1. Monthly simple returns of Decile 2 and Decile 10 portfolios.

(a) The Ljung-Box statistics, using \( m = 12 \), give \( Q(12) = 17.18 \) with p-value 0.14 and \( Q(12) = 47.71 \) with p-value \( 3.51 \times 10^{-6} \). Therefore, at the 5% level, there are no serial correlations in Decile 2 returns, but there exist serial correlations in the Decile 10 returns.

(b) Even though the Ljung-Box test fails to indicate serial correlations in the Decile 2 returns, a careful examination of sample ACF of the returns indicates a marginally significant lag-1 ACF. Also, the t-test indicates the mean return is not zero. Thus, we consider an MA(1) model for the Decile 2 returns. The fitted model is

\[
    r_t = 0.0093 + a_t + 0.131a_{t-1}, \quad \sigma^2_a = 0.0022.
\]

Model checking fails to indicate any major violation. The model is reasonable.

(c) The point forecasts are

\[
    r_T(1) = -0.0013, \quad r_T(\ell) = 0.0093, \quad \ell > 1.
\]

2. Log yields of Moody’s AAA bonds. Let \( r_t \) be the logged yield. The sample ACFs of \( r_t \) are all close to 1, indicating a unit root. Figure 1 shows the sample ACF and PACF of the differenced series \( (1 - B)r_t \). The plots indicate either an MA(1) model [from ACF] and an AR(2) model [from PACF] for \( (1 - B)r_t \). The \texttt{ar} command also shows an AR(3) model for \( r_t \). Consequently, we have either an ARIMA(0,1,1) model or an ARIMA(2,1,0) model. We fitted the two competing models. Model checking shown in Figures 2 and 3 suggest that ARIMA(2,1,0) model fits the data slightly better. The fitted model is

\[
(1 - 0.37B + 0.16B^2)(1 - B)r_t = a_t, \quad \sigma^2_a = 0.00046.
\]

The model checking also indicates possible outliers in the data.
3. Monthly log AAA bond yields. Since exponential smoothing corresponds to an ARIMA(0,1,1) model, we use the model to produce forecasts. We re-fit the ARIMA(0,1,1) model using the data at the forecast origin November 2010, which corresponds to $t = 1103$. The 1-step to 12-step ahead point forecasts are 1.594 and the associated forecast standard errors are increasing from 0.0213 to 0.0989. This is a feature of the exponential smoothing (or ARIMA(0,1,1) model. Specifically, at the forecast origin $T$, the 1-step ahead forecast is

$$r_T(1) = E(r_{T+1}|F_T) = r_T - \theta a_T, \quad e_T(1) = r_{T+1} - r_{T}(1) = a_{T+1}. $$

The 2-step ahead forecast is, via direct calculation,

$$r_T(2) = E(r_{T+2}|F_T) = r_T(1), \quad e_T(2) = a_{T+2} + (1 - \theta_1)a_{T+1}. $$

The 3-step ahead forecast is

$$r_T(3) = E(r_{T+3}|F_T) = r_T(2) = r_T(1), \quad e_T(3) = a_{T+3} + (1 - \theta_1)a_{T+2} + (1 - \theta_1)a_{T+1}. $$

Therefore, the point forecasts are the same, but the variances of forecast errors are increasing.

4. Let $r_t$ be the log AAA bond yields and $x_t$ the log BAA bond yields. Following the same procedure as that of interest-rate example in the lecture note, we obtain the following results:

(a) The least squares linear regression

$$r_t = -0.359 + 1.081x_t + \epsilon_t,$$

is not adequate because the residuals have strong serial correlations.

(b) The differenced data gave

$$(1 - B)r_t = 0.642(1 - B)x_t + \epsilon_t. $$

The ACF of the residuals indicate the residuals follow an MA(1) model.

(c) The final model is

$$(1 - B)r_t - 0.623(1 - B)x_t = a_t + 0.333a_{t-1}, \quad \sigma^2_a = 0.00022. $$

Model checking indicates that the final model is reasonable. See Figure 4. There are possible outliers in the data.

5. Johnson and Johnson’s quarterly earnings. Let $r_t$ be the log earnings. The ACF of $(1 - B)(1 - B^4)r_t$ does not show significant serial correlations except a minor one at
lag-1. Therefore, we employ the model \((1 - B)(1 - B^4)r_t = (1 - \theta B)a_t\). The fitted model is
\[
(1 - B)(1 - B^4)r_t = a_t - 0.325a_{t-1}, \quad \sigma_a^2 = 0.001.
\]
The standard error of the MA coefficient is 0.142. This model fits the data well. See the model checking in Figure 5. To produce 1-step to 10-step out-of-sample forecasts at the forecast origin 2008.IV, which is \(t = 68\). We re-fit the model using the first 68 data points, then use the fitted model to produce forecasts and the standard errors of forecasts. The results are then used to produce forecast plot. [You may use foreplot.R]. See Figure 6

Figure 1: Sample ACF and PACF of the differenced AAA bond yields.
Figure 2: Model checking of the ARIMA(0,1,1) model for the log AAA bond yields.

Figure 3: Model checking of the ARIMA(2,1,0) model for the log AAA bond yields.
Figure 4: Model checking for the regression model with time series errors of Problem 4.

Figure 5: Model checking for the seasonal model of Problem 5.
Figure 6: Forecasting plot of Problem 5. Solid line denotes actual observations, circles are point forecasts, and dashed lines indicate 95% interval forecasts.