factors are consistently profitable. We forecast the expected returns on portfolios which incorporate these factors and construct a set of views. The Black-Litterman model takes these views and constructs a set of expected returns on each asset. Although we manage many portfolios for many clients, using different benchmarks, different targeted risk levels, and different constraints on the portfolios, the same set of expected returns from the Black-Litterman model is used throughout. Even though the final portfolios may look different due to the differences in benchmarks, targeted risk levels and constraints, all portfolios are constructed to be consistent with the same set of views, and all will have exposures to the same set of historically profitable return-generating factors.


The Black-Litterman asset allocation model transforms these practical concerns into easy-to-understand problems. Using the expected returns as a starting point, the Black-Litterman model provides the appropriate weights on the portfolios, based on the views expressed about the expected returns on each of the portfolios. The model includes the contribution of investor views to each portfolio, which are translated into expected returns.

The Black-Litterman model is based on two key assumptions: the market model and the covariance model. The market model is used to generate expected returns for individual assets, while the covariance model is used to estimate the relationships between the assets.

Once the expected returns and covariances have been estimated, the Black-Litterman model can be used to generate an optimal portfolio that reflects the investor's views. The model provides a framework for incorporating subjective views into the portfolio optimization process, allowing investors to incorporate their views into the portfolio optimization process.
Sachs Asset Management’s prior written consent.
in light of their experience, circumstances and financial resources.
assess the exposure to risk. Investors should carefully consider whether such investments are suitable for them.
Transactions in over-the-counter derivatives involve additional risks as there is no market on which to close out
originally invested and may possibly result in unquantifiable further loss exceeding the amount invested.
certain investments, including futures, swaps, forwards, certain options and derivatives, whether on or off
large movement, unfavourable as well as favourable, in the price of the warrant or similar security. In addition,
investments, including warrants and similar securities, often involve a high degree of gearing or leverage so that
in exchange rates may cause the value of an investment to increase or decrease. Some investments may be
Past performance is not a guide to future performance and the value of investments and the income derived from
Information contained herein is believed to be reliable but no warranty is given as to its completeness or
might be relevant to the subscription, purchase, holding, exchange, redemption or disposal of any investments.

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### Table 1: Annualized volatilities, market-capitalization weights, and equilibrium expected returns for the equity markets in the seven countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Annualized Volatility</th>
<th>Weight (%)</th>
<th>Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.12</td>
<td>27.1</td>
<td>5.5</td>
</tr>
<tr>
<td>Canada</td>
<td>0.488</td>
<td>0.4</td>
<td>0.664</td>
</tr>
<tr>
<td>France</td>
<td>0.478</td>
<td>0.6</td>
<td>0.664</td>
</tr>
<tr>
<td>Germany</td>
<td>0.515</td>
<td>0.8</td>
<td>0.861</td>
</tr>
<tr>
<td>Japan</td>
<td>0.668</td>
<td>0.3</td>
<td>0.653</td>
</tr>
<tr>
<td>UK</td>
<td>0.652</td>
<td>0.3</td>
<td>0.206</td>
</tr>
<tr>
<td>USA</td>
<td>0.779</td>
<td>0.7</td>
<td>0.653</td>
</tr>
</tbody>
</table>

---

Second, investment managers tend to think in terms of weights in a portfolio rather than balancing expected returns against the contribution to portfolio risk—the relevant margin in the Markowitz framework. When in practice most managers find that the effort required to specify expected returns is much too high. To be sure, the investor could simply shift the expected return for Germany up by 2.5% and assume her view that Germany will outperform European equities by 5% per year. Since our investor does not have such a view, she then shifts the expected return for Germany up by 2.5% and assumes she has invented a German market bubble that will last forever. However, the same level of risk aversion parameter results in much more extreme weights, with a 33.5% weight on the United Kingdom and a 94.8% weight on Germany. What happens when the investor does not have a strong belief about the equity market in the United Kingdom? She sets the sum of market capitalization-weighted expected returns for Germany and the United Kingdom to be 5% higher than the weighted average expected return for the rest of the world. To be sure, this approach also results in very extreme portfolio weights, with a 94.8% weight on Germany! The weight for France now is -94.8 percent! Many different ways to translate the view to expected returns.

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Expected Returns

Goldman Sachs Investment Management

Model Portfolios

<table>
<thead>
<tr>
<th>Country</th>
<th>Weight (%)</th>
<th>Expected Return (%)</th>
<th>Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>20.3</td>
<td>2.2</td>
<td>6.9</td>
</tr>
<tr>
<td>France</td>
<td>24.8</td>
<td>5.2</td>
<td>8.4</td>
</tr>
<tr>
<td>UK</td>
<td>20.0</td>
<td>12.4</td>
<td>6.8</td>
</tr>
<tr>
<td>USA</td>
<td>18.7</td>
<td>61.5</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Table 1: Annualized volatilities, market-capitalization weights, and equilibrium expected returns for the equity markets in Canada, France, the United Kingdom, and the United States.

Throughout our examples, we use 25. as the risk aversion parameter representing the world average risk tolerance. Translating Views into Starting from Equal Expected Returns

Specifying a Starting Point

Starting from Equal Expected Returns

Black and Litterman also demonstrated the shortcomings of several other methods for specifying a starting point for expected returns. They pointed to the instability in the derived portfolio weights when using the traditional mean-variance framework. Chart 1A shows that using the equal expected returns as the neutral starting point, the portfolio weights are very sensitive to small changes in the expected returns. For example, the investor could simply shift the expected return for Germany by 1% and get a portfolio that is very different from the equal expected returns portfolio.

Second, investment managers tend to think in terms of weights in a portfolio rather than balancing expected returns against the contribution to portfolio risk—the relevant margin in the Markowitz framework. When investment managers try to optimize using the Markowitz approach, they usually find that the portfolio weights returned by the optimizer (when not overly constrained) are very sensitive to small changes in the expected returns and constraints that lead to reasonable answers does not lead to a stable portfolio.

The following example demonstrates the unstable behavior of the optimal portfolio weights using optimizers. The investor has only one view about the markets: German equity will outperform European equities by 5% per year. Since our investor does not have a complete set of expected returns for all markets, she starts by setting Germany to be 5% higher than the weighted average equilibrium expected return for the other European markets. This is the investor's view.
Appendix B

1. There are \( N \) assets in the market. The market portfolio (equilibrium portfolio) is \( \mu^* \), the covariance of the returns is \( \Sigma \). The expected returns is \( \mu \). The covariance of the returns is \( \delta \). The CAPM prior distribution for the expected returns is \( \delta \). The equilibrium expected returns is the neutral reference point of the Black-Litterman model.

2. The unconstrained optimal portfolio is \( \Lambda \mu^* + \omega \). For the new case of \( \omega \), the investor should invest in the market portfolio first, then deviate from the market weights by adding weights on portfolios representing her views. The investor has already tried to translate these views into the expected returns. Can she possibly do better?

3. The Black-Litterman asset allocation model addresses those practical issues. Starting from equilibrium expected returns, the model generates expected returns in some mysterious way, we have presented a method to understand the intuition of the model. With the new market weights by adding weights on portfolios representing her views, the investor can always use the expected returns (generated by the optimization package to obtain the optimal portfolio. Unlike a standard mean-variance optimization, the Black-Litterman model, if properly implemented, will always generate an optimal portfolio whose weights are generated by the optimization package. Instead of treating the Black-Litterman asset allocation model as a framework has had surprisingly little impact. Why is that the case? We cite practical issues.

4. The Black-Litterman approach provides both a reference point for expected return assumptions as well as a mechanism for interacting with market expectations. The investor can express views or statements about the expected returns of arbitrary portfolios, and the market combines the views with equilibrium, producing both the set of expected returns of assets as well as the optimal portfolio weights.

5. In the current Black-Litterman model, the traditional mean-variance approach the user inputs a complete set of expected returns, and the portfolio optimizes to generate the optimal portfolio. However, Black-Litterman approaches the portfolio optimization problem with expected excess return over the one-period risk-free rate 20.0%. The Black-Litterman model along with the covariance matrix) in a portfolio mean-variance optimization, the Black-Litterman model, if properly implemented, will always generate an optimal portfolio whose weights are consistent with the preceding discussion.

6. Starting from equilibrium expected returns, the investor can express views or statements about the expected returns of arbitrary portfolios, and the market combines the views with equilibrium, producing both the set of expected returns of assets as well as the optimal portfolio weights.
Appendix C

1. Given the expected returns $\mu$ and the covariance matrix $\Sigma$, the unconstrained maximization problem $\max w^\prime \mu - \frac{1}{2} w^\prime \Sigma w$ has a solution of $w^\ast = -\delta \mu \Sigma^{-1}$.

2. Given the covariance matrix $\Sigma$, the minimum variance portfolio is $w_m = -\Sigma^{-1} \iota$, where $\iota$ is a vector with all elements being one.

3. The solution to the risk-constrained optimization problem, $\max w^\prime \mu$ subject to $\sigma^2 = w^\prime \Sigma w$ can be expressed as $w^\ast = -\delta \mu \Sigma^{-1} + \frac{1}{\sigma^2} \Sigma^{-1} \iota$, where $\delta$ and $\sigma^2$ are chosen in the way both risk and budget constraints are satisfied.

4. The risk-, budget-, and beta-constrained optimization problem can be formulated as $\max w^\prime \mu$ subject to $\sigma^2 = w^\prime \Sigma w$ and $\iota^\prime w = 1$. Its solution has the form $w^\ast = a + b \Sigma^{-1} \iota$, where $a$, $b$, and $\sigma^2$ are chosen in the way all these constraints are satisfied.

References


The Intuition Behind Black-Litterman Model Portfolios

In this article and as our title suggests, we demonstrate a method for understanding the intuition behind the Black-Litterman asset allocation model.

To do this, we use examples to show the difference between the traditional mean-variance optimization process and the Black-Litterman process. We show that the mean-variance optimization process, while academically sound, can produce results that are extreme and not particularly intuitive. In contrast, we show that the optimal portfolios generated by the Black-Litterman process have a simple, intuitive property:

- The unconstrained optimal portfolio is the market equilibrium portfolio plus a weighted sum of portfolios representing an investor’s views.
- The weight on a portfolio representing a view is positive when the view is more bullish than the one implied by the equilibrium and other views.
- The weight increases as the investor becomes more bullish on the view as well as when the investor becomes more confident about the view.

December 1999