Midterm

ChicagoBooth Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signature: Name: ID:

Notes:
- Open notes and books.
- For each question, write your answer in the blank space provided.
- Manage your time carefully and answer as many questions as you can.
- The exam has 10 pages and the R output has 9 pages. Total is 19 pages. Please check to make sure that you have all the pages.
- For simplicity, ALL tests use the 5% significance level.
- Round your answer to 3 significant digits.
- This is a 2-hour exam (120 minutes).

Problem A: (34 pts) Answer briefly the following questions. Each question has two points.

1. Describe two improvements of the EGARCH model over the GARCH volatility model.
2. Describe two methods that can be used to infer the existence of ARCH effects in a return series, i.e., volatility is not constant over time.

3. Consider the IGARCH(1,1) volatility model: \( a_t = \sigma_t \epsilon_t \) with \( \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \). Often one pre-fixes \( \alpha_0 = 0 \). Why? Also, suppose that \( \alpha_0 = 0 \) and the 1-step ahead volatility prediction at the forecast origin \( h \) is 16.2\% (annualized), i.e., \( \sigma_h(1) = \sigma_{h+1} = 16.2 \) for the percentage log return. What is the 10-step ahead volatility prediction? That is, what is \( \sigma_h(10) \)?

4. (Questions 4 to 8) Consider the daily log returns of Amazon stock from January 3, 2007 to April 27, 2012. Some summary statistics of the returns are given in the attached R output. Is the expected (mean) return of the stock zero? Why?

5. Let \( k \) be the excess kurtosis. Test \( H_0 : k = 0 \) versus \( H_a : k \neq 0 \). Write down the test statistic and draw the conclusion.

6. Are there serial correlations in the log returns? Why?

7. Are there ARCH effects in the log return series? Why?

8. Based on the summary statistics provided, what is the 22-step ahead point forecast of the log return at the forecast origin April 27, 2012? Why?
9. Give two reasons that explain the existence of serial correlations in observed asset returns even if the true returns are not serially correlated.

10. Give two reasons that may lead to using moving-average models in analyzing asset returns.

11. Describe two methods that can be used to compare different models for a given time series.

12. (Questions 12 to 14) Let $r_t$ be the daily log returns of Stock A. Assume that $r_t = 0.004 + a_t$, where $a_t = \sigma t \epsilon_t$ with $\epsilon_t$ being iid N(0,1) random variates and $\sigma^2_t = 0.017 + 0.15a^2_{t-1}$. What is the unconditional variance of $a_t$?

13. Suppose that the log price at $t = 100$ is 3.912. Also, at the forecast origin $t = 100$, we have $a_{100} = -0.03$ and $\sigma_{100} = 0.025$. Compute the 1-step ahead forecast of the log price (not log return) and its volatility for Stock A at the forecast origin $t = 100$.

14. Compute the 30-step ahead forecast of the log price and its volatility of Stock A at the forecast origin $t = 100$.
15. Asset volatility has many applications in finance. Describe two such applications.

16. Suppose the log return $r_t$ of Stock A follows the model $r_t = a_t$, $a_t = \sigma_t \epsilon_t$, and $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$, where $\epsilon_t$ are iid N(0,1). Under what condition that the kurtosis of $r_t$ is 3? That is, state the condition under which the GARCH dynamics fail to generate any additional kurtosis over that of $\epsilon_t$.

17. What is the main consequence in using a linear regression analysis when the serial correlations of the residuals are overlooked?
Problem B. (30 pts) Consider the daily log returns of Amazon stock from January 3, 2007 to April 27, 2012. Several volatility models are fitted to the data and the relevant R output is attached. Answer the following questions.

1. (2 points) A volatility model, called \( m_1 \) in R, is entertained. Write down the fitted model, including the mean equation. Is the model adequate? Why?

2. (3 points) Another volatility model, called \( m_2 \) in R, is fitted to the returns. Write down the model, including all estimated parameters.

3. (2 points) Based on the fitted model \( m_2 \), test \( H_0 : \nu = 5 \) versus \( H_a : \nu \neq 5 \), where \( \nu \) denotes the degrees of freedom of Student-t distribution. Perform the test and draw a conclusion.

4. (3 points) A third model, called \( m_3 \) in R, is also entertained. Write down the model, including the distributional parameters. Is the model adequate? Why?

5. (2 points) Let \( \xi \) be the skew parameter in model \( m_3 \). Does the estimate of \( \xi \) confirm that the distribution of the log returns is skewed? Why? Perform the test to support your answer.

6. (3 points) A fourth model, called \( m_4 \) in R, is also fitted. Write down the fitted model, including the distribution of the innovations.
7. (2 points) Based on model $m_4$, is the distribution of the log returns skewed? Why? Perform a test to support your answer.

8. (2 points) Among models $m_1, m_2, m_3, m_4$, which model is preferred? State the criterion used in your choice.

9. (2 points) Since the estimates $\hat{\alpha}_1 + \hat{\beta}_1$ is very close to 1, we consider an IGARCH(1,1) model. Write down the fitted IGARCH(1,1) model, called $m_5$.

10. (2 points) Use the IGARCH(1,1) model and the information provided to obtain 1-step and 2-step ahead predictions for the volatility of the log returns at the forecast origin $t = 1340$.

11. (2 points) A GARCH-M model is entertained for the percentage log returns, called $m_6$ in the R output. Based on the fitted model, is the risk premium statistical significant? Why?

12. (3 points) Finally, a GJR-type model is entertained, called $m_7$. Write down the fitted model, including all parameters.

13. (2 points) Based on the fitted GJR-type of model, is the leverage effect significant? Why?
Problem C. (14 pts) Consider the quarterly earnings per share of Abbott Laboratories (ABT) stock from 1984.III to 2011.III for 110 observations. We analyzed the logarithms of the earnings. That is, \( x_t = \ln(y_t) \), where \( y_t \) is the quarterly earnings per share. Two models are entertained.

1. (3 points) Write down the model \( m_1 \) in R, including residual variance.

2. (2 points) Is the model adequate? Why?

3. (3 points) Write down the fitted model \( m_2 \) in R, including residual variance.

4. (2 points) Model checking of the fitted model \( m_2 \) is given in Figure 1. Is the model adequate? Why?

5. (2 points) Compare the two fitted model models. Which model is preferred? Why?

6. (2 points) Compute 95% interval forecasts of 1-step and 2-step ahead log-earnings at the forecast origin \( t = 110 \).
**Problem D.** (22 pts) Consider the growth rate of the U.S. weekly regular gasoline price from January 06, 1997 to September 27, 2010. Here growth rate is obtained by differencing the log gasoline price and denoted by $g_t$ in R output. The growth rate of weekly crude oil from January 03, 1997 to September 24, 2010 is also obtained and is denoted by $p_t$ in R output. Note that the crude oil price was known 3 days prior to the gasoline price.

1. (2 points) First, a pure time series model is entertained for the gasoline series. An AR(5) model is selected. Why? Also, is the mean of the $g_t$ series significantly different from zero? Why?

2. (2 points) Write down the fitted AR(5) model, called $m_2$, including residual variance.

3. (2 points) Since not all estimates of model $m_2$ are statistically significant, we refine the model. Write down the refined model, called $m_3$.

4. (2 points) Is the refined AR(5) model adequate? Why?

5. (2 points) Does the gasoline price show certain business-cycle behavior? Why?

6. (3 points) Next, consider using the information of crude oil price. Write down the linear regression model, called $m_4$, including $R^2$ and residual standard error.
7. (2 points) Is the fitted linear regression model adequate? Why?

8. (3 points) A linear regression model with time series errors is entertained and insignificant parameters removed. Write down the final model, including all fitted parameters.

9. (2 points) Model checking shows that the fitted final model has no residual serial correlations. Based on the model, is crude oil price helpful in predicting the gasoline price? Why?

10. (2 points) Compare the pure time series model and the regression model with time-series errors. Which model is preferred? Why?
Figure 1: Model checking for model \textbf{m2} of Problem C.

\textbf{R output:} edited

```
##### Problem A #### Amazon daily log returns
> getSymbols("AMZN")
> dim(AMZN)
[1] 1341 6
> head(AMZN)
   AMZN.Open AMZN.High AMZN.Low AMZN.Close AMZN.Volume AMZN.Adjusted
2007-01-03 38.68 39.06 38.05 38.70 12405100 38.70
   ....
2007-01-10 37.49 37.70 37.07 37.15  6527500 37.15
> rtn=diff(log(as.numeric(AMZN$AMZN.Adjusted)))
> basicStats(rtn)

   rtn
nobs  1340.000000
NAs   0.000000
Minimum -0.136759
Maximum  0.238621
Mean    0.001320
Median  0.000268
LCL Mean -0.000309
UCL Mean  0.002949
```
Variance 0.000924
Stdev 0.030398
Skewness 1.065340
Kurtosis 9.874977

> Box.test(rtn,lag=10,type='Ljung')
Box-Ljung test
data: rtn
X-squared = 10.6878, df = 10, p-value = 0.3824

> Box.test(rtn^2,lag=10,type='Ljung')
Box-Ljung test
data: rtn^2
X-squared = 39.2401, df = 10, p-value = 2.304e-05

##### Problem B ########################################
> pp=pacf(rtn^2)
> pp$acf
  [1,] 0.1150486318   %%% Lag-1 is larger than others
  [2,] 0.0084316679
  [3,] 0.0007132578
  [4,] 0.0261869924
  [5,] 0.0448622758

> m1=garchFit(~garch(1,0),data=rtn,trace=F)
> summary(m1)

Title: GARCH Modelling
Call: garchFit(formula = ~garch(1, 0), data = rtn, trace = F)
Mean and Variance Equation:
  data ~ garch(1, 0)
  [data = rtn]
Conditional Distribution: norm
Error Analysis:

  Estimate  Std. Error   t value  Pr(>|t|)
 mu        1.804e-03   7.866e-04    2.294    0.0218  *
 omega     7.577e-04   3.536e-05   21.428   < 2e-16 ***
 alpha1    1.883e-01   3.891e-02    4.840    1.3e-06 ***
---

Standardised Residuals Tests:

  Statistic   p-Value
 Jarque-Bera Test R  Chi^2  7950.315  0
 Shapiro-Wilk Test R  W  0.9015266  0
 Ljung-Box Test   R  Q(10)  8.114605  0.6176434
 Ljung-Box Test   R  Q(20) 24.58853  0.2176286
Ljung-Box Test $R^2$ Q(10) 3.992687 0.9476763
Ljung-Box Test $R^2$ Q(20) 8.754246 0.9855828

Information Criterion Statistics:

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<td>m2</td>
<td>-4.196024</td>
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> m2=garchFit(~garch(1,0),data=rtn,trace=F,cond.dist="std")
> summary(m2)

Title: GARCH Modelling
Call: garchFit(formula = ~garch(1, 0), data = rtn, cond.dist = "std", trace = F)

Mean and Variance Equation:

data ~ garch(1, 0); [data = rtn]

Conditional Distribution: std

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| mu       | 4.907e-04  | 6.260e-04 | 0.784    | 0.433169 |
| omega    | 7.463e-04  | 8.204e-05 | 9.098    | < 2e-16 *** |
| alpha1   | 2.026e-01  | 5.844e-02 | 3.467    | 0.000526 *** |
| shape    | 3.562e+00  | 3.664e-01 | 9.721    | < 2e-16 *** |

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Standardised Residuals Tests:

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<td>Ljung-Box Test $R$ Q(20)</td>
<td>24.43411 0.2239435</td>
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<tr>
<td>Ljung-Box Test $R^2$ Q(10)</td>
<td>3.4091 0.970095</td>
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<td>7.570487 0.9943495</td>
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> m3=garchFit(~garch(1,0),data=rtn,trace=F,cond.dist="sstd")
> summary(m3)

Title: GARCH Modelling
Call: garchFit(formula =~garch(1, 0), data=rtn, cond.dist = "sstd", trace = F)

Mean and Variance Equation:

data ~ garch(1, 0); [data = rtn]

Conditional Distribution: sstd
Error Analysis:

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|----------|
| mu        | 1.698e-03  | 6.964e-04 | 2.437   | 0.014791 * |
| omega     | 1.066e-05  | 5.789e-06 | 1.841   | 0.065549 . |
| alpha1    | 4.143e-02  | 1.228e-02 | 3.374   | 0.000741 *** |
| beta1     | 9.495e-01  | 1.512e-02 | 62.793  | < 2e-16 *** |
| skew      | 1.101e+00  | 4.298e-02 | 25.604  | < 2e-16 *** |
| shape     | 3.714e+00  | 3.869e-01 | 9.600   | < 2e-16 *** |

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<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.451153</td>
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<td>Ljung-Box Test R^2 Q(20)</td>
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<td>-4.412357</td>
<td>-4.431789</td>
<td>-4.424492</td>
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> m4=garchFit("garch(1,1),data=rtn,trace=F,cond.dist="sstd")
> summary(m4)

Title: GARCH Modelling
Call: garchFit(formula = "garch(1,1), data=rtn, cond.dist = "sstd", trace = F)

Mean and Variance Equation:
- data ~ garch(1, 1); [data = rtn]
Conditional Distribution: sstd

Error Analysis:

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|----------|
| mu        | 1.162e-03  | 7.387e-04 | 1.573   | 0.11581 |
| omega     | 7.418e-04  | 8.114e-05 | 9.142   | < 2e-16 *** |
| alpha1    | 2.081e-01  | 5.950e-02 | 3.497   | 0.00047 *** |
| skew      | 1.065e+00  | 3.904e-02 | 27.278  | < 2e-16 *** |
| shape     | 3.591e+00  | 3.737e-01 | 9.609   | < 2e-16 *** |

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Standardised Residuals Tests:

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<td>Ljung-Box Test R Q(10)</td>
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<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>19.32646</td>
</tr>
</tbody>
</table>
Ljung-Box Test : R^2 Q(10) 2.374727 0.9925782
Ljung-Box Test : R^2 Q(20) 3.763233 0.9999719

Information Criterion Statistics:
  AIC    BIC    SIC    HQIC
-4.485511 -4.462226 -4.485551 -4.476787

> source("Igarch.R")
> m5=Igarch(rtn,volcnt=T)
Estimates:  3.858622e-05 0.85
Maximized log-likehood: -2785.96
Coefficient(s):

    Estimate Std. Error t value  Pr(>|t|)  
omega 3.85862e-05 7.88909e-06 4.89109 1.0028e-06 ***
beta  8.50000e-01 2.63702e-02 32.23338 < 2.22e-16 ***

> names(m5)
[1] "par" "volatility"

> vol=m5$volatility
> length(rtn)
[1] 1340
> rtn[1340]
[1] 0.1462254
> vol[1340]
[1] 0.02108403

> source("garchM.R")
> rtn=rtn*100
> m6=garchM(rtn,type=2)
Maximized log-likehood: 3342.168
Coefficient(s):

    Estimate Std. Error t value Pr(>|t|)    
mu   0.5160324  0.5747823 0.89779  0.36929884
gamma -0.1119610  0.2000722 -0.55960  0.57575039
omega 0.7569616  0.2187081 3.46106  0.00053805 ***
alpha 0.0522117  0.0150660 3.46552  0.00052920 ***
beta  0.8658176  0.0352917 24.53316 < 2.22e-16 ***

> m7=garchFit(~aparch(1,1),data=rtn,trace=F,cond.dist="sstd",delta=2,include.delta=F)
> summary(m7)
Title: GARCH Modelling

Call: garchFit(formula =~aparch(1, 1), data =rtn, delta=2, cond.dist= "sstd",
   include.delta = F, trace = F)

Mean and Variance Equation:
   data ~ aparch(1, 1); [data = rtn]
Conditional Distribution: sstd

Error Analysis:

|                | Estimate   | Std. Error | t value | Pr(>|t|) |
|----------------|------------|------------|---------|----------|
| mu             | 1.407e-03  | 6.827e-04  | 2.061   | 0.039276 * |
| omega          | 7.583e-06  | 4.454e-06  | 1.703   | 0.088623 . |
| alpha1         | 3.622e-02  | 9.691e-03  | 3.738   | 0.000186 *** |
| gamma1         | 4.776e-01  | 9.727e-02  | 4.910   | 9.11e-07 *** |
| beta1          | 9.533e-01  | 1.243e-02  | 76.685  | < 2e-16 *** |
| skew           | 1.098e+00  | 4.374e-02  | 25.099  | < 2e-16 *** |
| shape          | 3.846e+00  | 4.049e-01  | 9.499   | < 2e-16 *** |

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<td>16.7532</td>
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<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.704707</td>
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<td>-4.493662</td>
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Problem C

```r
> da=read.table("q-abt-earns.txt",header=T)
> head(da)
FPEDATS MEASURE FPI ACTUAL
1 19840930 EPS 6 0.0475
6 19851231 EPS 6 0.0738
> abt=da$ACTUAL
> plot(abt,type='l')
> abt=log(abt) ### Log earnings
> m1=arima(abt,order=c(0,1,1),seasonal=list(order=c(0,1,1),period=4))
> m1
```

15
Call: arima(x = abt, order = c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 4))

Coefficients:
  ma1   sma1
       -0.5652  -0.1834

s.e.  0.1281  0.0830

sigma^2 estimated as 0.001608: log likelihood = 186.64, aic = -367.28

> Box.test(m1$residuals, lag = 12, type = 'Ljung')

Box-Ljung test
data: m1$residuals
X-squared = 25.7627, df = 12, p-value = 0.01159

> m2 = arima(abt, order = c(0, 1, 3), seasonal = list(order = c(0, 1, 0), period = 4))
> m2

Call: arima(x = abt, order = c(0, 1, 3), seasonal = list(order = c(0, 1, 0), period = 4))

Coefficients:
  ma1   ma2   ma3
-0.4428 -0.0613 -0.2853

s.e.  0.0929  0.1081  0.0902

sigma^2 estimated as 0.001435: log likelihood = 192.42, aic = -376.84

> c1 = c(NA, 0, NA)
> m2 = arima(abt, order = c(0, 1, 3), seasonal = list(order = c(0, 1, 0), period = 4), fixed = c1)
> m2

Call: arima(x = abt, order = c(0, 1, 3), seasonal = list(order = c(0, 1, 0), period = 4), fixed = c1)

Coefficients:
  ma1   ma2   ma3
-0.4696  0 -0.3121

s.e.  0.0817  0.0754

sigma^2 estimated as 0.00144: log likelihood = 192.26, aic = -378.52

> tsdiag(m2, gof = 16)
> predict(m2, 4)

$pred
Time Series:
Start = 110
End = 113
Frequency = 1
$se$

Time Series:
Start = 110
End = 113
Frequency = 1

[1] 0.37479267 0.01878729 0.22603233 0.27821808

### Problem D ###

```r
> da=read.table("w-gasoline.txt")
> da1=read.table("w-petroprice.txt",header=T)
> head(da1)
    Mon Day Year World US
1   1   3 1997   23.18  22.90
....
> gt=diff(log(da[,1]))
> pt=diff(log(da1$US))
> cor(gt,pt)
[1] 0.5795378
> m1=ar(gt,method="mle")
> m1$order
[1] 5
> t.test(gt)

One Sample t-test

data:  gt
t = 1.3062, df = 715, p-value = 0.1919
alternative hypothesis: true mean is not equal to 0

> m2=arima(gt,order=c(5,0,0),include.mean=F)
> m2

Call: arima(x = gt, order = c(5, 0, 0), include.mean = F)

Coefficients:
           ar1      ar2      ar3     ar4     ar5
     0.5073   0.0788   0.1355  -0.0360  -0.0862
s.e. 0.0372  0.0417  0.0415  0.0417  0.0372

sigma^2 estimated as 0.0003262: log likelihood = 1857.85, aic = -3703.71
```

```r
> c1=c(NA,NA,NA,0,NA)
> m3=arima(gt,order=c(5,0,0),include.mean=F,fixed=c1)
> m3

Call: arima(x = gt, order = c(5, 0, 0), include.mean = F, fixed = c1)
17`
Coefficients:

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<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
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<tr>
<td>s.e.</td>
<td>0.0370</td>
<td>0.0418</td>
<td>0.0385</td>
<td>0</td>
<td>0.0330</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 0.0003265: log likelihood=1857.48, aic = -3704.96

> Box.test(m3$residuals, lag=14, type='Ljung')

Box-Ljung test
data: m3$residuals
X-squared = 10.2668, df = 14, p-value = 0.7424

> p1=c(1,-.5036,-.0789,-.1220,0,.1009)

> p1
[1] 1.0000 -0.5036 -0.0789 -0.1220 0.0000 0.1009

> mm=polyroot(p1)

> mm
[1] 1.355223+0.42689i -0.404194+1.55485i -0.404194-1.55485i 1.355223-0.42689i
[5] -1.902059+0.00000i

> Mod(mm)
[1] 1.420867 1.606530 1.606530 1.420867 1.902059

> m4=lm(gt~-1+pt)

> summary(m4)

Call: lm(formula = gt ~ -1 + pt)

Coefficients:

|     | Estimate | Std. Error | t value | Pr(>|t|) |
|-----|----------|------------|---------|----------|
| pt  | 0.28703  | 0.01507    | 19.05   | <2e-16 *** |

---

Residual standard error: 0.01839 on 715 degrees of freedom
Multiple R-squared: 0.3366, Adjusted R-squared: 0.3357

> Box.test(m4$residuals, lag=10, type='Ljung')

Box-Ljung test
data: m4$residuals
X-squared = 273.2459, df = 10, p-value < 2.2e-16

> m5=ar(m4$residuals, method="mle")

> m5$order
[1] 6

> m6=arima(gt, order=c(6,0,0), xreg=pt, include.mean=F)

> m6
Call: arima(x = gt, order = c(6, 0, 0), xreg = pt, include.mean = F)
Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>ar6</th>
<th>pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.e.</td>
<td>0.3953</td>
<td>0.1634</td>
<td>0.0946</td>
<td>0.0297</td>
<td>-0.0873</td>
<td>-0.0525</td>
<td>0.1927</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 0.0002524: log likelihood = 1949.61, aic = -3883.21

> m6=arima(gt,order=c(5,0,0),xreg=pt,include.mean=F)
> m6
Call:arima(x = gt, order = c(5, 0, 0), xreg = pt, include.mean = F)
Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.e.</td>
<td>0.4022</td>
<td>0.1621</td>
<td>0.0899</td>
<td>0.0209</td>
<td>-0.1086</td>
<td>0.1914</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 0.0002531: log likelihood = 1948.62, aic = -3883.23

> c2=c(NA,NA,NA,0,NA,NA)
> m7=arima(gt,order=c(5,0,0),xreg=pt,include.mean=F,fixed=c2)
> m7
Call:arima(x = gt, order = c(5,0,0), xreg = pt, include.mean = F, fixed=c2)
Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.e.</td>
<td>0.4037</td>
<td>0.1642</td>
<td>0.0961</td>
<td>0</td>
<td>-0.1014</td>
<td>0.1911</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 0.0002532: log likelihood = 1948.48, aic = -3884.95

> Box.test(m7$residuals,lag=10,type='Ljung')
Box-Ljung test

<p>| | | | | | | |</p>
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</thead>
</table>

data: m7$residuals
X-squared = 4.7748, df = 10, p-value = 0.9057