ChicagoBooth Honor Code:  
*I pledge my honor that I have not violated the Honor Code during this examination.*

Signature:  
Name:  
ID:  

Notes:  
- Open notes and books.
- For each question, write your answer in the blank space provided.
- Manage your time carefully and answer as many questions as you can.
- The exam has 8 pages and the R output has 8 pages. Total is 16 pages. Please check to make sure that you have all the pages.
- For simplicity, ALL tests use the 5% significance level.
- Round your answer to 3 significant digits.

Problem A: (30 pts) Answer briefly the following questions. Each question has two points.

1. Describe the leverage effect of stock returns and give two classes of volatility models that can model the leverage effect.

2. Describe two ways that a GARCH model can induce heavy tails.

3. Write down an airline model for a quarterly time series. Why is the model useful in practice?
4. **For questions 4 to 8**, consider the daily adjusted closing prices of Johnson and Johnson (JNJ) stock from January 3, 2005 to April 26, 2011. We compute the daily log returns of JNJ from the price. Some summary statistics of the log returns are attached. Let \( p_t \) be the log closing price of JNJ. Is there evidence to support the hypothesis that \( p_t \) follows a random walk model with drift? Why?

5. Use Jacque-Bera statistics to test the null hypothesis that the log returns are normally distributed versus the alternative that they are not. What is the value of the test statistic? Draw your conclusion.

6. What is the standard deviation of the mean, i.e. SE Mean, for the log returns?

7. Let \( \rho_i \) be the lag-\( i \) ACF of the log returns. Based on the output, test \( H_0 : \rho_2 = 0 \) versus \( H_a : \rho_2 \neq 0 \). What is the test statistic and draw your conclusion.

8. Test \( H_0 : \rho_1 = \rho_2 = \cdots = \rho_{10} = 0 \) versus \( H_a : \rho_i \neq 0 \) for some \( 1 \leq i \leq 10 \). What is the test statistic and draw your conclusion.

9. **For Questions 9 to 11**, consider the quarterly AR(2) model

\[
 r_t = -0.7 + 1.1 r_{t-1} - 0.4 r_{t-2} + a_t, \quad \text{Var}(a_t) = 16.
\]

Is the model weakly stationary? Why?

10. Does the model imply the existence of business cycles? If yes, what is the average length of the cycles?
11. Suppose that $r_{300} = 7$ and $r_{299} = 5$. What is the 1-step ahead prediction of $r_{301}$ at the forecast origin $T = 300$? Is zero in the 95% confidence interval of the prediction?

12. Describe two methods that can be used to test the hypothesis that a return series follows a skew distribution. Hint: one uses data directly and the other uses volatility modeling.

13. Why is the usual $R^2$ (coefficient of determination) not a good measure of the goodness of fit in a linear regression model?

14. Consider the CAPM model $r_t = \alpha + \beta r_{m,t} + \epsilon_t$, where $r_t$ and $r_{m,t}$ are the excess returns of a stock and market, respectively. Based on the volatility modeling discussed in class, describe a method to estimate the time-varying beta.

15. Describe two methods that can be used to identify an AR model for a time series.
**Problem B.** (30 pts) Consider the daily log returns of Google stock from January 4, 2005 to April 26, 2011. The relevant R output is attached. Preliminary analysis indicates that we cannot reject the null hypothesis that the mean of the return is zero. We cannot reject the hypothesis that there are no serial correlations in the return series, at the 1% level with Q(10). Thus, we proceed with volatility modeling. Answer the following questions.

1. (2 points) Is there any ARCH effect in the daily log returns? Why?

2. (3 points) A volatility model, called m1 in R, is entertained. Write down the fitted model, including the mean equation. Is the model adequate? Why?

3. (3 points) Another volatility model, called m2 in R, is fitted to the returns. Write down the model, including the degrees of freedom.

4. (2 points) Based on the fitted model m2, test $H_0 : \nu = 5$ versus $H_a : \nu \neq 5$, where $\nu$ denotes the degrees of freedom of Student-\(t\) distribution. Perform the test and draw a conclusion.

5. (3 points) A third model, called m3 in R, is also entertained. Write down the model, including the distributional parameters. Is the model adequate? Why?

6. (2 points) Let $\xi$ be the skew parameter in model m3. Test $H_0 : \xi = 1$ versus $H_a : \xi \neq 1$. Perform the test and draw a conclusion.

7. (3 points) Again, focus on model m3 and use the last data point as the forecast origin. Compute 95% interval forecasts for the 1-step and 2-step ahead prediction of the log return.
8. (3 points) A fourth model, called m4 in R, is also fitted. Write down the fitted model, including the distribution of innovations.

9. (2 points) Based on model m4, is the leverage effect significant? Why?

10. (2 points) Among the volatility models entertained so far, which model is preferred? State the criterion used in your choice.

11. (3 points) Since the estimates \( \hat{\alpha}_1 + \hat{\beta}_1 \) is close to 1, we consider an IGARCH(1,1) model. Write down the fitted IGARCH(1,1) model.

12. (2 points) Use the IGARCH(1,1) model and the information provided to obtain 1-step ahead prediction for the log return and its volatility at the forecast origin \( t = 1086 \).

Problem C. (18 pts) Consider the quarterly earnings per share of Wal-Mart Stores from 1992 to 2010 for 76 observations. We analyzed the logarithms of the earnings. That is, \( x_t = \ln(y_t) \), where \( y_t \) is the quarterly earnings per share. Two models are entertained. Model checking statistics of the two models are given in Figures 1 and 2.

1. (3 points) Write down the model m1 in R, including residual variance.
2. (2 points) Is the model adequate? Why?

3. (3 points) Write down the fitted model \textbf{m2} in R, including residual variance.

4. (2 points) Is the model adequate? Why?

5. (2 points) Compare the two models. Based on in-sample fit, which model is preferred? Why?

6. (2 points) Based on 1-step ahead out-sample prediction, which model is preferred? Why?

7. (2 points) Based on 2-step ahead out-sample prediction, which model is preferred? Why?

\textbf{Problem D.} (22 pts) Consider the daily natural gas demand for a major northern city in the U.S. in one winter. It is well-known that gas demand depends on weather, such as temperature and windspeed, and weekly activities. Let \( y_t \) be the gas demand, \( x_{1t} \) be the degrees of heating days, defined as \( x_{1t} = 65 - \frac{(\text{daily high temperature} + \text{daily low temperature})}{2} \), \( x_{2t} = x_{1,t-1} \) (the lagged value of \( x_{1t} \)), \( x_{3t} \) the windspeed and \( x_{4t} \) the dummy variable for weekend. Preliminary analysis indicates starting with a linear regression model with all four explanatory variables.

1. (4 points) Write down the fitted linear regression model, (called \textbf{m1} in R), including R-square and the residual standard error.
2. (2 points) Is the linear regression model adequate? Why?

3. (5 points) Based on the ACF of the residuals, we employ an MA(7) model for the residuals. Initial estimates show some insignificant parameters in the model so that we further refine the model by removing insignificant parameters. Write down the refined model, called \texttt{m3} in R, including residual variance.

4. (2 points) Based on the output provided, are there any serial correlations in the residuals of model \texttt{m3}? Write down the null hypothesis of the test statistic and draw a conclusion.

5. (5 points) As an alternative, we employ a seasonal model for the residuals of linear regression in question 1. This model is called \texttt{m4}. Write down the fitted model, including residual variance.

6. (2 points) Between the two models (\texttt{m3} vs \texttt{m4}), which one is preferred? Why?

7. (2 points) Does the demand for natural gas decrease over the weekend? Why?
R output: edited

*** Problem A: Daily Johnson and Johnson stock returns ***

> head(JNJ)

<table>
<thead>
<tr>
<th>JNJ.Open</th>
<th>JNJ.High</th>
<th>JNJ.Low</th>
<th>JNJ.Close</th>
<th>JNJ.Volume</th>
<th>JNJ.Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-01-03</td>
<td>63.35</td>
<td>63.55</td>
<td>62.69</td>
<td>62.90</td>
<td>7859500</td>
</tr>
</tbody>
</table>

.....

> jnjp=log(as.numeric(JNJ[,6]))
> jnjrtn=diff(jnjp)
> basicStats(jnjrtn)

<table>
<thead>
<tr>
<th>jnjrtn</th>
</tr>
</thead>
<tbody>
<tr>
<td>nobs</td>
</tr>
<tr>
<td>NAs</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>SE Mean</td>
</tr>
<tr>
<td>LCL Mean</td>
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<tr>
<td>UCL Mean</td>
</tr>
<tr>
<td>Variance</td>
</tr>
<tr>
<td>Stdev</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Kurtosis</td>
</tr>
</tbody>
</table>

> m1=acf(jnjrtn)  <====== compute ACF
> m1$acf

[,1]
[1,] 1.00000000000
[2,] -0.0672245605
[3,] -0.1322490520

Figure 1: Model check for model m1.
Figure 2: Model checking for model \textbf{m2}.

\begin{verbatim}
[4,] 0.1097188528
[5,] -0.0043070506
[6,] -0.0503395269
[7,]  0.0404258278

> Box.test(jnjrtn,lag=10,type='Ljung')
Box-Ljung test
data: jnjrtn
X-squared = 66.2359, df = 10, p-value = 2.35e-10

**** Problem B:  Daily log returns of Google stock ***********************
> acf(rtn)  # log returns of Google stock.
> Box.test(rtn,lag=10,type='Ljung')
Box-Ljung test
data: rtn
X-squared = 21.1222, df = 10, p-value = 0.02026

> t.test(rtn)
One Sample t-test
data: rtn
t = 0.1765, df = 1085, p-value = 0.8599
alternative hypothesis: true mean is not equal to 0

> Box.test(rtn^2,lag=10,type='Ljung')
Box-Ljung test
data: rtn^2
\end{verbatim}
X-squared = 87.266, df = 10, p-value = 1.865e-14

> m1=garchFit(~garch(1,1),data=rtn,trace=F)
> summary(m1)
Title:  GARCH Modelling
Call:  garchFit(formula = ~garch(1, 1), data = rtn, trace = F)

Mean and Variance Equation:
  data ~ garch(1, 1) [data = rtn]

Conditional Distribution: norm

Error Analysis:

| Estimate  | Std. Error  | t value | Pr(>|t|) |
|-----------|-------------|---------|----------|
| mu        | 7.333e-04   | 5.548e-04 | 1.322   | 0.186273 |
| omega     | 4.586e-06   | 1.370e-06 | 3.347   | 0.000818 *** |
| alpha1    | 3.835e-02   | 6.779e-03 | 5.657   | 1.54e-08 *** |
| beta1     | 9.525e-01   | 7.722e-03 | 123.352 | < 2e-16 *** |

Standardised Residuals Tests:

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<thead>
<tr>
<th>Statistic</th>
<th>p-Value</th>
</tr>
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<tbody>
<tr>
<td>Jarque-Bera Test R Chi^2</td>
<td>2783.098</td>
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<tr>
<td>Shapiro-Wilk Test R W</td>
<td>0.9231201</td>
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<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>11.00794</td>
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<td>Ljung-Box Test R Q(20)</td>
<td>20.28717</td>
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<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>2.755296</td>
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<td>Ljung-Box Test R^2 Q(20)</td>
<td>6.72687</td>
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Information Criterion Statistics:

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<tr>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
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<tbody>
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<td>-4.982992</td>
<td>-4.964612</td>
<td>-4.983019</td>
<td>-4.976035</td>
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</tbody>
</table>

> m2=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std")
> summary(m2)
Title:  GARCH Modelling
garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std", trace = F)

Mean and Variance Equation:
  data ~ garch(1, 1) [data = rtn]

Conditional Distribution: std

Error Analysis:

| Estimate  | Std. Error  | t value | Pr(>|t|) |
|-----------|-------------|---------|----------|
| mu        | 1.074e-03   | 4.394e-04 | 2.444   | 0.0145 * |
| omega     | 2.498e-06   | 1.542e-06 | 1.620   | 0.1052 |

10
alpha1 $4.764 \times 10^{-2}$ $1.134 \times 10^{-2}$ $4.200$ $2.67 \times 10^{-5}$ ***
beta1 $9.511 \times 10^{-1}$ $1.014 \times 10^{-2}$ $93.803$ < $2 \times 10^{-16}$ ***
shape $3.709 \times 10^{0}$ $4.363 \times 10^{-1}$ $8.500$ < $2 \times 10^{-16}$ ***
---
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<table>
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<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
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<td>Ljung-Box Test R Q(20)</td>
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<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.317534</td>
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<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>7.32288</td>
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<td>-5.180516</td>
<td>-5.157541</td>
<td>-5.180558</td>
<td>-5.171819</td>
</tr>
</tbody>
</table>

> m3=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="sstd")
> summary(m3)

Title: GARCH Modelling

garchFit(formula = ~garch(1, 1), data = rtn, cond.dist="sstd",trace=F)

Mean and Variance Equation:

 data ~ garch(1, 1) [data = rtn]

Conditional Distribution: sstd

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|---------|
| mu | 1.203e-03 | 5.194e-04 | 2.315 | 0.0206 * |
| omega | 2.453e-06 | 1.551e-06 | 1.582 | 0.1137 |
| alpha1 | 4.860e-02 | 1.173e-02 | 4.142 | 3.44e-05 *** |
| beta1 | 9.509e-01 | 1.013e-02 | 93.833 | < 2e-16 *** |
| skew | 1.025e+00 | 4.289e-02 | 23.893 | < 2e-16 *** |
| shape | 3.660e+00 | 4.348e-01 | 8.417 | < 2e-16 *** |

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<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>10.59270</td>
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<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>19.26610</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.345845</td>
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<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>7.30709</td>
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<td>-5.151462</td>
<td>-5.179093</td>
<td>-5.168596</td>
</tr>
</tbody>
</table>

> predict(m3,4)
meanForecast  meanError standardDeviation
1 0.001202503 0.02103282 0.02103282
2 0.001202503 0.02108562 0.02108562
3 0.001202503 0.02113826 0.02113826
4 0.001202503 0.02119074 0.02119074

> m4=garchFit(~aparch(1,1),data=rtn,trace=F,delta=2,include.delta=F)
> summary(m4)
Title: GARCH Modelling
garchFit(formula=~aparch(1,1),data=rtn,delta=2,include.delta=F,trace=F)

Mean and Variance Equation:
   data ~ aparch(1, 1) [data = rtn]

Conditional Distribution: norm

Error Analysis:

|      | Estimate | Std. Error | t value | Pr(>|t|) |
|------|----------|------------|---------|----------|
| mu   | 5.803e-04| 5.473e-04  | 1.060   | 0.289050 |
| omega| 3.937e-06| 9.502e-07  | 4.143   | 3.43e-05 *** |
| alpha1| 1.301e-02| 4.014e-03  | 3.240   | 0.001194 ** |
| gamma1| 1.000e+00| 3.030e-01  | 3.301   | 0.00964 *** |
| beta1| 9.647e-01| 4.430e-03  | 217.784 | < 2e-16 *** |

---

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<th>p-Value</th>
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<tbody>
<tr>
<td>Jarque-Bera Test</td>
<td>R Chi^2</td>
<td>2592.154</td>
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<tr>
<td>Shapiro-Wilk Test</td>
<td>R W</td>
<td>0.9255044</td>
</tr>
<tr>
<td>Ljung-Box Test R</td>
<td>Q(10)</td>
<td>10.78645</td>
</tr>
<tr>
<td>Ljung-Box Test R</td>
<td>Q(20)</td>
<td>18.90509</td>
</tr>
<tr>
<td>Ljung-Box Test R^2</td>
<td>Q(10)</td>
<td>1.806153</td>
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<td>Ljung-Box Test R^2</td>
<td>Q(20)</td>
<td>5.468889</td>
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Information Criterion Statistics:

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<tbody>
<tr>
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<td>-4.984699</td>
<td>-5.007717</td>
<td>-4.998978</td>
</tr>
</tbody>
</table>

> plot(m4)
> source("Igarch.R")
> m6=Igarch(rtn)
Maximized log-likehood: -2706.858

Coefficient(s):

|      | Estimate  | Std. Error | t value | Pr(>|t|) |
|------|-----------|------------|---------|----------|
| mu   | 6.90156e-04| 5.56977e-04| 1.23911 | 0.2153043 |
| omega| 2.69091e-06| 8.74247e-07| 3.07798 | 0.0020841 ** |
| beta1| 9.54877e-01| 7.43291e-03| 128.46618 | < 2.2e-16 *** |
---
> length(rtn)
[1] 1086
> rtn[1086]
[1] 0.01469016
> m6$volatility[1086]
[1] 0.02113778

***** Problem C: Wal-Mart quarterly earnings ****************************
> da=read.table("q-earns-wmt9210.txt",header=T)
> head(da)
   Day Mon Year  earns
 1   19   3 1992  0.13
 2   11   5 1992  0.09
   ....
> dim(da)
[1] 76  4
> wmt=log(da$earns)
> m0=acf(diff(diff(wmt),4))
> m0$acf
[1,] 0.2096642 -0.2147662 -0.1481573  0.2393299 -0.3359907  0.0181023  0.0464477 -0.0474110  0.1567407 -0.1067436
> basicStats(diff(diff(wmt),4))
  diff.diff.wmt..4
  nobs         71.000000
  NAs          0.000000
  Mean        -0.002807
  SE Mean      0.007182
  LCL Mean     -0.017130
  UCL Mean     0.011517
  Variance     0.003662
  Stdev        0.060514
> m1=arima(wmt,order=c(0,1,1),seasonal=list(order=c(0,1,1),period=4))
> m1
arima(x=wmt,order=c(0,1,1),seasonal=list(order=c(0,1,1),period = 4))
Coefficients:
 ma1  sma1
> tsdiag(m1,gof=16)
> m2=arima(wmt,order=c(0,1,1),seasonal=list(order=c(1,1,0),period=4))
> m2
arima(x=wmt,order=c(0,1,1),seasonal=list(order=c(1,1,0),period=4))

Coefficients:
    ma1 sar1
  -0.3081 -0.3646

s.e.  0.1247  0.1173

sigma^2 estimated as 0.002932: log likelihood = 105.96, aic = -205.92
> tsdiag(m2,gof=16)

> source("backtest.R")
> backtest(m1,wmt,50,2)
[1] "RMSE of out-of-sample forecasts"
[1] 0.04511365 0.05751392
> backtest(m2,wmt,50,2)
[1] "RMSE of out-of-sample forecasts"
[1] 0.04487461 0.05686482

**** Problem D: Daily demand of Natural Gas *******************************
> da=read.table("d-gassendout.dat",header=T)
> head(da)
gas dhd dhdm1 wspeed wkend
1 227 32 30 12 1
.....
> dim(da)
[1] 63  5
> m1=lm(gas~dhd+dhdm1+wspeed+wkend,data=da)
> summary(m1)

Coefficients:
                     Estimate  Std. Error    t value  Pr(>|t|)  
(Intercept)     1.8581       11.5561       0.161    0.87282
 dhd          5.8742        0.2905     20.219 < 2e-16 ***
 dhdm1        1.4052        0.2928      4.799  1.16e-05 ***
 wspeed        1.3154        0.5787      2.273  0.02675 *
 wkend      -15.8571       5.3344     -2.973   0.00429 **
---

Residual standard error: 18.32 on 58 degrees of freedom
Multiple R-squared: 0.9521, Adjusted R-squared: 0.9488

> Box.test(m1$residuals, lag=14, type='Ljung')
Box-Ljung test
data: m1$residuals
X-squared = 49.1806, df = 14, p-value = 8.369e-06
> m1acf=acf(m1$residuals)
> m1acf$acf
[,1]
[1,] 1.00000000
[2,] 0.51514249
[3,] 0.27614439
[4,] 0.25870432
[5,] 0.27702979
[6,] 0.10484658
[7,] 0.26511473
[8,] 0.35066692
[9,] 0.09238026
> m1pacf=pacf(m1$residuals)  ===== PACF of residuals
> m1pacf$acf
[,1]
[1,] 0.51514249
[2,] 0.01466401
[3,] 0.15110560
[4,] 0.11923855
[5,] -0.14493573
[6,] 0.32408261
[7,] 0.10540219
[8,] -0.29272433
[9,] -0.02825233

> yt=da$gas; x1=da$dhd; x2=da$dhm1; x3=da$wspeed; x4=da$wkend

> m2=arima(yt, order=c(0,0,7), xreg=data.frame(x1,x2,x3,x4))
> m2
arima(x = yt, order = c(0, 0, 7), xreg = data.frame(x1, x2, x3, x4))
Coefficients:

                   ma1      ma2      ma3      ma4      ma5      ma6      ma7 intercept
s.e. 0.1387 0.1526 0.1640 0.1519 0.1481 0.1676 0.1573 13.8925
x1 0.6392 0.2707 0.1285 0.3922 -0.0871 0.2645 0.3350 15.1845
x2 0.1048 0.1511 0.1640 0.1519 0.1481 0.1676 0.1573 13.8925
x3 0.2587 0.2761 0.2587 0.2761 0.2587 0.2761 0.2587 13.8925
x4 0.1048 0.1511 0.1640 0.1519 0.1481 0.1676 0.1573 13.8925
s.e. 5.7982 1.1359 1.3470 1.1359 1.3470 1.1359 1.3470 13.8925

sigma^2 estimated as 150.4: log likelihood = -250.21, aic = 526.41
```r
> c1 = c(NA, NA, 0, NA, 0, NA, NA, NA, NA, NA, NA)
> m3 = arima(yt, order = c(0, 0, 7), xreg = data.frame(x1, x2, x3, x4), fixed = c1, include.mean = F)
> m3

arima(x = yt, order = c(0, 0, 7), xreg = data.frame(x1, x2, x3, x4), include.mean = F, fixed = c1)

Coefficients:
ma1  ma2  ma3  ma4  ma5  ma6  ma7     x1    x2    x3     x4
0.7066 0.2470 0.3580 0.2711 0.2790 5.9415 1.2959 1.3746
s.e. 0.1299 0.1272 0.1078 0.1316 0.1372 0.1872 0.1888 0.2257

sigma^2 estimated as 155.5: log likelihood = -251.22, aic = 522.44

> Box.test(m3$residuals, lag = 14, type = 'Ljung')

Box-Ljung test

data: m3$residuals
X-squared = 7.5962, df = 14, p-value = 0.9093

> tsdiag(m3, gof = 21)
> m4 = arima(yt, order = c(1, 0, 0), seasonal = list(order = c(1, 0, 0), period = 7),
>           xreg = data.frame(x1, x2, x3, x4), include.mean = F)
> m4

arima(x = yt, order = c(1, 0, 0), seasonal = list(order = c(1, 0, 0), period = 7),
      xreg = data.frame(x1, x2, x3, x4), include.mean = F)

Coefficients:
ar1  sar1   x1    x2    x3     x4
0.5359 0.3677 5.7651 1.4732 1.2819 -9.5947
s.e. 0.1065 0.1171 0.1999 0.1953 0.3637 6.0260

sigma^2 estimated as 189.5: log likelihood = -255.28, aic = 524.56

> Box.test(m4$residuals, lag = 14, type = 'Ljung')

Box-Ljung test

data: m4$residuals
X-squared = 7.4463, df = 14, p-value = 0.9161