Solutions to Homework Assignment #2

   (a) See Figure 1. The reduction in variability is clearly seen.
   (b) The Ljung-Box test is $Q(12) = 142.17$ with p-value close to zero. Thus, there are serial correlations in the GNP growth rates.
   (c) One-sample t-test is 22.70 with p-value also close to zero. The mean of GNP growth rate is significantly different from zero at the 5% level.
   (d) Yes, the augmented DF test is $-1.75$ with p-value 0.40. Therefore, we cannot reject the null hypothesis of having a unit root.

2. Again, consider the U.S. quarterly GNP growth rates of Problem 1.
   (a) The selected AR order is 9. After removing the insignificant estimates, the fitted model is
   \[ x_t = r_t - 0.0159 \\
   x_t = 0.42x_{t-1} + 0.20x_{t-2} - 0.11x_{t-3} - 0.10x_{t-5} + 0.18x_{t-9} + a_t, \]
   where $\hat{\sigma}^2 = 8.93 \times 10^{-5}$. Model checking indicates the fitted model is reasonable.
   (b) Yes, the AR(9) polynomial contains complex roots.
   (c) The 1-step to 4-step point forecasts are 0.0115, 0.0128, 0.0149, and 0.0148, respectively. The associated interval forecasts are
   \[ 0.0115 \pm 1.96(0.009), 0.0128 \pm 1.96(0.010), 0.0149 \pm 1.96(0.011), 0.0148 \pm 1.96(0.011). \]

   (a) The Ljung-Box test shows $Q(12) = 46.06$ with p-value $6.77 \times 10^{-6}$. Therefore, there are serial correlations in the log returns.
   (b) ACF indicates an MA(1) model. The fitted model is
   \[ r_t = 0.011 + a_t + 0.212a_{t-1}, \]
   where $\hat{\sigma}^2 = 0.0029$. The model checking indicates some large residuals (in absolute value). However, there is no serial correlations in the residuals.
   (c) The point predictions are $-0.010, 0.011, 0.011$, and $0.011$. The mean reverting feature is clearly seen.
4. Outliers

(a) The largest outlier, in absolute value, is at \( t = 5 \). The refined model is
\[
r_t = 0.011 - 0.336o5 + a_t + 0.228a_{t-1},
\]
where \( \hat{\sigma}^2 = 0.0028 \).

(b) The 2 largest outliers occurred at \( t = 5 \) and \( t = 574 \). The refined model is
\[
r_t = 0.011 - 0.337o5 - 0.309o574 + a_t + 0.226a_{t-1},
\]
where \( \hat{\sigma}^2 = 0.0026 \).

(c) Based on the two models with outliers, we see that the outliers only have small effects on the fitted model when the sample size is large.

5. Consider the daily close VIX index of CBOE from January 2, 2002 to July 1, 2011.

(a) Yes, there are serial correlations in the change log series. The Ljung-Box test is \( Q(10) = 75.77 \) with p-value close to zero.

(b) We start with a simple MA(2) model. The mean is not included because it is not significant based on the one-sample t-test. The model is
\[
c_t = a_t - 0.122a_{t-1} - 0.111a_{t-2},
\]
where \( \hat{\sigma}^2 = 38.08 \). Model checking indicates some further serial correlations at lag 6. Thus, we consider an MA(6) model. With insignificant parameters removed, the MA(6) model is
\[
c_t = a_t - 0.124a_{t-1} - 0.111a_{t-2} - 0.045a_{t-4} - 0.064a_{t-6},
\]
where \( \hat{\sigma}^2 = 37.79 \). Model checking indicates that the fitted model is reasonable.
Figure 1: Time plot of U.S. quarterly GNP growth rates from 1947 to 2011.