Solutions to Homework Assignment #3


(a) ACF of the data shows a significant lag-1 correlation. The mean equation is then
\[ r_t = 0.004 + a_t + 0.154a_{t-1}, \quad \sigma^2 = 0.00078, \]
where the estimates are significantly different from zero.

(b) Yes, the MA(1) model is adequate. See Figure 1 for model checking.

(c) Based on the fitted MA(1) model, the 1-step and 2-step point forecasts are 0.0025 and 0.00399, respectively. The 95% interval forecasts are \((-0.0524, 0.0574)\) and \((-0.0515, 0.0595)\), respectively.

2. Monthly market liquidity measure of Professors Pastor and Stambaugh.

(a) Based on the PACF, an AR(5) model is entertained. The fitted model is
\[(1 - 0.062B - 0.186B^2 - 0.131B^3 - 0.011B^4 - 0.110B^5)(x_t + 0.031) = a_t, \]
where \(\sigma^2 = 0.0036\).

(b) The model is reasonable except for a possible (negative) outlier. See the standardized residual plot in Figure 2.

(c) The outlier occurs at \(t = 303\). Define an indicator variable
\[ I_t^{(303)} = \begin{cases} 
1, & \text{if } t = 303 \\
0, & \text{otherwise.} 
\end{cases} \]
The model with outlier \(I_t^{(303)}\) is
\[(1 - 0.066B - 0.188B^2 - 0.131B^3 - 0.016B^4 - 0.126B^5)(x_t + 0.0299 + 0.426I_t^{(303)}) = a_t, \]
where \(\sigma^2 = 0.00324\).

(d) The coefficient with the smallest \(t\)-ratio (in absolute value) is lag-4 AR coefficient. Fixing \(\phi_4 = 0\), we obtain the model
\[(1 - 0.068B - 0.192B^2 - 0.132B^3 - 0.127B^5)(x_t + 0.0299 + 0.425I_t^{(303)}) = a_t, \]
where \(\sigma^2 = 0.00324\).
3. Quarterly earnings per share of Microsoft. Let \( x_t \) be the logarithm of earnings.

(a) The ACFs of \( x_t \), diff\((x_t)\), and diff(diff\((x_t),4)\) suggest an airline model. The fitted model is

\[
(1 - B)(1 - B^4)x_t = (1 - 0.48B)(1 - 0.71B^4)a_t, \quad \sigma^2 = 0.0212.
\]

Model checking is shown in Figure 3. The model is reasonable except for some possible outliers.

(b) The fitted model is

\[
(1 - B)x_t = (1 - 0.27B)(1 + 0.39B^4)a_t, \quad \sigma^2 = 0.0257.
\]

(c) The airline model is preferred because it has a smaller AIC value and its \( \sigma^2 \) is also smaller.

(d) The out-of-sample forecasting performance also selects the airline model. It has lower RMSE and MAE.

4. Monthly Fama-Bliss bond yields with maturities 1 and 3 years.

(a) The fitted linear regression model is

\[
y_{3t} = 0.713 + 0.941y_{1t} + \epsilon_t, \quad \sigma = 0.529
\]

with \( R^2 = 96.91\% \). The model is not adequate because its residuals have strong serial correlations.

(b) The fitted model is

\[
d_{3t} = 0.736d_{1t} + \epsilon_t, \quad \sigma = 0.18,
\]

with \( R^2 = 79.63\% \). We may take the first difference because the residuals of the model in part (a) has strong serial correlations. See Figure 4.

(c) No, the model is not adequate because its residuals have some serial correlations. The AIC criterion selects an AR(5) model. The model then becomes

\[
(1 + 0.118B + 0.085B^2 - 0.033B^3 + 0.089B^4 + 0.153B^5)(d_{3t} - 0.744d_{1t}) = a_t,
\]

where \( \sigma^2 = 0.031 \). This model is reasonable; see Figure 5. One may remove the lag-3 AR coefficient, but this is not critical.

(d) The bond yields are related in the change series. Basically, we have

\[
y_{3t} - y_{3,t-1} = 0.743(y_{1t} - y_{1,t-1}) + x_t,
\]

where \( x_t \) follows an AR(5) model.

5. Consider again the bond yields of Problem 4.
Figure 1: Model checking of the MA(1) model for hml returns

(a) The fitted AR(6) model is

\[(1 - 0.874B - 0.033B^2 - 0.118B^3 + 0.124B^4 + 0.064B^5 - 0.146B^6)(y_{3t} - 1.642 - 0.747y_{1t}) = a_t,\]

where \(\sigma^2 = 0.0306\).

(b) The refined model is

\[(1 - 0.89B - 0.133B^3 + 0.155B^4 - 0.114B^6)(y_{3t} - 1.634 - 0.749y_{1t}) = a_t,\]

where \(\sigma^2 = 0.0307\).

(c) This refined model is adequate. See Figure 6.

(d) There are two real solutions.

(e) The maximum value of the inverses is 0.987, which is close to 1. This provides justification for taking the difference. Note that the coefficient 0.749 of \(y_{1t}\) is close to 0.744 when the differenced series are used.
Figure 2: Model checking of an AR(5) model for liquidity

Figure 3: Model checking of the Airline model for MSFT earnings
Figure 4: Residual ACF of the simple linear regression between bond yields

Figure 5: Model checking model regression model with time-series errors for bond yields: differenced data
Figure 6: Model checking model regression model with time-series errors for bond yields