There are several ways to write an Exponential GARCH, or EGARCH, model. Different software packages use different parameterizations. We discuss three representations here and show their relation in the simplest case. Let $r_t$ be the return series satisfying $r_t = \mu_t + a_t$, where $\mu_t$ is the predictable part of the return conditioned on information available at time $t - 1$. Write $a_t = \sigma_t \epsilon_t$ as in the textbook. For simplicity, we assume $\epsilon_t$ is standard normal.

**Model A**: Nelson (1991) and textbook.
A simple EGARCH(1,0) model is

$$\ln(\sigma_t^2) = \alpha_0 + \frac{g(\epsilon_{t-1})}{1 - \alpha_1 B}$$

$$g(\epsilon_{t-1}) = \theta \epsilon_{t-1} + \gamma (|\epsilon_{t-1}| - \sqrt{2/\pi}) = \theta \epsilon_{t-1} + \gamma (|\epsilon_{t-1}| - 0.8).$$

Multiplying the first equation by $(1 - \alpha_1 B)$ and plugging in $g(\epsilon_{t-1})$, we obtain

$$\ln(\sigma_t^2) = \alpha_* + \theta \epsilon_{t-1} + \gamma |\epsilon_{t-1}| + \alpha_1 \ln(\sigma_{t-1}^2)$$

where $\alpha_* = (1 - \alpha_1) \alpha_0 - 0.8 \gamma$. The parameter $\theta$ can be referred to as the leverage parameter of the model.

**Model B**: SCA formulation.
In the SCA package, the function $g(\epsilon_{t-1})$ is defined as

$$g^*(\epsilon_{t-i}) = (|\epsilon_{t-i}| - \sqrt{2/\pi}) - v_i \epsilon_{t-i}, \quad i = 1, 2, \ldots.$$  

An EGARCH(1,1) model in SCA is defined as

$$\ln(\sigma_t^2) = d_0 + c_1 g^*(\epsilon_{t-1}) + d_1 \ln(\sigma_{t-1}^2).$$

Plugging in the definition of $g^*(\epsilon_{t-1})$, we have

$$\ln(\sigma_t^2) = d_0 + d_1 \ln(\sigma_{t-1}^2) + c_1 [(|\epsilon_{t-1}| - \sqrt{2/\pi}) - v_1 \epsilon_{t-1}]$$

$$= (d_0 - c_1 \sqrt{2/\pi}) + c_1 |\epsilon_{t-1}| + (-c_1 v_1) \epsilon_{t-1} + d_1 \ln(\sigma_{t-1}^2).$$

Consequently, the EGARCH(1,1) model in SCA is equivalent to the EGARCH(1,0) model in the textbook with

$$\alpha_* = d_0 - c_1 \sqrt{2/\pi}, \quad c_1 = \gamma, \quad -c_1 v_1 = \theta, \quad d_1 = \alpha_1.$$

**Model C**: S-plus formulation
In S-plus, the model is also called an EGARCH(1,1) model, but is written as

$$\ln(\sigma_t^2) = \omega_0 + \omega_1 (|\epsilon_{t-1}| + \delta \epsilon_{t-1}) + \beta_1 \ln(\sigma_{t-1}^2).$$

The parameter $\delta$ is called the leverage parameter of the model.
Comparing Eqs. (1) and (3), we see that

\[ \alpha_0 = \omega_0, \quad \alpha_1 = \beta_1, \quad \omega_1 = \gamma, \quad \theta = \omega_1 \delta. \]

The key point here is that the leverage parameters are related by \( \theta = \omega_1 \delta \).

To see the asymmetric response, Model A can be written as

\[
\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \begin{cases} 
(\gamma + \theta)\epsilon_{t-1} & \text{if } \epsilon_{t-1} \geq 0 \\
(\gamma - \theta)(-\epsilon_{t-1}) & \text{if } \epsilon_{t-1} < 0.
\end{cases}
\]

For Model B, we have

\[
\ln(\sigma_t^2) = \omega_0 + \beta_1 \ln(\sigma_{t-1}^2) + \begin{cases} 
\omega_1(1 + \delta)\epsilon_{t-1} & \text{if } \epsilon_{t-1} \geq 0 \\
\omega_1(1 - \delta)(-\epsilon_{t-1}) & \text{if } \epsilon_{t-1} < 0.
\end{cases}
\]