A measurement approach to conservatism and earnings management

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**Abstract**

This paper formalizes a two-step representation of accounting measurement and uses it to formalize a general rationale for conservatism as a measurement principle. A transaction's economic substance manifests itself in characteristics of the transaction, and an accounting rule is a mapping from transaction characteristics to an accounting report. Managers who have stakes in the accounting report are able to influence transaction characteristics. Such earnings management is ex post rational for managers but ex ante inefficient. To safeguard against such ex post opportunism, the optimal ex ante accounting rule is conservative in the sense that it requires more verification of the transaction characteristics favorable to managers. Thus, this rationale for conservatism is as general as the managers' ability and incentive to inflate transaction characteristics. By opening the black box of accounting measurement, the two-step representation also formalizes some classic accounting concepts, such as relevance, reliability, verifiability, verification, and accounting-motivated transactions.

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1. Introduction

When facing uncertainty, accounting measurement guided by conservatism requires more verification to recognize gains than to recognize losses. The long and pervasive influence of conservatism on accounting practice is evident by even a casual inspection of accounting practice and has been systematically documented in empirical studies (see Watts, 2003a, 2003b for a review).

Despite the persistent and pervasive influence of conservatism, the general value of conservatism as a measurement principle remains controversial in the theoretical literature and policy discourse. The core of the controversy might be understood as follows. In this literature, accounting measurement is treated as a black box that emanates an accounting report with certain statistical properties. With this reduced-form representation, conservatism is defined as trading an increase in the false negative (type II) error for an equal amount of decrease in the false positive (type I) error in the accounting report. The evaluation of conservatism is then conveniently transformed to an evaluation of the economic consequences of the report's measurement errors. As a result, the theories arrive at the conclusion that conservatism is
efficient if and only if the false negative error is less costly than the false positive error, a common thread underlying the results in many papers on conservatism (e.g., Gigler et al., 2009; Lu et al., 2011; Nan and Wen, 2011; Caskey and Hughes, 2012). In this framework, the generality of conservatism as a measurement principle is questionable to the extent that accounting reports are used in various settings in which the comparison of costs of measurement errors could go either direction. The controversy ensues.

Notwithstanding the empirical evidence, the lack of theoretical support for conservatism seems to be influencing standard setters both in the United States and around the world. Conservatism as a measurement principle was recently eliminated from the FASB and IASB’s joint conceptual framework (FASB, 2010), which guides the making of future accounting standards. Given the enormous implications of the topic, Leuz (2001) has called for the reconciliation of this stark discrepancy between practice and theories, a theme echoed by Guay and Verrecchia (2006), Gigler et al. (2009), and Lambert (2010).

A careful examination of the discrepancy between theories and empirical works reveals the inadequacy of the reduced-form representation of accounting measurement. For example, one rationale for conservatism, discussed in most empirical works (e.g., Ball, 2001; Watts, 2003a; Kothari et al., 2010), views conservatism as a differential verification requirement and emphasizes managers’ opportunistic influence on accounting measurement as the main justification, but neither feature shows up in the above framework that treats accounting measurement as a black box emanating an accounting report with certain statistical properties.

Opening the black box of accounting measurement invites a number of interesting questions. Conceptually, accounting measurement converts a firm’s transactions and events into accounting reports by accounting rules, with the aim to capture the economic substance of the transactions. What is an accounting rule? How does an accounting rule relate the economic substance of a transaction to an accounting report? What are the major frictions in the process? What instruments does a rule designer control to influence the quality of an accounting report? What do we mean by a conservative accounting rule? As an institutional feature of accounting measurement, conservatism cannot be fully understood without answering these questions.

To answer these questions, I formalize a two-step representation of accounting measurement. This representation and its importance for understanding the design of accounting rules have long been noted in the classical accounting literatures (e.g., Ijiri, 1975; Watts and Zimmerman 1986; Ball, 1989; Leuz, 1998b). First, the state of nature, or the economic substance of a transaction, manifests itself in various characteristics of the transaction. Second, an accounting rule is defined as a mapping from transaction characteristics to an accounting report. While the goal of the rule is to measure the transaction’s economic substance as accurately as possible, the design of the rule is subject to the restriction that it can only be written on transaction characteristics.

This restriction introduces a natural representation of two major frictions in accounting measurement. First, even in the absence of managers’ influence, the correlation between a transaction’s economic substance and its characteristics is likely to be imperfect. This correlation might be termed as a transaction characteristic’s relevance. Second, firms could engage in activities to influence the characteristics of a transaction without improving its economic substance. These activities might be termed as earnings management and a transaction characteristic’s vulnerability to earnings management could be interpreted as one measure of its reliability. Earnings management ranges from outright fabrication of evidence to the sophisticated accounting-motivated transactions such as off-balance-sheet financing activities. Such earnings management is ex post rational for managers but ex ante inefficient.

Facing these two frictions, the rule designer could use at least three instruments to influence the properties of the report. First, what transaction characteristics are admitted to the rule? Second, how much verification is required before the transaction characteristics are accepted? Finally, what is the evidence threshold above which an accounting treatment is accorded? While all the three instruments could be related to conservatism, I focus on the verification requirement. An accounting rule is conservative if it requires more verification of a positive transaction characteristic than of a negative one (e.g., Ball, 2001; Watts, 2003a; Kothari et al., 2010).1

By committing to more verification of transaction characteristics favorable to managers, the ex post benefit of earnings management diminishes and so does the incentive to engage in earnings management. This asymmetric verification requirement takes the form of conservatism when managers prefer better accounting reports. Thus, conservatism as a measurement principle is as general as managers’ ability and incentive to inflate transaction characteristics.

This paper makes two contributions. First, it contributes to the literature on conservatism. By opening the black box of accounting measurement, the paper formalizes the notion that conservatism serves as ex ante safeguards against managers’ ex post opportunistic influence on accounting measurement, which is widely cited in empirical works as the

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1 To make the representation more concrete, consider as an example a typical revenue recognition rule. The economic substance the rule aims to capture is the revenue earning process in a transaction. The characteristics of the transaction include cash receipt, product delivery, terms of warranty, customer credit worthiness, and so on. These characteristics are imperfectly correlated with the revenue earning process, and vulnerable to managers’ influence. For example, even if the firm has not earned revenue from the transaction, the manager may be able to accelerate cash receipt or product delivery through “channel stuffing.” A revenue recognition rule has at least three components. It identifies the subset of transaction characteristics to be used, imposes a verification requirement for each transaction characteristic, and prescribes a threshold of evidence above which revenue is recognized. It is conservative if it requires more verification of transaction characteristics that lead to revenue recognition (good news) than those that do not lead to revenue recognition (bad news).
contracting explanation of conservatism (e.g., Watts, 2003a). It suggests that the reduced-form treatment of accounting measurement is responsible for the controversy about the general value of conservatism as a measurement principle discussed in the second paragraph of this Introduction. For most specifications of verification technologies discussed in the paper, the optimal accounting rule could be conservative even if the false negative error is more costly than the false positive error. Moreover, the optimal accounting rule is always conservative if the costs of measurement errors are equal. In other words, a conservative rule is the path to a “neutral” accounting report in presence of managers’ opportunism.

The simplicity of this rationale for conservatism matches the persistent and pervasive influence conservatism has on accounting practice. Accounting rules are conservative as long as managers have incentive and ability to inflate transaction characteristics. Rewarding managers for good accounting performance is not only a general prescription from incentive theories but also a common feature of many real-world institutions. Empirically, most accounting frauds or irregularity involve inflated accounting reports.

The paper’s second contribution is to clarify two related questions in the context of accounting standard setting. The first concerns the demand for accounting: given the properties of an accounting report, how is it optimally used and what are its economic consequences? The second concerns the supply of accounting: what are the optimal properties of an accounting rule that generates an accounting report with the targeted properties? While answers to both questions provide guidance for accounting standard setting, the first question has received most attention in the literature since Demski (1973). Various models have greatly advanced our understanding of the demand for accounting information by looking at how an accounting report with a given property affects market and non-market interactions, including valuation in capital markets, communication in product markets, and contracting within a firm. In this endeavor, accounting measurement is often treated as a black box that emanates a signal with certain statistical properties.

While this convenient representation could be justified on the ground of focusing on the first question, it could become misleading when the results from this framework are extrapolated as direct answers to the second question about the design of accounting rules. The two questions differ to the extent that the properties of an accounting report differs from those of an accounting rule that generates the report. This distinction is absent in the reduced-form representation, but it is for real for accounting standard setting. Suppose an accounting report with certain properties has been established as desirable (from answering the first question), we are still left with the second question of designing the measurement process that generates such a report. After all, it is this process that determines the actual properties of accounting reports and that is presumably the core of accounting as an independent discipline. To the extent that the properties of an accounting report are influenced only indirectly by the design of accounting rules, the answer to the first question cannot substitute for the answer to the second.

This paper illustrates the importance of differentiating these two questions. Even though conservatism is, at least implicitly, interpreted as a property of an accounting rule in the previous literature, it is nonetheless defined as a property of an accounting report, partially due to the reduced-form representation’s inability to differentiate the two. With the two-step representation in this paper, conservatism is explicitly defined as a property of an accounting rule that generates the report. When we shift the focus from the first to the second question, a simple yet general rationale for conservatism as a measurement principle emerges. Thus, it is critical to look into the accounting measurement process in order to better guide accounting standard setting. The two-step representation of accounting measurement provides one useful framework to open the black box of accounting measurement.

Note that there is a large literature on biased performance measure and earnings management. However, most of that literature focuses on how the use of an accounting report (the demand side) is affected by earnings management (see Lambert, 2001 for a survey and see Glover et al., 2005; Arya and Glover, 2008 for recent examples). In contrast, this paper focuses on how the design of an accounting rule, or the production of an accounting report (the supply side), is constrained by earnings management. A review of literatures on conservatism and other efforts to open the black box of accounting measurement is deferred to Section 5.

The rest of the paper proceeds as follows. Section 2 describes the model. In Section 3, I show that the optimal accounting rule is conservative as long as the manager’s one-sided earnings management is not contractible. Section 4 considers several extensions, including different verification technologies, renegotiation, and multi-period contracting. Section 5 provides a review of related literatures. Section 6 discusses the model’s empirical and policy implications. Section 7 concludes.

2. A model of accounting measurement

To evaluate conservatism as a measurement principle, we require a setting in which the demand for accounting reports arises endogenously. I choose a corporate financing setting in which an accounting-based covenant is used. It will become clear that the rationale for conservatism goes beyond this specific setting. After describing the demand for an accounting-

\footnote{Watts and Zimmerman (1990) summarize it concisely: “Reacting to the incentive of managers to exercise accounting discretion opportunistically, the accepted set includes ‘conservative’ (e.g., lower of cost or market) and ‘objective’ (e.g., verifiable) accounting procedures.”}
based debt covenant, I elaborate the structure of the accounting measurement process. Because the focus of the paper is on the latter, I keep the former part deliberately simple.

### 2.1. A setting with “the difficulty of selective intervention”

An owner–manager (henceforth the manager or the firm) with initial wealth of $A$ has an indivisible project that requires an initial investment $I$ at date 0. At date 1 after the firm is financed, the state of nature, $o_1$, is realized. It is either Good or Bad with probability $q_G$ and $q_B = 1 - q_G$, respectively, i.e., $o \in \{G, B\}$. A firm in state $o$ is referred to as firm $o$. After the realization of the state, one of the two actions has to be taken, i.e., $a \in \{a_M, a_F\}$. The project’s payoff depends on both state $o$ and action $a$. In state $o$, action $a_M$ yields the manager a private benefit $X_{o_M} > 0$ and a risky cash flow $Y_{o_M}$ at date 2.

<table>
<thead>
<tr>
<th>State $o$</th>
<th>Measure $r$</th>
<th>Probability $q_G(1-q_B^G)$</th>
<th>Action taken $a_M$</th>
<th>Financier’s payoff $pD$</th>
<th>Manager’s payoff $p(Y-D)+X_{o_M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>$G$</td>
<td>$q_G(1-q_B^G)$</td>
<td>$a_M$</td>
<td>$pD$</td>
<td>$p(Y-D)+X_{o_M}$</td>
</tr>
<tr>
<td>$G$</td>
<td>$B$</td>
<td>$q_B^Gq_B^J$</td>
<td>$a_F$</td>
<td>$L$</td>
<td>$0$</td>
</tr>
<tr>
<td>$B$</td>
<td>$G$</td>
<td>$q_B^J(1-q_B^G)$</td>
<td>$a_M$</td>
<td>$0$</td>
<td>$X_B$</td>
</tr>
<tr>
<td>$B$</td>
<td>$B$</td>
<td>$q_B^J(1-q_B^G)$</td>
<td>$a_F$</td>
<td>$L$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

I assume that the manager is wealth constrained ($A < I$) and has to finance the difference $I-A$ through a standard debt contract at date 0. The financier provides $I-A$ at date 0 and in return receives a prioritized payment up to the face value $D \leq Y$ at date 2. I call $D$ face value or interest rate interchangeably. Both the manager and the financier are risk neutral and the risk free rate is 0. The lending market is assumed to be competitive at date 0. As a result, the financier’s individual rationality (IR) condition binds in equilibrium and the surplus to the manager also measures the efficiency of the contract.

To simplify the notation, I assume that $L < D$ in equilibrium so that all the cash flow from action $a_F$ goes to the financier. In addition, I assume that $pD < L$ in equilibrium. Otherwise, if $pD \geq L$, the first best is achieved by giving the control right to the financier unconditionally (e.g., through a short-term debt contract or equity) and there is no need for a covenant. As a result of these two assumptions, the payoffs to the manager and the financier ($M,F$) under the combination of states and actions are simplified in Table 1 (the second and third columns are explained later).

This simple setting creates a classic problem of “the difficulty of selective intervention” as in Williamson (1985) and Aghion and Bolton (1992). While the socially optimal action is state-contingent, i.e., $a^*(o = G) = a_M$ and $a^*(o = B) = a_F$, it is clear from Table 1 that the manager prefers $a_M$ and the financier prefers $a_F$ in both states, resulting in the demand for selective intervention. One way to implement the selective intervention is to use a state-contingent covenant that allocates the control right to the financier in and only in the Bad state. The implementation of such a covenant calls for the measurement of the state at date 1. That is, a state-contingent covenant is implemented as a measurement-contingent covenant in practice. Denote the measurement of the state at date 1. That is, a state-contingent covenant is implemented as a measurement-contingent covenant.

We now turn to the design of the accounting rule at date 0 that generates accounting report $r$ at date 1 that settles the measurement-contingent covenant.

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3 As standard in the literature, the difference between cashflow and private benefit is that private benefit cannot be paid out to the financier. That is, $X_{o_M}$ is non-pledgable. Since the manager has spent time in initiating, developing, and implementing the project, he cares not only about the cashflow of the project but also about other non-monetary aspects such as social objectives, employee relationship, reputation, etc. Further, the manager may also accumulate skills and human capital from implementing the project in his own way that could improve his value in the labor market in the future. Most of these benefits are non-pledgable.

4 The equilibrium value of $D$ could be restricted by parameters, including $p$, $Y$, and $A$.

5 Such accounting-based covenants are widely used in debt contracts, see, e.g., Watts and Zimmerman (1986), Christensen and Nikolaev (2012), and Tan (forthcoming).
2.2. A two-step representation of accounting measurement

To open the black box of accounting measurement, I use the following two-step representation. First, the state of nature \( \omega \), interpreted as a transaction's economic substance, manifests itself in the transaction's characteristics \( t \). Second, an accounting rule is a mapping from transaction characteristics \( t \) to accounting report \( r \).

As discussed in the Introduction, the design of an accounting rule involves at least three instruments: the set of admissible transaction characteristics, the verification requirement, and the evidence threshold. This paper focuses on verification because the differential verification requirement is probably the most common interpretation of conservatism in the empirical literature (e.g., Watts and Zimmerman, 1983; Watts, 2003a; Kothari et al., 2010). Gao (2012) examines the third instrument and provides a rationale for the pervasive use of binary classifications and thresholds in accounting rules.

In particular, I assume that \( t \) is a scaler and binary. It is either positive \( P \) (good news) or negative \( N \) (bad news). To capture the idea that the mapping from \( \omega \) to \( t \) could be noisy even in the absence of the manager's influence, I assume that

\[
\Pr(t = P | \omega = G) = \phi_C \quad \text{and} \quad \Pr(t = N | \omega = B) = \phi_B. \quad \phi_C \in \{0, 1\} \quad \text{and} \quad \phi_B \in \{0, 1\}
\]

are the relevance of the transaction characteristic. The verification requirement is operationalized as a vector \( \gamma_t \), \( t \in \{P, N\} \). If transaction characteristic \( t \) is presented, an accounting rule (verification requirement) \( \gamma_t \) requires that \( t \) be verified with probability \( \gamma_t \). The technology of verification is specified later.

In this two-step representation, accounting report \( r \) relates to state \( \omega \) through transaction characteristic \( t \) and the verification process \( \gamma_t \). While the goal of accounting measurement is to capture \( \omega \) as accurately as possible, the rule can only be written on \( t \). This restriction introduces a natural representation of earnings management.

Earnings management is modeled as the manager's activities to inflate \( t \). Between date 0 and date 1, that is, after signing the contract but before the realization of the state, the manager can increase by \( \beta \in [0, \phi_B] \) the probability that the Bad state (transaction) exhibits the positive characteristic \( t = P \). Thus, with earnings management, the mapping from \( \omega \) to \( t \) is altered to \( \Pr(t = P | \omega = G, \beta) = \phi_C + \beta \) and \( \Pr(t = P | \omega = B, \beta) = 1 - \phi_B + \beta \). The private cost of earnings management is \( hKK(\beta) \), \( h > 0 \). A higher \( h \) means that it is more costly for the manager to manipulate transaction characteristics. Thus, \( h \) might be interpreted as the reliability (or "hardness" of Ijiri 1975) of a transaction characteristic. \( K(\beta) \) has the standard properties of a cost function: it is increasing and convex with \( K(0) = K'(0) = 0 \), and \( K'(\phi_B) \) is sufficiently large. Further, \( \frac{d}{d\beta} \left[ K(\beta) \right] = hK''(\beta)K''(\beta) > 0 \), which sets a bound on the speed at which \( K'' \) increases. For example, the standard quadratic cost function \( K(\beta) = \frac{\beta^2}{2} \) with \( h \) properly restricted satisfies these assumptions.

That transaction characteristics could be compromised by earnings management generates the demand for verification. There could be at least two views about the role verification plays in accounting measurement. The narrow view is that verification attests to the authenticity of transaction characteristics, such as whether an invoice is fabricated or not. To understand this view, we could interpret the above specification of earnings management in a different way. Let an indicator variable \( s \) denote the success of earnings management. \( s = 1 \) indicates that the manager succeeds in generating the positive transaction characteristic in the Bad state and \( s = 0 \) indicates otherwise. The probability that \( s = 1 \) is \( \beta \). The narrow view of verification amounts to a claim that verification reveals the realization of \( s \) in the model. In contrast, the broad view equates verification with information production in general. That is, verification reveals the realization of the state \( \omega \). I start with the narrow definition in the baseline model and examine the broad interpretation and various other specifications of verification technology in Section 4.

In the baseline model, if verification is conducted, \( s \) is revealed perfectly, earnings management is undone, and the authentic transaction characteristic is measured accordingly. If verification is not conducted, then the transaction characteristic is taken at its face value. Verification incurs a cost \( \delta > 0 \) when it is conducted. Denote \( \tau \) as the total probability that verification is conducted under accounting rule \( \gamma_t \) : \( \tau = \Pr(t = P | \gamma_t) + \Pr(t = N | \gamma_t) \). Thus, the total expected cost of verification is \( \delta\tau \). Because \( \delta\tau \) is known at date 0, I assume it is financed from the financier at date 0.

Finally, this two-step representation enables us to separate two critical concepts: the property of an accounting report versus the property of an accounting rule that generates the report. Define the (conditional) measurement errors of accounting report \( r \) as

\[
Q^b_C = \Pr(r = b | \omega = G) = 1 - \phi_C, \\
Q^b_B = \Pr(r = b | \omega = B) = 1 - \phi_B + \beta(1 - \gamma_t).
\]
Definition 1. An accounting rule \(\gamma_t\), \(t \in \{P,N\}\), is conservative if it requires more verification of \(t=P\) than of \(t=N\), i.e.,
\[
\gamma_t - \gamma_N > 0.
\]

Definition 2. An accounting report \(r\) is neutral if the measurement errors are equal, i.e., \(Q^b_C = Q^b_B\).

Footnote 1 in page 2 provides a concrete example of this two-step representation of accounting measurement, which could also be depicted as follows:

\[
\begin{array}{cc}
\text{firm influence} & \text{rule design} \\
\hline
\text{(earnings management)} & \text{(e.g., verification)} \\
\end{array}
\]

In sum, the timeline of the events is summarized as follows:

1. At date 0, the manager and the financier sign a debt contract with face value \(D\), an accounting-based covenant in which the manager retains control right if and only if \(r=g\) and the accounting rule \((\gamma_p,\gamma_N)\) that generates \(r\).
2. At date \(\frac{1}{2}\) (between 0 and 1), the manager chooses the level of earnings management \(\beta\).
3. At date 1, the state is realized, transaction characteristic \(t\) occurs, accounting report \(r\) is generated according to the accounting rule \((\gamma_p,\gamma_N)\), the covenant is settled, and the action is taken.
4. At date 2, the project pays out and payment is made.

3. The main results

3.1. Preliminary analysis

The model is solved with backward induction. At date 1, the contracting parties’ expected payoffs depend on both the state \(\omega\) and its accounting measurement \(r\). The probabilities of the combination of \((\omega, r)\) are summarized in Table 1 in page 4. Denote the financier’s date-0 expectation of earnings management as \(\hat{\beta}\) and the manager’s actual choice of earnings management as \(\beta^*\). The contracting problem at date 0 can be formulated as the following maximization problem, labeled as Problem 1

\[
\begin{align*}
\max_{(D;\gamma_p,\gamma_N)} & \quad V(D;\gamma_p,\gamma_N) = q_G(1-Q^b_C)(p(Y-D)+X_G) + q_B Q^b_B(\beta^*)X_B - hK(\beta^*) \\
\text{subject to} & \quad I-A+\delta t \leq q_G(1-Q^b_C)pD + q_G Q^b_C L + q_B(1-Q^b_B(\hat{\beta}))L \\
& \quad \beta^* = \arg\max_{\beta} q_G(1-Q^b_C)(p(Y-D)+X_G) + q_B Q^b_B(\beta)X_B - hK(\beta) \\
& \quad \beta^* = \hat{\beta} \tag{IC} \quad \text{(rational expectations)} \\
& \quad \gamma_t \in [0,1], \quad t \in \{P,N\}.
\end{align*}
\]

The manager chooses face value \(D\) and accounting rule \(\gamma_t\) at date 0 to maximize his expected payoff, subject to the financier’ break-even condition, the anticipated ex post earnings management, the requirements of rational expectations and of \(\gamma_t\) being a probability. The objective function \(V\) is the manager’s expected payoff at date 0, calculated as the inner product of the third and fifth columns of Table 1 with \(\beta\) being replaced by \(\beta^*\) net of the cost of earnings management \(hK(\beta^*)\). The right-hand side of Constraint IR is the financier’s expected payoff from the debt contract, calculated as the inner product of the third and fourth columns of Table 1 with \(\beta\) being replaced by \(\hat{\beta}\). Constraint IC describes the manager’s earnings management decision at date \(\frac{1}{2}\) taking \(D\) and \(\gamma_t\) as given. Finally, the rational expectations require that the financier’s conjecture about the manager’s ex post earnings management be consistent with the manager’s actual choice.

The assumption of a competitive market for financiers at date 0 assures that Constraint IR binds in equilibrium and we could solve for the expression of \(D\) as a function of \(\beta^*\) and \(\gamma_t\). Substituting \(D\) into the objective function and imposing the requirement \(\beta^* = \beta^{**}\) in equilibrium, we rewrite Problem 1 as Problem 2

\[
\begin{align*}
\max_{(\gamma_p,\gamma_N)} & \quad V(\gamma_p,\gamma_N) = V^{FB} - q_B Q^b_B(\beta^*)A^{\text{over}} - q_G Q^b_C A^{\text{under}} - hK(\beta^*) - \delta t(\beta^*) \\
\text{subject to} & \quad \beta^* = \arg\max_{\beta} q_G(1-Q^b_C)(p(Y-D)+X_G) + q_B Q^b_B(\beta)X_B - hK(\beta) \\
& \quad \gamma_t \in [0,1], \quad t \in \{P,N\}.
\end{align*}
\]
\[ V^{FB} = q_B(X_C + pY) + q_BL - I + A \] is the manager's initial wealth plus the first-best firm value when the socially optimal actions are implemented in both states. Expression (3) states that, compared with \( V^{FB} \), the actual firm value \( V \) is reduced by four terms. \( q_BQ_{BM}^{a\text{Over}} \) and \( q_BQ_{BM}^{a\text{Under}} \) are the costs resulting from the suboptimal actions that are induced by the measurement errors \( Q_{BM}^{a\text{Over}}(\beta^*) \) and \( Q_{BM}^{a\text{Under}} \). \( hK(\beta) \) and \( \delta(\beta) \) are the resources consumed by earnings management and verification.

### 3.2. Benchmark: contractible earnings management

To highlight the role of non-contractible earnings management, we first look at the benchmark in which ex post earnings management could be contracted upon ex ante. Ex ante and ex post refer to the timing of signing the contract. Thus, the IC constraint is dropped and \( \beta \) becomes a choice variable in Problem 2. Then the firm value could be rewritten as

\[
V^{BM}(\beta, \gamma) = V^{FB} - q_BA^{\text{Over}}(1 - \phi_B + \beta(1 - \gamma_p)) - q_BA^{\text{Under}}(1 - \phi_C) - hK(\beta) - \delta(\beta).
\]

**Lemma 1.** When earnings management is contractible, in equilibrium,

1. there is no earnings management, i.e., \( \beta^{BM} = 0 \);
2. the optimal verification requirement is \( \gamma^{BM} = 0 \).

**Lemma 1** is straightforward from inspecting Eq. (4). The effect of the verification of \( t=N \) on firm value is \( \partial V^{BM} / \partial \gamma_N = -\delta(\partial d/d\gamma_N) < 0 \) for any \( \beta \) and \( \gamma \). It consumes resources without any benefit, because \( t=N \) is not manipulated by the manager, hence \( \gamma^{BM} = 0 \). The effect of earnings management on firm value is captured by \( \partial V^{BM} / \partial \beta = -hK' - q_BA^{\text{Over}}(1 - \gamma_p) - \delta q_B(\partial d/d\gamma_p) \). Earnings management consumes real resource (the first term). When it succeeds in generating \( t=P \), it either enables the manager to take suboptimal action \( a_M \) in the Bad state (the second term) or invokes costly verification (the third term). Thus, the optimal earnings management is 0. With \( \beta^{BM} = 0 \), the effect of the verification of \( t=P \) on firm value is \( \partial V^{BM} / \partial \gamma_p = -\delta(\partial d/d\gamma_p) < 0 \). Without earnings management, the verification of \( t=P \) consumes resources without any benefit, just like the verification of \( t=N \). Thus, \( \gamma^{BM} = 0 \) and \( \epsilon^{BM} = 0 \).

### 3.3. The design of ex ante accounting rule with non-contractible earnings management

In practice, earnings management is rarely contractible. Instead, the manager's choice of earnings management is governed only by his IC condition in Problem 2. In particular, the manager takes the interest rate \( D \) and accounting rule \( \gamma \) as given at the time of earnings management. The optimal earnings management \( \beta^* \) satisfies the following first-order condition:

\[
q_B(1 - \gamma_p)X_b = hK'(\beta^*).
\]

From the manager's perspective, the left-hand side is the marginal benefit of earnings management at date \( \frac{1}{2} \). By generating the positive transaction characteristic that is measured as good, earnings management allows the manager to retain the control right in the Bad state and receive the private benefit \( X_b \). The right-hand side is the marginal cost of earnings management borne by the manager.

**Lemma 2.** When earnings management is not contractible,

1. earnings management exists in equilibrium (i.e., \( \beta^* > 0 \)) unless \( \gamma_p = 1 \);
2. the verification of the positive transaction characteristic is more useful in mitigating earnings management than the verification of the negative transaction characteristic, i.e., \( \partial \beta^* / \partial \gamma_p < \partial \beta^* / \partial \gamma_N = 0 \).

Part 1 of **Lemma 2** is proved by inspecting Eq. (5). It suggests that earnings management is ex post rational for the manager. After signing the contract (ex post), it is the accounting report \( r \), not the state \( \omega \), that settles the covenant. The manager could keep out the external intervention as long as the state is measured as good by the pre-specified accounting rule. Thus, it is ex post rational for the manager to spend resources inflating transaction characteristics.

Part 2 of **Lemma 2** is obtained by differentiating Eq. (5) with respect to \( \gamma_N \) and \( \gamma_p \). \( \partial \beta^* / \partial \gamma_N = 0 \) and \( \partial \beta^* / \partial \gamma_p = -q_BX_b/hK' < 0 \). The verification of \( t=N \) does not affect earnings management; in contrast, the verification of \( t=P \) detects earnings management and prevents the manager from obtaining the preferred report, rendering earnings management less attractive to the manager. This asymmetry in the value of verification in deterring earnings management has immediate consequences for the design of the accounting rule.

**Proposition 1.** If earnings management is not contractible and verification cost \( \delta \) is sufficiently small, the optimal accounting rule is conservative.
To see the intuition of Proposition 1, we verify that $\gamma_N^t = 0$ and $\gamma_p^t > 0$ for a sufficiently small $\delta$. The effect of the verification of $t = N$ on firm value is $\partial V / \partial \gamma_N = -\delta d(t^* / d\gamma_N)$, which is negative for any $\beta$ and $\gamma_t$. It is the same as in the benchmark case because $\gamma_N$ does not affect earnings management. Thus, the optimal accounting rule sets $\gamma_N^t = 0$. Because $\gamma_N^t = 0$ holds for any $\beta$ and $\gamma_p$, Problem 2 can be treated as a maximization problem with one choice variable $\gamma_p^t$. The first-order condition of $\gamma_p^t$ could be expressed as

$$\frac{\partial V}{\partial \gamma_p^t} = q_B^t D^{Over} \beta^t + (q_B^t D^{Over} (1 - \gamma_p^t)) \frac{\partial \beta^t}{\partial \gamma_p^t} - \delta \frac{d_t}{d_t}.$$  

Eq. (6) reveals the effects of the verification of $t = P$ on firm value through its interaction with earnings management. First, given earnings management $\beta^t$, the verification of $t = P$ reduces the measurement error induced by earnings management (the first term). Second, the verification of $t = P$ reduces the manager’s incentive to engage in earnings management in the first place, as captured by the term $\partial \beta^t / \partial \gamma_p < 0$. This reduction in earnings management improves firm value because it reduces the measurement error ($q_B^t D^{Over} (1 - \gamma_p^t)$) and saves the cost of earnings management ($hK^t$). Finally, the total cost of verification also interacts with earnings management through $\tau$, the probability that the verification of $t = P$ is triggered. This interaction is complex and analyzed in detail in Appendix A. Because the first two effects are positive, the verification of $t = P$ always occurs if it is not prohibitively costly. That is, for a sufficiently small $\delta$, $\partial V / \partial \gamma_p^t |_{\gamma_p^t = 0} > 0$ and thus $\gamma_p^t > 0$. The combination of $\gamma_N^t = 0$ and $\gamma_p^t > 0$ proves Proposition 1 that $c^t = \gamma_p^t - \gamma_N^t = \gamma_p^t > 0$. Further, this result of $c^t > 0$ is independent of the specifics of the underlying economic problem, namely, $q_B^t D^{Under}$ and $q_B^t D^{Over}$. 

Proposition 1 is strikingly simple and general. While it might be straightforward after the accounting measurement process is articulated in the model, this result is at the heart of the heated debate on conservatism that has been influencing the accounting standard setting around the world. As discussed in the second paragraph of Introduction, much controversy about conservatism is predicated on the reduced-form representation that treats accounting measurement as a black box emanating a report with certain statistical properties. As a result, the property of an accounting report cannot be differentiated from the property of an accounting rule that generates the report. In the context of the model, the reasoning underlying the controversy could be understood as follows. One looks at the accounting report $r$, asks what are the optimal $Q_B^t$ and $Q_G^t$ if $Q_B^t + Q_G^t$ is constrained to be a constant, and arrives at the conclusion that $Q_B^t > Q_G^t$ if and only if $q_B^t D^{Over} > q_G^t D^{Under}$. Based on this exercise, one argues further that standard setters who serve a broad range of constituents should take an agnostic view of $q_B^t D^{Over} = q_G^t D^{Under}$. Therefore, the optimal accounting report is neutral. This conclusion is then further extrapolated as that the optimal accounting rule that generates the report is neutral.

While this reasoning is correct in describing the optimal property of accounting report $r$, it is misleading in describing the optimal property of the accounting rule that generates $r$. With the reduced-form representation of accounting measurement as an exogenous statistical process in the previous literature, conservatism is defined as a property of accounting report $r$, i.e., $Q_B^t > Q_G^t$. In contrast, in this paper, the two-step representation of accounting measurement allows us to define conservatism explicitly as a property of an accounting rule that generates $r$. Conservatism requires more verification for good news ($t = P$) than for bad news ($t = N$), i.e., $\gamma_p^t > \gamma_N^t$. Proposition 1 states that even if the economic problem calls for an accounting report $r$ with the property of $Q_B^t = Q_G^t$, the optimal accounting rule that generates such a report is conservative. A conservative rule is the path to a neural accounting report in the presence of managers’ opportunism. This simple shift of perspective, made possible by the articulation of the two-step representation of accounting measurement, clarifies the major controversy on conservatism.

We conduct comparative statics to derive empirical predictions about the determinants of conservatism. Proposition 1 relies only on $\gamma_p^t > 0$. In the Appendix, it is shown that $\gamma_p^t$ is unique and interior under the following assumption, under which all the comparative statics are conducted.

**Assumption 1.** $\delta$ is sufficiently small and $h$ is sufficiently large.

**Corollary 1.** Under Assumption 1, the optimal accounting rule is more conservative if

1. it is easier for the manager to manipulate, e.g., $h$ is smaller;
2. the manager’s incentive to engage in earnings management is higher, e.g., $q_B^t$ is larger;
3. the consequence of earnings management is more severe, e.g., $A^{Over}$ is larger.

Corollary 1 is proved by differentiating the first-order condition for $\gamma_p^t$ with respect to relevant parameters. The optimal level of conservatism increases as the ex post earnings management becomes more severe a problem, either because it is cheaper or more attempting for the manager or more costly for the contracting parties as a whole. This heightens the contention of the paper that conservatism arises as an ex ante response to the manager’s ex post opportunistic earnings management.

The effect of private benefit $X_B$ on conservatism is ambiguous in the model. On one hand, an increase in $X_B$ makes earnings management more attractive to the manager, which demands a more conservative rule as a counteraction. On the other hand, an increase in $X_B$ reduces the inefficiency resulting from the manager obtaining the control right in the Bad state, which leads to a lower conservatism. The net effect of $X_B$ on conservatism is thus determined by these two opposing effects.
3.4. Verifiability, reliability and relevance

One major benefit of opening the black box of accounting measurement is that it helps to formally define some commonly used concepts in accounting. In the model, the cost of verification $\delta$ might be interpreted as the verifiability of transaction characteristics. A smaller $\delta$ indicates that a transaction characteristic is more verifiable. Similarly, $h$ could be interpreted as the reliability of a transaction characteristic and $\phi_i, i \in \{G, B\}$ as the relevance of a transaction characteristic. With these terminologies, we could ask the question how the verifiability, reliability and relevance of a transaction characteristic affects the firm value.

**Corollary 2.** Under Assumption 1 and $K(\beta) = \beta^2 / 2$, the firm value increases in the verifiability, reliability, and relevance of a transaction characteristic, i.e., $dV^*/dh < 0$, $dV^*/d\delta > 0$ and $dV^*/d\phi_i > 0$ for $i \in \{G, B\}$. Further, there is a trade-off between the reliability and relevance of a transaction characteristic, i.e., $dh/d\phi_i < 0$, for $i \in \{G, B\}$.

**Corollary 2** is proved by the application of the envelop theorem. It formalizes some common intuition in accounting discourse. If a transaction characteristic becomes more verifiable, more reliable, or more relevant, the quality of accounting measurement improves and so does the firm value. When two transaction characteristics differ in both their relevance and reliability, we obtain the classic trade-off between relevance and reliability.

3.5. Conservatism and price protection through interest rate

So far the analysis has not exploited the structure of the underlying financing problem except that it entails the use of a measurement-contingent covenant. This independence attests to the generality of the rationale for conservatism.

With the specifics of the financing problem, we could also examine the interaction of the accounting rule and other parts of the contract, in this case, the interest rate. We could rewrite the financier’s break-even requirement at date 0 (binding IR condition in Problem 1) as

$$q_G\phi_GpD = I - A + \delta \tau - q_G(1 - \phi_G)L - q_B(\phi_B - \hat{\beta}(1 - \gamma_P))L. \quad (7)$$

**Proposition 2.** Under Assumption 1, the face value $D$ satisfies these properties:

1. *ceteris paribus*, it increases in the financier’s conjecture about earnings management $\hat{\beta}$, i.e., $\partial D / \partial \hat{\beta} > 0$;
2. in equilibrium, it is negatively associated with the level of conservatism, i.e., $dD^*/d\varepsilon^* < 0$.

Part 1 captures the notion of ex ante price protection by the financier. If the financier believes that the manager is more likely to engage in earnings management after contracting, which enables the manager to pursue his own interest at the financier’s expense, the financier demands a higher interest rate at date 0. As a result of the price protection, the manager bears the consequences of the ex post earnings management.

However, the ex ante price protection through the adjustment of interest rate does not eliminate the ex post opportunism. Because earnings management occurs after the interest rate is negotiated, the manager takes the interest rate as given when he chooses the level of earnings management. The first-order condition for the choice of earnings management (Eq. (5)) suggests that earnings management $\beta^*$ does not directly depend on the interest rate $D$. Therefore, the interest rate in the contract alone does not perfectly align the contracting parties’ preferences.

Part 2 of **Proposition 2** states that conservatism and interest rate are imperfect substitutes for financier “protection” in the contract. The financier demands a lower interest rate when the accounting rule in the covenant is more conservative. The reason is because conservatism increases the chance that the control right is transferred to the financier, i.e., $dPr(t = b)/d\gamma_P > 0$. The control right at date 1 is valuable and thus the financier is willing to receive less cashflow in return for more control right. In other words, the financier can be “protected” by either a higher interest rate or a more conservative accounting rule in the covenant.

4. Extensions

The baseline model is deliberately simplified to highlight the general rationale for conservatism and contrast it to results in the previous literature. In this section, I extend the model in various ways to show the robustness of the general rationale for conservatism.

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10 The joint conceptual framework of FASB and IASB lists relevance and faithful representation (reliability) as two fundamental qualitative characteristics, and verifiability as one of the enhancing qualitative characteristics. **Corollary 2** indicates that the three concepts should be at par with each other.
4.1. A broad view of verification as information production

The first extension considers a broad definition of verification. In the baseline model, verification is interpreted narrowly as the attestation to the authenticity of transaction characteristics. A broad definition equates verification with information production about the state \( \omega \).

In particular, I assume that when verification is not conducted, the transaction characteristic is accepted at its face value and that when verification is conducted, the state \( \omega \) is revealed and measured perfectly. With information production, the linear cost \( \partial \gamma \) is not sufficient to guarantee an interior choice of \( \gamma \). Thus, I assume that \( \gamma, t \in (P,N) \) incurs a cost of \( T(\gamma_t) \), with \( T(0) = T'(0) = 0 \). \( T' \geq 0 \), \( T'' > 0 \) and \( T'(1) = 0 \), and set \( \delta = 0 \). The measurement errors under this new verification technology are

\[
\begin{align*}
Q_b^b &= (1 - \phi_C)(1 - \gamma_N) = (1 - \phi_C)(1 - \gamma_N), \\
Q_b^\delta &= (1 - \phi_B + \beta)(1 - \gamma_P) = (1 - \phi_B + \beta(1 - \gamma_P)) - (1 - \phi_B)\gamma_P.
\end{align*}
\]

The Good state generates \( t = N \) with probability \( 1 - \phi_C \). When it is not verified, \( t = N \) is accepted as \( t = b \), hence \( Q_b^c \). \( Q_b^g \) has a similar interpretation except that \( t = P \) could result additionally from earnings management \( \beta \).

Compared with their counterparts in the baseline model (Eq. 1 and 2), \( Q_b^c \) and \( Q_b^g \) admit the additional components of \( - (1 - \phi_C)\gamma_N \) and \( -(1 - \phi_B)\gamma_P \). They reflect the information production of verification. In the baseline model, verification only mitigates the measurement bias introduced by earnings management; let us label it as the behavioral value of verification. With the broad definition, verification also reduces the measurement noise; let us label it as the technical value of verification. Conservatism is justified by the asymmetric behavioral value of verification in the baseline model, which is preserved in this extension. The technical value of verification depends on the specifics of the problem and is symmetric for \( t = P \) and \( t = N \). As a result, in a symmetric setting the optimal accounting rule is still strictly conservative.

Using the same procedure as in the baseline model, we could obtain the following results.

**Proposition 3.** When verification generates information about \( \omega \),

1. with contractible earnings management, \( \beta^{BM} = 0 \), \( c^{BM} \equiv c^{BM} \equiv 0 \) if and only if \( q_B A^{Over}(1 - \phi_B) > q_C A^{Under}(1 - \phi_C) \). \( \gamma^{BM} \) and \( \gamma^{BM} \) are determined by

\[
\begin{align*}
\frac{\partial A^{BM}}{\partial ^{BM}} &= q_C A^{Under}(1 - \phi_C) - T(\gamma^{BM}) = 0, \\
\frac{\partial A^{BM}}{\partial ^{BM}} &= q_B A^{Over}(1 - \phi_B) - T(\gamma^{BM}) = 0.
\end{align*}
\]

2. with non-contractible earnings management, \( \beta^* > 0 \), \( \frac{\partial \beta^*}{\partial \gamma_P} < \frac{\partial \beta^*}{\partial \gamma_N} = 0 \), \( \gamma^* > \gamma^{BM} \), and \( \gamma^* = \gamma^{BM} \). Thus, \( c^* > c^{BM} \).

3. In a symmetric case with \( q_B = q_C \), \( \phi_C = \phi_B \), and \( A^{Over} = A^{Under} \), \( \gamma^* > \gamma_N > \gamma_N > \gamma^{BM} > \gamma^{BM} > 0 \). Thus \( c^* > 0 \).

**Proposition 3** mirrors Lemmas 1 and 2 and Proposition 1. The asymmetric behavioral value of verification is preserved, as confirmed by \( \frac{\partial \beta^*}{\partial \gamma_P} < \frac{\partial \beta^*}{\partial \gamma_N} = 0 \). The only difference is that with the additional technical value of verification, the base level of verification is elevated in the benchmark case and carried over to the setting with non-contractible earnings management. Even though the technical value of verification depends on the specifics of the problem, it is symmetric in the specifics of the problem. Therefore, its presence does not alter the asymmetry of the total value of verification. In the case with symmetric costs of measurement errors, the optimal accounting rule is strictly conservative.

4.2. Noisy verification technologies

So far, the verification technologies are assumed to be costly but perfect. In this extension, I show that the rationale for conservatism is robust to noisy verification technologies. This extension also generates a result consistent with the notion that accounting measurement involves a trade-off of different measurement errors.

The setting is the same as in the previous subsection except that verification now reveals the state \( \omega \) imperfectly. Verification generates additional evidence (transaction characteristics) \( t' \) in the following way:

\[
Pr(t' = P|\omega = G) = Pr(t' = N|\omega = B) = \pi, \quad \pi \in \left(\frac{1}{2}, 1\right).
\]

To make verification not trivial, I assume that \( r \) is determined by \( t' \) when verification is conducted: \( r(t' = P) = g \) and \( r(t' = N) = b \). That is, \( \pi \) is sufficiently large. Note that the direct cost of verification is not necessary because verification is endogenously costly due to its imprecision. Subjecting a transaction characteristic to verification may result in its wrong measurement. As a result, we could set \( \delta = 0 \) and \( T = 0 \). For simplicity, I also assume that \( \phi_C = \phi_B = 1 \) so that the technical value of verification is absent. In addition, I assume that earnings management in the absence of verification is sufficiently severe so that the corner solution of non-verification at all is not optimal.
With this technology, the new measurement errors are
\[ Q_{Q}^{b} = \gamma_{p}(1-\pi), \]
\[ Q_{Q}^{g} = \beta(1-\gamma_{p}+\gamma_{p}(1-\pi)) + (1-\beta)\gamma_{N}(1-\pi) = \gamma_{N}(1-\pi) + \beta(1-\pi)\gamma_{p} - \gamma_{N}(1-\pi). \]

When \( t=P \) is subject to verification and fails to pass, \( Q_{Q}^{b} \) occurs. Similarly, \( Q_{Q}^{g} \) originates from two sources. First, with earnings management, the \( Bad \) state generates \( t=P \), which could be measured as \( good \) with probability \( 1-\gamma_{p}+\gamma_{p}(1-\pi) \). Second, even if the \( Bad \) state generates \( t=N \), it could still be measured as \( good \) when it is subject to verification but fails to pass, which occurs with probability \( \gamma_{N}(1-\pi) \).

Using the same procedures in the baseline model, we obtain the following results.

**Proposition 4.** When verification is noisy,

1. with contractible earnings management, \( \beta^{BM} = 0 \), and \( \gamma_{p}^{BM} = \gamma_{N}^{BM} = 0 \);
2. with non-contractible earnings management, \( \beta^{*} > 0 \), and \( \partial \beta^{*}/\partial \gamma_{p} < \partial \beta^{*}/\partial \gamma_{N} < 0 \);
3. in a symmetric case with \( q_{G} = q_{B} \) and \( A^{Under} = A^{Over} \), \( \gamma_{p}^{*} > \gamma_{N}^{*} = \gamma_{p}^{BM} = \gamma_{N}^{BM} = 0 \). Thus \( c^{*} > 0 \).

Again, Proposition 4 mirrors Lemmas 1 and 2 and Proposition 1. The asymmetric behavioral value of verification is preserved \((\partial \beta^{*}/\partial \gamma_{p} < \partial \beta^{*}/\partial \gamma_{N})\) and so does the rationale for conservatism. The only difference is that with the noisy verification technology, the endogenous cost of verification is related to the specifics of the problem. For example, because the verification of \( t=P \) could result in measurement error \( Q_{Q}^{g} \), the cost of \( \gamma_{p} \) depends on \( A^{Under} \), the consequence of the measurement error \( Q_{Q}^{g} \). Moreover, this cost of verification is symmetric for \( \gamma_{p} \) and \( \gamma_{N} \). As shown in the proof in the Appendix, the optimal verification requirement thus has two components, one for its asymmetric behavioral value and the other for its symmetric cost. Due to the asymmetric value of \( \gamma_{p} \) and \( \gamma_{N} \) in mitigating earnings management, the optimal accounting rule is still conservative when the costs associated with different measurement errors are symmetric.

### 4.3. Renegotiation

In the baseline model, the state-contingent covenant is not renegotiation proof but renegotiation is assumed away. Empirically, debt covenants are often renegotiated (e.g., Roberts and Sufi, 2009). Because at date 1 the financier and the manager may have information that is not captured by the accounting report used in the contract, a natural question is that whether the contracting parties could improve efficiency through renegotiation. Does the possibility of costless renegotiation after the settlement of the debt covenant preclude the value of using conservative accounting measurement rules? The answer is somewhat surprising: the possibility of ex post renegotiation intensifies earnings management and thus could make conservatism more attractive.

The only case in which renegotiation is possible is when the state is \( Bad \) but measured as \( good \).\(^{11}\) Without renegotiation, the manager would take action \( a_{fg} \), resulting in an ex post efficiency loss of \( A^{Over} \). Thus, the financier could “bribe” the manager to take action \( a_{f} \) by sharing with the manager some surplus from the saving of \( A^{Over} \). Denote the manager’s bargaining power as \( \mu \in [0,1] \) and consider a Nash bargaining solution. The manager’s payoff in the \( Bad \) state with \( r=g \) changes from \( X_{B} \) to \( X_{B} + \mu A^{Over} \). Anticipating the increased payoff in the \( Bad \) state with \( r=g \), the manager’s earnings management \( \beta^{*} \) is determined by the new first-order condition

\[ q_{B}(X_{B} + \mu A^{Over})(1-\gamma_{p}) = hK'(\beta^{*}). \tag{8} \]

Comparing it with its counterpart in the baseline model (Eq. (5)), it is straightforward that \( \beta^{*} \geq \beta^{*} \). In addition to receiving the private benefit, the manager also receives a fraction of the surplus resulting from the renegotiation. As a result, \( r=g \) becomes more valuable to the manager. As the marginal benefit of earnings management increases, the manager chooses a higher level of earnings management. Therefore, while renegotiation improves the ex post action taken at date 1, it intensifies ex ante earnings management. As a result, conservatism is even more attractive.\(^{12}\)

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\(^{11}\) If the state is \( Good \), there are two cases. In the case \( r=b \), renegotiation is not necessary because the manager takes \( a_{fg} \) efficiently. In the case \( r=b \), renegotiation is not feasible because the firm does not have any wealth to pay the financier and the private benefit is not segregable. If the state is \( Bad \) and measured as \( bad \), renegotiation is not necessary because the financier takes \( a_{f} \) efficiently.

\(^{12}\) Another possible solution combines a non-contingent contract with renegotiation. The firm retains control right by default and the financier could bribe the firm to take \( a_{f} \) in the \( Bad \) state. While this arrangement implements socially optimal actions in both states without inducing the cost of earnings management and verification, it does have one drawback. It requires a face value higher than \( b^{*} \) in the baseline model. Thus, if the cash flow \( Y \) is limited, there is a region in which this arrangement is not feasible but the contract in the baseline model is still feasible. See Aghion and Bolton (1992) for more details of results in this line.
4.4. Two types of conservatism

One limitation of this paper is that it focuses only on one aspect of a conservative rule, namely, the differential verification requirement. As such, all the results in the paper are obtained under the assumption that it is optimal to recognize a given transaction characteristic. Therefore, the model describes the “interior” form of conservatism but does not directly address the “corner solution” form of conservatism. Revenue recognition is an example of the interior form of measurement in which \( r \) could be either \( g \) or \( b \), depending on \( t \).

**Corollary 2** shows that the value of using a transaction characteristic depends on its quality in terms of verifiability, reliability, and relevance. Thus, it could be optimal not to use a transaction characteristic when the quality of a transaction characteristic deteriorates, leading to a corner solution. The corner solution corresponds to the more extreme types of conservatism in practice, such as expensing R&D in which \( r=b \) regardless of \( t \). It might be argued that the transaction characteristics indicative of the economic substance of R&D are often difficult to verify (a high \( \delta \)) and vulnerable to managers’ influence (a low \( h \)). As a result, it could be optimal not to capitalize any R&D expenditure. That is, \( r=b \) regardless of \( t \) and the verification of positive transaction characteristics that lead to \( r=g \) is absent. This type of conservatism requires a model that focuses on the first instrument of the design of measurement rules and is left to future research.

The classification of interior-form versus corner-solution-form conservatism also corresponds to the classification of conditional versus unconditional conservatism (e.g., Beaver and Ryan, 2005).

4.5. Does the value of conservatism reverse as the conservative bias for current contracts reverse in future?

One criticism of conservatism is that the conservative bias in current period leads to upward bias in future periods, as expressed by FASB: “Undertaking assets or overstating liabilities in one period frequently leads to overstating financial performance in later periods” (FASB 2010 BC3.28). It is interpreted that whatever value conservatism has in one period is inevitably reversed in others. This concern seems to be an important consideration when conservatism is eliminated from the FASB and IASB’s joint conceptual framework (FASB, 2010). In this extension, I show that when a conservative bias exists and reverses in the future, it does not diminish the efficiency of conservatism for contracting.\(^{13}\) The intuition relies on a careful differentiation of ex ante contractibility versus ex post observability of information.

To examine this issue, the model has to be extended to multiple periods. A simple way is to repeat the same stage game every period. Suppose every period the firm discovers a project that needs financing. All the payoffs of the projects across periods are independent and identically distributed. The firm enters into one new contract to finance the newly discovered project each period. The timeline could be depicted as follows:

<table>
<thead>
<tr>
<th>date</th>
<th>( t-1 )</th>
<th>( t )</th>
<th>( t+1 )</th>
<th>( t+2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project ( t )</td>
<td>Contract ( t ) signed</td>
<td>( o^t ) measured</td>
<td>( o^t ) realized</td>
<td>( o^{t+1} ) signed</td>
</tr>
<tr>
<td>Project ( t+1 )</td>
<td>Contract ( t+1 ) signed</td>
<td>( o^{t+1} ) measured</td>
<td>( o^{t+1} ) realized</td>
<td></td>
</tr>
</tbody>
</table>

Further, so far the state \( o^t \) has been assumed to be not contractible at date 0. Now I also assume that it is observable ex post at date 1. This assumption is implicitly behind the above criticism. If \( o^t \) is not observable at date 1, then we cannot ascertain the existence of the bias, rendering the criticism moot in the first place. Specifically, at date \( t \), the state of project \( t, o^t \), is measured as \( r^t(o^t) \). The bias of the accounting report for contract \( t \) is conservative if \( r^t(o^t) - o^t < 0 \). Similarly, at date \( t+1 \), the state of project \( t+1, o^{t+1} \), is measured as \( r^{t+1}(o^{t+1}) \). In addition, \( o^{t+1} \) is realized at date \( t+1 \). The realization of \( o^{t+1} \) then reverses the bias in the previous measurement \( r^t(o^t) \). That is, the reversal, \( -(r^t(o^t) - o^t) \), is added to the accounting report at date \( t+1 \). The aggregate accounting report at date \( t+1 \) is

\[
r^{t+1}(o^t, o^{t+1}) = r^{t+1}(o^{t+1}) + (o^t - r^t(o^t)).
\]

\( r^{t+1}(o^t, o^{t+1}) \) has an upward bias \( o^t - r^t(o^t) > 0 \) because contract \( t \) has a conservative bias, i.e., \( r^t(o^t) - o^t < 0 \). Under-measuring state \( o^t \) at date \( t \) leads to the over-measurement of state \( o^{t+1} \) at date \( t+1 \). In this sense, FASB’s observation is correct.

Does contract \( t+1 \) have to use \( r^{t+1}(o^t, o^{t+1}) \) as it is? From the contracting perspective, recall that at date \( t \) when contract \( t \) is settled, \( o^t \) is observed. In other words, the conservative bias \( r^t(o^t) - o^t \) is transparent. Therefore, contract \( t+1 \) could use a modified accounting report \( r_{\text{Modified}}^{t+1}(o^t, o^{t+1}) = r^{t+1}(o^t, o^{t+1}) - (o^t - r^t(o^t)) = r^{t+1}(o^{t+1}) \) to exclude the impact of the reversal of the conservative bias from contract \( t \). As a result, the conservative bias used in contract \( t \) is not carried over to contract \( t+1 \). FASB’s reason for eliminating conservatism from the conceptual framework is thus flawed from the contracting perspective.

Consider the example in which the outcome of R&D is not ex ante contractible but ex post observable. At the time the current contract was negotiated, it was excluded from measurement (or expensed) for contracting purpose because of its lack of reliability. However, one period later, after the state that determines the outcome of the R&D is observed but before

\(^{13}\) I operate under the assumption that there exists a conservative bias when a conservative rule is in place. This assumption itself is nebulous in that the benchmark for the bias is not specified. Because a conservative rule mitigates the manager’s aggressive influence, the direction of the net bias is not certain. See Section 4.4 for the discussion of the two different types of conservatism.
the actual benefit pays out, the contracting parties learn the magnitude of the conservative bias resulting from the expensing, and expect it to reverse when the actual benefit of the R&D pays out later. The key observation is that when the information that is not used in the previous contract due to its lack of reliability becomes observable ex post, it can be used for the new contract. That is, at the time of negotiating the new contract, the contracting parties could use the knowledge about the conservative bias from the previous contract to make appropriate adjustment to exclude the confounding effects of its expected reversal. As a result, due to the differential timing of the two contracts and the difference between ex ante contractibility and ex post observability of information, the conservative bias in the current contract does not affect the new contract.

5. Literature review

The two-step representation of accounting measurement, which separates the property of an accounting report from that of an accounting rule that generates the report, differentiates my paper from many models on conservatism. For example, Chen et al. (2007) also study the role of conservatism in dampening the insiders’ earnings management. In their model, efforts lead to economic earnings, which in turn are converted to reported earnings. Both earnings management and conservatism are defined as direct influence on the second mapping from economic earnings to reported earnings. The two-step representation adds transaction characteristics between economic earnings (economic substance) and reported earnings (accounting report). This allows me to define earnings management as the influence on the first step from economic substance to transaction characteristics, and define conservatism as a property of an accounting rule that maps from transaction characteristics to reported earnings. As a result, the two models are substantially different.

As discussed in the Introduction, with a reduced-form representation of accounting measurement and conservatism, many models essentially turn the evaluation of conservatism into a comparison of decision costs associated with different measurement errors. For example, Gigler et al. (2009) show that for a project that has been financed already, a false negative error is more costly than a false positive error. As a result, conservatism, defined as trading a higher false negative error for a lower false positive error, reduces debt contracting efficiency. Similarly, other papers explore frictions that alter the relative costs in different settings. For example, by adding a non-contractible ex post asset substitution problem that raises the cost of a false positive signal, Caskey and Hughes (2012) show that conservatism could be efficient when the asset substitution problem is sufficiently severe. Li (2010) introduces costs of renegotiation and Jiang (2011) brings in non-accounting information to find regions where conservatism could be efficient. Lu et al. (2011) introduce a value-enhancing expansion opportunity that is traded off with an asset substitution problem. In Nan and Wen (2011), conservatism could be efficient if the proportion of bad firms is large, which makes a false positive signal more costly.

It is worth noting that there are two types of agency problems in this context. One is the agency problem in the economic setting that creates the demand for accounting information, and the other is the agency problem with the accounting measurement process that directly affects the design of accounting rules. The models above, except (Chen et al., 2007), focus exclusively on the first friction to evaluate conservatism. Antle and Gjesdal (2001) and Beyer (2012) define conservatism in an articulated context of accounting measurement, expense recognition in the former and lower of cost or market in the latter. That the conditions for conservatism to be valuable identified in both papers rely heavily on the specifics of their economic settings that is, on the comparison of the relative costs of measurement errors, seems to be related to the lack of managers’ opportunistic influence on accounting measurement in their models. In Gox and Wagenhofer (2009, 2010) the main results are obtained without the second agency problem as well.

There are also models on conservatism in a principal-agent setting. Antle and Lambert (1988) motivate conservatism from the auditor’s asymmetric loss function and Antle and Nalebuff (1991) further endogenize the auditor’s preference from the strategic interaction between the auditor and the privately informed manager. In Kwon et al. (2001), conservatism loosens the limited liability of the agent and thus improves efficiency. Gigler and Hemmer (2001) model the link between the bias in accounting measurement and the incentives for the managers to issue voluntary disclosure. They argue that the concave earnings-return relation does not necessarily result from the conservatism in accounting. Bagnoli and Watts (2005) and Chen and Deng (2010) model conservatism as a signaling device to convey the manager’s private information.

Defining conservatism as differential verification requirements also relates this paper to the literature on costly state verification and conditional investigation. In Townsend (1979) the firm tends to claim a low report in an attempt to extract concessions from lenders who then respond with more verification of the low report. In my model, the firm has incentive to inflate the report so as to prevent intervention from the lenders, inducing more verification of a high report. Thus, Townsend (1979) is more appropriate for a short-term debt contract in which the report is the same as cash payment, whereas my model focuses on a long-term debt contract that demands accounting reports (earnings) to settle the covenant in the interim. As I argued in the introduction, as far as the use of accounting reports is concerned, users’ concerns tend to be more about the inflation of earnings than about the opposite. Baiman and Demski (1980) and Dye (1986) study a principal-agent model and conclude the optimal investigation policy depends on the agent’s utility function in general.

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14 In practice earnings and cash typically move in opposite directions. Empirical research finds firms that are doing well and expanding have reductions in cash and firms that are doing poorly and contracting have increases in cash (see, e.g., Dechow et al., 1998).
Christensen and Demski (2004) extend Baiman and Demski (1980) by allowing the agent to report the output. They highlight the asymmetric value of additional information in motivating agents’ effort and truth-telling, but do not explicitly model the accounting measurement process and the design of accounting rules.

6. Empirical and policy implications

The large empirical literature has relied on the contracting explanation of conservatism in Watts (2003a). By formalizing it, my model provides a formal justification for many empirical findings in the literature. In particular, it substantiates empirical predictions about the consequences and determinants of conservatism. When conservatism is considered as an institutional parameter, the model predicts that conservatism constrains the manager’s ex post opportunism and lowers the interest rate. When conservatism is viewed as a choice variable, the model predicts that conservatism level is higher if the reliability or hardness of a firm’s transactions is lower or if the agency cost associated with the manager is higher. Further, the cost of earnings management $h$ could also be broadly interpreted as the strength of other mechanisms that constrain the manager’s earnings management, such as reputation, corporate governance, and legal regimes. Thus, the model predicts that conservatism could be a substitute for these mechanisms. These predictions are consistent with the existing evidence such as Ball et al. (2000), Watts (2003b), Zhang (2008), Watts and Zuo (2011), and Kim et al. (forthcoming).

Second, the model strengthens the contracting explanation of conservatism. For example, the efficiency of conservatism does not rely on the assumption that the cost of overinvestment is larger than that of underinvestment. For another example, the financier’s posterior beliefs about the state do not play a direct role in the model, which has two important implications. First, earnings management in equilibrium could be observable ex post as long as it is not contractible ex ante. This equilibrium existence of transparent earnings management is empirically important. Many empirical studies of earnings management use direct proxies for earnings management and thus assume that earnings management is non-observable. These studies would be self-contradictory if they relied on a theory that requires earnings management to be non-observable. Second, that the contracting parties’ posterior beliefs do not affect the model directly implies that an accounting report is useful for contracting as long as it is correlated with the state; in particular, it does not have to be incrementally informative to contracting parties. This makes the contracting view of accounting measurement directly testable.

Finally, the model might have some implications for accounting standard setting. Arguably, one of the most difficult issues in standard setting is to deal with managers’ ex post opportunistic response to standards, as evident in standards for such controversial issues as consolidation, securitization, and leases. In the presence of this difficulty (non-contractibility of earnings management) and managers’ opportunistic incentives, my model shows that the optimal accounting rule is conservative even if the neutrality of accounting reports is the desirable goal. There is a difference between the properties of an accounting report and those of an accounting rule that generates the report. Even if it is agreed on that a neutral accounting report is desirable, the accounting rule, which is the domain of accounting standard setting, is conservative. This issues a cautionary note to the approach of pursuing neutral accounting reports via neutral accounting rules.

7. Conclusion

This paper formalizes the long-lasting intuition in accounting that conservatism serves as an ex ante safeguard against managers’ ex post opportunistic influence on accounting measurement. The representation features earnings management as the main friction and specifies instruments in the design of an accounting rule. With this framework articulated, it is easy to see that conservatism is optimal as long as the manager has the incentive and ability to inflate accounting reports. This rationale is more general than the requirement imposed on the comparison of the costs of measurement errors of the accounting report. Thus, the paper substantiates the generality of conservatism as measurement principle by shifting the focus from the properties of an accounting report to the properties of an accounting rule that generates the report.

In addition to its contribution to the debate on conservatism, this paper also illustrates the importance of opening the black box of accounting measurement. To understand institutional features of accounting practice, two questions could be asked. First, given a feature of accounting information, how is it used (optimally) and what are its economic consequences? Second, how is the accounting measurement process designed to generate accounting information with a targeted feature? The previous literature has devoted most attention to the first question and as a result dealt with the institutional features of accounting practice indirectly at most. This paper takes one step towards answering the second question and shows that the two-step representation of accounting measurement is a useful tool.

Finally, this paper focuses on contracting as the economic setting that calls for accounting measurement. Contracting setting has the nice feature that the contracting parties’ posteriors about the state do not directly interfere with the ex post settlement of the contract. The model could be extended to other economic settings that demand accounting measurement, such as informing capital markets. Gao (2012) models a general setting of the demand for accounting information.
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Appendix

Proof of Lemma 1 and 2, Proposition 1, and Corollary 1. By definition, the total probability of verification $\tau$ is

$$
\tau(\beta, \gamma) = \Pr(t = P|N) + \Pr(t = N|N) = (q_c - \phi_c + q_g(1 - \phi_g + \beta))\gamma_P + (q_c(1 - \phi_c) + q_g(\phi_g - \beta))\gamma_N.
$$

Differentiating Eq. (5) with respect to $\gamma_P$, we have

$$
\frac{d\tau}{d\gamma_P} = q_g(1 - \phi_g + \beta) > 0.
$$

Differentiating Eq. (5) with respect to $\gamma_N$, we have

$$
\frac{d\tau}{d\gamma_N} = (q_c(1 - \phi_c) + q_g(\phi_g - \beta)) > 0.
$$

The total probability of verification is maximized at $\gamma_P = 0$ and $\gamma_N = 0$. This proves Lemma 1.

For the benchmark case, $b$ is a choice variable at date 0. Because $dV^{bm}/d\gamma_P = -\delta dt/d\gamma_N < 0$ for any $\beta$ and $\gamma_P$, $\gamma^{bm}_N = 0$ for any $\beta$ and $\gamma_P$. Because $dV^{bm}/d\beta = -hK - q_g\Delta^{over}(1 - \gamma_P) - q_g\gamma_P < 0$, $\beta^{bm} = 0$ for any $\gamma_P$. Because $dV^{bm}/d\gamma_P = -\delta dt/d\gamma_P < 0$, $\gamma^{bm}_P = 0$. Thus, $e^{bm} = 0$. This proves Lemma 2.

From Eq. (5), we have $\beta^{*} > 0$ if $\gamma_P < 1$. Because Eq. (5) is independent of $\gamma_N$, $\beta^{*}/\gamma_P = 0$. Differentiating Eq. (5) with respect to $\gamma_P$, we have $\hat{\beta}^{*}/\gamma_P = -q_gX_B/hK'' < 0$. This proves Lemma 2.

For Proposition 1, I prove that $\gamma^{*}_N = 0$ and $\gamma^{*}_P > 0$ for a sufficiently small $\delta$. It is a straightforward application of Kuhn–Tucker theorem except that the cost of verification $\delta\tau$ in the objective function does not behave well: it is neither always increasing nor always convex in $\gamma_P$ to see the possibility of non-increasing,

$$
\frac{d\tau}{d\gamma_P} = \frac{d\tau}{\gamma_P} + \frac{d\tau}{\beta^*} \frac{\beta^*}{\gamma_P}.
$$

$\delta\tau/\gamma_P > 0$, $\delta\tau/\beta^* > 0$, but $\beta^*/\gamma_P < 0$. That is, the marginal cost of verification could be negative. The intuition is that an increase in $\gamma_P$ has both a direct and an indirect effect on $\tau$. The direct effect is that it triggers verification more often, holding earnings management constant, i.e., $\delta\tau/\gamma_P > 0$. The indirect effect is that it reduces earnings management, which decreases the probability of $t = P$ being presented and thus of verification being triggered, i.e., $\beta^*/\gamma_P < 0$. As a result, $d\tau/d\gamma_P$ could be negative. To see the possibility of non-convexity

$$
\frac{d^2\tau}{d\gamma_P^2} = q_g \left(2 + \gamma_P \frac{q_gX_BK''}{h(K'')^2}\right) \frac{\beta^*}{\gamma_P}
$$

could be clearly negative. A sufficiently small $\delta$ assures that the properties of the objective function are not dominated by $\delta\tau$.

With this, I proceed to prove $\gamma^{*}_N = 0$ and $\gamma^{*}_P > 0$. Because

$$
\frac{dV}{\gamma_P} = -\delta \frac{dt}{\gamma_P} = -\delta(q_c(1 - \phi_c) + q_g(\phi_g - \beta^*)) < 0,
$$

$\gamma^{*}_N = 0$ for any $\gamma_P$ and $\beta^*$. To prove $\gamma^{*}_P > 0$, I show that $dV/\gamma_P$ evaluated at $\gamma_P = 0$ is strictly positive

$$
\frac{dV}{\gamma_P} = q_g\Delta^{over}(1 - \gamma_P) + q_gX_BK'' - \delta \frac{dt}{\gamma_P} = q_g\left(\Delta^{over} + \frac{X_B}{K''}\right) - \delta \frac{dt}{\gamma_P}.
$$

The second equality utilizes the first-order condition for $\beta^*$ (Eq. (5)) and the expression of $\delta\tau/\gamma_P = -q_gX_B/hK''$. As mentioned in the text, the first term in Eq. (9) is the marginal benefit of $\gamma_P$ through its interaction with earnings management. From $\hat{\beta}^*/\gamma_P < 0$ and $(K'/K'') > 0$, we know that $\beta^*$ is maximized at $\gamma_P = 0$ and thus the first term in Eq. (9) is
maximized at \( \gamma_p = 0 \). The second term is the marginal verification cost of \( \gamma_p \), as analyzed above. Therefore, with a sufficiently small \( \delta \), \( \partial V / \partial \gamma_p |_{\gamma_p = 0} > 0 \) and \( \gamma_p^* > 0 \), hence **Proposition 1**.

To conduct comparative statics in order to prove **Corollary 1**, I first prove that \( \gamma_p^* \) is unique and interior under **Assumption 1**: \( \delta \) is sufficiently small and \( h \) is sufficiently high. At \( \gamma_p = 1 \), \( \beta^* = 0 \). Evaluating \( \partial V / \partial \gamma_p \) at \( \gamma_p = 1 \), we have

\[
\frac{\partial V}{\partial \gamma_p} |_{\gamma_p = 1} = -\delta \frac{dt}{dx_p} |_{\gamma_p = 1} = -\delta \left( \frac{\partial \tau}{\partial \gamma_p} + \frac{\partial \tau}{\partial \beta^*} \right) |_{\gamma_p = 1} = -\delta \left( q_c \phi_c + q_b (1 - \phi_b) - q_b X_B \right) / hK(0).
\]

If \( h \) is sufficiently high, the indirect effect of \( \gamma_p \) on \( \tau \) is mild. As a result, \( \partial V / \partial \gamma_p |_{\gamma_p = 1} > 0 \). Furthermore, a sufficiently small \( \delta \) assures that the first-order condition is both necessary and sufficient (because \( \gamma_p^* = 0 \) for any \( \gamma_p \), Problem 2 is effectively a single-variable maximization problem.) Together, **Assumption 1** assures that \( \gamma_p^* \) satisfying \( \partial V / \partial \gamma_p^* = 0 \) is unique and interior. With the first-order condition \( \partial V / \partial \gamma_p^* = 0 \) and \( \gamma_p^* = 0 \) for any \( \gamma_p \), the comparative statics for \( c^* \) are obtained by differentiating \( \partial V / \partial \gamma_p^* = 0 \) with respect to relevant parameters. That is, \( dc^* / di = d\gamma_p^* / di / i \) is a relevant parameter. For example, for \( i = h \),

\[
dc^* / dh = 
q_b \left( A_{\text{Over}} + (A_{\text{Over}} + X_B) \left( \frac{d^2}{dy_p} \right) \right) - \frac{\partial q_b}{\partial \gamma_p} \left( q_b \phi_c + q_b (1 - \phi_b) - q_b X_B \right) / hK(0) < 0.
\]

Other results could be obtained similarly. □

**Proof of Corollary 2.** The proof is by a straightforward application of the envelope theorem

\[
dc^* / dh = \frac{\partial V}{\partial \gamma_p} |_{\gamma_p = \gamma_p^*} = -q_b A_{\text{Over}} (1 - \gamma_p^*) \frac{\partial \beta^*}{\partial \gamma_p} + \frac{\beta^*}{2} - \delta q_b \gamma_p^* \frac{\partial \beta^*}{\partial \gamma_p} > 0,
\]

\[
dc^* / d\phi_c = \frac{\partial V}{\partial \phi_c} |_{\gamma_p = \gamma_p^*} = q_c A_{\text{Under}} - \delta (q_c \gamma_p^*),
\]

\[
dc^* / d\phi_b = \frac{\partial V}{\partial \phi_b} |_{\gamma_p = \gamma_p^*} = q_b A_{\text{Over}} (1 - \gamma_p^*) + \delta (q_b \gamma_p^*),
\]

\[
dc^* / d\partial = \frac{\partial V}{\partial \partial} |_{\gamma_p = \gamma_p^*} = -\tau < 0.
\]

\( dc^* / dh < 0 \) utilizes \( K(\beta) = \beta^* / 2 \) and \( dc^* / d\phi_c > 0 \) relies on \( \delta \) being sufficiently small. Applying the total differentiation to \( V^* \) with respect to \( \phi_i \) and \( h \), we have \( 0 = dc^* / dh = (dc^* / dh) dh + (dc^* / d\phi_c) d\phi_c + (dc^* / d\phi_b) d\phi_b \). Therefore, there is a trade-off between \( h \) and \( \phi_i \) because

\[
dc^* / d\phi_c = 0, \quad dh / d\phi_c = 0. \quad \Box
\]

**Proof of Proposition 2.** From Eq. (7), by treating \( \gamma \) and \( \tau \) as constants, we have \( q_c \phi_c p (\partial D / \partial \hat{\beta}) = q_b (1 - \gamma_p) L > 0 \). In equilibrium, \( \beta = \beta^* \) and neither \( \beta^* \) nor \( \gamma_p^* \) cannot be treated as constant. Substituting \( \gamma_p^* = 0 \) in to Eq. (7) and differentiating it with respect to \( c^* \) or \( \gamma_p^* \),

\[
q_c \phi_c p \frac{dD}{dc^*} = q_c \phi_c p \frac{dD}{d\gamma_p} = q_b (1 - \gamma_p) \frac{\partial \beta^*}{\partial \gamma_p} - \frac{\partial \beta^*}{\partial \gamma_p} L + \frac{\partial \beta^*}{\partial \gamma_p} \left( q_b (1 - \gamma_p) \frac{\partial \beta^*}{\partial \gamma_p} - q_b \beta^* \right) L + q_b \left( A_{\text{Over}} \beta^* + (A_{\text{Over}} + X_B) K^* \right)
\]

\[
= -q_b X_B \beta^* + q_b \left( (1 - \gamma_p) \frac{\partial \beta^*}{\partial \gamma_p} + K^* \right) L = -q_b X_B \beta^* < 0.
\]

The second step utilizes the first-order condition of \( \partial V / \partial \gamma_p^* = 0 \), the third the definition of \( A_{\text{Over}} = L - X_B \), and the last the expression of \( \partial \beta^* / \partial \gamma_p \) and Eq. (5). □

**Proof of Proposition 3.** With the new \( Q_{\phi_c}^* \) and \( Q_{\phi_b}^* \), Problem 2 could be rewritten as

\[
\max_{V \in [0,1]} V = V^{BM} - q_b A_{\text{Over}} (1 - \gamma_p) - q_c A_{\text{Under}} (1 - \gamma_p)(1 - \gamma_N) - q_b X_B (1 - \gamma_p) - hK(\beta^* - \gamma_p - T)
\]

subject to

\[
hK(\beta^* - \gamma_p) - q_b X_B (1 - \gamma_p)
\]

\( \gamma_p \in (0,1] \).

When earnings management is contractible, \( \partial V^{BM} / \partial \beta = -q_b A_{\text{Over}} (1 - \gamma_p) - hK(\beta^* - \gamma_p - T) < 0 \) for any \( \gamma_p \). Thus, \( \beta^{BM} = 0 \). The first-order conditions for \( \gamma_p \) are obtained accordingly

\[
\frac{\partial V^{BM}}{\partial \gamma_p} = q_c A_{\text{Under}} (1 - \gamma_p)(1 - \gamma_N) = 0,
\]
Because \( T'' > 0 \), \( \gamma^B_M > \gamma^B N \) if and only if \( q_B A^{Over} (1-\phi_B) > q_C A^{Under} (1-\phi_C) \).

When earnings management is not contractible, we have \( \partial \beta^* / \partial \gamma_p < \partial \beta^* / \partial \gamma_N = 0 \) by differentiating the IC condition. Thus, \( \gamma^*_N = \gamma^B_M \). In contrast, \( \partial V / \partial \gamma_p = q_B (A^{Over} + A^{Under} X_B) \left( \frac{K'}{K''} \right) - T(\gamma_p) \).

Because \( \partial^2 V / \partial \gamma^2_p = q_B (A^{Over} + A^{Under} X_B) (K'/K'') (\partial \beta^* / \partial \gamma_p) - T < 0 \), \( \partial V / \partial \gamma_p \) is decreasing in \( \gamma_p \). Evaluating \( \partial V / \partial \gamma_p \) at \( \gamma_p = \gamma^B_M \), we have

\[
\left. \frac{\partial V}{\partial \gamma_p} \right|_{\gamma_p = \gamma^B_M} = q_B \left( A^{Over} + A^{Under} X_B \right) \left( \frac{K'}{K''} \right) \left|_{\gamma_p = \gamma^B_M} > 0. \right.

Thus, \( \gamma^*_p > \gamma^B_M \) and \( c^* > c^B_M \). Further, in a symmetric case, it is easy to verify that \( \gamma^*_p > \gamma^*_N = \gamma^B_M = \gamma^B_N > 0 \) and thus \( c^* > 0 \). □

**Proof of Proposition 4.** With the newly defined \( Q^g_B \) and \( Q^b_C \), Problem 2 could be rewritten as

\[
\begin{align*}
\max & \quad V = V^B - q_B A^{Over} (\gamma_N (1-\pi) + \beta^* (1-\pi) \gamma_p - \gamma_N (1-\pi)) - q_C A^{Under} \gamma_N (1-\pi) - hK(\beta^*) \\
\text{subject to} & \\
& hK(\beta^*) = q_B X_B (1-\gamma_N (1-\pi)) \\
& \gamma_t \in [0,1].
\end{align*}
\]

When earnings management is contractible, \( \partial V^B / \partial \gamma_p = q_B A^{Over} (1-\gamma_N (1-\pi)) - hK(\beta^*) \), and \( \partial V^B / \partial \gamma_p = q_B A^{Under} (1-\gamma_N (1-\pi)) \), which implies \( \gamma^*_p > \gamma^*_N \) for the symmetric case of \( q_C = q_B \) and \( A^{Under} = A^{Over} \). We first prove \( \gamma^*_p \gamma^*_N < 1 \) by contradiction. If \( \gamma^*_p = \gamma^*_N = 1 \), then \( (1-\gamma^*_p) \gamma^*_N (1-\pi) = 0 \) and \( \beta^* = 0 \), which imply that \( \partial V / \partial \gamma^*_p |_{\gamma^*_p = 1} = -q_B A^{Over} (1-\gamma^*_N (1-\pi)) < 0 \) and \( \partial V / \partial \gamma^*_N |_{\gamma^*_N = 1} = -q_C A^{Under} (1-\gamma^*_N (1-\pi)) < 0 \), contradicting \( \gamma^*_N = \gamma^*_N = 1 \). Thus, \( \gamma^*_p \gamma^*_N < 1 \). This implies \( \beta^* > 0 \) and \( (\partial V / \partial \gamma^*_p - \partial V / \partial \gamma^*_N) |_{\gamma^*_p = \gamma^*_N} = q_B (A^{Over} \beta^* + (A^{Under} + X_B) K'/K'') (2P-1) > 0 \). Second, we prove that \( \gamma_p = \gamma_N = 0 \) is not the optimal solution by contradiction. If \( \gamma_p = \gamma_N = 0 \), then \( \beta^* = \beta^\# \equiv \arg \max hK(\beta^*) - q_B X_B \) is maximized. By the assumption that earnings management is sufficiently important, \( \partial V / \partial \gamma_p |_{\gamma_p = \gamma_N = 0, \gamma_N = 0} = q_B \left( A^{Over} \beta^0 + (A^{Under} + X_B) \left( \frac{K'}{K''} \right) \right) (1-\gamma_p) - q_B A^{Over} (1-\gamma_p) > 0 \), contradicting \( \gamma^*_N = 0 \). Finally, collecting \( \gamma^*_p \gamma^*_N < 1 \), \( \partial V / \partial \gamma^*_p \beta^* - \partial V / \partial \gamma^*_N \beta^* \) which means that \( \gamma^*_p > 0 = \gamma^*_N \) or \( \partial V / \partial \gamma^*_p \beta^* - \partial V / \partial \gamma^*_N \beta^* > 0 \), which means that \( \gamma^*_p = 1 > \gamma^*_N \). Thus, for the symmetric case, \( c^* > 0 \). □

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