Idiosyncratic Information, Moral Hazard and the Cost of Capital*

Pingyang Gao
The University of Chicago Booth School of Business
Pingyang.Gao@ChicagoBooth.edu

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Abstract

While much accounting information is idiosyncratic, economy-level factors such as accounting standards affect the quality of accounting information of many firms simultaneously. We study these two features of accounting information by embedding a moral hazard problem into a multi-firms economy with endogenous project financing decisions. Relative to the first-best benchmark without moral hazard, managers are exposed to idiosyncratic risk in our second-best equilibrium. We extend this insight to achieve two results. First, a firm-level improvement in accounting information reduces the firm’s cost of capital even though it does not affect the implied cost of capital inferred from unrestricted shares. Second, an economy-level improvement in

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idiosyncratic information quality reduces the risk premium for idiosyncratic risk but can increase the risk premium for systematic risk. Thus, its overall effect on a firm’s cost of capital depends on firms’ risk profiles.

**JEL recognition:** G12, G14, G31, M41

**Key Words:** cost of capital, project financing, moral hazard, risk premium, systematic risk, idiosyncratic risk
1 Introduction

Cost of capital is a key determinant of corporate project choices; as such, it affects the allocation of capital throughout the economy. Thus, an important issue in accounting research is whether accounting information affects the cost of capital and if so how. The empirical evidence on the relation between information quality and cost of capital has been at best mixed.\(^1\) While the mixed evidence is often attributed to empirical issues, such as measurement errors in proxies for the cost of capital and information quality, this paper explores possible theoretic explanations.

Much accounting information is idiosyncratic. On the one hand, modern asset pricing theories suggest that a project’s cost of capital is solely a function of its cash flow’s systematic risk; taken at face value, this implies that idiosyncratic accounting information does not affect the cost of capital.\(^2\) On the other hand, idiosyncratic accounting information plays an important role in mitigating agency problems, which intuitively makes a firm’s financing cheaper.

To fully understand the relation between idiosyncratic information quality and cost of capital, we combine both the asset pricing and agency views in this paper. We start with a simple moral hazard problem that creates the demand for idiosyncratic accounting information. We then embed it into a multi-firms asset pricing setting. There is a continuum of principals (investors) and agents (managers). Each investor is endowed with a project and decides whether to finance the project by issuing equity shares. If so, he offers a compensation contract to induce the manager to exert efforts. The manager’s efforts and random factors jointly determine the project’s cash flows. The accounting system provides a signal about the manager’s effort choice with

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\(^1\)Healy and Palepu (2001), Dechow, Ge, and Schrand (2010), Bertomeu and Cheynel (2015) and Leuz and Wysocki (2016) review this vast literature. They all come to a similar conclusion that the empirical evidence on the relation between information quality and the cost of capital is too diverse to reach a consensus.

idiosyncratic measurement errors. Finally, both investors and managers make their portfolio choices, and the stock prices are determined by the market clearing conditions. The project financing decisions, the compensation contracts, the stock prices and the portfolio choices are jointly determined in equilibrium. The (equity) cost of capital is defined as the required expected rate of return by stockholders to hold the firm’s equity shares. It consists of a risk-free rate of return normalized to 1 and a risk premium. Thus, the cost of capital in our model is equal to the risk premium the firm pays to shareholders for holding the firm’s shares.

In the absence of moral hazard (the first-best benchmark), the firm pays a fixed salary to the manager and issues all equity shares to fully diversified shareholders who demand only compensation for systematic risk. The firm’s cost of capital is thus equal to the risk premium for the firm’s systematic risk; it is not affected by the project’s idiosyncratic risk.

In the presence of moral hazard (the second-best equilibrium), the firm uses restricted equity shares as incentive provisions to the manager, which exposes her to idiosyncratic risk. The firm’s shares are divided into two categories, the unrestricted shares owned by well-diversified shareholders and the restricted shares owned by the manager. While the holders of the unrestricted shares demand only compensation for systematic risk, the manager with the restricted shares requires additional compensation for idiosyncratic risk. This additional compensation is a form of agency cost. Therefore, the project’s cost of capital consists of two components, the risk premium for its systematic risk and the risk premium for the idiosyncratic risk in the manager’s compensation contract. The agency problem increases the firm’s cost of capital by increasing the second component.

The managerial exposure to idiosyncratic risk is empirically significant. Holmstrom, Kroszner, and Sheehan (1999) report that in 1995 the mean and median equity ownership of directors and officers (D&O) of a sample of exchange-listed firms are 21.1% and
12.4%, respectively. Similarly, Fahlenbrach and Stulz (2009) find that the mean and median of D&O ownership are 22.3% and 14.8%, respectively, for the average sample period between 1988 and 2003. Moreover, the managerial exposure to idiosyncratic risk in other countries is even higher than that in the United States, as surveyed by Tirole (2006).

We then conduct comparative statics to examine the effects of information quality on the cost of capital. Our first result is that a firm-level improvement of the quality of idiosyncratic information reduces the risk premium for idiosyncratic risk but does not affect the risk premium for systematic risk. On the one hand, since the firm-level improvement does not affect the aggregate supply of systematic risk, it does not affect the risk premium for systematic risk. On the other hand, a firm-level improvement in information quality does mitigate the agency problem. It allows the firm to issue fewer restricted shares as incentive provisions, lowering the agency cost component of the cost of capital. Overall, a firm-level improvement in idiosyncratic information quality reduces the firm’s cost of capital.

Another salient feature of accounting information is that its quality is affected not only by individual firms’ choices but also by economy-level factors. For example, accounting standards and securities regulation simultaneously change the quality of many firms’ idiosyncratic accounting information. Our second result is that an economy-level improvement in the quality of idiosyncratic accounting information decreases the risk premium for idiosyncratic risk but can increase the risk premium for systematic risk. Accordingly, its overall effect on a firm’s cost of capital can go in either direction, depending on the firm’s risk profile. The intuition behind this perhaps surprising result is as follows. As the economy-level information quality improves, each firm’s agency cost is lower, which we label as the direct effect. All else equal, the lower cost of capital makes it easier for firms to finance their projects. Since the improvement occurs at the economy-level, all firms benefit from it (albeit with differing magnitude) and thus
more projects are financed at the same time. This increases the aggregate supply of risky assets and the demand for risk-bearing capacity. As a result, the risk premium for systematic risk increases, which we label as the indirect effect. The overall effect on a firm’s cost of capital is determined by the trade-off of these two effects. The direct effect depends on the firm’s idiosyncratic risk exposure while the indirect effect is a function of the firm’s systematic risk. Since a firm’s exposure to the two types of risk varies, the overall effect also varies across firms. In other words, as the economy-level information quality improves, a firm with larger systematic risk but smaller idiosyncratic risk may see its cost of capital increase while a firm with smaller systematic risk but larger idiosyncratic risk will see its cost of capital drop.

The paper provides two possible explanations for the mixed empirical evidence on the relation between information quality and cost of capital. First, the implied cost of capital is not the theoretically correct construct to look at as far as the consequences of the firm-level idiosyncratic information quality is concerned. Implied cost of capital, the most commonly used proxy for the cost of capital, is computed as the discount rate that equates a firm’s current stock price and expected future earnings in an equity valuation model. While it has many advantages as a proxy for expected returns, the implied cost of capital is conceptually the risk premium only for systematic risk. As such, there is no theoretical basis for a negative relation between idiosyncratic information quality and the implied cost of capital. Moreover, our model provides a remedy to address this systematic bias. The effect of idiosyncratic information quality on the cost of capital is theoretically captured by its effect on the agent’s exposure to idiosyncratic risk. An improvement in the firm-level information quality is associated with fewer restricted shares (lower idiosyncratic risk exposure for the agent).

The second explanation for the mixed evidence is confined only to the economy-level

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3For specifications and discussions of the implied cost of capital as a proxy for cost of capital, see, e.g., Frankel and Lee (1998), Botosan (1997), Botosan and Plumlee (2002), and Bertomeu and Cheynel (2015).
changes of information quality, such as those resulting from new accounting standards and/or IFRS and the Sarbanes-Oxley Act. Our second result explains that such improvement can either increase or decrease a firm’s cost of capital, depending on the firm’s risk profile. Moreover, our model predicts that such improvement leads to fewer restricted shares but the higher implied cost of capital.

Our paper contributes to the vast literature on the relation between agency problem and cost of capital. By embedding the agency problem into a multi-firms setting with endogenous project selection, we generate testable predictions about the effects of idiosyncratic accounting information on the cost of capital. In this respect, our analysis is related to four papers that investigate moral hazard within the context of asset pricing. Fischer (2000) studies the optimal compensation contract when the agent can trade in the capital market. Among other results, he shows that the combination of contracting only on output and allowing the agent to trade does not replicate the optimal contract unless the wealth effect from the realization of the controllable event is either absent or manageable from trading. In a “LEN” framework (Linear contracts, Exponential utility, Normally distributed performance measures), Christensen, Feltham, and Wu (2002) observe that, unlike the well-diversified principal, the manager is exposed to the firm-specific risk through the compensation contract and thus should be compensated accordingly. They use this insight to study the design of cost of capital charges in implementing residual income models in the presence of agency problems. Ou-Yang (2005) derives an equilibrium asset pricing model with moral hazard in a LEN model as well. He observes that idiosyncratic cash flow risk affects the stock price but not the dollar returns (risk premium). Bertomeu (2015) studies an agency model within a multi-firms setting. He shows that the average cost of capital is independent of agency friction. Since the optimal effort in his model is constant, the aggregate output is independent of agency frictions. While all these studies show that the firm’s cost of capital should include the agency cost in the form of risk premium for the idiosyncratic
risk the manager bears, none of them studies firms’ project selection and its implications for the effects of idiosyncratic information quality on the cost of capital, which is our focus. Bertomeu (2015) explain why a necessary condition for agency problems to affect the cost of capital is that they affect the aggregate risky assets. Our paper provides a specific mechanism through which this condition is endogenized.

Some papers examine the capital market consequences of firm-specific information in the absence of agency conflicts. Christensen and Feltham (1988) studies the role of firm-specific information in resource allocation, and Armstrong, Banerjee, and Corona (2009) examines the effect of information about firm-specific beta on a firm’s expected returns. Cheynel (2013) explores how a firm’s voluntary disclosure of firm-specific information affects the market risk premium. None of these papers, however, considers the role of accounting information in incentive provision.

The paper proceeds as follows. In Section 2 we describe our model of moral hazard in a diversified market setting, and then in Section 3 derive an equilibrium to our model. In Sections 4 and 5 we examine the economic consequences of a change in information quality at the firm-level and economy-level, respectively. In Section 6 we conclude. We provide proofs to lemmas and propositions stated in the paper in the Appendix A. Appendix B includes a table of all major notations.

2 The Model

We embed a moral hazard problem into a multi-firms asset pricing setting. Consider an economy with a continuum of investors and managers, each group with a mass of 1. We refer to an investor as “he” and a manager as “she” and denote them as \( i \in \{I, A\} \) (\( I \) for investors and \( A \) for agents), respectively. There are two dates. All actions take place at date 1 except consumption, which occurs at date 2. Each investor is endowed with a project that could generate uncertain cash flow at date 2 with the
assistance of a hired manager. The project is incorporated into a firm and we use the terms “project” and “firm” interchangeably. For a random variable, we denote “$x$” as a random variable, “$\bar{x}$” as its mean and “$x$” as its realization. For a choice variable, we denote “$x$” as the choice and “$x^*$” as its equilibrium value.

A representative project $j$ generates an uncertain gross cash flow $F_j$:

$$F_j = e_j F + \phi_j \xi_j + \gamma_j \eta,$$

where $F$ is the mean of cash flow and $e_j \in \{0, 1\}$ is the manager’s effort choice. If the manager exerts effort at a private cost $c$, $e_j = 1$; if the manager shirks at no cost, $e_j = 0$. $\phi_j \xi_j$ and $\gamma_j \eta$ are the project’s idiosyncratic and systematic components of cash flows, respectively. Both $\tilde{\xi}_j$ and $\tilde{\eta}$ are random variables with a standard normal distribution $N(0, 1)$. $\tilde{\xi}_j$ is independent of $\tilde{\eta}$ and $\tilde{\xi}_i$, for $i \neq j$. The non-negative constants $\phi_j$ and $\gamma_j$ represent the project’s exposure to these sources of risk.

Each investor first decides whether to finance his project by issuing equity shares in the capital market. We denote this financing decision as $H_j \in \{0, 1\}$. The investor chooses $H_j^* = 1$ if and only if the project generates positive net present value (hereafter NPV), which in turn depends on both the compensation contract and the capital market equilibrium. Since we match manager $j$ with investor $j$, $H_j$ also represents manager $j$’s employment status. If $H_j^* = 1$, we call the project viable and the manager employed.

If the project is viable, the investor offers a compensation contract to induce the manager to exert effort. The compensation consists of a base salary, an accounting-based bonus, and equity incentive shares:

$$\tilde{w}_j = a_j + b_j \tilde{Y}_j + s_j \tilde{G}_j.$$

$a_j$ is the base salary. $b_j \tilde{Y}$ is the accounting-based bonus. The firm’s accounting
system provides a signal $\tilde{Y}_j$ about the manager’s effort:

$$\tilde{Y}_j \equiv e_j F + \alpha_j \tilde{\varepsilon}_j. \quad (2)$$

$\alpha_j \tilde{\varepsilon}_j$ is the accounting system’s measurement error. $\tilde{\varepsilon}_j$ is a random variable with a standard normal distribution $N(0, 1)$ that is independent of all other random variables.\textsuperscript{4} The non-negative constant $\alpha_j$ is an inverse measure of information quality. $b_j$ is the accounting-based bonus coefficient. $a_j + b_j \tilde{Y}_j$ is paid out from the firm’s gross cash flow $\tilde{F}_j$.

The last component $s_j \tilde{G}_j$ is the equity incentive shares. The contract awards $s_j$ fraction of the firm’s shares to the manager. $\tilde{G}_j$ is the terminal cash flow (dividend) paid to shareholders at date 2, which equals the gross cash flow net of the manager’s compensation (other than the equity incentive shares):

$$\tilde{G}_j \equiv \tilde{F}_j - (a_j + b_j \tilde{Y}_j). \quad (3)$$

We conjecture and later verify that the optimal compensation contract does not allow the manager to short her own firm’s shares, similar to the commitment assumption in the principal-agent paradigm. We thus call the equity incentive shares restricted shares.

After awarding the manager with $s_j H_j$ fraction of shares, each firm issues the remaining $(1 - s_j) H_j$ shares to the capital market.\textsuperscript{5} These shares constitute the (trad-

\textsuperscript{4}While in reality an accounting system provides information that is both systematic and idiosyncratic in nature, the purpose of our model is to focus on the idiosyncratic component. Thus, we assume away the systematic component of the accounting signal $\tilde{Y}_j$. This is without loss of generality. Since there is a continuum of firms, the market portfolio (yet to be defined) perfectly reveals the systematic component of cash flow $\eta$ ex-post. Therefore, any accounting information about the systematic component is not incrementally valuable for contracting in the model.

\textsuperscript{5}We use $H_j$ as an indicator of the investor/manager types. Only viable firms ($H_j = 1$) issue shares and hire their managers. Non-viable firms with $H_j = 0$ don’t issue shares or hire managers. To avoid repetition in describing the actions of both viable and non-viable firms, we attach the decision $H_j$ to the corresponding variables to indicate the type of firms and managers.
able) market portfolio whose terminal cash flow is:

\[
\tilde{M} = \int (1 - s_j) H_j \tilde{G}_j dj = \tilde{M} + \sigma_M \tilde{\eta}.
\]  (4)

The market portfolio generates expected cash flow \( \tilde{M} \) and has systematic risk \( \sigma_M \tilde{\eta} \). It does not contain any idiosyncratic risk because of the diversification across a continuum of viable firms. Both \( \tilde{M} \) and \( \sigma_M \) are functions of the equilibrium variables, and their expressions are specified in expression 18 and 19 in the Appendix. Since all players take the market portfolio as given at the time of making their respective decisions, we have used the equilibrium variables directly in the definition of the market portfolio.

Both investors and managers make their own portfolio decisions. We conjecture and later verify they do not hold any individual stocks above and beyond the market portfolio (and the restricted equity shares for the employed managers) in equilibrium. Thus, each investor and manager hold \( x_j \) and \( z_j \) fraction of the market portfolio, respectively. Denote the price of the market portfolio and stock \( j \) as \( p_M \) and \( p_j \), respectively.

The investor sells the \( (1 - s_j) H_j \) fraction of shares at price \( p_j \), purchases \( x_j \) fraction of the market portfolio at price \( p_M \) that generates cash flow \( \tilde{M} \), and invests the remaining cash in the risk-free asset with rate of return normalized to 1. Similarly, the manager receives wage \( \tilde{w}_j H_j \) and purchases \( z_j \) fraction of the market portfolio. Therefore, the terminal wealth of investor \( j \) and manager \( j \) is

\[
\tilde{W}_j^I (H_j, a_j, b_j, s_j, x) = (1 - s_j) H_j p_j + x_j (\tilde{M} - p_M),
\]  (5)

\[
\tilde{W}_j^A (H_j, e_j, z_j) = \tilde{w}_j (e_j) H_j + z_j (\tilde{M} - p_M).
\]  (6)

Let \( U_i(W) \) represent the utility over wealth \( W \) of a representative player \( i \), \( i \in \{I, A\} \). Both investors and managers have negative exponential utility functions with
constant absolute risk aversion (CARA) coefficients $\frac{1}{r_I}$ and $\frac{1}{r_A}$, respectively. We exploit
the well-known result that when $\tilde{W}$ is normally distributed, the certainty equivalent of
the CARA utility, denoted as $V$, is equal to the expected wealth minus a risk premium.
Therefore, the certainty equivalents of the investor’s and the manager’s wealth are

$$V^I_j(H_j, a_j, b_j, s_j, x_j) = (1 - s_j) H_j p_j + x_j (\tilde{M} - p_M) - \frac{1}{2r_I} Var[\tilde{W}^I_j], \quad (7)$$

$$V^A_j(H_j, e_j, z_j) = \tilde{w}_j(e_j) H_j + z_j (\tilde{M} - p_M) - \frac{1}{2r_A} Var[\tilde{W}^A_j]. \quad (8)$$

To motivate the manager, the contract has to satisfy both her individual participation (IR) condition and incentive compatibility (IC) condition. The employed manager’s equilibrium certainty equivalent is $V^A_j(1, 1, z^*_j)$ and she incurs effort cost $c$. If
she declines the compensation contract, the manager enjoys her leisure with certainty equivalent $CE^0$ and can still invest in the capital market to obtain certainty equivalent $V^A_j(0, 0, z_j)$. Therefore, the manager’s IR condition is:

$$V^A_j(1, 1, z^*_j) - c \geq V^A_j(0, 0, z_j) + CE^0 \text{ for any } z_j. \quad (IR)$$

After the manager accepts the contract, i.e., $H^*_j = 1$, she compares her certainty equivalent with or without effort. Her IC condition is:

$$V^A_j(1, 1, z^*_j) - c \geq V_A(1, 0, z_j) \text{ for any } z_j. \quad (IC)$$

In both IR and IC, the manager’s certainty equivalent depends on her portfolio choice $z_j$. By allowing any $z_j$ on the right-hand sides of the conditions, we take into account the joint deviation possibility (that the manager adjusts her portfolio accordingly when she deviates to the off equilibrium path of shirking and/or not accepting the contract).
In sum, the representative investor’s decisions problems can be summarized as

$$\max_{\{H_j, a_j, b_j, s_j, x_j\}} V_j^I (H_j, a_j, b_j, s_j, x_j)$$  \hspace{1cm} \text{(Program I)}$$

s.t. IR; IC.

The representative manager’s decision is to choose $z_j$ to maximize $V_j^A(z_j)$ for any given contract $\{H_j, a_j, b_j, s_j, x_j\}$. Moreover, the stock prices $p_j$ and $p_M$ are determined by the market clearing condition that the demand for the market portfolio is equal to the supply:

$$\int x_j^* d_j + \int z_j^* d_j = 1. \hspace{1cm} \text{(9)}$$

The cost of capital is the risk-free rate of return plus a risk premium. Since the risk-free rate of return has been normalized to be 1, the cost of capital in our model $\Delta_j$ consists only of the risk premium.\(^6\)

$$\Delta_j \equiv NPV^{NoRisk} - NPV_j. \hspace{1cm} \text{(10)}$$

$NPV^{NoRisk}$ is the project’s NPV in the absence of risk ($\phi_j = \alpha_j = \gamma_j = 0$):

$$NPV^{NoRisk} \equiv F - (c + CE^0).$$

The managerial effort generates cash flow $F$ but it has an opportunity cost of $c + CE^0$. After we solve for the capital market equilibrium and the optimal compensation contract, we can obtain the project’s equilibrium NPV $j$. The investor then uses the positive NPV rule to decide whether to finance the project: $H_j^* = 1$ if and only if $NPV_j > 0$. Due to equation 10, this positive NPV rule is equivalent to a hurdle rate

\(^6\)This definition is consistent with the common interpretation of the cost of capital as a discount rate that equates expected cash flows and their present value. One could transform equation (10) into an expression that is tantamount to a discount rate: $\exp^{NPV_j} = \frac{\exp^{NPV^{NoRisk}}}{\exp^{\Delta_j}}$. $NPV^{NoRisk}$ is the firm’s expected free cash flows. The dollar risk premium, the proxy of the cost of capital used in some prior studies, is a special case of our definition in the absence of moral hazard.
The project is viable if and only if its cost of capital is lower than the hurdle rate $\text{NPV}^{\text{No Risk}}$.

The equilibrium solution concept is Nash Equilibrium since all choices are made at the date 1 at the same time. Until Section 5 when we study the economy-level information quality, we treat investors’ financing decisions as given. Thus, the aggregate systematic risk in the economy, which comprises of the systematic risk of all viable projects, is constant:

$$\bar{\gamma} \equiv \int \gamma_j H_j^* d_j.$$ 

All notations are defined in Table of Notations in Appendix B.

3 The equilibrium

In this section, we solve the model and discuss the equilibrium. We start with the first-best benchmark in the absence of the agency problem.

**Lemma 1** In the first-best benchmark when the effort is contractible,

1. the optimal contract pays a fixed salary and does not use incentive pays:

   $$s_j^{FB} = b_j^{FB} = 0;$$

2. both investors and managers hold the market portfolio according to their relative risk tolerance:

   $$x_j^{FB} = \frac{r_I}{r_A + r_I} \text{and} \ z_j^{FB} = \frac{r_A}{r_A + r_I};$$

3. the stock price is

   $$p_j^{FB} = \text{NPV}^{\text{No Risk}} - \Gamma_j, \text{ where } \Gamma_j \equiv \frac{\bar{\gamma}}{r_I + r_A} \gamma_j;$$
4. the project’s cost of capital is

\[ \Delta j^{FB} = \Gamma_j. \]

Lemma 1 is intuitive. In the absence of moral hazard, the optimal contract pays the manager a fixed salary and does not expose the manager to any risk. Both investors and managers diversify their portfolios by holding a fraction of the market portfolio that exposes them only to systematic risk. All idiosyncratic risk is fully diversified away. Investors and managers bear the systematic risk according to their relative risk tolerance. Since the market portfolio contains only systematic risk, the stock price contains only risk premium for systematic risk. Since no restricted shares are awarded as incentives, all shares are issued to the capital market. As a result, the market portfolio consists of all shares of viable firms and the project’s NPV is equal to its stock price.

Part 4 of Lemma 1 confirms the standard asset pricing result in the absence of moral hazard. On one hand, the aggregate risk in the economy \( \gamma \eta \) is fully captured in the market portfolio, that is, \( \gamma^{FB} = \sigma_M^{FB} \) and the Capital Asset Pricing Model (CAPM) applies. \( \frac{s^2}{r_I + r_A} \) is the risk premium to the market portfolio. The aggregate systematic risk in the economy is borne by the total risk tolerance \( r_I + r_A \), resulting in the per-unit risk premium for systematic risk \( \frac{Var[\eta]}{r_I + r_A} = \frac{s^2}{r_I + r_A} \). Moreover, each firm’s Beta is determined by its covariance with the market portfolio scaled by the variance of the market portfolio:

\[ \frac{Cov(G_j, M)}{Var[M]} = \frac{\gamma_j}{\gamma}. \]

As a result, the firm’s cost of capital is completely determined by its Beta and the risk premium for the market portfolio, that is, \( \Gamma_j = \frac{\gamma_j s^2}{\gamma (r_I + r_A)} \). On the other hand, both investors and managers bear the systematic risk according to their relative risk tolerance, \( \frac{r_I}{r_A + r_I} \gamma \eta \) and \( \frac{r_A}{r_A + r_I} \gamma \eta \), respectively. Their equilibrium exposure to the systematic risk is independent of their financing or employment decision.

Now we turn to the main case with the agency problem.

**Proposition 1** *In the presence of moral hazard when the effort is not contractible,*...
1. the optimal contract uses incentive pays:

\[ s_j^* > 0 \text{ and } b_j^* > 0; \]

2. both investors and managers hold the market portfolio:

\[ x_j^* = \frac{r_I}{r_A + r_I \sigma_M} \bar{\gamma}_j \text{ and } z_j^* = \frac{r_A}{r_A + r_I \sigma_M} s_j^* \gamma_j H_j^*; \]

3. the stock price is

\[ p_j^* = \bar{G}_j - \Gamma_j; \]

4. the project’s cost of capital is

\[
\begin{align*}
\Delta_j &= \Gamma_j + \Pi_j, \\
\Pi_j &= \frac{1}{2r_A} Var \left[ B_j^* \alpha_j \bar{\xi}_j + s_j^* \phi_j \bar{\xi}_j \right] = \frac{1}{2r_A} \frac{c^2 \alpha_j^2 \phi_j^2}{\alpha_j^2 + \phi_j^2}.
\end{align*}
\]

Proposition 1 should be contrasted with Lemma 1. Part 1 of Proposition 1 shows that the presence of moral hazard leads to the use of incentive pays in the optimal compensation contract. Specifically, both accounting signal \( \bar{Y}_j \) and cash flow \( \bar{G}_j \) are correlated with the manager’s effort \( e_j \). Thus, the optimal contract uses both an accounting-based bonus \( (b_j^* > 0) \) and restricted shares \( (s_j^* > 0) \). In sum, moral hazard affects the compensation contract.

Part 2 of Proposition 1 describes market participants’ portfolio choices. The investor’s portfolio choice is independent of his financing decision as \( x_j^* \) is not a function of \( H_j^* \). In contrast, the manager’s holding \( z_j^* \) depends on \( H_j^* \). Relative to her unemployed peers (with \( H_j^* = 0 \)), the employed manager reduces her holding of the market portfolio by \( \frac{s_j^* \gamma_j}{\sigma_M} \). Therefore, moral hazard also affects the manager’s equilibrium portfolio choice.
Part 1 and Part 2 of Proposition 1 jointly determine the equilibrium allocation of risk in the economy. The investor and the manager’s equilibrium wealth (after substituting the equilibrium variables into equation 5 and 6) is

\[ \tilde{W}_j^I = \bar{W}_j^I - \frac{r_I}{r_A + r_I} \tilde{c}_j, \]  
\[ \tilde{W}_j^A = \bar{W}_j^A + \left( B_j^\star \alpha_j \tilde{e}_j + s_j^\star \phi_j \tilde{\xi}_j \right) H_j^* + \left( s_j^\star \gamma_j H_j^* + z_j^\star \sigma_M \right) \tilde{c}. \]  

\[ (13) \]

\[ (14) \]

\( \tilde{W}_j^I \) and \( \tilde{W}_j^A \) are the mean of their respective wealth. Compared with Lemma 1, the investor’s equilibrium risk exposure is the same as in the first-best benchmark. First, he is exposed only to the systematic risk through her holding of the market portfolio. He does not bear any idiosyncratic risk in equilibrium. Second, the fraction of the systematic risk he bears in equilibrium is proportional to his relative risk tolerance \( \frac{r_I}{r_A + r_I} \).

The manager’s equilibrium risk exposure, however, differs from the first-best benchmark. In the presence of moral hazard, the employed manager is exposed to idiosyncratic risk, as captured by the second component of equation 14. It is absent for the unemployed manager (with \( H_j^* = 0 \)). The employed manager is exposed to idiosyncratic risk through the incentive pays in her compensation contract. The accounting-based bonus exposes her to the measurement error in the accounting signal \( \alpha_j \tilde{e}_j \). \( B_j^\star \equiv (1 - s_j^\star) b_j^\star \) measures the effective accounting-based bonus because the bonus is paid out by the firm of which the manager owns \( s_j^\star \) fraction. The restricted shares impose the idiosyncratic cash flow risk \( \phi_j \tilde{\xi}_j \) on the manager. Relative to the first-best benchmark, the presence of moral hazard distorts the sharing of idiosyncratic risk in the economy. Moral hazard leads to concentration of idiosyncratic risk on the (employed) managers.

How does moral hazard affect the manager’s equilibrium exposure to systematic risk? The last component of equation 14 is her equilibrium exposure to systematic
risk. For the unemployed manager, the exposure comes from her holding of the market portfolio and it is $z_j^*(H^*_j = 0)\sigma_M \tilde{\eta} = \frac{r_A}{r_A + r_I} \tilde{\gamma} \tilde{\eta}$. She bears a fraction of the systematic risk in the economy proportional to her relative risk tolerance. For the employed manager, the exposure comes from both her compensation and portfolio choice. On the one hand, the restricted shares in her compensation expose her to the systematic risk $s_j^* \gamma_j H^*_j \tilde{\eta}$. On the other hand, Part 2 of Proposition 1 shows that she adjusts her portfolio choice in response to her compensation risk (as $z_j^*$ is a function of her employment status $H^*_j$). Substituting $z_j^*$ into equation 14, we can verify that the employed manager’s net equilibrium exposure to systematic risk is also $\frac{r_A}{r_A + r_I} \tilde{\gamma} \tilde{\eta}$, the same as her unemployed peers and as her exposure in the first-best benchmark. Therefore, investors and managers bear the systematic risk according to their relative risk tolerance in equilibrium, regardless of their financing decisions and employment status. In other words, moral hazard does not affect the equilibrium sharing of the systematic risk in the economy.

Therefore, we obtain the equilibrium risk allocation in the economy. Unlike in the first-best benchmark, the employed manager in the presence of moral hazard bears idiosyncratic risk. In both the first-best and second-best equilibria, the aggregate systematic risk $\tilde{\gamma}$ is optimally shared by investors and managers according to their relative risk tolerance, regardless of their financing decisions and employment status. In this sense, moral hazard distorts the sharing of idiosyncratic risk but does not affect the sharing of systematic risk. Table 1 summarizes this equilibrium risk allocation.

<table>
<thead>
<tr>
<th>Table 1: The Equilibrium Risk Allocation in the Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endowment/Wage</strong></td>
</tr>
<tr>
<td>Investor $j$</td>
</tr>
<tr>
<td>Manager $j$</td>
</tr>
</tbody>
</table>

Part 3 of Proposition 1 shows that the stock price of the unrestricted shares only compensates their holders for systematic risk, just as in the first-best benchmark. This
is because they can diversify their portfolios, as we have seen in Part 2. Thus, moral hazard does not increase the risk premium for the unrestricted shares. However, this does not mean that moral hazard has no impact on the firm’s project selection. Part 4 of Proposition 1 confirms that moral hazard indeed increases the cost of capital through the agency cost. Moreover, $\Pi_j$ is exactly the compensation to the manager for bearing the idiosyncratic risk $\left(B_j^* \alpha_j \tilde{\xi}_j + s_j^* \phi_j \tilde{\xi}_j \right)$, as demonstrated by its calculation in equation 12. Moral hazard leads to the manager’s equilibrium exposure to idiosyncratic risk and thus the cost of moral hazard is equal to the compensation to the manager for bearing the idiosyncratic risk. Therefore, we call $\Pi_j$ the agency cost or the risk premium for idiosyncratic risk interchangeably.

The combination of Part 3 and Part 4 of Proposition 1 reveals the subtlety of the mechanism through which moral hazard affects the cost of capital. In the presence of moral hazard, the firm issues both unrestricted and restricted shares to finance the project. The unrestricted shares pay their holders only the risk premium for systematic risk, while the restricted shares have to compensate their holders for the concentrated (non-diversified) idiosyncratic risk. Thus, the unrestricted shares are less costly than the restricted shares. Moral hazard makes the financing more costly by increasing the use of restricted shares (not by increasing the risk premium for the unrestricted shares).

In sum, relative to the first-best benchmark, the presence of moral hazard does not affect the equilibrium allocation of systematic risk; instead, it distorts the sharing of idiosyncratic risk and leads to the manager’s exposure to idiosyncratic risk. Moral hazard increases the cost of capital because the manager has to be compensated for bearing the idiosyncratic risk.
The Firm-level Information Quality and Cost of Capital

In this section, we consider the effect of a firm-level improvement in the quality of idiosyncratic accounting information on the firm’s cost of capital. We examine a change in $\alpha_j$ while holding all other firms’ information quality constant (and recall that we hold $\bar{\gamma}$ constant until Section 5). An individual firm could affect its firm-level quality of accounting information. For example, it can choose more informative accounting methods, use more precise accounting estimates, commit to more forthcoming disclosure or guidance, and bond themselves to regimes with more stringent requirements or stronger enforcement.

**Proposition 2** An improvement in the firm-level information quality reduces the firm’s cost of capital. It does not affect its risk premium for systematic risk but reduces its risk premium for idiosyncratic risk. That is, $\frac{\partial \Delta_j}{\partial \alpha_j} = \frac{\partial \Pi_j}{\partial \alpha_j} \leq 0$ and $\frac{\partial \Gamma_j}{\partial \alpha_j} = 0$.

The proof is straightforward from the expression of the cost of capital $\Delta_j$ in equation 11. The cost of capital consists of the risk premium for systematic risk $\Gamma_j$ and the agency cost $\Pi_j$ (which is equal to the risk premium for the idiosyncratic risk). An improvement in idiosyncratic accounting information quality mitigates the agency problem and thus lowers the component of the cost of capital that compensates the manager for idiosyncratic risk, i.e., $\Pi_j$.

To better understand the intuition, we consider the effect of information quality on the optimal compensation contract. The key is to observe that as accounting information becomes more precise, the optimal compensation contract tilts more towards the accounting-based bonus and less towards restricted shares. The incentive compatibility condition (derived in equation 32 in the Appendix) requires that $(B_j^* + s_j^*) F = c$. When the manager chooses to work at cost $c$, the firm’s cash flow.
increases by $F$ and her compensation increases by $(B_j^* + s_j^*)F$ from the effective accounting-based bonus and $s_j^*F$ from the restricted shares. In other words, the accounting-based bonus and restricted shares are substitutes as incentive instruments. Moreover, both instruments are costly. Recall the expression of the agency cost in equation 12: \( \Pi_j = \frac{1}{2r_A} Var \left[ B_j^* \alpha_j \tilde{z}_j + s_j^* \phi_j \xi_j \right] \). As idiosyncratic accounting information quality improves (a smaller \( \alpha_j \)), it becomes less costly to use the accounting-based bonus as incentive provisions. This reduces the agency cost in two ways. First, fixing the weights on the two instruments \( (B_j^*, s_j^*) \), the agency cost is lower. Second, the optimal compensation contract shifts weight from restricted shares to accounting-based bonus, pushing the overall cost of incentive provisions further lower.

In contrast, since the agency problem does not affect the risk premium for systematic risk in the first place, an improvement in idiosyncratic accounting information does not affect the risk premium the firm pays for its systematic risk.

Proposition 2 provides one theoretical explanation for the mixed empirical evidence on the relation between information quality and cost of capital we have discussed in the Introduction; it also offers one remedy. The empirical literature uses the implied cost of capital as a proxy for the cost of capital. The implied cost of capital corresponds to \( \Gamma_j \) in our model and thus systematically underestimates a firm’s real cost of capital by the agency cost \( \Pi_j \). Because idiosyncratic accounting information only reduces \( \Pi_j \), failing to account for the agency cost component in the firm’s cost of capital implies that improvements in information quality may appear to yield no benefit. Moreover, our exact characterization of the bias also offers empirical solution to measure the bias. Even though the bias \( \Pi_j \) is not directly observable, we can characterize it as a function of observables (see the derivation in equation 36 in Appendix A):

\[
\Pi_j = \frac{c}{F} \frac{s_j^* \phi_j^2}{2r_A}.
\]
Corollary 1 The implied cost of capital from unrestricted shares \((\Gamma_j)\), biases downward the true cost of capital the firm actually pays to finance a project \((\Delta_j)\). The magnitude of the bias is \(\Pi_j\). It increases in the size of the restricted shares \(s^*\), the idiosyncratic cash flow risk \(\phi_j\), and decreases in the manager’s risk tolerance \(r_A\).

5 The Economy-level Information Quality and Cost of Capital

Another salient feature of accounting information is that its quality is affected by both firm choices and by the economy-level factors. Accounting standards, legal and regulatory enforcement and social norms can affect the quality of many firms’ idiosyncratic information quality, albeit of differing magnitude. To capture this feature, we posit the following structure for accounting measurement error \(\alpha_j\):

\[
\alpha_j \equiv \lambda_j \alpha + \delta_j. \tag{15}
\]

\(\alpha\) captures the economy-level factors that affect the measurement errors in the accounting signal. The non-negative parameter \(\lambda_j\) measures a firm’s exposure to \(\alpha\). Cross-sectional differences in \(\lambda_j\) reflect the notion that the economy-level factors could have differential effects on individual firms’ information quality. \(\delta_j\) is the firm-level component of information quality we have studied in the previous section. Note that the firm’s accounting information, \(\tilde{Y}_j \equiv eF + \alpha_j \tilde{\varepsilon}_j\), remains idiosyncratic through \(\alpha_j \tilde{\varepsilon}_j = (\lambda_j \alpha + \delta_j) \tilde{\varepsilon}_j\), even though its quality shares a common source with other firms through \(\alpha\).

Our next goal is to consider the effect of the economy-level information quality on the cost of capital. To do so, we endogenize firms’ decision to finance its project, \(H_j\). Each firm (investor) decides to finance its project, i.e., \(H^*_j = 1\), if and only if its cost of
capital $\Delta_j$ is below the hurdle rate $NPV^{NoRisk}$. When the information quality changes in the firm-level, it does not change the economy's aggregate systematic risk $\bar{\gamma}$ due to the atomistic nature of each individual firm, as we have studied in the previous section. However, when the information quality changes in the economy-level, it simultaneously affects many firms’ financing decisions and thus can change the aggregate systematic risk in the economy $\bar{\gamma}$. In equilibrium, firms’ financing decisions and the aggregate supply of risky asset are jointly determined.

We first characterize the case when all existing firms continue to exist as the information quality improves in the economy-level.

**Proposition 3** As the information quality improves in the economy-level ($\alpha$ becomes smaller), for an existing firm:

1. its risk premium for idiosyncratic risk decreases ($\frac{\partial j}{\partial \alpha} \geq 0$);
2. its risk premium for systematic risk increases ($\frac{\partial r_j}{\partial \alpha} \leq 0$); and
3. the cost of capital could either increase or decrease.

Part 1 of Proposition 3 is straightforward from Proposition 2 because an improvement in the economy-level information quality leads to an improvement in the firm-level information, i.e., $\frac{\partial j}{\partial \alpha} = \lambda_j \geq 0$. This reduces the agency cost, as we have discussed in Proposition 2.

Part 2 of Proposition 3 is novel and somewhat surprising. As the quality of idiosyncratic accounting information improves in the economy-level, the risk premium for an existing firm’s systematic risk increases. The intuition for the result is as follows. By Part 1 of Proposition 3, an improvement in the economy-level information quality reduces the agency cost. All else equal (holding aggregate risk $\bar{\gamma}$ constant), the lower agency cost leads to the lower cost of capital and makes more projects viable. But as more projects become viable, the aggregate systematic risk in the economy grows. This
increases the demand for risk-bearing capacity. Since the latter is constant, firms have to raise the risk premium to attract investors to provide their risk-bearing capacity. By endogenizing firms’ project financing decisions and the aggregate supply of risky assets, we have established a connection between risk premium for systematic risk and the quality of idiosyncratic information.

To make the arguments more rigorous, consider the effects of a change in information quality $\alpha$ on the cost of capital $\Delta_j$:

$$\frac{\partial \Delta_j}{\partial \alpha} = \frac{\gamma_j}{r_I + r_A} \frac{\partial \gamma_j}{\partial \alpha} + \frac{\partial \Pi_j}{\partial \alpha} \frac{\partial \alpha_j}{\partial \alpha}.$$  

(16)

There are two effects. First, without endogenizing the financing decisions, an improvement in the quality of idiosyncratic information both in the firm-level and in the economy-level unambiguously reduces the cost of capital, as suggested by Proposition 2 and Part 1 of Proposition 3. Mathematically, holding the aggregate systematic risk constant ($\frac{\partial \gamma_j}{\partial \alpha} = 0$), we have $\frac{\partial \Delta_j}{\partial \alpha} = \frac{\partial \Pi_j}{\partial \alpha} \frac{\partial \alpha_j}{\partial \alpha} \geq 0$. We label this channel the direct effect. Second, an improvement in the economy-level information quality endogenizes firms’ financing decisions, affects the aggregate supply of risky assets ($\frac{\partial \gamma_j}{\partial \alpha} \neq 0$), and thus affects the risk premium for systematic risk. This is captured by the first term in equation 16 and we can further prove that $\frac{\partial \Delta_j}{\partial \alpha} \leq 0$. Suppose $\frac{\partial \gamma_j}{\partial \alpha} > 0$. Then $\frac{\partial \Delta_j}{\partial \alpha} > 0$. As $\alpha$ decreases, the cost of capital drops for all firms and the set of viable firms is larger. This implies that the aggregate systematic risk $\bar{\gamma} = \int \gamma_j H_j^* dj$ has to increase (i.e., $\frac{\partial \gamma_j}{\partial \alpha} < 0$), resulting in a contradiction. Therefore, an improvement in the economy-level information quality has to increase the risk premium for systematic risk. We label this channel as the indirect effect. This proves part 2 of Proposition 3.

The indirect effect, resulting from firms’ endogenous financing decisions, is an externality the influx of new projects impose on the existing firms. As the economy’s risk-bearing capacity is scarce, firms have to pay a risk premium for the systematic risk.
associated with their projects. As the quality of idiosyncratic information improves, firms respond endogenously by increasing their investment. The new projects bring in more systematic risk into the economy, increase the demand for risk-bearing capacity, and thus push up the risk premium for systematic risk. As a result, the existing firms have to pay a higher risk premium for systematic risk as well.

Part 3 of Proposition 3 states that the overall effect of a change in economy-level information quality on the cost of capital for an existing firm is the sum of the direct and indirect effects. An improvement directly reduces the agency cost but indirectly pushes up the risk premium for systematic risk. Intuitively, a firm with higher exposure to idiosyncratic risk benefits more from the positive direct effect, and a firm with higher exposure to systematic risk is affected more by the negative indirect effect. As a result, an existing firm could be worse off overall if it has high exposure to systematic risk and/or low exposure to idiosyncratic risk. To the extent that firms’ risk profiles, i.e., \( \{ \gamma_j, \lambda_j, \phi_j, \alpha_j \} \), differ cross-sectionally, the effects of a change in economy-level information quality will have differential effects on firms’ cost of capital.

Proposition 3 examines only the case for existing firms and shows that the cost of capital for some existing firms can increase as a result of higher economy-level information quality. One of its natural implication is that the economy-level information quality generates a selection effect. In particular, as \( \alpha \) becomes smaller, some firms that were viable may be no longer viable. When these firms exit from the economy, the aggregate supply of risky assets can shrink, which in turn affects the cost of capital of all other firms. This selection effect contrasts with some standard results in the agency literature, as shown in the following results.

**Proposition 4** As the economy-level information quality improves, the sets of viable projects does not necessarily expand. Taking the improvement to the extreme, the set of viable projects in the first-best benchmark may not subsume its counterpart in the second-best equilibrium.
In a standard principal-agent setting, the set of viable projects grows as moral hazard decreases. This reflects the strict gain in efficiency through the reduction in agency cost. In contrast, this intuition does not hold any longer with the endogenous project selection in our model, as demonstrated in Proposition 4. The set of projects that are financed in equilibrium does not necessarily expand as moral hazard becomes less severe in our model. In other words, the indirect effect could be strong enough to dominate the direct effect. As a result, an equilibrium with higher information quality does not Pareto dominate an equilibrium with lower information quality.

Both Proposition 3 and Proposition 4 demonstrate possibility results. The general characterization of the cost of capital and of the equilibrium set of viable projects is difficult in our model. Each firm’s risk profile has four dimensions \( \{\lambda_j, \delta_j, \gamma_j, \phi_j\} \) and it is difficult to solve the fixed-point problem between firms’ financing decisions and the aggregate supply of risky assets. To illustrate both Proposition 3 and Proposition 4, we now impose more structure to make the explicit characterization tractable.

Consider an economy consisting of two types of firms, \( j \in \{1, 2\} \), with equal mass \( \frac{1}{2} \). Both types of firms have the same idiosyncratic cash flow risk \( (\phi_1 = \phi_2 = \phi_0 > 0) \). Type-1 firms have smaller exposure to systematic risk but larger exposure to the economy-level information quality, i.e., \( \gamma_1 < \gamma_2 \) and \( \lambda_1 > \lambda_2 \). We normalize \( \lambda_2 = 0 \) and \( \delta_1 = \delta_2 = 0 \). With this structure, we simplify the model from four-dimensions to two-dimensions.

We focus on the interesting case when the hurdle rate \( NPV^{NoRisk} \) is intermediate: \(^7\)

\[
\frac{(\gamma_1 + \gamma_2) \gamma_2}{2 \left( r_A + r_I \right)} > NPV^{NoRisk} > \frac{\gamma_2^2}{2 \left( r_A + r_I \right)}. \tag{17}
\]

Denote the viable fractions of type-1 and type-2 firms as \( \kappa_1 \) and \( \kappa_2 \), respectively. It

\(^7\)If \( NPV^{NoRisk} > \frac{(\gamma_1 + \gamma_2) \gamma_2}{2 \left( r_A + r_I \right)} \), then all firms are viable in the absence of moral hazard. If \( NPV^{NoRisk} < \frac{\gamma_2^2}{2 \left( r_A + r_I \right)} \), then no firms are viable when moral hazard is sufficiently severe.
is useful to explicitly write out the aggregate systematic risk in the economy:

$$\bar{\gamma} (\kappa_1, \kappa_2; \alpha) = \frac{\gamma_1 \kappa_1 + \gamma_2 \kappa_2}{2}.$$ 

$\bar{\gamma}$ is also a function of $\alpha$ because the equilibrium set of viable projects $(\kappa_1^*, \kappa_2^*)$ are functions of information quality $\alpha$.

The close-form solution of the cost of capital and its components $(\Delta_j, \Gamma_j, \Pi_j)$, the set of viable projects $(\kappa_1^*, \kappa_2^*)$, and their comparative statics with respect to $\alpha$ can be characterized. The following corollary presents the main results we would like to highlight. The thresholds $\alpha$ and $\tilde{\alpha}$ and constant $\kappa \in (0, 1)$ are defined in the Appendix.

**Corollary 2** As the economy-level information quality improves (as $\alpha$ decreases), the equilibrium set of viable projects $(\kappa_1^*, \kappa_2^*)$ and the cost of capital change as follows:

**Table 2: $\alpha$ and the Cost of Capital**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\kappa_1^*$</th>
<th>$\kappa_2^*$</th>
<th>$\frac{\partial \gamma}{\partial \alpha}$</th>
<th>$\frac{\partial \Delta_1}{\partial \alpha}$</th>
<th>$\frac{\partial \Delta_2}{\partial \alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0, \alpha]$</td>
<td>1</td>
<td>$\kappa$</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$[\alpha, \tilde{\alpha})$</td>
<td>$\frac{\partial \kappa_1^*}{\partial \alpha} &lt; 0$</td>
<td>1</td>
<td>$-$</td>
<td>0</td>
<td>$-$</td>
</tr>
<tr>
<td>$[\tilde{\alpha}, \infty)$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

The key to the proof of Corollary 2 is to keep track of the evolution of the aggregate risk $\bar{\gamma}$ as $\alpha$ changes. We relegate the details to the proof in the Appendix. Here we explain it in the context of illustrating both Proposition 3 and Proposition 4.

We first illustrate Proposition 3 with three possibilities about the effects of $\alpha$ on the cost of capital. The most surprising possibility is that $\frac{\partial \Delta_2}{\partial \alpha} < 0$: higher information quality increases the cost of capital of type-2 firms. This occurs when $\alpha \in [\alpha, \tilde{\alpha})$. As $\alpha$ decreases, more type-1 firms become viable (i.e., $\frac{\partial \kappa_1^*}{\partial \alpha} < 0$) due to their larger exposure to idiosyncratic risk. Their influx to the capital market increases the aggregate systematic risk $\bar{\gamma}$ (i.e., $\frac{\partial \bar{\gamma}}{\partial \alpha} < 0$). Since type-2 firms have no idiosyncratic risk ($\lambda_2 = 0$), they do not directly benefit from the improvement ($\frac{\partial \Pi_2}{\partial \alpha} = 0$). Nonetheless, they have
to pay a higher premium for systematic risk and thus their cost of capital has increased 
\[
(\frac{\partial \Delta_2}{\partial \alpha} = \frac{\partial \gamma_2}{\partial \alpha} < 0).
\]

In contrast, in the same regime of \(\alpha \in [\underline{\alpha}, \bar{\alpha}]\), the same improvement does not affect the cost of capital of type-1 firms. On the one hand, a lower \(\alpha\) reduces the agency cost of a type-1 firm. On the other hand, a lower \(\alpha\) also leads to a larger set of viable type-1 firms \(\left(\frac{\partial \gamma_1}{\partial \alpha} < 0\right)\), which in turn increases the aggregate supply of risky assets \(\left(\frac{\partial \gamma}{\partial \alpha} < 0\right)\). As a result, both the direct and indirect effects are present. They perfectly offset each other, resulting in no net effect on the cost of capital.

Finally, it is straightforward to identify cases in which the improvement in the economy-level information quality reduces the cost of capital. For example, this occurs for type-1 firms when \(\alpha \in [0, \alpha)\), i.e., \(\frac{\partial \Delta_1}{\partial \alpha} > 0\). In this region, all type-1 firms are viable and a fixed fraction of type-2 firms are viable. As a result, the aggregate supply of risky assets \(\bar{\gamma}\) is stuck at the corner and not sensitive to a marginal change in \(\alpha\). Hence, the indirect effect is absent. A decrease in \(\alpha\) then strictly reduce the cost of capital of type-1 firms.

We can also illustrate Proposition 4 with Table 2. As \(\alpha\) decreases, the agency problem becomes less severe. As a result, more type-1 firms become viable but (weakly) fewer type-2 firms stay viable. The set of viable projects does not unambiguously expand. In particular, all type-2 firms are viable in the second-best case when \(\alpha > \underline{\alpha}\), but some of them are not viable any longer in the first-best benchmark when \(\alpha = 0\). As moral hazard becomes less severe, firms’ viability may be undermined.

6 The Conclusion

This study aims at understanding the relation between idiosyncratic information quality and the cost of capital in the presence of moral hazard. We integrate a moral hazard problem into a multi-firms economy with endogenous project financing decisions. De-
spite the forces of diversification, moral hazard distorts the equilibrium sharing of idiosyncratic risk by exposing the manager to idiosyncratic risk but does not affect the allocation of systematic risk in the economy. As a result, the cost of capital includes both risk premium for systematic risk and the agency cost. The latter manifests as a risk premium for the idiosyncratic risk imposed on the manager. Armed with this insight, we make two points. First, an improvement in the firm-level idiosyncratic information quality reduces the agency cost component of the cost of capital but does not affect the risk premium for systematic risk. Second, an improvement in the economy-level information quality reduces the agency cost but can increase the risk premium for systematic risk. Its net effect on the firm’s cost of capital depends on the firm’s exposure to both systematic and idiosyncratic risk.

Our main results provide theoretical explanations for the mixed empirical evidence on the relation between information quality and cost of capital. First, the implied cost of capital, a popular proxy used in the empirical literature, corresponds to the risk premium for systematic risk in the model and thus is a biased proxy for a firm’s true cost of capital. A firm-level improvement in information quality does not affect the implied cost of capital.\textsuperscript{8} To fully capture the effect of such an improvement, our model predicts a reduction in the size of restricted shares a firm awards to the management as incentive provisions. Second, an improvement in the economy-level idiosyncratic information quality can either increase or decrease the cost of capital. Moreover, it reduces the size of restricted shares but can increase the implied cost of capital.

Our model also highlights the qualitative differences in the economic consequences of information quality in the firm-level versus in the economy-level. The latter generates a general equilibrium effect (indirect effect) that could differ qualitatively from the

\textsuperscript{8}This prediction is consistent with the survey results in Graham and Harvey (2001). They find that private firms and small public firms with higher managerial ownership are less likely to use systematic risk to estimate their cost of equity capital than their large, public counterparts. They attribute the difference to implementation problems for private and small firms. An alternative explanation from our analysis is that systematic risk is less representative of the cost of equity capital for firms whose fraction of unrestricted shares is smaller.
direct effect. When we examine the economic consequences of accounting standards or regulations (such as Sarbanes-Oxley Act, IFRS, and Regulation FD), the economic insights derived from a single-firm setting can lead to incorrect inferences.

The paper has made some critical assumptions to demonstrate the main results. First and foremost, the paper has adopted the linear–exponential–normal (LEN) formulation for the agency problem. An immediate justification for this choice is its tractability. The original formulation in Holmstrom (1979) cannot be pushed very far to study more complicated and interesting agency issues. Our main research question entails a model with both an agency component and a multi-firms asset pricing component. The LEN formulation generates closed-form solutions to both components and makes it possible to answer the research question. A more theoretical justification for the adoption of the LEN formulation comes from Holmstrom and Milgrom (1987). By allowing the agent to choose efforts after observing previous outcomes continuously, they show that the optimal contract to this relaxed agency problem is equivalent to one in which the agent selects a single-effort and the principal restricts himself to a linear contract. It has been controversial to extend their approach to settings with multi-tasks, multi-periods, and/or correlated multiple performance measures (e.g., Lambert (2001), Hemmer (2004), Christensen and Feltham (2005) and Christensen, Sabac, and Tian (2010)). However, these concerns do not seem to directly apply to our model, because we have only a single-action (single-period) and conditionally independent performance measures, a setting close to the original setting in Holmstrom and Milgrom (1987). Nonetheless, our overall model with the multi-firms asset pricing component departs significantly from Holmstrom and Milgrom (1987) and we are unable to derive the linear contract as optimal in our setting. As a result, it is important to highlight the potential impact of the linearity assumption on our main results. Had we relaxed the linearity assumption, ceteris paribus, the Mirrlees result (Mirrlees (1974)) that the agency cost approaches 0 asymptotically would apply given the normality assumption.
However, the insight that information quality reduces the agency cost seems more general than the LEN formulation. It can be proved, for example, by invoking the results (Proposition 1) from Kim (1995), since our information structure satisfies his MPS (mean-preserving-spread) criterion. Moreover, there seem no obvious reasons why our second main result would be qualitatively affected by the relaxation of the linearity assumption.

Second, the CARA utility function is also critical not only for the compensation contract but also for the pricing of risk. This assumption assumes away the wealth effect and leads to the constant risk-taking capacity in the economy. While it makes the intuition of our second main result transparent, one might argue that it is not fully realistic. What would happen if we relax this assumption? While we are unable to solve the model, one may conjecture that an improvement in the economy-level information quality would also change the aggregate risk-bearing capacity in the economy, which in turn affects the risk premium for systematic risk. That is, $\frac{\partial (r_A + r_I)}{\partial \alpha} \neq 0$. As a result, the net effect of $\alpha$ on the risk premium for systematic risk would depend on the trade-off of these two channels: $\frac{\gamma_j}{r_I + r_A} \frac{\partial \bar{r}}{\partial \alpha} + \gamma_j \bar{\gamma} \frac{\partial}{\partial \alpha} \left( \frac{1}{r_I + r_A} \right)$. What is necessary for the indirect effect to arise is that the effect of $\alpha$ on risk tolerance does not perfectly offsets its effect on the aggregate risk $\bar{\gamma}$. In that sense, the indirect effect is not solely driven by the CARA utility function.

Finally, given the LEN formulation, the linear contract with accounting-based bonus and restricted shares are optimal in our model when the manager is allowed to choose her own portfolio. There are three independent sources of uncertainty in the model: the measurement error, the idiosyncratic cash flow risk, and the systematic cash flow risk. The manager’s portfolio choice allows her to choose exposure to the systematic cash flow risk, and the compensation contract with an accounting-based bonus and restricted shares expose her to the accounting measurement error and the idiosyncratic cash flow risk. In other words, the three instruments, offered by the compensation
contract and the trading option, span the entire space of uncertainty.

Appendix A: the proofs

Proof. of Proposition 1 and Lemma 1

We first prove Proposition 1 and Lemma 1 is then proved as a special case of Proposition 1.

We characterize our economy by the exogenous parameters \{\phi_j, \gamma_j, \alpha_j\}_{j \in [0,1]}, c, CE^0, r_A, r_I, and an equilibrium to our economy by the endogenous variables \{H_j^*, a_j^*, b_j^*, s_j^*, x_j^*, z_j^*, p_j\}_{j \in [0,1]}.

We omitted the star for \(p_j\) for convenience as no confusion arises. In using this definition of equilibrium, we have employed the standard approach of “conjecture-and-verify.” We have conjectured that the following results are part of the equilibrium. We will solve for the equilibrium and come back to verify that these results indeed hold in equilibrium.

Result 1: the optimal contract requires that the manager does not short her own firm’s stock.

Result 2: investors and managers do not hold any individual firms’ shares above and beyond their composition in the market portfolio (and the employed manager’s restricted shares from her compensation contract) in equilibrium.

The solution concept is Nash equilibrium. We solve the equilibrium in four steps. First, we solve the capital market equilibrium \{x_j^*, z_j^*, p_j\}, holding constant the financing and compensation decisions \{H_j^*, a_j^*, b_j^*, s_j^*\}. Second, we then take the capital market equilibrium \{x_j^*, z_j^*, p_j\} and the optimal financing decision \(H_j^*\) as given and solve the optimal compensation contract within each firm \{a_j^*, b_j^*, s_j^*\}. Third, we solve for the optimal financing decision \(H_j^*\). Finally, we verify the two results above indeed hold in equilibrium.

Step 1: fixing the financing decisions and the compensation contracts, the (tradable) market portfolio consists of all unrestricted shares as defined in equation 4. Some algebra together with the definition of idiosyncratic risk yields the following expressions:

\[
\widetilde{M} = \int (1 - s_j^*)\bar{G}_j^*H_j^*dj \tag{18}
\]

\[
\sigma_M = \int (1 - s_j^*)\gamma_jH_j^*dj. \tag{19}
\]

\(\bar{G}_j^*\) is the expected net cash flow from a viable firm in equilibrium:

\[
\bar{G}_j^* = \bar{G}_j - b_j^*\alpha_j\bar{\zeta}_j + \phi_j\bar{\xi}_j + \gamma_j\bar{\eta}, \tag{20}
\]

\[
\bar{G}_j^* = F(1 - b_j^*) - a_j^*. \tag{20}
\]

The investor’s wealth and certainty equivalent are given in equation 5 and 7 in the text, respectively. Substituting \(\widetilde{M}\), we can rewrite his certainty equivalent as

\[
V_j^I = (1 - s_j) p_j H_j + x_j(\widetilde{M} - p_M) - \frac{1}{2r_I} x_j^2 \sigma_M^2. \tag{21}
\]
Here we use the variables without “*” as they are choices we will study. We follow this convention whenever no confusion arises.

The manager’s wealth and certainty equivalent are given in equation 6 and 8 in the text, respectively. To rewrite her certainty equivalent, we first rewrite wage \( \tilde{w}_j(e_j) \) in equation 1 as a function of the risk sources:

\[
\tilde{w}_j = a_j + b_j \tilde{Y}_j + s_j \tilde{G}_j = \tilde{w}_j^A(e_j) + B_j \alpha_j \tilde{\xi}_j + s_j \phi_j \tilde{\xi}_j + s_j \gamma_j \tilde{\eta}_j.
\]

\( B_j \equiv (1 - s_j) b_j \) is the normalized (effective) coefficient of the accounting-based bonus. When the firm pays the manager $1 bonus, the net pay to the manager is \( (1 - s_j) \) since the manager owns \( s_j \) fraction of the firm’s shares. \( \tilde{w}_j^A(e_j) \) is the manager’s expected wage, which depends on her effort choice \( e_j \):

\[
\tilde{w}_j(e_j) = e_j (s_j + B_j) F + (1 - s_j) a_j. \tag{22}
\]

Moreover, the compensation contract exposes her to all three sources of risk, the idiosyncratic measurement error \( \tilde{\xi}_j \), the idiosyncratic cash flow risk \( \tilde{\xi}_j \), and the systematic risk \( \gamma_j \tilde{\eta}_j \). The exposures are measured by \( B_j, s_j \) and \( s_j \), respectively. Her terminal wealth is thus

\[
\tilde{W}_j^A = \tilde{w}_j H_j + z_j (\tilde{M} - p_M) = \tilde{w}_j^A H_j + z_j (\tilde{M} - p_M) + \left( B_j \alpha_j \tilde{\xi}_j + s_j \phi_j \tilde{\xi}_j \right) H_j + \left( s_j \gamma_j H_j + z_j \sigma_M \right) \tilde{\eta}_j. \tag{23}
\]

Thus, her certainty equivalent can be rewritten as

\[
V_j^A(H_j, e_j, z_j) = \tilde{w}_j^A H_j + z_j (\tilde{M} - p_M)
- \frac{1}{2r_A} \left( H_j^2 \text{Var}[B_j \alpha_j \tilde{\xi}_j + s_j \phi_j \tilde{\xi}_j] + (s_j \gamma_j H_j + z_j \sigma_M)^2 \right). \tag{24}
\]

We can now solve for the portfolio decisions by the investor and the manager. The investor’s optimal portfolio choice \( x_j^* \) is characterized by the first-order condition (differentiating equation 21 with respect to \( x_j \)) evaluated at the equilibrium contracts and financing decisions:

\[
\frac{\partial V_j^I}{\partial x_j} \bigg|_{x_j = x_j^*} = (\tilde{M} - p_M) - \frac{1}{r_I} x_j^* \sigma_M^2 = 0. \tag{25}
\]

Similarly, the manager’s optimal portfolio choice \( z_j^* \) is characterized by the first-order condition (differentiating equation 24 with respect to \( z_j \)) evaluated at the equi-
librium contracts and financing decisions:

\[ \frac{\partial V^A_j}{\partial z_j} |_{z_j = z_j^*} = (M - p_M) - \frac{1}{r_A} s_j^* \gamma_j H_j^* \sigma_M - \frac{1}{r_A} z_j^* \sigma_M^2 = 0. \]  

(26)

Combining the two first-order conditions with the market clearing condition in equation 9, we can solve \( p_M \) as

\[ p_M = \bar{M} - \frac{\bar{\gamma}}{r_I + r_A} \sigma_M. \]  

(27)

By the definition of the market portfolio in equation 4, an individual firm’s price can be written as

\[ p_j = \bar{G}_j - \frac{\bar{\gamma}}{r_I + r_A} \gamma_j. \]  

(28)

Substituting \( p_M \) back to the investor and manager’s respective first-order condition (equation 25 and 26), we can solve \( x_j^* \) and \( z_j^* \) as

\[ x_j^* = \frac{r_I (M - p_M)}{\sigma_M^2} = \frac{r_I}{r_I + r_A} \frac{\bar{\gamma}}{\sigma_M}, \]  

(29)

\[ z_j^*(H_j^*) = \frac{r_A (M - p_M)}{\sigma_M^2} - \frac{s_j^* \gamma_j}{\sigma_M} H_j^* = \frac{r_A}{r_I + r_A} \frac{\bar{\gamma}}{\sigma_M} - \frac{s_j^* \gamma_j}{\sigma_M} H_j^*. \]  

(30)

Step 2: Given the functional form of \( p_j \) and \( p_M \), we solve for the optimal compensation contract. From equation 24, we can obtain the expressions for the manager’s certainty equivalent in various settings, including \( V^A_j(1, 1, z_j^*(1)) \), \( V^A_j(0, e_j, z_j^*(0)) \) and \( V^A_j(0, e_j, z_j^*(0)) \). Substituting these expressions into the IR condition (expression IR) and using the argument that the IR condition binds in equilibrium, we can determine the base salary \( a_j^* \) as follows:

\[ - (1 - s_j) a_j^* = (s_j + B_j) F - s_j \Gamma_j - \frac{1}{2r_A} \text{Var}[B_j \alpha_j \tilde{\xi}_j + s_j \phi_j \tilde{\xi}_j] - c - CE^0. \]  

(31)

We have used the notation \( \Gamma_j = \frac{\bar{\gamma} \gamma_j}{r_I + r_A} \) here.

Similarly, we can do the substitution and simply the manager’s IC condition (expression IC) as

\[ (s_j + B_j) F - c \geq 0. \]  

(32)

Having simplified the IR and IC conditions, we are ready to solve the representative investor’s optimal compensation decisions defined in Program 1 under the condition that he decides to finance the project: \( H_j^* = 1 \). We can write out the investor’s certainty equivalent by substituting into equation 21 \( x_j^* \) from equation 29, \( p_j \) from equation 28 and \( a_j^* \) from equation 31. Denoting \( \mu \) as the Lagrangian multiplier for the IC condition.
and making a change of variable $B_j = (1 - s_j)b_j$, the Lagrangian function of investor $j$’s optimization problem is

$$L(B_j, s_j, \mu) = (1 - s_j)p_j + x_j^*(\bar{M} - p_M) - \frac{1}{2r_I}x_j^*\sigma^2_M + \mu((s_j + B_j)F - c)$$

$$= -\frac{\text{Var}[B_j\phi_j^* + s_j\phi_j^*]}{2r_A} + \mu((s_j + B_j)F - c) + K$$

$K$ is a constant:

$$K \equiv F - c - CE^0 - \Gamma_j + \frac{r_I}{2} \left(\frac{\bar{\gamma}}{r_I + r_A}\right)^2.$$ 

Taking the first-order conditions with respect to $\{B_j, s_j, \mu\}$:

$$\frac{\partial L}{\partial B_j} = -\frac{1}{r_A}B_j^* + \mu F = 0;$$

$$\frac{\partial L}{\partial s_j} = -\frac{1}{r_A}s_j^*\phi^2 + \mu F = 0;$$

$$\frac{\partial L}{\partial \mu} = (s_j^* + B_j^*)F - c = 0.$$

Solving for the equations yields the solutions:

$$s_j^* = \frac{c\phi_j^2}{F(\alpha_j^2 + \phi_j^2)} \quad (33)$$

$$B_j^* = \frac{c\phi_j^2}{F(\alpha_j^2 + \phi_j^2)} \quad (34)$$

$$\mu = \frac{B_j^*\alpha_j^2}{r_A F} = \frac{s_j^*\phi_j^2}{r_A F} = \frac{c\phi_j^2\alpha_j^2}{r_A F^2(\alpha_j^2 + \phi_j^2)}. \quad (35)$$

We can also recover

$$b_j^* = \frac{B_j^*}{1 - s_j^*}.$$
We define the agency cost $\Pi_j$:

\[
\Pi_j = \frac{1}{2r_A} \text{Var} [B_j^* \alpha_j \xi_j + s_j^* \phi_j \xi_j] \\
= \frac{1}{2r_A} \frac{c^2 \phi_j^2 \alpha_j^2}{2} \\
= \frac{c}{2F} \frac{s_j^* \phi_j^2}{r_A}.
\]  

(36)

Substituting the optimal contract variables into price $p_j$, we have

\[
p_j = \bar{G}_j^* - \Gamma_j, \\
\bar{G}_j^* = F(1 - b_j^*) - a_j^* \\
= \frac{1}{1 - s_j}(F - s_j^* \Gamma_j - \Pi_j - c - CE^0).
\]

Step 3: Now we calculate the project’s NPV and solve for the financing decision $H_j^*$. In the absence of risk ($\alpha_j = \phi_j = \lambda_j = 0$), the project’s NPV is

\[
NPV^{NoRisk} = NPV_j|_{\alpha_j=\phi_j=\lambda_j=0} = F - (CE^0 + c).
\]

Using equation 21 and substituting in the equilibrium variables, we have

\[
NPV_j = V_j (H_j^* = 1) - V_j (H_j^* = 0) \\
= (1 - s_j^*)p_j \\
= F - \Gamma_j - \Pi_j - c - CE^0 \\
= NPV^{NoRisk} - \Gamma_j - \Pi_j.
\]

Therefore, the project’s cost of capital is

\[
\Delta_j = NPV^{NoRisk} - NPV_j = \Gamma_j + \Pi_j.
\]

The investor chooses $H_j^* = 1$ if and only if $NPV_j > 0$, which is equivalent to $\Delta_j < NPV^{NoRisk}$.

We can also prove Lemma 1 as a special case. In the absence of the agency problem, that is, if the manager’s effort is contractible, then the IC condition is dropped from the optimization. Following the same procedure, the optimal contract in this first-best
case is $b^F_j = s^F_j = 0$. We then have

\[
\begin{align*}
\tilde{G}^F_j &= F - c - CE^0 = NPV^{NoRisk}, \\
\tilde{p}^F_j &= NPV^{NoRisk} - \Gamma_j.
\end{align*}
\]

Therefore:

\[
\begin{align*}
NPV^F_j &= (1 - s^F_j)\tilde{p}^F_j = NPV^{NoRisk} - \Gamma_j, \\
\Delta^F_j &= \Gamma_j.
\end{align*}
\]

Step 4: to complete the proof, we verify the two conjectures we made at the beginning of this proof.

**Result 1**: the optimal contract requires that the manager does not short her own firm’s stock.

**Result 2**: investors and managers do not hold any individual firms’ shares above and beyond their composition in the market portfolio (and the employed manager’s restricted shares from her compensation contract) in equilibrium.

To verify them, we re-solve the investors and managers’ portfolio decisions by allowing them to choose any individual stocks. Each investor and manager holds $x_j$ and $z_j$ fraction of the market portfolio, respectively. In addition, they now can also choose to hold additional individual stock $i$. Denote investor and manager $j$’s holding of stock $i$ (above and beyond the market portfolio) as $X^i_j$ and $Z^i_j$, respectively. Result 1 is verified by proving $Z^i_j < 0$ for any $j = i$ and $H^*_j = 1$. That the employed manager would like to sell the incentive shares in absence of the restriction implies that the restriction binds in equilibrium. Result 2 is verified by proving that $X^i_j = 0$ for any $i$ and $j$ and that $Z^i_j = 0$ for any $j \neq i$ or $H^*_j = 0$.

We start with Result 1. Denote the employed manager’s demand for her own firm’s stock as $d_j = Z^j_i$ to ease the notation. With the additional holding $d_j$, her wealth becomes

\[
\tilde{w}_j H_j + z_j(\tilde{M} - p_M) + d_j \left(\tilde{G}_j - p_j\right).
\]

Her certainty equivalent in equation 24 needs to be rewritten as

\[
V^A_j = \tilde{w}_j + z_j \Gamma_d + d \Gamma_j - \frac{1}{2r_A} \left( (B^*_j - d_j B^*_j)^2 b_j^2 \alpha_j^2 + (s^*_j + d_j)^2 \phi_j^2 + ((d_j + s^*_j) \gamma_j + z_j \sigma_M)^2 \right),
\]

where we have used $\Gamma \equiv \frac{d^*_j}{r_A + r_I} \sigma_M$ for convenience. In absence of restriction, the manager’s first-order condition for $z_j$ is now

\[
\Gamma - \frac{1}{r_A} \left( (d^*_j + s^*_j) \gamma_j \sigma_M + z_j^* \sigma^2_M \right) = 0.
\]
The first-order condition for \( d_j \) is
\[
\Gamma_j - \frac{1}{r_A} \left( -(B^*_j - d^*_j b^*_j) b^*_j \alpha_j^2 + (s^*_j + d^*_j) \phi_j^2 + (d_j + s^*_j) \gamma_j^2 + z^*_j \sigma_M \gamma_j \right)
\]
\[
= \frac{-(B^*_j - d^*_j b^*_j) b^*_j \alpha_j^2 + (s^*_j + d^*_j) \phi_j^2}{r_A} = 0.
\]

The first equality has used the first-order condition for \( z^*_j \). Using the equilibrium relation between \( B^*_j \) and \( s^*_j \) (that is, \( B^*_j \alpha_j^2 = s^*_j \phi_j^2 \)), we can solve
\[
d^*_j = -s^*_j \left( \frac{1 - b^*_j}{b^*_j} \right) \phi_j^2 < 0.
\]

Therefore, the employed manager’s demand for her own firm’s shares is strictly negative in the absence of the restriction. In other words, the restriction that the employed manager cannot short her own firm’s shares binds in equilibrium.

The intuition is also clear from the first-order condition for \( d_j \): The employed manager is exposed idiosyncratic risk that is costly to her. In the absence of the contractual restriction, she would like to hedge that risk by shorting her own firm’s shares. Hence, the negative demand. Therefore, Result 1 is verified.

Finally, we verify Result 2 by proving that \( X^{si} \) is zero for any \( i \) and \( j \) and that \( Z^{si} \) is 0 for any \( j \neq i \) or \( H^*_j = 0 \). The proof for the investor’s demand \( X^{si} = 0 \) is the same as that for the unemployed manager’s. We thus only prove \( X^{si} = 0 \) to save space.

With the holding of \( (x_j, X^i_j) \), the investor’s certain equivalent in equation 21 needs to be rewritten as follows:
\[
V^I_j = (1 - s^*_j) p_j H^*_j + x_j \Gamma + X^i_j \Gamma_i - \frac{1}{2r_I} \left( X^{i2}_j (b^*_i \alpha_i^2 + \phi_i^2) + (X^i_j \gamma_i + x_j \sigma_M)^2 \right).
\]

The additional holding of \( X^i_j \) earns the investor a premium of \( X^i_j \Gamma_i \) but exposes him to both idiosyncratic risk \( X^{i2}_j (b^*_i \alpha_i^2 + \phi_i^2) \) and systematic risk \( (X^i_j \gamma_i + x_j \sigma_M)^2 \), for which he incurs a disutility. The first-order condition for \( x_j \) is now
\[
\Gamma - \frac{1}{r_I} (X^i_j \gamma_i \sigma_M + x^*_j \sigma^2_M) = 0.
\] (37)

The first-order condition for \( X^i_j \) is
\[
\Gamma_i - \frac{\gamma_i}{r_I} \left( X^i_j \gamma_i + x^*_j \sigma_M \right) - X^i_j \left( b^*_i \alpha_i^2 + \phi_i^2 \right) \left( \frac{1}{r_I} \right) = 0.
\]
After substituting \( x_j^* \) from equation 37, it becomes

\[
-X_j^{i*} \left( \frac{b_i^2 \sigma_i^2 + \phi_i^2}{r_I} \right) = 0.
\]

Therefore,

\[
X_j^{i*} = 0.
\]

Since \( X_j^{i*} = 0 \) for any \( H_j^* \), the investor in equilibrium has no incentive to hold any individual stock (above and beyond the market portfolio), regardless of his financing decision. The reason is also clear from its first-order conditions. The individual investor would demand compensation for the idiosyncratic risk, but the stock price compensates only for the systematic risk. Therefore, investors diversify by holding only the market portfolio. We thus have verified Result 2.

This completes the proof. ■

**Proof.** of Proposition 3, Proposition 4 and Corollary 2:

Part 1 and Part 2 of Proposition 3 are proved in the text. To prove Part 3 of Proposition 3 and Proposition 4, we only need to prove Corollary 2.

The aggregate risk in the economy is a function of the set of viable project \((\kappa_1, \kappa_2)\) and information quality \(\alpha\).

\[
\bar{\gamma} (\kappa_1, \kappa_2; \alpha) = \frac{\gamma_1 \kappa_1 + \gamma_2 \kappa_2}{2}.
\]

\(\alpha\) affects \(\bar{\gamma}\) through its effect on \(\kappa_1\) and \(\kappa_2\). The aggregate risk \(\bar{\gamma}\) has to be borne by the aggregate risk tolerance \(r_A + r_I\). Thus, the risk premium for systematic risk for each firm is

\[
\Gamma_j = \frac{\bar{\gamma} (\kappa_1, \kappa_2; \alpha)}{r_A + r_I} \gamma_j, \quad j \in \{1, 2\}.
\]

All else equal, \(\Gamma_j\) is increasing in both \(\kappa_1\) and \(\kappa_2\): the risk premium for systematic risk is higher as the set of viable projects expands.

The cost of capital of type-1 and type-2 firms are

\[
\Delta_1 (\kappa_1, \kappa_2; \alpha) = \frac{\gamma_1}{r_A + r_I} \bar{\gamma} (\kappa_1, \kappa_2; \alpha) + \Pi_1 (\alpha),
\]

\[
\Delta_2 (\kappa_1, \kappa_2; \alpha) = \frac{\gamma_2}{r_A + r_I} \bar{\gamma} (\kappa_1, \kappa_2; \alpha).
\]

All else equal, both \(\Delta_1\) and \(\Delta_2\) are increasing in either \(\kappa_1\) or \(\kappa_2\). Moreover, all else equal, \(\Delta_1\) is strictly increasing in \(\alpha\). We can take the difference between \(\Delta_1\) and \(\Delta_2\):

\[
\Delta (\kappa_1, \kappa_2; \alpha) \equiv \Delta_1 - \Delta_2 = \left( \frac{\gamma_1 - \gamma_2}{r_A + r_I} \right) \bar{\gamma} (\kappa_1, \kappa_2; \alpha) + \Pi_1 (\alpha).
\]

The first term is negative because \(\gamma_1 - \gamma_2 < 0\). Given that both types of firms are seeking financing in the same market, they face the same aggregate risk \(\bar{\gamma}\). Hence, the risk premium for systematic risk is always higher for type-2 firms than for type-1 firms.
On the other hand, the agency cost is always higher for type-1 firms than for type-2 firms. As a result, the comparison of their cost of capital depends on the comparison of both components.

Now we examine how the firms’ cost of capital varies with $\alpha$. We start from one extreme when $\alpha$ approaches 0. As $\alpha$ tends to 0,

$$\lim_{\alpha \to 0} \Delta (\kappa_1, \kappa_2; \alpha) \rightarrow \frac{\gamma_1 - \gamma_2 \hat{\gamma}}{r_A + r_I} (\kappa_1, \kappa_2; 0) < 0.$$ 

As the cost of capital is strictly smaller for type-1 firms than for type-2 firms, all type-1 firms have to be viable before any type-2 firms become viable. That is, if $\kappa_2 > 0$, then $\kappa_1 = 1$. By the regularity condition (inequality 17), we have

$$\kappa_1^* = 1, \kappa_2^* = \bar{\kappa}.$$ 

$\bar{\kappa} \in (0, 1)$ is constant determined by

$$\frac{\gamma_2}{r_A + r_I} - \frac{\gamma_1}{2} = NPV^{NoRisk}. \quad (38)$$

Accordingly, we have

$$\hat{\gamma} (1, \bar{\kappa}; 0) = \frac{\gamma_1 + \gamma_2 \bar{\kappa}}{2},$$

$$\Delta_1 = \frac{\gamma_1}{r_A + r_I} \left( \frac{\gamma_1 + \gamma_2 \bar{\kappa}}{2} \right) < NPV^{NoRisk},$$

$$\Delta_2 = \frac{\gamma_2}{r_A + r_I} \left( \frac{\gamma_1 + \gamma_2 \bar{\kappa}}{2} \right) = NPV^{NoRisk}.$$

Fixing $\kappa_1$ and $\kappa_2$, we have $\frac{\partial \Delta_1}{\partial \alpha} > 0$ and $\frac{\partial \Delta_2}{\partial \alpha} = 0$. As $\alpha$ increases from 0, $\Delta_1$ increases while $\Delta_2$ is constant. There exits $\alpha$ such that

$$\Delta_1 (1, \bar{\kappa}; \alpha) = \Delta_2 (1, \bar{\kappa}; \alpha) = NPV^{NoRisk}.$$ 

As $\alpha \leq \alpha$, $\Delta_1 (1, \bar{\kappa}; \alpha) \leq NPV^{NoRisk}$ and thus $\kappa_1^* = 1$ and $\kappa_2^* = \bar{\kappa}$. The definition of $\alpha$ also implies that

$$\frac{\gamma_1}{\gamma_2} NPV^{NoRisk} + \Pi_1 (\alpha) = NPV^{NoRisk}. \quad (39)$$

As $\alpha > \alpha$, $\Delta_1 (1, \bar{\kappa}; \alpha) > NPV^{NoRisk}$ and thus $(\kappa_1^* = 1, \kappa_2^* = \bar{\kappa})$ cannot be part of an equilibrium any longer, which implies that $\kappa_1^* < 1$. Moreover, since $\Delta_2 (\kappa_1, \bar{\kappa}; \alpha)$ is not affected by $\alpha$ but is increasing in $\kappa_1$, a reduction of $\kappa_1^*$ from 1 to be below 1 reduces $\Delta_2$. That is,

$$\Delta_2 (\kappa_1^*, \bar{\kappa}; \alpha) < \Delta_2 (1, \bar{\kappa}; \alpha) = NPV^{NoRisk} \text{ for } \kappa_1^* < 1.$$
That $\Delta_2 (\kappa_1^*, \bar{\kappa}) < NPV^{NoRisk}$ implies that more type-2 firms become viable and $\kappa_2^*$ increases. Thus, as $\alpha > \underline{\alpha}$, $\kappa_2^* > \bar{\kappa}$. Moreover, we can prove $\kappa_2^* = 1$ by contradiction. Suppose $\kappa_2^* < 1$ (and $\kappa_1^* \in (0, 1)$). Then it must be that

$$\Delta_1 (\kappa_1^*, \kappa_2^*; \alpha > \underline{\alpha}) = \frac{\gamma_1}{r_A + r_I} \bar{\gamma} + \Pi_1 (\alpha) = NPV^{NoRisk},$$

$$\Delta_2 (\kappa_1^*, \kappa_2^*; \alpha > \underline{\alpha}) = \frac{\gamma_2}{r_A + r_I} \bar{\gamma} = NPV^{NoRisk}.$$

Combining the two implies that

$$\frac{\gamma_1}{\gamma_2} NPV^{NoRisk} + \Pi_1 (\alpha; \alpha > \underline{\alpha}) = NPV^{NoRisk},$$

which contradicts equation 39 (and $\Pi_1 (\alpha; \alpha > \underline{\alpha}) > \Pi_1 (\underline{\alpha})$). Finally, if $\kappa_1^* = 0$, then by the regularity condition (inequality 17), we have $\kappa_2^* = 1$ as well. Therefore, $\kappa_2^* = 1$ when $\alpha > \underline{\alpha}$.

In addition, $\kappa_1^* > 0$ is determined by

$$\Delta_1 (\kappa_1^*, 1; \alpha) = \frac{\gamma_1}{r_A + r_I} \bar{\gamma} (\kappa_1^*, 1; \alpha) + \Pi_1 (\alpha) = NPV^{NoRisk}.$$

To the extent that $\kappa_1^* > 0$, we can differentiate it by $\alpha$ and obtain

$$\frac{\partial \kappa_1^*}{\partial \alpha} = - \frac{\partial \Pi_1 (\alpha)}{\partial \alpha} / \left( \frac{\partial \bar{\gamma} (\kappa_1^*, 1; \alpha)}{\partial \kappa_1^*} \frac{\gamma_1}{r_A + r_I} \right) < 0.$$

As $\alpha$ keeps increasing, type-1 firms are less likely to be viable. Therefore, there exists another threshold $\bar{\alpha}$ such that

$$\Delta_1 (0, \kappa_2; \bar{\alpha}) = \frac{\gamma_1}{r_A + r_I} \bar{\gamma} (0, 1) + \Pi_1 (\bar{\alpha}) = NPV^{NoRisk}.$$

When $\alpha > \bar{\alpha}$, the agency cost of type-1 firms is so large that no type-1 firms are viable, i.e., $\kappa_1^* = 0$. As a result, as $\alpha > \bar{\alpha}$, we have

$$\bar{\gamma} (0, 1) = \frac{\kappa_2}{2},$$

$$\Delta_1 = \frac{\gamma_1}{r_A + r_I} \frac{\gamma_2}{2} + \Pi (\alpha) > NPV^{NoRisk},$$

$$\Delta_2 = \frac{\gamma_2}{r_A + r_I} \frac{\gamma_2}{2} < NPV^{NoRisk}.$$

In sum, we have obtained the following results.
1. If $\alpha \in [0, \alpha)$, $\kappa_1^* = 1$, $\kappa_2^* = \bar{\kappa}$ and $\bar{\gamma}(\kappa_1^*, \kappa_2^*) = \frac{\gamma_1 + \gamma_2}{2}.$

2. If $\alpha \in [\alpha, \bar{\alpha})$, $\kappa_1^*$ is determined by equation 41, $\kappa_2^* = 1$ and $\bar{\gamma}(\kappa_1^*, \kappa_2^*) = \frac{\gamma_1 \kappa_1^* + \gamma_2}{2}.$ Moreover, $\frac{\partial \kappa_1^*}{\partial \alpha} < 0.$

3. If $\alpha \in [\bar{\alpha}, \infty)$, $\kappa_1^* = 0$ and $\kappa_2^* = 1$ and $\bar{\gamma}(\kappa_1^*, \kappa_2^*) = \frac{2\gamma_2}{2}.$

The comparative statics for each component of the cost of capital can be obtained accordingly.
This completes the proof. ■
Appendix B: Table of Notations

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References


