Technical Appendix

to accompany

Political Uncertainty and Risk Premia

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This online Technical Appendix provides proofs and additional theoretical results in support of the analysis presented in the paper. The contents are as follows:

- Section 1: Proofs (Page 1)
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1. Proofs

Proof of Lemma 1: The same argument leading to equation (IA.8) in the Internet Appendix of Pastor and Veronesi (2012) implies that conditional on policy $n$, $n = 0, 1, ..., N$, being chosen at time $\tau$, aggregate capital is given by

$$B_T = B_{\tau} e^{(\mu + g - \frac{1}{2} \sigma^2)(T - \tau) + \sigma(Z_T - Z_{\tau})}$$

Thus, exploiting $W_T = B_T$ we have

$$E_\tau \left[ \frac{W_T^{1-\gamma}}{1 - \gamma} \mid \text{policy } n \right] = \frac{B_{\tau}^{1-\gamma}}{1 - \gamma} e^{(1 - \gamma)(T - \tau) \mu_g + \frac{1}{2} (1 - \gamma)^2 (T - \tau)^2 \sigma_{g,n}^2 + (\mu - \frac{1}{2} \sigma^2) (T - \tau) (1 - \gamma)}$$

It follows immediately that

$$E_\tau \left[ \frac{W_T^{1-\gamma}}{1 - \gamma} \mid \text{policy } n \right] > E_\tau \left[ \frac{W_T^{1-\gamma}}{1 - \gamma} \mid \text{policy } m \right]$$

if and only if

$$\tilde{\mu}_n = \mu_g^n + \frac{1}{2} (1 - \gamma) (T - \tau) \sigma_{g,n}^2 > \mu_g^m + \frac{1}{2} (1 - \gamma) (T - \tau) \sigma_{g,m}^2 = \tilde{\mu}_m$$

Q.E.D.

Proof of Proposition 1. The government chooses policy $n \in \{0, 1, ..., N\}$ if and only if for all $m \neq n$, $m = 0, 1, ..., N$,

$$E_\tau \left[ \frac{C^n W_T^{1-\gamma}}{1 - \gamma} \mid \text{policy } n \right] > E_\tau \left[ \frac{C^m W_T^{1-\gamma}}{1 - \gamma} \mid \text{policy } m \right]$$

where recall that $C^0 = 1$. The same calculations as in Lemma 1 lead to the inequality

$$\mu_g^n - \frac{\sigma_{g,n}^2}{2} (T - \tau) (\gamma - 1) - \frac{c^n}{(\gamma - 1)(T - \tau)} > \mu_g^m - \frac{\sigma_{g,m}^2}{2} (T - \tau) (\gamma - 1) - \frac{c^m}{(\gamma - 1)(T - \tau)}$$

(B1)
The claim follows from the definitions of \( \tilde{\mu}^n \) and \( \tilde{c}^n \) in equations (15) and (23). Q.E.D.

**Proof of Corollary 1.** Immediate from Proposition 1 and equations (16) and (17).

**Proof of Corollary 2.** As of time \( t \), we have for each \( n = 1, \ldots, N \)

\[
\tilde{c}^n \sim N \left( \bar{c}_n^0, \sigma^2_{\bar{c}_n} \right) \tag{B2}
\]

Recall from Proposition 1 that policy \( n \in \{1, \ldots, N\} \) is chosen if and only if

\[
\begin{align*}
\tilde{\mu}^n - \tilde{c}^n &> \tilde{\mu}^m - \tilde{c}^m & m \neq n, m = 1, \ldots, N \tag{B3} \\
\tilde{\mu}^n - \tilde{c}^n &> x_\tau, \tag{B4}
\end{align*}
\]

where we define

\[
x_\tau \equiv \tilde{\mu}^0 = \tilde{g}_\tau - \frac{\tilde{\sigma}^2_\tau}{2} (T - \tau) (\gamma - 1). \tag{B5}
\]

Therefore, the conditional probability at \( t \) that policy \( n \) is chosen at \( \tau \) is given by

\[
p_t^n = \Pr \left( \frac{\tilde{\mu}^n - \tilde{c}^n > \tilde{\mu}^m - \tilde{c}^m \text{ for } m \neq n}{\tilde{\mu}^n - \tilde{c}^n > x_\tau} \right)
\]

\[
= \int_{-\infty}^{\infty} \Pr \left( \frac{\tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{\mu}^m \text{ for } m \neq n}{\tilde{\mu}^n - \tilde{c}^n > x_\tau} \right) \frac{\phi_{\tilde{c}^n}(\tilde{c}^n)}{d\tilde{c}^n}
\]

\[
= \int_{-\infty}^{\infty} \Pi_{m \neq n} \Pr (\tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{\mu}^m \mid \tilde{c}^n) \Pr (\tilde{\mu}^n - \tilde{c}^n > x_\tau \mid \tilde{c}^n) \phi_{\tilde{c}^n}(\tilde{c}^n) d\tilde{c}^n
\]

where we used the fact that \( \tilde{c}^m \)'s are independent of each other as well as of \( x_\tau \). Moreover, from the definition of \( x_\tau = \tilde{g}_\tau - \frac{\tilde{\sigma}^2_\tau}{2} (T - \tau)(\gamma - 1) \) (see equation (16)) we have \( x_\tau \mid \tilde{g}_t \sim N \left( \tilde{g}_t - \frac{\tilde{\sigma}^2_\tau}{2} (T - \tau)(\gamma - 1), \tilde{\sigma}^2_\tau - \tilde{\sigma}^2_\tau \right) \).

We note two properties:

1. As \( \tilde{g}_t \rightarrow \infty \), then \( p_t^n \rightarrow 0 \) for all \( n \in \{1, \ldots, N\} \), as \( \Phi_x (\tilde{\mu}^n - \tilde{c}^n \mid \tilde{g}_t) \rightarrow 0 \).

2. As \( t \rightarrow \tau \) we have

\[
\Phi_x (\tilde{\mu}^n - \tilde{c}^n \mid \tilde{g}_t) = \int_{\tilde{\mu}^n - \tilde{c}^n}^{\tilde{\mu}^n - \tilde{c}^n} \phi_x (x \mid \tilde{g}_t) dx \rightarrow 1_{\{x_\tau < \tilde{\mu}^n - \tilde{c}^n\}} \tag{B6}
\]

so that

\[
p_t^n = \int_{-\infty}^{\infty} \Pi_{m \neq n} \Phi_x (\tilde{\mu}^n - \tilde{c}^n \mid \tilde{g}_t) d\tilde{c}^n
\]

\[
\rightarrow \int_{-\infty}^{\infty} \Pi_{m \neq n} \left( 1 - \Phi_x (\tilde{\mu}^n + \tilde{\mu}^m - \tilde{c}^m) \right) d\tilde{c}^n
\]

\[
= p_\tau^n
\]
Q.E.D.

Proof of Lemma A1. Using the same arguments as to obtain equation (IA.20) in the Internet Appendix of Pastor and Veronesi (2012), after the announcement of policy \(n\) at time \(\tau_+\), the state price density is given by

\[
E_{\tau_+}[\pi_T|\text{policy } n] = \pi_{\tau_+}^n = \lambda^{-1} B_{\tau_+}^{-\gamma} e^{-\gamma \mu_0^n(T-\tau)} e^{\left(-\gamma \mu + \frac{1}{2} \gamma (\gamma +1) \sigma^2\right)(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2 \sigma_{\gamma,n}^2} \quad (B7)
\]

Therefore, using also \(B_{\tau} = B_{\tau_+}\), the state price density at \(\tau\) is

\[
\pi_{\tau} = \sum_{n=0}^{N} p^n_{\tau} \pi_{\tau_+}^n = \lambda^{-1} \sum_{n=0}^{N} p^n_{\tau} B_{\tau}^{-\gamma} e^{-\gamma \mu_0^n(T-\tau)} e^{\left(-\gamma \mu + \frac{1}{2} \gamma (\gamma +1) \sigma^2\right)(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2 \sigma_{\gamma,n}^2}
\]

Using the definition \(\mu_{\gamma}^0 = \tilde{\gamma}_{\tau}\) in equation (18) and the condition

\[
p^n_{\tau} = 1 - \sum_{n=1}^{N} p^n_{\tau} \quad (B8)
\]

we can rewrite the state price density at \(\tau\) as

\[
\pi_{\tau} = \lambda^{-1} B_{\tau}^{-\gamma} e^{\left(-\gamma \mu + \frac{1}{2} \gamma (\gamma +1) \sigma^2\right)(T-\tau) - \gamma \tilde{\gamma}_{\tau}(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2 \sigma_{\gamma}^2} \times
\]

\[
\times \left(1 + \sum_{n=1}^{N} p^n_{\tau} e^{-\gamma (\mu_{\gamma}^0 - \tilde{\gamma}_{\tau})(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2 (\sigma_{\gamma,n}^2 - \sigma_{\gamma}^2) - 1}\right)
\]

Similarly, after the announcement of policy \(n, n=0,1,\ldots,N\), at time \(\tau_+\), we have

\[
E_{\tau_+}[B_{\tau}^{-\gamma} B_{\tau}^i|\text{policy } n] = N_{\tau_+}^{i,n} = B_{\tau_+}^{-\gamma} B_{\tau}^i \times e^{\left((1-\gamma) \mu_0^i(T-\tau) + (1-\gamma) \mu + \frac{1}{2} \gamma (\gamma -1) \sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \sigma_{\gamma,n}^2} \quad (B9)
\]

Therefore, we have

\[
E_{\tau}[B_{\tau}^{-\gamma} B_{\tau}^i] = \sum_{n=0}^{N} p^n_{\tau} N_{\tau_+}^{i,n} = B_{\tau}^{-\gamma} B_{\tau}^i \times e^{\left((1-\gamma) \mu_0^i(T-\tau) + (1-\gamma) \mu + \frac{1}{2} \gamma (\gamma -1) \sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \sigma_{\gamma,n}^2}
\]

\[
\times \left(1 + \sum_{n=1}^{N} p^n_{\tau} \left(e^{\left((1-\gamma) (\mu_{\gamma}^0 - \tilde{\gamma}_{\tau})(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 (\sigma_{\gamma,n}^2 - \sigma_{\gamma}^2) - 1\right)\right}\right)
\]

\[3\]
The claim follows from taking the ratio \( M_i^* = \frac{E_{\tau}[\pi_{\tau} B_{i}^*]}{\pi_{\tau}} = \frac{E_{\tau}[\lambda^{-1} B_{i}^* B_{i}^*]}{\pi_{\tau}} \). Q.E.D.

**Proof of Lemma A2.** From (B7) and (B9) we obtain that if policy \( n, n = 0, 1, ..., N \), is selected at \( \tau^+ \), then

\[
M_{\tau^+} = \frac{E_{\tau^+}[B_{\tau}^* B_{\tau}^*|\text{policy } n]}{E_{\tau^+}[B_{\tau}^*|\text{policy } n]} = B_{\tau^+} e^{(\mu - \gamma \sigma^2 + \mu_n^2)(T - \tau) + \frac{1}{2}(T - \tau)^2 \sigma^2, (B10)}
\]

Q.E.D.

**Proof of Proposition 2:** To prove this proposition, we need three lemmas:

**Lemma B1.** From (B7) and (B9) we obtain that if policy \( n, n = 0, 1, ..., N \), is selected at \( \tau^+ \), then

\[
M_{\tau^+} = \frac{E_{\tau^+}[B_{\tau}^* B_{\tau}^*|\text{policy } n]}{E_{\tau^+}[B_{\tau}^*|\text{policy } n]} = B_{\tau^+} e^{(\mu - \gamma \sigma^2 + \mu_n^2)(T - \tau) + \frac{1}{2}(T - \tau)^2 \sigma^2, (B10)}
\]

Q.E.D.

**Proof of Lemma B1:** From Lemma A5 in Pastor and Veronesi (2012), we have that \( b_{\tau} = \log(B_{\tau}) \) and \( \hat{g}_{\tau} \) have the conditional joint distribution

\[
\left( \frac{b_{\tau} - b_t}{\hat{g}_{\tau}} \right) \sim N \left( \frac{E_{\tau}[\Delta b_{\tau}]}{E_{\tau}[\hat{g}_{\tau}]}, \left( \frac{V_{b}}{C_{g,b}} \right) \right)
\]

where

\[
E_{\tau}[\Delta b_{\tau}] = \left( \mu + \hat{g}_t - \frac{1}{2} \sigma^2 \right) (\tau - t)
\]

\[
E_{\tau}[\hat{g}_{\tau}] = \hat{g}_t
\]

\[
V_{b} = (\tau - t)^2 \sigma_t^2 + \sigma^2 (\tau - t)
\]

\[
V_{\hat{g}} = \sigma_t^2 - \sigma_{\hat{g}}^2
\]

\[
C_{g,b} = \sigma_t^2 (\tau - t)
\]

We now see that \( b_{\tau} - b_t \) and \( \hat{g}_{\tau} \) are perfectly correlated. In fact,

\[
Corr = \frac{\sigma_{\hat{g},b}}{\sqrt{V_{b} V_{\hat{g}}}} = \frac{\sigma_{\hat{g},b}}{\sigma_t^2 (\tau - t)}
\]

Using the fact that

\[
\sigma_{\hat{g}}^2 = \frac{1}{\sigma_t^2} + \frac{1}{\sigma^2 (\tau - t)} = \frac{\sigma_t^2 \sigma^2}{\sigma^2 + \sigma_t^2 (\tau - t)}
\]

we find

\[
Corr = \frac{\sigma_{\hat{g},b}}{\sqrt{((\tau - t)^2 \sigma_t^2 + \sigma^2 (\tau - t)) (\sigma_t^2 - \sigma_{\hat{g}}^2)}}
\]

\[
= \frac{\sigma_{\hat{g},b}}{\sigma_t^2 (\tau - t)}
\]

\[
= \frac{\sigma_{\hat{g},b}}{\sigma_t^2 (\sigma^2 + \sigma_t^2 (\tau - t)) - \sigma_t^2 \sigma^2}
\]

\[
= \frac{\sigma_{\hat{g},b}}{\sigma_t^2 (\tau - t)}
\]

\[
= \frac{1}{\sqrt{(\tau - t) (\sigma_t^2)^2 (\tau - t)}} = 1
\]
It follows that we can write
\[
\Delta b_t = E_t[\Delta b_t] + \{\tilde{g}_t - E_t[\tilde{g}_t]\} \frac{C_{b,\tilde{g}}}{V_{\tilde{g}}} = E_t[\Delta b_t] + \{\tilde{g}_t - E_t[\tilde{g}_t]\} \sqrt{\frac{V_b}{V_{\tilde{g}}}}
\]
\[
= E_t[\Delta b_t] + \{\tilde{g}_t - E_t[\tilde{g}_t]\} \left(\frac{\sigma_t^2 (\tau - t)}{\sigma_t^2 - \sigma_{\tau}^2}\right) = E_t[\Delta b_t] + \{\tilde{g}_t - E_t[\tilde{g}_t]\} \left[\frac{\sigma^2}{\sigma_t^2} + (\tau - t)\right]
\]
where we also used the equality
\[
\frac{\sigma_t^2 - \sigma_{\tau}^2}{\sigma_t^2} = \frac{\sigma_t^2 - \frac{\sigma_{\tau}^2}{\sigma_t^2} \sigma^2}{\sigma_t^2 + \frac{\sigma_{\tau}^2}{\sigma_t^2} \sigma^2} = \frac{(\sigma_t^2)^2 (\tau - t)}{\sigma_t^2 + \sigma_{\tau}^2 (\tau - t)}
\]

From the definition of \(x_\tau\), it also follows that \(x_\tau - E_t[x_\tau] = \tilde{g}_t - E_t[\tilde{g}_t]\). Q.E.D.

**Lemma B2**: The conditional distribution of \(\Delta b_t = b_t - b_t = \log (B_t/B_t)\) conditional on time-\(t\) information and policy \(n\) being chosen at time \(\tau\) is

\[
f(\Delta b_t | S_t, n \text{ at } \tau) = \frac{\phi_{\Delta b_t}(\Delta b_t)}{p_t^n} \int_{-\infty}^{\tilde{c}^n - E_t[x_\tau] - (\Delta b_t - E_t[\Delta b_t])} \frac{\partial^2}{\partial t^2} e^{-\frac{t^2}{2}} \prod_{m \neq n} (1 - \Phi_{\tilde{c}^n} (\tilde{c}^m - \tilde{m}^n + \tilde{m}^m)) \phi_{\tilde{c}^n}(\tilde{c}^m) d\tilde{c}^m
\]

(B14)

where \(\phi_{\Delta b_t}(\Delta b_t)\) is the normal density with mean \(E_t[\Delta b_t] = (\mu + \tilde{g}_t - \frac{1}{2} \sigma^2)(\tau - t)\) and variance \(V_b = (\tau - t)^2 \sigma_t^2 + \sigma^2(\tau - t)\). In addition, \(E_t[x_\tau] = \tilde{g}_t - \frac{\sigma_t^2}{\gamma} (T - \tau)(\gamma - 1)\).

Note that \(f(\Delta b_t | S_t, \kappa \text{ at } \tau)\) does not depend on the current value of log capital, \(b_t\), hence the conditional dependence only on \(S_t\) and time \(t\).

**Proof of Lemma B2**. The conditional CDF is

\[
F_{\Delta b_t}(\Delta b_t | S_t, \kappa = \tilde{c}^n - \tilde{m}^n + \tilde{m}^m < \tilde{c}^m \text{ for } m \neq n) = \frac{\Pr(\Delta b_t < \Delta b_t, x_\tau < \tilde{m}^n - \tilde{c}^n, \tilde{c}^n - \tilde{m}^n + \tilde{m}^m < \tilde{c}^m \text{ for } m \neq n | S_t)}{\Pr(x_\tau < \tilde{m}^n - \tilde{c}^n, \tilde{c}^n - \tilde{m}^n + \tilde{m}^m < \tilde{c}^m \text{ for } m \neq n | S_t)}
\]

(B16)

The denominator is just \(p_t^n\) from Corollary 2. Consider the numerator. From Lemma B1:

\[
\Delta b_t - E_t[\Delta b_t] = \{x_\tau - E_t[x_\tau]\} (\sigma_t^2 + (\tau - t))
\]

which implies

\[
x_\tau = E_t[x_\tau] + \{\Delta b_t - E_t[\Delta b_t]\} \frac{\sigma_t^2}{\sigma_t^2 + \sigma_{\tau}^2 (\tau - t)}
\]

Thus, the joint distribution can be written as

\[
\Pr(\Delta b_t < \Delta b_t, x_\tau < \tilde{m}^n - \tilde{c}^n, \tilde{c}^n - \tilde{m}^n + \tilde{m}^m < \tilde{c}^m | S_t)
\]
\[
\frac{\partial^2}{\partial^2 (\tau - t)} \left( \frac{\bar{c}^m - \mu^m + \bar{c}^n}{\sigma^2 + \sigma_t^2 (\tau - t)} \right) < \bar{\mu}^n - \bar{c}^n | S_t \)
\]
\[
= \int_{-\infty}^{\Delta b} \int_{-\infty}^{\Delta b} \int_{-\infty}^{\Delta b} \frac{\partial^2}{\partial^2 (\tau - t)} \left( \frac{\bar{c}^m - \mu^m + \bar{c}^n}{\sigma^2 + \sigma_t^2 (\tau - t)} \right) \Pi_{m \neq n} \left[ 1 - \Phi_{\bar{c}^m} (\bar{c}^m - \mu^m + \bar{c}^n) \right] \phi_{\bar{c}^m} (\bar{c}^m) d\bar{c}^n \phi_{\Delta b_r} (\Delta b_r) d\Delta b_r
\]

where we exploited the independence across \( \bar{c}^m \) and with respect to \( \Delta b_r \). Substituting into (B16) and taking the first derivative with respect to \( \Delta b \), we obtain the density (B15). Q.E.D.

**Lemma B3:** The distribution of \( \hat{g}_r \) conditional on time-\( t \) information and no new policy being chosen at time \( \tau \) is

\[
f (\hat{g}_r | \text{no policy change at } \tau) = \frac{\phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t)}{p_t^0} \Pi_{n=1}^{N} \left( 1 - \Phi_{\bar{c}^n} \left( \bar{\mu}^n - \hat{g}_r + \frac{\sigma^2}{2} (T - \tau) (\gamma - 1) \right) \right)
\]

(B17)

where \( \phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) \) is the conditional normal density of \( \hat{g}_r \), namely, \( N(\hat{g}_r, \sigma^2_t - \sigma^2_r) \).

**Proof of Lemma B3:** The conditional CDF is given by

\[
F_{\hat{g}_r} (g | \text{no policy change at } \tau)
= F_{\hat{g}_r} (g | x_{\tau} > \bar{\mu}^n - \bar{c}^n \text{ for all } n)
= \frac{\Pr (\hat{g}_r < g \& \hat{g}_r > \bar{\mu}^n - \bar{c}^n + \frac{\sigma^2}{2} (T - \tau) (\gamma - 1) \text{ for all } n)}{\Pr (x_{\tau} > \bar{\mu}^n - \bar{c}^n \text{ for all } n)}
= \int_{-\infty}^{\infty} \Pr (\hat{g}_r < g \& \hat{g}_r > \bar{\mu}^n - \bar{c}^n + \frac{\sigma^2}{2} (T - \tau) (\gamma - 1) \text{ for all } n | \hat{g}_r) \phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) d\hat{g}_r
\]

Taking the first derivative with respect to \( g \), we obtain the density (B17). Q.E.D.

**Proof of Proposition 2:** We know that

\[
\pi_t = E_t [\pi_{\tau^+}] = \sum_{n=0}^{N} E_t [\pi_{\tau^+} | n] p_t^n
\]

(B18)
Note that for \( n = 1, \ldots, N \)
\[
E_t [\pi_{\tau+} | n \text{ at } \tau] = E_t \left[ \lambda^{-1} B_{r+}^{-\gamma} e^{-\gamma \mu_g (T-\tau)} e^{-\gamma \mu_g (T-\tau) + \frac{\sigma_g^2}{2} (T-\tau)^2} | n \text{ at } \tau \right]
\]
\[
= \lambda^{-1} e^{-\gamma \mu_g (T-\tau) + \frac{\sigma_g^2}{2} (T-\tau)^2} e^{-\gamma b_t} E_t \left[ e^{-\gamma (b_t - b_t)} | n \text{ at } \tau \right]
\]
\[
= \lambda^{-1} B_{r+}^{-\gamma} e^{-\gamma \mu_g (T-\tau) + \frac{\sigma_g^2}{2} (T-\tau)^2} e^{-\gamma b_t} \times
\int_{-\infty}^{\infty} e^{-\gamma \Delta b_t} f(\Delta b_t, S_t, n \text{ at } \tau) \, d\Delta b_t
\]
Similarly, for \( n = 0 \) we have
\[
E_t [\pi_{\tau+} | 0 \text{ at } \tau] = E \left[ \lambda^{-1} B_{r+}^{-\gamma} e^{-\gamma \hat{g}_t (T-\tau)} e^{-\gamma \mu_g (T-\tau) + \frac{\sigma_g^2}{2} (T-\tau)^2} | 0 \text{ at } \tau \right]
\]
\[
= \lambda^{-1} e^{-\gamma \mu_g (T-\tau) + \frac{\sigma_g^2}{2} (T-\tau)^2} e^{-\gamma b_t} \times
E_t \left[ \frac{\gamma}{\tau} \left( E_t[\Delta b_t] + \hat{g}_t - E_t[\hat{g}_t] \right) \right] \times
\int_{-\infty}^{\infty} e^{-\gamma \Delta b_t} f(\Delta b_t | 0 \text{ at } \tau) \, d\Delta b_t
\]
The result follows from comparing the terms in equations (28) and (A1) with the ones above, and defining in this proposition \( \mu_g^0 = \hat{g}_t \) and \( \sigma_g^2, \delta_0^2, \delta_0^2 = \sigma_g^2 \). Q.E.D.

**Proof of Proposition 3:** The result follows from an application of Ito’s Lemma to equation (28), and recalling that \( \pi_t \) is a martingale, and thus \( E_t[d\pi_t/\pi_t] = 0 \). Q.E.D.

**Proof of Corollary 3:** The probability that the old policy will be retained is
\[
p_t^0 = \Pr \left( \hat{\mu}^n - \hat{c}^n < \hat{g}_t - \frac{\sigma_g^2}{2} (T-\tau) (\gamma - 1) \text{ for all } n > 0 \right)
\]
\[
= \int_{-\infty}^{\infty} \Pr \left( \hat{\mu}^n - \hat{c}^n < \hat{g}_t - \frac{\sigma_g^2}{2} (T-\tau) (\gamma - 1) \text{ for all } n > 0 | \hat{g}_t \right) \phi \left( \hat{g}_t | \hat{g}_t \right) \, d\hat{g}_t
\]
\[
= \int_{-\infty}^{\infty} \prod_{n=1}^{N} \left[ 1 - \Phi \left( \hat{\mu}^n - \hat{g}_t + \frac{\sigma_g^2}{2} (T-\tau) (\gamma - 1) \right) \right] \phi \left( \hat{g}_t | \hat{g}_t \right) \, d\hat{g}_t
\]
Thus, for a given \( \sigma_g^2 \), for \( n = 1, \ldots, N \), we have that as \( \hat{g}_t \to -\infty \), \( p_t^0 \to 0 \) and \( dp_t/d\hat{g}_t \to 0 \). From the proof of Proposition 2, for a sufficiently small \( \hat{g}_t \), the state price density converges to
\[
\pi_t = E_t [\pi_{\tau+}] \to \sum_{n=1}^{N} p_t^n E [\pi_{\tau+} | n \text{ at } \tau+]
\]
\[ \lambda^{-1} B_t^{-\gamma} e^{(-\gamma \mu + \frac{1}{2}\gamma(\gamma+1)\sigma^2)^2 (T-\tau)} \sum_{n=1}^{N} p_t^n e^{-\gamma \mu_n^b (T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \sigma_n^2} E \left[ e^{-\gamma \Delta b_r} \middle| S_t, n \text{ at } \tau \right] \]

(B19)

As \( g_t \) declines so that \( p_t^0 \rightarrow 0 \), we have \( E \left[ e^{-\gamma \Delta b_r} \middle| S_t, n \text{ at } \tau \right] \rightarrow E \left[ e^{-\gamma \Delta b_r} \middle| S_t \right] \). Indeed, note that in the proof of the distribution of \( \Delta b_r \) in Lemma B2, we have

\[ F (\Delta b_r \middle| S_t, n \text{ at } \tau) = \Pr \left( \Delta b_r < \Delta b_1 \middle| S_t, \bar{\mu}^m - \bar{\sigma}^m < \bar{\mu}^n - \bar{\sigma}^n \text{ for } m \neq n \right) \]

The statement \( p_t^0 = 0 \) implies that the event \( x_\tau < \bar{\mu}^m - \bar{\sigma}^m \) is certain to be realized. Thus:

\[ F (\Delta b_r \middle| S_t, n \text{ at } \tau) = \Pr (\Delta b_r < \Delta b_1 \middle| S_t, \bar{\mu}^m - \bar{\sigma}^m < \bar{\mu}^n - \bar{\sigma}^n \text{ for } m \neq n) = \Pr (\Delta b_r < \Delta b_1 \middle| S_t) \]

the last step due to the independence of \( \bar{\sigma}^m, \bar{\sigma}^n \) from \( \Delta b_r \) process. We then obtain

\[ \pi_t = \lambda^{-1} B_t^{-\gamma} e^{(-\gamma \mu + \frac{1}{2}\gamma(\gamma+1)\sigma^2)^2 (T-\tau)} \sum_{n=1}^{N} p_t^n e^{-\gamma \mu_n^b (T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \sigma_n^2} \]

\[ = \lambda^{-1} B_t^{-\gamma} e^{(-\gamma \mu + \frac{1}{2}\gamma(\gamma+1)\sigma^2)^2 (T-\tau)} \sum_{n=1}^{N} p_t^n e^{-\gamma \mu_n^b (T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \sigma_n^2} \]

\[ \times e^{-\gamma ((\mu+\gamma_t - \frac{1}{2}\gamma^2)(\tau-t) + \gamma^2 ((\tau-t)^2 \sigma_t^2 + (\tau-t)^2 \sigma^2) \sum_{n=1}^{N} p_t^n e^{-\gamma \mu_n^b (T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \sigma_n^2} \]

where the last step follows from Lemma B2:

\[ \Delta b_r \middle| S_t \sim N \left( \left( \mu + \gamma_t - \frac{1}{2} \sigma^2 \right) (\tau-t), (\tau-t)^2 \sigma_t^2 + (\tau-t)^2 \sigma^2 \right) \]

It follows that for \( \gamma_t \) sufficiently small, \( \Omega (S_t) \) converges to

\[ \Omega (S_t) = e^{-\gamma ((\mu+\gamma_t - \frac{1}{2}\gamma^2)(\tau-t) + \gamma^2 ((\tau-t)^2 \sigma_t^2 + (\tau-t)^2 \sigma^2) \sum_{n=1}^{N} p_t^n e^{-\gamma \mu_n^b (T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \sigma_n^2} \]

Taking the first derivative with respect to \( \gamma_t \) and dividing by \( \Omega (S_t) \), we find that as \( \gamma_t \rightarrow -\infty \),

\[ \sigma_{\pi,0} = \frac{1}{\Omega (S_t)} \frac{\partial \Omega (S_t)}{\partial \gamma_t} \sigma_t^2 \sigma^{-1} \rightarrow -\gamma (\tau-t) \sigma_t^2 \sigma^{-1} \]

proving the first part of the statement of Corollary 3.

To prove the second part of the statement of Corollary 3, from property 1 in the proof of Corollary 2, for a given distribution of \( \bar{\sigma}^m \), we have \( p_t^0 \rightarrow 1 \) as \( \gamma_t \rightarrow \infty \). It follows that the state price density converges to one that assigns zero probability to a policy change:

\[ \pi_t \rightarrow E_t [\pi_{\tau+} \middle| 0 \text{ at } n] = \lambda^{-1} E_t [B_T^{-\gamma} \middle| 0 \text{ at } n] \]

\[ = \lambda^{-1} B_T^{-\gamma} e^{-\gamma \bar{\theta}(T-t) e^{(-\gamma \mu + \frac{1}{2}\gamma(\gamma+1)\sigma^2)(T-t)+\frac{2}{T} (T-t)^2 \sigma_t^2} \]

(B21)
It follows that in this case, for \( \hat{g}_t \) sufficiently large, \( \Omega(S_t) \) converges to

\[
\Omega(S) = e^{-(\mu + \frac{1}{2}\gamma(\gamma+1)\sigma^2)(\tau-t)}e^{-\gamma\hat{g}_t(T-t)+\frac{1}{2}\gamma^2(T-t)^2\hat{\sigma}^2_t}
\]

Taking the first derivative with respect to \( \hat{g}_t \) and dividing by \( \Omega(S_t) \), we find that as \( \hat{g}_t \to \infty \),

\[
\sigma_{\pi,0} = \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial \hat{g}_t} \hat{\sigma}^2_t \sigma^{-1} \to -\gamma(T-t)\hat{\sigma}^2_t \sigma^{-1}
\]

Q.E.D.

**Proof of Corollary 4:** From expression (B21), we see that the state price density does not depend on any \( \hat{c}_t^n \). Hence, we have \( \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S)}{\partial \hat{c}_t^n} = 0 \). Q.E.D.

**Proof of Proposition 4:** The proof is identical to the proof of Proposition 2, except that we have to calculate

\[
E_t \left[ \pi_{\tau+M_{\tau+}^i} \right] = \sum_{n=0}^{N} p^n_t E_t \left[ \pi_{\tau+M_{\tau+}^i} | n \text{ at } \tau \right]
\]

From (B9), for \( n = 1, \ldots, N \):

\[
E_t \left[ \pi_{\tau+M_{\tau+}^i} | n \text{ at } \tau \right] = \lambda^{-1} E_t \left[ N_{\tau+}^i | n \text{ at } \tau \right]
\]

\[
= \lambda^{-1} E_t \left[ B_{\tau+}^{-\gamma} B_{\tau+}^i \times e^{(1-\gamma)\mu^\gamma(\tau-T)} e^{(1-\gamma)\mu^\gamma(\gamma(\gamma-1)\sigma^2)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\hat{\sigma}^2_{\tau,n}} | n \text{ at } \tau \right]
\]

\[
= \lambda^{-1} e^{(1-\gamma)\mu^\gamma(\tau-T)} e^{(1-\gamma)\mu^\gamma(\gamma(\gamma-1)\sigma^2)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\hat{\sigma}^2_{\tau,n}} E_t \left[ e^{-\gamma b_T+i\gamma T} | n \text{ at } \tau \right]
\]

Now, recall

\[
\frac{B_{\tau}^i}{B_{\tau}^i} = \frac{B_T}{B_{\tau}} e^{\frac{1}{2}(\gamma(\gamma-1)\sigma^2)(T-\tau)+\sigma_1(Z_{\tau}^i-Z_{\tau}^i)}
\]

which implies

\[
e^{b_{\tau}^i} = e^{b_{\tau}^i+b_T-b_{\tau}-\frac{1}{2}\sigma_1^2(T-\tau)+\sigma_1(Z_{\tau}^i-Z_{\tau}^i)}
\]

For \( n = 1, \ldots, N \), we then have:

\[
E_t \left[ \pi_{\tau+M_{\tau+}^i} | n \text{ at } \tau \right] = \lambda^{-1} B_{\tau}^{-\gamma} B_{\tau}^i e^{(1-\gamma)\mu^\gamma(\tau-T)} e^{(1-\gamma)\mu^\gamma(\gamma(\gamma-1)\sigma^2)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\hat{\sigma}^2_{\tau,n}}
\]

\[
E_t \left[ e^{(1-\gamma)\Delta b_T} | n \text{ at } \tau \right]
\]

\[
= \lambda^{-1} B_{\tau}^{-\gamma} B_{\tau}^i e^{(1-\gamma)\mu^\gamma(\tau-T)} e^{(1-\gamma)\mu^\gamma(\gamma(\gamma-1)\sigma^2)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\hat{\sigma}^2_{\tau,n}}
\]

\[
\int e^{(1-\gamma)\Delta b_T} f(\Delta b_T|S_T, n \text{ at } \tau) d\Delta b_T
\]

Similarly, for \( n = 0 \), we have:

\[
E_t \left[ \pi_{\tau+M_{\tau+}^i} | 0 \text{ at } \tau \right] = \lambda^{-1} E_t \left[ N_{\tau+}^i | 0 \text{ at } \tau \right]
\]
\[
\begin{align*}
\lambda^{-1} E_t \left[ B_t^{-\gamma} B_t^2 \times e^{(1-\gamma)\hat{g}_t(T-t)} e^{((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\sigma^2)(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2} \right] & \text{ at } \tau \\
= \lambda^{-1} e^{((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\sigma^2)(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2} E_t \left[ e^{-\gamma b_t + b_t + (1-\gamma)\hat{g}_t(T-t)} \right] & \text{ at } \tau \\
= \lambda^{-1} B_t^{-\gamma} B_t^2 e^{((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\sigma^2)(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2} E_t \left[ e^{(1-\gamma)\Delta b_t + (1-\gamma)\hat{g}_t(T-t)} \right] & \text{ at } \tau \\
= \lambda^{-1} B_t^{-\gamma} B_t^2 e^{((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\sigma^2)(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2} \\
\times E_t \left[ e^{(1-\gamma) \left( E_t[\Delta b_t] + \{\ddot{g}_t - E_t[\ddot{g}_t]\} \sqrt{\frac{1}{\gamma}} \right) + (1-\gamma)\hat{g}_t(T-t)} \right] & \text{ at } \tau \\
= \lambda^{-1} B_t^{-\gamma} B_t^2 e^{((1-\gamma)\hat{g}_t(T-t) + ((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\sigma^2)(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2} \\
\times \int e^{(1-\gamma) \left( E_t[\Delta b_t] + \{\ddot{g}_t - E_t[\ddot{g}_t]\} \sqrt{\frac{1}{\gamma}} \right) + (1-\gamma)(\hat{g}_t - \ddot{g}_t)(T-t)} f (\ddot{g}_t | S_t, 0 \text{ at } \tau) \, d\ddot{g}_t
\end{align*}
\]

The result follows from comparing the terms in equations (35), (A1), and (A2) with the ones above, and defining in this proposition \( \mu_0^g = \ddot{g}_t \) and \( \sigma_{g,0}^2 = \ddot{\sigma}_t^2 \). Q.E.D.

**Proof of Proposition 5.** The claim follows from an application of Ito’s Lemma to the price \( M_t^i \) in Proposition 4, and the equilibrium restriction \( \mu^*_M = -Cov_t \left( \frac{dM_t^i}{M_t^i}, \frac{d\pi_t}{\pi_t} \right) \). Q.E.D.

**Proof of Corollary 5.** The proof is identical to that of Corollary 3, except that we analyze the limiting behavior of \( E_t \left[ \pi_{\tau+} + M_{\tau+}^i \right] \). Comparing the proofs of Proposition 2 and Proposition 5, we see that the same arguments discussed in Corollary 3 apply for \( E_t \left[ \pi_{\tau+} + M_{\tau+}^i \right] \). In particular, as \( \ddot{g}_t \to -\infty \), we have that for \( \ddot{g}_t \) sufficiently small, \( H(S_t) \) converges to

\[
H(S_t) = e^{(1-\gamma)(\mu + \frac{1}{2}\sigma^2)(\tau-t) + \frac{1}{2}(1-\gamma)^2(\tau-t)^2 \sigma^2 + (\tau-t)^2 \sigma^2} \sum_{n=1}^{N} p^n t e^{(1-\gamma)\mu^* g(S_t) + \frac{1}{2}(1-\gamma)^2(\tau-t)^2 \sigma^2_{g,n}}
\]

Hence,

\[
\frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \ddot{g}_t} \to (1-\gamma)(\tau-t)
\]

Therefore, as \( \ddot{g}_t \to -\infty \), we have

\[
\sigma_{M,0} = \left( \frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \ddot{g}_t} - \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial \ddot{g}_t} \right) \ddot{\sigma}_t^2 \sigma^{-1} \to (\tau-t)\ddot{\sigma}_t^2 \sigma^{-1}
\]

Similarly, as \( \ddot{g}_t \to \infty \), we have that for \( \ddot{g}_t \) sufficiently large, \( H(S_t) \) converges to

\[
H(S_t) = e^{(1-\gamma)(\mu + \frac{1}{2}(1-\gamma)\sigma^2)(\tau-t) + (1-\gamma)\hat{g}_t(T-t) + \frac{1}{2}(1-\gamma)^2(T-t)^2 \sigma_t^2}
\]

which implies

\[
\frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \ddot{g}_t} \to (1-\gamma)(T-t)
\]
Hence, as $\hat{g}_t \to \infty$, we obtain

$$\sigma_{M,0} = \left( \frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \hat{g}_t} - \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial \hat{g}_t} \right) \sigma^2 \sigma^{-1} \to (T - t) \sigma^2 \sigma^{-1}$$

Q.E.D.

**Proof of Corollary 6.** The proof is identical to that of Corollary 4. In particular, from expressions (B21) and (B25), neither $\Omega(S_t)$ nor $H(S_t)$ depend on any $\hat{c}_i$. Thus, both $\partial \Omega(S_t)/\partial \hat{c}_i \to 0$ and $\partial H(S_t)/\partial \hat{c}_i \to 0$, which implies $\sigma_{M,n} \to 0$ for all $n$. Q.E.D.

**Proof of Proposition 6.** From Lemmas A1 and A2, the gross announcement return from announcing policy $n$ is

$$1 + R^n(\hat{g}_t) = e^{(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} \times \left( 1 + \sum_{n=1}^N p^n \left( e^{-\gamma(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} - 1 \right) \right) \left( 1 + \sum_{n=1}^N p^n \left( e^{(1 - \gamma)(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} - 1 \right) \right)$$

Similarly, recalling the notation $\mu^0 = \tilde{\mu}_t$ and $\sigma_{g,0} = \tilde{\sigma}_t$, from Lemma A1 and A2 the gross announcement return from announcing no policy change is

$$1 + R^0(\hat{g}_t) = \left( 1 + \sum_{n=1}^N p^n \left( e^{-\gamma(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} - 1 \right) \right) \left( 1 + \sum_{n=1}^N p^n \left( e^{(1 - \gamma)(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} - 1 \right) \right)$$

Therefore, we can write more compactly, for $n = 1, ..., N$,

$$1 + R^n(\hat{g}_t) = e^{(\mu^n - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)} \times (1 + R^0(\tilde{\mu}_t))$$

(B26)

From equations (15) and (16) in the paper, we have

$$\mu^n_g - \tilde{\mu}_t = (\tilde{\mu}_t - \tilde{\mu}^0) + \frac{(\sigma^2_{g,n} - \tilde{\sigma}^2_t)}{2} (T - \tau) (\gamma - 1)$$

For the exponent in equation (B27), we therefore obtain

$$(\mu^n_g - \tilde{\mu}_t)(T - \tau) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t) = (\tilde{\mu}^n - \tilde{\mu}^0)(T - \tau) + \frac{(\sigma^2_{g,n} - \tilde{\sigma}^2_t)}{2} (T - \tau)^2 (\gamma - 1) + \frac{1 - 2\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t) = (\tilde{\mu}^n - \tilde{\mu}^0)(T - \tau) + \frac{\gamma}{2}(T - \tau)^2 (\sigma^2_{g,n} - \tilde{\sigma}^2_t)$$

(B27)
The claim of Proposition 6 then follows immediately. Q.E.D.

**Proof of Corollary 7.** Immediate from Proposition 6. Q.E.D.

**Proof of Corollary 8.** Immediate from Corollary 7. Q.E.D.

**Proof of Proposition 7.** The expression for the jump risk premium follows immediately from

\[ J(S_\tau) = \sum_{n=0}^{N} p^n_\tau R^n(x_\tau) \]

where \( R^n(x_\tau) \) are given in Proposition 6. We now see that

\[ J(S_\tau) = -Cov_\tau \left( \frac{M^n_\tau}{M^n_\pi} - 1, \frac{\pi^n_\tau}{\pi^n_\tau} - 1 \right) = - \{ E_\tau [J_M J_\pi] - E_\tau [J_\pi] E_\tau [J_M] \} \]

where, if policy \( n \) is chosen, we denote \( J^n_M = \frac{M^n_\tau}{M^n_\pi} \) and \( J^n_\pi = \frac{\pi^n_\tau}{\pi^n_\tau} \). Recall from Proposition 6 that

\[ J^n_M = 1 + R^n(x_\tau) = e^{(\mu^n - x_\tau)(T-\tau) - \frac{1}{2}(T-\tau)^2(\sigma^2_{y,n} - \sigma^2_0)} \times \left( 1 + \sum_{\kappa=1}^{N} p^\kappa_\tau \left( e^{-\gamma(T-\tau)(\mu^n - x_\tau) + \sigma^2_{y,n} - \sigma^2_0} + \right) \frac{1}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi e^{(1-\gamma)(T-\tau)(\mu^n - x_\tau) - 1})} \right) \]

We can compute a similar expression now for the stochastic discount factor. From the expressions for \( \pi^n_\tau \) and \( \pi^n_\pi \) in the proof of Lemma A1, it follows that for \( n = 1, \ldots, N \)

\[ J^n_\pi(x_\tau) = \frac{\pi^n_\tau}{\pi^n_\pi} = e^{-\gamma(\mu^n - x_\tau)(T-\tau) + \sigma^2_{y,n} - \sigma^2_0} J^n_0(\pi_\pi)(x_\tau) \]

where

\[ J^n_0(x_\tau) = \frac{\pi^n_\tau}{\pi^n_\pi} = \frac{1}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi \left( e^{-\gamma(\mu^n - x_\tau)(T-\tau) + \sigma^2_{y,n} - \sigma^2_0} + \right) \frac{1}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi e^{(1-\gamma)(T-\tau)(\mu^n - x_\tau) - 1})} \right) \}

This implies that

\[ J^n_\pi(x_\tau) J^n_M(x_\tau) = \frac{e^{(1-\gamma)(\mu^n - x_\tau)(T-\tau)}}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi e^{(1-\gamma)(T-\tau)(\mu^n - x_\tau) - 1})} \text{ for } n = 1, \ldots, N \]

\[ J^0_\pi(x_\tau) J^0_M(x_\tau) = \frac{1}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi e^{(1-\gamma)(T-\tau)(\mu^n - x_\tau) - 1})} \]

It follows that

\[ E_\tau [J^n_\pi(x_\tau) J^n_M(x_\tau)] = \sum_{\kappa=1}^{N} p^\kappa_\pi \left\{ \frac{e^{(1-\gamma)(\mu^n - x_\tau)(T-\tau)}}{(1 + \sum_{\kappa=1}^{N} p^\kappa_\pi e^{(1-\gamma)(T-\tau)(\mu^n - x_\tau) - 1})} \right\} \]
\[
+ \left(1 - \sum_{\kappa=1}^{N} p_{\tau}^{\kappa}\right) \frac{1}{\left(1 + \sum_{\kappa=1}^{N} p_{\tau}^{\kappa} (e^{(1-\gamma)(T-\tau)}(\tilde{\mu}^{x-\tau} - 1))\right)}
\]

\[
= \frac{1}{\left(1 + \sum_{\kappa=1}^{N} p_{\tau}^{\kappa} (e^{(1-\gamma)(T-\tau)}(\tilde{\mu}^{x-\tau} - 1))\right)} \left(\sum_{\kappa=1}^{N} p_{\tau}^{\kappa} e^{(1-\gamma)(\tilde{\mu}^{x-\tau})(T-\tau)} + 1 - \sum_{\kappa=1}^{N} p_{\tau}^{\kappa}\right)
\]

\[
= 1
\]

Similarly,

\[
E_{\tau} [J_{\pi} (x_{\tau})] = \sum_{\kappa=1}^{N} p_{\tau}^{\kappa} \left\{ \frac{e^{-\gamma \tilde{\mu}^{x}(T-\tau)} e^{\tilde{\mu}^{2} \sigma_{\pi}^{2}(T-\tau)^2}}{e^{-\gamma x_{\tau}(T-\tau)+\tilde{\mu}^{2} \sigma_{\pi}^{2}(T-\tau)^2}} J_{0}^{\pi} (x_{\tau}) \right\} + \left(1 - \sum_{\kappa=1}^{N} p_{\tau}^{\kappa}\right) J_{0}^{\pi} (x_{\tau})
\]

\[
= \left[\sum_{\kappa=1}^{N} p_{\tau}^{\kappa} e^{-\gamma (\tilde{\mu}^{x}-x_{\tau})(T-\tau)} e^{\tilde{\mu}^{2} (\sigma_{\pi}^{2}(T-\tau)^2 + 1 - \sum_{\kappa=1}^{N} p_{\tau}^{\kappa})} J_{0}^{\pi} (x_{\tau})\right]
\]

\[
= 1
\]

Thus, we finally obtain

\[
J(x_{\tau}) = -\text{Cov}_{\tau} (J_{M}, J_{\pi}) = - \{ E_{\tau} [J_{M}J_{\pi}] - E_{\tau} [J_{M}] E_{\tau} [J_{\pi}] \}
\]

\[
= E_{\tau} [J_{M}] - 1
\]

Q.E.D.
2. Political Risk Premium: Additional Results

In this section we collect some additional results on the political risk premium in the limiting case of \( \hat{g}_t \to -\infty \). Recall that in this limiting case, the probability of retaining the old policy converges to zero (i.e., \( p^0_t \to 0 \)).

**Proposition B1**: As \( \hat{g}_t \to -\infty \), then, for every \( m = 1, \ldots, N \)

\[
\frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial c^m} = \sum_{n=1}^{N} w^n_{1,t} \left( \frac{1}{p^n_t} \frac{\partial p^n_t}{\partial c^m} \right); \quad \frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial c^m} = \sum_{n=1}^{N} w^n_{2,t} \left( \frac{1}{p^n_t} \frac{\partial p^n_t}{\partial c^m} \right);
\]

where the weights, which sum to one, are given by

\[
w^n_{1,t} = \frac{p^n_t e^{-\gamma \mu^n_H (T-\tau) + \frac{1}{2} \sigma^2_n (T-\tau)^2} \sigma^n_{g,n}}{\sum_{n=1}^{N} p^n_t e^{-\gamma \mu^n_H (T-\tau) + \frac{1}{2} \sigma^2_n (T-\tau)^2} \sigma^n_{g,n}}
\]

\[
w^n_{2,t} = \frac{p^n_t e^{(1-\gamma) \mu^n_H (T-\tau) + \frac{1}{2} (1-\gamma)^2 (T-\tau)^2} \sigma^n_{g,n}}{\sum_{n=1}^{N} p^n_t e^{(1-\gamma) \mu^n_H (T-\tau) + \frac{1}{2} (1-\gamma)^2 (T-\tau)^2} \sigma^n_{g,n}}
\]

**Proof.** Follows immediately from equations (B20) and (B24). Q.E.D.

Note that the weights are larger for policies with lower means \( \mu^n_g \) and higher uncertainties \( \sigma^n_{g,n} \). Since all of the \( \hat{c}^m_l \)'s are independent of each other and \( \sum_{n=1}^{N} p^n_t = 1 \), we have (i) \( \frac{\partial p^n_t}{\partial c^m} < 0 \) (i.e., an increase in the perceived political cost of policy \( n \) decreases the probability of its adoption); and (ii) \( \frac{\partial p^n_t}{\partial c^m} > 0 \) for \( m \neq n \) (i.e., an increase in the political cost of policy \( m \) increases the probability of adopting a different policy \( n \)).

We now specialize the above result to the case with only two policies, which we denote by \( H \) and \( L \). In that case, we obtain analytical solutions:

**Proposition B2.** For \( N = 2 \) and \( \hat{g}_t \to -\infty \), we have

\[
\frac{1}{\Omega} \frac{\partial \Omega}{\partial \tilde{c}^H} = -\int_{\gamma-1}^{(T-\tau)} \phi_{\tilde{c}H} (\tilde{c} - \tilde{\mu}^H + \tilde{\mu}^L) \phi_{\tilde{c}L} (\tilde{c}) \, d\tilde{c} \frac{1}{(\gamma-1)(T-\tau)} \left( G_1^{-1} + p_H^t \right)
\]

\[
\frac{1}{H} \frac{\partial H}{\partial \tilde{c}^L} = -\int_{\gamma-1}^{(T-\tau)} \phi_{\tilde{c}L} (\tilde{c} - \tilde{\mu}^H + \tilde{\mu}^L) \phi_{\tilde{c}H} (\tilde{c}) \, d\tilde{c} \frac{1}{(\gamma-1)(T-\tau)} \left( G_2^{-1} + p_H^t \right)
\]

where \( \phi_{\tilde{c}H} (\cdot) \) is the normal density with mean \( \hat{c}_t / (\gamma - 1)(T-\tau) \) and variance

\( \left( \hat{\sigma}^2_{c,t} - \sigma^2_{c,t} \right) / (\gamma - 1)^2 (T-\tau)^2 \), and

\[
G_1 = e^{-\gamma (\mu^H - \mu^L)(T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 (\sigma^2_{g,H} - \sigma^2_{g,L})} - 1
\]

\[
G_2 = e^{(1-\gamma)(\mu^H - \mu^L)(T-\tau) + \frac{1}{2} (1-\gamma)^2 (T-\tau)^2 (\sigma^2_{g,H} - \sigma^2_{g,L})} - 1
\]

**Proof.** With \( N = 2 \), we have \( p^L_t = 1 - p^H_t \). We prove only the expression for \( \frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \tilde{c}^m} \), \( m = H, L \), as the other case with \( H(S_t) \) can be proved analogously. Using (B20), we have
Thus, recalling the definition of $G$, we obtain

$$\Omega(S_t) \propto e^{-\gamma \hat{g}_t(t-T) + \frac{1}{2} \gamma^2 (t-T)^2 \hat{\sigma}_t^2} \left[ (1 - p_t^H) e^{-\gamma \mu_t^L(T-T) + \frac{1}{2} \gamma^2 (T-T)^2 \sigma_{g,L}^2} + p_t^H e^{-\gamma \mu_t^H(T-T) + \frac{1}{2} \gamma^2 (T-T)^2 \sigma_{g,H}^2} \right],$$

where the constant of proportionality is independent of any state variables. Thus,

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial \bar{c}^H} = \frac{\partial \Omega}{\partial \bar{c}^L} \left[ (1 - p_t^H) e^{-\gamma \mu_t^L(T-T) + \frac{1}{2} \gamma^2 (T-T)^2 \sigma_{g,L}^2} + p_t^H e^{-\gamma \mu_t^H(T-T) + \frac{1}{2} \gamma^2 (T-T)^2 \sigma_{g,H}^2} \right]$$

Using the definition of $G_1$ we obtain

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial \bar{c}^H} = \frac{\partial \Omega}{\partial \bar{c}^L} = \frac{\partial \Omega}{\partial \bar{c}^L} = \frac{\partial \Omega}{\partial \bar{c}^L} = \frac{\partial \Omega}{\partial \bar{c}^L}$$

We finally compute the sensitivity of probabilities to expected costs. As $\hat{g} \to -\infty$, we have $p_t^0 \to 0$, and we find that $p_t^H$ and $p_t^L$ converge to

$$p_t^H \to \int_{-\infty}^{\infty} \left( 1 - \Phi_{\bar{c}^L} \left( \hat{c}^H - \hat{\mu}_t^H + \hat{\mu}_t^L \right) \right) \phi_{\bar{c}^H} \left( \hat{c}^H \right) \, d\hat{c}^H$$

$$p_t^L \to \int_{-\infty}^{\infty} \left( 1 - \Phi_{\bar{c}^H} \left( \hat{c}^L - \hat{\mu}_t^L + \hat{\mu}_t^H \right) \right) \phi_{\bar{c}^L} \left( \hat{c}^L \right) \, d\hat{c}^L$$

Thus, recalling

$$\bar{c}^t | S_t = \frac{c^t}{(\gamma - 1)(T-t)} \sim N \left( \frac{\bar{c}_t^t}{(\gamma - 1)(T-t)}, \frac{\tilde{\sigma}_{c,t}^2}{(\gamma - 1)^2 (T-t)^2} \right)$$

and letting $V(t, \tau) = \frac{\tilde{\sigma}_{c,t}^2}{(\gamma - 1)(T-t)}$, we have

$$\Phi_{\bar{c}^H} \left( \hat{c}^H - \hat{\mu}_t^H + \hat{\mu}_t^L \right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi V(t, \tau)}} e^{-\frac{1}{2V(t, \tau)} \left( \hat{c}^t \right)^2} \, d\hat{c}^t$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2V(t, \tau)} \left( \hat{c}^t \right)^2} \, d\hat{c}^t$$

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Thus, we obtain

$$\frac{\partial \Phi_{cL}}{\partial \tilde{c}^H} \left( \hat{c}^H - \tilde{\mu}^H + \tilde{\mu}^L \right) = -\frac{1}{(\gamma - 1) (T - \tau)} \frac{1}{\sqrt{V(t, \tau)}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left[ \left( \frac{\hat{c}^H - \tilde{\mu}^H + \tilde{\mu}^L}{\sqrt{V(t, \tau)}} \right) \right]^2} d\tilde{c}$$

which gives

$$\frac{\partial p_t^H}{\partial \tilde{c}^L} = \int_{-\infty}^{\infty} -\frac{\partial \Phi_{cL}}{\partial \tilde{c}^L} \phi_{cL} \left( \hat{c}^H - \tilde{\mu}^H + \tilde{\mu}^L \right) d\tilde{c}^H$$

Similarly, using $p_t^L = 1 - p_t^H$, we also obtain

$$\frac{\partial p_t^L}{\partial \tilde{c}^L} = -\frac{1}{(\gamma - 1) (T - \tau)} \int_{-\infty}^{\infty} \phi_{cL} \left( \hat{c}^L - \tilde{\mu}^L + \tilde{\mu}^L \right) \phi_{cL} \left( \hat{c}^L \right) d\tilde{c}^L$$

This yields the expressions

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial \tilde{c}^H} = \frac{-\int \phi_{cL} \left( \hat{c} - \tilde{\mu}^L + \tilde{\mu}^H \right) \phi_{cL} \left( \hat{c} \right) d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)}$$

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial \tilde{c}^L} = \frac{\int \phi_{cL} \left( \hat{c} - \tilde{\mu}^H + \tilde{\mu}^L \right) \phi_{cL} \left( \hat{c} \right) d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)}$$

An identical argument, using (B24), proves the claim for $\frac{1}{H} \frac{\partial H}{\partial \tilde{c}^m}$, for $m = H, L$. Q.E.D.

This proposition allows us to obtain sufficient conditions for the signs for the sensitivity of $\Omega(S_t)$ and $H(S_t)$ to $\tilde{c}^n$, with $m = H, L$. In particular,

**Corollary B1:** The following two conditions hold:

$$G_1 > 0 \text{ if and only if } \left( \mu^H - \mu^L \right) < \frac{1}{2} \gamma (T - \tau) \left( \sigma_{g,H}^2 - \sigma_{g,L}^2 \right)$$

$$G_2 > 0 \text{ if and only if } \left( \mu^H - \mu^L \right) < \frac{1}{2} (\gamma - 1) (T - \tau) \left( \sigma_{g,H}^2 - \sigma_{g,L}^2 \right)$$

It follows that if both $G_1 > 0$ and $G_2 > 0$, then

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial \tilde{c}^H} < 0; \frac{1}{\Omega} \frac{\partial \Omega}{\partial \tilde{c}^L} > 0; \frac{1}{H} \frac{\partial H}{\partial \tilde{c}^H} < 0; \frac{1}{H} \frac{\partial H}{\partial \tilde{c}^L} > 0$$

**Proof.** We have

$$G_1 = e^{-\gamma \left( \mu^H - \mu^L \right)(T-\tau) + \frac{1}{2} \gamma^2 (T-\tau)^2 \left( \sigma_{g,H}^2 - \sigma_{g,L}^2 \right)} - 1 > 0$$
iff
\[(\mu_g^H - \mu_g^L) < \frac{1}{2}\gamma (T - \tau) \left(\sigma^2_{g, H} - \sigma^2_{g, L}\right)\]

Similarly,
\[G_2 = e^{(1-\gamma)(\mu_g^H - \mu_g^L)(T-\tau)+\frac{1}{2}(1-\gamma)(T-\tau)^2(\sigma^2_{g, H} - \sigma^2_{g, L})} - 1 > 0\]
iff
\[(\mu_g^H - \mu_g^L) < \frac{1}{2} (\gamma - 1) (T - \tau) \left(\sigma^2_{g, H} - \sigma^2_{g, L}\right)\]

The remaining part follows from Proposition B2. Q.E.D.

The conditions in this corollary are intuitive. Consider the case in which \(\sigma^2_{g, H} > \sigma^2_{g, L}\), that is, \(H\) is a riskier policy than \(L\). In that case, the conditions of this corollary state that if the risky policy \(H\) does not have a sufficiently higher expected impact than policy \(L\) to offset its higher risk (i.e., if \(\mu_g^H - \mu_g^L\) is not large enough), then an increase in the political cost of policy \(H\) is good news, as it decreases marginal utility (\(\frac{1}{\Omega} \frac{\partial \Omega}{\partial c_H} < 0\)) and increases expected utility (which is proportional to \(\frac{1}{\Omega} \frac{\partial \Omega}{\partial c_H} > 0\), since \(\gamma > 1\)). The opposite sign holds for an increase in the cost of the less risky policy \(L\).

When \(\mu_g^H - \mu_g^L\) is high enough to violate both conditions, then \(G_1 < 0\) and \(G_2 < 0\). In that case, the signs of the partial derivatives are ambiguous because they depend on the signs of \(G_1^{-1} + p^H_1\) and \(G_2^{-1} + p^H_2\). If the probability of the risky policy is small, then an increase in the cost of the risky policy actually increases marginal utility and decreases expected utility, due to the even smaller chance of adopting a high growth policy.

We continue with a discussion of the iso-utility case, in which we obtain even clearer predictions for the signs of the partial derivatives.

**Corollary B2:** In the iso-utility case (\(\bar{\mu}^L = \bar{\mu}^H\)), the limiting expressions when \(\hat{g} \to -\infty\) simplify to

\[
\frac{1}{\Omega} \frac{\partial \Omega}{\partial c_H} = -\int \phi_{\hat{g}}(\hat{c}) \phi_{\hat{g}}(\hat{c}) d\hat{c} \left(\frac{\gamma - 1}{(T - \tau)} G_1^{-1} + p^H_1\right) = -\frac{1}{\Omega} \frac{\partial \Omega}{\partial c_L} \\
\frac{1}{H} \frac{\partial H}{\partial c_H} = \frac{1}{H} \frac{\partial H}{\partial c_L} = 0
\]

where using \((\mu_g^H - \mu_g^L) = \frac{1}{2} (\sigma^2_{g, H} - \sigma^2_{g, L}) (T - \tau) (\gamma - 1)\) we have the simpler expression for \(G_1\) given by

\[G_1 = e^{\frac{1}{2} (\sigma^2_{g, H} - \sigma^2_{g, L})(T-\tau)^2} - 1 > 0\]

In this case, \(G_1 > 0\), and therefore an increase in the political cost of the high-risk policy \(H\) is always good news as it decreases the marginal utility of the representative agent. An increase in the cost of policy \(H\) of course does not change expected utility, as both policies yield the same utility.

We conclude by deriving a closed-form expression for the political risk premium in the iso-utility case when \(\hat{g}_t \to -\infty\). That is equation (A5) in the paper.
Corollary B3: If \( N = 2, \mu^L = \mu^H, \) and \( \hat{g}_t \to -\infty, \) then
\[
\sigma_{\pi,H} = \frac{1}{\Omega} \frac{\partial \Omega}{\partial \gamma} \sigma_{c,t}^2 h^{-1} \to - \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \sigma_{c,t}^2 h^{-1}
\]
\[
\sigma_{\pi,L} = \frac{1}{\Omega} \frac{\partial \Omega}{\partial \Gamma} \sigma_{c,t}^2 h^{-1} \to \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \sigma_{c,t}^2 h^{-1}
\]
and similarly
\[
\sigma_{M,H} = \left( \frac{1}{H \frac{\partial \Omega}{\partial \gamma}} - \frac{1}{\Omega \frac{\partial \Omega}{\partial \gamma}} \right) \sigma_{c,t}^2 h^{-1} \to \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \sigma_{c,t}^2 h^{-1}
\]
\[
\sigma_{M,L} = \left( \frac{1}{H \frac{\partial \Omega}{\partial \Gamma}} - \frac{1}{\Omega \frac{\partial \Omega}{\partial \Gamma}} \right) \sigma_{c,t}^2 h^{-1} \to - \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \sigma_{c,t}^2 h^{-1}
\]
Thus
\[
\text{Political Risk Premium} = -\sigma_{\pi,H} \sigma_{M,H} - \sigma_{\pi,L} \sigma_{M,L}
\]
\[
\to 2 \left( \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \right)^2 \sigma_{c,t}^4 h^{-2}
\]
This is equation (A5) in the paper.

Given our parameters \( \gamma = 5, (T - \tau) = 10, \sigma_{g,H} = .03, \sigma_{g,L} = .01, \) equal priors \( c_H, c_L \sim N \left( -\frac{1}{2} \sigma_c^2, \sigma_c^2 \right) \) with \( \sigma_c = .1, \) so that \( \sigma_{c,t}^2 = \sigma_c^2 = .1^2 \) at time \( t_0 = \tau - 1, \) and \( p_H = 0.5, \) we obtain \( G_1 = e^{.2 \left( \sigma_{g,H}^2 - \sigma_{g,L}^2 \right) (T - \tau)^2} - 1 = .2214, \) and \( \int \phi_\omega (\tilde{c})^2 \, d\tilde{c} = 126.1566 \) [where \( \tilde{c} \sim N \left( -\frac{1}{2} \sigma_c^2 / (\gamma - 1) (T - \tau))^2, -\frac{\sigma^2_{\gamma L} - \sigma^2_{\gamma H}}{(\gamma - 1) (T - \tau)} \right) \)] with \( \sigma_{c,\tau} = 0.0447. \) Thus,
\[
\left( \frac{\int \phi_{\gamma H} (\tilde{c}) \phi_{\gamma L} (\tilde{c}) \, d\tilde{c}}{(\gamma - 1) (T - \tau) \left( G_1^{-1} + p_t^H \right)} \right) = 0.6287
\]
and hence
\[
\text{Political Risk Premium} = 2 \times (0.6287)^2 \times .1^4 / .05^2 = 3.16\%
\]
This is indeed the value that we see in Figure 3 in the paper.
3. Model Extension: Different Signal Precisions

This extension, described in Section 7.1 in the paper, allows the precisions of political signals to vary across policies. In the two-policy case, we have

\[
\begin{align*}
    ds_t^H &= c^H \ dt + h^H \ dZ_{c,t}^H, \\
    ds_t^L &= c^L \ dt + h^L \ dZ_{c,t}^L,
\end{align*}
\]

with \( h^H \neq h^L \). The figure described in the paper is shown below. The political risk premium has two components, one driven by learning about \( c^H \) (shown in yellow at the top) and the other driven by learning about \( c^L \) (shown in red, second from the top).

Figure B1. The equity risk premium and its components: Different precisions of political signals.

This extension, described in Section 7.2 in the paper, introduces an additional persistent source of variation in profitability that is unrelated to government policy. In this appendix, we first fill in the details of the calculations that are omitted from the paper. We then show the plots that summarize the main results.

We modify the profitability process from the paper’s equation (1) as follows:

\[ d\Pi_t^i = (\mu_t + g_t) \, dt + \sigma \, dZ_t^i + \sigma_1 \, dZ_t^i, \]

where \( \mu_t \) follows the mean-reverting process

\[ d\mu_t = \beta (\bar{\mu} - \mu_t) \, dt + \sigma_\mu \, dZ_{\mu,t}. \]

Agents do not observe \( \mu_t \), but they learn about it by observing \( d\Pi_t^i \) and an additional signal:

\[ dS_t = \mu_t \, dt + \sigma_S \, dZ_{S,t}. \quad (B30) \]

Above, \( \beta, \bar{\mu}, \sigma_\mu, \) and \( \sigma_S \) are known constants, and \( dZ_{\mu,t} \) and \( dZ_{S,t} \) are Brownian motions uncorrelated with all others.

4.1. Learning

The set of unknown quantities includes political costs, \( \mu_t \), and \( g_t \). Learning about political costs proceeds in the same way as in the paper. Below, we characterize learning about \( \mu_t \) and \( g_0 \). Recall that \( g_0 = g^0 \) is constant until time \( \tau \).

We assume that the prior distribution for \( \mu_t \) and \( g^0 \) at time 0 is jointly normal:

\[ \left( \begin{array}{c} \mu_0 \\ g^0 \end{array} \right) \sim N \left( \begin{array}{c} \hat{\mu}_0 \\ \hat{g}_0 \end{array} , \begin{array}{cc} \hat{\sigma}_{\mu,0}^2 & \hat{\sigma}_{\mu g,0} \\ \hat{\sigma}_{\mu g,0} & \hat{\sigma}_{g,0}^2 \end{array} \right) . \]

Standard arguments for Bayesian learning in continuous time yield the following:

**Proposition B3:** The posterior distribution at any time \( t \leq \tau \) is given by

\[ \left( \begin{array}{c} \mu_t \\ g_0 \end{array} \right) \sim N \left( \begin{array}{c} \hat{\mu}_t \\ \hat{g}_0 \end{array} , \begin{array}{cc} \hat{\sigma}_{\mu,t}^2 & \hat{\sigma}_{\mu g,t} \\ \hat{\sigma}_{\mu g,t} & \hat{\sigma}_{g,t}^2 \end{array} \right) , \]

where the posterior means follow

\[ d\hat{\mu}_t = \beta (\bar{\mu} - \hat{\mu}_t) \, dt + \sigma^{-1} (\hat{\sigma}_{\mu,t}^2 + \hat{\sigma}_{\mu g,t}) \, d\hat{Z}_{1,t} + \sigma_S^{-1} \hat{\sigma}_{\mu,t} \, d\hat{Z}_{2,t} \]

\[ (B31) \]

\[ d\hat{g}_t = \sigma^{-1} (\hat{\sigma}_{\mu g,t} + \hat{\sigma}_{g,t}^2) \, d\hat{Z}_{1,t} + \sigma_S^{-1} \hat{\sigma}_{g,t} \, d\hat{Z}_{2,t} \]

\[ (B32) \]
and the posterior variances and covariances follow
\[
\begin{align*}
\frac{d\hat{\sigma}^2_{\mu,t}}{dt} &= -2\beta\hat{\sigma}^2_{\mu,t} + \sigma^2_{\mu} - \left(\sigma^{-2} \left(\hat{\sigma}^2_{\mu,t} + \hat{\sigma}_{\mu g,t}\right) + \sigma^{-2} \left(\hat{\sigma}^2_{\mu,t}\right)^2\right) \\
\frac{d\hat{\sigma}_{\mu g,t}}{dt} &= -\beta\hat{\sigma}_{\mu g,t} - \left(\sigma^{-2} \left(\hat{\sigma}^2_{\mu,t} + \hat{\sigma}_{\mu g,t}\right) \left(\hat{\sigma}_{\mu g,t} + \hat{\sigma}_{g t}\right) + \sigma^{-2}\hat{\sigma}_{\mu,t}^2\hat{\sigma}_{\mu g,t}\right) \\
\frac{d\hat{\sigma}^2_{g t}}{dt} &= -\left(\sigma^{-2} \left(\hat{\sigma}_{\mu g,t} + \hat{\sigma}_{g t}^2\right) + \sigma^{-2}\hat{\sigma}_{\mu g,t}^2\hat{\sigma}_{g t}\right).
\end{align*}
\]

Above, the new Brownian motions \(d\hat{Z}_{1,t}, d\hat{Z}_{2,t}\) reflect expectation errors:
\[
\begin{align*}
\hat{\mu}_{t} &= \beta (\hat{\mu}_{t} - \hat{\mu}_{t}) dt + \sigma^{-1} \left(\hat{\sigma}^2_{\mu,t} + \hat{\sigma}_{\mu g,t}\right) d\hat{Z}_{1,t} \\
\hat{g}_{t} &= \sigma^{-1} \left(\hat{\sigma}_{\mu g,t} + \hat{\sigma}^2_{g t}\right) d\hat{Z}_{1,t},
\end{align*}
\]
and the posterior means become instantaneously perfectly correlated. In particular, news about aggregate profitability \(d\hat{Z}_{1,t}\) affects both \(\hat{\mu}_{t}\) and \(\hat{g}_{t}\). The magnitudes of these effects depend on the degree of uncertainty about \(\hat{\mu}_{t}\) and \(\hat{g}_{t}\). If \(\hat{\sigma}^2_{\mu,t}\) is small relative to \(\hat{\sigma}^2_{g t}\), the update on \(\hat{g}_{t}\) is larger than the update on \(\hat{\mu}_{t}\), and vice versa. In addition, \(\hat{\mu}_{t}\) is affected by mean reversion. A higher value of \(\beta\) implies stronger mean reversion and, consequently, a relatively weaker influence of \(d\hat{Z}_{1,t}\). Furthermore, a higher \(\beta\) implies lower uncertainty about \(\hat{\mu}_{t}\).

### 4.2. The Government’s Policy Decision

We show below that the government’s decision rule in this extended model is very similar to the decision rule in the basic model in the paper. For each \(n = 0, ..., N\), we redefine the utility score of policy \(n\) at time \(\tau\) as follows:
\[
\hat{\mu}^n = \mu^n_g - \frac{\hat{\sigma}^2_{g,n}}{2} \left(\gamma - 1\right) \left(T - \tau\right) - \left(\gamma - 1\right) Q_1(\tau, T) \hat{\sigma}_{\mu g,n},
\]
where
\[
Q_1(t, T) = \frac{1 - e^{-\beta(T-t)}}{\beta}.
\]
This utility score is the same as the utility score in the paper, except for one additional term, \((\gamma - 1)Q_1(\tau, T)\hat{\sigma}_{\mu g,n}\). This term involves \(\hat{\sigma}_{\mu g,n}\), the covariance between \(\mu_t\) and \(g^n\) as of time \(\tau\). For \(n = 0\), this covariance is given by the posterior covariance \(\hat{\sigma}_{\mu g,\tau}\) because some learning about \(g^0\) takes place before time \(\tau\). For \(n > 0\), this is a prior covariance.

**Proposition B4:** The government chooses policy \(n\) at time \(\tau\) if and only if the following condition holds for all policies \(m \neq n, m \in \{0, 1, \ldots, N\}\):

\[
\bar{\mu}^n - \bar{c}^n > \bar{\mu}^m - \bar{c}^m,
\]

where

\[
\bar{c}^n = \frac{c^n}{(\gamma - 1)(T - \tau)} \quad n = 0, 1, \ldots, N.
\]

The government’s decision rule is thus identical to the rule in Proposition 1 in the paper, except for the slightly redefined utility score.

**Corollary B4:** A policy change occurs at time \(\tau\) if and only if

\[
\hat{g}_\tau < \max_{n \in \{1, \ldots, N\}} \{\bar{\mu}^n - \bar{c}^n\} + \frac{\hat{\sigma}_\tau^2}{2}\gamma Q(\tau; T) - \gamma Q_1(\tau; T)\hat{\sigma}_{\mu g,\tau}.
\]

This rule is the same as in Corollary 1 the paper, except for the extra term at the end, which reflects the persistent variation in \(\mu_t\) introduced here.

### 4.3. Stock Prices

First, we establish the pricing results immediately after the policy decision at time \(\tau\).

**Proposition B5:** The stochastic discount factor at time \(\tau^+\) conditional on policy \(n\) being chosen is given by

\[
\pi_{\tau^+}^n = E_{\tau^+}[B_{\tau^+}\gamma | n \text{ at } \tau] = B_{\tau^+}^{-\gamma} \exp\left\{\frac{1}{2}\gamma(1 + \gamma)\sigma^2(T - \tau) - \gamma\bar{\mu}(T - \tau) + \sigma^2\frac{\gamma^2}{2}Q(\tau; T) - \gamma Q_1(\tau; T)(\bar{\mu}_t - \bar{\mu})
\right.
\]

\[
- \gamma \mu_g^n(T - \tau) + \frac{1}{2}\gamma Q_1(\tau, T)^2\hat{\sigma}_{\mu,\tau}^2 + \frac{1}{2}\gamma^2(T - \tau)^2\hat{\sigma}_{g,n}^2 + \gamma^2 Q_1(\tau, T)(T - \tau)\hat{\sigma}_{\mu g,\tau}\}
\]

where \(Q_1(t, T)\) is given above and \(Q(t, T)\) is equal to

\[
Q(t; T) = \left[(T - t) + \frac{1 - e^{-2\beta(T-t)}}{2\beta} - 2Q_1(t; T)\right].
\]

**Proposition B6:** The M/B at time \(\tau^+\) conditional on policy \(n\) being chosen is

\[
\left(\frac{M}{B}\right)_{\tau^+}^n = \exp\left\{-\gamma\sigma^2(T - t) + \bar{\mu}(T - \tau) + \sigma^2\frac{1}{2}(1 - 2\gamma)Q(t; T) + Q_1(\tau; T)(\bar{\mu}_t - \bar{\mu})
\right.
\]

\[
+ \mu_g^n(T - \tau) + \frac{1}{2}(1 - 2\gamma)Q_1(\tau, T)^2\hat{\sigma}_{\mu,\tau}^2 + \frac{1}{2}(1 - 2\gamma)(T - \tau)^2\hat{\sigma}_{g,n}^2
\]

\[
+ (1 - 2\gamma)Q_1(\tau, T)(T - \tau)\hat{\sigma}_{\mu g,\tau}\}
\]

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Next, we provide the pricing results before time $\tau$, which are the focus of this paper.

**Proposition B7:** The stochastic discount factor at time $t < \tau$ is given by

$$\pi_t = B_t^{-\gamma} \exp \left\{ \frac{1}{2} \gamma (1 + \gamma) \sigma^2 (T - \tau) - \gamma \mu (T - \tau) + \sigma^2 \frac{\gamma^2}{2} Q (\tau; T) + \frac{1}{2} \gamma^2 Q_1 (\tau, T)^2 \frac{\sigma^2}{\mu} \tau \right\} \Omega (S_t, t),$$

where

$$\Omega (S_t, t) = E \left[ e^{-\gamma \Delta b_\tau - \gamma Q_1 (\tau; T) (\mu - \pi) - \gamma \mu^2 (T - \tau) + \frac{1}{2} \gamma^2 (T - \tau)^2 \frac{\sigma^2}{\mu} + \frac{1}{2} \gamma^2 Q_1 (\tau, T)^2 \frac{\sigma^2}{\mu} \tau} \mid S_t \right],$$

$\Delta b_\tau = b_\tau - b_t$, and the state variables are $S_t = [\hat{\mu}, \hat{\sigma}, \hat{\pi}, \hat{\gamma}, c_1, t, \ldots, c_N, t].$

The dynamics of the stochastic discount factor are as follows:

$$\frac{d\pi_t}{\pi_t} = \sigma_{\pi,1,t} d\tilde{Z}_{1,t} + \sigma_{\pi,2,t} d\tilde{Z}_{2,t} + \sum_{n=1}^{N} \sigma_{\pi, c, n, t} d\tilde{Z}_{c, n, t}.$$

where

$$\sigma_{\pi,1,t} = \left[ -\gamma \sigma + \frac{1}{\Omega \partial_{\hat{\mu}} \sigma} \sigma^{-1} (\hat{\sigma}_{\mu} + \hat{\sigma}_{\mu g, t}) + \frac{1}{\Omega \partial_{\hat{\sigma}} \sigma} \sigma^{-1} \hat{\sigma}_{\mu g, t} \hat{\sigma}_{\mu g, t} \right],$$

$$\sigma_{\pi,2,t} = \left[ \frac{1}{\Omega \partial_{\hat{\mu}} \sigma} \sigma^{-1} \hat{\sigma}_{\mu} \hat{\sigma}_{\mu} + \frac{1}{\Omega \partial_{\hat{\sigma}} \sigma} \sigma^{-1} \hat{\sigma}_{\mu g, t} \hat{\sigma}_{\mu g, t} \right],$$

$$\sigma_{\pi, c, n, t} = \left[ \frac{1}{\Omega \partial_{\hat{\sigma}}} \hat{\sigma}_{c, n, t} \hat{\sigma}_{c, n, t} \right].$$

The first variable, $\sigma_{\pi,1,t}$, represents the price of risk associated with the economic shocks from our basic model, namely, shocks associated with surprises in aggregate profitability. The third variable, $\sigma_{\pi, c, n, t}$, is the price of risk associated with the political shocks from our basic model. The second variable, $\sigma_{\pi,2,t}$, is new—it captures the additional economic shocks associated with the signal $dS_t$.

**Proposition B8:** The M/B at time $t < \tau$ is given by

$$\frac{M_t}{B_t} = \exp \left\{ -\gamma \sigma^2 (T - \tau) + \mu (T - \tau) + \sigma^2 \frac{1}{2} (1 - 2 \gamma) Q (\tau; T) + \frac{1}{2} (1 - 2 \gamma) Q_1 (\tau, T)^2 \frac{\sigma^2}{\mu} \tau \right\} \frac{H (S_t)}{\Omega (S_t)},$$

where

$$H (S_t) = E \left[ e^{(1-\gamma) \Delta b_\tau + (1-\gamma) Q_1 (\tau; T) (\mu - \pi) + (1-\gamma) \mu^2 (T - \tau) + \frac{1}{2} (1-\gamma)^2 (T - \tau)^2 \frac{\sigma^2}{\mu} + (1-\gamma)^2 Q_1 (\tau, T)^2 \frac{\sigma^2}{\mu} \tau} \mid S_t \right].$$

The stock return process is given by

$$\frac{dM_t}{M_t} = \mu_{M,t} dt + \sigma_{M1,t} d\tilde{Z}_{1,t} + \sigma_{M2,t} d\tilde{Z}_{2,t} + \sum_{n=1}^{N} \sigma_{M,c,n,t} d\tilde{Z}_{c,n,t}.$$
simplicity, because there is no obvious reason to make a different assumption. We vary the paper. We show those plots below.

From 1% to 5%, 10%, and infinity. We construct two types of plots, which are described in

\[ \hat{\sigma}_{g,n} \]

\[ \sigma \]

choose normal distributions. We calibrate the model for the two-policy case and the parameter values in Table 1. We also set the prior variance \( \hat{\sigma}_{g,s} = \hat{\sigma}_{g} \) and the prior covariance \( \hat{\sigma}_{g,n} = 0 \) for all \( n \). We set this prior covariance between \( \mu \) and \( g^n \) equal to zero, for simplicity, because there is no obvious reason to make a different assumption. We vary \( \sigma_S \) from 1% to 5%, 10%, and infinity. We construct two types of plots, which are described in the paper. We show those plots below.

\[
\sigma_{M,1,t} = \sigma + \left( \frac{1}{H} \frac{\partial H}{\partial \mu_t} - \frac{1}{\Omega} \frac{\partial \Omega}{\partial \mu_t} \right) \sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) + \left( \frac{1}{H} \frac{\partial H}{\partial \mu_t} - \frac{1}{\Omega} \frac{\partial \Omega}{\partial \mu_t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right).
\]

\[
\sigma_{M,2,t} = \left( \frac{1}{H} \frac{\partial H}{\partial \mu_t} - \frac{1}{\Omega} \frac{\partial \Omega}{\partial \mu_t} \right) \sigma^{-1} \hat{\sigma}_{\mu}^2 + \left( \frac{1}{H} \frac{\partial H}{\partial \mu_t} - \frac{1}{\Omega} \frac{\partial \Omega}{\partial \mu_t} \right) \sigma^{-1} \hat{\sigma}_{g,t}.
\]

\[
\sigma_{M,c,n,t} = \left( \frac{1}{H} \frac{\partial H}{\partial c^n} - \frac{1}{\Omega} \frac{\partial \Omega}{\partial c^n} \right) \hat{\sigma}_{c,n,t} h^{-1}.
\]

The risk premium has the same expression as in the paper, with one additional term:

\[
\mu_{M,t} = \sigma_{M,1,t} \sigma_{\pi,1,t} + \sigma_{M,2,t} \sigma_{\pi,2,t} + \sum_n \sigma_{M,c,n,t} \sigma_{\pi,c,n,t}.
\]

Given the large number of state variables, we compute \( \Omega (S_t) \) and \( H (S_t) \) by Monte Carlo integration. This method is efficient because we know the joint distribution of the relevant stochastic variables, namely, \( \Delta b, \mu, \mu^t, \mu^s, \mu^g, \sigma, \Sigma \). (The variable \( \mu^g \) appears in the relevant expectations because \( \mu^0 = \mu^g \).) This joint distribution as of time \( t < \tau \) is given by

\[
\left( \begin{array}{c}
\Delta b \\
\mu_t \\
\mu^g \\
\mu^s \\
\mu^g \\
\sigma \\
\Sigma \\
\end{array} \right) \sim N \left( \left( \begin{array}{c}
\hat{\mu}_t - \mu \\
\hat{\mu}_s - \mu \\
\hat{\mu}_g - \mu \\
\hat{\mu}_s - \mu \\
\hat{\mu}_g - \mu \\
\sigma \\
\Sigma \\
\end{array} \right), \left( \begin{array}{c}
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\end{array} \right) \right), \int \Sigma (s) \Sigma (s)' ds
\]

\[
\Sigma (s) = \left( \begin{array}{cc}
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) & 0 \\
\sigma^{-1} \left( \hat{\sigma}_{\mu,0}^2 + \hat{\sigma}_{\mu,t} \right) & \sigma^{-1} \left( \hat{\sigma}_{g,0}^2 \right) \\
\end{array} \right).
\]

We can compute \( \int \Sigma (s) \Sigma (s)' ds \) numerically, construct a large number of draws of \( \Delta b, \mu, \mu \) and \( \mu \) from their joint distribution, and calculate the relevant expectations by averaging across those draws. We also simulate the political costs \( \hat{c}_n \) by drawing them from their normal distributions.

We calibrate the model for the two-policy case and the parameter values in Table 1. We choose \( \bar{\mu} = 10\% \) for consistency with Table 1. We also choose \( \beta = 0.35 \) and \( \sigma = 2\% \), which correspond to estimates of the mean-reverting process for aggregate profitability reported in Pástor and Veronesi (2006). We set the prior variance \( \hat{\sigma}_{\mu,0}^2 = \sigma_{\mu} \) and the prior covariance \( \hat{\sigma}_{\mu,n} = 0 \) for all \( n \). We set this prior covariance between \( \mu \) and \( g^n \) equal to zero, for simplicity, because there is no obvious reason to make a different assumption. We vary \( \sigma_S \) from 1% to 5%, 10%, and infinity. We construct two types of plots, which are described in the paper. We show those plots below.
The first plot described in the paper is shown below. The political risk premium is shown in yellow at the top. The new (fourth) component of the risk premium, driven by the signals $dS_t$ from equation (B30), is shown in red, second from the top.

Figure B2. The equity risk premium and its components: Policy-unrelated business cycles.
The second plot described in the paper is shown below. The value of $\sigma_S = 5\%$, and the other parameters are described in the paper.

![Figure B3. Stock prices and risk premia: Policy-unrelated business cycles. $\sigma_S = 5\%$](image-url)
Finally, the plot below is analogous to the plot on the previous page, except that $\sigma_S \to \infty$. This plot is very similar to the previous one, which is why we do not elaborate on the small differences induced by $\sigma_S$ in the paper.

![Graphs](image)

**Figure B4.** Stock prices and risk premia: Policy-unrelated business cycles. $\sigma_S \to \infty$

**REFERENCES**
