GSB Honor Code:

I pledge my honor that I have not violated the Honor Code during this examination.

Signature:  
Name:  
UC. ID:  

Notes:

- Open notes and books. The exam has eight (8) pages and seven (7) pages of output.
- Write your answer in the blank space provided for each question.
- Manage your time carefully and answer as many questions as you can.
- Unless stated otherwise, \( \{a_t\} \) is a sequence of iid Gaussian random variables with mean zero and positive-definite covariance matrix \( \Sigma_a \). This assumption applies to the univariate case too.
- For simplicity, ALL tests use the 5% significance level.

Problem A: (45 points; 3 points per question) Answer briefly the following questions.

1. (For problems 1 to 4). Consider the following \( k \)-dimensional linear time series \( z_t \)

\[
\phi(B)z_t = \phi_0 + \theta(B)a_t,
\]

where \( \phi(B) = I - \sum_{i=1}^p \phi_i B^i \) and \( \theta(B) = I - \sum_{i=1}^q \theta_i B^i \) are two left-coprime matrix polynomials such that \( \text{rank}[\phi_p, \theta_q] = k \). What is the necessary and sufficient condition that \( z_t \) is an invertible series?

2. Assume that \( z_t \) is stationary, derive moment equations of \( z_t \) for \( \Gamma_\ell \), where \( \ell > q \) and \( \Gamma_\ell \) is the lag-\( \ell \) autocovariance matrix of \( z_t \).
3. Given the model, what is the 3-step ahead forecast error of \( z_t \) at the forecast origin \( t = n \)?

4. Given the model, what is the covariance matrix of 3-step ahead forecast errors of the series?

5. (For Problems 5 to 7). Suppose that the 2-dimensional process \( z_t \) follows the VARMA(2,1) model

\[
(I - \phi_1 B - \phi_2 B^2)z_t = c + (I - \theta_1 B)a_t,
\]

where \( c \) is a constant vector. State a sufficient condition under which the model reduces to a transfer function model.

6. Derive the AR representation for the \( z_t \) process.

7. What is the univariate ARMA model for \( z_{1t} \), the first component of \( z_t \)?

8. Consider the VMA(3) model \( z_t = a_t - \theta_1 a_{t-1} - \theta_3 a_{t-3} \). Obtain the autocovariance matrices \( \Gamma_j \) of \( z_t \) for \( j = 0, 1, 2 \) and 3.

9. Describe a procedure to test for Granger causality.
10. **(For Problems 10 to 12)** Consider the 2-dimensional VAR(1) model

\[ z_t = \begin{bmatrix} 0.3 & 0 \\ -0.6 & 0.9 \end{bmatrix} z_{t-1} + a_t, \quad \Sigma_a = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}. \]

Write down a simultaneous equation for the second component \( z_{2t} \).

11. Write down a transfer function model for the two series.

12. Obtain the first 3 lags of the impulse response function that shows the effect of the input variable on the output variable.

13. **(For Problems 13 to 15).** Suppose that \( z_t \) follows the model

\[ z_t = \begin{bmatrix} 0.94 & -1.12 \\ -1.12 & -0.74 \end{bmatrix} z_{t-1} + \begin{bmatrix} -0.48 & 0.04 \\ 0.04 & -0.42 \end{bmatrix} z_{t-2} + a_t. \]

Show that both \( z_{1t} \) and \( z_{2t} \) are unit-root nonstationary.

14. Write down an Error-Correction Model for the system.

15. Obtain the co-integrating vector for the system in which the first element is 1.
**Problem B.** (15 points) consider the monthly unemployment rates of Illinois and Indiana from January 1976 to March 2009. The data are seasonally adjusted and obtained from the Federal Reserve Bank at St Louis. SCA output is attached that fits a VAR(4) model to the data.

1. (3 points) Why a VAR(4) model is selected?

2. (5 points) After removing insignificant estimates, a final model is obtained. Write down the final fitted model, including the residual covariance matrix.

3. (3 points) Based on the fitted model, what is the relationship between the unemployment rates of Illinois and Indiana?

4. (4 points) Provide 95% interval forecasts for the July 2009 unemployment rates of Illinois and Indiana at the forecast origin March 2009.
Problem C. (10 points) Again, consider the monthly unemployment rates of Illinois and Indiana from January 1976 to March 2009. The data are seasonally adjusted and obtained from the Federal Reserve Bank at St Louis. S-Plus and R outputs are attached.

1. (4 points) Are the two unemployment series unit-root nonstationary? Why?

2. (6 points) Are the two time series co-integrated? Why? If yes, what is the co-integrating vector? If not, why?
**Problem D.** (10 points) Consider a univariate AR($p$) model $(1 - \phi_1 B - \cdots - \phi_p B^p)z_t = a_t$.

1. What is the necessary and sufficient condition that $z_t$ is weakly stationary?

2. (4 points) Express the model for $z_t$ as a $p$-dimensional VAR(1) model, say $Z_t = \Phi Z_{t-1} + b_t$. Give the definitions of $Z_t$, $\Phi$ and $b_t$.

3. Show that the necessary and sufficient condition for the weakly stationarity of $z_t$ is that all eigenvalues of $\Phi$ are less than 1 in modulus.

4. Show that $\sum_{i=1}^{p} \phi_i = 1$ if $z_t$ has a unit root.
Problem E. (20 points total, 5 points each question) Simple derivation.

1. Consider the random-walk time series \((1 - B)z_t = a_t\). Let \(z_0\) be the initial value of the series and \(S_t = \sum_{i=1}^{t} a_i\) be the partial sum of the first \(t\) innovations. Derive the limiting distributions of (a) \(T^{-3/2} \sum_{t=1}^{T} z_t\) and (b) \(T^{-1} \sum_{t=1}^{T} z_{t-2} a_t\) as \(T \to \infty\). (You may use the results available in the lecture notes.)
2. Consider the univariate process

\[ z_t = z_{t-4} + a_t - \theta a_{t-4}, \]

where \( z_j = 0 \) for \( j \leq 0 \) and \( |\theta| < 1 \). Suppose that the sample \( \{z_1, \ldots, z_T\} \) is available. Derive the limiting distributions of (a) \( T^{-2} \sum_{t=1}^{T} z_{t-4}^2 \) and (b) \( T^{-1} \sum_{t=1}^{T} z_{t-4} a_t \) as \( T \to \infty \).
SCA output: output edited.

input year,mom,day,il. file "unem-il.txt"

input year,mon,day,id. file "unem-in.txt"

miden il,id. no ccm. arfts 1 to 12.

TIME PERIOD ANALYZED . . . . . . . . . . . . 1 TO 399
EFFECTIVE NUMBER OF OBSERVATIONS (NOBE). . . 399

SERIES   NAME   OBSERVATIONS
         MEAN   STD. ERROR
 1       IL     6.7080   1.8195
 2       ID     5.8449   2.2189

========== STEPWISE AUTOREGRESSION SUMMARY =========

I RESIDUAL I EIGENVAL. I CHI-SQ I I SIGNIFICANCE
LAG I VARIANCES I SIGMA I TEST I AIC I OF PARTIAL AR COEFF.
--+-+----------+----------+---------+----------+----------------------
1 I .337E-01 I .261E-01 I 2859.62 I -6.265 I + +
   I .638E-01 I .714E-01 I I I I - +
--+-+----------+----------+---------+----------+----------------------
2 I .290E-01 I .243E-01 I 64.62 I -6.415 I - -
   I .601E-01 I .648E-01 I I I I - -
--+-+----------+----------+---------+----------+----------------------
3 I .280E-01 I .239E-01 I 18.89 I -6.444 I - .
   I .586E-01 I .626E-01 I I I I - .
--+-+----------+----------+---------+----------+----------------------
   I .585E-01 I .626E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
5 I .267E-01 I .227E-01 I 6.00 I -6.458 I .
   I .585E-01 I .625E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
6 I .266E-01 I .224E-01 I 5.26 I -6.452 I .
   I .582E-01 I .623E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
7 I .265E-01 I .224E-01 I 4.40 I -6.444 I .
   I .575E-01 I .617E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
8 I .264E-01 I .222E-01 I 3.07 I -6.432 I .
   I .574E-01 I .616E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
9 I .263E-01 I .221E-01 I 2.72 I -6.420 I .
   I .574E-01 I .616E-01 I I I I .
--+-+----------+----------+---------+----------+----------------------
10 I .259E-01 I .218E-01 I 6.99 I -6.419 I -
1.571E-01 I .612E-01 I I I . .

11 I .258E-01 I .217E-01 I 3.32 I -6.408 I . .
I .568E-01 I .609E-01 I I I . .

12 I .257E-01 I .216E-01 I 3.61 I -6.398 I . .
I .565E-01 I .606E-01 I I I . .

NOTE: CHI-SQUARED CRITICAL VALUES WITH 4 DEGREES OF FREEDOM ARE
5 PERCENT: 9.5 1 PERCENT: 13.3

MTSMODEL m1. SERIES IL ,ID. MODEL IS @
(1 - p1*B-p2*B**2 - p3*B**3 - p4*B**4)SERIES = CNST + NOISE. @

mestim m1. hold resi(r1,r2)

SUMMARY FOR THE MULTIVARIATE ARMA MODEL

<table>
<thead>
<tr>
<th>SERIES</th>
<th>NAME</th>
<th>MEAN</th>
<th>STD DEV</th>
<th>DIFFERENCE ORDER(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IL</td>
<td>6.7080</td>
<td>1.8195</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>ID</td>
<td>5.8449</td>
<td>2.2189</td>
<td></td>
</tr>
</tbody>
</table>

NUMBER OF OBSERVATIONS = 399 (EFFECTIVE NUMBER = NOBE = 395)

FINAL MODEL SUMMARY WITH CONDITIONAL LIKELIHOOD PARAMETER ESTIMATES

----- CONSTANT VECTOR (STD ERROR) -----
  0.120 ( 0.036 )
  0.115 ( 0.052 )

----- PHI MATRICES -----

ESTIMATES OF PHI( 1 ) MATRIX AND SIGNIFICANCE
  1.166 .122 + +
  .075 1.178 . +

STANDARD ERRORS
  .052 .036
  .075 .052

ESTIMATES OF PHI( 2 ) MATRIX AND SIGNIFICANCE
  -.092 -.098 ..
  .087 -.094 ..

STANDARD ERRORS
  .080 .055
  .116 .080

ESTIMATES OF PHI( 3 ) MATRIX AND SIGNIFICANCE
  .004 .076 ..
  -.241 -.049 - .

STANDARD ERRORS
  .080 .056
.116 .082
ESTIMATES OF PHI(4) MATRIX AND SIGNIFICANCE
-.133 -.057 -.  
.054 -.024 .  
STANDARD ERRORS
.051 .039
.074 .056
-----------------------
ERROR COVARIANCE MATRIX
-----------------------
1 2
1 .027341
2 .011659 .057745
--
miden r1,r2.

TIME PERIOD ANALYZED . . . . . . . . . . . . 5 TO 399
EFFECTIVE NUMBER OF OBSERVATIONS (NOBE) . . 395
SERIES NAME MEAN STD. ERROR
1 R1 0.0000 0.1654
2 R2 0.0000 0.2403
SAMPLE CORRELATION MATRIX OF THE SERIES
1.00
0.29 1.00
CROSS CORRELATION MATRICES IN TERMS OF +,-,.  
LAGS 1 THROUGH 6
. . . . . . . . . . . .
LARGS 7 THROUGH 12
. . . . + . . . . . .
LARGS 13 THROUGH 18
. . . . . . . . . . . .
LARGS 19 THROUGH 24
. . . . . . . . . . . .
--
mestim m1. hold resi(r1,r2) <= revised model with zero constraints.

SUMMARY FOR THE MULTIVARIATE ARMA MODEL
SERIES NAME MEAN STD DEV DIFFERENCE ORDER(S)
1 IL 6.7080 1.8195
2 ID 5.8449 2.2189
NUMBER OF OBSERVATIONS = 399 (EFFECTIVE NUMBER = NOBE = 395)
FINAL MODEL SUMMARY WITH CONDITIONAL LIKELIHOOD PARAMETER ESTIMATES

----- CONSTANT VECTOR (STD ERROR) -----
  0.128 (  0.035 )
  0.138 (  0.051 )

----- PHI MATRICES -----
ESTIMATES OF PHI( 1 ) MATRIX AND SIGNIFICANCE
  1.107  .132  +  +
  .000  1.202  .  +

STANDARD ERRORS
  .024  .035
  --  .049

ESTIMATES OF PHI( 2 ) MATRIX AND SIGNIFICANCE
  .000  -.086  .  -
  .202  -.183  +  -

STANDARD ERRORS
  --  .036
  .069  .052

ESTIMATES OF PHI( 3 ) MATRIX AND SIGNIFICANCE
  .000  .000  .
  -.237  .000  -

STANDARD ERRORS
  --  --
  .065  --

ESTIMATES OF PHI( 4 ) MATRIX AND SIGNIFICANCE
  -.165  .000  -
  .000  .000  .

STANDARD ERRORS
  .021  --
  --  --

ERROR COVARIANCE MATRIX

--

miden r1,r2.

TIME PERIOD ANALYZED . . . . . . . . . . . . . . 5 TO 399
EFFECTIVE NUMBER OF OBSERVATIONS (NOBE). . . 395

SERIES   NAME   MEAN   STD. ERROR
  1  R1   0.0000   0.1661
  2  R2   0.0000   0.2414

NOTE: THE APPROX. STD. ERROR FOR THE ESTIMATED CORRELATIONS BELOW
IS (1/NOBE**.5) = 0.05032

SAMPLE CORRELATION MATRIX OF THE SERIES
1.00
0.29 1.00

CROSS CORRELATION MATRICES IN TERMS OF +,-,. LAGS 1 THROUGH 6

<table>
<thead>
<tr>
<th></th>
<th>.</th>
<th>.</th>
<th>.</th>
<th>.</th>
<th>.</th>
<th>.</th>
<th>.</th>
<th>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
| LAGS 7 THROUGH 12
|   | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| LAGS 13 THROUGH 18
|   | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| LAGS 19 THROUGH 24
|   | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |

--

mfore m1. nofs 4.
-----------------------------
4 FORECASTS, BEGINNING AT ORIGIN = 399
-----------------------------

<table>
<thead>
<tr>
<th>SERIES:</th>
<th>IL</th>
<th>ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME</td>
<td>FORECAST</td>
<td>STD ERR</td>
</tr>
<tr>
<td>400</td>
<td>9.523</td>
<td>0.166</td>
</tr>
<tr>
<td>401</td>
<td>9.882</td>
<td>0.257</td>
</tr>
<tr>
<td>402</td>
<td>10.145</td>
<td>0.341</td>
</tr>
<tr>
<td>403</td>
<td>10.354</td>
<td>0.430</td>
</tr>
</tbody>
</table>

--

S-Plus and R output:

```r
> module(finmetrics)
> da=read.table("unem-il.txt")
> il=da[,4]
> da1=read.table("unem-in.txt")
> id=da1[,4]
> z=cbind(il,id)
> z=data.frame(z)
> VAR.fit=VAR(z,max.ar=12,criterion="AIC")
> VAR.fit$ar.order
[1] 4
> coint.uc=coint(z,lags=3)
> coint.uc

Call:
coint(Y = z, lags = 3)
```
Trend Specification:
H1(r): Unrestricted constant

Trace tests significant at the 5% level are flagged by ‘ +‘.
Trace tests significant at the 1% level are flagged by ‘++‘.
Max Eigenvalue tests significant at the 5% level are flagged by ‘ *‘.
Max Eigenvalue tests significant at the 1% level are flagged by ‘**‘.

Tests for Cointegration Rank:

<table>
<thead>
<tr>
<th>Model</th>
<th>Eigenvalue</th>
<th>Trace Stat</th>
<th>95% CV</th>
<th>99% CV</th>
<th>Max Stat</th>
<th>95% CV</th>
<th>99% CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(0)++**</td>
<td>0.0519</td>
<td>24.1797</td>
<td>15.4100</td>
<td>20.0400</td>
<td>21.0612</td>
<td>14.0700</td>
<td>18.6300</td>
</tr>
<tr>
<td>H(1)</td>
<td>0.0079</td>
<td>3.1185</td>
<td>3.7600</td>
<td>6.6500</td>
<td>3.1185</td>
<td>3.7600</td>
<td>6.6500</td>
</tr>
</tbody>
</table>

> coint.uc$coint.vectors
   il    id
   coint.1  -1.4141606 1.1185284
   coint.2   0.1348843 0.3576502

> coint.rc=coint(z,lags=3,trend="rc")
> coint.rc

Call:
coint(Y = z, lags = 3, trend = "rc")

Trend Specification:
H1*(r): Restricted constant

Tests for Cointegration Rank:

<table>
<thead>
<tr>
<th>Model</th>
<th>Eigenvalue</th>
<th>Trace Stat</th>
<th>95% CV</th>
<th>99% CV</th>
<th>Max Stat</th>
<th>95% CV</th>
<th>99% CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(0)+ **</td>
<td>0.0525</td>
<td>24.5813</td>
<td>19.9600</td>
<td>24.6000</td>
<td>21.2932</td>
<td>15.6700</td>
<td>20.2000</td>
</tr>
<tr>
<td>H(1)</td>
<td>0.0083</td>
<td>3.2882</td>
<td>9.2400</td>
<td>12.9700</td>
<td>3.2882</td>
<td>9.2400</td>
<td>12.9700</td>
</tr>
</tbody>
</table>

> coint.rc$coint.vectors
   il    id    Intercept*
   coint.1  -1.4067061 1.11069375   3.08022295
   coint.2  -0.0913903 0.37983854   3.04772247
   coint.3  0.1756982 -0.03318965  -0.01606589

**** R output ****
> source("coint.R")
> da=read.table("unem-il.txt")
> il=da[,4]
> da1=read.table("unem-in.txt")
> id=da1[,4]
> z=cbind(il,id)
> source("VARorder.R")
> m1=VARorder(z,maxp=12)
Order selected by aic = 6
Order selected by bic = 2
Order selected by hq = 3

> m2 = coint(z, 5)
[1] "Rank, Eigenvalue, trace, & max-eig"
   rnk  ev  lktr  lkmx
[1,] 0 0.03732 17.66 14.95
[2,] 1 0.00688  2.71  2.71
> names(m2)
[1] "trace"  "maxeig"  "eigvalue"  "eigvector"
> m2$eigvector
  [,1]       [,2]
[1,] 0.8034939 -0.1672056
[2,] -0.5953130  0.9859221

> library(fUnitRoots)
> adfTest(il, 6)
Title: Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 6
  STATISTIC:
    Dickey-Fuller: -0.1733
    P VALUE: 0.5611

> adfTest(id, 6)
Title: Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 6
  STATISTIC:
    Dickey-Fuller: 0.0938
    P VALUE: 0.6462