Midterm

ChicagoBooth Honor Code:  
I pledge my honor that I have not violated the Honor Code during this examination.

Signature: ___________________________  Name: ___________________________  ID: ___________________________

Notes:

• Open notes and books. Exam time: 120 minutes.

• You may use a calculator or a PC. However, turn off Internet connection and cell phones. Internet access and phone communication are strictly prohibited during the exam.

• The exam has 8 pages and the R output has 11 pages. Please check that you have all 19 pages.

• For each question, write your answer in the blank space provided.

• Manage your time carefully and answer as many questions as you can.

• For simplicity, if not specifically given, use 5% Type-I error in hypothesis testings.

• Round your answer to 3 significant digits.

Problem A: (30 pts) Answer briefly the following questions. Each question has two points.

1. Give two situations under which returns of an asset follow an MA(1) model.

2. Describe two ways by which a GARCH(1,1) model can introduce heavy tails.
3. (Questions 3 to 6): Suppose that the asset return $r_t$ follows the model

$$
\begin{align*}
  r_t &= 0.005 + a_t \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{iid } t^*_6 \\
  \sigma_t^2 &= 0.09 + 0.10a_{t-1}^2 + 0.855\sigma_{t-1}^2.
\end{align*}
$$

Compute the mean and variance of $r_t$, i.e., $E(r_t)$ and $\text{Var}(r_t)$.

4. Express the volatility model in an ARMA(1,1) formation using $\eta_t = a_t^2 - \sigma_t^2$.

5. Describe two nice characteristics of the volatility model for $r_t$.

6. Suppose further that $a_{100} = -0.02$ and $\sigma_{100}^2 = 0.16$. Compute the 1-step ahead prediction (both mean and volatility) of the return at the forecast origin $t = 100$.

7. Give two nice features of the exponential GARCH model that the standard GARCH model does not have.

8. Give two empirical characteristics of daily asset returns.
9. **(Questions 9 - 11):** Let $p_t$ be the log price of an asset at time $t$. Assume that the log price follows the model

$$p_t = 0.001 + p_{t-1} + a_t, \quad a_t \sim iid \ N(0, 0.16),$$

where $N(\mu, \sigma^2)$ denotes normal distribution with mean $\mu$ and variance $\sigma^2$. Assume further that $p_{200} = 4.551$. Compute the 95% interval forecast for $p_{201}$ at the forecast origin $t = 200$.

10. Compute the 2-step ahead point forecast and its standard error for $p_{202}$ at the forecast origin $t = 200$.

11. What is the 100-step ahead forecast for $p_{300}$ at the forecast origin $t = 200$?

12. Describe two methods for comparing two different models for a time series $z_t$.

13. **(Questions 13-15):** Suppose that the quarterly growth rates $r_t$ of an economy follows the model

$$r_t = 0.006+0.168r_{t-1}+0.338r_{t-2}−0.189r_{t-3}+a_t, \quad a_t \sim iid \ N(0, .0016).$$

What is the expected growth rate of $r_t$?


15. What is the average length of business cycles of the economy, if any?
Problem B. (45 points) Consider the daily log returns $r_t$, in percentages, of the NASDAQ index for a certain period of time with 1841 observations. Answer the following questions, using the attached R output.

1. (2 points) Let $\mu$ be the expected value of $r_t$. Test $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$. Obtain the test statistic and draw your conclusion.

2. (2 points) Is the distribution of $r_t$ skew? Why?

3. (2 points) Does the distribution of $r_t$ have heavy tails? Why?

4. (2 points) Let $\rho_1$ be the lag-1 ACF of $r_t$. Test $H_0 : \rho_1 = 0$ versus $H_a : \rho_1 \neq 0$. The sample lag-1 ACF is $-0.086$. Obtain the test statistic and draw your conclusion.

5. (2 points) An MA(1) model is fitted. Write down the fitted model, including $\sigma^2$ of the residuals.

6. (2 points) Is there any ARCH effect in the residuals of the fitted MA(1) model? Why?

7. (2 points) An ARMA(0,1)+GARCH(1,1) model with Gaussian innovations was entertained for $r_t$. See m2. Write down the fitted model.
8. (2 points) Is the fitted ARMA(0,1)+GARCH(1,1) model adequate? Why?

9. (3 points) A GARCH(1,1) model with standardized Student-\(t\) innovations was fitted. See m3. Write down the fitted model.

10. (3 points) To further improve the model, a GARCH model with skew standardized Student-\(t\) innovations was considered. See m4. Write down the fitted model.

11. (2 points) Based on the fitted model m4. Does the return \(r_t\) follow a skew distribution? Why? Perform proper test and draw the conclusion.

12. (2 points) Based on the model m4. Obtain 1-step and 2-step 95\% interval forecasts for \(r_t\) at the forecast origin \(t = 1841\).

13. (2 points) An IGARCH model is fitted to \(r_t\). See m5. Write down the fitted model, including mean equation.

14. (2 points) Based on the output provided, is the IGARCH model adequate? Why?
15. (2 points) From the IGARCH model, we have \( r_{1841} = -0.0955 \) and \( \sigma_{1841} = 0.745 \). Compute the 1-step ahead forecast of \( r_t \) and its volatility at the forecast origin \( t = 1841 \)?

16. (3 points) A GARCH-M model is entertained for the \( r_t \) series. See \( \textbf{m6} \). Write down the fitted model.

17. (2 points) Based on the fitted model \( \textbf{m6} \), is the risk premium statistically significant? Why?

18. (4 points) A Threshold GARCH model with standard Student-\( t \) innovations is considered. See \( \textbf{m7} \). Write down the fitted model.

19. (2 points) Based on the fitted TGARCH model \( \textbf{m7} \), is the leverage effect statistically significant? Why?

20. (2 points) Among models \( \{ \textbf{m7, m4, m2} \} \), which one is preferred? Why?
Problem C. (12 points) The consumption of natural gas in northern American cities depends heavily on temperature and daily activities. Let $y$ be the daily sendout of natural gas and DHD be the degrees of heating days defined as $DHD = 65^\circ F$ minus daily average temperature. Also, let $x_1 = \text{DHD}$, $x_2 = \text{lag-1 DHD}$, $x_3 = \text{windspeed}$, and $x_4 = \text{the indicator variable for weekend}$. The data were collected for 63 days. Statistical analysis is included in the attached R output. Answer the following questions.

1. (2 points) A multiple linear regression is applied. Write down the fitted model, including the $R^2$.

2. (3 points) Model checking shows that the residuals have serial correlations, and the AIC selects an AR(8) model. After removing insignificant parameters, we have the model $\text{n5}$. Write down the fitted model. Is the model adequate? Why?

3. (3 points) Let $\phi_i$ be the lag-$i$ AR coefficient. It is seen that $\phi_1 \times \phi_7 \approx 0.2$, which is not far away from $-\phi_8$, especially in view of its standard error. This is indicative of a multiplicative model. Therefore, we fit a seasonal model. Denoted by $\text{n6}$. Write down the fitted seasonal model. Is the model adequate? Why?

4. (2 points) Compare models $\text{n5}$ and $\text{n6}$. Which one is preferred? Why?

5. (2 points) The weekend effect is rather significant in the multiple linear regression model of Question 1, but it is not so in the seasonal model. Why?
**Problem D.** (13 points) Consider the monthly log return of Decile 10 portfolio of CRSP from 1961.1 to 2014.12 with \( T = 648 \). The returns include dividends. Let \( r_t \) denote the monthly log return. Answer the following questions based on the attached R output.

1. (2 points) The ACF of \( r_t \) shows \( \hat{\rho}_1 = 0.203 \) and \( \hat{\rho}_{12} = 0.127 \). Test \( H_0 : \rho_{12} = 0 \) versus \( H_a : \rho_{12} \neq 0 \). Compute the test statistic and draw the conclusion.

2. (2 points) The \( \hat{\rho}_{12} \) is likely due to the January effect. To remove \( \rho_{12} \), one can use a simple linear regression with January dummy variable. The fitted model is \( r_t = 0.0064 + 0.06926 \text{Jan}_t + \epsilon_t \). Let \( \tilde{r}_t = r_t - 0.06928 \text{Jan}_t \) be the adjusted log returns of Decile 10 portfolio. Several models were entertained for \( \tilde{r}_t \); see model \( g_1, g_2, g_3 \) and \( g_4 \). Which model is preferred? Why?

3. (2 points) Write down the fitted model \( g_4 \).

4. (2 points) Based on the fitted model \( g_4 \). Is the leverage effect significant? Why?

5. (3 points) The average of fitted volatility is 0.0614 and the 1% quantile of the residuals of model \( g_4 \) is \(-0.171\). Compute the ratio \( \frac{\sigma_t^2(-0.171)}{\sigma_t^2(0.171)} \) of model \( g_4 \), where \( \sigma_t^2(a_t) \) denotes the conditional variance of the series when innovation is \( a_t \).

6. (2 points) Consider the forecasts of model \( g_4 \). Why is the 1-step ahead forecast of \( \tilde{r}_t \) different from multi-step ahead forecasts?
### Problem B #########

```r
> getSymbols("^IXIC", from="XXXX", to='''XXXX'"
[1] "IXIC"
> rtn=diff(log(as.numeric(IXIC[,6]))) * 100
> require(fBasics)
> basicStats(rtn)
```

```r
rtn
nobs 1841.000000
Mean 0.036063
Median 0.099857
Sum 66.391926
SE Mean 0.035029
LCL Mean -0.032637
UCL Mean 0.104763
Variance 2.258907
Stdev 1.502966
Skewness -0.245339
Kurtosis 6.831855
```

```r
> m0=acf(rtn)
> m0$acf[2]
[1] -0.08602153
> m1=arima(rtn,order=c(0,0,1),include.mean=F)
> m1
Call: arima(x = rtn, order = c(0, 0, 1), include.mean = F)
Coefficients:
ma1
-0.0948
s.e. 0.0243
sigma^2 estimated as 2.241: log likelihood = -3354.9, aic = 6713.79
```

```r
> Box.test(m1$residuals,lag=10,type='Ljung')
Box-Ljung test
data: m1$residuals
X-squared = 12.9361, df = 10, p-value = 0.2273
```

```r
> Box.test(m1$residuals^2,lag=10,type='Ljung')
Box-Ljung test
data: m1$residuals^2
X-squared = 1507.765, df = 10, p-value < 2.2e-16
```

```r
> require(fGarch)
> m2=garchFit(~arma(0,1)+garch(1,1),data=rtn,trace=F)
> summary(m2)
Title:  GARCH Modelling
Call:
  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rtn, trace = F)
```
Mean and Variance Equation:
\[
data \sim \text{arma}(0, 1) + \text{garch}(1, 1)
\]
\[
[\text{data} = \text{rtn}]
\]
Conditional Distribution: norm

Error Analysis:

| Estimate     | Std. Error | t value | Pr(>|t|) |
|--------------|------------|---------|----------|
| mu           | 0.091456   | 0.022416| 4.080    | 4.50e-05 *** |
| ma1          | -0.036450  | 0.025636| -1.422   | 0.155       |
| omega        | 0.028193   | 0.007026| 4.013    | 6.00e-05 *** |
| alpha1       | 0.104304   | 0.013844| 7.534    | 4.93e-14 *** |
| beta1        | 0.880167   | 0.014468| 60.835   | < 2e-16 ***  |

Log Likelihood:
\[-2911.304\] normalized: -1.581371

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera Test</td>
<td>R Chi^2</td>
<td>93.64382</td>
</tr>
<tr>
<td>Shapiro-Wilk Test</td>
<td>R W</td>
<td>0.9860011</td>
</tr>
<tr>
<td>Ljung-Box Test</td>
<td>R Q(10)</td>
<td>7.810015</td>
</tr>
<tr>
<td>Ljung-Box Test</td>
<td>R Q(20)</td>
<td>15.9162</td>
</tr>
<tr>
<td>Ljung-Box Test</td>
<td>R^2 Q(10)</td>
<td>17.04423</td>
</tr>
<tr>
<td>Ljung-Box Test</td>
<td>R^2 Q(20)</td>
<td>27.94681</td>
</tr>
<tr>
<td>LM Arch Test</td>
<td>R TR^2</td>
<td>16.9636</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.168174</td>
<td>3.183161</td>
<td>3.168160</td>
<td>3.173700</td>
</tr>
</tbody>
</table>

> m3=garchFit(\~garch(1,1),data=rtn,trace=F,cond.dist="std")
> summary(m3)
Title: GARCH Modelling
Call:
garchFit(formula = \~garch(1,1), data = rtn, cond.dist="std", trace = F)

Mean and Variance Equation:
\[
data \sim \text{garch}(1, 1)
\]
\[
[\text{data} = \text{rtn}]
\]
Conditional Distribution: std

Error Analysis:

| Estimate     | Std. Error | t value | Pr(>|t|) |
|--------------|------------|---------|----------|
| mu           | 0.119076   | 0.022341| 5.330    | 9.82e-08 *** |
| omega        | 0.025339   | 0.008176| 3.099    | 0.00194 **   |
| alpha1       | 0.106679   | 0.017710| 6.024    | 1.70e-09 *** |
beta1 0.882429 0.017503 50.417 < 2e-16 ***
shape 7.030196 1.254542 5.604 2.10e-08 ***
---
Log Likelihood:
-2894.18 normalized: -1.572069

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>11.29792 0.334783</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>19.92898 0.4623801</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>15.97748 0.100279</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>28.08922 0.1073041</td>
</tr>
<tr>
<td>LM Arch Test R TR^2</td>
<td>16.01562 0.1905215</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.149571</td>
<td>3.164557</td>
<td>3.149556</td>
<td>3.155097</td>
</tr>
</tbody>
</table>

> m4=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="sstd")
> summary(m4)

Title: GARCH Modelling
Call:
garchFit(formula="garch(1,1),data=rtn,cond.dist="sstd",trace=F)

Mean and Variance Equation:
\[
data \sim garch(1, 1)
\]
[\text{data = rtn}]

Conditional Distribution: sstd

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| mu       | 0.090509   | 0.022901| 3.952    | 7.75e-05 *** |
| omega    | 0.023284   | 0.007639| 3.048    | 0.0023 **    |
| alpha1   | 0.103543   | 0.016509| 6.272    | 3.57e-10 *** |
| beta1    | 0.884482   | 0.016813| 52.607   | < 2e-16 ***  |
| skew     | 0.873450   | 0.027496| 31.767   | < 2e-16 ***  |
| shape    | 8.123793   | 1.660601| 4.892    | 9.98e-07 *** |
---
Log Likelihood:
-2884.789 normalized: -1.566968

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>11.36986 0.3294374</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>19.82094 0.4691798</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>15.70359 0.1084377</td>
</tr>
</tbody>
</table>
Ljung-Box Test  R^2  Q(20)  27.38012  0.1249042

Information Criterion Statistics:

AIC  BIC  SIC  HQIC  
3.140455  3.158439  3.140434  3.147086

> predict(m4,5)
   meanForecast  meanError  standardDeviation
1 0.09050898  0.7804844  0.7804844
2 0.09050898  0.7906612  0.7906612
3 0.09050898  0.8005892  0.8005892
4 0.09050898  0.8102788  0.8102788
5 0.09050898  0.8197398  0.8197398

> source("Igarch.R")
> m5=Igarch(rtn)
Estimates:  0.9224943
Maximized log-likelihood:  2939.672

Coefficient(s):
   Estimate Std. Error t value Pr(>|t|)
beta 0.92249434  0.00841949 109.567  < 2.22e-16 ***

> names(m5)
[1] "par" "volatility"
> vol5=m5$volatility
> rtn[1841]
[1] -0.09549699
> vol5[1841]
[1] 0.744785
> resi=rtn/vol5
> Box.test(resi,lag=10,type="Ljung")
   Box-Ljung test
   data: resi
   X-squared = 13.1735, df = 10, p-value = 0.2141
> Box.test(resi^2,lag=10,type="Ljung")
   Box-Ljung test
   data: resi^2
   X-squared = 21.4537, df = 10, p-value = 0.01814

> source("garchM.R")
> m6=garchM(rtn,type=1)
Maximized log-likelihood:  -2915.346

Coefficient(s):
   Estimate Std. Error t value Pr(>|t|)
mu 0.07601900  0.03238316 2.34749  0.018901 *
gamma 0.01408378 0.02140652 0.65792 0.510589
omega 0.02845490 0.00715017 3.97961 6.9027e-05 ***
alpha 0.10499661 0.01401392 7.49231 6.7724e-14 ***
beta 0.87913601 0.01471637 59.73864 < 2.22e-16 ***
---
> m7=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std",leverage=T)
> summary(m7)
Title: GARCH Modelling
Call:
garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std",
leverage = T, trace = F)

Mean and Variance Equation:
data ~ garch(1, 1)
[data = rtn]
Conditional Distribution: std

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| mu       | 0.078450   | 0.022038| 3.560    | 0.000371 *** |
| omega    | 0.032456   | 0.007679| 4.226    | 2.37e-05 *** |
| alpha    | 0.052368   | 0.016678| 3.140    | 0.001690 **   |
| gamma    | 0.992492   | 0.280199| 3.542    | 0.000397 *** |
| beta     | 0.874213   | 0.016132| 54.192   | < 2e-16 ***   |
| shape    | 7.960959   | 1.584444| 5.024    | 5.05e-07 ***  |
---
Log Likelihood:
-2861.373 normalized: -1.554249

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>10.42453</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>18.25502</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>18.89127</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>30.74011</td>
</tr>
<tr>
<td>LM Arch Test R TR^2</td>
<td>20.56976</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.115017</td>
<td>3.133000</td>
<td>3.114995</td>
<td>3.121648</td>
</tr>
</tbody>
</table>

##### Problem C #######
> da=read.table("JW74.DAT",header=T)
> head(da)
Sendout DHD DHDm1 Windspeed Weekend
1 227 32 30 12 1
2 236 31 32 8 1
dim(da)
[1] 63 5

n1 = lm(Sendout ~ DHD + DHDm1 + Windspeed + Weekend, data = da)
n1

Call:
  lm(formula = Sendout ~ DHD + DHDm1 + Windspeed + Weekend, data = da)

summary(n1)

Call:
  lm(formula = Sendout ~ DHD + DHDm1 + Windspeed + Weekend, data = da)

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
  (Intercept)          1.8581     11.556  0.1616   0.8728
  DHD                   5.8742      0.290  20.2194  < 2e-16 ***
  DHDm1                 1.4052      0.292  4.7989  1.16e-05 ***
  Windspeed             1.3154      0.579  2.2733   0.0268 *
  Weekend               -15.8571     5.334 -2.9732   0.0043 **
---
Residual standard error: 18.32 on 58 degrees of freedom
Multiple R-squared: 0.9521, Adjusted R-squared: 0.9488
F-statistic: 288.1 on 4 and 58 DF, p-value: < 2.2e-16

Box.test(n1$residuals, lag = 10, type = 'Ljung')

Box-Ljung test

  data: n1$residuals
  X-squared = 48.1743, df = 10, p-value = 5.768e-07

acf(n1$residuals)

y = da[,1]
X = da[, -1]
pacf(n1$residuals)

n2 = ar(n1$residuals, method = "mle")
n2$order
[1] 8

n3 = arima(y, order = c(8, 0, 0), xreg = X)
n3

Call: arima(x = y, order = c(8, 0, 0), xreg = X)

Coefficients:
         ar1     ar2     ar3     ar4     ar5     ar6     ar7     ar8
s.e.   0.1323 0.1401 0.1465 0.1352 0.1268 0.1565 0.1367 0.1287
intercept 17.0627 5.6777 1.2451 1.2691 -14.8311
s.e. 14.9832 0.2226 0.2180 0.3755 8.1959

sigma^2 estimated as 161.3: log likelihood = -250.66, aic = 529.31
> c1=c(NA,0,0,NA,NA,0,NA,NA,0,NA,NA,NA,NA)
> n4=arima(y,order=c(8,0,0),xreg=X,fixed=c1)
> n4
Call: arima(x = y, order = c(8, 0, 0), xreg = X, fixed = c1)

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>ar6</th>
<th>ar7</th>
<th>ar8</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5229</td>
<td>0.1069</td>
<td>0</td>
<td>0.2252</td>
<td>-0.1715</td>
<td>0</td>
<td>0.3746</td>
<td>-0.2835</td>
<td>0</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.1069</td>
<td>0.1069</td>
<td>0</td>
<td>0.1231</td>
<td>0.1152</td>
<td>0</td>
<td>0.1121</td>
<td>0.1184</td>
<td>0</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 177.1: log likelihood = -253.33, aic = 526.67
> c1=c(NA,0,0,0,0,NA,NA,0,NA,NA,NA,NA,NA)
> n5=arima(y,order=c(8,0,0),xreg=X,fixed=c1)
> n5
Call: arima(x = y, order = c(8, 0, 0), xreg = X, fixed = c1)

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>ar6</th>
<th>ar7</th>
<th>ar8</th>
<th>intercept</th>
<th>DHD</th>
<th>DHDm1</th>
<th>Windspeed</th>
<th>Weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5362</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.3677</td>
<td>-0.2613</td>
<td>0</td>
<td>5.7669</td>
<td>1.4751</td>
<td>1.3192</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.1064</td>
<td>0.1064</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1165</td>
<td>0.1210</td>
<td>0</td>
<td>0.1946</td>
<td>0.1897</td>
<td>0.3630</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 188.1: log likelihood = -255.07, aic = 526.13
> Box.test(n5$residuals,lag=10,type='Ljung')

Box-Ljung test
data: n5$residuals
X-squared = 7.2835, df = 10, p-value = 0.6984

> n6=arima(y,order=c(1,0,0),seasonal=list(order=c(1,0,0),period=7),xreg=X,include.mean=F)
> n6
Call: arima(x = y, order = c(1, 0, 0), seasonal = list(order = c(1, 0, 0), period = 7), xreg = X, include.mean = F)

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>sar1</th>
<th>DHD</th>
<th>DHDm1</th>
<th>Windspeed</th>
<th>Weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5359</td>
<td>0.3677</td>
<td>5.7651</td>
<td>1.4732</td>
<td>-9.5947</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td>0.1065</td>
<td>0.1171</td>
<td>0.1999</td>
<td>0.1953</td>
<td>0.3637</td>
<td>6.0260</td>
</tr>
</tbody>
</table>
sigma^2 estimated as 189.5: log likelihood = -255.28, aic = 524.56

> Box.test(n6$residuals,lag=10,type='Ljung')

Box-Ljung test

data: n6$residuals
X-squared = 6.5785, df = 10, p-value = 0.7645

### Problem D ###

> da=read.table("m-dec12910-6114.txt",header=T)
> head(da)

<table>
<thead>
<tr>
<th>date</th>
<th>dec1</th>
<th>dec2</th>
<th>dec9</th>
<th>dec10</th>
</tr>
</thead>
<tbody>
<tr>
<td>19610131</td>
<td>0.058011</td>
<td>0.068040</td>
<td>0.096754</td>
<td>0.087303</td>
</tr>
<tr>
<td>19610228</td>
<td>0.029241</td>
<td>0.042879</td>
<td>0.056564</td>
<td>0.060040</td>
</tr>
<tr>
<td>19610330</td>
<td>0.025896</td>
<td>0.025270</td>
<td>0.060563</td>
<td>0.073311</td>
</tr>
<tr>
<td>19610428</td>
<td>0.005667</td>
<td>0.000877</td>
<td>0.011911</td>
<td>0.025753</td>
</tr>
<tr>
<td>19610531</td>
<td>0.019208</td>
<td>0.037392</td>
<td>0.046248</td>
<td>0.052023</td>
</tr>
<tr>
<td>19610630</td>
<td>-0.024670</td>
<td>-0.025332</td>
<td>-0.050651</td>
<td>-0.052041</td>
</tr>
</tbody>
</table>

> dec10=da$dec10
> jan=rep(c(1,rep(0,11)),54)
> m0=acf(dec10)
> m0$acf[c(2,13)]

[1] 0.2031250 0.1271504

> m1=lm(dec10~jan)
> m1

Call: lm(formula = dec10 ~ jan)

Coefficients:
   (Intercept)      jan
         0.006396   0.069263

> adj10=dec10-0.069263*jan
> acf(adj10)
> g1=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F)
> summary(g1)

Title: GARCH Modelling
Call:
garchFit(formula = ~arma(0, 1) + garch(1, 1), data = adj10, trace = F)

Mean and Variance Equation:
  data ~ arma(0, 1) + garch(1, 1)
  [data = adj10]

Conditional Distribution: norm

Error Analysis:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>Std. Error</td>
<td>t value</td>
<td>Pr(&gt;</td>
<td>t</td>
</tr>
<tr>
<td>mu</td>
<td>0.0070817</td>
<td>0.0027669</td>
<td>2.559</td>
<td>0.01048 *</td>
</tr>
</tbody>
</table>
ma1  0.2340229  0.0429797  5.445  5.18e-08 ***
omega  0.0002646  0.0001126  2.350  0.01879 *
alpha1  0.0856456  0.0272147  3.147  0.00165 **
beta1  0.8484757  0.0468131 18.125  < 2e-16 ***

Log Likelihood:
900.7496   normalized:  1.390046

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera Test</td>
<td>R Chi^2</td>
<td>309.9221</td>
</tr>
<tr>
<td>Shapiro-Wilk Test</td>
<td>R W</td>
<td>0.9706054</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td></td>
<td>2.717944</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td></td>
<td>14.40794</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td></td>
<td>1.275305</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td></td>
<td>4.427128</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.764659</td>
<td>-2.730139</td>
<td>-2.764777</td>
<td>-2.751268</td>
</tr>
</tbody>
</table>

> g2=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F,cond.dist="std")
> summary(g2)

Title: GARCH Modelling
Call:
garchFit(formula =~arma(0,1)+garch(1,1),data=adj10,cond.dist ="std", trace = F)

Mean and Variance Equation:
  data ~ arma(0, 1) + garch(1, 1)
  [data = adj10]

Conditional Distribution: std

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|---------|
| mu       | 0.0071115  | 0.0024374 | 2.918   | 0.00353 ** |
| ma1      | 0.2148526  | 0.0395801 | 5.428   | 5.69e-08 *** |
| omega    | 0.0002084  | 0.0001204 | 1.730   | 0.08363 . |
| alpha1   | 0.1182122  | 0.0434399 | 2.721   | 0.00650 ** |
| beta1    | 0.8366444  | 0.0583986 | 14.326  | < 2e-16 *** |
| shape    | 5.4767582  | 1.1049796 | 4.956   | 7.18e-07 *** |

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td></td>
<td>3.092116</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td></td>
<td>14.15442</td>
</tr>
</tbody>
</table>

17
Ljung-Box Test $R^2$ Q(10) 2.437512 0.9917554
Ljung-Box Test $R^2$ Q(20) 4.907722 0.9997601
LM Arch Test R TR^2 3.07941 0.9949569

Information Criterion Statistics:

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.844666</td>
<td>-2.803241</td>
<td>-2.844835</td>
<td>-2.828596</td>
</tr>
</tbody>
</table>

> g3=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F,cond.dist="std", leverage=T)
> summary(g3)

Title: GARCH Modelling
Call:
garchFit(formula = ~arma(0, 1)+garch(1,1),data = adj10, cond.dist = "std", leverage = T, trace = F)

Mean and Variance Equation:
\[ \text{data} \sim \text{arma}(0, 1) + \text{garch}(1, 1) \]

Conditional Distribution: std

Error Analysis:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| mu 0.0060302 | 0.0024882 | 2.423 | 0.0154 * |
| ma1 0.2224592 | 0.0400198 | 5.559 | 2.72e-08 *** |
| omega 0.0003067 | 0.0001572 | 1.950 | 0.0511 . |
| alpha1 0.1294974 | 0.0495784 | 2.612 | 0.0090 ** |
| gamma1 0.3189026 | 0.1376621 | 2.317 | 0.0205 * |
| beta1 0.7898273 | 0.0729151 | 10.832 | < 2e-16 *** |
| shape 5.7288196 | 1.1917451 | 4.807 | 1.53e-06 *** |

---

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>3.126521</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>16.21446</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.290507</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>5.845771</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.852305</td>
<td>-2.803976</td>
<td>-2.852535</td>
<td>-2.833556</td>
</tr>
</tbody>
</table>

> g4=garchFit(~arma(0,1)+aparch(1,1),data=adj10,trace=F,cond.dist="std", delta=2,include.delta=F)
> summary(g4)

Title: GARCH Modelling
Call:
garchFit(formula = ~arma(0, 1) + aparch(1, 1), data = adj10,
    delta = 2, cond.dist = "std", include.delta = F, trace = F)

Mean and Variance Equation:
data ~ arma(0, 1) + aparch(1, 1)
[data = adj10]

Conditional Distribution: std

Error Analysis:

|      | Estimate | Std. Error | t value | Pr(>|t|) |
|------|----------|------------|---------|----------|
| mu   | 0.0060302| 0.0024882  | 2.423   | 0.0154 * |
| ma1  | 0.2224592| 0.0400198  | 5.559   | 2.72e-08 *** |
| omega| 0.0003067| 0.0001572  | 1.950   | 0.0511 . |
| alpha| 0.1294974| 0.0495784  | 2.612   | 0.0090 ** |
| beta1| 0.3189026| 0.1376621  | 2.317   | 0.0205 * |
| beta2| 0.7898273| 0.0729151  | 10.832  | < 2e-16 *** |
| shape| 5.7288196| 1.1917451  | 4.807   | 1.53e-06 *** |

---

Standardised Residuals Tests:

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>3.126521</td>
<td>0.9782877</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>16.21446</td>
<td>0.7032337</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>3.290507</td>
<td>0.9737376</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>5.845771</td>
<td>0.9990903</td>
</tr>
<tr>
<td>LM Arch Test R TR^2</td>
<td>4.740345</td>
<td>0.9660941</td>
</tr>
</tbody>
</table>

Information Criterion Statistics:

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>BIC</th>
<th>SIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.852305</td>
<td>-2.803976</td>
<td>-2.852535</td>
<td>-2.833556</td>
</tr>
</tbody>
</table>

> predict(g4,4)

<table>
<thead>
<tr>
<th>meanForecast</th>
<th>meanError</th>
<th>standardDeviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0.013232245</td>
<td>0.05388801</td>
<td>0.05388801</td>
</tr>
<tr>
<td>2 0.006030204</td>
<td>0.05619847</td>
<td>0.05490500</td>
</tr>
<tr>
<td>3 0.006030204</td>
<td>0.05715694</td>
<td>0.05583665</td>
</tr>
<tr>
<td>4 0.006030204</td>
<td>0.05803645</td>
<td>0.05669161</td>
</tr>
</tbody>
</table>

> v4=volatility(g4)
> r4=residuals(g4)
> mean(v4)

[1] 0.06144349

> quantile(r4,prob=c(0.01,0.99))

1% 99%

-0.1712219 0.1462078