Collateral Constraints and Macroeconomic Asymmetries

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Abstract

A simple macroeconomic model with collateral constraints displays strong asymmetric responses to changes in house prices. When house prices are high, collateral constraints are slack, and the response of aggregate consumption, hours and output to a housing wealth shock is positive but small. When house prices are low, collateral constraints are tight, and the response of consumption to a given change in housing values is negative and large. This finding is corroborated using national VAR analysis, state and MSA level data. Our results imply that wealth effects computed in normal times might severely underpredict the response of the economy to large house price declines, and that public policies aimed at helping the housing market may be far more effective during protracted housing downturns.

KEYWORDS: Housing, Collateral Constraints, Occasionally Binding Constraints.

JEL CODES: E32, E44, E47, R21, R31

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*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System. Replication codes that implement our solution technique for DSGE models with occasionally binding constraints (irreversible capital, zero bound, occasionally binding borrowing constraints) using an add-on to Dynare are available upon request. Stedman Hood and Walker Ray performed superb research assistance. Supplemental material is available at http://www2.bc.edu/matteo-iacoviello/research.htm.

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1 Introduction

Accounts of the recent financial crisis attribute a central role to the collapse in housing wealth and to financial frictions in explaining the sharp contraction in consumption and overall economic activity.\footnote{For instance, see Mian and Sufi (2010) and Hall (2011).} Prior to the crisis, however, the increase in housing wealth associated with the steady increase in house prices between 2001 and 2006 seems to have had much less effect in boosting consumption. Taken together, these observations appear hard to reconcile with the notion that the importance of housing collateral for macroeconomic aggregates is constant over the business cycle, and instead point to an asymmetry in the relationship between housing prices and economic activity. In this paper, we argue that the sensitivity of macroeconomic aggregates to movements in housing wealth can be large when housing wealth is low, and small when housing wealth is high. We develop this argument in a quantitative general equilibrium model, and confirm its predictions against U.S. data.

Our main story goes as follows. When housing wealth is high, collateral is abundant, and the sensitivity of borrowing and spending to changes in house prices is positive but not large. Conversely, when housing wealth is low, collateral constraints are tight, and borrowing and expenditures move with house prices in a more dramatic fashion. As a consequence, the effects of movements in housing prices are larger and more severe when housing wealth is low than when it is high. The empirical analysis overwhelmingly supports our model’s findings that the fallout from a persistent decline in housing prices is much more severe than the boost to activity from an increase.

The model used in this paper is borrowed from Iacoviello and Neri (2010). It is an estimated DSGE model that allows for numerous empirically-realistic nominal and real rigidities as in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). In addition, the model encompasses a housing sector. On the supply side, a separate sector produces new homes using capital, labor, and land. On the demand side, households consume housing services and can use housing as collateral for loans. In characterizing the properties of the model, we focus on a shock to households preferences for housing. When house prices decline, household wealth is reduced, collateral constraints become binding, and the effective share of credit-constrained households increases. In contrast, house price increases relax households’ borrowing constraints. We employ a non-linear solution technique that allows us to capture asymmetric effects of shocks depending on whether the shocks push housing wealth up or down.\footnote{Iacoviello and Neri (2010) solve a model of this kind using a first-order perturbation method. As a result, the importance of credit-constrained agents remains constant and the effects of shocks that move house prices is symmetric for increases and decreases.} A simple moment-matching exercise shows that the data prefer a version of the model that can generate a response of consumption and hours to house prices that is three times larger when house prices are low than when they are high.

Figure 1 offers a first look at national house prices. It shows the evolution of U.S. house prices over the period 1975-2012. To highlight their correlation properties, the top panel superimposes the

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1 For instance, see Mian and Sufi (2010) and Hall (2011).
2 Iacoviello and Neri (2010) solve a model of this kind using a first-order perturbation method. As a result, the importance of credit-constrained agents remains constant and the effects of shocks that move house prices is symmetric for increases and decreases.
time series of U.S. house prices and of U.S. aggregation consumption expenditures. The correlation coefficient is 0.55, a value substantial but not extreme. The bottom panel is a scatterplot of changes in consumption and house prices. It highlights that most of the positive correlation seems to be driven by periods when house prices are below average, both during the 1992-1993 period, and during the 2007-2009 recession. When periods with house price decreases (the solid, magenta line) are included, there is a strong positive correlation between consumption and house prices. However, excluding periods with declines in house prices results in almost no correlation between consumption and house prices.

We test the prediction of the model that house price changes should have asymmetric effects using both national and regional data. We proceed in two steps. First, we estimate a VAR that includes U.S. consumption and house prices. Each equation in the VAR allows for separate house price terms, depending on whether house prices are high or low. Estimates of the VAR parameters based on data generated by the model imply a strong asymmetry in the response of consumption to innovations in house prices, depending on whether the shock to house prices is positive or negative. These population estimates are remarkably consistent with estimates obtained using aggregate U.S. data.

In the second step, we use regional data. The task of isolating the asymmetric effect of changes in house prices using only national data may be fraught with difficulty. Barring the Great Recession, house price declines have been rare at the national level. In addition, knowing what would have happened to economic activity had house prices not changed raises challenging identification issues. Accordingly, we use panel and cross-sectional regressions at the regional level. Regional data exhibit greater variation in house prices. Moreover, at the regional level, we can use instruments that other studies have found useful in isolating exogenous changes in house prices. In doing so, we verify that the asymmetries uncovered using national aggregate data are even more pronounced when using regional data.³

Our analysis builds on an expanding literature that has linked changes in measures of economic activity, such as consumption and employment, to changes in house prices. Recent contributions include Case, Quigley, and Shiller (2005), Campbell and Cocco (2007), Mian and Sufi (2011), Midrigan and Philippon (2011), Mian, Rao, and Sufi (2012), Liu, Wang, and Zha (2011), and Abdallah and Lastrapes (2012). The emerging consensus from this literature points towards an important role for housing as collateral in influencing both consumption and employment. However, such literature has not recognized that such a channel implies asymmetric relationships for house price increases and declines with other measures of aggregate activity. Furthermore, our uncovering of statistically significant differences for house price increases and declines, as theory predicts,³

³ We are fully aware of the notion that housing prices are endogenous both in theory and in the data. Our modeling strategy attributes most of the variation in house prices to shocks to housing preferences (as in recent work by Liu, Wang, and Zha (2011) and Iacoviello and Neri (2010)). Part of our empirical analysis looks for instruments for house price changes in a way to isolate housing preference shocks from other shocks that are more likely to jointly move both housing and other endogenous variables, as done by Mian and Sufi (2011).
provides more cogent support for the hypothesis that the housing collateral channel has played an important role in linking house price fluctuations to other key measures of economic activity. In addition, an important contribution of this paper is that we analyze this asymmetry not only empirically, but also theoretically in the context of a quantitative equilibrium model.\footnote{The idea that borrowing constraints may introduce asymmetric responses of consumption to shocks is a well-known result in macroeconomics. For instance, Jappelli and Pistaferri (2010) observe that if households are credit constrained, they will cut consumption strongly when hit by a negative transitory shock but will not react much to a positive one.}

To the best of our knowledge, Case, Quigley, and Shiller (2005) and Case, Quigley, and Shiller (2011) first highlighted the possibility, using U.S. state-level data, that house prices could have asymmetric effects on consumption. Their 2011 paper, in particular, finds in some specifications that declines in housing market wealth have had negative and somewhat larger effects upon consumption than previous increases. Our analysis extends their work by considering a larger set of variables and regional detail, by tying the results to a full-blown estimated equilibrium model, and by illustrating the policy relevance of this asymmetry. Our paper is also related to the work of Lustig and van Nieuwerburgh (2010): they find that in times when US housing collateral is scarce nationally, regional consumption is about twice as sensitive to income shocks, but the channel they emphasize is the extent to which risk-sharing between regions changes over time.

The rest of the paper proceeds as follows: Section 2 presents a simple intuition for why collateral constraints imply an asymmetry in the relationship between house prices and consumption using a partial equilibrium model. Section 3 considers an empirically-validated general equilibrium model. Section 4 highlights properties of the general equilibrium model and matches them against an asymmetric VAR estimated on aggregate U.S. data. Section 5 presents additional evidence on asymmetries in the relationship between house prices and other measures of economic activity based on state and MSA-level data. Section 6 considers an experiment which highlights how the same policy – a transfer to indebted borrowers – can have opposite effects depending on whether house prices are high or low. Section 7 concludes.

# 2 Collateral Constraints and Asymmetries: A Basic Model

To fix ideas regarding the fundamental asymmetry introduced by collateral constraints, it is useful to work through a simple model and analyze its implications for the size of the response of consumption to changes in housing prices. Throughout this section, we sidestep obvious general equilibrium considerations and assume that the price of housing is exogenous: we relax all these assumptions in the DSGE model of the next section. Consider the problem of a household that has to choose profiles for goods consumption $c_t$, housing $h_t$, and borrowing $b_t$. The utility of the household is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t (\log c_t + j \log h_t)$$

(1)
where $E_0$ is the conditional expectation operator. The budget and borrowing constraints are given by:

$$c_t + q_t h_t = y + b_t - R b_{t-1} + q_t (1 - \delta_h) h_{t-1};$$  \hspace{1cm} (2)

$$b_t \leq mq_t h_t,$$  \hspace{1cm} (3)

where $R$ denotes the gross one-period interest rate, and $\beta$ is assumed to satisfy the restriction that $\beta R < 1$, so that in a steady state without shocks the borrowing constraint is binding and leverage (the ratio of debt to housing wealth) is at its upper bound given by $m$. The price of housing, $q_t$, is assumed to follow an AR(1) stochastic process, and income $y$ is exogenously fixed and normalized to one. Housing, which depreciates at rate $\delta_h$, is used as collateral for borrowing, and $q_t h_t$ is the value of collateral. The parameter $m$ denotes the maximum loan-to-value ratio. Letting $\mu_t$ be the Lagrange multiplier on the borrowing constraint, the consumption Euler equation is:

$$\frac{1}{c_t} = \beta R E_t \left( \frac{1}{c_{t+1}} \right) + \mu_t. $$ \hspace{1cm} (4)

In a steady state, $\bar{\mu} > 0$, and $\bar{\mu} = \bar{y} - ((R - 1) m - \delta_h) \bar{q}\bar{h}$. Solving this equation forward and log-linearizing around the steady state, one obtains the following expression for consumption in percent deviation from steady state, $\bar{c}_t$:

$$\bar{c}_t = -\frac{1 - \beta R}{\bar{\mu}} E_t (\mu_t - \bar{\mu} + \beta R (\mu_{t+1} - \bar{\mu}) + \beta^2 R^2 (\mu_{t+2} - \bar{\mu}) + ...). $$ \hspace{1cm} (5)

Expressing the Euler equation as above shows that consumption depends negatively on current and future expected borrowing constraints. As shown by equation 3, increases in $q_t$ will loosen the borrowing constraint. So long as they keep $\mu_t$ positive, increases and decreases in $q_t$ will have roughly symmetric effects on $c_t$. However, large enough increases in $q_t$ imply a fundamental asymmetry. The multiplier $\mu_t$ cannot fall below zero. Consequently, large increases in $q_t$ can bring $\mu_t$ to its lower bound and will have proportionally smaller effects on $c_t$ than decreases in $q_t$. Intuitively, an impatient borrower prefers a consumption profile that is declining over time. A large temporary increase in house prices will enable such a profile (high $c$ today, low $c$ tomorrow) without borrowing all the way up to the limit.

More formally, the household’s state at time $t$ is its housing $h_{t-1}$, debt $b_{t-1}$ and the current realization of the house price $q_t$, and the optimal decision are given by the consumption choice $c(q, h, b)$, the housing choice $h'(q, h, b)$ and the debt choice $b'(q, h, b)$ that maximize expected utility subject to 2 and 3, given the house price process. Figure 2 illustrates the optimal leverage and the consumption function obtained from the model above given the parameter values calibrated and estimated in the next section.\footnote{Figure 2 shows the policy functions obtained solving the partial equilibrium model described in this section using standard global methods. For the general equilibrium model described below in Section 3, we approximate the}
the household to a region where the borrowing constraint is not binding. When the constraint is not binding, consumption becomes less sensitive to changes in house prices. Instead, when the household is borrowing constrained, with leverage is at its maximum level—something that happens when house prices are low and initial stock debt is high—the sensitivity of consumption to changes in house prices becomes large.

3 The Full Model

To quantify the importance of the asymmetric relationship between house prices and consumption, we now embed the basic ideas of Section 2 in an empirically validated general equilibrium model. The model is borrowed from Iacoviello and Neri (2010). It builds on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) by allowing for two sectors, a housing sector and non-housing a sector, as well as financial frictions and borrowing collateralized by housing following Iacoviello (2005).

On the supply side, firms in the housing sector produce new homes using capital, labor and land. Firms in the non-housing sector produce intermediate consumption and investment goods using capital and labor. The non-housing sector features nominal price rigidities. Both sectors have nominal wage rigidities and real rigidities in the form of imperfect labor mobility, capital adjustment costs and variable capital utilization.

On the household side, there is a continuum of agents in each of two groups that display different discount factors. Households in the group with the higher discount factor are dubbed “patient,” the other “impatient.” Patient households accumulate housing and own the productive capital of the economy. They make consumption and investment decisions and supply labor to firms and funds to both firms and impatient households. Impatient households work, consume, and accumulate housing. Their higher impatience pushes them to borrow. In the non-stochastic steady state, their housing collateral constraint is binding.

Below, we sketch the key features of the model. Appendix B provides the list of all necessary conditions for an equilibrium.

solution using the methods described in Appendix C (for the simple model of this section, Appendix A compares the properties of the solution methods). The parameter values match those of Table 1 and are: \( \beta = 0.988, j = 0.12, m = 0.925, R = 1.01, \delta = 0.01 \). The resulting steady-state housing wealth to quarterly income ratio is 6.1, close to the housing wealth to income ratio for impatient households in the steady state of the extended model. Finally, the house price process is described by an AR(1) process of the form

\[
\log q_t = \rho_q \log q_{t-1} + \varepsilon_{q,t}
\]

where \( \rho_q = 0.96 \) and \( \varepsilon_{q,t} \) is a zero mean, i.i.d. process with standard deviation equal to 0.0169, in order to match a standard deviation of the quarterly growth rate of house prices equal to 1.71 percent, as in the data.
3.1 Households

Within each group of patient and impatient households, a representative household maximizes:

$$E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left( \Gamma_c \ln (c_t - \varepsilon c_{t-1}) + j_t \ln h_t - \frac{\tau_t}{1 + \eta} \left( n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{1+\gamma} \right);$$  \hspace{1cm} (6)

$$E_0 \sum_{t=0}^{\infty} (\beta' G_C)^t z_t \left( \Gamma_c' \ln (c_t' - \varepsilon' c'_{t-1}) + j_t \ln h_t' - \frac{\tau_t}{1 + \eta'} \left( n_{c,t}'^{1+\xi'} + n_{h,t}'^{1+\xi'} \right)^{1+\gamma'} \right).$$  \hspace{1cm} (7)

Variables accompanied by the prime symbol refer to patient households. $c$, $h$, $n_c$, $n_h$ are consumption, housing, hours in the consumption sector and hours in the housing sector. The discount factors are $\beta$ and $\beta'$. By definition, $\beta' < \beta$. The terms $z_t$, $j_t$, and $\tau_t$ capture shocks to intertemporal preferences, labor supply, and housing preferences, respectively. The shocks follow:

$$\ln z_t = \rho_z \ln z_{t-1} + u_{z,t}, \quad \ln j_t = (1 - \rho_j) \ln j + \rho_j \ln j_{t-1} + u_{j,t}, \quad \ln \tau_t = \rho_{\tau} \ln \tau_{t-1} + u_{\tau,t},$$  \hspace{1cm} (8)

where $u_{z,t}$, $u_{j,t}$, $u_{\tau,t}$ and are i.i.d. processes with variances $\sigma_z^2$, $\sigma_j^2$, and $\sigma_{\tau}^2$. Above, $\varepsilon$ measures habits in consumption and $G_C$ is the growth rate of consumption along the balanced growth path. The scaling factors $\Gamma_c = (G_C - \varepsilon) / (G_C - \beta \varepsilon G_C)$ and $\Gamma_c' = (G_C - \varepsilon') / (G_C - \beta' \varepsilon G_C)$ ensure that the marginal utilities of consumption are $1/c$ and $1/c'$ in the non-stochastic steady state.

Patient households accumulate capital and houses and make loans to impatient households. They rent capital to firms, choose the capital utilization rate; in addition, there is joint production of consumption and business investment goods. Patient households maximize their utility subject to:

$$c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t + p_{l,t} l_t - b_t = \frac{w_{c,t} n_{c,t}}{X_{w,c,t}} + \frac{w_{h,t} n_{h,t}}{X_{w,h,t}}$$
$$+ \left( R_{c,t} z_{c,t} + \frac{1 - \delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + \left( R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t-1} + p_{b,t} b_{t-1} - \frac{R_{l,t-1} b_{t-1}}{\pi_t}$$
$$+ (p_{l,t} + R_{l,t}) l_{t-1} + q_t (1 - \delta_h) h_{t-1} + Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1}. \hspace{1cm} (9)$$

Patient agents choose consumption $c_t$, capital in the consumption sector $k_{c,t}$, capital $k_{h,t}$ and intermediate inputs $k_{b,t}$ (priced at $p_{b,t}$) in the housing sector, housing $h_t$ (priced at $q_t$), land $l_t$ (priced at $p_{l,t}$), hours $n_{c,t}$ and $n_{h,t}$, capital utilization rates $z_{c,t}$ and $z_{h,t}$, and borrowing $b_t$ (loans if $b_t$ is negative) to maximize utility subject to (9). The term $A_{k,t}$ captures investment-specific technology shocks, thus representing the marginal cost (in terms of consumption) of producing capital used in the non-housing sector. Loans are set in nominal terms and yield a riskless nominal return of $R_l$. Real wages are denoted by $w_{c,t}$ and $w_{h,t}$, real rental rates by $R_{c,t}$ and $R_{h,t}$, depreciation rates by $\delta_{kc}$ and $\delta_{kh}$. The terms $X_{w,c,t}$ and $X_{w,h,t}$ denote the markup (due to monopolistic competition in the labor market) between the wage paid by the wholesale firm and the wage paid to the households,
which accrues to the labor unions (we discuss below the details of nominal rigidities in the labor market). Finally, \( \pi_t = P_t/P_{t-1} \) is the money inflation rate in the consumption sector, \( Div_t \) are lump-sum profits from final good firms and from labor unions, \( \phi_t \) denotes convex adjustment costs for capital, \( z \) is the capital utilization rate that transforms physical capital \( k \) into effective capital \( zk \) and \( a (\cdot) \) is the convex cost of setting the capital utilization rate to \( z \).

Impatient households do not accumulate capital and do not own finished good firms or land (their dividends come only from labor unions). In addition, their maximum borrowing \( b'_t \) is given by the expected present value of their home times the loan-to-value (LTV) ratio \( m_t \):

\[
\begin{align*}
&c'_t + q_th'_t - b'_t = w'_{c,t}n'_{c,t}/X'_{w,c,t} + w'_{h,t}n'_{h,t}/X'_{w,h,t} + q_t (1 - \delta_h) h'_{t-1} - R_{t-1}b'_{t-1}/\pi_t + Div'_t; \\
&b'_t \leq m_tE_t \left( \frac{q_{t+1}h'_{t+1}\pi_{t+1}}{R_t} \right). 
\end{align*}
\]

Departing slightly from Iacoviello and Neri (2010), we also allow for shocks to the LTV ratio governed by an auto-regressive process.

### 3.2 Firms

To allow for nominal price rigidities, the models differentiates between competitive flexible price/wholesale firms that produce wholesale consumption goods and housing using two distinct technologies, and a final good firm (described below) that operates in the consumption sector under monopolistic competition. Wholesale firms hire labor and capital services and purchase intermediate goods to produce wholesale goods \( Y_t \) and new houses \( IH_t \). They solve:

\[
\begin{align*}
&\max \frac{Y_t}{X_t} + q_tIH_t - \left( \sum_{i=c,h} w_{i,t}n_{i,t} + \sum_{i=c,h} w'_{i,t}n'_{i,t} + \sum_{i=c,h} R_{i,t}z_{i,t}k_{i,t-1} + R_{i,t}l_{i,t-1} + p_{b,t}k_{b,t} \right). 
\end{align*}
\]

Above, \( X_t \) is the markup of final goods over wholesale goods. The production technologies are:

\[
\begin{align*}
Y_t &= \left( A_{c,t} \left( n^\alpha_{c,t} n^{1-\alpha}_{c,t} \right) \right)^{1-\mu_c} (z_{c,t}k_{c,t-1})^{\mu_c}; \\
IH_t &= \left( A_{h,t} \left( n^\alpha_{h,t} n^{1-\alpha}_{h,t} \right) \right)^{1-\mu_h-\mu_l} (z_{h,t}k_{h,t-1})^{\mu_h} k_{b,t}^{\mu_l} k_{l,t-1}^{\mu_l}.
\end{align*}
\]

In (13), the non-housing sector produces output with labor and capital. In (14), new homes are produced with labor, capital, land and the intermediate input \( k_b \). The terms \( A_{c,t} \) and \( A_{h,t} \) measure productivity in the non-housing and housing sector, respectively.
3.3 Nominal Rigidities and Monetary Policy

There are Calvo-style price rigidities in the non-housing consumption sector and wage rigidities in both sectors. The resulting consumption-sector Phillips curve is:

\[
\ln \pi_t - \eta \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \eta \ln \pi_t) - \varepsilon \ln (X_t/X) + u_{p,t} \tag{15}
\]

where \( \varepsilon = \frac{(1-\theta_C)(1-\beta G_C \theta_C)}{\eta} \). Above, cost shocks \( u_{p,t} \) are allowed to affect inflation independently from changes in the markup. These shocks have zero mean and variance \( \sigma^2_p \).

Wage setting is modeled in an analogous way. Households supply homogeneous labor services to unions. The unions differentiate labor services as in Smets and Wouters (2007), set wages subject to a Calvo scheme and offer labor services to wholesale labor packers who reassemble these services into the homogeneous labor composites \( n_c, n_h, n'_c, n'_h \). Wholesale firms hire labor from these packers. Under Calvo pricing with partial indexation to past inflation, the pricing rules set by the union imply four wage Phillips curves that are isomorphic to the price Phillips curve.

Monetary policy follows an interest rate rule that responds to inflation and GDP growth:

\[
R_t = R_{t-1}^{rR} \pi_t^{(1-r_R)r_R} \left( \frac{GDP_t}{GDP_{t-1}} \right)^{(1-r_R)r_Y} \overline{r}^{1-r_R} \frac{u_{R,t}}{s_t}. \tag{16}
\]

GDP is the weighted average of output in the two sectors with nominal share weights fixed at their values in the non-stochastic steady state. The term \( \overline{r} \) is the steady-state real interest rate; \( u_{R,t} \) is a monetary shock with variance \( \sigma^2_R \); \( s_t \) is a stochastic process with high persistence capturing long-lasting deviations of inflation from its steady-state level.\(^6\)

3.4 Market Clearing Conditions and Equilibrium

The goods market produces consumption, business investment and intermediate inputs. The housing market produces new homes \( IH_t \). The equilibrium conditions are:

\[
C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{h,t} = Y_t - \phi_t; \tag{17}
\]

\[
H_t - (1 - \delta_h) H_{t-1} = IH_t, \tag{18}
\]

together with the loan market equilibrium condition. Above, \( C_t = c_t + c'_t \) is aggregate consumption, \( H_t = h_t + h'_t \) is the aggregate stock of housing, and \( IK_{c,t} = k_{c,t} - (1 - \delta_{kc}) k_{c,t-1} \) and \( IK_{h,t} = k_{h,t} - (1 - \delta_{kh}) k_{h,t-1} \) are the two components of business investment. Total land is fixed and normalized to one.

A competitive equilibrium consists of sequence of prices and allocations such that, taking prices as given, the allocations solve the optimization problems for households and firms, the markets

\(^6\)In the sensitivity analysis we later consider the case when the monetary authority is constrained by the zero lower bound on interest rates.
clear, and the monetary policy rule and borrowing constraints are satisfied at all times, given initial
conditions and sequences of shocks. In practice, computation of our equilibrium is made difficult
by the presence of two occasionally binding constraints, the collateral constraint on borrowing on
the one hand, and the nonnegativity constraint on the nominal interest rate on the other. We
circumvent this problem by using a piece-wise linear solution approach to find the equilibrium
allocations for a given sequence of unforeseen shocks.\(^7\) Appendix C provides details on the solution
method.

### 3.5 Calibration

Iacoviello and Neri estimate the model with full information Bayesian methods on U.S. data running
from 1965:Q1 to 2006:Q4 and including 10 observed series: real consumption, real residential
investment, real business investment, real house prices, nominal interest rates, inflation, hours and
wage inflation in the consumption sector, hours and wage inflation in the housing sector. We
set parameters based on the mean of the posterior distributions estimated by Iacoviello and Neri
(2010). For completeness, their estimates of the model behavioral parameters are reported again
in the left column of Table 1.\(^8\)

Some parameter choices are based on information complementary to the estimation sample. These parameters are: the discount factors $\beta, \beta'$, the weight on housing in the utility function $j$, the technology parameters $\mu_c, \mu_h, \mu_l, \mu_b, \delta_h, \delta_{kc}, \delta_{kh}$, the steady-state gross price and wage markups $X, X_{wc}, X_{wh}$, the loan-to-value (LTV) ratio $m$ and the persistence of the inflation objective shock $\rho_s$. Values for all the calibrated parameters are reported in the right column of Table 1.

We depart from the estimates in Iacoviello and Neri (2010) for the following parameters. We set
$m$, the steady-state value of the loan-to-value ratio, equal to 0.925, a parameter that more closely
aligns with data from the 1980s and onwards. The wage share of credit constrained households,
$\lambda$, is estimated by Iacoviello and Neri (2010) to be around 20 percent. We set $\lambda$ at 40 percent in
the non-stochastic steady state. When the model is solved with first-order perturbation methods,
$\lambda$ remains constant. With the solution method advocated in this paper, shocks that increase the
value of the housing collateral can make the borrowing constraint slack. Hence, $\lambda$ is time-varying
and it only provides an upper bound on the fraction of credit-constrained agents.

A key parameter for the asymmetries we highlight is the discount factor of the impatient agents
$\beta'$. Very low values of this parameter imply that impatient agents never escape the borrowing
constraint. Then, the model has no asymmetries, regardless of the size of the shocks. Conversely,
when $\beta'$ takes on higher values, closer to discount factor of patient agents, smaller increases in house

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\(^7\) This approach is common in the literature on the zero lower bound on nominal interest rates. For instance, see Eggertsson and Woodford (2003) and Bodenstein, Guerrieri, and Gust (2010).

\(^8\) Iacoviello and Neri (2010) provide an extensive discussion of both the estimation method and results, including the relative importance of different sources of fluctuations. Given our different focus on highlighting asymmetries implied by collateral constraints, we did not reproduce their estimation results concerning the parameters of the model governing the exogenous stochastic processes.
prices suffice to make the borrowing constraint slack (even though the constraint is expected to bind in the long run). We set $\beta'$ equal to 0.988, based on the moment matching exercise described below.

4 Results of the Full Model

First, we complete the calibration of the model through a model matching exercise. Second, we use a simple non-linear VAR to investigate the asymmetric relationship between house prices and consumption. The VAR implied by population moments from our model captures asymmetric responses of consumption to house price increases and declines. The VAR estimated on the observed data sample is consistent with its model counterpart.

4.1 A Moment Matching Exercise

We use the model to generate data conditional on two sources of stochastic variation: an AR(1) process that governs the loan-to-value ratio, $m_t$; and a shock to housing preferences $j_t$. We single these two shocks out because several studies have suggested that movements in housing demand and credit market shocks may play an important role in driving housing prices and aggregate consumption. Another advantage of these two shocks is that the housing demand shock primarily drives housing prices and, to the extent that there are strong collateral channels, affects consumption as well. The shock to the loan-to-value ratio affects consumption relatively more, since it influences the short-term resources that borrowers use to finance consumption. We choose the standard deviations of the two shocks and the discount factor of the impatient agents in order to optimize the model’s ability to account for the volatility of consumption and house prices and their correlation. Importantly, we do not impose any requirements on the model’s ability to fit higher moments in the data, such as asymmetries in the responses to shocks. The metric used in our optimization procedure is $\mathcal{L}(ss)$, where $ss$ is the vector including estimates of $\sigma_j, \sigma_m, \beta'$ and $\mathcal{L}(ss)$ is given by

$$\mathcal{L}(ss) = (\hat{mm} - f(ss)) \tilde{V}^{-1} (\hat{mm} - f(ss))'$$. \hspace{1cm} (19)

Here, $\hat{mm}$ is a $3 \times 1$ vector that includes the sample standard deviation of quarterly consumption growth and quarterly real house price growth, as well as their correlation. The $3 \times 3$ matrix $\tilde{V}$ is the identity matrix. Finally, $f(ss)$ is a $3 \times 1$ vector with moments analogous to the ones in $\hat{mm}$ but implied by the model in population (with all other parameters set as described in the calibration section above). The parameter values that minimize $\mathcal{L}(ss)$ are $\sigma_j = 0.0825$, $\sigma_m = 0.0205$, and $\beta' = 0.988$.

As a cross check, the standard deviations of quarterly consumption growth and house price growth implied by the model in population are 0.66 and 1.71 percent, very close to their observed sample counterparts of 0.63 and 1.77 percent. The correlation of consumption growth and house
price growth implied by the model in population is 0.42, also close to its observed sample counterpart of 0.39.

4.2 A Nonlinear VAR

With the estimates above, we use model-generated data on consumption and housing prices to fit a two-variable nonlinear VAR. Each equation in the VAR regresses linearly detrended consumption and house prices on: a constant, the linearly detrended consumption, and distinct terms for positive and negative lagged deviations of housing prices from a linear trend.\textsuperscript{9} Innovation to each equation are orthogonalized using a Cholesky scheme: we treat model and data symmetrically, by imposing an ordering scheme such that a “house price shock” affects contemporaneously both house prices and consumption.

Figure 3 shows population estimates from the model (the thin lines) against estimates for U.S. data running from 1975 to 2011 (the thick lines) and 95% bootstrap confidence bands. The top panels focus on innovations to house prices that yield about a 2 standard deviation increase in house prices. The bottom panels show responses to an innovation that brings about a 2 standard deviation decline in house prices. Strikingly, model and data appear in substantial agreement: the response of consumption to a large house price decline is nearly twice as large as that to a large house price increase of equal magnitude, in the model as in the data. Furthermore, for the estimates based on observed data, we compute confidence intervals for the difference between the peak response of the absolute value of consumption to the positive and negative innovations. We confirm that this difference is statistically different from zero at standard significance levels. Accordingly, we fail to reject the null hypothesis of asymmetric responses.

4.3 Responses to Positive and Negative Shocks

To illustrate the fundamental source of the asymmetry in the model, Figure 4 considers the effects of a shock to housing preferences, the process $j_t$ in Equation (8), which we interpret as a shock to housing demand. Between periods 1 and 10, a series of innovations to $j_t$ are set to bring about a decline in house prices of 10 percent.\textsuperscript{10} Thereafter, the shock follows its autoregressive process. In this case, the decrease in house prices reduces the collateral capacity of constrained households. Accordingly those households can borrow less and are forced to curtail their non-housing consumption even further in order to comply with the borrowing constraint. On balance, the decline in aggregate consumption is close to 2.5 percent. The new-Keynesian channels in the

\textsuperscript{9} In other words, the right-hand side variables in the VAR are (aside from the constant term) the lag of $c_t$, max($q_t, 0$), and min($q_t, 0$), where $c_t$ and $q_t$ denote the log deviations of consumption and house prices from their respective linear trends.

\textsuperscript{10} Iacoviello and Neri (2010) find that house preference shocks are one of the key determinants of house price movements at business cycle frequencies. Similarly, Liu, Wang, and Zha (2011) highlight that a shift in housing demand in a credit-constrained economy can lead to large fluctuations in land prices, an produce a broader impact on hours worked and output.
model imply that the large decline in aggregate consumption translates into a large decline in the firms’ demand for labor. In equilibrium, the drop in hours worked comes close to reaching 2.5 percent below the balanced growth path.

Unforseen to the agents in the model, in period 51 a series of innovations for the shock to housing preferences brings about a 10 percent increase in house prices over the next 10 quarters. Recalling the partial equilibrium model described in Section 2, an increase in house prices can relax borrowing constraints. After only two quarters, the borrowing constraint for the representative impatient household becomes slack. The Lagrange multiplier in the households’ utility maximization problem bottoms out at zero. In period 61, the shock to housing preferences starts following its autoregressive process and house prices begin to decline. The borrowing constraint remains slack for another couple of quarters, but even as house prices are well above their balanced growth path, the borrowing constraint starts binding again (and its Lagrange multiplier takes on positive values).

When the constraint becomes slack, the borrowing constraint channel remains operative only in expectation. Thus, impatient households discount that channel more heavily the longer the constraint is expected to remain slack. As a consequence, the response of consumption to the large house price increases considered in the figure is not as dramatic as the reaction to house price declines of an equal magnitude. At peak, the increase both in consumption and hours worked is about 0.5 percent, much smaller than the response to the house price decline.

Figure 5 plots the peak response of consumption to a house demand shock as a function of the change in house prices induced by the same shock. The figure also shows the relationship between the peak elasticity of consumption to housing wealth as a function of the peak impact to housing wealth. Prosaically, the former is defined as the ratio of the peak response of aggregate consumption to the peak response of house wealth, the latter as the peak response of the value of the housing stock. In our model, if borrowing constraints were always binding, this elasticity would be constant, regardless of the change in house prices. However, because large increases in house prices can make the borrowing constraint slack, they affect consumption less and less in proportion. Mechanically, the peak impact on consumption elasticity of a housing demand shock continues to decline because our solution algorithm attributes a longer duration to the regime with slack borrowing constraints when the house price increases become larger.

4.4 Sensitivity Analysis

Figure 6 considers again the peak impact of consumption relative to the peak impact on house prices of a housing demand shock. For ease of comparison, the blue solid line reproduces the benchmark results shown in Figure 5. In addition, Figure 6 considers two alternative calibrations. The dashed black line, labeled “High Impatience” focuses on a lower discount factor for impatient agents, setting $\beta'$ equal to 0.98. Focusing on the bottom panel of the figure, with greater impatience, larger increases in house prices are required to relax the borrowing constraint. Accordingly, the peak elasticity of consumption to housing wealth remains constant for larger increases in housing

13
wealth than under the benchmark calibration. Moreover, even when the borrowing constraint is
eventually relaxed by larger underlying housing demand shocks, the constraint is expected to stay
slack for a shorter period than under the benchmark. These differences are also reflected in the top
panel. The flattening out of the response of consumption to increases in housing wealth becomes
less pronounced.

The dot-dashed, red lines in Figure 6 show results for a lower value of the LTV ratio, with
$m$ equal to 0.75. When increases in housing wealth make the borrowing constraint slack, there
are little differences between the benchmark and the results under this alternative calibration. If
anything, for large increases in house prices, the response of consumption is stronger, since the
borrowing constraint is likely to be less slack, and the collateral effect stronger, for low values of
the LTV ratio. However, when housing wealth declines, the collateral effect is smaller, and the
decrease in borrowing is less pronounced. Accordingly, lower values for $m$ also imply a flattening
of the response of consumption to increases in housing wealth and a compression of the asymmetry
that we have highlighted so far.

Moving in the opposite direction, Figure 7 considers a mechanism that can enhance the asym-
metric response of consumption to housing demand shocks. In addition to the baseline model
already considered in Figure 5, it considers a variant of the model, labeled “ZLB”, that allows for
another occasionally binding constraint, namely the zero lower bound on the policy interest rate.
In that case, the Monetary policy rule becomes:

$$R_t = \max \left[ 1, R_{t-1}^{r_R} \left( \frac{GDP_t}{GDP_{t-1}} \right) \left( 1 - r_R \right)^R \pi_t \left( 1 - r_R \right)^R \frac{u_{R,t}}{s_t} \right].$$

In the ZLB case, sufficiently large price declines can bring the gross policy rate $R_t$ to 1 (equivalently,
the net policy rate hits zero). With mechanisms familiar from the literature on the effects of
aggregate demand shocks in a liquidity trap, the spillover effects of contractionary housing demand
shocks onto aggregate consumption become amplified. At the zero lower bound with constant
nominal rates, declines in inflation can bring up real interest rates and deepen the contractionary
effects of the shock. We pick up this theme again below when discussing our estimates from panel
regressions on regional data.

5 Regional Evidence on Asymmetries

The results of our theoretical model and the evidence from the vector autoregressions at the national
level motivate additional empirical analysis that we conduct using a panel of data from U.S. states
and Metropolitan Statistical Areas (MSA). The obvious advantage of these data is that variation
in housing prices and economic activity is greater at the regional than at the aggregate level, as
documented for instance by Del Negro and Otrok (2007), who find a large degree of heterogeneity

\footnote{For instance, see Christiano, Eichenbaum, and Rebelo (2011).}
across states in regard to relative importance of the national factors. The use of regional data also allays the concern that little can be learned using national data, given the rarity of declines in house prices at the national level.

In order to set the stage, Figure 8 shows changes in house prices and changes in employment in the service sector, auto sales, electricity consumption, and mortgage originations in 2005 and 2008 for all the 50 U.S. states and the District of Columbia. For each state there are two dots in each panel: the green dot (concentrated in the north–east region of the graph) shows the lagged percent change in house prices and the percent change in the indicator of economic activity in 2005, at the height of the housing boom. The red dot represents analogous observations for the 2008 period, in the midst of the housing crash. Fitting a piecewise linear regression to these data yields a correlation between house prices and activity that is smaller when house prices are high. This evidence on asymmetry is bolstered by the large cross-sectional variation in house prices across states over the period in question.

5.1 State-Level Evidence

We use annual data from 1990 to 2011 from the 50 U.S. states and the District of Columbia on house prices and measures of economic activity. We choose measures of economic activity to match our model counterparts for consumption, employment and credit.

Our main specification takes the following form:

\[
\Delta \log y_{i,t} = \alpha_i + \gamma_t + \beta_{POS} \sum_{i} \Delta \log h_{t-1} + \beta_{NEG} (1 - \mathbb{I}_{i,t}) \Delta \log h_{t-1} + \delta X_{i,t-1} + \varepsilon_{i,t} \tag{21}
\]

where \( y_{i,t} \) is an index of economic activity and \( h_{t-1} \) is the inflation-adjusted house price index in state \( i \) in period \( t \); \( \alpha_i \) and \( \gamma_t \) represent state and year fixed effects; and \( X_{i,t} \) is a vector of additional controls. We interact changes in house prices with a state-specific indicator variable \( \mathbb{I}_{i,t} \) that takes value 1 when house prices are high, and value 0 when house prices are low. We classify house prices as high in a particular state when house prices are above a state-specific linear trend estimated for the 1975-2010 period. Using this approach, the fraction of states with high house prices is about 20 percent in the 1990s, rising gradually to peak at 100 percent in 2005 and 2006, and dropping to 27 percent in 2010. Our results were similar using a different definition of \( \mathbb{I}_{i,t} \) that takes value 1 when real house price inflation is positive. In our baseline specification, we use one-year lags of house prices and other controls to control for obvious endogeneity concerns. Our results were also little changed when instrumenting current or lagged house prices with one or more lags.

\[12\] In the sample period we analyze, the first principal component for annual house price growth accounts for 64 percent of the variance of house prices across the 50 U.S. states and the District of Columbia. The corresponding numbers for employment in the service sector, auto sales, electricity consumption, and mortgage originations are respectively 73, 90, 44, and 89 percent.

\[13\] An analogous relationship is more tenuous for house prices and employment in the manufacturing goods sector. Most goods are traded and are less sensitive to local house prices than services.
Tables 2 to 5 present our estimates when the indicators of economic activity $y_{i,t}$ are employment in the service sector, automobile sales, electricity usage and mortgage originations respectively.

Table 2 presents the results for our preferred measure of regional economic activity, namely employment in the non-tradeable service sector. We choose this measure (rather than, say, total employment) since U.S. states (and MSAs) heavily trade with each other, so that employment in sectors that mainly produce for the local economy better isolates the local effects of movements in local house prices. The first two columns do not control for time effects. They show that the asymmetry is strong and economically important, and that house prices matter, at statistically conventional levels, both when high and when low. After controlling for time effects in the third column, the coefficient on high house prices is little changed, but the coefficient on low house prices is lower. A large fraction of the decline in house prices in our sample took place against the background of the zero lower bound on policy interest rates. As discussed in the model results, the zero lower bound is a distinct source of asymmetry for the effect of change in house prices. Time fixed effects allow us to parse out the effects of the national monetary policy reaching the zero lower bound and, in line with our theory, they compress the elasticity of employment to low house prices. In the last two columns, after adding additional variables, the only significant coefficient is the one on low house prices. In column five, the coefficient on “high house prices” is positive, although is low and not significantly different from zero. The coefficient on “low house prices,” instead, is positive and significantly different from zero. Taken at face value, these results imply that house prices only matter for economic activity when they are low. The difference in the coefficient on low and high house prices is significantly different from zero, with a p-value of 0.014.

Table 3 reports our results when our measure of activity is retail automobile sales. Auto sales are an excellent indicator of local demand, since autos are almost always sold to state residents, and since durable goods are notoriously very sensitive to changes in economic conditions. After adding lagged car sales and personal income as controls, the coefficients on low and high house prices are both positive; the coefficient on low house prices is nearly four times as large, and the p-value of the difference between low and high house prices is 0.11.

Table 4 reports our results using residential electricity usage as a proxy for consumption. Even though electricity usage only accounts for 3 percent of total consumption, we take electricity usage to be a useful proxy for nondurable consumption. Most economic activities involve the use of electricity which cannot be easily stored: moreover, the flow usage of electricity may even provide a

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14 The BLS collects state-level employment data by sectors broken down according to NAICS (National Industry Classification System) starting from 1990. According to this classification (available at http://www.bls.gov/ces/cessuper.htm), the goods-producing sector includes Natural Resources and mining, construction and manufacturing. The service-producing sector includes wholesale trade, retail trade, transportation, information, finance and insurance, professional and business services, education and health services, leisure and hospitality and other services. A residual category includes unclassified sectors and public administration. We exclude from the service sector wholesale trade (which on average accounts for about 6 percent of total service sector employment) since wholesale trade does not necessarily cater to the local economy.

15 Da and Yun (2010) show that using electricity to proxy for consumption produces asset pricing implications that are consistent with consumption-based capital asset pricing models.
better measures of the utility flow derived from a good than the actual purchase of the good. Even in cases when annual changes in weather conditions may affect year-on-year consumption growth, their effect can be easily filtered out using state-level observations on heating and cooling degree days, which are conventional measures of weather-driven electricity demand. We use these weather measures as controls in all specifications reported. As the table shows, in all regressions low house prices affect consumption growth more than high house prices. After time effects, lagged income growth and lagged consumption growth are controlled for (last column), the coefficient on high house prices is 0.11, the coefficient on low house prices is nearly twice as large at 0.18, and their difference is statistically larger than 0 at the 10 percent significance level.

Because the effects of low and high house prices on consumption work in our model through tightening or relaxing borrowing constraints, it is important to check whether measures of leverage also depend asymmetrically on house prices. Table 5 shows how mortgage originations at the state level respond to changes in house prices. We choose mortgage originations because they are available for a long time period, and because they better measure the flow of new credit to households than the stock of existing debt. As the table shows, mortgage originations depend asymmetrically on house prices too, as in our model where the effect of house prices on consumption and employment works through the asymmetric effect on borrowing that changes in house price produce.

We note here that the aggregated state-level series that we use as proxy for consumption tracks consumption from the National Income and Product Accounts rather well. Over the sample period, the correlation between NIPA motor vehicle consumption growth (about 1/3 of total durable expenditure) and retail auto sales growth is 0.89; and the correlation between services consumption growth and electricity usage growth is 0.54.

5.2 MSA-Level Evidence

Tables 6 and 7 present the results of evidence across MSAs. MSAs account for about 80 percent of the population and of employment in the entire United States. In Table 7, the results from the MSA-level regressions are similar qualitatively and quantitatively to those at the state level.

A legitimate concern with the panel and time-series regressions discussed so far is that the correlation between house prices and economic activity could be due to some omitted factor that simultaneously drives both house prices and economic activity. Even if this were the case, our regressions would still be of independent interest, since they would support the idea – even in absence of a causal relationship – that the comovement between housing prices and economic activity is larger when house prices are low, as predicted by the model.

To support claims of causality, one needs to isolate exogenous from endogenous movements in house prices. In Table 7, we follow the methodology and insight of Mian and Sufi (2011) and the data from Saiz (2010) in an attempt to distinguish an independent driver of housing demand. The insight is to use the differential elasticity of housing supply at the MSA level as an instrument for housing prices, so as to disentangle movements in housing prices due to general changes in
economic conditions from movements in the housing market that are directly driven by shifts in housing demand in a particular area. Because such elasticity is constant over time, we cannot exploit the panel dimension of our dataset, and instead use the elasticity in two separate periods by running two distinct regressions of car sales on house prices. The first regression is for the 2003-2007 housing boom period, the second for the 2007-2011 housing bust period. In practice, we rely on the following differenced instrumental variable specifications

\[
\log h_{pt} - \log h_{ps} = \theta + \delta \text{ Elasticity} + \varepsilon \\
\log car_t - \log car_s = \delta + \beta (\log h_{pt} - \log h_{ps}) + u
\]

(22) (23)

where \( s = 2003 \) and \( t = 2007 \) in the first set of regressions, and \( s = 2007 \) and \( t = 2011 \) in the second set.

The first stage regressions show that elasticity is a powerful instrument in driving house prices, with an \( R^2 \) from the first stage regression around 0.15 in the first period, and to 0.2 in the second.\(^{16}\) The second stage regression, when run across the two separate sub-periods, shows how car sales respond to house prices only in the second period. In the 2003 – 2007 period, the elasticity of car sales to house prices is close to, and not statistically different from zero. In the 2007 – 2011 period, in contrast, this elasticity rises to 0.53, and is significantly different from zero.\(^{17}\)

6 A Policy Experiment: Debt Relief and Borrowing Constraints

So far, our theoretical and empirical results show that movements in house prices can produce asymmetries that are economically and statistically important. Next, we consider whether these asymmetries are also important for gauging the effects of policies aimed at the housing market in the context of a deep recession. To illustrate our ideas, we choose a simple example of one such policy, a lump-sum transfer from patient (saver) households to impatient (borrower) households. For instance, this policy could mimic voluntary debt relief from the creditors, or a scheme where interest income is taxed and interest payments are subsidized in lump-sum fashion, so that the net effect is a transfer of resources from the savers to the borrowers.

We consider this experiment against two different baselines. In one case, housing prices are assumed to be declining, in the other case, housing prices are assumed to be increasing. The baseline housing price changes are brought about by the same preference shocks considered in

\(^{16}\) The F statistics on the first stage regressions are 40.4 and 71.6 for the first and the second period respectively, well above the conventional threshold of 10 for evaluating weak instruments.

\(^{17}\) Using ZIP-code level data from 2007 to 2009 and a similar methodology, Mian, Rao, and Sufi (2012) find a large elasticity (0.74) of auto sales to housing wealth during the housing bust, in line with our findings. Importantly, they also find that this elasticity is smaller in zip codes with a high fraction of non-housing wealth to total wealth. One interpretation of their result – in line with our model – is that households in zip codes with high non-housing wealth might be, all else equal, less likely to face binding borrowing constraints during periods of housing price declines because they can use other forms of wealth to support their consumption plans. Using PSID data, Dynan (2012) also finds that high leverage appears to be associated with weak consumption growth.
Figure 4 and discussed at length above. Accordingly, we do not need to repeat a description of the baseline at this point.

Figure 9 shows the cumulative response of housing prices to the baseline housing preference shocks and to two transfer shocks from saver households to borrower households. Both transfer shocks are unforeseen. They are sized at the same 1 percent of steady-state total consumption in both cases. Each transfer is governed by an auto-regressive process of order 1, with coefficient equal to 0.5. The first transfer starts in period 10. A series of unforeseen innovations to the shock process phases in the transfer, until it reaches a peak of 1 percent of steady-state consumption. Then, the auto-regressive component of the shock reduces the level of the transfer back to 0. The first transfer happens against a background of housing price declines and tight borrowing constraints. The second transfer, starting in period 50, mimics the first but happens against a baseline with housing price increases and slack borrowing constraints.

The top left panel of Figure 9 shows housing prices in deviation from their steady-state level. The path shown is almost identical to the one in Figure 4 because the transfer shocks only have a negligible effect on housing prices. The transfer payments are timed to coincide with the series of housing preference shocks that reduce housing prices.

The remaining panels in Figure 9 show responses of key variables to the transfer shock in deviation from the baseline path that obtains with the housing preference shock only. Thus, those panels isolate the partial effects of the transfer shocks. The consumption response of borrower households is dramatically different depending on the baseline variation in housing prices. When housing prices decline, the borrowing constraint is tight and the marginal propensity to consume of borrower households is elevated. When housing prices increase, the borrowing constraint becomes slack and the marginal propensity to consume of borrower households drops down closer to that for saver households. In reaction to the lump-sum transfer, consumption of the savers declines less, and less persistently, against a baseline of housing price declines. In that case, there are expansionary spillover effects from the increased consumption of borrowers to aggregate hours worked and output. Taking together the responses of savers and borrowers, the partial effects of the transfer on aggregate consumption are sizable when housing prices are low, and negligible when housing prices are elevated. As a consequence, actions such as mortgage relief can almost pay for themselves through their expansionary effects on aggregate economic activity in a scenario of severely binding borrowing constraints.

7 Conclusions

Our empirical and theoretical results suggest that policy measures aimed at the housing market have the potential of producing outsize spillovers to aggregate consumption in periods when collateral constraints are tight, either because of large declines in house prices or because credit supply standards have been made more stringent. These spillovers are likely to be larger than those that
one can estimate in samples dominated by house price increases, because these periods can severely
underpredict the sensitivity of consumption to movements in housing wealth.

Numerous recent papers with an empirical focus have emphasized the importance of household
debt and the housing market in understanding the 2007-2009 recession. Our model provides a
framework to analyze these results; to make sense of why household debt seems to matter more
during severe recessions; and to better assess the costs and benefits of alternative policies aimed at
restoring the efficient functioning of the housing market.

Throughout the paper, our story has mainly focused on the role of housing as collateral for
households, and on the effects of changes in housing wealth on consumption. However, to the
extent that fixed assets are used for collateral by entrepreneurs or firms, the asymmetries that we
emphasize here should be also relevant for the behavior of fixed investment.\textsuperscript{18}

\textsuperscript{18} See for instance Chaney, Sraer, and Thesmar (2012) and Adelino, Schoar, and Severino (2013).
References


Da, Z. and H. Yun (2010). Electricity consumption and asset prices. *Available at SSRN 1608382*. [16]


Figure 1: House Prices and Consumption in U.S. National Data

House Prices: Loan Performance National House Price Index (SA), Haver Analytics, USLPH-PIS@USECON, divided by the GDP deflator (DGDP@USECON). Consumption: Real Personal Consumption Expenditures (CH@USECON). In the bottom panel, consumption growth and house price growth are expressed in deviation from their sample mean.
Figure 2: House Prices and Consumption in a Partial Equilibrium Model

Note: Optimal leverage and consumption as a function of the housing price for different levels of debt, low, average and high, when housing is at its nonstochastic steady-state value. In the top panel, low levels of house prices move the household closer to the maximum borrowing limit given by $m = 0.925$. This is more likely to happen at high levels of debt (thick line). In the bottom panel, the higher house prices are, the more likely is the household not to be credit constrained, and the consumption function becomes flatter. At high levels of debt, the household is constrained for a larger range of realizations of housing prices, and the consumption function is steeper when house prices are low.
Figure 3: Estimates from Asymmetric VAR vs Model

Note: Top row: Impulse Responses to a 2 standard error increase in house prices. Bottom row: Impulse Response to a 2 standard error decrease in house prices. Horizontal axis: quarters from the shock; vertical axis: percentage deviation from the unshocked path. Data VAR run using quarterly data for inflation-adjusted house prices and consumption (linearly detrended) from 1975 to 2011. Model VAR run using observations generated from a model simulation of 500 periods using parameters of Table 1.
Figure 4: Impulse Responses to Positive and Negative House Price Shocks in model with occasionally binding borrowing constraints

Note: The simulation shows the dynamic response of macroeconomic variables to two housing preference shocks. In period 1, a decline in housing demand causes house prices to drop by around 10 percent after 8 quarters. In period 50, an increase in housing demand causes house price to rise by around 10 percent. The variables are plotted in red when the collateral constraint is slack.
Figure 5: Response of Consumption to Positive and Negative Changes in Housing Prices in the DSGE model

Note: The top panel plots the maximum response of consumption relative to the zero baseline following a housing price shock of size given by the x-axis. The bottom panel plots the maximum elasticity of consumption to housing wealth given a housing wealth shock of size given by the x-axis. The housing price and wealth shocks are caused by a housing preference shock.
Figure 6: Response of Consumption to Positive and Negative Changes in Housing Prices in the DSGE model. Sensitivity Analysis.

Note: The top panel plots the maximum response of consumption relative to the zero baseline following a housing price shock of size given by the x-axis, in the benchmark model (Benchmark), in a model with higher borrowers impatience, and in a model with a lower Loan-to-Value Ratio. The bottom panel plots the maximum elasticity of consumption to housing wealth given a housing wealth shock of size given by the x-axis. The housing price and wealth shocks are caused by a housing preference shock.
Figure 7: Response of Consumption to Positive and Negative Changes in Housing Prices in the DSGE model. Allowing for Zero Lower Bound

Note: The top panel plots the maximum response of consumption relative to the zero baseline following a housing price shock of size given by the x-axis, in the baseline model without zero lower bound on nominal interest rates (NO ZLB) and in a model with the zero lower bound constraint (ZLB). The bottom panel plots the maximum elasticity of consumption to housing wealth given a housing wealth shock of size given by the x-axis. The housing price and wealth shocks are caused by a housing preference shock.
Figure 8: House prices and Economic Activity by State

Note: Each panel shows house price growth and activity growth across US states in 2005 and 2008. The “fitted” line shows the fitted values of a regression of activity growth on house prices growth broken down into positive and negative changes.
Figure 9: A Transfer from Lenders to Borrowers Against a Background of Low and High Housing Prices

Note: The figure shows the effects of two unexpected lump-sum transfers from savers to borrowers each sized at 1 percent of steady-state total consumption. The first transfer (periods 10 to 19) happens against a baseline of low house prices and tight collateral constraints. The second transfer (periods 50 to 59) happens against a baseline of high house prices and slack collateral constraints. Both housing price changes in the baseline stem from a housing preference shock. The responses of consumption, hours, consumption of savers, and consumption of borrowers are shown in deviation from the baseline to isolate the partial effect of the transfer shocks. The variables are plotted in red when the collateral constraint is slack.
Table 1: Parameter Values

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</tr>
<tr>
<td>φ_{k,h}adj.cost, capital for houses</td>
<td>δ_{kh}capital depreciation, goods</td>
<td>10.90</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>ρ_{j}AR(1) housing demand shock</td>
<td>δ_{kh}capital depreciation, houses</td>
<td>0.96</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>r_{π}inflation response Taylor rule</td>
<td>Xprice markup</td>
<td>1.44</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>r_{Y}output response Taylor rule</td>
<td>X_{wc}wage markup, goods</td>
<td>0.52</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>θ_{p}Calvo price stickiness</td>
<td>X_{wh}wage markup, houses</td>
<td>0.83</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>t_{π}Calvo price indexation</td>
<td></td>
<td>0.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ_{w,c}Calvo wage stickiness goods</td>
<td>αsavers wage share</td>
<td>0.79</td>
<td>0.60***</td>
<td></td>
</tr>
<tr>
<td>t_{w,c}Calvo wage index. goods</td>
<td>mloan-to-value ratio</td>
<td>0.08</td>
<td>0.925**</td>
<td></td>
</tr>
<tr>
<td>θ_{w,h}Calvo wage stickiness houses</td>
<td>r_{R}inertia, Taylor rule</td>
<td>0.91</td>
<td>0.70**</td>
<td></td>
</tr>
<tr>
<td>t_{w,h}Calvo wage index. houses</td>
<td>ρ_{m}AR(1), LTV shock</td>
<td>0.40</td>
<td>0.95*</td>
<td></td>
</tr>
<tr>
<td>ζCapital Utilization convexity</td>
<td></td>
<td>0.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ_{AC}goods technology trend</td>
<td>β'discount borrower</td>
<td>0.0032</td>
<td>0.988***</td>
<td></td>
</tr>
<tr>
<td>γ_{AH}housing technology trend</td>
<td>σ_{j}st.dev housing pref. shock</td>
<td>0.0008</td>
<td>0.0825***</td>
<td></td>
</tr>
<tr>
<td>γ_{AH}investment technology trend</td>
<td>σ_{m}st.dev LTV shock</td>
<td>0.0027</td>
<td>0.0205***</td>
<td></td>
</tr>
</tbody>
</table>

Note: Parameters denoted with a * were not present in the original model of Iacoviello and Neri (2010). Parameters denoted with ** are calibrated differently. Parameters denoted with *** are estimated in Section 4.
<table>
<thead>
<tr>
<th>% Change in Employment ($\Delta emp_t$)</th>
<th>$\Delta hp_{t-1}$</th>
<th>$\Delta hp_{high\ t-1}$</th>
<th>$\Delta hp_{low\ t-1}$</th>
<th>$\Delta emp_{t-1}$</th>
<th>$\Delta income_{t-1}$</th>
<th>pval difference</th>
<th>Time effects</th>
<th>Observations</th>
<th>States</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.14***</td>
<td>0.07*** 0.08*** 0.03* 0.02</td>
<td>0.24*** 0.12*** 0.08*** 0.07***</td>
<td>0.26*** 0.23***</td>
<td>0.07**</td>
<td>0.000 0.100 0.013 0.017</td>
<td>no no yes yes yes</td>
<td>1071 1071 1071 1020 1020</td>
<td>51 51 51 51 51</td>
<td>0.12 0.16 0.66 0.72 0.73</td>
</tr>
</tbody>
</table>

Note: Regressions using annual observations from 1991 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***, **, *: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: $\Delta hp$ is the inflation–adjusted (using the GDP deflator) percent change in the FHFA House Price Index. $\Delta emp$ is the percent change in employment in the Non-Tradeable Service Sector which includes: Retail Trade, Transportation and Utilities, Information, Financial Activities, Professional and Business Services, Education and Health Services, Leisure and Hospitality, and Other Services (source: BLS Current Employment Statistics: Employment, Hours, and Earnings - State and Metro Area). $\Delta income$ is the percent change in the inflation–adjusted state-level disposable personal income (source: Bureau of Economic Analysis).
Table 3: State Level: Auto Sales and House Prices

<table>
<thead>
<tr>
<th>% Change in Auto Sales ($\Delta auto_t$)</th>
<th>$\Delta hp_{t-1}$</th>
<th>$\Delta hp_{high_{t-1}}$</th>
<th>$\Delta hp_{low_{t-1}}$</th>
<th>$\Delta income_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.24***</td>
<td>-0.05 0.16*** 0.11*** 0.07**</td>
<td>0.62*** 0.33*** 0.27** 0.20**</td>
<td>0.34***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.04) 0.04 0.03 0.03</td>
<td>(0.05) 0.06 0.11 0.09</td>
<td>(0.11)</td>
</tr>
<tr>
<td>pval difference</td>
<td>0.000 0.040 0.137 0.155</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time effects</td>
<td>no no yes yes yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>969 969 969 918 918</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>States</td>
<td>51 51 51 51 51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.02 0.06 0.86 0.87 0.88</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: State-level Regressions using annual observations from 1992 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***, **, *: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: $\Delta auto$ is the percent change in inflation-adjusted auto sales, "Retail Sales: Motor vehicle and parts dealers" from Moody’s Analytics Database. Auto sales data are constructed with underlying data from the US Census Bureau and employment statistics from the BLS. The two Census Bureau surveys are the quinquennial Census of Retail Trade, a subset of the Economic Census, and the monthly Advance Retail Trade and Food Services Survey. See Table 2 for other variable definitions.
Table 4: State Level: Electricity Consumption and House Prices

<table>
<thead>
<tr>
<th>Δhp_{t-1}</th>
<th>0.11*** (0.02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δhp_{high,t-1}</td>
<td>0.03  0.09***  0.14***  0.12*** (0.02) (0.02) (0.03) (0.03)</td>
</tr>
<tr>
<td>Δhp_{low,t-1}</td>
<td>0.24***  0.16***  0.22***  0.19*** (0.03) (0.03) (0.04) (0.04)</td>
</tr>
<tr>
<td>Δelec_{t-1}</td>
<td>-0.41*** -0.41*** (0.02) (0.02)</td>
</tr>
<tr>
<td>Δincome_{t-1}</td>
<td>0.15*** (0.05)</td>
</tr>
<tr>
<td>pval difference</td>
<td>0.000  0.105  0.058  0.090</td>
</tr>
<tr>
<td>Time effects</td>
<td>no  no  yes  yes  yes</td>
</tr>
<tr>
<td>Weather Controls*</td>
<td>yes  yes  yes  yes  yes</td>
</tr>
<tr>
<td>Observations</td>
<td>1071  1071  1071  1020  1020</td>
</tr>
<tr>
<td>States</td>
<td>51  51  51  51  51</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.04  0.04  0.08  0.12  0.12</td>
</tr>
</tbody>
</table>

Note: State-level Regressions using annual observations from 1990 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***, **, *: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: Δelec is the percent change in Residential Electricity Consumption (source: the U.S. Energy Information Administration’s Electric Power Monthly publication. Electricity Power Annual: Retail Sales - Total Electric Industry - Residential Sales, NSA, Megawatt-hours). See Table 2 for other variable definitions. All regressions in the Table control separately for number of heating and cooling degree days in each state (source: U.S. National Oceanic and Atmospheric Administration’s National Climatic Data Center).
Table 5: State Level: Mortgage Originations and House Prices

<table>
<thead>
<tr>
<th></th>
<th>% Change in Mortgage Originations ($\Delta mori_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta hp_{t-1}$</td>
<td>$1.10^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
</tr>
<tr>
<td>$\Delta hp_{high_{t-1}}$</td>
<td>$-0.41^{<em>}$  $1.08^{</em><strong>}$  $1.46^{</strong><em>}$  $1.54^{</em>**}$</td>
</tr>
<tr>
<td></td>
<td>(0.24)  (0.16)  (0.21)  (0.33)</td>
</tr>
<tr>
<td>$\Delta hp_{low_{t-1}}$</td>
<td>$3.13^{<em><strong>}$  $1.85^{</strong></em>}$  $2.53^{*<strong>}$  $2.67^{</strong>}$</td>
</tr>
<tr>
<td></td>
<td>(0.59)  (0.68)  (0.90)  (1.11)</td>
</tr>
<tr>
<td>$\Delta mori_{t-1}$</td>
<td>$-0.20^{<em><strong>}$ $-0.20^{</strong></em>}$</td>
</tr>
<tr>
<td></td>
<td>(0.02)  (0.02)</td>
</tr>
<tr>
<td>$\Delta income_{t-1}$</td>
<td>-0.63</td>
</tr>
<tr>
<td></td>
<td>(1.04)</td>
</tr>
<tr>
<td>pval difference</td>
<td>0.000  0.211  0.160  0.181</td>
</tr>
</tbody>
</table>

| Time effects      | no  no  yes  yes  yes |
| Observations      | 1020 1020 1020 969 969 |
| States            | 51 51 51 51 51 |
| R-squared         | 0.01 0.03 0.58 0.53 0.53 |

Note: State-level Regressions using annual observations from 1992 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***, **, *: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: $\Delta mori$ is the percent change in “Mortgage originations and purchases: Value” from the U.S. Federal Financial Institutions Examination Council: Home Mortgage Disclosure Act. See Table 2 for other variable definitions.
Table 6: MSA Level: Employment in Services and House Prices

<table>
<thead>
<tr>
<th></th>
<th>% Change in Employment (Δ\text{emp}_t)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆\text{hp}_t−1 0.134*** (0.006)</td>
<td>∆\text{hp}_\text{high}_t−1 0.104*** (0.008)</td>
<td>∆\text{hp}_\text{low}_t−1 0.183*** (0.009)</td>
<td>0.049*** (0.008)</td>
<td>0.044*** (0.010)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>∆\text{emp}_t−1 0.033 (0.041)</td>
<td>∆\text{income}_t−1 0.040** (0.019)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

pval difference 0.0000 0.0003 0.0001 0.0001

Time effects no no yes yes yes
Observations 5390 5390 5390 5147 5147
MSA 262 262 262 262 262
R-squared 0.09 0.10 0.37 0.39 0.40

Note: State–level Regressions using annual observations from 1992 to 2011 on 262 MSAs (102 MSAs were dropped since they had incomplete or missing data on employment by sector). Robust standard errors in parenthesis. ***,**,,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: see Table 2.
Table 7: MSA Level: Auto Registrations and House Prices

<table>
<thead>
<tr>
<th></th>
<th>2003-2007 (Housing Boom)</th>
<th>2007-2011 (Housing Bust)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>-3.80***</td>
<td>4.31***</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(0.51)</td>
</tr>
<tr>
<td>∆hp</td>
<td>-0.02</td>
<td>0.54***</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Method</td>
<td>OLS</td>
<td>OLS</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>Observations</td>
<td>254</td>
<td>254</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.14</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.41</td>
</tr>
</tbody>
</table>

Note: Regressions using Housing supply Elasticity at the MSA level as an instrument for housing prices in a regression of MSA car registrations on MSA house prices. ***,**,,: Coefficients statistically different from zero at 1, 5 and 10% confidence level, respectively. The housing supply elasticity is taken from Saiz (2010) and measures limits on real-estate development due to geographic factors that affect the amount of developable land, as well as factors like zoning restrictions. The elasticity data are available for 269 cities: we dropped 15 areas because they were covering primary metropolitan statistical areas (PMSA), which are portions of metropolitan areas, rather than complete MSAs.

Data Sources: Car Registrations are retail (total less rental, commercial and government) auto registrations from Polk Automotive Data. ∆car is the percent change in car registrations. See Table 2 for other data sources.
A Accuracy of the Occbin Solution Method for the Basic Model

The model described in Section 2 is simple enough as to be solved with both our occbin algorithm, described in Section C, and with standard global solution methods. Among these standard methods, we focus on value function iteration since it is reliable, and well understood. Overall, we find that key aspects of the global solution obtained through value function iteration are matched by the solution from the occbin algorithm. A key advantage of the occbin algorithm is that it can easily handle the solution of a model, such as the one described in Section 3, for which the curse of dimensionality renders standard global solution methods infeasible.

In Figures 1 and 2 we compare the simulated paths for house prices, consumption, leverage and debt using alternative solution methods. In Figure 1, we consider impulse responses to negative and positive house price shocks. In Figure 2, we generate a realization of house prices drawing shock innovations for 50 periods from the stochastic AR(1) process described in the text.

The “occbin/piecewise linear” lines are computed using our method. The “nonlinear stochastic” lines refer to the nonlinear model solution obtained using global methods (value function iteration) under the assumption that the agents know and act upon the future distribution of the random shocks. The “nonlinear deterministic” lines refer to the perfect foresight case, solved using global methods under the assumption that agents ignore the future variance of shocks (that is, each period they expect that future shock innovations will equal zero with probability one). Finally, the “linear” lines refer to the model solved used brute force linearization under the – counterfactual – assumption that the borrowing constraint is always binding.

As can be seen from the figures, the nonlinear methods (value function iteration) and the occbin/piecewise linear method deliver very similar dynamics for the variables of interest. The similarity of the simulation paths causes the business cycle statistics (reported in Table 1) to be in broad agreement for those two methods. As expected, leverage and debt are on average lowest in the full stochastic case, since buffer stock motives – ignored by construction or by design in the other cases – cause agents to save more and reduce indebtedness. However, our method – which combines first-order perturbation solutions under two different regimes – comes remarkably close to matching the dynamics of the full nonlinear method under perfect foresight. As first-order perturbation solutions ignore the possibility of future shocks, it is not surprising that our occbin method would not be able to capture precautionary motives present in the full stochastic non-linear solution. Accordingly, we consider the comparison with a perfect-foresight non-linear solution as more apt. By contrast, the linearized solution that assumes that the constraint is always binding cannot capture the asymmetry of consumption and grossly overestimates its volatility.

As a different metric to judge to accuracy of our solution method, the last column of Table 1

---

1 Our state variables are the level of debt, the housing stock and the house price process. We discretize the AR(1) house price process with using Tauchen’s method (Tauchen (1986)) with 115 grid points. We pick a solution range for housing and debt between −60 and +60 percent of their steady state values, discretized over 100 points for debt and 110 points for housing (we use interpolation between grid points and we fine tune the grid by putting more mass in points of the state space that are visited more frequently in a given simulation). In between iterations, we use Howard’s improvement step. We verified that increasing the number of grid points did not materially change any of the results.
also reports the welfare cost (in percentage of lifetime consumption) for a household of using the approximated policy functions instead of the correct one in order to solve its problem. The welfare cost of using the piecewise policy function is small (about 0.02% of lifetime consumption), and is one order of magnitude smaller than the cost of using the linearized policy function.
B Summary of the Equilibrium Conditions of the Extended Model

We summarize here the equations describing the equilibrium of the extended model. Let \( u_c \) denote the marginal utility of consumption, \( u_{nc} \) (or \( u_{nh} \)) the marginal disutility of working in the goods (housing) sector, and \( u_h \) the marginal utility of housing (with analogous definitions holding for impatient households). We drop the \( t \) subscript to denote the steady-state value of a particular variable. The budget constraint for patient households is:

\[
c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + q_t h_t + p_t l_t - b_t = \frac{w_{c,t}}{X_{wc,t}} n_{c,t} + \frac{w_{h,t}}{X_{wh,t}} n_{h,t} - \phi_t
\]

\[
+ \left( R_{c,t} z_{c,t} + \frac{1 - \delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + (R_{h,t} z_{h,t} + 1 - \delta_{kh}) k_{h,t-1} + p_b t k_{b,t} - \frac{R_t b_{t-1}}{\pi_t}
\]

\[
+ (p_{l,t} + R_{l,t}) l_{t-1} + q_t (1 - \delta_h) h_{t-1} + Div_t - \frac{a(z_{c,t})}{A_{k,t}} k_{c,t-1} - a(z_{h,t}) k_{h,t-1}.
\]

(A.1)

The first-order conditions for patient households are:

\[
u_{c,t} q_t = u_{h,t} + \beta GCE_t \left( u_{c,t+1} + (1 - \delta_h) \right)
\]

(A.2)

\[
u_{c,t} = \beta GCE_t \left( u_{c,t+1} + R_t / \pi_{t+1} \right)
\]

(A.3)

\[
u_{c,t} \left( 1 + \frac{\partial \phi_{c,t}}{\partial k_{c,t}} \right) = \beta GCE_t u_{c,t+1} \left( R_{c,t+1} z_{c,t+1} - \frac{a(z_{c,t+1}) + 1 - \delta_{kc}}{A_{k,t+1}} - \frac{\partial \phi_{c,t+1}}{\partial k_{c,t}} \right)
\]

(A.4)

\[
u_{c,t} \left( 1 + \frac{\partial \phi_{h,t}}{\partial k_{h,t}} \right) = \beta GCE_t u_{c,t+1} \left( R_{h,t+1} z_{h,t+1} - a(z_{h,t+1}) + 1 - \delta_{kh} - \frac{\partial \phi_{h,t+1}}{\partial k_{h,t}} \right)
\]

(A.5)

\[
u_{c,t} w_{c,t} = u_{nc,t} X_{wc,t}
\]

(A.6)

\[
u_{c,t} w_{h,t} = u_{nh,t} X_{wh,t}
\]

(A.7)

\[
u_{c,t} \left( p_{l,t} - 1 \right) = 0
\]

(A.8)

\[
R_{ct} A_{kt} = a'(z_{ct})
\]

(A.9)

\[
R_{ht} = a'(z_{ht})
\]

(A.10)

\[
u_{c,t} p_{l,t} = \beta GCE_t u_{c,t+1} \left( p_{l,t+1} + R_{l,t+1} \right).
\]

(A.11)

The budget and borrowing constraint for impatient households are:

\[
c'_t + q_t h'_t = \frac{w'_{c,t}}{X'_{wc,t}} n'_{c,t} + \frac{w'_{h,t}}{X'_{wh,t}} n'_{h,t} + b'_t - \frac{R_{t-1} b'_{t-1}}{\pi_t} + q_t (1 - \delta_h) h'_{t-1} + Div'_t
\]

(A.12)

\[
b'_t \leq m_t E_t \left( \frac{q_{t+1} h'_t \pi_{t+1}}{R_t} \right)
\]

(A.13)
and the first-order conditions are:

\[ u_{c',t} q_t = u_{h',t} + \beta' G_C E_t \left( u_{c',t+1} (q_{t+1} (1-\delta)) \right) + E_t \left( \lambda_t \frac{m q_{t+1} \pi_{t+1}}{R_t} \right) \]  
\[ u_{c',t} = \beta' G_C E_t \left( u_{c',t+1} \frac{R_t}{\pi_{t+1}} \right) + \lambda_t \]  
\[ u_{c',t} w_{c,t}' = u_{n c',t} X_{wc,t}' \]  
\[ u_{c',t} w_{h,t}' = u_{n h',t} X_{wh,t}' \]  

where \( \lambda_t \) denotes the multiplier on the borrowing constraint, which is greater than zero in a neighborhood of the equilibrium.

The production technologies are:

\[ Y_t = (A_{c,t} (n_{c,t}^\alpha n_{c,t}^{1-\alpha}))^{1-\mu_c} (z_{c,t} k_{c,t-1})^{\mu_c} \]  
\[ IH_t = (A_{h,t} (n_{h,t}^\alpha n_{h,t}^{1-\alpha}))^{1-\mu_h} (z_{h,t} k_{h,t-1})^{\mu_h} \]  

The first-order conditions for the wholesale goods firms are:

\[ (1-\mu_c) \alpha Y_t = X_t w_{c,t} n_{c,t} \]  
\[ (1-\mu_c) (1-\alpha) Y_t = X_t w_{c,t} n_{c,t}' \]  
\[ (1-\mu_h - \mu_t - \mu_b) \alpha q_t IH_t = w_{h,t} n_{h,t} \]  
\[ (1-\mu_h - \mu_t - \mu_b) (1-\alpha) q_t IH_t = w_{h,t} n_{h,t}' \]  
\[ \mu_c Y_t = X_t R_{c,t} z_{c,t} k_{c,t-1} \]  
\[ \mu_h q_t IH_t = R_{h,t} z_{h,t} k_{h,t-1} \]  
\[ \mu_t q_t IH_t = R_{t,t} k_{t-1} \]  
\[ \mu_b q_t IH_t = p_{h,t} k_{b,t} \]  

The Phillips curve is:

\[ \ln \pi_t - \epsilon_\pi \ln \pi_{t-1} = \beta G_C \left( E_t \ln \pi_{t+1} - \epsilon_\pi \ln \pi_t \right) - \epsilon_\pi \ln \left( X_t / X \right) + u_{\pi,t}. \]  

Denote with \( \omega_{i,t} \) nominal wage inflation, that is, \( \omega_{i,t} = \frac{w_{i,t} \pi_t}{w_{i,t-1}} \) for each sector/household pair. The four wage equations are:

\[ \ln \omega_{c,t} - \epsilon_{wc} \ln \pi_{t-1} = \beta G_C \left( E_t \ln \omega_{c,t+1} - \epsilon_{wc} \ln \pi_t \right) - \epsilon_{wc} \ln \left( X_{wc,t} / X_{wc} \right) \]  
\[ \ln \omega_{c,t}' - \epsilon_{wc} \ln \pi_{t-1} = \beta' G_C \left( E_t \ln \omega_{c,t+1}' - \epsilon_{wc} \ln \pi_t \right) - \epsilon_{wc}' \ln \left( X_{wc,t} / X_{wc} \right) \]  
\[ \ln \omega_{h,t} - \epsilon_{wh} \ln \pi_{t-1} = \beta G_C \left( E_t \ln \omega_{h,t+1} - \epsilon_{wh} \ln \pi_t \right) - \epsilon_{wh} \ln \left( X_{wh,t} / X_{wh} \right) \]  
\[ \ln \omega_{h,t}' - \epsilon_{wh} \ln \pi_{t-1} = \beta' G_C \left( E_t \ln \omega_{h,t+1}' - \epsilon_{wh} \ln \pi_t \right) - \epsilon_{wh}' \ln \left( X_{wh,t} / X_{wh} \right) \]
where \( \varepsilon_{wc} = (1 - \theta_{wc}) (1 - \beta G C \theta_{wc}) / \theta_{wc}, \varepsilon_{wc}' = (1 - \theta_{wc}) (1 - \beta' G C \theta_{wc}) / \theta_{wc} \),
\( \varepsilon_{wh} = (1 - \theta_{wc}) (1 - \beta G C \theta_{wc}) / \theta_{wh} \) and \( \varepsilon_{wh}' = (1 - \theta_{wh}) (1 - \beta' G C \theta_{wh}) / \theta_{wh} \).

The Taylor rule is:
\[
R_t = \max \left( 1, (R_{t-1})^{r_R} \pi_t^{1-r_R} \left( \frac{GDP_t}{G C GDP_{t-1}} \right)^{r_Y (1-r_R)} \left( \frac{u_{R,t}}{s_t} \right)^{-r_R} \right) \tag{A.33}
\]

where \( GDP_t \) is the sum of the value added of the two sectors, that is \( GDP_t = Y_t - k_{h,t} + qI H_t \).

Two market-clearing conditions are
\[
\begin{align*}
C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{h,t} &= Y_t - \phi_t \tag{A.34} \\
h_t + h_t' - (1 - \delta_h) (h_{t-1} + h_{t-1}') &= IH_t. \tag{A.35}
\end{align*}
\]

By Walras’ law, \( b_t + b_t' = 0 \). Finally, total land is normalized to unity:
\[
l_t = 1. \tag{A.36}
\]

In equilibrium, dividends paid to households equal respectively:
\[
Div_t = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}} w_{c,t} n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}} w_{h,t} n_{h,t}
\]
\[
Div_t' = \frac{X_{wc,t}' - 1}{X_{wc,t}'} w_{c,t}' n_{c,t}' + \frac{X_{wh,t}' - 1}{X_{wh,t}'} w_{h,t}' n_{h,t}'.
\]

In addition, the functional forms for the capital adjustment cost and the utilization rate are:
\[
\phi_t = \frac{\phi_{kc}}{2G_{IK_c}} \left( \frac{k_{c,t}}{k_{c,t-1}} - G_{IK_c} \right)^2 k_{c,t-1} \left( 1 + \gamma_{AK} \right)^t + \frac{\phi_{kh}}{2G_{IK_h}} \left( \frac{k_{h,t}}{k_{h,t-1}} - G_{IK_h} \right)^2 k_{h,t-1}
\]

\[
a(z_{c,t}) = R_c \left( \bar{\omega} z_{c,t}^2 / 2 + (1 - \bar{\omega}) z_{c,t} + (\bar{\omega} / 2 - 1) \right)
\]
\[
a(z_{h,t}) = R_h \left( \bar{\omega} z_{h,t}^2 / 2 + (1 - \bar{\omega}) z_{h,t} + (\bar{\omega} / 2 - 1) \right)
\]

where \( R_c \) and \( R_h \) are the steady-state values of the rental rates of the two types of capital. In the estimation of the model, we specify our prior for the curvature of the capacity utilization function in terms of \( \zeta = \bar{\omega} / (1 + \bar{\omega}) \). With this change of variables, \( \zeta \) is bounded between 0 and 1, since \( \bar{\omega} \) is positive.

Equations A.1 to A.36, together with the values for \( IK_c, IK_h, GDP_t, \phi_t, a(z), Div_t \) and \( Div_t' \) and the laws of motion for the exogenous shocks, define a system of 36 equations in the following variables: \( c, h, k_c, k_h, n_c, n_h, b, l, z_c, z_h, c', h', n_c', n_h', b', IH, Y, q, R, \pi, \lambda, X, w_c, w_h, w_c', w_h', X_{wc}, X_{wh}, X_{wc}', X_{wh}', R_c, R_h, R_t, p_b, \) and \( p_t \).

After detrending the variables by their balanced growth trends, we use the methods described in Guerrieri and Iacoviello (2013) to solve the model subject to the two occasionally binding constraints given by equations A.13 and A.33.
C  The Solution Method

We use a piece-wise linear solution approach to find the equilibrium allocations for a given sequence of unforeseen shocks. Such method deals with the problem of computing decision rules that approximate the equilibrium well both when borrowing constraint binds and when it does not (similar reasoning applies to the nonnegativity constraint on the interest rate, an issue that we return to at the end of this section).

The economy features two regimes: a regime when collateral constraints bind; and a regime in which they do not, but are expected to bind in the future. With binding collateral constraints, the linearized system of necessary conditions for an equilibrium can be expressed as

\[ A_1 E_t X_{t+1} + A_0 X_t + A_{-1} X_{t-1} + Bu_t = 0, \]  

where \( A_1, A_0, \) and \( A_{-1} \) are matrices of coefficients conformable with the vector \( X \) collecting the model variables in deviation from the steady state for the regime with binding constraints; and where \( u \) is the vector collecting all shock innovations (and \( B \) is the corresponding conformable matrix). Similarly, when the constraint is not binding, the linearized system can be expressed as

\[ A_1^* E_t X_{t+1} + A_0^* X_t + A_{-1}^* X_{t-1} + B^* u_t + C^* = 0, \]

where \( C^* \) is a vector of constants. When the constraint binds, we use standard linear solution methods to express the decision rule for the model as

\[ X_t = \mathcal{P} X_{t-1} + Qu_t. \]

When the collateral constraints do not bind, we use a guess-and-verify approach. We shoot back towards the initial conditions, from the first period when the constraints are guessed to bind again. For example, if the constraints do not bind in \( t \) but are expected to bind the next period, the decision rule for period \( t \) can be expressed, starting from 2 and using the result that \( E_t X_{t+1} = \mathcal{P} X_t \), as:

\[ X_t = - (A_1^* \mathcal{P} + A_0^*)^{-1} (A_{-1}^* X_{t-1} + B^* u_t + C^*). \]

We proceed in a similar fashion to compute the allocations for the case when collateral constraints are guessed not to bind for multiple periods or when they are foreseen to be slack starting in periods beyond \( t \). As shown by equation 4, the model dynamics when constraints are not binding depend both on the current regime (through the matrices \( A_1^*, A_0^* \) and \( A_{-1}^* \)) and on the expectations of future regimes when constraints will bind again (through the matrix \( \mathcal{P} \), which is a nonlinear function of

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2 This approach is common in the literature on the zero lower bound on nominal interest rates. For instance, see Eggertsson and Woodford (2003) and Bodenstein, Guerrieri, and Gust (2010).

3 If one assumes that the constraints are not expected to bind in the future, the regime with slack borrowing constraints becomes unstable, since borrowers' consumption falls over time and their debt rises over time until it reaches the debt limit, which contradicts the initial assumption.
the matrices $A_1$, $A_0$ and $A_{-1}$).\textsuperscript{4}

It is tedious but straightforward to generalize the solution method described above for multiple occasionally binding constraints.\textsuperscript{5} The extension is needed to account for the zero lower bound (ZLB) on policy interest rates as well as the possibility of slack collateral constraints. In that case, there are four possible regimes: 1) collateral constraints bind and policy interest rates are above zero, 2) collateral constraints bind and policy interest rates are at zero, 3) collateral constraints do no bind and policy interest rates are above zero, 4) collateral constraints do not bind and policy interest rates are at zero. Apart from the proliferation of cases, the main ideas outlined above still apply.


\textsuperscript{5} For an array of models, Guerrieri and Iacoviello (2013) compare the performance of the piece-wise perturbation solution described above against a dynamic programming solution obtained by discretizing the state space over a fine grid. Their results show that this solution method efficiently and quickly computes a solution that closely mimics the (perfect-foresight) nonlinear solution.
References


Guerrieri, L. and M. Iacoviello (2013). A toolkit to solve models with occasionally binding constraints easily. Manuscript, Federal Reserve Board. [6, 8]


Figure 1: Accuracy of Our Method: Impulse Responses

Note: Impulse Responses of the basic model to a negative house price shock in period 10 and a positive house price shock in period 50.
Figure 2: Accuracy of Our Method: Simulated Time Series

Note: Simulation of the basic model for 50 periods using identical realizations for the exogenous random shock to house prices.
Table 1: Accuracy of the Solution Method

<table>
<thead>
<tr>
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<th>Log Consumption Correlations</th>
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<tr>
<td></td>
<td>st.dev</td>
<td>skewness</td>
<td>log q, log c</td>
<td>log q, log b</td>
<td>mean</td>
</tr>
<tr>
<td>Linear</td>
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<td>0.03</td>
<td>0.40</td>
<td>0.00</td>
<td>0.925</td>
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<tr>
<td>Piecewise Linear (occbin)</td>
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<td>-1.17</td>
<td>0.54</td>
<td>-0.60</td>
<td>0.911</td>
</tr>
<tr>
<td>Nonlinear perfect foresight</td>
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<td>0.53</td>
<td>-0.58</td>
<td>0.910</td>
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<tr>
<td>Nonlinear stochastic</td>
<td>3.7%</td>
<td>-1.30</td>
<td>0.65</td>
<td>-0.71</td>
<td>0.896</td>
</tr>
</tbody>
</table>

Note: Selected properties of the basic model using different solution algorithms. These properties are based on the outcomes of a simulation of 5,000 observations using identical realizations for the exogenous random shocks.