Solutions to Homework Assignment #6

This assignment is concerned with Value at Risk and Expected Shortfall. Assume that the tail probability of interest is 1% and the portfolio consists of two stocks, namely Amazon (AMZN) and Coca Cola (KO). The sample period is from January 5, 2004 to May 18, 2012. The data can be downloaded from Yahoo via the \texttt{quantmod} package. Use the adjusted closing prices to compute the daily log returns. For Questions 1 to 4, assume that you hold (long position) both stocks valued at $1 million each.

1. Consider the AMZN stock only.

   - The IGARCH estimation shows that the $\beta$ estimate is essentially 1. This is due to the fact that the RiskMetrics model is mis-specified. In any case, the program continues to provide volatility estimate. [The error message is concerned the estimate of the standard error of $\beta$.] Based on the result, we have $\text{VaR} = 0.0665$ and $\text{ES} = 0.0762$ for the log returns when the tail probability is 1%. Therefore, for holding $1$ million of the stock, we have $\text{VaR} = 66,500$ and $\text{ES} = 76,200$.

   - The mean return is not significantly different from zero so we omit it. The fitted Gaussian GARCH(1,1) model is

     \[
     r_t = a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1)
     \]

     \[
     \sigma_t^2 = 8.13 \times 10^{-5} + 0.044 a_{t-1}^2 + 0.856 \sigma_{t-1}^2.
     \]

     Based on the model, we have $\text{VaR} = 63,004$ and $\text{ES} = 72,182$.

   - The fitted GARCH(1,1) model with $t$-innovations are

     \[
     r_t = a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t^{3.691}
     \]

     \[
     \sigma_t^2 = 5.401 \times 10^{-6} + 0.02 a_{t-1}^2 + 0.972 \sigma_{t-1}^2.
     \]

     Based on the model, we have $\text{VaR} = 102,202$ and $\text{ES} = 145,805$.

2. The estimates are $\xi = 0.371(0.096)$, $\sigma = 0.0165(0.0016)$, and $\mu = 0.0327(0.0019)$, where the number in parentheses are standard error. Based on the fitted parameters, we have $\text{VaR} = 67,346$. For the next 10-trading days, we have $\text{VaR} = 158,398$. 

3. With threshold 3%, the estimates (standard errors) are $\xi = 0.343(0.105)$ and $\beta = 0.0144(0.0018)$. The VaR and ES are $74,708$ and $120,037$, respectively.

For threshold 4.5%, the estimates (standard errors) are $\xi = 0.458(0.206)$ and $\beta = 0.0165(0.0038)$. The risk measures are VaR = $72,949$ and ES = $126,919$, respectively.

Comparing the results, we see that the risk measures are not sensitive to the choices of the two thresholds.

4. RiskMetrics: For KO stock, we have VaR = $18,439$. For the combined position, we use the sample correlation $0.343$ and obtain $\sqrt{(66500)^2 + (18439)^2 + 2(0.343)(66500)(18439)} = $74,856.

For time-varying correlation, we fitted IGARCH models to the sum and difference of two log returns to obtain the time-varying covariance, which is $1.189 \times 10^{-4}$. The correlation is $0.562$. The VaR for the combined position is then $\sqrt{(66500)^2 + (18439)^2 + 2(0.562)(66500)(18439)} = $78,361.

5. The AMZN stock is in short position so that we use the log returns. The sample correlation then is $-0.343$. In this case, the VaR is $\sqrt{(66500)^2 + (18439)^2 + 2(-0.343)(66500)(18439)} = $62,619 for the RiskMetrics approach.

For GARCH(1,1) with Student-\(t\) innovations, the fitted model for KO stock returns is

\[
\begin{align*}
    r_t &= a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{5.296}^* \\
    \sigma_t^2 &= 1.775 \times 10^{-6} + 0.068 a_{t-1}^2 + 0.918 \sigma_{t-1}^2. 
\end{align*}
\]

Based on the model, we have VaR = $29,410$. For the AMZN stock, we have (from problem 1), VaR = $102,202$. Therefore, using the sample correlation coefficient, we have VaR = $\sqrt{(102202)^2 + (29410)^2 + 2(-0.343)(102202)(29410)} = $96,168.