Objective: To gain experience in using alternative approaches to volatility calculation, neural network, and model comparison.

Due Date: May 24, 2011

1. The goal is to study the stock returns and volatility of the bailout firm AIG. Focus on the period from January 3, 2004 to April 30, 2011. You may download the daily open, high, low, close prices and related information from Yahoo via the quantmod package. Note that there was a 1-for-20 reverse stock split on June 30, 2009.

Use the data to construct the variance estimates $\sigma^2_i$ of Section 3.15.2 of the textbook for $i = 0, 1, 2, 3, 5,$ and $6$. These volatilities are for the stock price. Take the square root transformation to obtain volatility series. Multiple the volatility by $\sqrt{252}$ to obtain annualized volatility. Obtain mean, median, maximum, and minimum of each of the six annualized volatility series.

2. Again, consider the AIG stock return, but focus on the volatility of the log returns. Use the data and $n = 63$ to compute the Yang and Zhang (2000) variance estimate $\hat{\sigma}^2_{yz}$ of Section 3.15.2 of the textbook. Obtain a time plot of the estimated (and annualized) volatility series (square-root of annualized variance). Build a time series model to produce 1-step to 5-step ahead volatility forecasts at the forecast origin April 30, 2011. Hint: You can consider the log series of the volatility in modeling.

3. Consider the monthly stock returns of 3M company and the S&P composite index from January 1961 to December 2010. Data are in the file m-3msp6110.txt. Define the direction of price movement of 3M stock as follows:

$$M_t = \begin{cases} 
1 & \text{if } r_t > 0 \\
0 & \text{otherwise}
\end{cases}$$

Similarly, we can define the direction for market movement using the S&P composite index returns. Denote the direction by the market by $S_t$.

(a) Fit a linear logistic regression model for $P(M_t = 1)$ using $M_{t-1}, S_{t-1}, M_{t-2}, S_{t-2}$. Based on the model, can past price movements of either the stock or the market predict the future 3M price movement? Why?
(b) Use \((M_{t-i}, S_{t-i})\) for \(i = 1\) and \(2\) to build a 4-2-1 look-forward network with direct link for \(P(M_t = 1)\). Write down the fitted model.

(c) Divide the sample into modeling and forecasting subsamples with the latter consisting of the last 63 observations. Use 1-step ahead prediction to compare the previous two models. You can treat \(\bar{M}_t = 1\) if \(P(M_t = 1) > 0.5\).

4. Consider the tick-by-tick trade data of the 3M stock from October 4 to October 8, 2010.

- Use the data within the normal trading hours only, i.e. from 9:30 am to 4:00 pm Eastern time, to construct a series of intraday 5-minute log returns. If there is no trading within a 5-minute interval, assume that the log return is zero. If there are multiple trades in a 5-minute interval, use the last trade to obtain the price for that interval. Plot the log return series.
- Are there any serial correlations in the intraday 5-minute log return series? Use \(Q(10)\) to perform the test.
- Use 5-minute intraday log returns to compute the realized volatility for each of the five trading days.
- Use 1-minute intraday log returns to compute the realized volatility for each of the five trading days.

5. Again, consider the tick-by-tick trade data of 3M stock from October 4 to October 10, 2010.

- Construct the series of the number of trades within a 5-minute intervals. Use data in the normal trading hours only.
- Compute the ACF of the constructed time series, say from lag 1 to lag 250. Is there any evidence of diurnal pattern? [No formal test is needed. Simply comment on the ACF plot.]

**Reading assignments:** Sections 4.1, 5.1 to 5.4 of the textbook.