Disclosure Quality, Cost of Capital, and Investor Welfare

Pingyang Gao
The University of Chicago

ABSTRACT: One might expect that disclosure quality improves investor welfare by reducing cost of capital. This study shows that the argument is valid only in limited circumstances. Based on a production economy with perfect competition among investors, the analysis demonstrates three points. First, cost of capital could increase with disclosure quality when new investment is sufficiently elastic. Second, there are plausible conditions under which disclosure quality reduces the welfare of current and/or new investors. Finally, cost of capital could move in opposition to the welfare of either current or new investors as disclosure quality changes.

Keywords: cost of capital; disclosure; welfare; real effects.

I. INTRODUCTION

Regulators and public firms are concerned about the welfare impact of ex ante disclosure policies. Because it is difficult to empirically measure investor welfare, a great deal of recent efforts have focused on the relation between disclosure quality and cost of capital, as an intermediate step to the ultimate goal of understanding the welfare impact of disclosure policies. For example, Arthur Levitt, the former chairman of the Securities and Exchange Commission, has claimed, “The truth is, high [accounting] standards lower the cost of capital. And that’s a goal we share” (Levitt 1998, 82). This reasoning, as well as similar arguments pervasive in policy discussions, has been frequently cited as the motivation for studying the relation between disclosure quality and cost of capital. One

This study is part of my dissertation at Yale University. I sincerely thank my advisers, Rick Antle, John Geanakoplos, Brian Mittendorf, and Shyam Sunder (Chair), for their guidance and encouragement. I also thank Jeremy Bertomeu, Ron Dye, Thomas Hemmer, Bjorn Jorgensen, Steven Kachelmeier, Christian Leuz, Dong Lou, Richard Sansing, Haresh Sapra, Phillip Stocken, Ro Verrecchia, Winnie Wen, and two anonymous referees for many helpful comments and discussions. The work has been substantially improved by the constructive suggestions I have received from the workshops at Carnegie Mellon University, Columbia University, Baruch College–CUNY, Dartmouth College, Duke University, New York University, Northwestern University, The Pennsylvania State University, The University of Chicago, University of Houston, University of Minnesota, University of Toronto, University of Pennsylvania, Yale University, and the AAA 2008 Financial Accounting and Reporting Section meeting. In addition, I am grateful for the generous financial support of the Deloitte Foundation and The University of Chicago Booth School of Business.

Editor’s note: Accepted by Steven Kachelmeier.
interpretation of this reasoning is that cost of capital summarizes the impact of disclosure quality on investor welfare. I explicitly examine this interpretation.

Moreover, even on the relation between disclosure quality and cost of capital, there has been a gap between the empirical evidence and theoretical research. While empirical findings on the relation have been mixed, as surveyed by Healy and Palepu (2001) and Leuz and Wysocki (2007), most theoretical studies have examined a competitive, pure exchange economy and predicted that disclosure quality monotonically reduces cost of capital. Although empirical challenges may have contributed to the inconsistent empirical findings, such as the self-selection problem and measurement errors in proxies for cost of capital and disclosure quality, my model provides a theoretical explanation by introducing the investment effect of disclosure.

In sum, I address two questions. First, how does disclosure quality affect cost of capital, current investors’ welfare, and new investors’ welfare when disclosure influences a firm’s investment decisions? Second, how well does cost of capital represent the welfare of current and/or new investors as disclosure quality changes? I first construct an economy in which disclosure affects a firm’s investment decisions by influencing investors’ valuations. I then identify necessary and sufficient conditions under which disclosure quality reduces cost of capital and improves the welfare of current and new investors. Finally, I compare these conditions to show that they are neither equivalent nor do they subsume each other. Therefore, cost of capital is not always synchronous with the welfare of either current or new investors in the analysis of the economic consequences of disclosure quality.

The results serve as a caveat for the usefulness of cost of capital, defined as risk premium per dollar investment, in representing investor welfare in complex settings. When a single risk-averse owner cares only about the mean and variance of a project’s final cash flow and holds the investment until it pays off, she prefers the project with lower cost of capital, i.e., lower variance provided that the projects have the same means (e.g., Hadar and Russell 1969). In such a setting, one might reasonably expect that cost of capital moves in parallel with the owner’s welfare. But as one moves toward a more complicated environment in which the investment is influenced by disclosure and current investors divest their holdings subsequent to disclosure, the effectiveness of cost of capital as a measure of welfare diminishes and cost of capital does not always remain consistent with investor welfare.

One complexity of the environment is the investment effect of disclosure. Disclosure affects a firm’s investment by revealing its information to the market. Such disclosure influences investors’ beliefs and valuations, which in turn guide the firm’s investment decisions. The firm’s investment decisions affect the stock price, and the stock price feeds back into the firm’s investment choices, both being determined consistently in a rational expectations equilibrium.

The investment effect of disclosure is instrumental in its impact on cost of capital. Disclosure reduces investors’ uncertainty about the firm’s marginal profitability, such that investors would like to pay a higher price for the firm’s shares on average. In a pure exchange economy, higher price implies lower cost of capital, and thus disclosure quality monotonically reduces cost of capital. However, when new investment is possible, as in my model, resolution of uncertainty of the firm’s marginal profitability also guides the firm to adjust its investment level. As a result, both the total investment and the marginal profitability of per unit investment are affected by disclosure quality. A priori, it is not clear whether the cost of capital, a measure of per dollar risk premium, is higher or lower in
equilibrium. The first result of my analysis shows that cost of capital increases with disclosure quality if and only if the adjustment cost of new investment is sufficiently low and the prior expected profitability of existing investment is sufficiently high.

The investment effect is also important for the welfare consequences of disclosure quality. In a pure exchange economy, disclosure only allocates the risk between current and new investors by substituting the price risk for the cash flow risk. For current investors, disclosure creates a trade-off between a higher average level and a higher volatility of the stock price. When current investors are sufficiently risk-averse relative to new investors, high disclosure quality makes current investors worse off by preventing them from transferring more risk to new investors. In a production economy, however, disclosure also facilitates the firm’s investment decisions and thus improves current investors’ welfare. The second result of the model demonstrates that current investors are worse off with higher disclosure quality if and only if current investors are sufficiently risk-averse relative to new investors and the adjustment cost of new investment is sufficiently high.

Finally, disclosure quality makes new investors better off only in a production economy. New investors gain surplus from trading by contributing their risk tolerance to the market. Early resolution of uncertainty reduces the amount of risk left in the market and thus decreases the demand for risk-taking capacity. As a result, new investors gain less surplus from bearing risk for current investors. In the presence of the investment effect, the overall risk of the firm’s cash flow is a function of both the risk of per-unit investment and the total investment. While disclosure reduces the risk of per-unit investment, it could increase the total investment. The third result of the analysis shows that disclosure reduces new investors’ welfare if and only if both the adjustment cost of new investment and the level of existing investment are sufficiently high.

The above analysis reveals that the economic forces behind the impacts of disclosure quality on cost of capital, current investors’ welfare, and new investors’ welfare are different and do not subsume each other. Therefore, cost of capital does not measure well the welfare of either current or new investors. In particular, disclosure quality could increase cost of capital when it increases the overall risk of the firm’s cash flow. Such an endogenous increase in risk could benefit current investors if it is accompanied by a simultaneous increase in the level of the firm’s cash flow, and could benefit new investors because it makes their risk tolerance more valuable.

The main contribution of the study is twofold. First, it links cost of capital to investor welfare and shows that cost of capital does not always move parallel with investor welfare. Given the prominent empirical focus on the relation between disclosure quality and cost of capital, these results help to clarify the connection between this literature and the broad policy debate on the reform of financial reporting and disclosure regulation. The other contribution is to identify the relation between disclosure quality and cost of capital in the presence of the investment effect of disclosure. This result may help sort out the mixed empirical findings on the relation between disclosure quality and cost of capital.

My analysis is couched in a setting of perfect competition among investors to highlight the role of the investment effect of disclosure. Every atomistic investor conjectures that his or her demand does not affect the price, and in equilibrium investor beliefs are fulfilled. As a result, (il)liquidity is never an issue. Alternatively, in models of imperfect competition, such as Diamond and Verrecchia (1991) and Lambert and Verrecchia (2009), disclosure serves to reduce cost of capital and potentially enhance welfare by improving liquidity and ameliorating adverse selection. This is a promising direction to further connect cost of capital to investor welfare.
Section II develops the model and studies the effect of disclosure quality on the distribution of the firm’s cash flow. Section III examines and compares the effects of disclosure quality on cost of capital, current investors’ welfare, and new investors’ welfare. Section IV discusses the related literatures and some assumptions. Section V explores empirical implications of the results. Section VI concludes. All proofs are in the Appendix.

II. THE MODEL AND EQUILIBRIUM

This section describes and solves the model. It is a disclosing-and-then-trading model that allows disclosure to influence the firm’s investment. After solving for the unique equilibrium, I discuss three properties of the equilibrium and in particular examine how disclosure quality changes the characteristics of the distribution of the firm’s cash flow.

The Model

I study a large economy to allow for risk sharing in a competitive market. The number of risky assets (firms) per capita is finite, although the number of both investors and risky assets could be infinite. Thus, I could describe the model in terms of per capita without loss of generality. The risky shares of a representative firm are traded between current and new investors after disclosure, and the number of shares per capita is normalized to be one. There is also a risk-free asset, which acts as a numeraire with a normalized return of one. Figure 1 describes the time line of events.

At $t = 1$, the firm, which has $m$ units of existing investment, discloses a public signal about its marginal profitability, according to a pre-specified disclosure policy. Investors’ prior belief about the marginal profitability is characterized by a mean $\mu_0$ plus a future innovation $\tilde{\mu}$. Before disclosure, they perceive that $\mu$ has a prior distribution of $N\left(0, \frac{1}{\alpha}\right)$. The disclosure, denoted by $\tilde{y}$, provides all investors with an unbiased estimator of $\mu$, and takes the following form:

$$\tilde{y} = \tilde{\mu} + \tilde{\epsilon}, \tilde{\epsilon} \sim N\left(0, \frac{1}{\beta}\right),$$

where $\tilde{\epsilon}$ is independent of $\tilde{\mu}$. $\beta$ is the disclosure quality and the main variable of interest. As $\beta$ increases, the disclosure conveys better information to investors about the firm’s

---

**FIGURE 1**
The Time Line of Events

<table>
<thead>
<tr>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm discloses a signal according to a stipulated quality.</td>
<td>Firm makes new investment; current investors sell all shares to new investors and consume.</td>
<td>Investment pays off; new investors collect the proceeds and consume.</td>
</tr>
</tbody>
</table>

---

1 Although I only study a single-firm economy, the analysis is robust to diversification in a multi-firm economy. Based on Lambert et al. (2007), it will become clear later that I can interpret the variance of the firm’s cash flow here as the covariance of the firm’s cash flow with the cash flow of the market portfolio. The Bayes update rule for covariance is similar to that for variance, and thus the major proofs can go through with covariance.
marginal profitability. The model focuses on the consequences of different levels of $\beta$ for cost of capital and investor welfare.

At $t = 2$, the firm makes an additional investment $k$ to maximize its expected stock price, and current investors then sell their shares to new investors. The net cash flow from $k$ units of new investment takes a quadratic form. Thus, new investors perceive that the firm’s cash flow is as follows:

$$
\tilde{F} = m(\mu_0 + \tilde{\mu}) + k\tilde{\mu} - \frac{z}{2}k^2.
$$  \hspace{1cm} (1)

For new investors, $\tilde{F}$ is the stochastic net cash flow at $t = 3$, if the firm has $m$ units of existing investment and makes $k$ units of new investment at $t = 2$. The first component $m(\mu_0 + \tilde{\mu})$ is the cash flow from the existing investment. The other component, $k\tilde{\mu} - \frac{z}{2}k^2$, is the net cash flow from the new investment $k$. $z$ is the adjustment cost of new investment, capturing the degree to which disclosure quality influences the firm’s investment.\(^2\) Thus, $m$, $\mu_0$, and $z$ are fixed parameters, $k$ is the firm’s choice variable, and $\tilde{\mu}$ is the only source of uncertainty in the firm’s cash flow.

After the firm makes the new investment, current investors sell all of their shares to new investors in a competitive market, consume the proceeds, and leave the market. Based on the firm’s disclosure and new investment, new investors submit their demands for the firm’s shares. Market clearing yields the stock price, which is the market valuation of the firm’s stochastic cash flow\(^3\).

At $t = 3$, the firm’s investment pays off, the firm is liquidated, and new investors consume.

Both current and new investors have CARA utility functions; that is, $U(W_i) = -\exp\left(-\frac{W_i}{\tau_i}\right)$, $i \in \{c, n\}$. $\tau_c$ and $\tau_n$ are the coefficients of risk tolerance of current and new investors, respectively. The subscripts $c$ and $n$ represent current investors and new investors, respectively.

In sum, the two decisions to be made are how much new investment to make by the firm (or equivalently by current investors) and how many shares new investors want to bid. All parties observe all parameters, including existing investment level $m$, new investment level $k$, and the public signal $y$. For expositional ease, I assume that all parameters are well defined except in three special economies defined later.

**Equilibrium Trading Price and Optimal Investment**

A rational expectations equilibrium is a pair of a trading price function $p(y)$ and an investment function $k(y)$, such that, for any signal $y$, the pair $(k(y), p(y))$ satisfies:

1. given $k(y)$, $p(y)$ clears the market;
2. given $p(y)$, $k(y)$ maximizes $p(y)$.

---

\(^2\) $z$ could also be interpreted as a measure of the general economic outlook. By rescaling and rewriting the net cash flow from new investment as $k^2\tilde{\mu} - k^2$, becomes a measure of the profitability of new investment.

\(^3\) Although I describe the investment and trading as two sequential steps, the order does not matter because rational expectations guarantee that the firm’s investment decisions and new investors’ valuation decisions are consistent in equilibrium.
Lemma 1: For any signal $y$, the unique equilibrium $(k(y), p(y))$ is as follows:

$$
k(y) = \frac{E[\hat{\mu}|y]}{z + \frac{2}{\tau_n} \text{Var}[\hat{\mu}|y]} - \frac{2}{\tau_n} \text{Var}[\hat{\mu}|y] - m,

$$
p(y) = \frac{1}{\tau_n} \text{Var}[\hat{F}(y, k(y))].

I examine three properties of this equilibrium: the investment effect, the risk allocation effect, and the overall effects of disclosure quality on the distribution of the firm’s cash flow. First, the adjustment cost of new investment $z$ measures the intensity of the investment effect. One proxy for the impact of disclosure on the firm’s investment decisions is the unconditional variance of the firm’s new investment ($\text{Var}[k(y)]$).

$$
\text{Var}[k(y)] = \frac{\text{Var}[E[\hat{\mu}|y]]}{\left(z + \frac{2}{\tau_n} \text{Var}[\hat{\mu}|y]\right)^2} = \frac{\text{Var}[\hat{\mu}] - \text{Var}[\hat{\mu}|y]}{\left(z + \frac{2}{\tau_n} \text{Var}[\hat{\mu}|y]\right)^2}.
$$

As disclosure quality $\beta$ increases, the remaining uncertainty about the marginal profitability $\text{Var}[\hat{\mu}|y]$ dissipates and the firm’s investment becomes more aggressive. Since $\text{Var}[k(y)]$ decreases monotonically in the adjustment cost $z$ (given disclosure quality $\beta$), $z$ measures the degree of the investment effect. When $z$ is infinitely large, the optimal investment level is always zero and the economy becomes a pure exchange economy.

Second, the risk allocation effect of disclosure quality, similar to that in Dye (1990), is at work because the firm has $m$ units of existing investment. Since the investment effect interacts with the risk allocation effect, I focus on the “residual” risk allocation effect by keeping the total investment fixed. In such a pure exchange economy, as defined later in Definition 1, disclosure quality does not eliminate the risk of the firm’s cash flow; instead, it only allocates the risk between current and new investors. One can decompose the total risk, i.e., $\text{Var}[\hat{\mu}]$, into two components, $\text{Var}[E[\hat{\mu}|y]]$, and $\text{Var}[\hat{\mu}|y]$. The first component is tantamount to the ex ante uncertainty of the trading price (price risk), while the second component characterizes the remaining uncertainty of the firm’s cash flow (cash flow risk). Current investors bear the price risk, and new investors take the cash flow risk in return for a risk premium. Disclosure quality substitutes the price risk for the cash flow risk. Figure 2 illustrates this risk allocation effect in the pure exchange economy.

I prefer the label “risk allocation” to “risk sharing.” The essence of risk sharing in the sense of Wilson (1968) is that trading reduces the total risk by diversifying the idiosyncratic risk. Optimal risk sharing requires that all investors hold the same portfolio under certain conditions. Such risk sharing exists in an model where contemporary investors trade among themselves. However, in a model where one group of investors sell to another, at any level

\[ \text{Var}[\hat{x}_1] + \text{Var}[\hat{x}_2] \leq \frac{1}{\tau_n} \text{Var}[\hat{F}(y, k(y))] \]

\[ \text{Var}[\hat{x}_1] + \text{Var}[\hat{x}_2] = \frac{1}{\tau_n^2} \left( \text{Var}[\hat{x}_1 + \hat{x}_2] + 2\text{Cov}[\hat{x}_1, \hat{x}_2] \right) \]
of disclosure, the risk current investors face is independent of that new investors take. Thus, disclosure allocates the risk between current and new investors, but does not reduce total risk.

The presence of both the investment and risk allocation effects and their interaction complicate the model. While I prove the main results for the general model, I also analyze three special economies to enhance the intuition for the general results. The first two special economies, the pure exchange economy and the economy with constant return to scale (the CRTS economy), represent two extreme cases of the investment effect; the third special economy, the economy without existing investment, isolates the investment effect from the risk allocation effect.

**Definition 1:** The pure exchange economy is an economy in which the firm cannot change investment after disclosure. Mathematically, it is achieved by setting the adjustment cost of new investment $z$ to infinity. In addition, I normalize $m$ to one unit in this case. Thus, $\tilde{F}_{pe} = \lim_{z \to \infty, m \to 1} \tilde{F} = \mu_0 + \tilde{\mu}.$ The subscript $pe$ stands for pure exchange.

**Definition 2:** The CRTS economy is an economy in which the firm’s new investment exhibits the property of constant return to scale. It is achieved by setting the adjustment cost of new investment $z$ to zero. Thus, $\tilde{F}_{crts} = \lim_{z \to 0} \tilde{F} = m(\mu_0 + \tilde{\mu}) + k\tilde{\mu}.$ The subscript $crts$ stands for constant return to scale.

**Definition 3:** The economy without existing investment is an economy in which the firm does not have existing investment before disclosure. It is achieved by setting the existing investment level $m$ to zero. Thus, $\tilde{F}_{we} = \lim_{m \to 0} \tilde{F} = k\tilde{\mu} - \frac{z}{2} k^2.$ The subscript $we$ stands for without endowment.

Finally, disclosure quality affects the ex ante distribution of the firm’s cash flow. The firm’s cash flow conditional on the disclosure $y$, $\tilde{F}|y$, is normally distributed and depends on the realization of the signal $y$. To focus on the impact of ex ante disclosure quality, I look at the unconditional mean and variance of the firm’s cash flow before the disclosure, denoted as $M$ and $V$, respectively. The unconditional stock price, denoted as $P$, is then a function of $M$ and $V$. 

---

**FIGURE 2**

The Risk Allocation Effect

<table>
<thead>
<tr>
<th>Total Risk (Var[\tilde{\mu}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Risk (Var[E[\tilde{\mu}</td>
</tr>
<tr>
<td>Disclosure Quality $\beta$</td>
</tr>
</tbody>
</table>
\[ M = E[E[\tilde{F}|y]]; \]  
\[ V = E[Var[\tilde{F}|y]]; \]  
\[ P = E[p(y)] = M - \frac{V}{\tau_n}. \]

\(M, V, \) and \(P\) are taken expectations with respect to disclosure \(y\). For simplicity, I call \(E[\tilde{F}|y]\) and \(Var[\tilde{F}|y]\) the conditional mean and variance of the firm’s cash flow, \(\tilde{M}\) and \(\tilde{V}\) the mean and the variance of the firm’s cash flow, and \(P\) the stock price, whenever there is no confusion.

**Lemma 2 (Disclosure Quality and Cash Flow Distribution):** As disclosure quality improves, both the mean of the firm’s cash flow (\(M\)) and the stock price (\(P\)) increase, but the variance of the firm’s cash flow (\(V\)) increases if and only if the adjustment cost of new investment is sufficiently low (\(z < z^*\)). The cutoff \(z^*\) is given in Expression (A-15) in the Appendix.

With the investment effect, disclosure quality changes both the mean and variance of the firm’s cash flow, and the variance of the firm’s cash flow could increase with disclosure quality. Therefore, the investment effect is important for the economic consequences of disclosure quality, given that all the main variables of interest—cost of capital, current investors’ welfare, and new investors’ welfare—are related to the characteristics of the entire distribution of the firm’s cash flow. The importance becomes more obvious when Lemma 3 reveals that the impact of disclosure quality on the distribution of the firm’s cash flow varies dramatically in three special economies.

**Lemma 3 (Disclosure Quality and Cash Flow Distribution in the Special Economies):** As disclosure quality improves:

1. in the pure exchange economy, the mean is constant, the variance decreases, and the price increases;
2. in the CRTS economy, the mean, the variance, and the stock price all increase;
3. in the economy without existing investment, the mean and the stock price increase, but the variance increases if and only if the adjustment cost is sufficiently low \(z < \frac{2}{(\beta - \alpha)\tau_n}\).

Table 1 summarizes Lemma 2 and 3 and Figure 3 is a figurative illustration of Lemma 3.

**III. MAIN RESULTS**

Having characterized the equilibrium, I conduct comparative statics in this section to addresses the main research questions. I first identify necessary and sufficient conditions under which disclosure quality reduces cost of capital and improves the welfare of current and new investors. I then compare these conditions to show that they are not equivalent, nor do they subsume each other, as summarized in Table 2. Therefore, cost of capital does
TABLE 1
Effects of Disclosure Quality on the Mean and Variance of the Firm’s Cash Flow

<table>
<thead>
<tr>
<th>Economies (Cash Flow $F$)</th>
<th>Mean $M$</th>
<th>Variance $V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Exchange ($\mu_0 + \hat{\mu}$)</td>
<td>Constant</td>
<td>Decrease</td>
</tr>
<tr>
<td>CRTS ($m(\mu_0 + \hat{\mu}) + k\hat{\mu}$)</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>No Endowment $k\hat{\mu} - \frac{z}{2}k^2$</td>
<td>Increase</td>
<td>Increase/Decrease</td>
</tr>
<tr>
<td>General Economy $m(\mu_0 + \hat{\mu}) + k\hat{\mu} - \frac{z}{2}k^2$</td>
<td>Increase</td>
<td>Increase/Decrease</td>
</tr>
</tbody>
</table>

FIGURE 3
The Impacts of $\beta$ on $M$ and $V$ in Three Special Economies

TABLE 2
Effects of Disclosure Quality on Cost of Capital and Investors’ Welfare

<table>
<thead>
<tr>
<th>Economies</th>
<th>Cost of Capital</th>
<th>Current Investors’ Welfare</th>
<th>New Investors’ Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Exchange</td>
<td>Decrease</td>
<td>Increase/Decrease</td>
<td>Decrease</td>
</tr>
<tr>
<td>CRTS</td>
<td>Increase</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>No Endowment</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase/Decrease</td>
</tr>
<tr>
<td>General Economy</td>
<td>Increase/Decrease*</td>
<td>Increase/Decrease**</td>
<td>Increase/Decrease***</td>
</tr>
</tbody>
</table>

*, **, *** Conditions differ from and do not subsume each other.

not always summarize the impact of disclosure quality on the welfare of either current or new investors.

Disclosure Quality and Cost of Capital

Cost of capital is defined as the expected return on the firm’s equity. Importantly, it is the cost of capital for new investors to invest in the firm’s stocks, not the cost of capital for the firm to undertake new investment.

$$E[\hat{R}] = \frac{M - P}{P}. \tag{5}$$

Cost of capital could be characterized as either the conditional or unconditional expected returns on equity; the latter is a more appropriate characterization in this analysis because it measures cost of capital over all possible information realizations. Empirically,
this unconditional expected return $E[\tilde{R}]$ is the value-weighted average of the conditional expected returns $E[\tilde{R}|y]$, representing the obtainable return for an investor who invests in the same firm over time or simultaneously in many similar firms. In addition, unlike conditional expected return, $E[\tilde{R}]$ is always positive if $\mu_0 > \tilde{\mu}_0$:

$$E[\tilde{R}] = \frac{2\alpha zm^2 - \beta \tau_n}{4\alpha m + 2\alpha^2 zm\tau_n + 2\alpha \beta zm\tau_n}.$$ Note that $\tilde{\mu}_0$ is independent of the signal $y$.

**Proposition 1 (Disclosure Quality and Cost of Capital):** As disclosure quality improves, cost of capital decreases if and only if the adjustment cost of new investment is sufficiently high ($z > z^*$) or the prior expected profitability of the firm’s existing investment is sufficiently low ($\mu_0 < \mu^*_0$). $z^*$ is the same as that in Lemma 2, and $\mu^*_0$ is given in Expression (A-18) in the Appendix.

The intuition behind Proposition 1 centers on the impacts of disclosure quality on the characteristics of the distribution of the firm’s cash flow (Lemma 2). Cost of capital measures the per-dollar risk premium. The size of the overall risk premium increases with the variance of the firm’s cash flow, and the scaling variable (i.e., the stock price) increases with the prior expected profitability of the firm’s existing investment. Disclosure quality could increase cost of capital if it increases the variance and the variance grows faster than the stock price. A sufficiently low adjustment cost guarantees the increasing variance and a sufficiently high prior expected profitability further ensures that the per-dollar variance is increasing. Thus, Proposition 1 extends the relation between disclosure quality and cost of capital to a production economy and confirms the conjecture in Lambert et al. (2007, 410).

Cost of capital could be rewritten as a function of the variance-mean ratio of the firm’s cash flow by plugging Equation (4) into Equation (5).

$$E[\tilde{R}] = \frac{1}{\tau_n \frac{V}{M} - 1}. \quad (6)$$

Cost of capital increases monotonically with the variance-mean ratio $\left(\frac{V}{M}\right)$ and decreases with new investors’ risk tolerance ($\tau_n$). Using “” to denote the partial derivative with respect to $\beta$, cost of capital increases with disclosure quality if and only if the sign of the following partial derivative is positive.

---

5 $E[R] = \frac{M - P}{P} = \int_{-\infty}^{\infty} E[F|y] \frac{p(y)}{p(y)} \phi(y)dy = \int_{-\infty}^{\infty} E[R|y] \frac{p(y)}{p(y)} \phi(y)dy$ where $\phi(y)$ is the probability density function of $\tilde{y}$. Note that the equal-weighted return, $E \left[ \frac{E[F|y] - p(y)}{p(y)} \right]$, is difficult to calculate because of the appearance of the random variable $y$ in the denominator.
\[
\frac{\partial E[\bar{R}]}{\partial \beta} = \frac{MM'\tau_n}{(\tau_n M - V)^2} \left( \frac{V'}{M'} - \frac{V}{M} \right). \tag{7}
\]

When does \( \frac{\partial E[\bar{R}]}{\partial \beta} > 0 \)? First, a positive \( V' \) is a necessary condition for the derivative to be positive. All variables in Equation (7) are always positive except \( V' \).\(^6\) If \( V' < 0 \), then \( \frac{V'}{M'} < 0 \), and disclosure quality monotonically reduces cost of capital. By Lemma 2, \( V' < 0 \) is equivalent to the condition that the adjustment cost of the firm’s new investment is sufficiently high \( (z > z^*) \). This explains the condition about the adjustment cost \( z \) in Proposition 1.

Second, when \( V' > 0 \), the sign of the derivative is determined solely by the sign of the difference between two variance-mean ratios, \( \frac{V'}{M'} - \frac{V}{M} \). The economic intuition of these two ratios is as follows. Consider a marginal increase in disclosure quality that causes an incremental change in the firm’s cash flow. The firm’s new cash flow becomes a weighted average of the pre-change cash flow with a variance-mean ratio of \( \frac{V}{M} \) and the incremental cash flow with a variance-mean ratio of \( \frac{V'}{M'} \). If the variance-mean ratio of the incremental cash flow \( \left( \frac{V'}{M'} \right) \) is greater than that of the pre-change cash flow \( \left( \frac{V}{M} \right) \), then the new (weighted average) variance-mean ratio becomes greater and cost of capital increases.\(^7\)

Finally, to explain the determinants of the variance-mean ratios, note that the prior expected profitability \( \mu_0 \) does not change the incremental cash flow, but affects the pre-change cash flow by altering the mean \( M \). All else equal, when \( \mu_0 \) is greater, \( M \) is greater, and thus \( \frac{V}{M} \) is smaller. When \( \mu_0 \) is large enough, \( \frac{V'}{M'} \) exceeds \( \frac{V}{M} \) and \( \frac{\partial E[\bar{R}]}{\partial \beta} > 0 \). This completes the intuition of Proposition 1. Further, the intuition that disclosure quality influences cost of capital through its impact on the variance-mean ratio of the firm’s cash flow becomes more transparent in three special economies, as noted before.

**Corollary 1 (Disclosure Quality and Cost of Capital in the Special Economies):** As disclosure quality improves, cost of capital decreases in the pure exchange economy and in the economy without existing investment, but increases in the CRTS economy.

---

\(^6\) Under condition \( \mu_0 > \hat{\mu}_0 \), the price \( P \) is positive. Since the variance \( V \) is positive, \( M = P + \frac{1}{\tau_n} V \) is also positive. Finally, Lemma 2 proves that \( M' \) is positive.

\(^7\) Another interpretation of the result is to rewrite Equation (7) as \( \frac{\partial E[\bar{R}]}{\partial \beta} = \frac{MV\tau_n}{(\tau_n M - V)^2} \left( \frac{V}{V} - \frac{M'}{M} \right) \). The impact of disclosure quality on cost of capital is determined by the disclosure quality elasticity of variance and mean.
In the pure exchange economy, the variance-mean ratio is \( \left( \frac{V}{M} \right)_{pe} = \lim_{z \to \infty, m \to 1} \frac{V}{M} = \frac{\text{Var}[\tilde{\mu}|y]}{\mu_0} \). Disclosure quality monotonically reduces \( \text{Var}[\tilde{\mu}|y] \) and thus reduces cost of capital. Disclosure quality does not change the mean but always reduces the conditional variance of the firm’s cash flow, resulting in a decreasing variance-mean ratio.

When the investment effect is present, disclosure quality affects both the mean and variance of the firm’s cash flow. As a result, the impact of disclosure quality on cost of capital becomes more subtle. In the economy without existing investment, the variance-mean ratio is \( \left( \frac{V}{M} \right)_{we} = \lim_{m \to 0} \frac{V}{M} = \frac{2\tau_n}{4 + \frac{z\tau_n}{\text{Var}[\tilde{\mu}|y]}} \). Disclosure quality also monotonically reduces \( \text{Var}[\tilde{\mu}|y] \) and thus reduces the cost of capital. In this economy, the mean of the firm’s cash flow monotonically increases with disclosure quality, while the variance is not monotonic. The mean grows faster than the variance. As a result, disclosure quality also decreases the variance-mean ratio.\(^8\)

The CRTS economy provides an example in which the variance outpaces the mean. In this economy, the variance-mean ratio is \( \left( \frac{V}{M} \right)_{crts} = \lim_{z \to 0} \frac{V}{M} = \frac{\tau_n}{2 + \frac{m\mu_0\tau_n}{\text{Var}[\tilde{\mu}]} \frac{V_{crts}}{V_{crts}}} \). Disclosure quality monotonically increases \( V_{crts} \) and thus increases the cost of capital. As disclosure quality improves, both the mean and variance increase, but the variance grows faster than the mean, leading to an increasing variance-mean ratio.

In sum, disclosure quality affects the cost of capital through its impact on the variance-mean ratio of the firm’s cash flow. In the presence of the investment effect, disclosure quality affects both the mean and variance of the firm’s cash flow. As a result, there are plausible conditions under which disclosure quality increases the cost of capital.

**Disclosure Quality and Current Investors’ Welfare**

I define investor welfare as investors’ *ex ante* expected utility: the utility after the disclosure quality has been set, but before the signal comes out. In particular, current investors’ welfare is as follows:

\[
E[U(W_c)] = E[E[U(W_c)|y]] = H_1\exp(H_2),
\]

where \( H_1 \) and \( H_2 \) are given in Expressions (A-21) and (A-22) in the Appendix. The complexity of \( H_1 \) and \( H_2 \) results from the investment effect, which induces a non-normal distribution of \( p(y) \).

**Proposition 2 (Disclosure Quality and Current Investors’ Welfare):** As disclosure quality improves, current investors are better off if and only if they are sufficiently risk-tolerant relative to new investors \( \left( \tau_c > \frac{\tau_n}{2} \right) \) or the adjustment cost of new investment is sufficiently low \( (z < z^*) \).

The cutoff is characterized in Expression (A-23) in the Appendix.

\(^8\) Lambert et al. (2007) analyze a similar example of the production economy without existing investment.
Remark 1 (Cost of Capital and Current Investors’ Welfare): In the analysis of the economic consequences of disclosure quality, as disclosure quality changes, cost of capital does not always move in parallel with current investors’ welfare.

The intuition for Proposition 2 and Remark 1 lies in the dual effect of disclosure quality of facilitating investment and allocating risk. On one hand, disclosure better aligns the firm’s investment decisions with the market’s expectations, which enhances current investors’ welfare. On the other hand, disclosure quality also allocates the risk between current and new investors by resolving the uncertainty before current investors transfer it to new investors. Whether this risk allocation effect improves current investors’ welfare depends on the relative risk tolerance of current and new investors, as in Dye (1990). When current investors are sufficiently risk-averse and the improvement in investment is marginal, disclosure quality could reduce current investors’ welfare.

The special economies provide transparent intuition. The pure exchange economy illustrates the welfare consequences of the risk allocation effect; the economy without existing investment reveals the welfare impact of the investment effect; and all three special economies are informative about the discrepancy between cost of capital and current investors’ welfare in Remark 1.

Corollary 2 (Disclosure Quality and Current Investors’ Welfare in the Special Economies): As disclosure quality improves, current investors are better off in the pure exchange economy if and only if they are sufficiently risk-tolerant \( \tau_c > \frac{\tau_n}{2} \), and they are always better off in both the economy without existing investment and the CRTS economy.

In the pure exchange economy, disclosure divides the firm’s risk into price risk and cash flow risk. Disclosure quality reduces cash flow risk but increases price risk, creating a trade-off for current investors’ welfare. Formally, the certainty equivalent of the welfare of current investors is as follows:

\[
(CE_{c})_{pe} = P - \frac{1}{2\tau_c} \text{Var}(p(y));
\]

\[
= \mu_0 - \frac{1}{\tau_n} \text{Var}[\tilde{\mu} | y] - \frac{1}{2\tau_c} \text{Var}[E[\tilde{\mu} | y]]; \tag{10}
\]

\[
= \mu_0 - \frac{1}{2\tau_c} \alpha + \left( \frac{1}{2\tau_c} - \frac{1}{\tau_n} \right) \text{Var}[\tilde{\mu} | y]. \tag{11}
\]

Equation (9) indicates that current investors care about not only the average level, but also the ex ante uncertainty of their ex post wealth \( p(y) \). Equation (10) shows that current investors suffer from both cash flow risk and price risk, constituting the initial risk of the firm’s cash flow (\( \text{Var}[\tilde{\mu}] \)). Disclosure quality substitutes the price risk for the cash flow risk. Equation (11) reveals the trade-off for current investors. Loosely speaking, one more unit of price risk costs current investors a relative disutility of \( \frac{1}{2\tau_c} \) utile, and one more unit of
The Accounting Review January 2010
American Accounting Association

cash flow risk costs \( \frac{1}{\tau_n} \) utile. Thus, the reduction in cash flow risk improves current investors’ welfare if and only if \( \tau_c > \frac{\tau_n}{2} \). When it is relatively more expensive to bear the risk by themselves, current investors would rather pay new investors to assume this. In this case, by resolving uncertainty before trading, disclosure quality prevents current investors from transferring more risk to new investors and makes current investors worse off.

Having analyzed how disclosure quality affects current investors’ welfare in the pure exchange economy, I now discuss the intuition behind Remark 1 that cost of capital is a complete measure of current investors’ welfare only in limited circumstances. Cost of capital is synchronous with the welfare of current investors when the reduction of cost of capital arises from the reduction of new investors’ risk aversion or when current investors are sufficiently risk-tolerant. But when current investors are sufficiently risk-averse and have to sell their holdings to new investors subsequent to disclosure, disclosure quality could reduce cost of capital as well as current investors’ welfare.

In the pure exchange economy, cost of capital measures only the average level of current investors’ ex post wealth, but does not capture the ex ante uncertainty of their wealth. Disclosure quality affects both cost of capital and the ex ante uncertainty of their wealth simultaneously. To see this effect, it is helpful to decompose current investors’ certainty equivalent into three components:

\[
(CE)_{pe} = \nu_0 - \frac{E[\tilde{R}]}{1 + E[\tilde{R}]} \nu_0 - \frac{1}{2\tau_c} \text{Var}[E[\tilde{\mu} | y]].
\] (12)

The first component is the expected value of the firm’s cash flow, the second component is the risk discount current investors concede to new investors, and the last is current investors’ utility loss from price risk. One case in which cost of capital moves in parallel with current investors’ welfare is when the change in cost of capital arises from the change in new investors’ risk tolerance. For example, since \( E[\tilde{R}] = \frac{1}{\tau_n} \), an increase in new investors’ risk tolerance \( (\tau_n) \) reduces cost of capital \( E[\tilde{R}] \) (the second component), but keeps the price risk \( \text{Var}[E[\tilde{\mu} | y]] \) (the third component) unchanged. As a result, the decrease in cost of capital improves the welfare of current investors by reducing the risk premium new investors require.

However, this analysis focuses on the relation between cost of capital and investor welfare intermediated by disclosure quality. In this case, new investors’ risk tolerance is not affected by disclosure quality. Lower cost of capital is associated with higher current investors’ welfare only if current investors are sufficiently risk-tolerant. Lower cost of capital caused by higher disclosure quality reduces the risk premium current investors pay to new investors (the second component in Equation (12)), but the benefit comes at the cost of increased exposure to price risk (the third component in Equation (12)). When current investors are sufficiently risk-tolerant, the saving in risk premium dominates their disutility from the increased exposure to the price risk. As a result, they are better off as the cost of capital decreases. But when current investors are sufficiently risk-averse, cost of capital is no longer synchronous with current investors’ welfare.
In the economy without existing investment, there is no risk before disclosure, and thus the risk allocation effect is muted. Current investors’ welfare is \( (E[U(W_{c})])_{we} = \lim_{m \to 0} \) 

\[ E[U(W_{c})] = \frac{1}{\sqrt{1 + \frac{2}{\tau_{c}} P_{we}}} \]

Disclosure quality affects current investors’ welfare only through its impact on the level of their wealth. Since disclosure quality monotonically increases the stock price, the investment effect enhances current investors’ welfare. Furthermore, cost of capital is not a comprehensive measure of the impact of disclosure quality on the level of current investors’ wealth. Since \( P = \frac{M}{1 + E[R]} \), disclosure quality improves the mean and reduces cost of capital at the same time. Therefore, cost of capital is not the only channel for disclosure quality to influence current investors’ welfare. This observation substantiates Remark 1.

In the CRTS economy, the risk allocation is also absent because the firm could always undo the existing investment: \( k(y) = -m + \frac{\tau_{y}}{2} \frac{E[\mu | y]}{Var[\mu | y]} \). Current investors’ welfare is

\[ (E[U(W_{c})])_{crts} = \lim_{z \to 0} E[U(W_{c})] = -\frac{\exp \left( -\frac{m\mu_{0}}{\tau_{c}} \right)}{\sqrt{1 + \frac{2(P_{crts} - m\mu_{0})}{\tau_{c}}}} \]

only about the level of their wealth \( P \), implying that current investors benefit from better disclosure. Moreover, Remark 1 is more evident in the CRTS economy because both cost of capital and current investors’ welfare increase with disclosure quality. While disclosure quality increases cost of capital, it also improves the mean of the firm’s cash flow. The welfare gain from the increased mean dominates the welfare loss from the increased cost of capital, leading to the overall improved welfare for current investors.

In summary, current investors care about both the level and the \textit{ex ante} uncertainty of their wealth. Cost of capital does not capture the \textit{ex ante} uncertainty and is only associated with one component of the level of current investors’ wealth. Disclosure quality could affect cost of capital and other components of current investors’ welfare simultaneously. Therefore, only in some limited cases does cost of capital move in parallel with current investors’ welfare as disclosure quality changes. Beyond these cases, cost of capital becomes less effective as a measure of current investors’ welfare.

\textbf{Disclosure Quality and New Investors’ Welfare}

Similarly, the welfare of new investors is their \textit{ex ante} expected utility:

\[ E[U(W_{n})] = -E[E[U(W_{n})|y]] \]

\[ = -E \left[ \exp \left( -\frac{1}{2\tau_{n}} Var[F|y] \right) \right] = N_{1}\exp(N_{2}), \quad (13) \]

where \( N_{1} \) and \( N_{2} \) are given in Expressions (A-26) and (A-27) in the Appendix.
**Proposition 3 (Disclosure Quality and New Investors’ Welfare):** As disclosure quality improves, new investors are better off if and only if one of the following two conditions holds:

1. If the initial disclosure quality is low ($\beta < \alpha$), then either the adjustment cost of new investment is sufficiently low ($z < z_n^*$) or the level of existing investment is sufficiently low ($m < m_n^*$);
2. If the initial disclosure quality is high ($\beta > \alpha$), then either the adjustment cost of new investment is sufficiently low ($z < z_n^*$), or the adjustment cost of new investment is modest (i.e., $z_n^* < z < z_n^{**}$) and the level of existing investment is sufficiently low ($m < m_n^*$).

Proposition 3 is illustrated in Figure 4. Disclosure quality increases new investors’ welfare in the grid areas. The cut-offs, $z_n^*$, $z_n^{**}$, and $m_n^*$, are given in Expressions (A-28), (A-29), and (A-30) in the Appendix.

**Remark 2 (Cost of Capital and New Investors’ Welfare):** In the analysis of the economic consequences of disclosure quality, as disclosure quality changes, cost of capital does not always move in parallel with new investors’ welfare.

**Remark 3 (The Tension between Current and New Investors):** Current and new investors could have conflicting demands for disclosure quality.

There are two elements in the intuition behind Proposition 3. First, new investors’ conditional expected utility increases with the cash flow risk ($\text{Var}[F \cdot y]$). Second, they are averse to the *ex ante* uncertainty of their conditional expected utility.

The first element may seem surprising, although it is evident in Equation (13) that $E[U(W_n)]$ increases with $\text{Var}[F|y]$. Not only do new investors gain positive surplus from trading in a competitive market, but also the size of the surplus increases with the cash flow risk they bear. The following analysis reveals that the source of new investors’ trading surplus is their risk aversion. Further, the cash flow risk increases their surplus because it makes their demand less price sensitive and reduces competition among themselves.

Since new investors are risk-averse, their demand ($D$) is downward-sloping, as shown in the Appendix (Equation (A-3)).

---

**FIGURE 4**

**The Impact of Disclosure Quality on New Investors’ Welfare**

![Figure showing the impact of disclosure quality on new investors’ welfare.](image)
\[ D = \tau_n \frac{E[\hat{F}|y] - p(y)}{\text{Var}[\hat{F}|y]}. \]

The downward-sloping demand yields diminishing marginal utility \( \left( \text{MU} = E[\hat{F}|y] - \frac{\text{Var}[\hat{F}|y]}{\tau_n} D \right) \), which in turn determines the price \( p(y) \) in equilibrium. As a result, the price is lower than the average utility \( \left( \text{AU} = E[\hat{F}|y] - \frac{\text{Var}[\hat{F}|y]}{2\tau_n} D \right) \). The gap between MU and AU, \( \frac{\text{Var}[\hat{F}|y]}{2\tau_n} D \), is the source of new investors’ surplus from trading in a competitive market. In contrast, when new investors become risk-neutral in the limit, the gap between MU and AU dissipates and \( p(y) = E[\hat{F}|y] \). As a result, new investors always pay a “fair” price for whatever they get, consistent with the notion that trading is a fair game for risk-neutral investors in a competitive market. New investors’ welfare is not directly affected by whatever impacts disclosure has on the mean of the future cash flow \( M \) because the price adjustment absorbs such impacts.

Competition among new investors does affect and reduce the surplus they gain from trading, but in a subtle way. When the cash flow risk \( \text{Var}[\hat{F}|y] \) is higher, new investors’ demand for the firm’s shares in Equation (14) becomes less sensitive to price. That is, the slope of the demand curve \( -\frac{\text{Var}[\hat{F}|y]}{\tau_n} \) becomes much steeper. Less sensitive demand amounts to less competition among new investors, increasing the gap between MU and AU and thus the surplus accruing to new investors.

The forced nature of current investors’ sales increases, but does not generate, new investors’ surplus. When current investors have to leave the market after disclosure, the total risk-taking capacity in the market shrinks, reducing the overall competition for the firm’s shares and improving the reward for risk-taking by new investors. However, even if current investors stay in the market, new investors still gain surplus from trading, although both the gap between MU and AU and the equilibrium holding of new investors become smaller, leading to lower surplus for new investors.

Second, new investors’ welfare is the probability-weighted average of their conditional expected utility. Since they are risk-averse, new investors are averse to the ex ante uncertainty of their conditional expected utility. From the perspective of new investors before disclosure, one realization of \( y \) leads to one conditional distribution of the firm’s cash flow \( \hat{F}|y \) with differential mean and variance. Thus, disclosure creates ex ante uncertainty of the conditional cash flow risk \( \text{Var}[\hat{F}|y] \) and the ex ante uncertainty of new investors’ conditional expected utility. The special economies make the intuition more transparent and illustrate Remarks 2 and 3.

**Corollary 3 (Disclosure Quality and New Investors’ Welfare in the Special Economies):** As disclosure quality improves, new investors are always worse off in the pure exchange economy, always better off in the CRTS economy, and better off in the economy without existing investment if and only if the adjustment cost of new investment is sufficiently low

\[ \varepsilon < \frac{2}{(\beta - \alpha)\tau_n}. \]
In the pure exchange economy, new investors’ welfare is \( \left( E[U(W_n)] \right)_{pe} = \lim_{z \to \infty, m \to 1} E[U(W_n)] = -\exp \left( -\frac{1}{2\tau_n} \text{Var}[\tilde{\mu} | y] \right) \). Disclosure quality decreases new investors’ welfare by leaving less risk for them to assume through trading. Moreover, cost of capital does not link disclosure quality to new investors’ welfare, either. High disclosure quality reduces cost of capital by materializing more risk before trading, but this early resolution of uncertainty reduces the demand for new investors’ risk tolerance and thus reduces their welfare. Therefore, cost of capital is a comprehensive measure of the welfare of neither current nor new investors. Finally, Remark 3 is also evident in the pure exchange economy. While current investors’ preference for disclosure quality depends on their relative risk tolerance, new investors always prefer lower disclosure quality.

In the economy without existing investment, new investors’ welfare increases monotonically with the variance of the firm’s cash flow because \( \left( E[U(W_n)] \right)_{we} = \lim_{m \to 0} E[U(W_n)] = \frac{1}{\sqrt{1 + \frac{1}{\tau_n^2} V_{we}}} \). Since the variance of the firm’s cash flow is not monotonic, new investors benefit from better disclosure quality when the variance is increasing, that is, when \( z < \frac{2}{(\beta - \alpha)\tau_n} \), according to Lemma 3. Moreover, this economy provides another example of the discrepancy between cost of capital and new investors’ welfare. Driven by the decreasing variance-mean ratio, cost of capital decreases with disclosure quality. In contrast, new investors’ welfare is driven by the variance alone, and thus could either increase or decrease with disclosure quality. Finally, this economy also lends support to Remark 3.

When \( z > \frac{2}{(\beta - \alpha)\tau_n} \), current investors favor more transparency while new investors prefer less.

In the CRTS economy, new investors’ welfare also increases with the variance of the firm’s cash flow because \( \left( E[U(W_n)] \right)_{crts} = \lim_{z \to 0} E[U(W_n)] = \frac{1}{\sqrt{1 + \frac{1}{\tau_n^2} V_{crts}}} \). Since the variance of cash flow increases with disclosure quality in the CRTS economy, new investors are better off with better disclosure quality. Moreover, Remark 2 is striking in the CRTS economy. New investors’ welfare increases with disclosure quality. So does the cost of capital. One can rewrite new investors’ welfare as a function of cost of capital: \( \left( E[U(W_n)] \right)_{crts} = \frac{1}{\sqrt{1 + \frac{m\mu_0}{\tau_n} \frac{E[\tilde{R}]}{1 - E[\tilde{R}]} V_{crts}}} \). Since \( E[\tilde{R}] \in (0,1) \), current investors’ welfare increases with cost of capital. Disclosure quality increases both the mean and variance of the firm’s cash flow, but the variance outpaces the mean. As a result, as disclosure quality improves, the cost of capital increases because of the increasing variance-mean ratio, and new investors’ welfare also increases because of the increasing variance. Thus, disclosure quality does not summarize the impact of disclosure quality on new investors’ welfare. Finally, although disclosure quality increases the welfare of both current and new investors, it does so through different channels. Current investors benefit from the improved mean of the firm’s cash flow.
flow but suffer from the accompanying increase in the variance, whereas new investors benefit only from the increase in the variance.

In all three special economies, new investors’ preference for disclosure quality can be summarized by the impact of disclosure quality on the average level of their conditional expected utility (or the variance of the firm’s cash flow $V$). When the risk allocation effect interacts with the investment effect, the $\text{ex ante}$ uncertainty of the cash flow risk ($\text{Var}[\tilde{F}|y]$) also plays a role in new investors’ welfare. In sum, new investors earn surplus from risk-bearing in equilibrium. Their welfare is determined by both the average level and the $\text{ex ante}$ uncertainty of the amount of the cash flow risk they could absorb from trading, and cost of capital is only partially associated with the first component. Since disclosure quality changes both components of the welfare of new investors, the cost of capital does not always move in concert with new investors’ welfare as disclosure quality changes.

IV. DISCUSSION AND RELATED LITERATURES

This section discusses two key assumptions of the model, perfect competition and forced divestiture, in the context of the related literatures. My study links the cost of capital, an important empirical construct in accounting, to the broad economic literature on the efficiency of disclosure quality. A central result about the role of information in a capital market is that the combination of pure exchange, constant absolute risk aversion (CARA), and perfect competition leaves little room for studying investor welfare. Subsequent research introduces private information acquisition (e.g., Diamond 1985), relaxes the assumption of perfect competition (e.g., Kyle 1985; Diamond and Verrecchia 1991), or incorporates production use of information (e.g., Kunkel 1982; Christensen and Feltham 1988; Pae 1999, 2002). This study examines the effect of disclosure on a firm’s investment decisions.

The focus on the investment effect of disclosure entails the first key assumption: perfect competition among investors. My model is couched in a setting in which trade in a firm’s shares is perfectly competitive. In a perfect competition setting, each individual investor is assumed to conjecture that his/her demand has no effect on price, and in equilibrium this conjecture is sustained. Because markets are perfectly liquid by construction, (il)liquidity issues never arise and adverse selection is not priced. Alternatively, in models of imperfect competition, disclosure serves to reduce the cost of capital and potentially enhance welfare by improving liquidity and ameliorating adverse selection (e.g., Diamond and Verrecchia 1991). A recent study by Lambert and Verrecchia (2009) explores this reasoning further. They decompose the cost of capital into three additive risk factors: market risk, (pure) liquidity risk, and adverse selection risk arising from the imperfect competition among investors. It will be interesting to see how the welfare consequences of disclosure quality change in their imperfect competition setting.

The investment effect of disclosure in my analysis draws heavily on the research about the real effects of disclosure in capital market. A firm’s disclosure influences investors’ perceptions which in turn guide the firm’s real decisions, and both the investors’ valuation and the firm’s real decisions are consistently determined in a rational expectations equilibrium. This notion, developed by Kanodia (1980), has been used to study the effect of

---

periodical performance reports (e.g., Kanodia and Lee 1998), measuring intangibles (e.g., Kanodia et al. 2004), and accounting for derivatives (e.g., Melumad et al. 1999; Kanodia et al. 2000; Sapra 2002; Sapra and Shin 2007). While these studies typically focus on the firm’s ex post disclosure choices, I examine the firm’s ex ante disclosure policy and thus abstract from the signaling and communication game.

The focus on ex ante disclosure quality entails the second key assumption: forced divestiture. In this model, current investors have to sell all of their shares of the firm after disclosure. While liquidity-motivated trade is a common feature of models that involve information and trade, its role in this analysis is less benign than that of simply ensuring that market-clearing prices are not fully revealing (e.g., Grossman and Stiglitz 1980; Diamond and Verrecchia 1981). By virtue of the fact that current investors are required to divest their shares after disclosure, it is in their best interest to make an investment choice subsequent to disclosure that maximizes the firm share price that is determined by the preference and belief of new investors. Thus, the investment choice subsequent to disclosure is not necessarily the one that current investors would prefer if they had the opportunity to retain their ownership in the firm. This modeling choice apparently contributes to the discrepancy between disclosure quality and investor welfare.

If current investors keep an exogenous portion of their holdings after disclosure, then the investment decision will reflect the weighted average of the preferences of current and new investors. This alternative does not qualitatively affect the main conclusions of the analysis due to the continuity of the results. However, if current investors (or the firm on their behalf) and new investors have differential access to information, then relaxing the assumption of forced sale complicates the model because the current investors’ divestiture decisions and/or the firm’s investment choice could potentially convey information. In this case, the effects of disclosure on cost of capital and investor welfare could differ from that in my model.

Finally, by studying the investment effect of disclosure quality, this study contributes to the large literature that has focused on the relation between cost of capital and disclosure quality in a pure exchange economy (e.g., Easley and O’Hara 2004; Yee 2006; Hughes et al. 2007; Lambert et al. 2007, 2008). A common theme in the previous literature is that disclosure reduces cost of capital by reducing the conditional variance (or covariance) of the firm’s future cash flow because the mean of the firm’s cash flow is fixed. One exception is Lambert et al. (2007), who also study the indirect effect of disclosure. They point out that cost of capital may increase with disclosure quality if disclosure changes both the mean and variance of the firm’s cash flow. However, they do not link this result directly to disclosure quality. Building on their insight, I study the investment effect and identify conditions for both a positive and negative relation between disclosure quality and cost of capital. Another literature starting from Lucas (1978) approaches the topic through the connection between information and investors’ intertemporal consumption decision (e.g., Veronesi 2000; Yee 2007). Instead, my study focuses on the firm’s investment decisions.

V. EMPIRICAL IMPLICATIONS

With the caveat that cost of capital is not always synchronous with investor welfare, the current results provide one potential theoretical explanation for the mixed empirical evidence on the relation between disclosure quality and cost of capital. A typical empirical specification for testing the relation between disclosure quality and cost of capital is as follows:

\[ E[\hat{R}_i] = \phi X_i + \gamma \beta_i + \varepsilon_i, \]

The Accounting Review January 2010
American Accounting Association
where $X_i$ is a vector of known risk characteristics and $\beta_i$ is a proxy of a firm’s (or country’s) disclosure quality. The sign of $\gamma$ is the variable of interest. Due to the nonlinearity of the impact of $z$ and $\mu_0$ on the relation between disclosure quality and cost of capital, one cannot directly interact these terms with $\beta_i$ in the above regression. One way to sort out the sign of $\gamma$ is to run the regression in the subsamples sorted by $z$ and $\mu_0$. My model predicts that the sign of $\gamma$ is negative except for the subsample of low $z$ and high $\mu_0$.

My model also predicts the determinants of a firm’s choice of ex ante disclosure quality. For example, according to the “bonding hypothesis” (e.g., Coffee 2002; Stulz 1999), a firm can cross-list to commit itself to higher disclosure quality. The current model predicts that firms with lower adjustment cost $z$ and relatively risk-tolerant existing investors, such as growth firms, are more likely to seek cross-listing to maximize their existing investors’ welfare. Moreover, the cross-listing firms on average receive high valuation. Other ways for a firm to commit include the firm’s choices of going public or staying private and of what accounting standards and policies firms choose. The same predictions about cross-listing could also be applied directly to these corporate choices.

Finally, theoretical research has adopted many definitions of cost of capital that correspond to distinct empirical measurements. First, cost of capital in my model, as well as in Lambert et al. (2007) and Hughes et al. (2007), is derived from the price obtained from trading after the firm reveals its disclosure, while Diamond and Verrecchia (1991) and Christensen et al. (2008) examine some transformation of the price before disclosure but after the disclosure quality has been set. Thus, the former set of assumptions corresponds to an association study, while the latter is more suitable for an event study. Second, the first group of studies transform the post-disclosure price in different ways to generate construct of cost of capital. My study uses the weighted average of the conditional returns ($E[R]$), Hughes et al. (2007) adopt the equal weighted unscaled risk premium, and Lambert et al. (2007) draw inferences directly from conditional returns ($E[R|y]$).

VI. CONCLUSION

This study examines the notion that disclosure quality improves investor welfare by reducing the cost of capital. I model a production economy in which disclosure changes a firm’s investment. I identify necessary and sufficient conditions under which disclosure quality reduces cost of capital and improves the welfare of current and new investors. I then show that these conditions are neither equivalent nor do they subsume each other. Therefore, cost of capital does not summarize the impact of disclosure quality on the welfare of either current or new investors. In addition, disclosure quality creates a tension between current and new investors.

With the caveat that they are derived under the assumption of perfect competition, these results may help reconcile the mixed empirical evidence on the relation between disclosure quality and cost of capital, inform empirical efforts to measure the economic consequences of accounting disclosure, and add to the ongoing debate on the reform of financial reporting and disclosure regulation.

APPENDIX

The following proofs involve Lemma 4 and 5. They are proved first.

**Lemma 4:** $f(\cdot)$ is a continuous function, $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$, and $f'(\cdot)$ is the first derivative of $f(\cdot)$. If $f'(x) > 0(<0)$, then $y > f(x) \Leftrightarrow x < (>f^{-1}(y))$. 

The Accounting Review January 2010
American Accounting Association
Proof of Lemma 4

\[ y > f(x) \iff y - f(x) > 0 \]
\[ \iff f(f^{-1}(y)) - f(x) > 0. \]

By the Lagrange Mean Value theorem, there exists \( x_0 \in (x, f^{-1}(y)) \) if \( x < f^{-1}(y) \), or \( x_0 \in (f^{-1}(y), x) \) if \( x > f^{-1}(y) \) such that:

\[ f(f^{-1}(y)) - f(x) = f'(x_0)(f^{-1}(y) - x) > 0. \]

Since \( f'(x_0) > (<>0, thus:

\[ f^{-1}(y) - x > (<>0 \iff x < (>)f^{-1}(y). \]

Lemma 5: Suppose \( x \) is a normally distributed random variable with mean zero and variance \( \sigma^2 \). If \( 1 - 2a\sigma^2 > 0 \), then:

\[ E[\exp(ax^2 + bx + c)] = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp(ax^2 + bx + c) \exp \left( -\frac{1}{2} \frac{x^2}{\sigma^2} \right) dx \]

Proof of Lemma 5

\[ E[\exp(ax^2 + bx + c)] = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp \left( -\frac{1}{2} \frac{x^2}{\sigma^2} \right) \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} \right) dx \]

\[ = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} \right) \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} \right) \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} + c \right) \int_{-\infty}^{\infty} \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} \right) \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} + c \right) dx \]

\[ \times \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} \right) dx \]

\[ = \frac{1}{\sqrt{1 - 2a\sigma^2}} \exp \left( -\frac{1}{2} \frac{b^2\sigma^2}{1 - 2a\sigma^2} + c \right). \]

Proof of Lemma 1

First, one solves for the price \( p(y) \), given new investors’ conjecture about the firm’s investment decision \( k(y) \). Given their knowledge of \( \beta \) and \( y \), and their conjecture \( k(y) \),
new investors perceive that the firm’s cash flow has a normal distribution with mean $E[\tilde{F} | (y, k(y))]$ and $\text{Var}[\tilde{F} | (y, k(y))]$:

$$E[\tilde{F} | (y, k(y))] = m(\mu_o + E[\tilde{\mu} | y]) + kE[\tilde{\mu} | y] - \frac{z}{2} k^2; \quad (A-1)$$

$$\text{Var}[\tilde{F} | (y, k(y))] = (m + k^2) \text{Var}[\tilde{\mu} | y]. \quad (A-2)$$

A representative new investor $i$ chooses her demand $D_i$ to maximize her expected utility by solving the following program $(P1)$:

$$(P1) \max_{D_i} E[U(W_i) | (y, k(y))]$$

$$= -E \left[ \exp \left( -\frac{1}{\tau_n} D_i (\tilde{F} - p) \right) | (y, k(y)) \right]$$

$$= -\exp \left( -\frac{D_i}{\tau_n} \left( E[\tilde{F} | (y, k(y))] - p(y) - \frac{D_i}{2\tau_n} \text{Var}[\tilde{F} | (y, k(y))] \right) \right).$$

The first-order condition gives the optimal demand function:

$$D^*_i = \frac{\tau_n E[\tilde{F} | (y, k(y))] - p(y)}{\text{Var}[\tilde{F} | (y, k(y))]}. \quad (A-3)$$

The second-order condition is equivalent to $-\frac{\text{Var}[\tilde{F} | (y, k(y))]}{\tau_n} < 0$, guaranteeing that $D^*_i$ is the maximum solution.

Since the per capita supply of shares is normalized to one unit, market clearing requires that:

$$1 = \int_0^1 D^*_i di = \frac{\tau_n E[\tilde{F} | (y, k(y))] - p(y)}{\text{Var}[\tilde{F} | (y, k(y))]}. \quad (A-4)$$

Thus, one gets the trading price:

$$p(y) = E[\tilde{F} | (y, k(y))] - \frac{1}{\tau_n} \text{Var}[\tilde{F} | (y, k(y))].$$

Second, anticipating the trading price $p(y)$, the firm makes the investment decision $k(y)$ to maximize the stock price by solving the following program.

$$(P2) \max_{k(y)} E[p(y) | y] = E[\tilde{F} | (y, k(y))] - \frac{1}{\tau_n} \text{Var}[\tilde{F} | (y, k(y))]$$

$$= m\mu_o + (m + k(y))E[\tilde{\mu} | y] - \frac{z}{2} k^2(y) - \frac{(m + k(y))^2 \text{Var}[\tilde{\mu} | y]}{\tau_n}. \quad (A-5)$$
The first-order condition gives the optimal new investment function:

\[
k(y) = \frac{E[\tilde{\mu}|y]}{z + \frac{2}{\tau_n} \text{Var}[\tilde{\mu}|y]} - \frac{\frac{2}{\tau_n} \text{Var}[\tilde{\mu}|y]}{z + \frac{2}{\tau_n} \text{Var}[\tilde{\mu}|y]} m.
\] (A-5)

The second-order condition is equivalent to \(-z - \frac{2 \text{Var}[\tilde{\mu}|y]}{\tau_n} < 0\), guaranteeing that \(k(y)\) is the maximum solution. Therefore:

\[
p(y) = E[\tilde{F}|y] - \frac{1}{\tau_n} \text{Var}[\tilde{F}|y];
\] (A-6)

\[
E[\tilde{F}|y] = m\mu_0 - \frac{2zm^2 \text{Var}[\tilde{\mu}|y]^2}{(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} + \frac{zm\tau_n(z\tau_n + 4\text{Var}[\tilde{\mu}|y])}{(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} E[\tilde{\mu}|y]
+ \frac{\tau_n(z\tau_n + 4\text{Var}[\tilde{\mu}|y])}{2(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} E[\tilde{\mu}|y]^2;
\] (A-7)

\[
\text{Var}[\tilde{F}|y] = \frac{z^2m^2\tau_n^2 \text{Var}[\tilde{\mu}|y]}{(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} + \frac{2zm^2 \text{Var}[\tilde{\mu}|y]}{(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} E[\tilde{\mu}|y]
+ \frac{\tau_n^2 \text{Var}[\tilde{\mu}|y]}{(z\tau_n + 2\text{Var}[\tilde{\mu}|y]^2)} E[\tilde{\mu}|y]^2.
\] (A-8)

**Proof of Lemma 2**

The expected mean, variance, and price are:

\[
M = E[E \tilde{F}|y] = m\mu_0 - \frac{4\alpha zm^2 + \beta \tau_n (4 + (\alpha + \beta)z\tau_n)}{2\alpha(2 + z\tau_n(\alpha + \beta))^2};
\] (A-9)

\[
V = E[\text{Var}[\tilde{F}|y]] = \frac{(\beta + \alpha^2 z^2 m^2 + \alpha \beta z^2 m^2)\tau_n^2}{\alpha(2 + z\tau_n(\alpha + \beta))^2};
\] (A-10)

\[
P = E \left[ E \tilde{F}|y] - \frac{1}{\tau_n} \text{Var}[\tilde{F}|y] \right] = M - \frac{1}{\tau_n} V.
\] (A-11)

\[
\frac{\partial M}{\partial \beta} = \frac{\tau_n(8 + \alpha z(8zm^2 + 6\tau_n + \beta z\tau_n^2))}{2\alpha(2 + \alpha z\tau_n + \beta z\tau_n)^3} > 0;
\]

\[
\frac{\partial P}{\partial \beta} = \frac{\tau_n(2 + \alpha z(2zm^2 + \tau_n))}{2\alpha(2 + \alpha z\tau_n + \beta z\tau_n)^2} > 0.
\]
By taking the partial derivative of Equation (A-10) with respect to \( \beta \) and solving for \( \frac{\partial V}{\partial \beta} > 0 \), one gets the solution \( \{ \alpha < \alpha^*(z), \beta < \beta^*(z) \} \), where:

\[
\alpha^*(z) = \frac{2zm^2 + \tau_n}{2z^2 m^2 \tau_n} + \frac{1}{2} \sqrt{\frac{4z^2 \tau_n^2 + 12z m^2 \tau_n + \tau_n^2}{z^4 m^2 \tau_n}}, \tag{A-12}
\]

\[
\beta^*(z) = \frac{2 + 2\alpha \alpha \tau_n + \alpha \beta \tau_n - \alpha^2 \tau_n^2}{\tau_n + \alpha \beta \tau_n^2 m^2 \tau_n} \tag{A-13}
\]

Both the cutoffs \( \alpha^*(z) \) and \( \beta^*(z) \) are decreasing in \( z \), because:

\[
\frac{\partial \alpha^*(z)}{\partial z} = -\frac{1}{z^2 m^2 \tau_n} \left( zm^2 + \tau_n + \frac{(2z^2 m^4 + 9z^2 m^2 \tau_n + \tau_n^2)}{\sqrt{4z^2 m^4 + 12z m^2 \tau_n + \tau_n^2}} \right) < 0;
\]

\[
\frac{\partial \beta^*(z)}{\partial z} = -\frac{2(1 + 2\alpha \tau_n + \alpha \beta \tau_n)(zm^2 + 2\tau_n)}{\tau_n(z + \alpha \beta \tau_n)^2} < 0.
\]

Thus, by Lemma 4:

\[
\{ \alpha < \alpha^*(z), \beta < \beta^*(z) \} \iff \{ z < z_\alpha(\alpha), z < z_\beta(\beta) \}
\]

\[
\iff z < z^* \tag{A-14}
\]

where \( z_\alpha(\alpha) \) and \( z_\beta(\beta) \) are the inverse functions of \( \alpha^*(z) \) and \( \beta^*(z) \) evaluated at \( \alpha \) and \( \beta \), respectively, and:

\[
z^* = \min(z_\alpha(\alpha), z_\beta(\beta)) \tag{A-15}
\]

Therefore, \( \frac{\partial M}{\partial \beta} > 0, \frac{\partial P}{\partial \beta} > 0, \frac{\partial V}{\partial \beta} > 0 \iff \{ z > z^* \}. \]

**Proof of Lemma 3**

Lemma 3 is proved by taking the limit \( (z \to \infty, z \to 0, m \to 0) \) of the conditions in Lemma 2.

**Proof of Proposition 1**

Cost of capital is defined as:

\[
E[\tilde{R}] = \frac{M - P}{P}. \tag{A-16}
\]

By Equations (A-9), (A-10), and (A-11):

\[
E[\tilde{R}] = \frac{2(\beta + \alpha \beta \tau_n + \alpha \beta \tau_n^2 m^2)\tau_n}{(2 + \alpha \beta \tau_n + \beta \tau_n)(\beta \tau_n + 2\alpha \beta \tau_n \mu_0 + 2\alpha m(2\mu_0 + \beta \tau_n \mu_0 - zm))}. \tag{A-17}
\]
By taking the partial derivative of Equation (A-17) with respect to $\beta$, and solving for $\frac{\partial E[\tilde{R}]}{\partial \beta} > 0$ with respect to $\{\alpha, \beta, \mu_0\}$, one obtains the solution $\{\alpha < \alpha^*(z), \beta < \beta^*(z), \mu_0 > \mu_0^*\}$, where $\alpha^*(z)$ and $\beta^*(z)$ are the same as in Expressions (A-12) and (A-13), and:

$$
\mu_0^* = \frac{z(\beta^2 \tau_n^2 + \alpha m^2(4 + 4\alpha z m^2 + \tau_n) + (\alpha + \beta)^2 z^2 \tau_n)}{2\alpha m(2 + \alpha z \tau_n + \beta z \tau_n)(2 + (\alpha - \beta) z \tau_n + \alpha z^2 m^2(2 - (\alpha + \beta) z \tau_n))},
$$

(A-18)

According to the proof of Lemma 2:

$$
\{\alpha < \alpha^*(z), \beta < \beta^*(z), \mu_0 > \mu_0^*\} \Leftrightarrow \{z < z^*, \mu_0 < \mu_0^*\}.
$$

Therefore, $\frac{\partial E[\tilde{R}]}{\partial \beta} < 0 \Leftrightarrow \{\{z > z^*\}, \{\mu_0 < \mu_0^*\}\}$. \hfill \qed

**Proof of Corollary 1**

Corollary 1 is proved by taking the limit $(z \rightarrow \infty, z \rightarrow 0, m \rightarrow 0)$ of the conditions in Proposition 1. \hfill \qed

**Proof of Proposition 2**

Current investors’ welfare is

$$
E[U(W_c)] = E[E[U(W_c)|y]] = -E\left[ E\left[ \exp\left(-\frac{1}{\tau_c} p\right)|y\right]\right];
$$

(A-19)

$$
= H_1 \exp(H_2);
$$

(A-20)

$$
H_1 = \frac{1}{\sqrt{1 + \frac{\beta \tau_n}{\alpha \tau_c(2 + \alpha z \tau_n + \beta z \tau_n)}}};
$$

(A-21)

$$
H_2 = \frac{m^2(2\alpha z \tau_c + \beta z \tau_n)}{2\tau_c(2\alpha \tau_c + \beta \tau_n + \alpha^2 z \tau_c \tau_n + \alpha \beta z \tau_n \tau_c)} - \frac{m \mu_0}{\tau_c}.
$$

(A-22)

The step from Equation (A-19) to Equation (A-20) involves Lemma 5. By taking the partial derivative of Equation (A-20) with respect to $\beta$ and solving for $\frac{\partial E[U(W_c)]}{\partial \beta} > 0$, one gets $\left\{\left\{\tau_c \geq \frac{\tau_n}{2}\right\}, \left\{\tau_c < \frac{\tau_n}{2}, m < m^*(z)\right\}\right\}$, where:

$$
m^*(z) = \sqrt{(2 + \alpha z \tau_n)(2\alpha \tau_c + \beta \tau_n + \alpha^2 z \tau_c \tau_n + \alpha \beta z \tau_n \tau_c)}/(\alpha^2 z \tau_n(2 + \alpha z \tau_n + \beta \tau_n)).
$$
When \( \tau_c < \frac{\tau_n}{2} \), \( m_c^*(z) \) decreases in adjustment cost \( z \) because:

\[
\frac{\partial m_c^*(z)}{\partial z} = -H \sqrt{\frac{\alpha^2 z^2 (\tau_n - 2\tau_c)(2 + \alpha z \tau_n + \beta z \tau_n)}{(2 + \alpha z \tau_n)(2\alpha \tau_c + \beta \tau_n + \alpha^2 z \tau_c \tau_n + \alpha \beta z \tau_c \tau_n)}} < 0
\]

where \( H \) is a positive constant when \( \tau_c < \frac{\tau_n}{2} \). By Lemma 4 and denoting \( z^* \) as the inverse function of \( m_c^*(z) \) evaluated at \( m \), one has:

\[ m < m_c^*(z) \iff z < z^*_c. \]  

(A-23)

Therefore, \( \frac{\partial E[U(W_n) - E[U(W_n)|y]]}{\partial \beta} > 0 \iff \left\{ \left\{ \tau_c = \frac{\tau_n}{2} \right\}, \left\{ \tau_c < \frac{\tau_n}{2}, z < z^*_c \right\} \right\}. \]

\( \blacksquare \)

**Proof of Corollary 2**

Corollary 2 is proved by taking the limit \( (z \to \infty, z \to 0, m \to 0) \) of the conditions in Proposition 2.

\( \blacksquare \)

**Proof of Proposition 3**

New investors’ welfare is:

\[
E[U(W_n)] = E[E[U(W_n)|y]]
\]

\[
= -E\left[ E \left[ \exp \left( -\frac{1}{\tau_n} (\hat{F} - p) \right) | y \right] \right] ; \quad (A-24)
\]

\[
= N_1 \exp(N_2) ; \quad (A-25)
\]

\[
N_1 = \frac{1}{\sqrt{1 + \frac{\beta}{\alpha(2 + \alpha z \tau_n + \beta z \tau_n)^2}} ;} \quad (A-26)
\]

\[
N_2 = -\frac{\alpha(\alpha + \beta) z^2 m^2}{2(\beta + \alpha^3 z^2 \tau_n + 2 \alpha^2 z \tau_n(2 + \beta z \tau_n) + \alpha(2 + \beta z \tau_n)^2) ;} \quad (A-27)
\]

The last step from Equations (A-24) to (A-25) involves Lemma 5. By taking the partial derivative of Equation (A-25) with respect to \( \beta \) and solving for \( \frac{\partial E[U(W_n)]}{\partial \beta} > 0 \). The solution consists of four regions: \( \{ \beta > \alpha, z < z^*_n \}, \{ \beta > \alpha, z^*_n < z < z^*_m, m < m^*_n \}, \{ \beta \leq \alpha, z < z^*_n \}, \) and \( \{ \beta \leq \alpha, z > z^*_n, m < m^*_n \} \) where:
\[ z^*_{ni} = \frac{\sqrt{3}}{(\alpha + \beta)\tau_n}; \quad (A-28) \]

\[ z^{**}_{ni} = \frac{2}{(\beta - \alpha)\tau_n}; \quad (A-29) \]

\[ m^*_n = \frac{(2 + \alpha z\tau_n - \beta z\tau_n)(4\alpha + \beta + \alpha z\tau_n(\alpha + \beta)(4 + z\tau_n(\alpha + \beta))}{\alpha^2 z^2(2 + \alpha z\tau_n + \beta z\tau_n)(\alpha^2 z^2 \tau_n^2 + 2\alpha \beta z^2 \tau_n^2 + \beta^2 z^2 \tau_n^2 - 3)}. \quad (A-30) \]

**Proof of Corollary 3**

Corollary 3 is proved by taking the limit \((z \to \infty, z \to 0, m \to 0)\) of the conditions in Proposition 3.

**REFERENCES**


