Plots are not shown in the solutions. However, they can be produced by repeating the R commands used in the this solution.

1. Log series of U.S. monthly housing starts: Let \( x_t \) be the log series for the monthly housing starts. The fitted model is

\[
(1 - B)(1 - B^{12})x_t = (1 - 0.33B + 0.12B^3)(1 - 0.87B^{12})a_t, \quad \sigma_a^2 = 0.00798.
\]

All estimates are significant at the 5% level and model checking shows that this model is adequate.

The 1-step ahead forecasts are 3.85, 3.94, 4.00, 4.02, 3.97, 3.93, 3.89, 3.87, 3.69 and 3.55, respectively. The 95% interval forecasts are:

Lower bounds: 3.67 3.73 3.76 3.74 3.66 3.59 3.52 3.48 3.27 3.12

2. Monthly Decile 2 returns

(a) Regression model

\[
d_{2t} = 0.0069 + 0.077J_t + a_t + 0.298a_{t-1}, \quad \sigma_a^2 = 0.00330.
\]

This model fits the data well. The January effect is significant.

(b) Pure time series model

\[
(1 - 0.999B^{12})(d_{2t} - 0.0132) = (1 + 0.296B)(1 - 0.985B^{12})a_t,
\]

where \( \sigma_a^2 = 0.00334 \). The near cancellation of the seasonal factors indicates a deterministic seasonality. This also leads to the finding of January effect. For the residuals, we have \( Q(12) = 7.90, Q(24) = 18.56 \) and \( Q(36) = 30.38 \). These Ljung-Box statistics all have high p-value.


The key conclusion is that there is no January effect. We can use either of the two models employed for Decile 2 returns.

For regression model, the fitted model is

\[
d_{9t} = 0.0101 + 0.0088J_t + a_t + 0.165a_{t-1}, \quad \sigma_a^2 = 0.00234.
\]
The t-ratio for January effect is \( t = \frac{0.0088}{0.0064} = 1.375 \), which is less than 1.96.

For pure time series model, we have

\[
d_{gt} = 0.0109 + a_t + 0.167a_{t-1}, \quad \sigma^2_a = 0.00235.
\]

Model checking indicates that the model fits the data well. Since the model does not contain any seasonal component, there is no January effect.

4. Let \( y_t \) and \( x_t \) be the growth rate series of PCE and DSPI, respectively. The residuals of the simple linear regression model \( y_t = 0.0046 + 0.186x_t + \epsilon_t \) show certain serial correlations. Thus, we employ a regression model with time-series errors. The fitted model is

\[
y_t = 0.0045 + 0.213x_t + (1 - 0.208B + 0.069B^4 + 0.075B^5 + 0.191B^6)a_t,
\]

where \( \sigma^2_a = 2.799 \times 10^{-5} \). Model checking indicates that the model is adequate. Note that some of the MA coefficients are between 1 and 2 of their standard errors. They can not further removed if necessary.

5. Using AR models, we obtain

\[
(1 + 0.156B - 0.07B^3 - 0.07B^4 - 0.18B^5 - 0.14B^6 - 0.08B^8)(y_t - 0.0057) = a_t,
\]

where \( \sigma^2_a = 2.914 \times 10^{-5} \). This model is also adequate.

Applying backtest program, for the pure time-series model, the 1-step and 2-step ahead RMSE are 0.004557 and 0.00436, respectively. For the regression model, they are 0.004587 and 0.00431. Thus, for 1-step ahead prediction, the pure time-series model is slightly preferred. For 2-step ahead prediction, the regression model is selected. Overall, the difference between the two models is small.