This assignment is concerned with Value at Risk and Expected Shortfall. Assume that the tail probability of interest is 1% and the portfolio consists of two stocks, namely American International Group (AIG) and International Business Machines (IBM). The sample period is from January 3, 2001 to April 30, 2011. The data can be downloaded from Yahoo via the quantmod package. Use the adjusted closing prices to compute the daily log returns. For Questions 1 to 4, assume that you hold (long position) both stocks valued at $1 million each.

1. Consider the AIG stock only.
   - Calculate the VaR of your position for the next trading day using the RiskMetrics method on April 30, 2011. You must estimate the IGARCH(1,1) model without constant to obtain the parameter value needed in the calculation. What is the associated expected shortfall?
   - Build a GARCH(1,1) model for the log return series with Gaussian innovations. What is the VaR based on the fitted model for the next trading day? What is the corresponding expected shortfall?
   - Build a GARCH(1,1) model with Student-$t$ innovations for the log return series. What is the VaR for the next trading day based on the fitted model? What is the corresponding expected shortfall?

2. Again, consider the daily log returns of AIG stock. The goal here is to use EVT to estimate risk. Using blocks of size 21, fit a generalized extreme value distribution to a proper return series. Write down the estimates and their standard errors. Compute the 1% VaR of your financial position based on the fitted parameters. What is the 1% VaR of your financial position for the next 10 trading days?

3. Again, consider the log returns of the AIG stock. Fit a generalized Pareto distribution to the return series with threshold 4.0%. Based on the fitted model, what is the 1% VaR of your position? What is the associated expected shortfall? Repeat the analysis using threshold 8.0%. Are the results sensitive to the choice of thresholds?
4. Consider now the log returns of IBM stock. Calculate the VaR using RiskMetrics method. Also, what is the VaR for the combined position of AIG and IBM stocks? Next, what is the VaR for the combined position of AIG and IBM if we entertain time-varying correlations between the two stocks. More specifically, use \( \text{Cov}(X, Y) = 0.25[\text{Var}(X + Y) - \text{Var}(X - Y)] \) to compute the time-varying covariances. That is, fit IGARCH models as before to \( X + Y \) and \( X - Y \) to obtain the time-varying covariance. The volatilities of \( X \) and \( Y \) were obtained in the prior questions.

5. Consider a new portfolio that holds a short position of $1 million in AIG stock, but a long position of $1 million in IBM stock. Obtain the VaR (at the 1% level) of the portfolio for the next trading day on April 30, 2011 if (a) RiskMetrics is used and (b) GARCH(1,1) models with Gaussian innovations are used.