Idiosyncratic Information, Moral Hazard and Risk Premium*

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Abstract

While much accounting information is idiosyncratic in nature, economy-wide factors such as accounting standards affect the quality of idiosyncratic accounting information of many firms simultaneously. We study these two features of accounting information by embedding a parsimonious, moral hazard problem into the framework of a multi-firm economy in which project choices are endogenous to accounting information. In our model moral hazard distorts the sharing of idiosyncratic risk but does not affect the sharing of systematic risk. We extend this insight to achieve two results. First, the reduction in the risk premium for idiosyncratic risk is not captured by the risk premium of traded shares. Second, an economy-wide improvement in idiosyncratic information quality reduces the risk premium for idiosyncratic risk but increases the risk premium for systematic risk. Thus, its overall effect on the total risk premium depends on firms’ risk profiles.

JEL recognition: G12, G14, G31, M41

Key Words: project financing, moral hazard, risk premium, systematic risk, idiosyncratic risk

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1 Introduction

Idiosyncratic accounting information plays an important role in mitigating agency problems,\(^1\) which intuitively makes the firm’s financing easier and cheaper. Yet, much of the accounting literature suggests that even in the presence of agency problems idiosyncratic accounting information doesn’t affect cost of capital (e.g., Ou-Yang (2005) and Bertomeu (2015)).\(^2\) In this paper, we present a model to reconcile this discrepancy. Our key departure from the prior work is that we explicitly model firms’ investment decisions that are endogenously affected by accounting information. Even though the empirical works on accounting information and cost of capital are often motivated by the observation that cost of capital affects firms’ project choices and capital budgeting, most theoretical works have treated the supply of risky assets as given. This paper fills this void.

In the model, accounting information affects firms’ investment decisions by mitigating managers’ moral hazard problems. The solution to the moral hazard problem distorts the sharing of idiosyncratic risk, but has no effect on the sharing of the systematic risk (Baiman and Demski (1980)). In the absence of moral hazard, a firm issues traded shares to share risk. In the presence of moral hazard, the firm issues both traded shares to generic investors to share appropriately the risk of the firm, and a restricted equity interest in the firm to the manager to provide incentives. These incentives endogenously preclude the manager from trading his restricted equity interest in the firm, resulting in the distortion of risk sharing. However, because the realization of systematic risk is revealed ex post (by the performance of the market portfolio, for example) while the realization of idiosyncratic risk is commingled with the manager’s action, the incentive contract saddles the manager with exclusively idiosyncratic risk. As a result, moral hazard in a multi-firm setting leads to distortion in the sharing of idiosyncratic risk, but not systematic risk. The endogenously concentrated idiosyncratic risk commands a risk premium, which makes financing more expensive for a firm.

While the insight that moral hazard distorts the sharing of idiosyncratic risk but has no

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\(^1\)e.g., Holmstrom (1979), Baiman and Demski (1980), Watts and Zimmerman (1986), Bushman and Smith (2001) and Tirole (2006).

\(^2\)In the absence of agency problems, asset pricing models suggest that idiosyncratic accounting information doesn’t affect cost of capital (e.g., Hughes, Liu, and Liu (2007), Lambert, Leuz, and Verrecchia (2007) and Caskey, Hughes, and Liu (2015)).
effect on systematic risk sharing might be relatively straightforward, we extend this insight to examine the economic consequences of the two features of accounting information discussed before. First, an improvement in the quality of idiosyncratic information facilitates a firm’s financing, but this effect is not captured by the risk premium for traded shares. The first part is intuitive. Because idiosyncratic accounting information is informative about the manager’s action, it enables the firm to substitute an accounting-based bonus for a restricted equity interest. The substitution reduces the restricted equity interest, improves the sharing of idiosyncratic risk, and reduces the cost to the firm to finance its project. The second part that this benefit of idiosyncratic information is not reflected in the risk premium implied for the traded shares comes from the result that moral hazard does not affect the sharing of the systematic risk in the first place. Investors of the traded shares are not restricted from hedging the idiosyncratic risk of their positions and thus in equilibrium the traded shares command a risk premium for exclusively systematic risk.

The significance of this result lies in its empirical implications. An improvement in the quality of idiosyncratic information may appear to yield no benefit if an econometrician focuses on the risk premium for traded shares. The wedge between the risk premium of traded shares and the total risk premium the firm has to pay for financing may account for the difficulty in establishing the empirical link between accounting information quality and the total risk premium. Further, the above mechanism suggests that an appropriate empirical measure for the economic consequences of idiosyncratic accounting information is the size of the traded shares, not their risk premium. To illustrate, consider two firms with an identical risk profile, Firm A and Firm B, in an identical circumstance where Firm A’s accounting system provides better idiosyncratic information. In equilibrium, the risk premiums of the traded shares of the two firms are the same; these risk premiums only compensate for systematic risk. The total risk premium Firm A pays to finance its project, however, is lower than that of Firm B. Idiosyncratic information does not manifest in the risk premium for traded shares, but rather in the composition of the firms’ equity interest. Firm A has a larger fraction of shares issued as traded shares and a smaller fraction issued as restricted equity interest.

Our second result is concerned about economy-wide factors that affect the quality of idio-
syncratic accounting information. For example, the rapid pace of the global convergence of accounting standards and harmonization of disclosure regulations could change simultaneously the quality of all firms’ accounting information. While idiosyncratic information does not affect the risk premium of systematic risk directly, an economy-wide change in the quality of idiosyncratic accounting information does have an indirect effect, and one whose direction is somewhat surprising. When the quality of economy-wide idiosyncratic information improves, firms’ costs to finance their projects drop due to the reduction of the risk premium for idiosyncratic risk. This reduction has two effects. First, it reduces the component of idiosyncratic risk premium of the project. Second, since all firms’ premium for idiosyncratic risk have been lower at the same time, the set of projects that will be financed are affected. Note that these two effects affect firms differently. The first effect depends on the firm’s idiosyncratic risk while the second effect on the firm’s systematic risk. To the extent that a firm’s exposure to the two types of risks varies, the two effects are different. This generates a composition effect. As the economy wide quality improves, a firm with larger systematic risk but smaller idiosyncratic risk could see its value drops while a firm with smaller systematic risk but larger idiosyncratic risk will see its value increases. To the extreme, the set of projects that are financed in equilibrium are different. In other words, accounting standards have both efficiency and redistributive effects. These results may aid cross-sectional studies that attempt to capture the economic consequences of an economy-wide change in accounting quality, such as the adoption of IFRS and Sarbanes-Oxley Act.

Our paper extends the large literature on the relation between agency problem and cost of capital. By embedding the agency problem into a multi-firm setting with endogenous project selections, we generate testable consequences of idiosyncratic accounting information for cost of capital. In this respect, our analysis is related to three papers that investigate moral hazard within the context of asset pricing. Fischer (2000) studies the optimal compensation contract when the agent can trade in the capital market. Among other results, he shows that the combination of contracting only on output and allowing the agent to trade doesn’t replicate the optimal contract unless the wealth effect from the realization of the controllable event is either absent or manageable from trading. Ou-Yang (2005) derives an equilibrium asset pricing model with moral hazard using a similar framework of ours, namely, the constant
absolute risk aversion (CARA) preference. He observes that idiosyncratic cash flow risk affects the stock price but not the dollar returns (risk premium). He doesn’t model project selections, a key focus of our study. Bertomeu (2015) studies an agency model within a multi-firm setting. He shows that the average cost of capital is independent of agency frictions. Since the optimal effort in his model is constant, the aggregate output is independent of agency frictions. In other words, like Ou-Yang (2005), Bertomeu (2015) doesn’t study project selection either. Bertomeu and Cheynel (2015) explain why a necessary condition for agency problems to affect cost of capital is that they affect the aggregate risky asset. Our paper provides a specific mechanism through which this condition is endogenized.

Some papers examine the capital market consequences of firm-specific information in the absence of agency conflicts. Christensen and Feltham (1988) studies the role of firm-specific information in resource allocation, and Armstrong, Banerjee, and Corona (2009) examines the effect of information about firm-specific beta on a firm’s expected returns. Cheynel (2013) explores how a firm’s voluntary disclosure of firm-specific information affects the market risk premium. None of these papers, however, considers the role of accounting information in incentive provision.

Our paper also complements the literature on the relation between cost of capital and accounting information. This literature has focused mainly on the valuation role of accounting in that accounting information helps investors to assess firms’ terminal cash flows. As a result, cost of capital is defined as the risk premium paid to external investors (or implied in traded shares). In contrast, accounting information serves a stewardship role in our model. Accordingly, cost of capital is defined as the total risk premium cost a firm pays to finance its project. Our notion of cost of capital is often adopted in corporate finance and more comprehensive for firms’ project selection decisions. Bertomeu and Cheynel (2015) provide an excellent discussion about the distinction between these two definitions of cost of capital.

The paper proceeds as follows. In Section 2 we describe our model of moral hazard in a diversified market setting, and then in Section 3 derive an equilibrium to our model. In Sections 4 and 5 we examine the economic consequences of a change in accounting quality at the firm level and economy-wide, respectively. In Section 6 we conclude. We provide proofs to lemmas and propositions stated in the paper in the Appendix.
2 Model

To study the economic consequences of idiosyncratic accounting information, we embed a parsimonious, moral hazard problem into a multi-firm setting. Specifically, we assume that a representative investor $j$ is endowed with project $j$, and incorporates this project into firm $j$ (we use the terms “project” and “firm” interchangeably). With the assistance of manager $j$ (whom investor $j$ hires), project $j$ generates an uncertain cash flow $\tilde{F}_j$:

$$\tilde{F}_j \equiv F \cdot \iota + \phi_j \tilde{\xi}_j + \gamma_j \tilde{\eta},$$

where $F$ is the mean of the firm’s cash flow and $\iota$ is an indicator function. If the manager exerts effort at a private cost $c$, $\iota = 1$; if the manager shirks at no cost, $\iota = 0$. We assume that $F$ is sufficiently large in relation to $c$ such that it is always optimal to motivate effort when the project is financed. The binary effort choice is employed so as to hold constant the expected cash flow generated from the project in equilibrium: that is, $E[\tilde{F}_j] = F$. Both $\tilde{\xi}_j$ and $\tilde{\eta}$ are random variables with a standard normal distribution: $\tilde{\xi}_j$ is the idiosyncratic risk of the project’s cash flow, and $\tilde{\eta}$ is the project’s systematic risk. By definition, $\tilde{\xi}_j$ is independent of $\tilde{\eta}$ and $\tilde{\xi}_i$, for $i \neq j$. The non-negative constants $\phi_j$ and $\gamma_j$ represent the project’s exposure to these sources of risk.

The accounting system provides information $\tilde{Y}_j$ about the manager’s effort:

$$\tilde{Y}_j \equiv F \cdot \iota + \alpha_j \tilde{\epsilon}_j,$$

where $\iota$ is the same indicator function defined above. The random variable $\tilde{\epsilon}_j$ also has a standard normal distribution that is independent of all other random variables; $\tilde{\epsilon}_j$ represents idiosyncratic measurement risk. The non-negative constant $\alpha_j$ is an inverse measure of the quality of the idiosyncratic accounting information.

While in reality an accounting system provides information that is both systematic and idiosyncratic in nature, the purpose of our model is to focus on the idiosyncratic component. Thus, we assume away the systematic component of the accounting signal $\tilde{Y}_j$. We use the expression “idiosyncratic risk” to describe both the idiosyncratic cash flow risk, $\phi_j \tilde{\xi}_j$, and the
idiosyncratic accounting measurement error, \( \alpha_j \tilde{\varepsilon}_j \), because both are sources of uncertainty in the manager’s compensation, as we shall soon see.

There are two dates. All actions take place at date 1 except cash flow realization and consumption, which occur at date 2. Where appropriate, we use \( I \) and \( A \) to represent the investor and agent (manager), respectively. Investors and managers have negative exponential utility functions (CARA) with (constant) tolerance for risk of \( r_I \) and \( r_A \), respectively.\(^3\) We exploit the well-known result that when wealth at date 2, \( \tilde{W}_i \), has a normal distribution, the certainty equivalent of an expected CARA utility has the simple form of \( CE(\tilde{W}_i) = E[\tilde{W}_i] - \frac{1}{2r_i} Var[\tilde{W}_i], i \in \{I, A\} \). In other words, maximizing expected utility is equivalent to maximizing the mean of wealth minus a discount for its risk.

We embed our moral hazard problem into a multi-firm setting to differentiate between idiosyncratic and systematic risks. We assume that there exists a continuum of investors and managers in the economy, where each group has a mass of 1. The multi-firm setting introduces two features into the model. First, it enlarges the space of compensation contracts to deal with moral hazard. The compensation contract investor \( j \) offers to manager \( j \) is contingent not only on firm \( j \)'s performance, but also the performance of every other firm in the economy. From the relative performance literature (Holmstrom (1982)), one can simplify the contract design without loss of generality by writing the contract solely on the market index. The market index is the best-diversified portfolio and thus yields the least amount of noise for relative performance evaluation. Second, both investors and managers are able to optimize their portfolios across the public shares of all firms. Because risk sharing between the investor and the manager could be arranged through either the compensation contract or their respective portfolio decisions, there are many equivalent ways to model contracting and portfolio decisions. We adopt the following one: we assume that manager \( j \) does not directly invest in firms and that his exposure to risk comes only from the compensation contract with

\(^3\)There are three reasons why CARA utility facilitates our analysis. First, with CARA utility there is no wealth effect; this enables us to focus exclusively on the effect of information quality on the risk premium. Second, CARA utility in conjunction with normally-distributed payoffs yields closed-form solutions for prices and risk premia. Third, CARA utility, normally-distributed payoffs, and linear contracts yield closed-form solutions for equilibrium compensation contracts. Because our model incorporates both firms’ share prices in a multi-firm setting and firms’ intra-firm contracts, closed-form solutions to both phenomena make our analysis considerably more tractable.
In particular, investor $j$ offers a linear contract to manager $j$:

$$\tilde{w}_j \equiv v_j + b_j \tilde{Y}_j + m_j \tilde{M} + s_j \tilde{G}_j, \quad (1)$$

where $v_j$ is the base salary, $b_j$ is the accounting-based bonus coefficient, $m_j$ is the coefficient on a market index, $\tilde{M}$, and $s_j$ is the percentage of the equity of firm $j$ awarded to the manager under the contract. The market portfolio $\tilde{M}$ has a mean of $E[\tilde{M}]$ and standard deviation of $\sigma_M$. The inclusion of the market-based pay $m_j$ is a form of relative performance evaluation. It also allows an investor to invest in the stock market on behalf of a manager, which frees us from modeling the manager’s portfolio decision. The expression $\tilde{G}_j \equiv \tilde{F}_j - (v_j + b_j \tilde{Y}_j + m_j \tilde{M})$ is the cash flow that is available for equity holders. We refer to $s_j$ as “restricted equity interest.” The restriction that the equity interest cannot be traded or otherwise redeemed before the manager exerts effort (in our model, before the firm liquidates at date 2) arises endogenously as part of the optimal contract. To facilitate the analysis, we also assume that the firm pays the base salary $v_j$, the accounting-based bonus $b_j \tilde{Y}_j$, and the market-based pay $m_j \tilde{M}$ before distributing the (residual) cash flow to shareholders. Because there is unlimited liability, the exact order of payments does not matter.

In addition to designing the compensation contract, investor $j$ also optimizes his portfolio across the equity of all firms and a risk-free asset whose return is normalized to be 1. We assume that compensation contracts are observable to all investors. By standard portfolio theory, after awarding manager $j$ with $s_j$ shares of equity interest, investor $j$ sells the remaining $1 - s_j$ shares of equity interest in his own firm and invests the total proceeds between the market portfolio and the risk-free asset. Denote investor $j$’s weight on the representative market portfolio as $x_j$, the per-share price of firm $j$’s equity share as $p_j$, and the per-share price of the representative market portfolio as $p_M$. Investor $j$’s budget at date 1 is $(1 - s_j)p_j$.

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1We thank a referee for this suggestion; it simplifies the model significantly.
2To facilitate the analysis, we also assume that the manager continues to invest through his matching investor even in the event that the project is not financed.
3Because it is a representative-agent model with a continuum of agents, unscaled aggregate variables all go to infinity. Thus, all the aggregate variables, including $\tilde{M}$, are expressed in per-share (per capita) terms: for example, $\tilde{M}$ is the representative, per capita market portfolio. In Section 5 we determine $E[\tilde{M}]$ and $\sigma_M$ endogenously.
4Recall that $\tilde{M}$ is the representative market portfolio (Footnote 6). Consequently, $p_M$ is the price of the representative market portfolio and $x_j$ is the fraction of the representative market portfolio, not the entire
He pays $x_j p_M$ for a fraction $x_j$ of the representative market portfolio and invests the remaining wealth in the risk-free asset (which generates a rate of return of 1). Thus, investor $j$’s wealth at date 2 is $\hat{W}_j = ((1 - s_j)p_j - x_j p_M) + x_j \hat{M} = (1 - s_j)p_j + x_j(\hat{M} - p_M) \tag{8}$. In addition, the $CE$ of investor $j$ is

$$CE(W_j) = (1 - s_j)p_j + x_j(E[\hat{M}] - p_M) - \frac{x_j^2}{2\nu^2 M^2}\sigma^2_M. \tag{2}$$

Note that $CE(W_j)$ has two components. First, investor $j$ receives the NPV of his project $j$, $(1 - s_j)p_j$. This value depends on both the compensation contract and the portfolio decisions of all investors. The compensation contract determines not only the fraction of equity shares available for public issuance, but also the residual cash flow $\hat{G}_j$ whose valuation is determined by the portfolio decisions of all investors. In addition, if this NPV is negative, then investor $j$ could choose not to finance the project. Denote $I_j$ as firm $j$’s financing decision. Until Section 5, we assume that $I_j = 1$ for any $j$ for ease of exposition. Second, investor $j$ also receives a surplus from trading in the market portfolio, $x_j(E[\hat{M}] - p_M) - \frac{x_j^2}{2\nu^2 M^2}\sigma^2_M$. This surplus originates from the risk-bearing capacity that investor $j$ contributes to the market.\footnote{Formally, let $a_j$ denote investor $j$’s investment in the risk-free asset. Then investor $j$ solves the following allocation problem:

$$\max_{a_j, x_j} a_j + x_j E[\hat{M}] - \frac{x_j^2}{2\nu^2 M^2}\sigma^2_M \text{ subject to } a_j = (1 - s_j)p_j - x_j p_M. \tag{9}$$

Because there is a surplus for bearing risk, manager $j$’s reservation utility in our setting is complicated. In a stand-alone moral hazard problem, manager $j$’s reservation utility is the opportunity cost of his effort-free labor. In our setting, the manager’s compensation contract incorporates his portfolio decision. Thus, the manager’s reservation utility has an additional component equivalent to the surplus he receives for the risk he bears through direct participation in the stock market.}

We summarize the problem as follows. Investor $j$ chooses $\{x_j, v_j, b_j, s_j, m_j\}$ to maximize $CE(W_j)$ in eqn. (2), subject to the capital market equilibrium that determines price $p_j$ and the manager’s IR (Individual Rationality) condition and IC (Incentive Compatibility) condition.
3 Equilibrium

In this section, we sketch the process that yields a solution to the problem we posit above. The solution requires determining simultaneously the optimal compensation contract and the capital market equilibrium. To be more specific, \( p_j \) in investor \( j \)'s objective function, i.e., \( CE(W_j) \), is an endogenous variable that is determined by the capital market equilibrium. At the same time, the capital market equilibrium is also determined by the compensation contract that affects both the generation and distribution of the cash flow. The solution process consists of two steps. First we fix the compensation contract to hold constant the distribution of the firm's cash flow \( \tilde{G}_j \) and the fraction of shares available for public issuance \( 1 - s_j \). This allows us to solve the capital market equilibrium, which expresses \( p_j \) as a function of the compensation contract \( \{v_j, b_j, s_j, m_j\} \). Then we substitute \( p_j \) into \( CE(W_j) \) and solve for the optimal compensation contract. The solution to the problem is complicated but the techniques are relatively standard. Thus, we relegate most details to the Appendix.

Lemma 1 Given the compensation contract \( \{v_j, b_j, s_j, m_j\} \), the per-share price of the traded shares of firm \( j \) is

\[
p_j = E[\tilde{G}_j] - \frac{1}{r} \text{Cov}[\tilde{G}_j, \tilde{M}].
\]

Lemma 1 is a standard asset-pricing result. The price of publicly traded shares equals the expected value of the residual cash flow available to equity holders minus a discount for risk whose size is determined by the covariance of the firm's cash flow and the market portfolio. The unusual feature here is that \( p_j \) is conditional on the compensation contract. In other words, the capital market is willing to pay \( p_j \) per share for the traded shares only if the firm sells a fraction \( 1 - s_j \) of shares as traded shares and retains the remaining \( s_j \) as restricted equity interest awarded to the manager. This is important for understanding the economic consequences of improving the quality of idiosyncratic information.

Plugging \( p_j \) into investor \( j \)'s \( CE(W_j) \), we obtain a standard moral hazard problem. Let \( CE(w_j(\cdot)) \) denote the certainty equivalent of manager \( j \)'s wealth.\(^{10}\) By comparing the man-
ager’s $CE$ with or without shirking, we obtain the manager’s IC condition:

$$(b_j(1 - s_j) + s_j)F - c \geq 0.$$  \hspace{1cm} (4)

Let $CE^0$ represent the manager’s reservation $CE$ in the absence of working for the firm. The manager’s IR condition is

$$CE(w_j(t = 1)) - c \geq CE^0.$$  \hspace{1cm} (5)

**Lemma 2** 1. When the manager’s effort is contractible, $b_j^{FB} = s_j^{FB} = 0$, where “FB” denotes “first best.”

2. When the manager’s effort is not contractible, $s_j = \frac{c}{F(1 + (\frac{s_j}{\alpha_j})^2)} > 0$ and $b_j = \frac{c - s_j}{1 - s_j} > 0$.

3. The agency cost (i.e., the reduction in firm value in the presence of moral hazard) is

$$\Pi_j = \frac{1}{2r \lambda} \frac{c^2}{F^2} \frac{\alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)}.$$

Part 1 of Lemma 2 is straightforward. In absence of moral hazard, the compensation contract imposes no idiosyncratic risk on the manager and thus the coefficients on the accounting-based bonus and restricted equity interest are both 0. In other words, the capital market is willing to purchase all shares of the firm at $p_j$ per share and all shares are publicly traded.

Part 2 of Lemma 2 is a direct application of the insight in Holmstrom (1979) that any information about the manager’s non-contractible action is useful as an incentive provision. Both the firm’s terminal cash flow and accounting signal are informative about the manager’s effort and thus both are utilized in the compensation contract. Recall that the IC condition for the manager is $(b_j(1 - s_j) + s_j)F - c \geq 0$. Thus, $s_j > 0$ and/or $b_j > 0$ imply that the manager’s expected compensation increases in his effort choice. The exact magnitudes of $s_j$ and $b_j$ are determined by their relative informativeness $\frac{\phi_j}{\alpha_j}$ and the scaled-cost parameter of the manager’s effort $\frac{c}{F}.$

Part 3 of Lemma 2 follows from the comparison of firm value with and without moral hazard. The agency cost of moral hazard is the reduction in firm value in the presence of moral hazard. An inspection of the expression reveals that the agency cost results exclusively from the project’s idiosyncratic risk, not its systematic risk. This result is the starting point
of our main results. The intuition for this important result is as follows. Recall that the manager’s exposure to risk, rearranged from his wealth using eqn. (1), is \( b_j(1 - s_j)\alpha_j\tilde{e}_j + s_j(\phi_j\tilde{\xi}_j + \gamma_j\tilde{\eta}) + (1 - s_j)m_j\sigma_M\tilde{\eta} \). The manager is exposed to idiosyncratic risk through the accounting-based bonus and the restricted equity interest, and to systematic risk through the restricted equity interest and the market-based pay. Because only the idiosyncratic component of the firm’s performance is indicative of the manager’s effort, the compensation contract allows the manager to hedge away the systematic risk of the restricted equity interest by adjusting the coefficient for the market-based pay. In contrast, for incentive provision purposes, the compensation contract does not permit the manager to hedge his exposure to the idiosyncratic risk associated with the accounting-based bonus and the restricted equity interest. As a result, the sharing of the idiosyncratic risk per se is suboptimal in equilibrium, whereas the sharing of systematic risk is. In other words, the cost of moral hazard manifests in the distortion in the sharing of idiosyncratic risk. This explains part 3 of Lemma 2: the agency cost of moral hazard is captured solely by the risk premium for the idiosyncratic risk the manager endogenously bears.

One can expand on the intuition for part 3 of Lemma 2 by making reference to the manager’s risk exposure in equilibrium. Using eqn. (1) and eqn. (11), the manager’s equilibrium exposure to systematic risk \( \tilde{\eta} \) is \( s_j\gamma_j\tilde{\eta} + (1 - s_j)m_j\sigma_M\tilde{\eta} = \frac{r_A}{r_I}\sigma_M\tilde{\eta} \). Note that it does not depend on the incentive coefficients \( \{b_j, s_j\} \) or any other characteristics of the firm. Because the manager can always trade publicly-traded equity, the sharing of systematic risk between managers and investors is determined solely by their relative risk tolerances \( \frac{r_A}{r_I} \). When \( s_j \) changes to provide incentives, \( m_j \) adjusts to ensure that \( s_j \) does not affect the manager’s total exposure to systematic risk. As a result, one can obtain the risk premium for systematic risk (i.e., the risk premium charged for publicly-traded equity) from Lemma 1 and eqn. (11):

\[
\Gamma_j \equiv (1 - s_j)(p_j - E[\tilde{G}_j]) = \frac{\gamma_j}{r_A + r_I}\gamma_j. \tag{6}
\]

The classic asset-pricing result that systematic risk alone commands a risk premium is valid for the firm’s publicly-traded equity. In contrast, the manager’s equilibrium exposure to idiosyncratic risk is \( b_j(1 - s_j)\alpha_j\tilde{e}_j + s_j\phi_j\tilde{\xi}_j; b_j > 0 \) and \( s_j > 0 \) mean that the manager
bears idiosyncratic risk in equilibrium. This endogenous concentration of idiosyncratic risk is costly and the manager requires a risk premium for it. The risk premium for concentrated idiosyncratic risk is

$$\Pi_j = \frac{1}{2r_A F^2} \frac{c^2 \alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)}.$$  \hspace{1cm} (7)

In presence of moral hazard, idiosyncratic risk exacerbates the incentive problem and thus makes the financing of the project more costly by $\Pi_j$. Thus, $\Pi_j$ has a dual identity. On the one hand, it represents the reduction in firm value in the presence of moral hazard. On the other hand, $\Pi_j$ also represents the risk premium for the endogenously concentrated, idiosyncratic risk imposed on the manager. This dual identity underlies the intuition for part 3 of Lemma 2.

Finally, from the firm’s perspective, the total risk premium the firm pays to finance the project consists of the risk premium for systematic risk and the idiosyncratic risk borne by the manager:

$$\Delta_j = \Gamma_j + \Pi_j = \frac{\bar{r}}{r_A + r_I} \gamma_j + \frac{1}{2r_A F^2} \frac{c^2 \alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)}.$$  \hspace{1cm} (8)

While part 3 of Lemma 2 is intuitive, it yields an important insight about the economic consequences of idiosyncratic accounting information. Specifically, moral hazard creates a wedge between the risk premium investors charge for publicly-traded equity ($\Gamma_j$) and the total risk premium the firm actually pays to finance the project ($\Delta_j$). The wedge, i.e., $\Pi_j$, manifests as a risk premium for idiosyncratic risk and reflects the shadow price for restrictions on the restricted-equity interest. As discussed in the Introduction, the observation that there exists a wedge was made as far back as in Diamond and Verrecchia (1982), Holmstrom (1982) and Ramakrishnan and Thakor (1982), and has been used subsequently to address issues related to executive compensation (e.g., Hall and Murphy (2002) and Ou-Yang (2005)). In the next section, we develop this basic observation to study the economic consequences of idiosyncratic information.

Finally, the intuition that underlies part 3 of Lemma 2 could be likened to “capital rationing,” a circumstance where a firm cannot borrow additional funds even though it is willing to pay the current interest rate. The reason is that more borrowing destroys the incentive for the firm to behave appropriately. As with credit rationing, firms in our model
are rationed for risk sharing. A firm cannot issue more traded shares even though it is willing to pay the prevailing risk premium, or an even higher premium than the one the market is asking for the existing traded shares. More traded shares (a smaller restricted equity interest) destroy the incentive for the manager to exert effort. In other words, moral hazard leads to the firm being rationed by quantity (i.e., the number of traded shares it could issue), not by price (i.e., the risk premium of the traded shares).

4 Firm-Level Information Quality

We attempt to capture the fact that both firm-level and economy-wide factors influence accounting information quality by positing the following structure for accounting measurement error $\alpha_j$:

$$\alpha_j = \lambda_j \alpha + \delta_j. \quad (9)$$

Here there are two sources of accounting measurement error: a common factor $\alpha$, and a firm-specific factor $\delta_j$. We interpret $\delta_j$ as a determinant of accounting quality that differs across firms within the same regime or industry. Alternatively, $\delta_j$ could be interpreted as factors that are under the control of firm $j$. Firms could choose more informative accounting methods, use more precise accounting estimates, commit to more forthcoming disclosure or guidance, and bond themselves to regimes with more stringent requirements or stronger enforcement. On the other hand, we interpret $\alpha$ as factors that affect a firm’s accounting quality outside the firm’s direct control. These factors could include economy-wide determinants of accounting quality such as accounting standards, legal and regulatory enforcement, social norms, and economic environment of an economy. The parameter $\lambda_j$ measures a firm’s exposure to the economy-wide factor $\alpha$. Cross-sectional differences in $\lambda_j$ capture the notion that an economy-wide factor could have differential effects on the quality of firms’ idiosyncratic accounting information.

Note that a firm’s accounting information, $\tilde{Y}_j = F\cdot t + \alpha_j \tilde{z}_j$, remains idiosyncratic through $\alpha_j \tilde{z}_j = (\lambda_j \alpha + \delta_j) \tilde{z}_j$, but shares a common source with other firms through $\alpha$. For example, when a new accounting standard becomes effective, this affects the quality of the idiosyncratic accounting information of all firms in the economy, albeit differentially.
In this section we consider the effect of cross-sectional differences through $\delta_j$. A decrease in $\delta_j$ improves the quality of idiosyncratic information at the firm level; this, in turn, reduces the risk premium the firm pays to finance its project, as stated in the following proposition.

**Proposition 1** As the quality of accounting information at the firm-level improves ($\delta_j$ becomes smaller), the risk premium for the systematic risk, $\Gamma_j$, remains the same, the risk premium for the idiosyncratic risk, $\Pi_j$, becomes smaller, and the total risk premium the firm pays to finance the project, $\Delta_j$, falls.

The proof is straightforward from the expression for $\Delta_j$, i.e., eqn. (8). To better understand the intuition, we consider the effect of idiosyncratic accounting information on the compensation contract. The key is to observe that as accounting information becomes more precise, the optimal compensation contract tilts more towards an accounting-based bonus and less towards restricted equity interest. Recall from Lemma 2, the restricted equity interest $s_j$ is increasing in $\alpha_j$ and the accounting-based bonus $b_j$ is decreasing in $s_j$. Thus, the accounting-based bonus and restricted equity interest are substitute incentive mechanisms. The restricted equity interest and bonus are both costly to the firm because of the inefficient sharing of idiosyncratic risk. However, as idiosyncratic accounting information quality improves, it becomes relatively cheaper to use the bonus as an incentive mechanism; this tilts the firm toward more bonus and less restricted equity interest until the exposures to $\tilde{\epsilon}_j$ and $\tilde{\xi}_j$ are equalized, i.e., $b_j(1 - s_j)\alpha_j = s_j\phi_j$. As a result, this improves idiosyncratic risk sharing and the risk premium for idiosyncratic risk falls. In contrast, because the compensation contract does not affect the sharing of systematic risk in the first place, it should be clear that the change in the compensation contract induced by an improvement in idiosyncratic information does not affect the risk premium for systematic risk.

While Proposition 1 is straightforward, its value is reflected mainly in its implications for the vast empirical literature that seeks to measure the economic consequences of accounting information. Proposition 1 implies that empirical proxies for a firm’s risk premium - proxies based on the price of traded shares, $\Gamma_j$ - may underestimate the total premium a firm pays to finance its project, $\Delta_j$, by the risk premium for idiosyncratic risk, $\Pi_j$. For example, consider two, ostensibly identical projects that have the same systematic and idiosyncratic cash flow
risks: we label the first project A and the second project B. While ostensibly identical, we assume that the accounting system of firm A provides better idiosyncratic information than the accounting system of firm B. If financed, both projects are expected to generate cash flow $F$. Thus, firm value is inversely related to the total risk premium the firm pays to finance the project. Lemma 2 and Proposition 1 establish that the total risk premium to finance project A ($\Delta_A$) is lower than that to finance project B ($\Delta_B$). In other words, project A is more likely to be financed than firm B. However, when an empiricist contemplates the risk premiums of the two projects, typically he or she only considers the prices $p_A$ and $p_B$ of the firms’ traded shares. Because the risk premiums the empiricist infers from the traded shares are $\Gamma_A$ and $\Gamma_B$, he or she will arrive at the conclusion that the two projects’ risk premiums are identical (i.e., $\Gamma_A = \Gamma_B$).

Because idiosyncratic accounting information only reduces $\Pi_j$, failing to account for idiosyncratic risk in the risk premium a firm pays implies that improvements in the quality of idiosyncratic information may appear to yield no benefit. This underestimate may account for the difficulty in establishing an empirical link between accounting information quality and the risk premium.

**Proposition 2** The implied risk premium from traded shares, $\Gamma_j$, biases downward the risk premium the firm pays to finance a project, $\Delta_j$. The magnitude of the bias, $\Pi_j$, increases in the size of the restricted equity interest $s_j$ and the idiosyncratic cash flow risk $\phi_j$, and decreases in the risk tolerance of the manager, $r_A$.

In our previous example, we observed that even though the risk premiums of the traded shares of the two firms are identical, firm A has a larger fraction of shares traded publicly than firm B (i.e., $1 - s_A > 1 - s_B$). Firm A’s better accounting information does reduce its cost of financing its project, but this is only reflected in a larger fraction of firm A’s equity interest being publicly traded, not in the lower risk premium implied in A’s publicly traded shares.

Restricted equity interest is a sign of distorted risk sharing resulting from intra-firm frictions including moral hazard. A proxy for restricted equity interest may be the ownership in a firm by top executives. As such, risk-sharing distortion could be a significant cost of
financing. For example, Holmstrom, Kroszner, and Sheehan (1999) reports that in 1995 the mean and median equity ownership of directors and officers (D&O) of a sample of exchange-listed firms are 21.1% and 12.4% (12.2% and 4.6% for NYSE sample), respectively. Similarly, Fahlenbrach and Stulz (2009) finds that the mean and median of D&O ownership for their sample are 22.3% and 14.8%, respectively, for the average sample period between 1988 and 2003. The concentration of ownership is even larger in other countries. Failure to account for the distortion in risk sharing in measures of risk premiums may explain why it has been difficult for empirical studies to identify the seemingly evident relation between information quality and the risk premium the firm pays to finance its project.

It is worth pointing out that idiosyncratic accounting information could affect aspects of firm value other than moral hazard, which is the focus of our study. Solutions to intra-firm agency frictions commonly involve the concentration of idiosyncratic risk. For example, the solution to adverse selection in Leland and Pyle (1977) involves the concentration of ownership. The solution to the free-rider problem in monitoring insiders also entails block holders holding undiversified portfolios. Hence, an improvement in idiosyncratic accounting information could also reduce the risk premium the firm pays to finance a project by acting as a substitute for these imperfect solutions.

5 Economy-wide Information Quality

A salient feature of accounting is that accounting information quality is governed by both the choices of individual firms and economy-wide factors such as accounting standards and legal regimes. Given the rapid pace of the global convergence of accounting standards and the harmonization of disclosure regulations, it is increasingly more important to understand how economy-wide changes in accounting quality affect the allocation of resources and the distribution of surplus.

Recall that the change in economy-wide information quality through $\alpha$, the common factor in the expression $\alpha_j = \lambda_j \alpha + \delta_j$, affects all firms (recall that we describe accounting information as $\tilde{Y}_j = a_j + \alpha_j \tilde{\tilde{e}}_j$). Thus, our next goal is to consider the economic consequences of a change in $\alpha$. To do so, we need to relax the assumption that all projects are financed.
in equilibrium. Instead, we now explicitly study a firm’s decision to finance its project, $I_j$.\footnote{By matching manager $j$ with project $j$, we also use $I_j$ to denote the employment status of the manager $j$ without loss of generality: that is, $I_j = 1$ means that manager $j$ is employed. The employment decision of a representative manager $j$ is redundant because his investor will always adjust his compensation contract in such a way that the manager is indifferent between taking the offer or not.} This decision makes both the supply of risky assets and, as a consequence, the market portfolio endogenous. Recall the definition $\gamma \equiv \int_{I_j=1} \gamma_j dj$; here, $\gamma$ is the systematic risk of all projects that are financed in equilibrium. Note that from the proof of Lemma 2 in Appendix, the systematic risk of the market portfolio is $\sigma_M = \frac{\nu_I}{r_l + \nu_A} \tilde{\gamma}$, which is proportional to $\tilde{\gamma}$. Because of the proportionality, we refer to $\tilde{\gamma}$ as the systematic risk of the market portfolio $\tilde{M}$.

The manager bears the other portion of systematic risk through the compensation contract, $s_j \gamma_j + (1 - s_j) m_j \sigma_M = \frac{\nu_A}{r_A + r_I} \tilde{\gamma}$ (see the expression for the manager’s wealth $\tilde{w}_j$ in eqn. (1)).

The firm’s decision as to whether to finance its project reduces to the NPV rule: $I_j = 1$ if firm value $(1 - s_j) p_j > 0$. Information quality affects firm value and thus affects the financing decision as well. When economy-wide information changes, this may simultaneously affect firms’ financing decisions; this, in turn, may lead to changes in the market portfolio. Because the risk premium for systematic risk is determined by the risk of the market portfolio (that is, $\Gamma_j = \frac{\nu_A}{r_A + r_I} \gamma_j$), the change in the market portfolio affects the risk premium for systematic risk. We summarize the effects of an economy-wide improvement in accounting quality in the following proposition.

**Proposition 3** As the quality of economy-wide accounting information improves ($\alpha$ becomes smaller), for an existing firm:

1. the risk premium for the firm’s idiosyncratic risk decreases ($\frac{\partial \Pi}{\partial \sigma} > 0$);
2. the risk premium for the firm’s systematic risk increases ($\frac{\partial \Pi}{\partial \alpha} < 0$); and
3. the total risk premium the firm pays to finance its project could either increase or decrease, depending on its sensitivity to the economy-wide factor $\alpha$ and its relative exposure to systematic and idiosyncratic risk.

Part 1 of Proposition 3 is straightforward from Proposition 1. As the quality of economy-wide accounting information improves, the improvement reduces the total idiosyncratic noise
in the accounting measure \(\alpha_j\) for all firms, which results in a lower risk premium for idiosyncratic risk.

Part 2 of Proposition 3 is novel and somewhat surprising. As the quality of economy-wide idiosyncratic accounting information improves, the improvement increases the risk premium for an existing firm’s systematic risk. The intuition for the result is as follows. Improvements in the quality of idiosyncratic accounting information reduce the risk premium for idiosyncratic risk (or agency cost), which leads to riskier projects becoming more profitable and thus more likely to be financed. As a result, the market portfolio becomes larger and the level of systematic risk in the economy grows. Because the improvement in the quality of idiosyncratic accounting information does not affect the total risk-bearing capacity of the economy, of necessity the risk premium for systematic risk has to increase.

This intuition is borne out by a simple proof. Consider the effect of a change in \(\alpha\) on \(\Delta_j\):

\[
\frac{\partial \Delta_j}{\partial \alpha} = \frac{\gamma_j}{r_I + r_A} \frac{\partial \gamma_j}{\partial \alpha} + \lambda_j \frac{\partial \Pi_j}{\partial \alpha}.
\]

With moral hazard, a firm’s total risk premium compensates for both systematic and idiosyncratic risk. Holding the market portfolio \(\gamma\) constant, Proposition 1 (similarly, part 1 of Proposition 3) predicts that the improvement in the quality of a firm’s idiosyncratic information reduces the risk premium for idiosyncratic risk, i.e., \(\frac{\partial \Pi_j}{\partial \alpha} > 0\). We label this the direct effect. When an economy-wide change takes place, the direct effect works on all firms simultaneously, i.e., \(\frac{\partial \gamma_j}{\partial \alpha} \neq 0\). We label this the indirect effect. What is the sign of the indirect effect? Suppose \(\frac{\partial \gamma_j}{\partial \alpha} \geq 0\). Then \(\frac{\partial \Delta_j}{\partial \alpha} > 0\) and all firms’ NPVs increase as economy-wide information quality improves. As a result, as \(\alpha\) decreases, the set of firms (i.e., \(\int_{J=1}^{\gamma_j} dj\)) that are financed is strictly larger, which implies that \(\bar{\gamma} = \int_{J=1}^{\gamma_j} dj\) increases (i.e., \(\frac{\partial \bar{\gamma}}{\partial \alpha} < 0\)); this leads to a contradiction. Therefore, \(\frac{\partial \gamma}{\partial \alpha} < 0\) and \(\frac{\partial \Pi_j}{\partial \alpha} < 0\). This proves part 2 of Proposition 3.

The indirect effect is an externality on existing firms arising from the influx of new firms. Firms (projects) that would otherwise have had negative NPVs become positive NPV projects as a result of the improvement in the quality of idiosyncratic information. These new firms enter the capital market and compete with existing firms for the market’s limited risk-bearing
capacity and hence push up the price for risk-bearing capacity (i.e., the risk premium for systematic risk).

Note that the indirect effect is not solely driven by the CARA utility function. The constant risk-taking capacity associated with the CARA utility function makes the intuition for Part 2 of Proposition 3 transparent. However, what is necessary for the result to hold is that investors’ risk-taking capacity doesn’t vary in perfect sync with changes in accounting quality.

Part 3 of Proposition 3 states that the overall effect of a change in economy-wide information quality on the total risk premium an existing firm pays to finance its project is the sum of the direct and indirect effects. An improvement in economy-wide idiosyncratic information quality reduces the risk premium for idiosyncratic risk (the direct effect, or part 1 of Proposition 3) but increases the risk premium for systematic risk (the indirect effect, or part 2 of Proposition 3). Intuitively, a firm with a higher exposure to idiosyncratic risk benefits more from the beneficial direct effect, and a firm with a higher exposure to systematic risk is hurt more by the detrimental indirect effect. Furthermore, existing firms could be worse off overall if they have high exposure to systematic risk and/or low exposure to idiosyncratic risk. To the extent that firms’ risk profiles, i.e., \( \{\gamma_j, \lambda_j, \phi_j, \alpha_j\} \), differ cross-sectionally, the effects of a change in economy-wide information quality will have differential effects on the cost to a firm to finance its projects. The proof of part 3 of Proposition 3 in the Appendix provides examples in which an increase in \( \alpha \) increases the overall risk premium a firm pays to finance its project.

Overall, Proposition 3 suggests that an increase in \( \alpha \) induces a composition effect. Firms that were not financed may be financed; firms that were financed before may not be financed any longer; and the composition of equilibrium firms being financed is determined by their risk exposure to both systematic and idiosyncratic risk.

Since a major departure of this paper from the literature is to endogenize the project selection decisions, the composition effect is thus one of the most important results. The following proposition reveals how the composition effect differentiates our model from a standard agency model with regard to welfare implications.
**Proposition 4** As economy-wide information quality improves, the sets of projects that are financed in equilibrium before and after the improvement may not subsume each other. Taking the improvement to the extreme, the sets of projects that are financed in equilibrium in the first- and second-best cases may not subsume each other.

Proposition 4 differs from the intuition in a standard principal-agent setting. In a standard principal-agent setting, the set of projects that are financed in equilibrium grows as moral hazard decreases; this reflects the strict gain in efficiency through the reduction in moral hazard. In contrast, the set of projects that are financed in equilibrium does not necessarily grow as moral hazard becomes less severe in our model. In other words, the indirect effect in our model could be strong enough to dominate the direct effect. As a result, an equilibrium with higher quality information does not Pareto dominate an equilibrium with lower quality information. The proof of Proposition 4 in Appendix provides such examples.

The presence of a strong indirect effect complicates empirical studies of the economic consequences of the adoption of new accounting standards, and in particular the convergence of global accounting standards. In addition to the issue of choosing an appropriate proxy for the risk premium discussed in the previous section, such tests need to capture the change in the risk premium for both idiosyncratic and systematic risks.

The presence of a strong indirect effect also suggests that accounting standards have both resource allocation and distributional consequences because firms share the same pool of risk-bearing capacity. This yields the prediction that some firms (or countries) with predictable characteristics could be worse off and opposed to accounting standards and disclosure regulations even if the standards improve the accounting quality of all firms. Those firms most likely to be opposed are ones with high systematic risk and low idiosyncratic risk. They do not benefit as much from improvement in idiosyncratic information quality, but share more of the negative externality in the form of an increased market risk premium.

6 Discussion and Conclusion

The purpose of this paper was to attempt to understand the economic consequences of idiosyncratic accounting information in the presence of agency problems. We integrate a moral
hazard problem into the framework of a multi-firm economy with endogenous project selection, so as to better understand how an improvement in a firm’s accounting information affects the cost to a firm to finance its projects/investments. The tension that arises in the context of such a model is that, on the one hand, accounting information about a firm is primarily idiosyncratic, whereas, on the other hand, contemporary asset pricing theories prescribe that a project’s risk premium is solely a function of the systematic risk of a project’s cash flow. Despite a surfeit of papers in the literature on (separately) moral hazard and the multi-firm asset pricing, to our knowledge no one has studied this issue.

In a context of our analysis, first we establish that moral hazard distorts the sharing of idiosyncratic risk, but has no effect on systematic risk sharing. Armed with this insight, we attempt to make two points. First, because a firm’s traded shares command a risk premium for exclusively systematic risk while idiosyncratic accounting information reduces the risk premium for idiosyncratic risk, an improvement in the quality of idiosyncratic information is unlikely to manifest in the risk premium of the firm’s traded shares. Second, when the quality of economy-wide idiosyncratic information improves, the costs for firms to finance their projects drop and more projects are financed simultaneously; as a result, the market portfolio becomes larger and each firm’s risk premium for systematic risk increases. Therefore, improvements in the quality of economy-wide idiosyncratic information could either increase or decrease the overall cost to a firm to finance its project. In other words, accounting standards may have both efficiency and redistributive effects.

Our analysis offers two chief empirical implications. First, an improvement in idiosyncratic information is unlikely to manifest in the risk premium of the firm’s traded shares, a popular proxy for cost of capital in both empirical and theoretical studies. We also predict that the risk premium of traded shares is lower than a firm’s cost of financing a project and that the discrepancy increases in the size of the restricted equity share. Second, an economy-wide improvement in the quality of idiosyncratic information decreases the risk premium for idiosyncratic risk but increases the risk premium for systematic risk. The overall cost for an existing firm to finance its project could either increase or decrease, depending on its risk profile. This insight could aid cross-sectional studies that attempt to capture the economic consequences of an economy-wide change in accounting quality, such as the adoption of IFRS.
While we make no attempt to test empirically our results, our claim that improvements in idiosyncratic information are unlikely to manifest in the risk premium of a firm’s traded shares comports well with the survey results in Graham and Harvey (2001). This paper finds that private firms and small public firms with higher managerial ownership are less likely to use systematic risk to estimate their cost of equity capital than their large, public counterparts. The paper attributes the difference to implementation problems for private and small firms. An alternative explanation from our analysis is that systematic risk is less representative of the cost of equity capital for firms whose fraction of traded shares is smaller.

Appendix

Proof of Lemma 1 and 2.

We assumed in the text that $I_j = 1$ for any $j$ until Section 5. In this Appendix, however, we prove a more general version of the model with endogenous financing decision $I_j$. The results up to Section 5 are obtained by simply setting $I_j = 1$.

We characterize our economy by the exogenous parameters $\{\phi_j, \gamma_j, \alpha_j\}_{j \in [0,1]}, c, CE^0, r_A, r_I$, and an equilibrium to our economy by the endogenous variables $\{I_j, v_j, b_j, s_j, m_j, x_j, p_j\}_{j \in [0,1]}$. We solve for the endogenous variables in three steps. First, we solve the capital market equilibrium $\{x_j, p_j\}$, holding constant the financing and compensation decisions (i.e., holding $\{I_j, v_j, b_j, s_j, m_j\}$ constant). Second, we then take the financing decision $I_j$ and the capital market equilibrium as constant and solve the compensation contract within firm $\{v_j, b_j, s_j, m_j\}$. Finally, we solve the financing decision $I_j$.

Fixing the financing decision $I_j = 1$ and the compensation contract $\{v_j, b_j, s_j, m_j\}$, the market portfolio that is available to investor $j$ is $\tilde{M} \equiv \int I_j = 1 (1 - s_j)\tilde{G}_j dj$. Some algebra together with the definition of idiosyncratic risk (i.e., $\int \tilde{\xi}_j dj = \int I_j = 1 \tilde{\xi}_j dj = 0$) yields

$$\tilde{M} = E[\tilde{M}] + \sigma_M \tilde{\eta}$$

where $E[\tilde{M}] \equiv \int I_j = 1 (1-s_j)(1-b_j)F-v_j dj$ and $\sigma_M \equiv \int I_j = 1 (1-s_j)\gamma_j dj$.

Plugging the definitions of $\tilde{Y}_j$, $\tilde{M}$, and $\tilde{F}_j$ into the manager $j$’s wealth $\tilde{w}_j(i)$ in eqn. (1)
yields the following certainty equivalent:

\[
CE(w_j(t)) = (1 - s_j)v_j + (s_j + b_j(1 - s_j))F_t + m_j(1 - s_j)E[\tilde{M}]
- \frac{1}{2r_A}((b_j(1 - s_j)\alpha_j)^2 + (s_j\phi_j)^2 + (s_j(\gamma_j - m_j\sigma_M) + m_j\sigma_M)^2).
\]  

(10)

The IC condition reduces to \(CE(w_j(t = 1)) - c \geq CE(w_j(t = 0))\), which we rearrange as \((s_j + b_j(1 - s_j))F \geq c\). The IR condition reduces to \(CE(w_j(t = 1)) - c \geq CE^0\). Note that the IR condition binds in equilibrium; we use this to determine \(v_j\):

\[
(1 - s_j)v_j = CE^0 + c - (s_j + b_j(1 - s_j))F - m_j(1 - s_j)E[\tilde{M}]
+ \frac{1}{2r_A}((b_j(1 - s_j)\alpha_j)^2 + (s_j\phi_j)^2 + (s_j(\gamma_j - m_j\sigma_M) + m_j\sigma_M)^2).
\]

Finally, the certainty equivalent of investor \(j\)'s wealth at date 2 is expressed in eqn. (2).

Denoting \(\mu\) as the Lagrange multiplier, the Lagrange function of investor \(j\)'s problem is

\[L(x_j, b_j, s_j, m_j, \mu) = (1 - s_j)p_j + x_j(E[\tilde{M}] - p_M) - \frac{1}{2r_I}x_j^2\sigma_M^2 + \mu((s_j + b_j(1 - s_j))F - c).\]

To solve the equilibrium, we first take the compensation contract as given and solve for \(p_j\) and \(p_M\) as functions of \(\{v_j, b_j, s_j, m_j\}\). The first-order condition for \(L\) with respect to \(x_j\) yields \(E[\tilde{M}] - p_M - \frac{1}{r_I}\sigma_M^2 = 0\). Because investors are symmetric in their tolerance for risk, they all hold the same portfolio. Market clearing implies that \(x_j = 1\). Thus, \(p_M = E[\tilde{M}] - \frac{1}{r_I}\sigma_M^2\).

An individual firm \(j\)'s per-share price \(p_j\) is determined as follows. Allow for the fact that initially the market is in equilibrium. Next, suppose that an atomless firm, firm \(j\), is offered to the market at per-share price \(p_j\). Firm \(j\) generates cash flow \(\tilde{G}_j\) for shareholders, but only \(1 - s_j\) of this cash flow is offered to the public market. A representative investor \(i\) who demands \(z\) of firm \(j\)'s share at \(p_j\) would have wealth of \(z(\tilde{G}_j - p_j) + (1 - s_i)p_i + (\tilde{M} - p_M)\) and a certainty equivalent of \(z(E[\tilde{G}_j] - p_j) + (1 - s_i)p_i + E[\tilde{M}] - p_M - \frac{1}{2r_I}(z^2Var[\tilde{G}_j] + Var[\tilde{M}] + 2zCov[\tilde{G}_j, \tilde{M}]\)). The first-order condition with respect to \(z\) generates \(E[\tilde{G}_j] - p_j - \frac{1}{r_I}(zVar[\tilde{G}_j] + Cov[\tilde{G}_j, \tilde{M}]\)). Because firm \(j\) is atomless, a representative investor only holds an infinitely small interest in firm \(j\). Thus, \(p_j\) is determined by taking the limit of the above
As the opportunity cost of the manager’s effort-free labor. Thus, Deﬁne as the surplus for bearing risk is 

\[ \text{the opportunity cost of the manager’s effort-free labor. Thus, Deﬁne as the surplus for bearing risk is} \]

\[ x_j(E[\tilde{M}] - p_M) - \frac{1}{2r_I} \frac{\tilde{r}_j^2 \sigma_M^2}{r_I + r_A} = \frac{r_I}{2(r_I + r_A)} \tilde{\gamma} \frac{\tilde{r}_j^2 \sigma_M^2}{r_I + r_A} \]

Second, the NPV of project \( j \) to investor \( j \) is

\[ (1 - s_j)p_j = F - (c + CE^0 - \frac{r_A}{r_I + r_A} \frac{\tilde{r}_j^2 \sigma_M^2}{2(r_I + r_A)}) - \frac{1}{2r_A} \frac{\alpha_j^2 \psi_j^2}{F^2(\alpha_j^2 + \phi_j^2)} - \frac{\gamma_j \tilde{\gamma}}{r_A + r_I} \]

Investor \( j \) receives a surplus for bearing systematic risk, \( \frac{r_A}{r_I + r_A} \frac{\tilde{r}_j^2 \sigma_M^2}{2(r_I + r_A)} \). This surplus is proportional to the investor’s risk tolerance relative to the manager’s. Similarly, if manager \( j \) participates directly in the capital market, he could also obtain a surplus of \( \frac{r_A}{r_I + r_A} \frac{\tilde{r}_j^2 \sigma_M^2}{2(r_I + r_A)} \) for bearing risk. Therefore, \( \frac{r_A}{r_I + r_A} \frac{\tilde{r}_j^2 \sigma_M^2}{2(r_I + r_A)} \) is the additional component of the manager’s reservation utility. See Footnote 9 for the discussion about this issue. Define \( CE^0 \equiv CE^0 - \frac{r_A}{r_I + r_A} \frac{\tilde{r}_j^2 \sigma_M^2}{2(r_I + r_A)} \) as the opportunity cost of the manager’s effort-free labor. Thus, \( CE^0 \) is not a function of \( \tilde{\gamma} \).

The NPV of project \( j \) is \( (1 - s_j)p_j \) and the net cash ﬂow of project \( j \) is \( F - (c + CE^0) \); that is, gross cash ﬂow \( F \) net of the manager’s cost \( (c + CE^0) \). Define \( \Gamma_j \equiv \frac{\gamma_j \tilde{\gamma}}{r_A + r_I} \) and
\[ \Pi_j = \frac{1}{2r_A} \frac{c^2 \alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)}. \] The risk premium the firm pays to finance the project is

\[ \Delta_j = \frac{\gamma_j \tilde{\gamma}}{r_A + r_I} + \frac{1}{2r_A} \frac{c^2 \alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)} = \Gamma_j + \Pi_j. \]

Moreover, if the manager’s effort is contractible, then the IC condition is dropped from the optimization. Following the same procedure, the optimal contract in this first-best case is

\[ b_j^{FB} = 0, \quad s_j^{FB} = 0, \quad \text{and} \quad m_j^{FB} = \frac{r_A}{r_I}. \]

The firm value in this first-best case is

\[ (1 - s_j^{FB})p_j^{FB} = \frac{1}{2r_A} \frac{c^2 \alpha_j^2 \phi_j^2}{(\alpha_j^2 + \phi_j^2)} = \Pi_j. \]

This proves Lemma 2.

Finally, the firm’s financing decision \( I_j \) is simple: \( I_j = 1 \) if and only if the project has a positive NPV (i.e., \( (1 - s_j)p_j > 0 \)).

**Proof of part 3 of Proposition 3.**

Part 3 is proved by showing that \( \frac{\partial \Delta_j}{\partial \alpha} \) could be either positive or negative. From eqn. (8),

\[ \frac{\partial \Delta_j}{\partial \alpha} > 0 \text{ if and only if } \frac{\gamma_j}{r_I + r_A} \frac{\partial \tilde{\gamma}}{\partial \alpha} + \lambda_j \frac{\partial \Pi_j}{\partial \alpha_j} > 0. \] (13)

We prove that the above condition is not an empty set by the construction of an example.

Suppose the economy consists of two types of firms, with parameters \( \{\lambda_j, \delta_j, \gamma_j, \phi_j\}, \quad j \in \{1, 2\} \). The mass of type-1 firm is \( h \) and of type-2 firm \( 1 - h \). Set \( \delta_1 = \delta_2 = 0, \phi_1 = \phi_2 = \phi_0 > 0, \gamma_1 < \gamma_2, \lambda_1 \leq \lambda_2 \). Define \( \hat{F} \equiv F - (c + CE^0) \) as the net cash flow of a project.

Assume that

\[ \frac{h \gamma_1 + (1 - h) \gamma_2 \gamma_1 + \Pi(\alpha_1)}{r_A + r_I} < \hat{F}, \quad \text{(Assumption 1)} \]

\[ \frac{h \gamma_1 + (1 - h) \gamma_2 \gamma_2 + \Pi(\alpha_2)}{r_A + r_I} > \hat{F} > \frac{h \gamma_1}{r_A + r_I} \gamma_2 + \Pi(\alpha_2). \quad \text{(Assumption 2)} \]

With these assumptions, we could prove that all type 1 firms and some interior fraction
of type 2 firms are financed. Denote $k$ as the fraction of type 2 firms that are financed. Then $\tilde{\gamma}(k) = h\gamma_1 + (1 - h)\gamma_2 k$. By putting $k$ explicitly as an argument, $\tilde{\gamma}(k)$ highlights the general equilibrium effect that the aggregate risk in the economy depends on firms’ financing decisions. Because $\tilde{\gamma}(k)$ is increasing in $k$, there exists a $k^*$ such that $\frac{\tilde{\gamma}(k^*)}{\gamma_1} + \frac{\Pi(\alpha_2)}{\lambda_2} = \hat{F}$ by Assumption 2. Differentiating it with respect to $\alpha$, we have $\frac{\partial \tilde{\gamma}(k^*)}{\partial \alpha} = -\frac{\partial \Pi(\alpha_2)}{\partial \alpha_2} \lambda_2 \frac{r_A + r_I}{\gamma_2}$.

Because $\frac{\partial \Pi(\alpha_2)}{\partial \alpha_2} > 0$, $\frac{\partial \tilde{\gamma}(k^*)}{\partial \alpha} < 0$. That is, more type-2 firms are financed as $\alpha$ decreases.

Further, for type-1 firms,

$$\frac{\partial \Delta_1}{\partial \alpha} = \frac{\gamma_1}{r_A + r_I} \frac{\partial \Pi(\alpha_1)}{\partial \alpha_1} \lambda_1 = \frac{\partial \Pi(\alpha_1)}{\partial \alpha_1} \lambda_1 - \frac{\gamma_1}{\gamma_2} \frac{\partial \Pi(\alpha_2)}{\partial \alpha_2} \lambda_2.$$ 

The second step applies $\frac{\partial \Delta_1}{\partial \alpha} = -\frac{\partial \Pi(\alpha_2)}{\partial \alpha_2} \lambda_2 \frac{r_A + r_I}{\gamma_2}$. As $\lambda_1 \to 0$, $\frac{\partial \Delta_1}{\partial \alpha} < 0$. As $\lambda_1 \to \lambda_2$, $\alpha_1 \to \alpha_2$ and $\frac{\partial \Delta_1}{\partial \alpha} = \frac{\partial \Pi(\alpha_2)}{\partial \alpha_2} \lambda_2 (1 - \frac{\gamma_1}{\gamma_2}) > 0$. Therefore, the condition in eqn. (13) is not an empty set.

**Proof of Proposition 4.**

The proof is also by construction. Following the two-type-firm economy in the previous proof, we now set $\delta_1 = \delta_2 = 0$, $\phi_1 = \phi_2 = \phi_0 > 0$, $\gamma_1 < \gamma_2$, and $\lambda_1 > \lambda_2 = 0$. Further, assume

$$\frac{\gamma_1 h + \gamma_2 (1 - h)}{r_A + r_I} \gamma_2 > \hat{F} > \frac{(1 - h)\gamma_2^2}{r_A + r_I} \text{ and } \Pi(\alpha_1) > \hat{F} > \frac{h\gamma_1^2}{r_A + r_I}.$$ 

The proof is completed by two claims: a) all type-1, but not all type-2, firms are financed in the first-best equilibrium; b) all type-2, but no type-1, firms are financed in the second-best equilibrium. Therefore, the sets of firms that are financed in the first- and second-best cases do not subsume each other.

In the first-best equilibrium, only systematic risk matters for project selection. Because $\gamma_1 < \gamma_2$, a necessary condition for any type-2 firm to be financed is that all type-1 firms are financed. Suppose no firms are financed. But then the risk premium for the first firm is 0 because $\tilde{\gamma}$ is 0. Therefore, at least some type-1 firms will be financed. But this implies that all type-1 firms are financed because $\Delta_1 \leq \frac{h\gamma_1^2}{r_A + r_I} < \hat{F}$. Finally, suppose that all type-1 and type-2 firms are financed. Then $\tilde{\gamma} = \gamma_1 h + \gamma_2 (1 - h)$. For a type-2 firm $\Delta_2 = \frac{\gamma_1 h + \gamma_2 (1 - h)}{r_A + r_I} \gamma_2 > \hat{F}$, leading to a contradiction. Thus, at least some type-2 firms are not financed. This proves
claim a.

In a second-best equilibrium that satisfies $\Pi(\alpha_1) > \hat{F}$, no type-1 firm is financed because the risk premium for its idiosyncratic risk is too high. Thus, $\Delta_2$ is 0 for the first type-2 firm to enter the market because $\gamma = 0$. Further, all type-2 firms are financed because $(1-h)\frac{\gamma_2}{r_A+r_I} < \hat{F}$. This proves claim b.

References


Tirole, J., 2006, The theory of corporate finance, Discussion paper HAL.