Assignment 3:
Hypothesis tests and confidence intervals

Do by week 5.

1. In 1960, census results indicated that the age at which American men first married had a mean of 23.3 years. It is widely suspected that young people today are waiting longer to get married. We want to find out if the mean age at first marriage has increased during the past 50 years. We plan to test our hypothesis by selecting a random sample of 40 men who married for the first time last year. The men in our sample married at an average age of 24.2 years, with a standard deviation of 5.3 years.

   (a) At a 5% level, test the hypothesis that average age of first marriage has increased in the past 50 years.

   (b) Give a symmetric 95% confidence interval for the difference in average marriage age between the two populations.

2. A SurveyUSA poll conducted on March 1, 2011 asked randomly sampled Los Angeles residents about their views on American vs. foreign-made products. One of the questions on the survey was “If an American-made product cost slightly more than a foreign-made product, which would you be more likely to buy?”. 81 out of the 166 respondents between the ages of 18 and 34, and 248 out of the 334 respondents 35 years and older said they would prefer the American-made product. We are interested to see if younger people are less likely to choose American-made products. Test this hypothesis at the 5% level.

3. A small poll of 40 randomly selected Kansans gave 21 responses in favor of Rick Santorum for the Republican presidential candidate.

   (a) Give a symmetric 90% confidence interval for the true proportion of Kansans in favor of Santorum.

   (b) Assuming that all Kansans favor either Santorum or Mitt Romney, does this poll suggest a statistically significant lead for Santorum at the 10% level?
4. A classroom of twenty-five individuals are asked aloud and in turn what their political party affiliation is; 16 answer Democrat and 9 answer Republican. There is a concern that lack of anonymity could have impacted the accuracy of the responses. To test the hypothesis that the responses were provided independently of one another, we count the number of times that adjacent responses differ. In the observed sequence of responses

\[(D, D, D, D, R, R, R, R, D, D, D, D, R, D, D, D, R, D, R, R, D, D, D)\]

this number is 6. By randomly shuffling these 25 observations, we get a distribution for the number of “switches” under the null hypothesis that the ordering of the responses was random, which has the following quantiles:

<table>
<thead>
<tr>
<th>Quantile</th>
<th>0.5%</th>
<th>1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>50%</th>
<th>60%</th>
<th>95%</th>
<th>97.5%</th>
<th>99%</th>
<th>99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>

(a) Is the p-value of the observed data under the hypothesis of independent responses above or below 5%? Explain why.

(b) Do you reject the null hypothesis of independent responses at the 1% level?

(c) If friends tend to share political affiliation and friends also tend to sit together, would the above test be able to answer the question of whether or not honest responses were elicited?
5. Your dad claims that he shoots better than 50% from the three point line. You bet him $100 that doesn’t, based on a challenge where he attempts 10 shots. After making 8 of his shots, he claims that you owe him $100. You say that you do not, because you cannot reject the null hypothesis that he shoots exactly 50%.

(a) Use the normal approximation to the binomial to compute the p-value of his performance under the null hypothesis that \( p = 0.5 \), against the alternative that \( p > 0.5 \).

(b) Compute the exact p-value of his observed performance (relative to the same test as above) using the binomial distribution.

(c) Use this scenario to argue why it is important to set the level of your test before you observe your data.

Hint: use the \( \text{pnorm()} \) and \( \text{pbinom()} \) or \( \text{dbinom()} \) functions in R.

6. Answer TRUE or FALSE. Provide a brief explanation of your reasoning.

(a) If a p-value is greater than 1%, we fail to reject.

(b) A ‘statistically significant effect’ means that the null hypothesis was not rejected.

(c) Assume you observe 30 i.i.d. observations from a normal distribution with unknown mean \( \mu \) and known variance \( \sigma^2 \). To decrease the width of your confidence interval for \( \mu \) by a factor of 4, you must collect 90 additional data points.

(d) A null hypothesis contained within the confidence interval implies a test statistic not contained in the corresponding rejection region.

(e) If a 98% confidence interval for a normal mean \( \mu \) is given by \((X - b, X + a)\) for a single observation of a random variable distributed \( X \sim \text{N}(\mu, \sigma^2) \), the corresponding 2% level rejection region for the null hypothesis \( H_0 : \mu = \mu_0 \) is \( X < \mu_0 - b \) and \( X > \mu_0 + a \).

(f) A confidence interval for mean \( \mu \) is centered at \( \mu \).