Assignment:

1. Consumer Sentiment of the University of Michigan.
   (a) See Figure 1.
   (b) Yes, there is a unit root in the series. There are two ways to see this. The first approach is to compute sample ACF of the series. The ACFs are high and decay slowly. The second approach is to perform unit-root tests. We perform augmented Dickey-Fuller tests with various options concerning the deterministic component (No constant, constant, time-trend), and all results fail to reject the null hypothesis of a unit root. The test results are shown in the R output.
   (c) Based on the one-sample t statistic, one cannot reject the null hypothesis of zero mean for the change series. The t-ratio is $-0.019$ with $p$ value 0.98.
   (d) The Ljung-Box statistics show $Q(12) = 24.11$ with $p$-value 0.020. Consequently, the null hypothesis is rejected at the 5% level. There is serial dependence in the change series.

2. The change series of consumer sentiment.
   (a) An AR(5) model is selected by the command.
   (b) Let $c_t$ be the change series. The fitted model is
   \[(1 + 0.0499B + 0.126B^2 + 0.083B^3 + 0.0498B^4 + 0.1107B^5)c_t = a_t,\]
   where $B$ is the backshift operator and the residual variance is 15.46. The model checking of the fitted AR(5) model is shown in Figure 2. The model appears to be adequate.
   (c) Yes, because the characteristic equation of the fitted AR(5) model contains complex roots.
   (d) The 1-step to 4-step ahead point forecasts are $-0.1039, -0.5744, 0.2852, 0.1059$, respectively. The 95% confidence interval forecasts are as follows: (i) lower limits: $-7.811, -8.291, -7.491$ and $-7.689$; (ii) upper limits: $7.603, 7.142, 8.061,$ and $7.901$.

3. The monthly change series of consumer sentiment of Problem 1.
   (a) The simplified model, by fixing the AR-1 and AR-4 coefficients to zero, is
   \[(1 + 0.119B^2 + 0.075B^3 + 0.107B^5)c_t = a_t, \quad \sigma^2 = 15.54.\]
   (b) Except for a possible outlier, the fitted model is adequate. See the model checking in Figure 3.
(c) For in-sample fitting, one can use the AIC criterion to compare models. In this particular instance, AIC for the simplified model is smaller (1291.17 vs 1293.17). Therefore, the simplified model is preferred.

(d) Yes, the simplified model continues to indicate the existence of business cycle in the data, because its characteristic equation has complicated solutions.

(e) Based on the backtest with 1-step ahead predictions, the simplified AR(5) model is slightly preferred. The RMSE and MAE of the forecasts for the simplified model are smaller; (3.9955, 3.1040) vs (3.9886, 3.0901).


(a) Yes, it contains a unit root. This is based on either the sample ACF of the duration or the unit-root test. The ACFs are high and decay very slowly. The augmented Dickey-Fuller test has a test statistic $-1.33$ with $p$-value 0.56. If one plots the time series, one can see an upward trend in the mean duration of unemployment.

(b) The one-sample $t$ test shows a $t$-ratio of 1.65 with $p$-value about 0.1. Thus, one cannot reject the null hypothesis of zero mean of the change series in duration at the 5% level.

(c) An AR(12) model is used for the change series. Model checking is shown in Figure 4. From the plots, except for one or two possible outliers, the model is adequate.

(d) The fitted AR(12) model is

$$
(1 + 0.14B - 0.11B^2 - 0.11B^3 - 0.12B^4 - 0.08B^5 - 0.08B^6 - 0.09B^7 + 0.06B^8
+ 0.07B^9 - 0.06B^{10} - 0.02B^{11} + 0.07B^{12})d_t = a_t,$$

where $r_t$ denotes the change series of mean duration and variance of $a_t$ is 0.297.

(e) The fitted seasonal model is

$$
(1 - 0.65B - 0.26B^2)(1 - 0.57B^{12})r_t = (1 - 0.80B)(1 - 0.74B^{12})a_t, \quad \sigma^2 = 0.293.
$$

Model checking is shown in Figure 5. The model appears to be adequate.

(f) Based on the in-sample fit, AIC selects the seasonal model as its has a lower value.

(g) Based on out-of-sample predictions, the seasonal model is also preferred. The seasonal model has smaller RMSE and MAE for the 1-step ahead predictions.


(a) Yes, there is significant serial correlation in the time series $r_t$. The Ljung-Box statistics give $Q(10) = 45.53$ with a small $p$-value.

(b) The AIC criterion selects an AR(11) model. The estimation results show some insignificant estimates. Simplifying the model by removing these estimates, we obtain the model

$$
(1 - 0.11B + 0.05B^2 - 0.10B^3 - 0.04B^4 + 0.03B^7 - 0.09B^8 + 0.09B^{11})r_t = a_t,
$$

where $\sigma^2 = 0.0018$. Model checking plots are given in Figure 6. Except for possible outliers, the model is adequate.

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Figure 1: Time series plot of the monthly consumer confidence of the University of Michigan from 1978 to August 2013.

(c) The fitted ARIMA(3,0,2) model is

\[(1 - 0.57B + 0.85B^2 - 0.17B^3)r_t = (1 - 0.47B + 0.78B^2)a_t, \quad \sigma^2 = 0.0018.\]

This model is also adequate. For the in-sample fitting, AIC prefers the ARIMA(3,0,2) model. This example shows that ARMA models may fare better than the AR models in some applications.
Figure 2: Model checking for an AR(5) model for the change series of monthly consumer sentiment.

Figure 3: Model checking for a simplified AR(5) model fitted to the change series of monthly consumer sentiment.
Figure 4: Model checking for an AR(12) model for the change series of monthly mean unemployment durations.

Figure 5: Model checking for a seasonal model fitted to the change series of mean unemployment durations.
Figure 6: Model checking for a simplified AR(11) model for the growth rates of weekly crude oil prices.