Solutions to Homework Assignment #6

1. Problem 6.2 of the textbook.

Answer: Take partial derivatives, we have
\[ \frac{\partial F_{t,T}}{\partial P_t} = e^{r(T-t)}, \]
\[ \frac{\partial F_{t,T}}{\partial t} = -r P_t e^{r(T-t)}, \]
\[ \frac{\partial^2 F_{t,T}}{\partial P_t^2} = 0. \]
Using Ito’s Lemma, we obtain
\[ dF_{t,T} = \left( e^{r(T-t)} \mu P_t - r P_t e^{r(T-t)} \right) dt + e^{r(T-t)} \sigma P_t dw_t, \]
\[ = (\mu - r) F_{t,T} dt + \sigma F_{t,T} dw_t. \]

2. Problem 6.10 of the textbook.

Answer: The lower bound of the put price is
\[ 47 \exp(-0.06 \times 0.5) - 44 \]
which is greater than the put price of $1.0. An arbitrageur can borrow $45 for six months to buy both the put option and the stock. At the end of six months, the arbitrageur repays the loan at $45 \exp(0.06 \times 0.5) = 46.37$. If \( P_T < 47 \), the arbitrageur exercises the option to sell the stock for $47, repays the loan, and makes a profit of \( 47 - 46.37 = 0.63 \). If \( P_T > 47 \), the arbitrageur discards the option, sells the stock to repay the loan and makes an even greater profit.

3. Problem 3.

- Calculate the VaR of your position for the next trading day using the RiskMetrics method, using \( \alpha = 0.98 \), \( r_{2770} = 4.792 \) and \( \sigma_{2770} = 1.869 \), where 2770 is the sample size. \( \sigma^2_{2771} = 0.98(1.869)^2 + 0.02(4.792)^2 = 3.883 \). The VaR for the log return is \( 2.326 \times \sqrt{3.883} = 4.5832 \). The VaR for the position is \( 1000000 \times 4.5832/100 = 45832 \).
- Based the 1-step ahead forecast of the fitted GARCH(1,1) model, we have mean \( = 0.1615 \) and variance \( = 9.488 \). Therefore, 1% quantile of the predicted log return is \( 0.1615 - 2.326 \times \sqrt{9.488} = -7.0032 \). The VaR for the position is \( 7.0032 \times 1000000/100 = 70032 \).
- Based on the forecast of the fitted GARCH(1,1) model with Student-\( t \) innovations, we have mean \( = 0.0668 \) and variance \( = 7.852 \). Also, the degrees of freedom is 4.75. The 1% quantile is \( 0.0668 - (3.44/\sqrt{4.75/2.75})\sqrt{7.852} = -7.268 \). The VaR of the position is \( 7.268 \times 1000000/100 = 72680 \).
4. Problem 4. For the case of block size \( n = 21 \), the estimates are \( k = -0.352 \), \( \sigma = 1.775 \) and \( \beta = 4.282 \). The standard errors are 0.081, 0.154, and 0.178, respectively. The 1% VaR is \( 4.282 + \frac{1.775}{-0.352}[1 - (-21 \ln(0.99))^{-0.352}] = 7.9584 \). The VaR for the position is \( $1000000 \times 7.9584/100 = $79584 \). For the POT method with threshold 4%, the VaR is \( $1000000 \times 8.544/100 = $85440 \). The expected shortfall is $124,600. The estimated parameters are \( \xi = 0.2742 \) and \( \beta = 1.5966 \). Both estimates are significantly different from zero at the 5% level.

5. Problem 5. For the AIG stock the prediction of the volatility is \( \sigma_{t+1}^2 = 0.95(0.636)^2 + 0.05(-0.306)^2 = 0.3890 \). The 1% quantile is then \( 2.36 \times \sqrt{0.3890} = 1.451 \). The VaR is then \( $500000 \times 1.451/100 = $7255 \). The VaR for the combined portfolio is

\[
\sqrt{45832^2 + 7255^2 + 2 \times (0.152)(45832)(7255)} = $47479.
\]