   (a) Plot both the daily VIX index and its log series on the same page.
   The plots are in Figure 1.
   (b) Is there a unit root in the log VIX series? Why?
   Yes, the series has a unit root. The ADF test, with lags = 11, shows the test statistic is -0.67 with p-value 0.40 so that the null hypothesis cannot be rejected. Note that I used the subcommand type='nc' in the ADF test because there should be no drift in the VIX index.
   (c) Let $z_t$ be the growth rate of VIX index, i.e. $z_t = \log(V_t) - \log(V_{t-1})$. Test $H_0 : E(z_t) = 0$ versus $H_a : E(z_t) \neq 0$. Draw your conclusion.
   The one-sample t-test shows $t = -0.024$ with p-value 0.98. As mentioned before, VIX should not have drift. [Otherwise, there exists arbitrage opportunity in VIX futures.]
   (d) Compute ACF and PACF of the $z_t$ series. Plot them on the same page.
   See Figure 2.
   (e) Consider the $z_t$ series. Test $H_0 : \rho_1 = \cdots = \rho_{10} = 0$ versus $H_a : \rho_i \neq 0$ for some $1 \leq i \leq 10$. Draw the conclusion.
   The Ljung-Box statistics give $Q(10) = 142.95$ with p-value close to zero. Therefore, there are serial correlations in the $z_t$ series.

2. Again, consider the growth rate series $z_t$ of Problem 1.
   (a) Use the ar command with subcommand method=''mle'' to identify an AR model for the $z_t$ series. Fit the specified AR model, perform model checking, and write down the fitted model.
   The order selected is 11. The fitted model is
   $$z_t = -.12 z_{t-1} - 0.09 z_{t-2} - 0.07 z_{t-3} - 0.07 z_{t-4} - 0.06 z_{t-5} - 0.06 z_{t-6} - 0.05 z_{t-7} - 0.04 z_{t-8} - 0.02 z_{t-9} + 0.04 z_{t-10} - 0.04 z_{t-11} + a_t$$
   with $\sigma^2 = 0.0037$. Model checking indicates the model is adequate. See Figure 3.
   (b) Next, return to the log series of VIX index, say $y_t = \log(V_t)$. Based on the AR model in part (a), fit a AR model for $y_t$. Use the fitted model to compute 1-step to 3-step ahead point forecasts of the log VIX index at the forecast origin March 31, 2015.
   The fitted model is the same as that of part 1. The forecasts are 2.715, 2.713, and 2.703, respectively. The forecast standard errors are 0.061, 0.081, and 0.095, respectively.
(c) Compute the 1-step to 3-step ahead 95% interval forecasts for $y_t$.

The 95% interval forecasts are given below:

```r
> p3=predict(m3,3)
> up=p3$pred+1.96*p3$se
> lb=p3$pred-1.96*p3$se
> pintv=cbind(lb,up)
> print(round(pintv,3))
```

Time Series:
Start = 6360
End = 6362
Frequency = 1
```
<table>
<thead>
<tr>
<th></th>
<th>lb</th>
<th>up</th>
</tr>
</thead>
<tbody>
<tr>
<td>6360</td>
<td>2.595</td>
<td>2.834</td>
</tr>
<tr>
<td>6361</td>
<td>2.553</td>
<td>2.872</td>
</tr>
<tr>
<td>6362</td>
<td>2.517</td>
<td>2.889</td>
</tr>
</tbody>
</table>
```

3. Consider, again, the log VIX index $y_t$ of Problem 2.

(a) Fit an ARIMA(1,1,1) model to $y_t$. Write down the fitted model.

The fitted model is

$$(1 - 0.73B)(1 - B)y_t = (1 - 0.84B)a_t, \quad \sigma^2 = 0.0037.$$

(b) Is the model adequate? Why? Which residual ACF is significantly different from zero, if any?

The model is not adequate. See Figure 4. The lag-10 ACF of the residuals is different from zero.

(c) Fit a refine model using the following R command:

```r
mm = arima(yt,order=c(1,1,1),seasonal=list(order=c(0,0,1),period=10))
```

Perform model checking. Is the model adequate? Why? Refer to this model as `mm`.

The fitted model is

$$(1 - 0.78B)(1 - B)y_t = (1 - 0.88B)(1 + 0.07B^{10})a_t, \quad \sigma^2 = 0.0037.$$

The model is adequate. See Figure 5.

(d) Compare the `mm` model with the AR model built in Problem 2 for $y_t$. In terms of in-sample fitting, which model is preferred? Why?

The model `mm` is preferred by the AIC criterion.

(e) Use `backtest` to compare the AR and `mm` models. You may use the initial forecast origin at $t = 6329$. Which model is preferred? Why?

Again, the `mm` model is preferred as its has lower RMSE (0.0539 versus 0.0540) and lower MAE (0.0409 versus 0.0411). The difference, however, is small.

(a) Does the $y_t$ series have a unit root? Why?
The ADF test with lags = 9 and type = ‘nc’ shows that the test statistic is $-0.67$ with p-value $0.404$ so that the unit-root hypothesis cannot be rejected at the 5% level.

(b) Focus on the change series of the claims, i.e. the first differenced series. Denote the change series by $r_t$. Is $r_t$ serially correlated? Why? [You may use $Q(10)$ statistics.]
The Ljung-Box statistics give $Q(10) = 241.96$ with p-value close to zero so that there are serial correlations in the $r_t$ series.

(c) Build an AR model for the $r_t$ series. Perform model checking using $gof = 24$. Is the model adequate? Why? Denote the model by $m_1$.
The fitted AR(9) model is
$$ r_t = -0.35r_{t-1} - 0.23r_{t-2} - 0.14r_{t-3} - 0.02r_{t-4} - 0.001r_{t-5} + 0.03r_{t-6} $$
$$ - 0.008r_{t-7} + 0.02r_{t-8} + 0.067r_{t-9} + a_t, \quad \sigma^2 = 298687172. $$
Model checking shows the model is adequate. See Figure 6.

(d) Refine the AR model by removing any estimate with $t$-ratio less than 1.65 in absolute value. Write down the refined model. Is it adequate? Why? Denote the refined model by $m_2$.
The simplified model is
$$ r_t = -0.35r_{t-1} - 0.23r_{t-2} - 0.14r_{t-3} + 0.061r_{t-9} + a_t, \quad \sigma^2 = 299204894. $$
The model is also adequate. See Figure 7.

(e) Use backtest with forecast origin $t = 2490$ to compare models $m_1$ and $m_2$. Draw your conclusion.
In this particular instance, the RMSE selects the model $m_1$ whereas the MAE prefers the model $m_2$.

(f) To obtain the forecasts of the claims, we like to use the $y_t$ series directly. Since $r_t = y_t - y_{t-1}$, an ARIMA($p,0,q$) for $r_t$ implies an ARIMA($p,1,q$) for $y_t$. Refit a model for $y_t$. Compare the estimates of AR coefficients with those of the model for $r_t$. Is there any difference? Why?
The parameter estimates remain unchanged, because differencing simply fixes the coefficient to 1.

(g) Based on the model for $y_t$, obtain 1-step to 3-step ahead forecasts for $y_t$ at the forecast origin March 21, 2015.
The forecasts are 286383, 289641 and 290092, respectively. The associated standard errors are 17283, 20613 and 22647, respectively.


(a) Obtain the time plot of the gold price.
See top panel Figure 8.

(b) Let $r_t$ be the log return of the daily gold price. Obtain the time plot of $r_t$.
See the bottom panel of Figure 8.
(c) Are there serial correlations in the $r_t$ series? You may use $Q(10)$ to draw the conclusion. Yes, the Ljung-Box statistics give $Q(10) = 45.83$ with $p$-value $1.5 \times 10^{-6}$.

(d) Build an AR model for $r_t$. Check the adequacy of the model. The AIC selects an AR(12) model and the fitted model is

$$r_t = -0.05r_{t-1} + 0.01r_{t-2} + 0.01r_{t-3} + 0.00r_{t-4} + 0.04r_{t-5} - 0.01r_{t-6} - 0.05r_{t-7} + 0.02r_{t-8} + 0.01r_{t-9} - 0.02r_{t-10} - 0.02r_{t-11} - 0.02r_{t-12} + a_t, \quad \sigma^2 = 1.096 \times 10^{-4}.$$  

The model seems adequate. See Figure 9.

(e) Remove any parameter of the AR model with $t$-ratio less than 1.645 in absolute value. Write down the final model. The final model is

$$r_t = -0.05r_{t-1} + 0.04r_{t-5} - 0.05r_{t-7} - 0.02r_{t-12} + a_t, \quad \sigma^2 = 1.098 \times 10^{-4}.$$

(f) Use the final model to compute 1-step to 3-step ahead forecasts of $r_t$ at the forecast origin March 31, 2015. The forecasts are $-9.59 \times 10^{-5}, 1.41 \times 10^{-4}, -2.47 \times 10^{-4}$ and the associated standard errors are .0105, .0105, .0105.
Figure 2: Sample ACF and PACF of the log returns of VIX index.

Figure 3: Model checking of an ARMA(11,0,0) model for the log returns of VIX index.
Figure 4: Model checking of an ARIMA(1,1,1) model for the log returns of VIX index.

Figure 5: Model checking of a seasonal ARIMA(1,1,1)\times(0,0,1)_{10} model for the log returns of VIX index.
Figure 6: Model checking of an ARIMA(9,1,0) model for the U.S. weekly initial jobless claims.

Figure 7: Model checking of a simplified ARIMA(9,1,0) model for the U.S. weekly initial jobless claims.
Figure 8: Time plots of daily gold price (top) and log return (bottom) from January 2, 1992 to March 31, 2015.

Figure 9: Model checking for an AR(12) model for the log returns of gold price.