Solutions to Homework Assignment #4

1. Daily log returns of Microsoft stock.

(a) We cannot reject the null hypothesis of zero mean, $t = 0.78$ with $p$-value 0.43. On the other hand, we easily reject the null hypothesis of no serial correlations. The Ljung-Box statistics give $Q(10) = 34.50$ with $p$-value 0.00015.

(b) An MA(2) model is entertained. The fitted model is

$$r_t = a_t - 0.073a_{t-1} - 0.044a_{t-2}, \quad \sigma^2_a = 0.00036.$$ 

Model checking indicates the model is adequate; there are no serial correlations in the residuals; see $Q(10) = 11.31$ with $p$-value 0.33; but there are ARCH effect in the residuals; see $Q(10) = 602.82$ with $p$-value close to zero for the squared residuals.

(c) The QQ-plot of the standardized residuals is shown in Figure 1, indicating deviations form normality. The fitted model is

$$r_t = 0.0038 + a_t - 0.044a_{t-1}, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

$$\sigma^2_t = 3.98 \times 10^{-6} + 0.048a^2_{t-1} + 0.939\sigma^2_{t-1}.$$ 

Except for the normality assumption, the model appears to fit the data well. All tests indicate both the mean and volatility equations are adequate.

(d) The fitted model is

$$r_t = 2.26 \times 10^{-4} + a_t - 0.044a_{t-1}, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t^*_{4.76}$$

$$\sigma^2_t = 1.54 \times 10^{-6} + 0.055a^2_{t-1} + 0.943\sigma^2_{t-1}.$$ 

Model checking indicates the model is adequate; see the R output and Figure 2.

(e) The forecasts are

<table>
<thead>
<tr>
<th>meanForecast</th>
<th>meanError</th>
<th>standardDeviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 9.586606e-05</td>
<td>0.01228494</td>
<td>0.01228494</td>
</tr>
<tr>
<td>2 2.259510e-04</td>
<td>0.01234370</td>
<td>0.01233197</td>
</tr>
<tr>
<td>3 2.259510e-04</td>
<td>0.01239048</td>
<td>0.01237870</td>
</tr>
<tr>
<td>4 2.259510e-04</td>
<td>0.01243696</td>
<td>0.01242514</td>
</tr>
</tbody>
</table>
2. Consider again the daily log returns of Microsoft stock in Problem 1.

(a) The fitted model is
\[ \sigma_t^2 = 0.027a_{t-1}^2 + 0.973\sigma_{t-1}^2. \]

(b) No, the Ljung-Box statistics of the standardized residuals give \( Q(10) = 14.46 \) with p-value 0.15.

(c) No, the Ljung-Box statistics give \( Q(10) = 0.47 \) with p-value close to 1.

(d) The volatility forecast is 0.0139 for all steps.

3. The monthly returns of Boeing (BA) stock from January 1961 to December 2013.

(a) The expected log return is not zero, because \( t \)-test gives \( t = 2.93 \) with p-value 0.004. The Ljung-Box statistics show that there is no serial correlation in the log return; \( Q(12) = 9.49 \) with p-value 0.66. There is, however, significant ARCH effect because \( Q(12) = 32.17 \) with p-value 0.001 for the squared residuals.

(b) The fitted model is
\[
\begin{align*}
    r_t &= 0.015 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1) \\
    \sigma_t^2 &= 0.00025 + 0.136a_{t-1}^2 + 0.844\sigma_{t-1}^2.
\end{align*}
\]

Except for the normality assumption, the model seems adequate. See Ljung-Box tests for standardized residual series and its squared series.

(c) The fitted model is
\[
\begin{align*}
    r_t &= 0.013 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{10, 8.88}^* \\
    \sigma_t^2 &= 0.00029 + 0.108a_{t-1}^2 + 0.864\sigma_{t-1}^2,
\end{align*}
\]

where 10 and 8.88 denote, respectively, the degrees of freedom and skew parameter. The model seems adequate.

(d) Based on result, we have \( t = (0.888 - 1)/0.06 = -1.87 \), which is less than 1.96 in absolute. Therefore, we cannot reject the null hypothesis that the log return has a symmetric distribution.

(e) The fitted model is
\[
\begin{align*}
    r_t &= 0.011 + 0.514\sigma_t^2 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1) \\
    \sigma_t^2 &= 0.00026 + 0.137a_{t-1}^2 + 0.842\sigma_{t-1}^2.
\end{align*}
\]

The risk premium parameter is not statistically significant at the 5% level. It has a \( t \)-ratio of 0.66, which is low.
(f) The fitted model is

\[ r_t = 0.013 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \]

\[ \sigma_t^2 = 0.0003 + 0.839 \sigma_{t-1}^2 + 0.074 a_{t-1}^2 + 0.117 a_{t-1}^2 N_{t-1} \]

where \( N_{t-1} = 1 \) if \( a_{t-1} \leq 0 \) and \( = 0 \), otherwise. From the output, the estimate 0.117 is significant at the 5% level.

4. The monthly log returns of the S&F composite index.

(a) The mean of the log return is significantly different from zero. Starting with Gaussian innovations, one ends up with a GARCH(1,1) model with skew Student-\( t \) innovations for the series. The fitted model is

\[ r_t = 6.15 \times 10^{-3} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{7.57,0.774}^* \]

\[ \sigma_t^2 = 9.62 \times 10^{-5} + 0.134 a_{t-1}^2 + 0.822 \sigma_{t-1}^2. \]

Model checking shows that the model is adequate. See the Ljung-Box statistics in the R output and the QQ-plot of Figure 4.

(b) The 1-step to 5-step ahead forecasts are

\[
\begin{array}{ccc}
\text{predict(m3,5)}
\end{array}
\]

\[
\begin{array}{ccc}
\text{meanForecast} & \text{meanError} & \text{standardDeviation} \\
1 & 0.006152749 & 0.03322843 & 0.03322843 \\
2 & 0.006152749 & 0.03393009 & 0.03393009 \\
3 & 0.006152749 & 0.03458728 & 0.03458728 \\
4 & 0.006152749 & 0.03520380 & 0.03520380 \\
5 & 0.006152749 & 0.03578300 & 0.03578300 \\
\end{array}
\]

(c) The fitted GJR model is

\[ r_t = 5.26 \times 10^{-3} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{7.71,0.772}^* \]

\[ \sigma_t^2 = 1.63 \times 10^{-4} + 0.079 (|a_{t-1}| - 0.694a_{t-1})^2 + 0.796 \sigma_{t-1}^2. \]

Based on the fitted model, the leverage effect is not significant at the 5% level.

5. The log return of daily exchange rate between Japanese Yen and U.S. Dollar from July 6, 2005 to April 18, 2014.

(a) The fitted model is

\[ r_t = a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{5.66}^* \]

\[ \sigma_t^2 = 4.0 \times 10^{-7} + 0.033 a_{t-1}^2 + 0.958 \sigma_{t-1}^2. \]

Model checking indicates the model is adequate.
Figure 1: Normal QQ-plot for the residuals of daily log return of MSFT with a Gaussian GARCH(1,1) model.

(b) Percentage log returns. Using APARCH model, the fitted model is

\[
  r_t = a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{5.71}^*
\]

\[
  \sigma_t^2 = 0.0052 + 0.033(|a_{t-1}| - 0.212a_{t-1})^2 + 0.953\sigma_{t-2}^2.
\]

The leverage parameter 0.212 is significantly different from zero so that the leverage effect is statistically significant.
Figure 2: Normal QQ-plot for the residuals of daily log return of MSFT with a GARCH(1,1) model with standardized Student-t innovations.

Figure 3: QQ-plot of the standardized residuals of a GARCH(1,1) model with skew Student-t innovations for the monthly log returns of Boeing stock.
Figure 4: QQ-plot of the standardized residuals of a GARCH(1,1) model with skew Student-$t$ innovations for the monthly log returns of S&P composite index.