Uncertainty, Risk, and Incentives: Theory and Evidence

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Uncertainty has qualitatively different implications than risk in studying executive incentives. We study the interplay between profitability uncertainty and moral hazard, where profitability is multiplicative with managerial effort. Investors who face greater uncertainty desire faster learning, and consequently offer higher managerial incentives to induce higher effort from the manager. In contrast to the standard negative risk-incentive trade-off, this “learning-by-doing” effect generates a positive relation between profitability uncertainty and incentives. We document empirical support for this prediction.

Key words: executive compensation; optimal contracting; learning; uncertainty; risk–incentive trade-off

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1. Introduction

A central prediction of the principal–agent theory is the negative trade-off between risk and incentives (Holmstrom and Milgrom 1987). Higher performance pay induces greater effort from the agent but increases risk, which in turn raises the compensation that must be paid to the agent for bearing risk. The greater the output risk, the higher the compensation for risk, leading to a lower performance pay to the risk-averse agent in the optimal contract. Yet, numerous studies over the past two decades find mixed empirical evidence on such a negative relation between risk and incentives. After reviewing more than two dozen empirical studies and concluding that evidence on the risk–incentive trade-off is inconclusive, Prendergast (2002) argues that in a more uncertain environment, the principal may want to delegate responsibilities to the agent, leading to a positive risk–incentive relation. Other leading explanations for this puzzle includes the idea of endogenous firm risk, where firms offer high-powered incentives to induce the agent to take risk (e.g., Core and Guay 1999, Edmans and Gabaix 2011a), or the view that risk does not affect incentives because, from the principal’s perspective, the cost of risk bearing is outweighed by the benefits of efforts, and thus risk is second order (e.g., Edmans et al. 2009, Edmans and Gabaix 2011b).

In this paper we offer another plausible theory to explain why the negative risk–incentive trade-off has received mixed empirical support. Empirically measured risk, which is essentially output performance variance, can come from either cash flow risk or project profitability uncertainty, or both. Specifically, in many types of economic environments with agency relationships, output performance not only consists of the agent’s effort plus some transitory random noise (i.e., cash flow risk), but also the project’s unobserved long-run profitability (i.e., profitability uncertainty).¹

We incorporate endogenous learning about the firm’s profitability uncertainty into the standard Holmstrom and Milgrom (1987) setting and show that

¹ Most of the existing principal–agent literature assumes that the productivity of managerial input is known. Our paper introduces the uncertainty on the productivity parameters in a simple two-period setting to study the relation between incentives, risk, and uncertainty. For other papers with learning in short-term contracting, see Murphy (1986) and Gibbons and Murphy (1992). Long-term optimal contracting with learning is much more technically challenging because of the hidden-state problem; see DeMarzo and Sannikov (2011), Prat and Jovanovic (2012), and He et al. (2013).
a potentially positive relation between uncertainty and incentives emerges. In a nutshell, besides the traditional risk channel, the learning channel implies that greater effort, induced by high-powered incentives, leads to more informative signals about uncertain project profitability, improving the firm’s future investment decisions. Moreover, somewhat surprisingly, even if one can perfectly separate risk from uncertainty, this learning channel may also overturn the traditional negative risk–incentive relation. Based on several widely used proxies for firm profitability uncertainty, we find empirical support for the positive uncertainty–incentive relation. This suggests that prior mixed empirical results in testing the negative risk–incentive trade-off may be attributable to a positive bias caused by omitting variables that are proxies for profitability uncertainty.

In this paper we develop a two-period investment model, in which the firm hires a manager to manage a project at the beginning of period 1. The project generates an output of $y_1 = \theta K_1^{1-\alpha}L_1^\alpha + \epsilon_1$, where $K_1$ is capital, $L_1$ is managerial labor (effort) input, and $\epsilon_1$ is exogenous cash flow shock. The parameter $\theta$ is the project’s marginal productivity or profitability. The key departure of our model from standard agency models is that profitability $\theta$ is unknown. Investors learn $\theta$ and then make future investment decisions. Both multiplicative labor with $\theta$ and additive cash flow noise $\epsilon_1$ are the drivers of our mechanism; they imply that a greater labor input can increase the information-to-noise ratio of the output signal $y_1$ based on Bayes’ rule. At period 2, the firm with a posterior belief of $\theta$ adjusts capital $K_2$ through investment, and resets labor input $L_2$.

To optimize over period 2 investment, investors desire faster learning about $\theta$ from period 1 output signal $y_1$. As a result, for a more informative signal $y_1$, high-powered incentives that induce greater effort from the manager are more preferable. Moreover, the higher the degree of uncertainty, the greater the reduction of the posterior variance of $\theta$, and thus the greater the benefit in inducing a higher period 1 effort. In other words, firms with uncertain profitability offer high-powered incentives to their managers for more informative signals to guide their investment policies. This mechanism is similar in spirit to the learning-by-doing literature (e.g., Jovanovic and Lach 1989, Jovanovic and Nyarko 1996, Johnson 2007). Because uncertainty in $\theta$ also increases the total volatility of output $y_1$ on the risk-averse manager, when the manager’s risk aversion is relatively high, the traditional negative risk–incentive effect dominates and leads to a standard negative uncertainty–incentive relation. However, when the manager’s risk aversion is relatively low, the learning-by-doing effect dominates and leads to a positive uncertainty–incentive relation. Moreover, the learning mechanism may also overturn the traditional negative risk–incentive relation. The higher the risk, the smaller the information-to-noise ratio, and the more the room to learn about the unknown profitability uncertainty. Thus offering high-powered incentives might be desirable.

We empirically test whether the uncertainty–incentive relation is positive in §3. Following Pastor and Veronesi (2003) and Korteweg and Polson (2010) we use firm age as our first proxy as older firms usually have lower uncertainty. We also use stock price reaction to earnings announcements (i.e., earnings response coefficient, or ERC) as another proxy for profitability uncertainty (Pastor et al. 2009). Intuitively, investors who are more uncertain about a company’s profitability should be more responsive to earnings surprises. Our other proxies for profitability uncertainty are tangibility and market-to-book ratio (Korteweg and Polson 2010), and analyst forecast error (Lang and Lundholm 1996). We then run panel regressions of pay–performance sensitivities (PPSs henceforth) on these uncertainty proxies and the risk proxy while controlling for other factors known to affect PPS. We find that firm age and tangibility are negatively related to PPS; ERC, market-to-book ratio, and analyst forecast error are positively related to PPS.

Several remarks are worth highlighting in interpreting our empirical results. First, we acknowledge that each individual proxy for uncertainty is imperfect; these proxies may reflect firm characteristics such as growth opportunities. For example, firms with more growth opportunities are often younger, have higher market-to-book ratios, and have more intangible assets. These firms are also harder to analyze and thus are associated with larger ERC and analyst forecast errors. Hence, in all regressions, we control for firm growth using analysts’ long-term earnings growth forecast. This is not a perfect solution to remove the effect of growth from the uncertainty

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2 Our paper, with the inclusion of learning, is different from Prendergast (2002) and some other papers (see, e.g., Zabojnik 1996, Baker and Jorgensen 2003, Peng and Röell 2013) that predict a possible positive relation between uncertainty and incentives. For example, Prendergast (2002) argues that in a more uncertain environment that the agent knows more than the principal, the positive value of delegating responsibilities to the agent may dominate the negative effect of risk on incentives, resulting in a positive relation between uncertainty and incentives. In contrast, our model has symmetric information along the equilibrium path, and learning is the key mechanism. Peng and Röell (2013) study optimal contracting when managers can manipulate firm performance. They find that uncertainty in managerial manipulation propensity may also lead to a positive uncertainty–incentive relation. Based on a different type of uncertainty, the mechanism in their paper is complementary to ours.
proxies, and the results in this paper need to be interpreted with this caveat in mind.

Second, for our analysis, it is important (to try) to separate uncertainty from risk. Fortunately, some uncertainty variables we use are positively correlated with firm volatility, whereas others (e.g., ERC) are negatively correlated with volatility. Examining all of the different uncertainty variables will help us separate the role of uncertainty from that of volatility. We do, however, acknowledge that the separation of uncertainty from risk in the paper is not perfect.

Third, in our model, profitability uncertainty is taken as exogenous, and firms design endogenous optimal incentive contracts as a response to uncertainty. It could well be possible that the causality goes the other way in practice; that is, incentive contracts affect managers’ choices of project uncertainty. This reverse causality problem exists even if we can measure uncertainty perfectly. Although we use fixed effects regressions in the robustness section to address the potential endogeneity problem due to time-invariant omitted variables, fixed effects can address neither the problem of time-variant omitted variables nor the reverse causality problem. In this paper we do not claim identification of causality, although we lag our uncertainty proxies by one year in our regression analysis in an attempt to mitigate the reserve causality issue. Because the incentive variables are persistent and some of the uncertainty proxies are forward looking, this treatment is far from perfect.

The contribution of this paper is to propose a new explanation for mixed empirical evidence on the negative risk–incentive trade-off. Our learning-based model suggests two reasons: first, the effect of risk on incentives may be confounded by the uncertainty effect if uncertainty is not captured in the model, and second, under learning, the risk–incentive relation becomes ambiguous. On the empirical side, we provide preliminary analysis to see whether the data is consistent with our model. Our analysis suggests that controlling for profitability uncertainty helps partially (if not fully) to restore the negative risk–incentive relation predicted by standard agency theories. Although the coefficients of the risk variable often become less positive or more negative after the uncertainty variables are incorporated in the empirical model, we acknowledge that our analysis cannot fully restore the negative risk–incentive trade-off, and thus is far from resolving Prendagast’s (2002) statement that the evidence on the risk–incentive trade-off is inconclusive. We further reiterate that our empirical methodology has several other limitations: our uncertainty proxies are not perfect, the separation of uncertainty from risk is not ideal, and our method does not allow us to establish causality. The attempt to rule out alternative explanations in the robustness section is suggestive rather than conclusive; we await future research on this topic.

The rest of this paper is organized as follows. Section 2 presents the model and its prediction of the positive relation between profitability uncertainty and incentives. Section 3 conducts empirical analysis. Section 4 concludes the paper. All proofs are in Appendix A.

2. The Model
2.1. The Setting
We consider a two-period investment model, where investment consists of capital and (managerial) labor inputs. The risk-free rate is zero. Investors are risk neutral, and managers are risk averse with exponential (constant absolute risk aversion, or CARA) preference. We interpret labor input as the manager’s effort. For simplicity, we assume that moral hazard only exists in the first period; the firm matures in the second period and therefore is no longer subject to agency issues.

The output in each period, before investment cost, is modeled as (similar to the standard Cobb–Douglas technology with constant returns to scale)

$$y_t = \theta K_t^{1-\lambda} L_t^\lambda + \epsilon_t,$$

where $K_t$ is capital level, $L_t$ is managerial labor input, $\lambda \in (0, 1)$ and $1 - \lambda$ are output elasticities of labor and capital, respectively, and $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$ is i.i.d. normally distributed. Importantly, $\theta$, which can be interpreted as project profitability or marginal productivity, is uncertain. Neither the firm nor the manager observes profitability $\theta$ directly, and they will learn $\theta$ from the realized output. At time 0, the common prior about profitability is $\theta \sim \mathcal{N}(\theta_0, \gamma_0)$, where $\theta_0 > 0$ and $\gamma_0 > 0$ are prior mean and variance, respectively.

At the beginning of period 1, the firm with a zero outside option decides whether or not to invest $K_1$.
Given $K_I$, investors hire a manager to provide labor input $L_1$, which is unobservable. We interpret $L_1$ as managerial effort, and investors offer the manager a compensation contract for proper incentives. We focus on the space of linear contracts. The contract $w_1(y; \alpha, \beta)$ takes the following form with fixed salary $\alpha$ and incentive $\beta$:

$$w_1(y; \alpha, \beta) \equiv \alpha + \beta y_1 = \alpha + \beta(\theta K_1^{1-\lambda} L_1^\lambda + \epsilon_1).$$

Here, the monetary cost for managerial labor $L_1$ is $(l/2)L_1^2$, where $l > 0$ is a positive constant. Therefore, the manager’s utility from accepting the contract $w_1(y; \alpha, \beta)$ and working $L_1$ is given by

$$U(L_1, w_1) = -\exp\left(-\alpha + \beta y_1 - \frac{l}{2} L_1^2\right), \quad (2)$$

where $a > 0$ is the manager’s risk-aversion coefficient. Finally, the manager has a reservation utility of $\hat{U}$ at time 0, which is normalized to $-1$ without loss of generality.

Suppose that the firm induces a labor input of $L_1^*$ from the period 1 manager. At the second period the firm makes capital investment and labor investment based on the updated posterior of profitability $\hat{\theta}_1$. For period 2 labor investment $L_2$, the firm hires another manager with the same cost function $(l/2)L_2^2$, and for simplicity, we assume away any agency problem at period 2 (as the firm’s operation becomes more routine). Capital investment is subject to standard (constant-return-to-scale) quadratic adjustment cost; given initial capital $K_I$, a (gross) investment of $I + (\kappa/2K_I)I^2$ leads to a new capital level of $K_1 + I$, where $\kappa > 0$ is a positive constant. As a result, investors at the beginning of period 2 will solve the following problem:

$$\max_{l_1, l_2} \left[ \theta(K_1 + I)^{1-\lambda} L_2^\lambda + \epsilon_2 - I - \frac{\kappa}{2K_1} I^2 - \frac{l}{2} L_1^2 \right] y_1, L_1^*.$$

We provide a summary of the model timeline as follows; see Figure 1.

1. At the beginning of $t = 1$, the firm decides whether to take the project. Its outside option is normalized to zero. Thus $(\hat{\theta}_0, \gamma_0)$ must be sufficiently favorable for the project to be adopted. This stage plays the only role to ensure that $\hat{\theta}_0 > 0$ (so maximizing expected output $\theta K_1^{1-\lambda} L_1^\lambda$ in (1) makes sense), an assumption that holds throughout this paper.

2. If the firm decides to take this project, investors hire one manager and offer him a linear contract $w_1 = \alpha + \beta y_1$, where $y_1 = \theta K_1^{1-\lambda} L_1^\lambda + \epsilon_1$ is the project’s output in period 1. Investors’ period 1 payoff is

$$y_1 - w_1 - K_1 = \theta K_1^{1-\lambda} L_1^\lambda + \epsilon_1 - \alpha - \beta y_1 - K_1.$$

3. Given the outcome $y_1$, investors update their belief about $\theta$ based on the prior $\theta \sim N(\hat{\theta}_0, \gamma_0)$.

4. At $t = 2$, the firm makes capital investment $I$ and labor investment $L_2$, so that $y_2 = \theta(K_1 + I)^{1-\lambda} L_2^\lambda + \epsilon_2$. The period 2 payoff is

$$\theta(K_1 + I)^{1-\lambda} L_2^\lambda + \epsilon_2 - I - \frac{\kappa}{2K_1} I^2 - \frac{l}{2} L_1^2.$$

2.2. Discussion of Modeling Assumptions

Before solving the model backward, we briefly discuss the key assumptions of the model. In particular, we highlight the necessary assumptions for the key model mechanism and discuss the assumptions made for technical convenience as well.

First, two features of production technology in Equation (1) are important: multiplicative specification between productivity $\theta$ and managerial labor input $L_1$, and additive cash flow noise $\epsilon_1$. Under this setting, a greater labor input can increase the information-to-noise ratio when investors learn the project’s profitability $\theta$ from the output signal $y_1$, using Bayes’ rule, resulting in a potentially positive uncertainty–incentive relation due to the learning-by-doing effect. If instead we assume that output is additive in profitability and labor so that $y = \theta + K_1^{1-\lambda} L_1^\lambda + \epsilon$, the learning-by-doing effect disappears. Our learning-by-doing effect also vanishes if we assume a multiplicative cash flow noise, i.e., $y = \theta K_1^{1-\lambda} L_1^\lambda \epsilon$. This disappearance occurs because

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For purely technical convenience, we follow Gaussian-learning framework where $\theta$ can be negative. Our results go through if we assume that $\hat{\theta}$ is lognormal. However, due to the principal’s option to abandon the project, $\hat{\theta}$ must be reasonably high for the project to be taken.
increasing effort does not reduce the posterior variance of the unknown parameter $\theta$ in these two alternative settings.

Second, the common prior on the unknown parameter $\theta$ indicates that the agent and the principal have the same information regarding $\theta$. It is possible that the agent knows $\theta$ more than the principal. This is especially true if $\theta$ captures the manager’s productivity type. Two questions arise under this asymmetric information scenario. The first question is whether the learning-by-doing effect remains. Typically, the mechanism design approach will first solicit information from the agent in an incentive-compatible manner, and then offer the agent some (potentially different) contract based on the agent’s truthful report. If the agent knows $\theta$ perfectly, then the principal will learn $\theta$ immediately, annihilating our learning-by-doing effect. Away from this extreme scenario, as long as there is uncertainty in $\theta$ (either because the agent does not know $\theta$ perfectly or the true $\theta$ varies over time), the principal’s learning-by-doing effect (that is orthogonal to soliciting the agent’s truthful report) remains.

Another question is whether information asymmetry leads to an ambiguous uncertainty–incentive relation. A thorough analysis of this question is unavailable. However, from another related angle, Sung (2005) allows for information asymmetry and endogenous project volatility in a setting similar to Holmstrom and Milgrom (1987) and finds that sometimes the higher the volatility, the higher the sensitivity of the contract. This effect may be complementary to our mechanism.

Third, the assumption of no agency issue in the second period is innocuous and for convenience only. As long as the period 2 managerial labor input has impact on the learning of profitability of period 3, period 2 incentives (if a moral hazard problem still persists) will share the same qualitative feature as period 1 incentives. The important assumption is that the old period 1 manager is replaced by a new manager in period 2, so that the incentive contract is short-term. With long-term employment relationship and endogenous learning, the manager can enjoy some endogenous information rent (because the manager who shirks at period 1 knows that the project actually is better than what investors believe), which makes analysis complicated. See DeMarzo and Sannikov (2011), Prat and Jovanovic (2012), or He et al. (2013).

In sum, our main mechanism goes through as long as (1) unknown profitability enters marginal labor productivity and (2) there is strictly positive cash flow noise that is not scaled with expected output. To highlight the insight, we have chosen to push these two assumptions to extremes so that $y = \theta K^{1-\lambda} L^\lambda + \epsilon$.

### 2.3. Learning and Investing in Period 2

Immediately after observing $y_1$ at period 1, investors update their belief about $\theta$. Given the optimal labor input $L_1^*$ implemented by the incentive contract at period 1, Bayes’ rule implies that the posterior of the project’s profitability is characterized by the posterior mean and posterior variance:

$$
\theta_1 = \mathbb{E}[\theta | y_1, L_1^*] = \theta_0 + \frac{y_1 K_1^{1-\lambda} (L_1^*)^\lambda}{\sigma_\epsilon^2} [y_1 - \theta_0 K_1^{1-\lambda} (L_1^*)^\lambda],
$$

$$
\gamma_1 = \text{Var}[\theta | y_1, L_1^*] = \frac{\gamma_0 \sigma_\epsilon^2}{\sigma_\epsilon^2 + \gamma_0 (K_1^{1-\lambda} (L_1^*)^\lambda)^2}.
$$

Intuitively, $y_1 - \theta_0 K_1^{1-\lambda} (L_1^*)^\lambda$ represents an unexpected shock from the output. If investors observe a positive unexpected shock $y_1 - \theta_0 K_1^{1-\lambda} (L_1^*)^\lambda > 0$, which serves a positive signal to the project profitability $\theta$, then Equation (3) says that they should update $\theta$ upwards. As we will see shortly, given period 1 output information, profitability estimate $\theta_1$ guides the firm’s investment decision at period 2; moreover, posterior variance $\gamma_1$ in Equation (4), which measures the precision of profitability estimate $\theta_1$, determines investment efficiency at period 2. Finally, posterior variance $\gamma_1$ negatively depends on $L_1^*$, thanks to the structure in Equation (1).

Without loss of generality, we set $\kappa = 1$ to simplify exposition. Solving the model backwards, at period 2 the firm makes capital investment and labor investment so that

$$
\max_{I_2, L_2} \mathbb{E} \left[ \theta(K_1 + I)^{1-\lambda} L_2^\lambda + \epsilon_2 - I - \frac{\kappa}{2K_1} I^2 - \frac{L_2^2}{2} \right],
$$

$$
= M\theta_1 + \frac{K_1}{2},
$$

where the constant $M \equiv \frac{1}{2} (\lambda / I)^\lambda (1 - \lambda)^{1-\lambda} K_1^{1-\lambda} > 0$. The investors’ period 2 value

$$
V_2(\theta_1) = M\theta_1 + \frac{K_1}{2}
$$

is a function of the period 1 posterior mean $\theta_1$. For instance, had the investors perfectly known $\theta$, they would have chosen

$$
I^* = (1 - \lambda)^{2-\lambda} (\lambda / I)^{2-\lambda} K_1^{(2-\lambda)/2} \theta - K_1
$$

$$
= (2M(1 - \lambda)K_1)^{1/2} \theta - K_1.
$$

However, due to imperfect information, they choose

$$
I^* = (2M(1 - \lambda)K_1)^{1/2} \theta - K_1,
$$

which deviates from the full-information benchmark (5).

Standing at time 0, the time 0 expected payoff from period 2 is given by

$$
\mathbb{E}[V_2(\theta_1)] = M(\gamma_0 - \gamma_1) + M\theta_0^2 + \frac{K_1}{2},
$$

which is decreasing in $\gamma_1$, the posterior variance of the unobserved profitability $\theta$. Intuitively, the lower the...
posterior variance \( \gamma_i \), the more precise the estimate of \( \theta \), and the more efficient the second period investment. Moreover, from Equation (4), \( \gamma_i \) decreases with effort \( L_i^* \). This decrease implies that, raising incentive \( \beta_i \) in period 1 improves the information content of period 1 output \( y_i \), and hence investors learn more about \( \theta \).

### 2.4. Optimal Contracting in Period 1

We now solve for the optimal linear contract in period 1. Here, investors offer a linear contract \( w_i = \alpha + \beta y_i \) to implement the optimal labor (effort) \( L_i^* \), and the optimal contract maximizes their expected total value (including both periods’ payoffs),

\[
\max_{\alpha, \beta, L_i^*} \mathbb{E}[y_i - w_i - K_i + V_2(\theta_i)], \tag{7}
\]

subject to the manager’s incentive compatibility and participation constraints:

\[
L_i^* = \arg\max_{L_i} \mathbb{E}\left[-\exp\left(-\alpha \left(w_i - \frac{1}{2} L_i^2\right)\right)\right] \quad \text{and} \quad \mathbb{E}\left[-\exp\left(-\alpha \left(w_i - \frac{1}{2} L_i^2\right)\right)\right] \geq \hat{U}.
\]

The following lemma gives the manager’s optimal labor (effort) input.

**Lemma 1.** A contract \( w_i = \alpha + \beta y_i \) implements labor \( L_i^* \) and satisfies the manager’s participation constraint if and only if \( L_1^* \) uniquely solves

\[
\lambda \beta \theta_i K_i^{1-\lambda} - \frac{1}{2} L_i^2 - a \gamma_0 \lambda \beta^2 K_i^{2(1-\lambda)} L_i^* = 0 \tag{8}
\]

and

\[
\alpha = -\lambda \beta \theta_i K_i^{1-\lambda} (L_i^*)^\lambda + \frac{1}{2} L_i^2 \\
+ \frac{1}{2} a \beta^2 (\gamma_0 K_i^{2(1-\lambda)} (L_i^*)^\lambda + \sigma_e^2). \tag{9}
\]

Essentially, Lemma 1 establishes an important link between implemented labor \( L_i^* \) and incentive loadings \( \beta \) in any incentive-compatible contracts, which allows the firm to choose implemented \( L_i^* \) to maximize its value function. In light of Lemma 1, we can replace the incentive compatibility and participation constraints in the investors’ problem by Equation (8) and Equation (9). Together with Equations (3), (4), and (6), we can rewrite the investors’ problem in Equation (7) (for details, see the proof of Lemma 1 in Appendix A) as

\[
L_i^* \in \arg\max_{L_i} \left[ \theta_i K_i^{1-\lambda} L_i^2 - \frac{1}{2} a \beta^2 (\gamma_0 K_i^{2(1-\lambda)} L_i^2) \right] \\
+ M \frac{\gamma_0 K_i^{2(1-\lambda)} L_i^2}{\sigma_e^2 + \gamma_0 K_i^{2(1-\lambda)} L_i^2} \\
s.t. \ 0 = -\lambda \beta \theta_i K_i^{1-\lambda} - \frac{1}{2} L_i^2 - a \gamma_0 \lambda \beta^2 K_i^{2(1-\lambda)} L_i^*.
\]

The first term in the investors’ value function is expected period 1 output, the second term is labor cost, the third term is the manager’s risk compensation, and the last term is the firm’s period 2 payoff. Once we derive the optimal effort level \( L_i^* \), the optimal contract (i.e., \( \alpha^* \) and \( \beta^* \)) is fully determined by Equations (8) and (9).

### 2.5. Positive Incentive–Uncertainty Relation

In our model, learning could induce a positive relation between incentives and uncertainty. This result is rooted in the fact that investors’ expected value of period 2 value, \( \mathbb{E}_0[V_2(\theta_i)] \), depends on learning about profitability \( \theta \) from period 1 output \( y_i \). As indicated by Equation (6), maximizing \( \mathbb{E}_0[V_2(\theta_i)] \) is equivalent to minimizing the posterior variance of \( \theta \), i.e., \( \gamma_i \). Because \( L_i^* \) is multiplicative with \( \theta \) in signal \( y_i \) as in Equation (1), implementing a higher effort \( L_i^* \) raises the informativeness of the period 1 signal \( y_i \) or, equivalently, reduces the posterior variance \( \gamma_i \). Essentially, this mechanism shares a spirit similar to the learning-by-doing literature. For example, Johnson (2007) shows that when return-to-scale in firm’s production function is unknown in advance, overinvestment relative to the full-information case becomes optimal, as overinvestment expedites learning about the unknown production function.

Presumably, this learning-by-doing effect is stronger in a more uncertain environment (i.e., a larger \( \gamma_0 \)). The effect is stronger because starting with a larger initial uncertainty \( \gamma_0 \), the reduction of the posterior variance will be more significant, which results in a greater benefit of inducing a higher effort; that is, based on Equation (4), we have

\[
\frac{\partial^2 (-\gamma_i)}{\partial L_i^2} > 0.
\]

In Figure 2, we plot \(-\gamma_i\) as a function of effort \( L_i^* \) for different levels of \( \gamma_0 \). As we can see, when \( \gamma_0 \) increases, the marginal benefit of raising effort \( L_i^* \) becomes greater. To implement a higher effort, a greater incentive \( \beta^* \) is needed, which results in a positive relation between uncertainty and incentives.

In Proposition 2 we formally prove the existence of such a positive uncertainty–incentive relation when the manager is sufficiently risk tolerant. Note that higher uncertainty also implies that the manager is bearing larger output volatility, and hence a higher incentive provision cost. Therefore, for the positive uncertainty–incentive relation to hold, the manager needs to be sufficiently risk tolerant so that the learning-by-doing effect is dominant.

**Proposition 2.** For sufficiently small risk-aversion coefficient \( a \), a positive relation exists between \( \beta^* \) and \( \gamma_0 \), i.e., \( d\beta^*/d\gamma_0 > 0 \).
Given that CEOs are relatively wealthy, it is reasonable to choose with the qualitative implications of our model on the relationship between uncertainty and incentives under realistic parameterizations.

Panel D shows the traditional negative trade-off between risk $\sigma_e^2$ and incentives $\beta^*$. In contrast, as predicted by Proposition 2, panel A shows a positive relation between profitability uncertainty $\gamma_0$ and incentive $\beta^*$ when the manager is relatively risk tolerant. Of course, uncertainty also raises the perceived volatility of output. When risk aversion is relatively high as in panel C, the traditional negative risk-incentive effects dominate, leading to a negative relation between incentives and uncertainty.

We observe another interesting result in panel B with $a = 0.5$. Here, because of the learning-by-doing effect, even the traditional risk–incentive relation becomes hump shaped. Notice that investors would like to reduce the posterior variance $\gamma_1$ in Equation (4), and $\partial(-\gamma_1)/\partial \gamma_1^*$ can be viewed as the marginal benefit of expediting learning through raising effort. The higher $\partial(-\gamma_1)/\partial \gamma_1^*$, the greater the incentive $\beta^*$ that investors would like to offer. Linking this benefit to output risk $\sigma_e^2$, in Appendix A we show that $\partial(\gamma_1)/\partial \gamma_1^*$, which explains the nonmonotone incentive-risk relation in panel B. This intuition is rooted in the fact that a higher $\sigma_e^2$ implies a lower information-noise ratio. When $\sigma_e^2 \geq \gamma_0 \eta K_1^{2(1-\lambda)}(L_1)^{2\lambda}$ so that we are on the right-hand side of the hump shape in panel B, the information-noise ratio is low, and there is plenty of room for learning. Here, the marginal benefit of expediting learning is positively related to the information-to-noise ratio. Hence, a greater $\sigma_e^2$ lowers the marginal benefit of learning $\partial(-\gamma_1)/\partial \gamma_1^*$, and consequently investors offer a lower-powered incentive contract. On the left-hand side of the hump shape where $\sigma_e^2 < \gamma_0 K_1^{2(1-\lambda)}(L_1)^{2\lambda}$, the opposite holds. This is because the information-noise ratio is already high and investors have learned a great deal about $\theta_1$ and a higher $\sigma_e^2$ lowers the information-to-noise ratio. This increases the room to learn, leading to a greater marginal benefit from learning. Taken together, panel B shows that a potential positive risk–incentive relation due to learning may overturn the traditional negative risk–incentive trade-off when the manager is sufficiently risk tolerant.

In sum, in addition to the leading alternative explanations surveyed in the introduction, our model provides another plausible explanation for why it is difficult to identify a negative risk–incentive trade-off in the data. According to our model, there could be two reasons. First, we might have a positive relation between uncertainty and incentives for small risk-aversion coefficients (panel A), and existing empirical analysis does not distinguish uncertainty from risk. Second, even if we can identify risk from uncertainty, with learning there is not necessarily a clear-cut relation between risk and incentives (panel B).

3. Empirical Analysis

In this section, we empirically test the prediction of a positive relation between uncertainty and incentives. We also investigate how this positive relation affects the traditional trade-off between risk and incentives. In §3.1, we describe our data, incentive and risk measures, and profitability uncertainty proxies. We then provide regression results in §3.2.

3.1. Data, Variables, and Summary Statistics

3.1.1. Data and Sample Selection. Our sample consists of a manager–firm matched panel data set...
3.1.2. Pay–Performance Sensitivity. The dependent variable in the paper is PPS, a standard variable used in the literature to measure managerial incentives. There are three PPS measures in the executive compensation literature. The first measure, dollar-to-dollar measure (PPS1) (Jensen and Murphy 1990), is equal to the dollar change in stock and option holdings for a one dollar change in firm value (see also Demsetz and Lehn 1985, Yermack 1995, Schaefer 1998, Palia 2001, Jin 2002, Aggarwal and Samwick 2003). This measure is essentially $\partial Wealth/\partial (Firm\_Value)$ (where Wealth is the chief executive officer’s (CEO)’s wealth) and is also called value-sensitivity or share of the money in Becker (2006). The second measure, dollar-to-percentage measure (PPS2) (Hall and Liebman 1998), is equal to the dollar change in stock and option holdings for a 1% change in firm value (see also Holmstrom 1992, Core and Guay 2002). The PPS2 measure is equal to $\partial Wealth/\partial \ln(Firm\_Value)$ and is also referred to as return sensitivity or money at stake by Becker (2006). The third measure, scaled wealth–performance sensitivity measure (PPS3) (Edmans et al. 2009), is equal to PPS2 divided by TDC1, where TDC1 is the total compensation of an executive.\(^6\) This incentive measure is similar to the percentage-to-percentage incentives from 1992 to 2008. This data set allows us to track the highest-paid executives of firms covered by ExecuComp through time. We merge the manager-level ExecuComp data with the firm-level annual accounting variables from Compustat, stock returns from the Center for Research in Security Prices (CRSP), corporate board information from RiskMetrics, and analyst forecast information from the Institutional Brokers’ Estimate System (I/B/E/S). We then remove the observations with incomplete data. We also winsorize the continuous variables that present obvious outliers by replacing the extreme values with the 1 and 99 percentile values. The main regressions are estimated based on our full sample, which includes 2,441 firms and 25,999 top executives.

\(^6\)The values of PPS3 for each individual executive are available from Alex Edmans’ website. We thank Alex Edmans for kindly sharing his data.

Notes. Parameters are as follows: \(l = 1.6, \kappa = 1, \theta_0 = 1, \lambda = 0.67, \) and \(K_r = 0.28.\) In panel A, we set \(a = 0.5, \sigma_r = 0.2, \) and \(\gamma_0 \in [0.2, 0.3].\) In panel B, we set \(a = 0.5, \gamma_0 = 0.25, \) and \(\sigma_r \in [0.05, 0.15].\) In panel C, we set \(a = 5, \sigma_r = 0.2, \) and \(\gamma_0 \in [0.2, 0.3].\) In panel D, we set \(a = 5, \gamma_0 = 0.25, \) and \(\sigma_r \in [0.05, 0.15].\)
The literature has proposed various explanations for this pattern, and therefore size may not be a clean profitability uncertainty variable for our purpose. For instance, in the Holmstrom (1989), Murphy (1985), Gibbons and Murphy (1992), Rosen (1992), and Peng and Röell (2008), but replaces flow compensation in the numerator of the Murphy (1985) measure with the change in the executives’ wealth.

3.1.3. Empirical Proxies for Profitability Uncertainty. Despite a large literature studying the effect of parameter uncertainty on asset prices and investment (see Pastor and Veronesi 2009 for a recent survey), separating uncertainty from risk is empirically challenging. In the existing literature, most of the studies (e.g., Pastor and Veronesi 2003, Pastor et al. 2009, Korteweg and Polson 2010) use imperfect proxies to test model implications. Following their footsteps, we use five profitability uncertainty proxies in our study. These proxies have been used in the existing literature; for detailed definitions of these proxies, see Appendix B. We do not use firm size as an uncertainty proxy, although it is proposed by such literature as Korteweg and Polson (2010). There exists a strong empirical relation between size and PPS; that is, firm size is negatively correlated with PPS1 and positively correlated with PPS2 (e.g., Edmans et al. 2009). We do, however, include firm size and (size)$^2$ as control variables in all of our regressions to capture the (potentially nonlinear) size effect.

Natural log of firm age. The first proxy that we employ is firm age. Previous studies such as Pastor and Veronesi (2003) and Korteweg and Polson (2010) use firm age as a proxy for profitability uncertainty. Uncertainty declines over a firm’s lifetime due to learning, and younger firms have higher uncertainty. Following Pastor and Veronesi (2003), we consider each firm as “born” in the year of its first appearance in the CRSP database. Specifically, we obtain the first occurrence of a valid stock price on CRSP, as well as the first occurrence of a valid market value in the CRSP/Compustat database, and take the earlier of the two. The firm’s age is assigned the value of one in the year in which the firm is born and increases by one in each subsequent year. As in Pastor and Veronesi (2003), we take the natural log of firm age. Log(Firm age) is concave in a firm’s plain age, and captures the idea that, regarding uncertainty, one year of age should matter more for young firms than for old firms.

Earnings response coefficient. We follow Pastor et al. (2009) and Cremers and Yan (2012) to use the stock price reaction to earnings announcements (i.e., earnings response coefficient). More specifically, ERC is the average of a firm’s previous 12 stock price reactions to quarterly earnings surprises. Intuitively, investors who are more uncertain about the profitability of a company should respond more strongly to earnings surprises. As noted in Pastor et al. (2009), the ERC measure is ideal to separate uncertainty from volatility because ERC is high when uncertainty is high and earnings volatility is low. When realized earnings are more precise, investors react more to earnings surprises, leading to a higher value of ERC. The shortcoming of the ERC measure is its measurement error. As a result, we also incorporate other empirical proxies of uncertainty in the analysis.

Market-to-book ratio. The third proxy for profitability uncertainty is the market-to-book ratio, which equals market value of equity plus the book value of debt, divided by total assets. Pastor and Veronesi (2003) show that aging in the life of a firm is accompanied by a decrease in the market-to-book ratio. According to Korteweg and Polson (2010), the market-to-book ratio is a proxy for firm growth opportunities, and such opportunities are inherently more difficult to value than the assets in place. As a result, the market-to-book ratio increases with uncertainty about firm profitability.

Tangibility. The fourth proxy is tangibility. Korteweg and Polson (2010) mention that firms with more tangible assets (property, plant, and equipment) are easier to value and thus are related to lower profitability uncertainty. We use net property, plant, and equipment scaled by firm total assets to measure tangibility.

Analyst forecast error. We also construct an analyst forecast error variable as a proxy of profitability uncertainty. Based on Bae et al. (2008) and Lang and

7 The literature has proposed various explanations for this pattern, and therefore size may not be a clean profitability uncertainty variable for our purpose. For instance, in the Holmstrom and Milgrom’s (1987) CARA-Normal framework, risk is measured in dollar returns. Then dollar-to-dollar PPS1 should be lower for larger firms with greater dollar variances in output. For the dollar-to-percentage PPS2 measure, the matching model in Gabaix and Landier (2008) suggests that pay increases with firm size. Since part of compensation is in variable pay, it suggests that PPS2 is positively correlated with firm size.

8 We also decide not to use some other uncertainty proxies found in the literature. Baker and Wurgler (2006) provide some proxies for hard-to-value stocks. Besides the variables we mention above, they mention that non-dividend-paying stocks are harder to value than dividend-paying stocks because the value of a firm with stable dividends is less subjective. As a result, dividend-paying firms possibly have lower uncertainty, and thus may be related to lower incentives. Our regressions control for dividend-paying indicator and do observe a consistent negative association between the dividend-paying indicator and PPS. An alternative explanation of the negative association is that firms with cash constraints (such as non-dividend-paying companies) might prefer restricted stock and options over cash compensation. As a result, a higher PPS is more likely to be observed among non-dividend payers (Jin 2002, Yermack 1995).

9 Pastor et al. (2009) also use a second ERC measure that is the negative of the regression slope of the firm’s last 20 quarterly earnings surprises on its abnormal stock returns around earnings announcements. We report in this paper the results from using the ERC1 variable. The results from the ERC2 variable are similar and available upon request.
Lundholm (1996), for each specific company in each fiscal year, we first obtain the absolute value of the forecast error made by each analyst, where forecast errors are defined as the difference between the forecast value and the actual value of earnings per share. We then use the median value of these absolute forecast errors, scaled by the absolute value of the actual earnings per share (EPS). Using the mean value of the absolute forecast errors gives similar results.\footnote{Another widely used measure based on I/B/E/S data is analysts’ forecast dispersion, which usually proxies for potential disagreement in the market. The difference between forecast dispersion and forecast error is that the latter considers the distance between EPS forecast and actual EPS, whereas the former considers the distance between EPS forecast and the mean forecast among analysts. The forecast error variable better captures profitability uncertainty studied in this paper. Consider the situation where two analysts issued the same EPS forecast of $5, and the actual EPS turns out to be $3. Then, in this example the forecast error will be 2 (which might result from large uncertainty), but the forecast dispersion is just 0.}

We end this section by pointing out that uncertainty is hard to measure and could be endogenous. We use five different proxies for uncertainty, hoping that establishing similar results for all of them can raise hurdles for other alternative explanations. Unfortunately, the five proxies we use can be all linked to firm growth. Fast-growing firms have higher marginal benefit of managerial effort and thus should have higher-powered incentives, which can also explain the positive uncertainty–incentive relation.\footnote{We thank an anonymous referee for this excellent point.} To address this issue at least partially, our control variables include the long-term earnings growth forecast from analysts, which gives a more precise measure of firm growth (relative to our five uncertainty proxies). Indeed, in the regressions, the coefficient on long-term earnings growth forecast is always significantly positive, suggesting the validity of this alternative mechanism.

### 3.1.4. The Risk Variable

Similar to the literature that tests the risk–incentive relation, we take stock return volatility as a measure of risk in our regression analysis. We measure stock return volatility as the standard deviation of daily log (percentage) returns over the past five years, which is then annualized by multiplying by the square root of 254 (Yermack 1995, Palia 2001). We acknowledge that this proxy for firm risk may be imperfect and can also capture profitability uncertainty. We also use the percentage rank of stock dollar return variance (Aggarwal and Samwick 1999, 2002, 2003; Garvey and Milbourn 2003; Jin 2002) in the empirical analysis, but obtain essentially the same results.

### 3.1.5. Control Variables

In the regressions, we include various control variables that could potentially affect the incentives a firm provides to its managers; see detailed definitions of all of the following variables in Appendix B. These control variables have been used in the empirical literature on the determinants of managerial incentives (Aggarwal and Samwick 2003, Core et al. 1999, Jin 2002, Palia 2001, etc.). As mentioned at the beginning of §3.1.3, since there is a well-established empirical pattern between incentives and firm size, we first include firm size and the square of firm size as controls. Following the literature, we also include profitability, the ratio of capital expenditure to total assets, advertising expenses scaled by total assets, a dummy variable that is set to one whenever advertising expenses are missing, firm leverage, and dividend payout indicator. We further control for corporate governance variables, which include the CEO chair indicator and the proportion of inside directors on the board. Manager-level variables, such as log(Tenure), the CEO indicator, and the female indicator, are also controlled in the regressions. Finally, year and industry effects are included to capture the time and industrial differences in the level of managerial incentives.

### 3.1.6. Summary Statistics and Correlations Between Variables

Table 1 contains summary statistics of the variables used in the regression analysis. For instance, the average (median) dollar-to-dollar measure of PPS1 is 1.13% (0.22%), suggesting that the average (median) dollar change in the sample executives’ stock and option holdings for a one thousand dollar change in firm value is $11.3 ($2.2). These summary numbers are consistent with those provided in the empirical literature such as in Core and Guay (1999), Palia (2001), and Yermack (1995). The statistics also imply a positive skewness in PPS, with a few companies having very high incentives.

The average, median, minimum, and maximum age of the sample firms are 26, 20, 1, and 84 years, respectively, similar to those reported in Pastor and Veronesi (2003). The firms in the sample have an average (median) earnings response coefficient of 4.44 (2.88), market-to-book ratio of 2.08 (1.51), tangibility of 0.29 (0.23), and total assets of $6.6 ($1.3) billion. The average analyst forecast error relative to the actual value is about 16%. In addition, the average (median) annual stock return volatility is 44% (39%).

Table 2 examines the pairwise correlations between the variables. Not surprisingly, the three PPS variables are positively correlated; the correlation coefficient between the dollar-to-dollar PPS1 and the dollar-to-return PPS2 is 0.55, and PPS1 (PPS2) is correlated with PPS3 at 0.21 (0.25). The PPS variables are in general negatively correlated with firm age and tangibility, and are positively correlated with the earnings response coefficient and the market-to-book ratio. The correlations between PPS2 and firm age are very low. The low correlations may be due to the fact that PPS2...
is PPS1 multiplied by market value of equity, and the negative relation between age and PPS1 is canceled out by the positive relation between age and market value. When we control for firm size in the model, the relation between PPS2 and firm age becomes negative and significant. PPS3 has a very low correlation (−0.03) with firm size, consistent with the property mentioned in Edmans et al. (2009) that the PPS3 measure is independent of firm size.

Table 2 also shows that the uncertainty proxy variables are correlated with each other, with the correlation between firm age and market to book being −0.23 and the correlation between firm age and tangibility being around 0.18. These correlations indicate that younger firms tend to be firms with more growth options and lower tangibility ratios. The table also reveals very low correlations between ERC and volatilities and between ERC and firm size, suggesting that ERC serves an ideal proxy variable that separates uncertainty from volatility and firm size. In contrast, the percentage return and dollar return volatilities have opposite signs in correlations

| Table 2 | Pairwise Correlations Between Variables |
|------------------|------------------|------------------|------------------|
|                  | Incentives       | Profitability uncertainty | Risk |
|                  | PPS1 | PPS2 | PPS3 | Age | ERC | M/B | Tang | Forerr | Vol | Dolvol | Size |
| PPS1             | 0.55 | 1    |      |      |      |      |      |      |      |      |      |
| PPS2             | 0.21 | 0.25 | 1    |      |      |      |      |      |      |      |      |
| Wealth–performance sensitivity (PPS3) |      |      |      |      |      |      |      |      |      |      |      |
| Log(Firm age) (Age) | −0.16 | 0.003 | −0.10 | 1    |      |      |      |      |      |      |      |
| Earnings response coefficient (ERC) | 0.04 | 0.05 | 0.07 | −0.06 | 1    |      |      |      |      |      |      |
| Market-to-book (M/B) | 0.08 | 0.20 | 0.19 | −0.23 | 0.07 | 1    |      |      |      |      |      |
| Tangibility (Tang) | −0.05 | −0.08 | −0.03 | 0.18 | −0.06 | −0.12 | 1    |      |      |      |      |
| Analyst forecast error (Forerr) | 0.02 | −0.04 | −0.01 | −0.04 | −0.05 | −0.04 | 0.01 | 1    |      |      |      |
| Stock return volatility (Vol) | 0.10 | −0.03 | 0.03 | −0.44 | −0.04 | 0.23 | −0.22 | 0.13 | 1    |      |      |
| Rank of dollar return volatility (Dolvol) | −0.14 | 0.29 | 0.09 | 0.23 | 0.01 | 0.16 | −0.06 | −0.12 | −0.12 | 1    |      |
| Firm size (Size) | −0.20 | 0.19 | −0.03 | 0.44 | −0.01 | −0.24 | 0.03 | −0.10 | −0.48 | 0.73 | 1    |
| Long-term growth forecast | 0.11 | 0.06 | 0.12 | −0.38 | 0.06 | 0.39 | −0.17 | 0.02 | 0.45 | −0.04 | −0.37 |

Note. Detailed definitions of the variables are in Appendix B.
with other variables. This is perhaps due to the fact that the dollar return volatility, which equals percentage return volatility multiplied by firm market value, captures the firm size effect.

3.2. Empirical Results
This section uses regression analysis to examine the effect of profitability uncertainty and risk on incentives. The main empirical model is as follows:

$$PPS_{ijt} = \alpha + \beta_1(Uncertainty\ proxies)_{ij,t-1} + \beta_2(Risk)_{ij,t-1} + \beta_3(Firm\ characteristics)_{ij,t-1} + \beta_4(Managerial\ characteristics)_{ij,t-1} + \beta_5(Year\ dummies)_{t} + \beta_6(Industry\ dummies)_{j} + \epsilon_{ijt}.$$  

(10)

In the equation, we use $i$ to denote manager, $j$ to denote firm, and $t$ to denote year. The dependent variable is pay–performance sensitivities. In the ordinary least squares (OLS) regressions, we control for industry effects using two-digit Standard Industrial Classification (SIC) indicator variables. In the firm–manager pair fixed effects regressions, we replace industry effects with firm–manager fixed effects in Equation (10), as the latter absorbs the former. We lag all the explanatory variables by one year to mitigate potential reverse causality issues, and later use the fixed effects model in robustness analysis to deal with the endogeneity problem caused by time-invariant unobservable factors. We acknowledge that lagging may not fully resolve endogeneity because serial correlations may exist in some uncertainty proxies (some of our proxies may be forward looking). We also note that the fixed effects model cannot deal with timevariant unobservable factors.

3.2.1. Main Results. Tables 3–5 report the OLS regression results, with each table having different PPS dependent variables. The $t$-statistics in these regressions are heteroskedasticity robust and are adjusted for clustering within firms. In all three tables, column (1) does not include any of the five uncertainty variables, columns (2)–(6) include one of the five uncertainty variables, and column (7) includes all five uncertainty variables.

Positive uncertainty–incentive relation. The results in Tables 3–5 show that firm age is negatively related to incentives (columns (2) and (7)), indicating that younger firms, i.e., firms with higher uncertainty, are associated with greater managerial incentives. Both the ERC and the market-to-book ratio are positively associated with the incentive variables in most regressions. The relation between tangibility and PPS is generally negative, suggesting that firms that have more tangible assets are associated with lower incentives. Firms with greater analyst forecast errors (that might be due to greater uncertainty) are weakly related to higher incentives.

All of these results indicate a positive relation between profitability uncertainty and incentives, consistent with our model when the manager’s risk aversion is relatively low. This positive relation is not only statistically significant but also economically important. Take column (7) in Tables 3–5 as examples. A one-standard-deviation decrease in log(Firm age), which is about 0.97 (i.e., firm age reduces by about three years), is associated with an increase of approximately 0.23% ($=0.97 \times 0.24$) in PPS1, 34.09 ($=0.97 \times 35.14$) in PPS2, and 11.72 ($=0.97 \times 12.08$) in PPS3. These increases in PPS are of similar magnitude to those of the median values of PPS. Other uncertainty variables have similar economic significance.

Reexamining the risk–incentive relation. The negative risk–incentive relation is a key prediction from standard agency theories, but with mixed empirical support from existing literature. From the point of view of this paper, the risk proxies used in the previous literature, namely, stock volatility and rank of dollar return volatility, could be contaminated by profitability uncertainty. If profitability uncertainty is positively related to incentives, then it is not surprising that previous research, in which the risk proxy captures both the cash flow risk $\sigma^2$ and the profitability uncertainty $\gamma_0$, finds an ambiguous risk–incentive relation.

The above reasoning suggests that in revealing the negative risk–incentive relation, it is important to control for uncertainty, because it helps correct for the positive bias potentially caused by omitting relevant variables that are proxies for profitability uncertainty. Our empirical results offer evidence for this implication. Compared with the specification that does not include the uncertainty proxies (i.e., columns (1) of Tables 3–5), when we include the uncertainty variables in the regressions (columns (7) of Tables 3–5), the relation between volatility and incentives becomes less positive or more negative. This pattern generally holds in other specifications considered in §3.2.2 for robustness checks.

Although our results do not fully restore the significantly negative risk–incentive relation from the data (possibly because of such reasons as endogenous matching between firm risk and CEO’s risk appetite, the learning-by-doing effect in panel D of Figure 3 of this paper, etc.), it should be safe to say that separating profitability uncertainty from cash flow risk is important when empirically examining the negative risk–incentive relation. Our results also indicate that it may be important to separate the effect
of profitability uncertainty from that of risk in other empirical studies.

### 3.2.2. Robustness Analysis

This section performs additional analysis to investigate the robustness of our empirical results.

Risk measured as dollar return volatility. In addition to measuring firm risk using the variance of stock percentage returns, we attempt to use a different measure of firm risk: volatility of stock dollar returns. Following Aggarwal and Samwick (1999, 2003) and Jin (2002), we use the percentage rank of the variance measured as dollar return volatility.
of dollar returns\footnote{According to Aggarwal and Samwick (1999, 2003), the use of the percentage ranks deals with potential outliers in the dollar return data and also allows the pay-performance incentives at different points in the distribution of firm risk to be easily compared. In the regressions, we also use an alternative transformation of the raw dollar return variance, namely, the logarithm of dollar return variance, and we find basically the same results.} and report results in panel A of Table 6. In column (1), we find that the rank of dollar return volatility is negative and significant, consistent with Aggarwal and Samwick (1999, 2002, 2003), Garvey and Milbourn (2003), and Jin (2002). In column (2), we include the uncertainty variables and find that greater profitability uncertainty is related to higher incentives. Moreover, the dollar return volatility (i.e., the risk proxy) continues to be negative and significant after including uncertainty variables. In columns (3)–(6), in which PPS2 and PPS3 are dependent variables, we continue to find that firms with greater uncertainty provide higher incentives to their executives. The effect of the risk variable is positive and significant when the uncertainty variables are

<table>
<thead>
<tr>
<th>Table 4</th>
<th>OLS Regression Results on the Effects of Profitability Uncertainty and Risk on Incentives (PPS2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: PPS2</td>
<td>(1)</td>
</tr>
<tr>
<td>Profitability uncertainty variables</td>
<td></td>
</tr>
<tr>
<td>Log(Firm age) (−)</td>
<td>—</td>
</tr>
<tr>
<td>ERC (+)</td>
<td>—</td>
</tr>
<tr>
<td>Market-to-book (+)</td>
<td>—</td>
</tr>
<tr>
<td>Tangibility (−)</td>
<td>—</td>
</tr>
<tr>
<td>Analyst forecast error (+)</td>
<td>—</td>
</tr>
<tr>
<td>Risk variable</td>
<td></td>
</tr>
<tr>
<td>Stock return volatility</td>
<td>−6.59</td>
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<tr>
<td>(−0.22)</td>
<td>(−1.23)</td>
</tr>
<tr>
<td>Control variables</td>
<td></td>
</tr>
<tr>
<td>Firm size</td>
<td>93.35***</td>
</tr>
<tr>
<td>Squared firm size</td>
<td>11.04***</td>
</tr>
<tr>
<td>(6.66)</td>
<td>(6.58)</td>
</tr>
<tr>
<td>Long-term growth forecast</td>
<td>8.39***</td>
</tr>
<tr>
<td>Profitability</td>
<td>294.99***</td>
</tr>
<tr>
<td>(6.05)</td>
<td>(5.74)</td>
</tr>
<tr>
<td>Capital expenditure</td>
<td>205.51***</td>
</tr>
<tr>
<td>(3.09)</td>
<td>(2.78)</td>
</tr>
<tr>
<td>Advertisement</td>
<td>584.96***</td>
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<tr>
<td>(2.99)</td>
<td>(3.07)</td>
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<tr>
<td>Advertisement missing indicator</td>
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<tr>
<td>(0.30)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>Leverage</td>
<td>−144.48***</td>
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<tr>
<td>(−5.22)</td>
<td>(−5.36)</td>
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<tr>
<td>Dividend-paying indicator</td>
<td>−43.39***</td>
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<tr>
<td>(−4.10)</td>
<td>(−2.99)</td>
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<tr>
<td>CEO chair indicator</td>
<td>20.04***</td>
</tr>
<tr>
<td>(2.84)</td>
<td>(2.89)</td>
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<tr>
<td>Fraction of inside directors</td>
<td>318.06***</td>
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<tr>
<td>(9.79)</td>
<td>(9.69)</td>
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<tr>
<td>Log(Tenure)</td>
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<tr>
<td>CEO indicator</td>
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<td>(32.33)</td>
<td>(32.33)</td>
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<td>Female indicator</td>
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<td>(−4.88)</td>
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<tr>
<td>Number of observations</td>
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</tr>
</tbody>
</table>

Notes. The dependent variable is the dollar-to-percentage measure (PPS2) of pay-performance sensitivity. Other information is the same as that in Table 3. *Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.
**Table 5** OLS Regression Results on the Effect of Profitability Uncertainty and Risk on Incentives (PPS3)

<table>
<thead>
<tr>
<th>Dependent variable: PPS3</th>
<th>(1)</th>
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<td>Yes</td>
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<td>116,115</td>
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Notes: The dependent variable is the percentage-to-percentage measure (i.e., wealth-performance sensitivity or PPS3) proposed by Edmans et al. (2009). In the regressions, PPS3 is winsorized at 99% to deal with outliers. Other information is the same as that in Table 3.

*Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.

excluded, but the effect becomes insignificant when the uncertainty variables are introduced to the model.

Median regressions. Following Aggarwal and Samwick (1999, 2003) and Jin (2002), we use median regressions to deal with outliers and right skewness in the compensation data. Results are reported in panel B (with risk measured by the percentage return volatility) and panel C (with risk measured by the rank of dollar return volatility) of Table 6. Both tables show that, in general, uncertainty is positively related to incentives. The coefficient on the risk variable becomes less positive or more negative if profitability uncertainty is captured in the model.

Fixed effects regressions. In panel D of Table 6, we deal with potential endogeneity issues by adding the firm–manager paired fixed effects in the regressions. For example, it is possible that some unobservable managerial attributes (e.g., risk aversions)
are correlated with the explanatory variables, such as firm age, and at the same time are correlated with the dependent variable, PPS. The firm–manager fixed effects may also capture time-invariant unobservable factors that potentially affect endogenous matching between the firm and the manager (Graham et al. 2012). We can see from panel D of Table 6 that the coefficients on the profitability uncertainty proxies continue to show a positive relation between profitability uncertainty and incentives.

Admittedly, the fixed effects specification can only address the potential endogeneity problem due to time-invariant omitted variables. Fixed effects cannot address the time-variant omitted variables, nor the reverse causality problem, where some of our proxies of uncertainty (e.g., market-to-book ratio) are forward looking and thus respond to tomorrow’s pay-performance sensitivity (recall that we have lagged uncertainty proxies by one year in regression).

Other robustness checks. Finally, the tables reported so far examine each top executive’s incentives. In an untabulated analysis, we also examine CEO incentives only, non-CEO incentives, and the average incentives for top executives in each individual company. We also examine the incentives from stock and options, separately. The results, omitted for brevity, provide the same implications as those reported here.

In addition, Pastor and Veronesi (2003) find that the market-to-book ratio increases with uncertainty about average profitability, especially for firms that pay no dividends. We interact the dividend-paying dummy with the uncertainty proxy variables and run regressions with interaction variables. The coefficients of the interaction variables are not significant, suggesting

Table 6: Robustness Analysis Results on the Effects of Profitability Uncertainty and Risk on Incentives

<table>
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<th>Dependent variable: PPS</th>
<th>Expected sign</th>
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<th>(2) PPS1</th>
<th>(3) PPS2</th>
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<td>–</td>
<td>–</td>
<td>–</td>
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<td>+</td>
<td>+</td>
<td>+</td>
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<td>+</td>
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<tr>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<tr>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>Yes</td>
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<td>Yes</td>
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that the positive relation between uncertainty and incentives does not vary significantly between firms that pay dividends and firms that do not.

In all, the empirical results that we obtain offer preliminary support to our theoretical prediction that profitability uncertainty is positively related to incentives.

4. Conclusion

This paper introduces profitability uncertainty into an agency model and investigates the relation between profitability uncertainty and incentives. Our model predicts a positive uncertainty–incentive relation, in contrast to the negative risk–incentive trade-off obtained in the extant literature. Using several proxies for profitability uncertainty, we find empirically that the data seem to be consistent with our theoretical prediction. Our analysis suggests that controlling for uncertainties helps partially to restore the negative risk–incentive relation predicted by standard agency theories. We acknowledge several limitations in our empirical analysis. Because of these limitations, the empirical results in this paper are suggestive rather than conclusive.

Table 6 (Continued)

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<th>Expected sign</th>
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<th>(2) PPS1</th>
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<td>80,642</td>
<td>92,424</td>
<td>80,425</td>
<td></td>
</tr>
<tr>
<td>Profitability uncertainty variables</td>
<td>Panel D: Fixed effects regressions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log(Firm age)</td>
<td>–</td>
<td>–</td>
<td>–0.42***</td>
<td>–</td>
<td>–71.50***</td>
<td>–</td>
<td>–29.06***</td>
</tr>
<tr>
<td>ERC</td>
<td>+</td>
<td>–</td>
<td>0.003***</td>
<td>–</td>
<td>0.20</td>
<td>–</td>
<td>0.25***</td>
</tr>
<tr>
<td>Market-to-book</td>
<td>+</td>
<td>–</td>
<td>–0.0001</td>
<td>–</td>
<td>54.85***</td>
<td>–</td>
<td>4.36***</td>
</tr>
<tr>
<td>Tangibility</td>
<td>–</td>
<td>–</td>
<td>–0.60***</td>
<td>–</td>
<td>13.57</td>
<td>–</td>
<td>6.25</td>
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<tr>
<td>Analyst forecast error</td>
<td>+</td>
<td>–</td>
<td>–0.01</td>
<td>–</td>
<td>1.68</td>
<td>–</td>
<td>–0.49</td>
</tr>
<tr>
<td>Risk variable</td>
<td>Stock return volatility</td>
<td>–</td>
<td>–</td>
<td>–0.52***</td>
<td>–</td>
<td>–150.40***</td>
<td>–</td>
</tr>
<tr>
<td>Control variables, year dummies, and firm–manager paired fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.81</td>
<td>0.82</td>
<td>0.72</td>
<td>0.74</td>
<td>0.69</td>
<td>0.70</td>
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<tr>
<td>Number of observations</td>
<td>119,365</td>
<td>100,835</td>
<td>119,365</td>
<td>100,835</td>
<td>117,238</td>
<td>99,730</td>
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</tbody>
</table>

Notes: Panel A contains OLS regressions with the volatility variable being dollar return volatility. Panel B contains median regression results. Panel C is median regressions with the volatility variable being dollar return volatility. Panel D is firm–manager paired fixed effect regression results, in which there is one fixed effect for each unique firm–manager combination. Unless mentioned, the return volatility is percentage return volatility. The dependent variable is the dollar–to–dollar measure (PPS1) of pay–performance sensitivity in columns (1) and (2), the dollar–to–percentage measure (PPS2) in columns (3) and (4), and the wealth–performance sensitivity (PPS3) in columns (5) and (6). All the specifications include the same control variables as those in Table 3, but to save space, the coefficient estimates on these control variables are not reported. All explanatory variables are lagged by one year. The sample includes all companies in ExecuComp and covers the period from 1992 to 2008. Detailed definitions of all the variables are in Appendix B. For median regressions, t-statistics derived from the bootstrapped standard errors (based on 20 replications) are in parentheses. For OLS (firm–manager fixed effect) regressions, heteroskedasticity robust t-statistics adjusting for clustering within companies (firm–manager pairs) are in parentheses.

*Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.
Acknowledgments
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Appendix A. Proofs
Proof of Lemma 1. Note that given \(\alpha, \beta, \) and \(L_1,\) the manager’s expected utility is
\[
\mathbb{E}[-e^{-a(y_0 - l/2)}] = -\exp\left[-a\left(\alpha + \beta \theta_0 K_1^{1 - \lambda} L_1^\lambda - \frac{1}{\lambda} L_1^\lambda \right) \right].
\]

Denote the above function by \(\tilde{U}(L_1).\) Its first-order condition is
\[
\frac{d\tilde{U}(L_1)}{dL_1} = \tilde{U}(L_1)(-a)(\alpha + \beta \theta_0 K_1^{1 - \lambda} L_1^\lambda - \frac{1}{\lambda} L_1^\lambda) - a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} L_1^{2(1 - \lambda)} - a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} L_1^{2(1 - \lambda)} - 1,\]
and its second-order condition is
\[
\frac{d^2\tilde{U}(L_1)}{dL_1^2} = (-a)^2 \tilde{U}(L_1)(\alpha + \beta \theta_0 K_1^{1 - \lambda} L_1^\lambda - \frac{1}{\lambda} L_1^\lambda) - 2a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} L_1^{2(1 - \lambda)} - 1\]
\[
+ 2(\lambda - 1)a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} L_1^{2(1 - \lambda)} < 0.
\]

The optimal \(L_1^*\) is determined by the first-order condition of the manager’s optimization problem; that is, it is the unique solution of the following equation:
\[
\lambda \theta_0 K_1^{1 - \lambda} - \lambda L_1^{2 - \lambda} - a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} L_1^{2(1 - \lambda)} = 0.
\]

The assumption \(\theta_0 > 0\) ensures a unique positive solution for \(L_1^*.\) The fixed salary \(\alpha\) is chosen to satisfy the manager’s participation constraint:
\[
\alpha = -\beta \theta_0 K_1^{1 - \lambda} L_1^\lambda + \frac{1}{\lambda} L_1^{\lambda} + \frac{1}{2} a\beta_0^2 (\gamma_0 K_1^{1 - \lambda} L_1^\lambda)^2 + \sigma_\theta^2
\]
\[
= -\frac{1}{a} \log(-\tilde{U});
\]

or, after substituting the expression of \(L_1^*\) and \(\tilde{U} = -1,\) we have
\[
\alpha = -\beta \theta_0 K_1^{1 - \lambda} L_1^\lambda + \frac{1}{\lambda} L_1^{\lambda} + \frac{1}{2} a\beta_0^2 (\gamma_0 K_1^{1 - \lambda} L_1^\lambda)^2 + \sigma_\theta^2.
\]

Proof of Proposition 2. We first prove that \(d^2\tilde{V}/d\gamma_0 > 0\) holds when the manager is risk neutral (i.e., \(a = 0\)); the statement in the proposition immediately follows in light of the continuity of the derivative \(d^2\tilde{V}/d\gamma_0\) in \(a.\) We can view the maximization problem in terms of implemented effort \(L_1^*:\)
\[
\frac{dL_1^*}{d\gamma_0} > 0, \quad (A1)
\]
and if higher effort is linked to higher incentives, which requires that
\[
\frac{dL_1^*}{d\beta} > 0, \quad (A2)
\]
then we obtain our desired result \(d^2\tilde{V}/d\gamma_0 > 0.\) Below we proceed to show that both Equations (A1) and (A2) hold.
From the incentive compatibility condition \(\lambda \theta_0 K_1^{1 - \lambda} - l(L_1)^{2 - \lambda} - a\gamma_0 \beta_0^2 K_1^{2(1 - \lambda)} (L_1)^\lambda = 0,\) we have
\[
\lambda \theta_0 K_1^{1 - \lambda} - l(2 - \lambda)(L_1)^{2 - \lambda} + \frac{dL_1^*}{d\beta} = 2a\gamma_0 \beta_0 K_1^{2(1 - \lambda)} (L_1)^\lambda = 0.
\]
Simplifying the above equation, we have
\[
\frac{dL_1^*}{d\beta} = \frac{\lambda \theta_0 K_1^{1 - \lambda} - 2a\gamma_0 \beta_0 K_1^{2(1 - \lambda)} (L_1)^\lambda}{(2 - \lambda)(L_1)^{2 - \lambda} + a\gamma_0 \beta_0 K_1^{2(1 - \lambda)} (L_1)^{1 - \lambda}}.
\]
Setting \(a = 0\) and noticing that \(\theta_0 > 0,\) we have
\[
\frac{dL_1^*}{d\beta} \bigg|_{a=0} = \frac{\lambda \theta_0 K_1^{1 - \lambda}}{(2 - \lambda)(L_1)^{2 - \lambda}} > 0.
\]
Now we use the supermodularity property to prove \(dL_1^*/d\gamma_0 > 0.\) To prove \(dL_1^*/d\gamma_0 > 0,\) it suffices to show that
\[
\frac{\partial^2 V(L, \gamma_0)}{\partial L \partial \gamma_0} \bigg|_{a=0} > 0.
\]
Recall that the time 0 expected payoff function is given by
\[
V(L, \gamma_0) = \theta_0 K_1^{1 - \lambda} L_1^\lambda - \frac{1}{2} L_1^{2 - \lambda} - 2a\beta_0^2 \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda} + \frac{M}{\sigma_\theta^2 + \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda}} - \frac{1}{2} \sigma_\theta^2 - \frac{1}{2} K_1.
\]
We thus have
\[
\frac{\partial^2 V(L, \gamma_0)}{\partial L \partial \gamma_0} = \lambda \theta_0 K_1^{1 - \lambda} L_1^\lambda - l(L_1)^{2 - \lambda} + 2\lambda M \sigma_\theta^2 K_1^{2(1 - \lambda)} L_1^{2 - \lambda} + 2 \lambda M \sigma_\theta^2 \frac{\gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda}}{(\sigma_\theta^2 + \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda})^2},
\]
and
\[
\frac{\partial^2 V(L, \gamma_0)}{\partial L \partial \gamma_0} = -2\lambda a^2 \beta_0^2 \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda} + 2\lambda M \sigma_\theta^2 K_1^{2(1 - \lambda)} L_1^{2 - \lambda} + 2 \lambda M \sigma_\theta^2 \frac{\gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda}}{(\sigma_\theta^2 + \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda})^2}.
\]
Therefore, when \(a = 0\) (and hence when \(a\) is sufficiently small),
\[
\frac{\partial^2 V(L, \gamma_0)}{\partial L \partial \gamma_0} = \frac{4 \lambda M \sigma_\theta^2 K_1^{2(1 - \lambda)} L_1^{2 - \lambda} + \gamma_0 \sigma_\theta^2}{(\sigma_\theta^2 + \gamma_0 K_1^{2(1 - \lambda)} L_1^{2 - \lambda})^3} > 0.
\]
This completes the proof. □

**Appendix to the Second Last Paragraph of Section 2.5.** We have
\[
\frac{\partial (-\gamma_L)}{\partial \sigma^2_L} = -\frac{\gamma_L^2(K_1^{-1} - (L_L)^2)}{(\sigma^2_L + \gamma_L K_1^{-1} - (L_L)^2)^2} = -\frac{\gamma_L^2}{(\sigma^2_L + \gamma_L K_1^{-1} - (L_L)^2)^2} \]

therefore \(\frac{\partial^2 (-\gamma_L)}{\partial L_L \partial \sigma^2_L}\) depends on the sign of
\[
\frac{\partial [\gamma_L^2/K_1^{-1} - (L_L)^2 + \gamma_L K_1^{-1} - (L_L)^4]}{\partial L_L^2} = \lambda [\gamma_L K_1^{-1} - (L_L)^2 - \sigma^2_L]/K_1^{-1} - (L_L)^{2k+1},
\]

**Appendix B. Definition of Variables**

**Firm-Level Variables**

*Firm age.* Based on Pastor and Veronesi (2003), we consider each firm as "born" in the year of its first appearance in the CRSP database. Specifically, we look for the first occurrence of a valid stock price on CRSP, as well as the first occurrence of a valid market value in the CRSP/Compustat database. We winsorize RC at 5% and 95% and average the winsorized quarterly RCs over the rolling three-year window. We winsorize RC at 5% and 95% and average the winsorized quarterly RCs (obtained from the I/B/E/S unadjusted actuals file) and the actual EPS values) made by each analyst, and then we use the mean value of these absolute forecast errors scaled by the absolute value of the actual EPS. Using the mean value of the absolute forecast errors or scaling by stock price per share gives similar results. The analyst forecast error variable is constructed from the I/B/E/S details database.

*Stock return volatility.* First, we obtain the standard deviation of daily log returns over the past five years, and then annualize the standard deviation by multiplying by the square root of 254. This is the percentage return volatility.

*Rank of dollar return volatility.* Dollar return volatility is equal to stock percentage return volatility multiplied by the beginning-of-year firm market value. This variable is measured in millions of dollars. Consistent with Aggarwal and Samwick (1999) and Jin (2002), we employ the percentage ranks of dollar return variance in our tests and these percentage ranks range from 0 (lowest risk) to 100 (highest risk).

*Firm size.* This variable is the natural log of total assets = \(\log(\text{AT}) = \log(\text{data6})\). Assets are measured in millions of dollars.

*Analysts' long-term growth forecast.* This variable comes from I/B/E/S analysts' forecast of long-term earnings growth (LTG in I/B/E/S). When multiple analysts give LTG forecasts about the same company during the same period, the median forecast is used.

*Profitability.* This variable is operating income before depreciation and amortization/total assets = \(\text{OIBDP}/\text{AT} = \text{data13}/\text{data6}\).

*Capital expenditure.* This variable is capital expenditures/total assets = \(\text{CAPX}/\text{AT} = \text{data128}/\text{data6}\).

*Advertisement.* This variable is advertising expense/total assets = \(\text{XAD}/\text{AT} = \text{data45}/\text{data6}\). This variable is set to 0 if it is missing, and an advertisement missing indicator is thus included in the regressions to deal with the missing advertisement issue.

*Advertisement missing indicator.* This variable is a dummy variable equal to 1 if the advertisement variable is missing.

*Leverage.* This variable is (long term debt + debt in current liabilities)/total assets = \(\text{DLTT} + \text{DLC}/\text{AT} = \text{data9 + data34}/\text{data6}\).

*Dividend-paying Indicator.* This variable is a dummy variable equal to 1 if dividends on common stock (data21 or DVC) are strictly positive and 0 otherwise.

*CEO Chair Indicator.* This is a dummy variable equal to 1 if the CEO of the company is also the board chairman and 0 otherwise.

*Fraction of inside directors.* This variable is the number of inside board directors divided by board size, where an inside director is defined as a director who is a current or former firm manager or one of his or her family members is a current or former firm manager.

**Manager-Level Variables**

*PPS1.* This is a dollar-to-dollar measure of pay–performance sensitivity. This variable measures the dollar change in stock and option holdings for a one dollar change in firm value. To estimate PPS1, first calculate a variable named Totaldelta, which is obtained from multiplying the
Black–Scholes hedge ratio by the shares in options owned by the executive. $PS_1$ in year $t$ is equal to an executive’s $Totaldelta$ over fiscal year $t$ divided by total number of shares outstanding (Compustat data item CSHO) of the company at the beginning of year $t$. The construction of $Totaldelta$ involves a lot of details (e.g., how to construct the Black–Scholes hedge ratio, how to deal with previously granted options, what to assume for expected life on the options, etc.), and we follow Appendix B in Edmans et al. (2009) in estimating the $Totaldelta$ variable. In the regressions, $PS_1$ is in percentages.

$PPS_2$. This is a dollar-to-percentage measure of pay–performance sensitivity. This variable measures the dollar change in stock and option holdings for a 1% change in firm value. $PPS_2$ in year $t$ is equal to $PS_1$ in year $t \times share\ price$ at the beginning of fiscal year $t \times total\ number\ of\ shares\ outstanding$ at the beginning of $t/100$, where share price is Compustat data item PRCC_F and total number of shares outstanding is Compustat data item CSHO. In the regressions, $PPS_2$ is in thousands of dollars.

$PPS_3$. This is the scaled wealth–performance sensitivity proposed in Edmans et al. (2009). It is available from Alex Edmans’ website (http://finance.wharton.upenn.edu/~aedmans/data.html). Specifically, this sensitivity measure equals the dollar change in executive wealth for a 100 percentage point change in firm value, divided by annual flow compensation (TDC1). This incentive measure is a variant of the percentage-to-percentage incentives used in Murphy (1985), Gibbons and Murphy (1992), and Rosen (1992), and replaces flow compensation in the numerator of the measure in Murphy (1985) with the change in the executives’ wealth. By considering the change in wealth, the scaled wealth–performance sensitivity captures the important incentives from changes in the value of previously granted stock and options. See Edmans et al. (2009) for details.

$Log(Tenure)$. This is the natural log of the number of years the manager has been with the company, which equals the difference between the year of the observation and the year when the individual joined the company. 

$CEO\ indicator$. This is a dummy variable that equals 1 if the manager is the CEO in a particular year and 0 if the manager is a non-CEO top executive. This dummy variable is time variant for a given individual because a specific manager could be a CEO in some years and a non-CEO in other years.

$Female\ indicator$: This is a dummy variable that equals 1 if the manager is a female and 0 otherwise.

References


