Heterogeneity, Job Creation, and Unemployment Volatility*

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Abstract
In this paper, I explore the impact of match-specific heterogeneity at the job creation margin on business cycle fluctuations. I show that this form of heterogeneity alone does not help to amplify labor market volatility, either under full or under asymmetric information. First, I show analytically that, under full information, heterogeneity has no first-order effect on the response of unemployment and job creation to productivity, and actually, tends to dampen the response of market tightness. Then, with a series of calibrations, I show that, with both full and asymmetric information, the model delivers labor market volatilities close to the representative-agent, full-information benchmark.

Keywords: unemployment, job creation, market tightness, heterogeneity, asymmetric information.

JEL Classification: E24, E32, J41, J63, J64.

1 Introduction
Whether search models can match the observed cyclical behavior of unemployment and vacancies in the U.S. economy is an issue that has recently received new attention. Shimer (2005) and Hall (2005a) have opened the debate by showing that the conventional search model à la Mortensen and Pissarides (Mortensen and Pissarides, 1994, and Pissarides, 2000) cannot account for the high responsiveness of unemployment to productivity shocks. The main reason is that, in that kind of model, wages absorb the shocks, reducing the response of firms’ profits and, hence, of job creation.

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Workers heterogeneity in terms of skills and ability is a pervasive element of labor markets. In particular, at the moment of hiring, a firm has to assess whether the worker is a good fit for the job. In this paper, I focus on this form of heterogeneity, which is relevant at the job creation margin, and I ask whether it can help to explain the cyclical behavior of labor markets.

Given my interest in the job creation margin, I make the simplifying assumption that workers only differ in terms of effort costs they have to incur at the beginning of the employment relationship. These costs capture both time and energy spent in training activities, and the disutility from moving to the new location and to adapt to the new environment. I could also include the effort spent in production as long as there are substantial sunk costs at the beginning of the match. This assumption, together with the absence of on-the-job mobility, ensures that heterogeneity does not affect the destruction margin. Although both endogenous separation and on-the-job mobility are relevant ingredients of the labor market,¹ this paper abstracts from these issues to investigate, in isolation, the role of an endogenous hiring margin.

In this paper, I analyze both the case of full information and the case in which workers privately observe their own effort costs. The main difference with the standard model is that the job creation rate can now be decomposed into two margins: the standard matching margin and an endogenous hiring margin generated by the heterogeneity element. When a good productivity shock hits the economy, the hiring margin becomes looser, in the sense that workers with higher costs are hired. On the one hand, this implies that both the expected cost of a matched worker and wages increase. However, on the other hand, firms’ profits can potentially increase because output is higher for each worker hired. When the information is asymmetric, there is an additional potential source of volatility. The firm has to pay the workers some rents in order to give them the incentive to reveal their information. Then, when there is a good productivity shock, there is more surplus to cover the rents and the distortion becomes smaller, generating a boost in job creation. Nevertheless, when the hiring margin increases, the rents that need to be paid to the workers are higher and asymmetric information could dampen the responsiveness of vacancies. Therefore, on theoretical grounds, heterogeneity together with asymmetric information may or may not amplify the response of the economy to productivity shocks. Under realistic parametric assumptions, however, amplification is absent or quantitatively not significant.

I construct a competitive search model,\(^2\) where workers are hit by match-specific idiosyncratic shocks. Employers and workers are both risk-neutral and \textit{ex-ante} homogeneous. Employers post contracts and workers direct their search towards them. When a match is formed, the worker's type is drawn randomly and can be either public information or private information of the worker. An employment contract is a mechanism that is incentive compatible, when the information is private, and satisfies a participation constraint on the worker's side. A worker cannot be forced to participate to the employment relationship, he can always quit and join the ranks of the unemployed.

First, I introduce the model with full information and I derive the main analytical result: in steady state heterogeneity does not amplify the response of the unemployment rate to changes in productivity. On the contrary, it tends to dampen the response of market tightness to productivity. Next, I turn to the model with asymmetric information and show that in steady state there is an additional effect on the response of market tightness to productivity, coming from the binding incentive compatibility constraint, which makes ambiguous the comparison with the standard model.

Finally, I show the results of a series of calibrations. I describe how I parametrize the model to match the facts of the U.S. labor market documented in Shimer (2005). This guarantees that the version of the model with degenerate distribution of idiosyncratic shocks corresponds to the standard model calibrated in Shimer (2005). Then, I explore the model with more general distributions for the idiosyncratic shocks, both under full and asymmetric information. Using different families of distributions, I show that heterogeneity at the job creation margin alone cannot help in amplifying the responsiveness of market tightness and unemployment to productivity, neither under asymmetric information.

A growing group of papers react to Shimer (2005) by attempting to generate an higher volatility for the unemployment rate.\(^3\) My paper contributes to this literature by showing that workers' heterogeneity at the creation margin alone cannot help in explaining business cycle fluctuations. This does not mean that heterogeneity cannot be part of a more complex explanation. Other papers explore the role of other sources of volatility and the interaction of heterogeneity with other crucial ingredients of the labor market. Among others, Hall (2005b)


\(^3\)Mortensen and Nagypal (2005) offer a comprehensive review of this literature together with an alternative calibration proposal.
constructs a model exhibiting wage rigidity, Menzio (2004) assumes employers have private information about productivity with a distribution of types that varies over the cycle, Nagypál (2004) combines workers’ heterogeneity, asymmetric information and on-the-job search. Brugemann and Moscarini (2007) is the closest in spirit to this paper. They investigate different wage determination alternatives in search models and their ability to match the response of unemployment rate to productivity shocks. In particular, they explore wage bargaining models with asymmetric information and construct theoretical bounds on the level of market tightness elasticity with respect to productivity that these models can generate. My analysis is complementary to their model, because I assume wage posting, focus on the role of heterogeneity, with both full and asymmetric information, and analyze the equilibrium dynamics using numerical exercises, while they perform a comparative static analysis. My results are consistent with their claim that asymmetric information cannot generate realistic levels of volatility.4

Hagedorn and Manovskii (2005) show a calibration where the value of non-market activity is much higher than Shimer (2005),5 generating an higher elasticity of market tightness to productivity. In my calibration I am careful to keep the value of non-market activity being less than half than labor income, in order to shut down that explanation and highlight the role of heterogeneity and information.

From a theoretical point of view, the model in this paper is related to Shimer and Wright (2004) and Moen and Rosen (2004) who model asymmetric information in competitive search model. More specifically, the model presented in this paper is the stochastic version of the dynamic model in Guerrieri (2007), which explores the dynamic efficiency properties of competitive search models under asymmetric information. As I explain in Section 2, apart from the introduction of an aggregate productivity shock into the basic environment, there is also another modeling difference for the sake of analytical tractability. In this paper, the worker’s type represents a sunk cost that the worker has to pay at the moment of hiring, such as a training or a mobility cost, instead of a flow cost, such as work effort. This change makes the model with aggregate shocks more tractable, because it allows to avoid endogenous separation, which, otherwise, would naturally emerge once a good shock hits the economy.

This paper is organized as follows. In Section 2, I introduce the model with ex-post hetero-

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4In Section 3, I show that their weak bound applies to the steady state version of my model.
5They calibrate the worker’s bargaining power and the non-market activity to match elasticity of wages and average firms’ profits.
geneous workers and full information, define and characterize the competitive search equilibrium. Moreover I illustrate some comparative statics results in the extreme case of no aggregate shock. In Section 3, I extend the model to the case of asymmetric information, define and characterize the competitive search equilibrium. In Section 4, I describe the calibration exercises and show the results of the main specifications. Finally, Section 5 concludes.

2 The model with Full Information

In this section I explore a stochastic directed search model with match-specific heterogeneity and full information. I show that when the distribution of the idiosyncratic shocks is degenerate the model boils down to the discrete version of Shimer (2005), where the Hosios condition is satisfied. When, instead, the distribution of idiosyncratic shocks is non-degenerate, then the reaction of market tightness to productivity shocks can in fact be dampened.

Environment

Consider an economy with infinite horizon and discrete time, populated by a continuum of measure 1 of workers and a large continuum of potential employers. Both workers and employers are ex-ante homogeneous and have linear preferences with discount factor $\beta$. Workers can search freely, while employers need to pay an entry cost $c$ to post a vacancy. When a match is formed, the worker has to face some training cost $x$, where $x$ is drawn randomly from the cumulative distribution function $\Phi(.)$, with support $X = [x, \bar{x}]$. I assume that the cumulative distribution function $\Phi(.)$ is differentiable, with $\varphi(.)$ denoting the associated density function. Any worker-employer match produces $p_t$ units of output at any time $t$ in which it is productive. Productivity $p_t$ follows a first order Markov process in discrete time, according to some distribution $G(p', p) = \Pr(p_{t+1} \leq p' | p_t = p)$ with finite support $P = [p_1, ..., p_N]$. The net surplus of the match is given by $p_t - x$ at time $t$, when the match is created, and $p_\tau$ at any future time $\tau \geq t$ in which the match is still productive. Both $x$ and $p$ are common knowledge.

At the beginning of each period $t$ the aggregate productivity shock $p_t$ is realized. Employers can be either productive or not, and workers can be either employed or unemployed. Non-productive employers can open a vacancy at a cost $c$, which entitles them to post an employment

\footnote{Note that productivity $p_t$ is common to all the matches existing at time $t$ even though created at different times $t - \tau$, for $\tau = 0, 1, ..., t$.}
contract contingent on the aggregate shock. A contract is a revelation mechanism, that is, a map \( C : X \times P \mapsto [0, 1] \times \mathbb{R}_+ \), specifying for each worker of type \( x \) matched when productivity is \( p_t \), the hiring probability \( e(x, p_t) \in [0, 1] \) and the expected net present value of wages \( \omega(x, p_t) \in \mathbb{R}_+ \). Each worker observes the contracts posted by active firms, the set \( C^P \subseteq C \), and chooses to search for a specific contract \( C \in C^P \). Then, matching takes place and, for each match, the draw \( x \) is realized. Conditional on the aggregate shock \( p_t \) and on his match-specific shock \( x \), the worker decides whether to participate to the employment relationship or not. If the worker does not meet an employer or is not hired, he remains unemployed, gets a non-transferable utility from leisure \( z \) and looks for another match next period. If the worker is hired, the parties are productive until separation, which happens according to a Poisson process with parameter \( s \). A part from the stochastic aggregate shock, the main difference with the environment in Guerrieri (2007) is about the idiosyncratic shock. In this paper \( x \) is a sunk cost that the worker suffers at the moment of the match, which I interpret as cost of training. In Guerrieri (2007), \( x \) represents the expected value of the disutility that the worker suffers each period in which he is productive, which I interpret as work effort. Once I introduce the aggregate shock, this change is crucial to make the model tractable, given that it ensures that at any point in time \( \tau > t \), it is never optimal to end the match created at time \( t \), as long as \( p_\tau > 0 \). If \( x \) was per-period work effort, when there are bad times the firm would like to fire workers with an high enough \( x \), generating an additional endogenous separation mechanism.

Trading frictions in the labor market are modeled through random matching. Employers and workers know that their matching probabilities will depend on the contract that they respectively post and seek. Each type of contract \( C \) is associated with a labor submarket, where a mass \( v(C) \) of employers posts contracts of type \( C \) and a mass \( u(C) \) of unemployed workers applies for jobs at firms offering that type of contract. I assume that each submarket is characterized by a constant returns to scale matching function \( m(v(C), u(C)) \) and by an associated “tightness” \( \theta(C) = v(C)/u(C) \).\(^7\) Hence, for each contract \( C \), I can define the function \( f(\theta) = m(\theta, 1) \), which represents the matching probability of a worker applying for \( C \). On the other hand, the matching probability for a firm posting \( C \) is represented by the non-increasing function \( f(\theta)/\theta \). The function \( f(\theta) : [0, \infty) \mapsto [0, 1] \) satisfies standard conditions:\(^8\)

\( \text{(i) } f(\theta) \leq \min \{\theta, 1\} \), \( \text{(ii) } f(\theta) \) is twice differentiable with \( f'(\theta) > 0 \) and \( f''(\theta) < 0 \).

\(^7\)In order to simplify the notation, from now on I am going to drop the dependence of \( u \), \( v \) and \( \theta \) on the contract \( C \), whenever it does not cause any confusion.

\(^8\)With discrete time, this condition ensures that both \( f(\theta) \) and \( f(\theta)/\theta \) are proper probabilities.
Bellman Values

Linear preferences imply that the wage profile over the life of the relationship is irrelevant for the analysis. This makes possible to specify the contract in terms of the net present value of wages. The utility of a worker of type $x$ who meets an employer when productivity is $p_t$, is given by

$$v(x, p_t) = e(x, p_t)[\omega(x, p_t) - x + V(p_t)] + [1 - e(x, p_t)]U(p_t),$$

where $V(p_t)$ represents the continuation value of employed workers net of wages and training cost when the aggregate shock is $p_t$ (from now on simply continuation utility of employed workers) and $U(p_t)$ represents the continuation value of unemployed workers at the same time.

The continuation value of employed workers, $V(p_t)$, represents just the discounted expected value of being separated and becoming unemployed, that is,

$$V(p_t) = \beta E[sU(p_{t+1}) + (1 - s)V(p_{t+1}) | p_t].$$

Moreover the continuation value of unemployed workers, $U(p_t)$, is given by

$$U(p_t) = \beta E\left[f(\theta(p_{t+1})) \int_x^\infty e(x, p_{t+1}) [\omega(x, p_{t+1}) - x + V(p_{t+1})] d\Phi(x) | p_t\right]$$

$$+ z + \beta E\left[1 - \int_x^\infty e(x, p_{t+1}) d\Phi(x) \right]U(p_{t+1}) | p_t \right].$$

Notice that, in order to make every type $x$ to participate, it must be that

$$v(x, p_t) \geq U(p_t) \text{ for any } x, p_t.$$  

Finally, the large number of potential firms ensures free entry and implies that the value of an open vacancy will be zero at each time, that is,

$$\beta \frac{f(\theta(p_t))}{\theta(p_t)} \int_x^\infty e(x, p_t) [T(p_t) - \omega(x, p_t)] d\Phi(x) = c.$$  

where

$$T(p_t) = p_t + \beta (1 - s) \int T(p_{t+1}) dG(p_{t+1} | p_t).$$
Competitive Search Equilibrium

In this section, I generalize the standard definition of the Competitive Search Equilibrium to an environment with aggregate shock. In order to simplify the analytical treatment, we adopt a definition in recursive terms.\(^9\)

It is possible to show that a recursive Competitive Search Equilibrium takes the following simple form. It is a set of contracts \(C^*(p)\) contingent only on the aggregate shock, a tightness function \(\Theta^*(p, C)\), that for any \(p\) and \(C(p) \in C(p)\) gives a market tightness \(\theta(p) \in \mathbb{R}_+ \cup \infty\), and a pair of continuation utility functions \(\{U^*(p), V^*(p)\}\) such that, given \(p\), employers maximize profits and workers apply optimally for jobs, taking as given the future values of being employed and unemployed and aware of the market tightness associated with each contract, even if not offered in equilibrium. Moreover, expected profits are driven to zero by free entry.

Next Proposition characterizes the stochastic symmetric competitive search equilibrium in recursive terms. In the rest of the analysis I adopt a recursive notation, dropping the \(t\) whenever this causes no confusion, and denoting a variable at time \(t+1\) with a prime.

**Proposition 1** If \(\{C^*(p), \Theta^*(p), U^*(p), V^*(p)\}\) is a Recursive Competitive Search Equilibrium, then any pair \((C^*(p), \theta^*(p))\) with \(C^*(p) \in C^*(p)\) and \(\theta^*(p) = \Theta^*(p, C^*(p))\) satisfies the following

(i) for a given pair of functions \(\{U^*(p), V^*(p)\}\), \(C^*(p) = [e^*(x,p), \omega^*(x,p)]_{x \in \Theta}\) and \(\theta^*(p)\) solve

\[
W(U^*(p), V^*(p), p) = \max_{C(p), \theta(p)} \beta f(\theta(p)) \int_{\mathbb{R}} e(x,p) [\omega(x,p) - x + V^*(p)] d\Phi(x) \\
+ \beta \left[ 1 - f(\theta(p)) \int_{\mathbb{R}} e(x,p) d\Phi(x) \right] U^*(p) \quad (P1)
\]

subject to \(e(x,p) \in [0,1]\), the individual rationality constraints (3) and the free-entry condition (4);

(ii) for a given pair \(\{C^*(p), \theta^*(p)\}\), the pair of functions \(\{U^*(p), V^*(p)\}\) evolve according to

\[
U^*(p) = z + \int W(U^*(p'), V^*(p'), p') dG(p'|p)
\]

\(^9\)The definition and the characterization of the Competitive Search Equilibrium both in the full information and in the asymmetric information case follow the analysis in Guerrieri (2007). The proof of Proposition 1 follows steps similar to Acemoglu and Shimer (1999) and Guerrieri (2007) and is therefore omitted.
and
\[ V^*(p) = \beta \int \left[ sU^*(p') + (1 - s)V^*(p') \right] dG(p'|p). \]

Conversely, if a pair of functions \( \{ C^*(p), \theta^*(p) \} \) solves the program \( P_4 \), then there exists an equilibrium \( \{ C^*(p), \Theta^*(p, \cdot), U^*(p), V^*(p) \} \) such that \( C^*(p) \in C^*(p) \) and \( \theta^*(p) = \Theta^*(p, C^*(p)) \).

Using pointwise maximization with respect to \( e(x) \), the trading area can be fully described by a cut-off value \( x(p) \) such that \( e(x, p) = 1 \) if \( x \leq x(p) \) and \( e(x, p) = 0 \), otherwise. Then, the equilibrium can be characterized for given \( p \) and given continuation utilities \( U(p) \) and \( V(p) \), by a pair of functions \( \hat{x}(p) \) and \( \theta(p) \) satisfying the first order conditions of the maximization problem \( P_1 \), that is,
\[ T(p) - \hat{x}(p) - [U(p) - V(p)] = 0 \]
\[ \beta f'(\theta(p)) \int_{x}^{\hat{x}(p)} \left[ T(p) - x - [U(p) - V(p)] \right] d\Phi(x) = c. \]

Comparative Statics

In this section, I look at some comparative statics to get a sense of the volatility of job creation implied by the model. Following Shimer (2005), I derive the elasticity of market tightness with respect to productivity, when there are no aggregate shocks. I first consider the case of a degenerate distribution for the idiosyncratic shocks and I show that, in this case, my model is isomorphic to the one analyzed in Shimer (2005) with random matching and Nash bargaining, where the Hosios condition is met. Next, I introduce a non-degenerate distribution of idiosyncratic shocks and show that the effective job creation depends not only on the equilibrium matching probability \( f(\theta(p)) \), but also on the equilibrium hiring decision, reflected in \( \Phi(\hat{x}(p)) \), which may amplify the impact of productivity shocks on market tightness and unemployment rate. However, I show that when the parameters of the model are properly re-calibrated, the presence of heterogeneity tends to dampen the response of market tightness to productivity and does not alter the response of job creation and, hence, unemployment.

Let assume that the productivity \( p \) is fixed and that the matching function takes a Cobb-Douglas form, so that \( f(\theta) = \mu \theta^{1-\alpha} \). Then, from the fixed point problem described in the second part of Proposition (1) we get that in equilibrium
\[ U(p) - V(p) = \frac{\delta \beta f(\theta(p)) \int_{x}^{\hat{x}(p)} [\delta p - x] d\Phi(x) + \delta z - \delta \theta(p) c}{1 + \delta \beta f(\theta(p)) \Phi(\hat{x}(p))}, \]

9
where $\delta = (1 - \beta (1 - s))^{-1}$ and the equilibrium $\hat{x} (p)$ and $\theta (p)$ solve Problem P1. Combining equation (8) with the first order conditions (6) and (7), after some algebra, the equilibrium can be characterized by the following two conditions

$$\hat{x} (p) = \delta (p - z) - \delta \left( \frac{\alpha}{1 - \alpha} \right) \theta (p) c,$$

$$\frac{1}{\delta \beta \Phi (\hat{x}) f (\theta) / \theta} + \alpha \theta = (1 - \alpha) \int_0^{\hat{x}(p)} \frac{[p - x / \delta - z] \Phi (x)}{c \Phi (\hat{x})} dx. \quad \text{(10)}$$

Notice that if I assume that the distribution of the idiosyncratic shocks is degenerate, and, for simplicity $x = 0$, then all matches are productive and the equilibrium can be characterized simply by

$$\frac{1}{\delta \beta f (\theta) / \theta} + \alpha \theta = (1 - \alpha) \frac{p - z}{c}.$$

This expression is exactly the discrete time version of the equilibrium equation of the baseline model in Shimer (2005), given that the firm’s matching probability $f (\theta) / \theta$ is equal to the queue length $q (\theta)$, and $(1 - \beta (1 - s)) / \beta \approx r + s$ where $r$ is the discount rate in the continuous time version. Then, the elasticity of market tightness with respect to productivity, $\varepsilon_{\theta p}$, is equal to

$$\varepsilon_{\theta p} = \frac{p}{p - z} \frac{1 - \beta (1 - s) + \alpha \beta f (\theta)}{\alpha [1 - \beta (1 - s) + \beta f (\theta)]}.$$ 

Can heterogeneity increase this elasticity? I now show that when the distribution of the idiosyncratic shocks is non-degenerate, there is an extra term in the expression for $\varepsilon_{\theta p}$, coming from the responsiveness of the hiring margin to productivity. However, I will show that when the model parameters are properly re-calibrated, this will not help to increase the equilibrium value of $\varepsilon_{\theta p}$.

Let me use the subscript $D$ to denote the model with a degenerate distribution of idiosyncratic shocks (and $x = 0$) and let me set the parameters $\mu_D, z_D, \alpha_D, c_D$ and $p, \beta, s$ at the corresponding values in Shimer (2005). In the degenerate case, the job creation rate $j_D$ is simply equal to the probability for a firm to find a worker, that is, $j_D (p) = f_D (\theta (p)) = \mu_D (1 - \alpha_D)$. When agents are heterogenous, the job creation rate comes from the combination of the standard finding probability and the additional hiring margin, that is, $j (p) = \mu (1 - \alpha) \Phi (\hat{x} (p))$ (I use no subscript for this general case). Moreover, the steady state level of unemployment is

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\footnote{The equivalence would work for any $x$ small enough, where the effective outside option for the worker is $z + x / \delta$, instead of $z$.}
given by

\[ u(p) = \frac{s}{s + j(p)}. \]  

Next, notice that the job creation rate and the unemployment rate in the model with heterogeneity are identical to the ones in the degenerate case, when \( p, \beta, s \) are the same in both models and the parameters \( \mu, z, \alpha \) and \( c \) satisfy:

\[ \mu \Phi(\hat{x}) = \mu_D, \]  
\[ E[x/\delta + z|x \leq \hat{x}] = z_D, \]  
\[ \alpha = \alpha_D, \]  
\[ c = c_D. \]

In the appendix I show that, when (12)-(15) hold, the derivative of the market tightness with respect to productivity in the model with idiosyncratic shocks coincides with the degenerate case.

**Proposition 2** Suppose conditions (12)-(15) are satisfied, then \( \theta' (p) = \theta'_D (p) \).

**Proof.** See Appendix. ■

The Proposition shows that keeping constant the elasticity of the matching function, \( 1 - \alpha \), the response of the hiring margin to productivity does not change the response of the market tightness. On the one hand, when the hiring margin increases, firms have an incentive to post more vacancies because the return per vacancy posted is higher. On the other hand, firms face a higher cost for vacancy posted because of two effects: a direct one, as the expected training cost for a matched worker has increased, and an indirect one, as the outside option of the matched worker has also increased. The Proposition shows that in equilibrium the increase of the benefit of a posted vacancy coincides exactly with the increase of its cost, and the response of the market tightness to productivity is not affected by the adjustment in the hiring margin.

Under the parametrization proposed the model matches the job creation rate in the baseline model (and, hence, in the data). However, the model does not match the elasticity of job creation to market tightness. As Shimer (2005) shows, there is an approximate loglinear relationship between market tightness and job creation rate in the data. In the degenerate model, where \( j(p) = \mu \theta (p)^{1-\alpha} \), the elasticity of the job creation rate to market tightness, \( d \log j/d \log \theta \), is simply equal to \( 1 - \alpha \) and can be estimated directly by regressing \( \log j \) on
log \theta. However, in the model with a non-degenerate distribution of idiosyncratic shocks \( j(p) = \mu \theta(p)^{1-\alpha} \Phi(\hat{x}(p)) \), which gives

\[
\frac{d \log j}{d \log p} = (1 - \alpha) \frac{d \log \theta(p)}{d \log p} + \frac{\varphi(\hat{x}(p)) d \log \hat{x}(p)}{\Phi(\hat{x}(p))} \frac{d \log \hat{x}(p)}{d \log p}. 
\]

By dividing this expression by \( d \log \theta/d \log p \), it follows that the elasticity of the job creation rate to market tightness corresponds to

\[
\frac{d \log j}{d \log \theta} = 1 - \alpha + \frac{\varphi(\hat{x}(p)) \hat{x}'(p)}{\Phi(\hat{x}(p)) \theta'(p)}. 
\]

This suggests to choose parameters \( \mu, z, \alpha \) and \( c \) such that

\[
\mu \theta^{1-\alpha} \Phi(\hat{x}) = \mu_D \theta^{1-\alpha_D}, \quad E[x/\delta + z|x \leq \hat{x}] = z_D \quad \alpha - \frac{\varphi(\hat{x}) \hat{x}'(p)}{\Phi(\hat{x}) \theta'(p)} \hat{x}(p) = \alpha_D, 
\]

\[
\frac{1 + \alpha \beta \delta \mu \theta^{1-\alpha} \Phi(\hat{x})}{1 - \alpha} = c_D \frac{1 + \alpha_D \beta \delta \mu_D \theta^{1-\alpha_D}}{1 - \alpha_D}. 
\]

With this parametrization the model can match not only the job creation rate and the unemployment rate, but also the elasticity of job creation to market tightness of the baseline economy.

**Proposition 3** Suppose conditions (17)-(20) are satisfied, then \( \theta'(p) < \theta'_D(p) \).

**Proof.** See Appendix.

This Proposition shows that once we parametrize the model properly, the responsiveness of the market tightness to productivity not only is not amplified, but it is actually dampened.

Finally, I look at the impact of heterogeneity on the responsiveness of job creation and unemployment rate to a productivity shock. First, notice that when the distribution of the idiosyncratic shocks \( x \) is non-degenerate then the response to productivity of the cut-off \( \hat{x}(p) \), that is, the hiring margin, is always positive. By differentiating condition (9) and using the expression for \( \theta'(p) \) derived in the Appendix (see proof of Proposition 3), it follows that

\[
\hat{x}'(p) = [1 - \beta (1 - s) + \beta f(\theta) \Phi(\hat{x})]^{-1} > 0. 
\]

Both the parametrizations presented above ensure that the job creation rate, and, thus, the unemployment rate, are the same as in the degenerate model. However, one can match the job
creation rate in two ways. In the first parametrization, I just shift the job creation function by
the constant factor $\mu$ in order to match the equilibrium level. In the second parametrization,
instead, I shift both $\mu$ and the elasticity $\alpha$ of the matching function, in order to match both the
equilibrium level and the equilibrium elasticity of job creation. Next Proposition shows that the
first parametrization amplifies the response of job creation to productivity and, consequently,
amplifies also the response of unemployment. However, under the proper parametrization (the
second one), these responses are both unchanged, once heterogeneity enters into the picture.

**Proposition 4** Suppose conditions (12)-(15) holds, then $j'(p) > j'_D(p)$ and $u'(p) < u'_D(p)$.
If instead conditions (17)-(20) are satisfied, then $j'(p) = j'_D(p)$ and $u'(p) = u'_D(p)$.

The results in this section highlight the importance of choosing the correct parametrization
to evaluate the impact of heterogeneity on job creation. I will show in Section 4 that the
dynamics of the stochastic model follow closely these analytical steady-state results.

### 3 Asymmetric information

In this section, I introduce asymmetric information, by assuming that the worker privately
observes his type. I show that in steady state there is an additional effect on the response
of market tightness to productivity, coming from the informational problem, which makes
ambiguous the comparison with the standard model. In chapter 4, I will show that, asymmetric
information can generate amplification in the fluctuations of the labor market, depending on
the distribution of the idiosyncratic shocks. However, these effects are typically small.

### Environment and Bellman Values

The economy is the same described in section 2, with the only difference that the training cost
$x$ is now private information of the worker and that $\varphi(x)$ satisfies the monotone hazard rate
condition, that is, $d [\Phi (x) / \varphi (x)] / dx > 0$. Given that $x$ is private information, a contract is
now a revelation mechanism, that is, a map $C : \Theta \times P \rightarrow [0, 1] \times \mathbb{R}_+$, specifying for each matched
worker when productivity is $p_t$, who reports type $\tilde{x}$, the hiring probability $e(\tilde{x}, p_t) \in [0, 1]$ and
the expected net present value of wages $\omega(\tilde{x}, p_t) \in \mathbb{R}_+$.\textsuperscript{11} Without loss of generality, by

\textsuperscript{11}In Guerrieri (2007), I show that this contract is without loss of generality, given that the firm would give a
zero transfer to workers that she decides not to hire.
invoking the Revelation Principle, I can restrict attention to the set $C$ of incentive-compatible and individually rational direct revelation mechanisms. Notice that I can restrict attention to this set of contracts, thanks to the assumption that unemployed workers are anonymous. All the unemployed workers searching for a job cannot be distinguished, so that contracts cannot be conditioned on the past employment history. Moreover, as in the full information case, linear preferences imply that the wage profile over the life of the relationship is irrelevant for the analysis.

The employment contract $C$ must be incentive compatible and individually rational, that is, it has to ensure that the worker reveals truthfully his type and chooses to participate in the employment relationship after the draw has been realized. The expected utility of a worker of type $x$ reporting type $\tilde{x}$ when productivity is $p_t$, is given by

$$v(x, \tilde{x}, p_t) = e(\tilde{x}, p_t)[\omega(\tilde{x}, p_t) - x + V(p_t)] + [1 - e(\tilde{x}, p_t)]U(p_t),$$

where $V(p_t)$ and $U(p_t)$, as in the case of full information, satisfy equations (1) and (2), and denote respectively the continuation value of employed workers net of wages and training cost and the continuation value of unemployed workers when productivity is $p_t$. Given $p_t$, an employment contract is *incentive-compatible (IC)* whenever it satisfies

$$v(x, x, p_t) \geq v(x, \tilde{x}, p_t) \text{ for all } x, \tilde{x} \in \Theta$$

and *individually rational (IR)* whenever

$$v(x, x, p_t) \geq U(p_t) \text{ for all } x \in \Theta.$$

Following Myerson (1981), I can reduce IC and IR to a monotonicity condition $e_x(x, p_t) < 0$ for all $x$, the IR binding for the worse type

$$v(x, x, p_t) \geq U(p_t), \quad (22)$$

and the following condition

$$v(x, x, p_t) = v(x, \overline{x}, p_t) + \int_{x}^{\overline{x}} e(y, p_t) \, dy \quad \forall x \in \Theta. \quad (23)$$

Finally, the large number of potential firms ensures free entry and implies that the value of an open vacancy will be zero at each time, that is,

$$\beta \frac{f(\theta(p_t))}{\Theta(p_t)} \int_{x}^{\overline{x}} [e(x, p_t) T(p_t) - \omega(x, p_t)] \, d\Phi(x) = c. \quad (24)$$

where $T(p_t)$ is defined in (5).
Competitive Search Equilibrium

This section generalizes the definition of the Competitive Search Equilibrium introduced in section 2 to an environment with asymmetric information.

It is possible to show that a recursive Competitive Search Equilibrium takes the same simple form of the full information case, except that now the equilibrium set of contracts contingent on the aggregate shock, \( C^* (p) \), is a set of incentive-compatible and individually-rational direct mechanisms.

Next Proposition is the natural extension of Proposition 1 and states the characterization of a stochastic symmetric competitive search equilibrium under asymmetric information in recursive terms.\(^{12}\)

**Proposition 5** If \( \{C^* (p), \Theta^* (p), U^* (p), V^* (p)\} \) is a Recursive Competitive Search Equilibrium, then any pair \( \{C^* (p), \theta^* (p)\} \) with \( C^* (p) \in C^* (p) \) and \( \theta^* (p) = \Theta^* (p, C^* (p)) \) satisfy the following

(i) for given pair of functions \( \{U^* (p), V^* (p)\}, \ C^* (p) = [e^* (x, p), \omega^* (x, p)]_{x \in \Theta} \) and \( \theta^* (p) \)

\[
W(U^* (p), V^* (p), p) = \max_{\theta^* (p)} \beta f (\theta (p)) \int e (x, p) [\omega (x, p) - x + V^* (p)] d\Phi (x)
\]

\[
+ \beta \left[ 1 - f (\theta (p)) \int e (x, p) d\Phi (x) \right] U^* (p) (P2)
\]

subject to \( e (x, p) \in [0, 1] \), the free-entry condition \((24)\), the constraints IC and IR reduced to the conditions \((22), (23)\) and \( e_x (x, p) < 0 \);

(ii) for given pair \( \{C^* (p), \theta^* (p)\} \), the pair of functions \( \{U^* (p), V^* (p)\} \) evolve according to

\[
U^* (p) = z + \int W(U^* (p'), V^* (p'), p') dG (p'|p)
\]

and

\[
V^* (p) = \beta \int [s U^* (p') + (1 - s) V^* (p')] dG (p'|p).
\]

\(^{12}\)Propositions 5 and 6 are natural extensions of the analysis in Guerrieri (2007) and therefore their proofs are omitted.
Conversely, if a pair of functions \( \{C^* (p), \theta^* (p) \} \) solves the program \( P4 \), then there exists an equilibrium \( \{C^* (p), \Theta^* (p), \Upsilon^* (p), \psi^* (p) \} \) such that \( C^* (p) \in \C^* (p) \) and \( \theta^* (p) = \Gamma^* (p, C^* (p)) \).

Proposition 5 shows that for given \( U (p) \) and \( V (p) \), a recursive symmetric equilibrium incentive-compatible and individually-rational contract \( C (p) \) and tightness \( \theta (p) \) must solve Problem \( P2 \). The next Proposition shows that the equilibrium can be equivalently described by a hiring function \( e (x, p) \) and a tightness \( \theta (p) \) that solve a simplified program \( P3 \). Given \( e (x, p) \) and \( \theta (p) \), an associated wage function \( \omega (x, p) \) can be constructed so that the incentive compatibility and participation constraints are satisfied.

**Proposition 6** For given \( U (p) \) and \( V (p) \), any couple of functions \( e (x, p) \) and \( \theta (p) \) which solve Problem \( P2 \), solve also

\[
W(U (p), V (p), p) = \max_{e(x,p), \theta(p)} \beta f (\theta (p)) \int e (x, p) [T (p) - x + V (p) - U (p)] d\Phi (x) \text{ (P3)} \\
+ \beta U (p) - \theta (p) c
\]

s.t.

\[
\beta f (\theta (p)) \int e (x, p) \left[ T (p) - x - \frac{\Phi (x)}{\varphi (x)} + V (p) - U (p) \right] d\Phi (x) \geq \theta (p) c \quad (25)
\]

Furthermore, for any pair of functions \( e (x, p) \) and \( \theta (p) \) which solve problem \( P3 \), there exists a function \( \omega (x, p) \) such that the contract \( C (p) = [e (x, p), \omega (x, p)]_{x \in \Theta, p \in Y} \) and \( \theta (p) \) solve problem \( P2 \).

In order to characterize the equilibrium, I proceed by studying the relaxed problem without the monotonicity assumption on \( e (x, p) \). Then, using pointwise maximization with respect to \( e (x, p) \), the trading area can be fully described by a cut-off value \( \hat{x} (p) \), as in the full information case, implying that the optimal \( e (x, p) \) is effectively non-increasing in \( x \). When the constraint of problem \( P3 \) is binding\(^{13}\), the equilibrium can be characterized, for given \( U (p) - V (p) \), by a set of functions \( \hat{x} (p) \), \( \theta (p) \) and \( \lambda (p) \) satisfying the first order conditions

\[
T (p) - \hat{x} (p) - \lambda (p) \frac{\Phi (\hat{x} (p))}{\varphi (\hat{x} (p))} - (U (p) - V (p)) = 0, \quad (26)
\]

\(^{13}\) A generalization of Lemma 2 in Guerrieri (2007) gives that the constraint is binding if and only if the cost of posting a vacancy \( c \) is strictly positive.
\[ \beta f' (\theta (p)) \int_{\mathbb{Z}}^\hat{x}(p) \left[ T(p) - x - \lambda (p) \frac{\Phi(x)}{\varphi(x)} - (U(p) - V(p)) \right] d\Phi(x) = c \]  

(27)

and the binding constraint (25). The variable \( \lambda (p) \) represents a normalized version of the shadow value of the informational rents.\(^{14}\) Notice that when \( \lambda (p) = 0 \), the constraint is slack and the full information allocation described in Section 2 is achieved. Clearly, for given \( U(p) - V(p) \), asymmetric information reduces job creation, as the surplus of the economy must cover not only the cost of vacancy creation but also the rents needed to extract information from the workers. As intuition suggests, \( \hat{x}(p) \) decreases with \( \lambda (p) \).

**Equilibrium with no Aggregate Shocks**

In order to give some intuition of the role of asymmetric information, let me fix \( p \) and consider the version of the model with no aggregate shocks.

From the fixed point problem described in the second part of Proposition (5), I obtain that in steady state \( U - V \) satisfies expression (8), where the equilibrium \( \hat{x} \) and \( \theta \) solve now Problem P3. Combining equation (8) with the binding constraint (25) and the first order conditions (26), (27), after some algebra, the equilibrium can be characterized by the following three conditions

\[ p - \frac{\hat{x}}{\delta} - \lambda \frac{\Phi(\hat{x})}{\varphi(\hat{x})} = -z + \theta c = \beta f(\theta) \int_{\mathbb{Z}}^\hat{x} \left( \hat{x} - x + \lambda \frac{\Phi(x)}{\varphi(x)} \right) d\Phi(x), \]

\[ \beta (1 - \alpha) f(\theta) \int_{\mathbb{Z}}^\hat{x} \left[ \hat{x} - x + \lambda \left( \frac{\Phi(x)}{\varphi(x)} - \frac{\Phi(\hat{x})}{\varphi(\hat{x})} \right) \right] d\Phi(x) = \theta c, \]

\[ \lambda = 1 - \left[ (1 - \alpha) \frac{\beta f(\theta)}{\theta} \int_{\mathbb{Z}}^\hat{x} \frac{\Phi(x)}{\varphi(x)} d\Phi(x) \right]^{-1} \alpha c. \]

Notice that when \( \lambda = 0 \) this system boils down to (6) and (7), as expected. The third equation shows that at a first order approximation \( \lambda \) decreases with \( \theta \), since the more vacancies are posted the higher becomes the expected surplus created per vacancy which cover the workers’ rents, and increases with \( \hat{x} \), because as the hiring margin increases more rents need to be paid to the workers and the distortion get worse. On the other hand, the first two equations show that as \( \lambda \) changes, market tightness and hiring margin are changing endogenously. Hence, the responsiveness of market tightness to productivity seems ambiguous. In any case, Section 4 shows that the impact of asymmetric information on volatility is not quantitatively significant.

\(^{14}\) Notice that \( \lambda \equiv \hat{\lambda} / (1 + \hat{\lambda}) \), where \( \hat{\lambda} \) is the Lagrangian multiplier attached to the constraint of problem P3. As in Guerrieri (2007), define \( v(x, x, p) - v(\mathbf{x}, \mathbf{x}, p) \) as the informational rent of a worker of type \( x \leq \mathbf{x} \), that is, the additional utility that such a worker must receive in order to reveal his own type.
Before turning to the calibration exercises, it is useful to notice that this model of wage determination is a special discrete time case of the general one considered in Brugemann and Moscarini (2007).\(^\text{15}\) It is easy to show that the weak upper bound on the elasticity of market tightness with respect to productivity derived in Brugemann and Moscarini (2007) applies.\(^\text{16}\) This comparative static analysis suggests that market tightness volatility may be not too high. In the next section, several numerical exercises will confirm that this is actually the case also once aggregate shocks are explicitly introduced in the model.

4 Unemployment volatility

In this section I analyze the cyclical behavior of unemployment and market tightness generated by the model, under full and asymmetric information, comparing them to the benchmark model with no idiosyncratic shocks. First I describe how I calibrate the different versions of the model. Then, I show the results.

Calibration

In order to investigate the role of heterogeneity and asymmetric information in explaining the U.S. unemployment rate volatility, I parametrize the model to match the facts of the US labor market documented in Shimer (2005).

I normalize a time period to be one quarter and then I set the discount factor \(\beta\) to .988, consistent with an annual discount factor of .953. I set the separation rate \(s\) equal to .1, consistent with jobs lasting about 2.5 years on average. I choose the stochastic process for labor productivity \(p\) following a discrete-time version of the strategy in Shimer (2005). I define an underlying stochastic variable \(y\), such that \(p_t = z + e^{yt} (p^* - z)\), where \(y_{t+1} = \rho y_t + \varepsilon_t\) with \(\rho \in (0, 1)\) and \(\varepsilon \sim N (0, \sigma^2)\). I approximate the AR(1) process for \(y\) with a 35-state Markov chain\(^{17}\) and normalize \(p^*\) to 1. I calibrate \(\rho\) and \(\sigma^2\) in order to match the volatility

\(^{15}\)I thank one of the referee to point this out.

\(^{16}\)The trading probability coincides with the job creation probability, the worker rents are \(\int_{-\infty}^{x} \Phi (x) dx\), and the firm rents are \(\int_{-\infty}^{x} \left[ T(p) - x - \frac{\Phi (p)}{\Phi (p - 1)} - (U(p) - V(p)) \right] d\Phi (x)\). In steady state \(T(p) - (U(p) - V(p)) = \alpha [p - (1 - \beta) U(p)]\). Substituting this into (26) and (27) and into the binding constraint (25), it is immediate that the trading probability, \(f(\theta) \Phi (\hat{x}(p))\) depends on \(p\) and \(1 - \beta) U(p)\) only through the expression \(\alpha [p - (1 - \beta) U(p)]\). Moreover, applying the implicit function theorem, it is easy to show that \(\hat{x}(p)\) is increasing in \(p\), which together with monotone hazard rate implies that worker rents are increasing in \(p\), that worker rents are increasing in the sense of Brugemann and Moscarini (2007), and that firms’ rents are non-increasing in \((1 - \beta) U(p)\).

\(^{17}\)I choose a 35-state Markov process as in Hagedorn and Manovskii (2006).
and autocorrelation of the resulting process for $\log p$ with the correspondent values in the data, reported in Shimer (2005), that is, respectively, .020 and .878.

I set the instantaneous utility of unemployed workers $z$ such that $z + E [x/\alpha | x \leq \hat{x}(p)]$ is approximately equal to .4 of the labor income. In my model the effective opportunity cost of being employed for a matched worker of type $x$ is $z + x/\alpha$ so that to calibrate $z$ equal to .4 of the labor income, would be equivalent to choose an effective higher value of leisure, in the direction of Hagerdon and Manovskii (2006). I want to shut down their mechanism to highlight the role of heterogeneity and information.

For the job-creation rate, $j_t$, I use the series constructed by Shimer (2005). He constructs the series for the job-creation rate from 1951 to 2003, using the series for the behavior of unemployment level and the short-term unemployment level constructed by the BLS from the CPS. In particular, $j_t$ is such that the number of unemployed next month is equal to the number of unemployed last month who did not find a job, that is, $(1 - j_t) u_t$, and workers who have lost the job between the beginning of this month and the end of it, denoted by $u_{t+1}^s$.

That is,

$$j_t = 1 - \frac{u_{t+1} - u_t^s}{u_t}.$$

Using this series, the monthly job-creation probability is .45, corresponding to a quarterly job-creation probability of .8336. As explained in section 2, in the model with idiosyncratic shocks, the job-creation probability corresponds to $f(\theta) \Phi(\hat{x})$ because unemployed workers in order to get a job have both to find an employer and being hired from him. Hence, I match

$$E [f(\theta) \Phi(\hat{x})] = .8336.$$  

I use the standard Cobb-Douglas matching function and set $f(\theta) = \mu \theta^{1-\alpha}$. Shimer (2005) estimates the elasticity parameter $1 - \alpha = .28$ with a first-order autoregressive residual using detrended data on the matching rate and the $v/u$ ratio for the U.S. between 1951 and 2003. When the distribution of the idiosyncratic shocks is degenerate, the job-creation probability coincides with the finding probability and the elasticity of job creation to market tightness is simply equal to $1 - \alpha$. However, when the distribution of the idiosyncratic shocks is not

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18 I thank Robert Shimer for making available the series. Notice that I call job creation rate what he calls finding rate, to make clear that in my model finding a match does not translate necessarily in being hired.

19 Notice that $u_{t+1}^s$ represents the number of workers unemployed for less than a month.

20 The hiring margin is trivial in the standard model, because it is optimal to hire any matched worker and the job-creation rate reduces simply to $f(\theta)$. 
degenerate and job-creation is equal to \( j_t = f(\theta_t) \Phi(\bar{x}_t) \), then the elasticity of job creation to market tightness is given in equation (16) and takes into account the fact that the hiring margin reacts to changes in the market tightness. I perform two different types of exercises: \textit{quasi-calibration} and \textit{proper calibration}. In the quasi-calibration, I keep \( 1 - \alpha \) equal to .28, and calibrate \( \mu \) in order to match expression (28). In the proper calibration, instead, I calibrate \( \alpha \) and \( \mu \) in order to match simultaneously expression (28) and the elasticity of job creation to market tightness, using the simulated series for market tightness and job creation generated by the model, that is,

\[
\frac{\text{Cov}(\log j_t, \log \theta_t)}{\text{Var}(\log \theta_t)} = .28.
\]

As in Shimer (2005), if I double \( c \) and multiply \( \mu \) by \( 2^{1-\alpha} \), then \( \theta \) is reduced by \( 1/2 \) and the rate at which firms find workers is doubled, but the worker's finding rate is not affected, that is, the scale of \( \theta \) is meaningless in the model. I set \( c = .2 \), in order to normalize the average \( \theta \) to 1 and make the calibration of the degenerate version of the model identical to the calibration of the discrete version of Shimer (2005).

Finally, I show that my results are relatively not sensitive to the choice of the distribution for the idiosyncratic shocks. I report the results using three families of distributions: Pareto, Uniform and Lognormal.

\section*{Results}

In this section, I report the main results coming from my calibration exercises. I use the calibrated parameter values to simulate the model and create artificial time series of unemployment and vacancy rates, market tightness and finding rate. Then, I compare the volatility of the simulated series obtained with the degenerate model to the ones obtained with the model with heterogeneity, both under full information and under asymmetric information. I report the results for different distributions of the idiosyncratic shock (Pareto, Uniform and Lognormal). Overall, I find that heterogeneity does not help much in amplifying the volatility of unemployment and vacancy rates in comparison with the benchmark model, even in the case of asymmetric information.

First, I show the results for the degenerate version of the model that matches the results in Shimer (2005). Table 1 reports the standard deviation for unemployment, vacancy, market tightness, job-creation rate and productivity in detrended logs.
Table 1. Degenerate model

<table>
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<td>.009</td>
<td>.027</td>
<td>.035</td>
<td>.010</td>
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Note: Results from simulating the model with degenerate distribution of idiosyncratic shocks. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Next, I compare these results with the model with heterogeneity when the distribution of the idiosyncratic shocks is Pareto. Table 2 reports the same statistics under full and asymmetric information, both for the quasi-calibration exercise and the full-calibration one.

Table 2. Pareto distribution of $x$

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<tbody>
<tr>
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<td>.010</td>
<td>.027</td>
<td>.035</td>
<td>.011</td>
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<tr>
<td>Proper calibration</td>
<td>.009</td>
<td>.027</td>
<td>.034</td>
<td>.010</td>
<td>.020</td>
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<tr>
<td>Asymmetric Information</td>
<td>Quasi-calibration</td>
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<td>.027</td>
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<td>Proper calibration</td>
<td>.009</td>
<td>.028</td>
<td>.035</td>
<td>.010</td>
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Note: Results from simulating the model with Pareto distribution of idiosyncratic shocks with parameters .4 and 3. In the full information case, the quasi-calibration requires $b = .327$ and $\mu = .851$, and the proper calibration requires $b = .329$, $\mu = .887$ and $1 - \alpha = .249$. In the asymmetric information case, the quasi-calibration requires $b = .330$ and $\mu = .874$, and the proper calibration requires $b = .332$, $\mu = .924$ and $1 - \alpha = .230$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

In the quasi-calibration exercise for the full information case I keep $1 - \alpha = .28$ and set $\mu = .851$ in order to match (28). Notice that $\mu$ has to increase in comparison to the benchmark $\mu = .834$ because when workers are heterogenous, a match does not translate automatically in job creation. Table 2 shows that both unemployment and job creation are slightly more volatile in comparison to the benchmark, as Proposition 4 suggests. This is due to the endogenous movements in the hiring margin which responds to changes in productivity. Moreover, consistently with Proposition 2, the market tightness volatility is approximately equal to the benchmark. However, the movements in unemployment and job creation are minuscule and, once I properly calibrate the model, by changing also $\alpha$ in order to match condition (29), even these small volatility gains approximately disappear and, as Proposition 3 suggests, the market tightness volatility actually decreases slightly.

When asymmetric information enters into the picture, there is an extra source of volatility, given that the distortion generated by the informational rents that the employers have to pay
to the workers is also responsive to productivity. Using the quasi-calibration, unemployment, market tightness, and job creation exhibit higher volatility, although the increase in volatility is minuscule. Moreover, once one considers the proper calibration exercise, once again, the increase in unemployment and job creation volatility virtually disappears and only the vacancy rate seems to be slightly more volatile than in the benchmark. Unemployment appears more volatile only if one looks at the fourth digit, unemployment goes from .0086, in the degenerate case, to .0087, in the proper calibration for the case of asymmetric information. Job creation goes only from .346 to .353. If follows that also asymmetric information, once calibrated properly, does not seem to have any relevant impact on volatility of unemployment, market tightness, or job creation.

Table 3 reports similar results for the case of a lognormal distribution for the idiosyncratic shocks.

<table>
<thead>
<tr>
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<th>Quasi-calibration</th>
<th>Proper calibration</th>
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<tr>
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<td>Asymmetric Information</td>
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Note: Results from simulating the model with Lognormal distribution of idiosyncratic shocks with parameters $\mu = 0.2$ and $\sigma^2$. In the full information case, the quasi-calibration requires $b = 0.252$ and $\mu = 0.836$, and the proper calibration requires $b = 0.252$, $\mu = 0.852$ and $1 - \alpha = 0.265$. In the asymmetric information case, the quasi-calibration requires $b = 0.253$ and $\mu = 0.856$, and the proper calibration requires $b = 0.256$, $\mu = 0.913$ and $1 - \alpha = 0.220$. All variables are reported in logs as deviations from an HP trend with smoothing parameter $10^{5}$.

Also in the case of a lognormal distribution for idiosyncratic shocks, heterogeneity does not seem to have any significant impact on labor market fluctuations, with or without full information. The table shows that, under full information, heterogeneity seems to have virtually no impact on the volatility of unemployment, job creation, or market tightness (this actually slightly drops again, from .0346 to .0345). As in the case of a Pareto distribution, under asymmetric information a quasi-calibration would deliver slight increase in volatilities. However, using the proper calibration, these increase almost disappear.

Finally, I check that I obtain similar results using the uniform distribution. It is interesting to notice that a proper calibration exercise imposes some restrictions on the parameters of the
Uniform distribution. In this paper the distribution of the idiosyncratic shocks is not pinned down by the data. However, this example shows that not all the distributions are consistent with the model. In fact, the range of the Uniform imposes a lower bound on the response of the job creation rate to the business cycle fluctuations, that is, on the matching parameter $1 - \alpha^D$ in the case of a degenerate distribution. Given that the data require $1 - \alpha^D = .28$, the set of the possible distributions consistent with the model is restricted. Let me set $\bar{x} = 0$. Then, for the model with asymmetric information, a proper calibration imposes an upper bound $\bar{x}$ of 1.25. Table 4 reports the results for both full and asymmetric information for $\bar{x} = 1.25$.

<table>
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<th>Table 4. Uniform distribution of $x$</th>
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<td>Proper calibration</td>
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<tr>
<td>.010</td>
</tr>
<tr>
<td>v .029</td>
</tr>
<tr>
<td>$\theta$ .036</td>
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<tr>
<td>$j$ .011</td>
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<tr>
<td>$p$ .020</td>
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</tbody>
</table>

Note: Results from simulating the model with Uniform distribution of idiosyncratic shocks with parameters 0 and 1.25. In the full information case, the quasi-calibration coincides with the proper calibration and requires $b = .321$ and $\mu = \mu^D$. In the asymmetric information case, the quasi-calibration requires $b = .323$ and $\mu = .882$, and the proper calibration requires $b = .328$, $\mu = .956$ and $1 - \alpha = .134$. All variables are reported in logs as deviations from an HP trend with smoothing parameter $10^5$.

The table shows that asymmetric information generates small increase in volatilities of unemployment, market tightness and job creation. As in the previous cases, these increases in volatility are even smaller with a proper calibration. Moreover, in this case, the results under full information are exactly the same as the degenerate case because the equilibrium hiring margin is at a corner, that is, $\hat{x} = \bar{x}$. To make the full information case interesting, Table 5 reports the results for the full information case with $\bar{x} = 1.62$.

<table>
<thead>
<tr>
<th>Table 5. Uniform distribution of $x$.</th>
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<tr>
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<tr>
<td><strong>Full Information</strong></td>
</tr>
<tr>
<td>Quasi-calibration</td>
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<tr>
<td>.009</td>
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<tr>
<td>v .027</td>
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<td>$\theta$ .035</td>
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<td>$p$ .020</td>
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</tbody>
</table>

Note: Results from simulating the model with Uniform distribution of idiosyncratic shocks with parameters 0 and 1.62. In the quasi-calibration $b = .301$ and $\mu = .833$. In the proper calibration $b = .301$, $\mu = .837$ and $\eta = .276$. All variables are reported in logs as deviations from an HP trend with smoothing parameter $10^5$. 23
Once again, the results show that heterogeneity under full information does not have virtually any impact on the volatility of unemployment, market tightness and job creation.

These calibration exercises suggest that the type of heterogeneity I have introduced in the standard search model cannot explain the high observed volatility of market tightness, unemployment and job creation.

5 Conclusions

This paper explores the role of heterogeneity and asymmetric information in explaining the cyclical behavior and the volatility of unemployment and vacancies in search models.

First, I have derived some steady state comparative statics. I have shown that, when information is symmetric, heterogeneity dampens the response of market tightness to productivity shocks and does not affect the response of job creation and, hence, of the unemployment rate. Then, I have performed some calibration exercises both for the case of symmetric and asymmetric information, choosing the parameter values in order to match some basic statistics for the post-war U.S. economy. I have considered idiosyncratic shocks distributed according to Uniform, Pareto and Lognormal distributions and I have shown that in all cases there is no significant amplification in the volatility of unemployment and vacancies. Attempts to explicitly calibrate the distribution of idiosyncratic shocks are left for future research.

The main result of the paper is a negative one: match-specific heterogeneity affecting the job creation margin does not amplify significantly the response of unemployment rate and vacancies to productivity shocks in comparison with the standard model. This is true whether or not one introduces asymmetric information.

Appendix

Proof of Proposition 2. First, rewrite equation (10) as the following implicit function

\[ h(x(p), \theta(p), p) = \frac{\int_{\bar{x}}^{\underline{x}(p)} [\bar{x} - \delta x] \Phi(x) \Phi(x)}{\Phi(\bar{x})} - \left[ \frac{1 - \beta (1 - s)}{(1 - \alpha) \beta f(\theta) \Phi(\bar{x})} + \frac{\alpha}{1 - \alpha} \right] c\theta = 0. \]

By total differentiating it, I get

\[ h_x(x(p), \theta(p), p)x'(p) + h_\theta(x(p), \theta(p), p)\theta'(p) + h_p(x(p), \theta(p), p) = 0. \]
Then,

\[ \theta'(p) = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)} - \frac{h_\theta(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)} \hat{x}'(p) \]

\[ = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)} \left[ 1 + \frac{h_\theta(\hat{x}(p), \theta(p), p)}{h_p(\hat{x}(p), \theta(p), p)} \hat{x}'(p) \right] , \]

where

\[ h_\theta(\hat{x}(p), \theta(p), p) = \left[ p - \frac{\hat{x}(p)}{\delta} - z - E \left[ p - \frac{x}{\delta} - z \mid x \leq \hat{x} \right] + \frac{1 - \beta (1 - s)}{(1 - \alpha) \beta f(\theta) \Phi(\hat{x})} \right] \frac{\varphi(\hat{x}(p))}{\Phi(\hat{x})} . \]

Using equation (10) I can rewrite

\[ h_\theta(\hat{x}(p), \theta(p), p) = p - \frac{\hat{x}(p)}{\delta} - z - \left( \frac{\alpha}{1 - \alpha} \right) c \theta, \]

which is equal to zero from the first equilibrium condition equation (9), implying that

\[ \theta'(p) = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)} . \]

Next, consider the case of a fixed \( \hat{x} = 0 \), then the equilibrium condition becomes

\[ h^D(\theta(p), p) \equiv p - z_D - \left[ \frac{1 - \beta (1 - s)}{(1 - \alpha) \beta f_D(\theta) \Phi(\hat{x})} + \frac{\alpha}{1 - \alpha} \right] c \theta = 0. \]

By total differentiating it, I obtain

\[ h_D^p(\theta(p), p) \theta'(p) + h_D^\theta(\theta(p), p) = 0, \]

so that

\[ \theta'(p) = -\frac{h_D^p(\theta(p), p)}{h_D^\theta(\theta(p), p)} . \]

Conditions (12)-(15) give that

\[ h_p(\hat{x}(p), \theta(p), p) = h_p^D(\theta(p), p) = 1, \]

\[ h_\theta(\hat{x}(p), \theta(p), p) = h_\theta^D(\theta(p), p) = -\frac{[1 - \beta (1 - s) + \beta f_D(\theta)] \alpha}{1 - \beta (1 - s) + \beta f_D(\theta) \alpha} \left( \frac{p - z_D}{\theta} \right) , \]

and, hence

\[ \frac{\theta'(p)}{\theta'_D(p)} = 1, \]

completing the proof.

**Proof of Proposition 3.** Following the first steps of the previous proof I can show that

\[ \theta'(p) = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)}, \]
If condition (14) is satisfied, then

\[ \theta_D' (p) = - \frac{h_p^D (\theta (p), p)}{h_\theta^D (\theta (p), p)}. \]

Now, conditions (17)-(20) give that

\[ h(\hat{x} (p), \theta (p), p) = p - z_D - \left[ \frac{1 - \beta (1 - s)}{\alpha (1 - s) + \beta f_D (\theta)} + \frac{1 - \alpha}{\alpha} \right] c \theta = 0. \]

It follows that

\[ h_\theta (\hat{x} (p), \theta (p), p) = - \left[ \frac{1 - \beta (1 - s) + \beta f_D (\theta)}{(1 - s) f_D (\theta)} \right] \alpha, \]

and, using the first order condition (10), I obtain

\[ h_p (\hat{x} (p), \theta (p), p) = h_p^D (\theta (p), p) = 1, \]
\[ h_\theta (\hat{x} (p), \theta (p), p) = \left[ \frac{1 - \beta (1 - s) + \beta f_D (\theta)}{(1 - s) f_D (\theta)} \right] \alpha p - z_D, \]
\[ h_\theta^D (\theta (p), p) = \left[ \frac{1 - \beta (1 - s) + \beta f_D (\theta)}{(1 - s) f_D (\theta)} \right] \alpha p - z_D. \]

After some algebra, this implies that

\[ \theta' (p) - \theta_D' (p) = \left[ \frac{1}{\alpha} - \frac{1}{\alpha_D} \right] \left[ \frac{1 - \beta (1 - s)}{(1 - s) f_D (\theta)} \right] \theta \]

Given that \( \hat{x}' (p) \) and \( \theta' (p) \) are positive, from (18), it follows that \( \alpha > \alpha_D \) and then

\[ \theta' (p) < \theta_D' (p), \]

completing the proof.

**Proof of Proposition 4.** From the definition of \( j (p) \) and \( j_D (p) \), it follows that

\[ j' (p) - j_D' (p) = f_D (\theta (p)) \frac{\varphi (\hat{x} (p))}{\Phi (\hat{x} (p))} \hat{x}' (p) - (\alpha - \alpha_D) f_D (\theta (p)) \theta (p)^{-1} \theta' (p). \]

If condition (14) is satisfied, then \( \alpha_D = \alpha \) and equation (21) implies that

\[ j' (p) - j_D' (p) = f_D (\theta (p)) \frac{\varphi (\hat{x} (p))}{\Phi (\hat{x} (p))} \hat{x}' (p) > 0. \]

If, instead, condition (19) holds, then

\[ \alpha - \alpha_D = \frac{\varphi (\hat{x})}{\Phi (\hat{x})} \frac{d \hat{x}}{d \theta}, \]

and \( j' (p) = j_D (p) \). Differentiating equation (11) gives

\[ u' (p) = - \frac{u (p)}{s + J (p)} j' (p). \]

It follows immediately that, under conditions (12)-(15),

\[ u' (p) = - j' (p) - \frac{u (p)}{s + J (p)} < - j_D' (p) \frac{u_D (p)}{s + J_D (p)} = u_D (p), \]

while under conditions (17)-(20), \( u' (p) = u_D' (p) \), completing the proof.
References


