1. Log return of CAT stock.
   
   - Yes, there is ARCH effect, because the $Q(m)$ statistics of the squared shocks to the log returns show $Q(10) = 41.55$ with p-value $9.02 \times 10^{-6}$.
   - The PACF of the squared shocks indicates an AR(3) model so that we start with an ARCH(3) model. The fitted model is
     \[ r_t = 0.00087 + a_t, \quad a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1), \]
     \[ \sigma_t^2 = 2.85 \times 10^{-4} + 0.048a_{t-1}^2 + 0.102a_{t-2}^2 + 0.086a_{t-3}^2. \]
     All parameter estimates are significant at the 5% level.
   - The fitted Gaussian GARCH(1,1) model is
     \[ r_t = 0.00084 + a_t, \quad a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1), \]
     \[ \sigma_t^2 = 0.026 \times 10^{-4} + 0.017a_{t-1}^2 + 0.975a_{t-1}^2. \]
     The $Q(m)$ statistics of the standardized residuals give $Q(10) = 11.47$, $Q(15) = 17.71$ and $Q(20) = 22.49$ with p-values all greater than 0.05 so that the mean equation is adequate. The $Q(m)$ statistics of the squared standardized residuals also fail to indicate any inadequacy in the volatility equation; see, for instance, $Q(10) = 0.31$ with p-value almost 1.
   - The fitted GARCH(1,1) model with Student-$t$ innovations is
     \[ r_t = 0.00085 + a_t, \quad a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim t_{6.41}, \]
     \[ \sigma_t^2 = 0.023 \times 10^{-4} + 0.023a_{t-1}^2 + 0.971a_{t-1}^2. \]
     - The GARCH(1,1) model with Student-$t$ innovations because it has the smallest AIC value (and largest likelihood value).

2. Daily log returns of the VW index.
   - No, the $Q(m)$ statistics give $Q(10) = 11.85$ with p-value 0.30. One cannot reject the null hypothesis that the first 10 lags of ACF are zero.
Yes, there is ARCH effect because $Q(10) = 702.2$ with p-value close to zero for the shock series of the index returns.

The fitted IGARCH(1,1) model with Student-$t$ innovations is

\[
    r_t = 0.57 \times 10^{-3} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{10.57},
\]

\[
    \sigma_t^2 = 5.66 \times 10^{-7} + 0.075 a_{t-1}^2 + 0.925 \sigma_{t-1}^2.
\]

All forecasts of the returns are $5.73 \times 10^{-4}$, and all volatility forecasts are $\sqrt{0.0001395} = 0.0118$.

3. Again, consider the daily log returns of CAT stock.

The fitted GARCH(1,1)-M model with Student-$t$ distribution is

\[
    r_t = 0.0013 - 1.417 \sigma_t^2 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{6.39},
\]

\[
    \sigma_t^2 = 0.219 \times 10^{-5} + 0.022 a_{t-1}^2 + 0.971 \sigma_{t-1}^2.
\]

No, the ARCH-in-mean parameter is not significantly different from zero at the 5% level.

The fitted GJR model is

\[
    r_t = 8.35 \times 10^{-4} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1),
\]

\[
    \sigma_t^2 = 2.75 \times 10^{-5} + (0.017 + 0.0011 N_{t-1}) a_{t-1}^2 + 0.974 \sigma_{t-1}^2.
\]

Applying the $Q(m)$ statistics to the standardized residuals and the squared standardized residuals shows that the fitted model is adequate. [All p-values are greater than 0.05.]

No, the leverage parameter is not statistically significant at the 5% level.

4. GE monthly log returns.

No, because $Q(10)$ of the returns is 11.06 with p-value 0.52.

The fitted GARCH(1,1) model with the generalized error distribution is

\[
    r_t = 1.01 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{ged},
\]

with parameter 1.69, and the volatility equation is

\[
    \sigma_t^2 = 2.22 + 0.071 a_{t-1}^2 + 0.868 \sigma_{t-1}^2.
\]

The 1-step to 5-step forecasts for the returns, in percentages, are all equal to 1.007. For the volatility, the 1-step to 5-step ahead forecasts are 5.06, 5.13, 5.19, 5.25 and 5.30, respectively.
5. The exchange rate series between Swiss Francs and U.S. Dollar.

- No, there is no serial correlations in the log returns of the exchange rate, because $Q(10) = 14.09$ with p-value 0.17.
- Yes, because $Q(10) = 250.61$ for the squared log returns. The p-value is close to zero.
- I multiplied the return series by 100 to obtain log returns in percentages. The fitted IGARCH(1,1) model for the RiskMetrics is

$$r_t = -0.012 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1),$$

$$\sigma_t^2 = 0.025a_{t-1}^2 + 0.976\sigma_{t-1}^2.$$  

Based on the $Q(m)$ statistics for the standardized residuals and the squared standardized residuals, the model is adequate because all tests have p-values greater than 0.05.

- The returns are in percentages. The 1-step to 4-step ahead forecasts for the return series are all equal to $-0.0121$. For the volatility forecasts, they are all 0.854.