Delegated Asset Management and Investment Mandates*

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Abstract

This paper develops a model to explain the widely used investment mandates in the institutional asset management industry based on two insights: First, giving a manager more investment flexibility weakens the link between fund performance and his effort in the designated market, and thus increases agency cost. Second, presence of tail risk (i.e., an asset with negatively skewed return) can further increase the agency cost if the manager is incentivized to pursue outside opportunities. These effects motivate narrow mandates to most fund managers except those with exceptional talents. Our model sheds light on capital immobility and market segmentation that are widely observed in financial markets, and highlights important effects of tail risk on institutional incentive structures.

Keywords: Institutional Frictions, Tail Risk, Fund-manager Compensation, Capital Immobility, Market Segmentation

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1 Introduction

The institutional asset management industry has experienced rapid growth in the last two decades. According to Wikipedia, in 2008 the global asset management industry managed a total of around $90 trillion, with $24 trillion in pension funds, $18.9 trillion in mutual funds, $18.7 trillion in insurance funds, and the rest in other funds, such as sovereign wealth funds, hedge funds, private equity funds and exchange traded funds. This sheer size already makes institutional investors a key player in the financial markets. Because of their distinctive institutional incentive structures, they have different preferences and investment characteristics from those of individual investors. The Bank for International Settlements initiated a working group on incentive structures in the institutional asset management industry. By interviewing more than 100 industry practitioners from 14 countries, the working group identified several general trends in this industry. One of them is the use of more stringent investment mandates, i.e., “a tiering and narrowing of investment mandates, enhanced by an increasing emphasis on relative performance measurement, narrowing tracking errors and more pervasive use of other investment constraints, such as limits on investing in specific securities or diversification rules.” (BIS report, 2003.) This trend is puzzling—from an investment efficiency perspective—because stringent mandates limit the fund managers’ ability to take advantage of investment opportunities outside their mandates.1 As we will discuss later, this trend can have important implications for asset market dynamics.

What causes the use of more stringent investment mandates? After all, a typical expertise related argument implies that managers with superior expertise in certain markets will voluntarily choose to invest in their specialized markets without the aid of mandates. Furthermore, which managers should be incentivized to pursue outside opportunities? How should they be compensated? In this paper, we provide an agency-based model to address these questions. Our model also incorporates tail risk, a widely recognized challenge to the modern financial markets, to analyze the effects of tail risk on funds’ incentive structures and investment strategies.

The asset management industry has a complex incentive structure. Financial service companies, such as Fidelity and TIAA-CREF, offer families of investment funds for investors to choose from and typically charge them a fixed fee based on asset under management. As

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1 Jame (2010) finds evidence that trades made by pension funds in non-S&P 500 stocks significantly outperform their trades in S&P 500 stocks, and that after controlling for risk and transaction costs tracking error constraints imposed on pension funds weaken the performance of their trades by roughly 30 basis points per year.
better fund performance tends to induce new fund inflows, the inflows provide implicit incentives for the fund family to hire a capable fund manager (e.g., Lakonishok, Shleifer and Vishny (1992), Brown, Harlow and Starks (2006), and Chevalier and Ellison (1997)). Despite the fixed fee charged by fund families, the majority part of individual fund managers’ compensation is a relative performance based bonus (BIS report 2003). Thus, the asset management industry builds on two layers of implicit and explicit incentives: one layer of implicit incentives from investors to fund families, and another layer of explicit incentives from fund families to fund managers. The literature has developed distinctive approaches to analyze the effects of these implicit and explicit incentives. The career-concern framework has been widely adopted to analyze the implicit incentives (e.g., Berk and Green (2004), Dasgupta and Prat (2008), Vayanos and Woolley (2008), Malliaris and Yan (2009), Makarov and Plantin (2010), Guerrieri and Kondor (2010), and Kaniel and Kondor (2010)); while the optimal contracting approach has been used to study the explicit incentives (e.g., Bhattacharya and Pfleiderer (1985), Heinkel and Stoughton (1994), Ou-Yang (2003), Palomino and Prat (2003), Cadenillas, Cvitanic, and Zapatero (2007), and Dybvig, Farnsworth, and Carpenter (2010)).

Our model adopts the optimal contracting approach to focus on explicit incentives of individual fund managers. This approach allows us to derive investment mandates, which are usually explicitly stated in fund prospectuses and enforced by managers’ compensation contracts. Specifically, we analyze a model with a risk neutral principal delegating capital to a risk averse fund manager. In light of the two-layered incentive structure discussed above, one can interpret the principal as a fund family, which hires the manager to manage one of its funds. Different from the aforementioned models, our model allows the manager to face investment opportunities in several markets instead of one. As such, the principal needs to motivate not only effort from the manager, but also investment choices across the markets. Specifically, the manager has expertise in a primary market and he can improve the precision of his private signal about the asset return in this market by exerting unobservable costly effort. Thus, the principal needs to incentivize the manager to work. In addition, the manager’s knowledge about financial markets also gives him a free signal about the asset return in another market, whose identity is known only to the manager. This free signal

\footnote{“The size of the bonus component in individual asset managers’ compensation varies considerably across countries. However, at least in some countries, there seems to be a general trend towards a higher share of variable compensation in total pay over recent years…. US managers can earn average bonuses of 100% and higher. In the United Kingdom, where the median fund manager will get a bonus of about 100%, exceptional asset managers can earn as much as six times their base salary in the form of bonuses.” (the BIS report, 2003, page 23)
is not as precise as his costly signal about the primary market, but is nevertheless useful. Thus, the principal also faces an additional issue on whether to incentivize the manager to take advantage of outside opportunities if there is no profitable opportunity in the primary market.

We also impose another important and realistic assumption that the incentive contract cannot be contingent upon the manager’s investment positions. In practice, it is difficult to find a single measure to summarize a fund’s investment positions as it typically holds many positions with different characteristics in many dimensions. More importantly, it is possible for a fund manager to obscure the origin of his performance through complex financial contracts with others and thus to game any compensation scheme that bases on his positions. The scandals of rogue traders, such as Nick Leeson of the bankrupted British bank Barings and Jerome Kerviel of French bank Societe Generale, vividly demonstrated that they were able to hide their positions from their supervisors for prolonged periods. Thus, it is realistic to consider incentive contracts based only on the fund performance and the primary market return.

A key insight of our model is that there is a conflict between ex post investment efficiency and ex ante incentive provision efficiency. Allowing the manager to take advantage of outside opportunities when he fails to find a good opportunity in the primary market is ex post efficient, but implementing this efficient strategy weakens the link between the fund performance and his unobservable effort in the primary market. This is because the manager can generate good performance either by effort in the primary market or by random luck (an opportunity unrelated to effort) from outside. In the language of Holmstrom (1979), implementing the efficient investment strategy reduces the ex ante incentive provision efficiency by making benchmarking more difficult.

Building on this tradeoff, our model shows that it can be optimal to confine the manager in his primary market depending on his cost of effort and outside investment opportunities. Intuitively, this holds true if his cost of effort is sufficiently high or if his free outside opportunities are only modest. The principal can implement such a strategy by imposing a tight limit on the manager’s tracking error of the primary market return. On the other hand, if the manager’s effort cost is sufficiently low or if his outside opportunities are sufficiently abundant, it is optimal to incentivize him to pursue opportunities both inside and outside the primary market by granting a sufficiently high limit on tracking error and by rewarding him for beating the primary market.

The incentive to pursue outside opportunities can also induce risk-seeking behavior—the
manager might seek tail risk even when he finds no good opportunity. As highlighted by Rajan (2010), active seeking of tail risk by many financial firms such as AIG and Lehman Brothers is a key contributing factor to the recent financial crisis and poses a great challenge to the ongoing reform of the financial industry’s risk management system and incentive structure. To analyze this important issue, we incorporate a market whose return has an unattractive mean but a negative skewness, i.e., it gives a modest positive return most of the time but a large negative return once in a while. This market is attractive to the manager because he gets compensated for the positive return with a high probability, and leaves the principal to bear the huge loss due to the manager’s limited liability. Our model shows that to prevent the manager from seeking unwarranted tail risk, the least costly contract will compensate him even if his performance is inferior but the bad performance can be traced to the poor return in his primary market. Despite the negative effect of such a payment in motivating his effort, it raises his opportunity cost of seeking tail risk. Through this payment and the necessary increases in other payments to offset its negative incentive effect, the presence of tail risk can substantially increase the agency cost. As a result, only managers with exceptional talents can have broad investment mandates.

Taken together, our model provides an agency-based explanation for funds with narrow investment mandates, together with a set of testable implications for varying degrees of investment flexibility across funds. For example, funds tend to face more stringent investment mandates when their managers have lower ability or when they work in more obscure markets that are difficult to analyze. In light of the easier accessibility of tail risk in the increasingly complex financial markets, our model also explains the aforementioned trend of narrowing investment mandates in the delegated asset management industry.

The widely used narrow investment mandates can have important implications for asset market dynamics. Duffie (2010) highlights capital immobility, i.e., capital often fails to flow to liquidity distressed markets that offer profitable opportunities, as an important factor in understanding asset market liquidity. According to our model, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations constrain most fund managers from moving capital into other liquidity distressed markets. Instead, the strategic decisions of allocating capital across different market segments are often left to the less informed investors themselves. As a result, the flow of capital is likely to be delayed. This explanation of capital immobility based on institutional constraints at the originating end of capital flow is distinct from the other explanations based on information barriers about asset fundamentals at the receiving end.
Narrow investment mandates can also explain the de facto segmentation of various asset markets from the broad financial markets in the absence of explicit regulatory and physical constraints on investment. For example, Bekaert, Harvey, Lundblad, and Siegal (2008) find that stock valuation in many emerging markets is significantly lower than that in the global financial markets after controlling for financial leverage and earnings volatility, even though the regulatory constraints on foreign investment in these countries had been largely lifted over the past few decades. Collin-Dufresne, Goldstein, and Martin (2001), Gabibix, Krishnamurthy, and Vigneron (2007), and Garleanu, Pedersen, and Poteshman (2009) provide evidence of risk premia for market-specific risk factors in the corporate bond market, mortgage-backed securities market and S&P 500 index option market. These findings are broadly consistent with our model in the sense that investors heavily rely on professional fund managers to invest in these markets and agency considerations can motivate narrow mandates on the fund managers. As a result, they are exposed to market specific risk. With fund managers likely being the marginal investor, these markets can exhibit premia for market specific risks and thus de facto segmentation.

Our paper adds to the literature on effects of agency frictions on financial market inefficiency. Shleifer and Vishny (1997) and Stein (2005) focus on agency risk in arbitrage trading—fund managers are reluctant to take on arbitrage positions because if asset prices deviate further away from fundamentals in the future, investors will withdraw money and thus causes forced liquidation. In contrast, our paper emphasizes that agency frictions can lead to narrow investment mandates, which limit fund managers’ ability to take advantage of profitable opportunities outside their mandates.

The significant effects of tail risk derived in our model differ from the effects of standard volatility risk in delegated asset management (e.g., Ou-Yang (2003), Palomino and Prat (2003), and Cadenillas, Cvitanic, and Zapatero (2007)). These models tend to find that even when fund managers can choose return volatility, pay for performance is still useful in aligning fund managers’ incentives with their investors. Our model shows that deterring active seeking of tail risk is more challenging and costly than volatility risk and motivates the use of narrow investment mandates.

Malliaris and Yan (2009) and Makarov and Plantin (2010) analyze fund managers’ risk-seeking incentives using career-concern models. As fund performance affects investors’ learning of a fund manager’s ability, convexity in the fund flow can induce the manager to seek risk. The emergence of such an incentive depends on the specifications of investors’ learning technology and investment environment. More importantly, the career-concern models are
not convenient for analyzing investors’ active deterrence of managers’ risk seeking because investors are typically passive in these models. In contrast, our model shows that the incentive to seek tail risk is an inherent by-product of incentivizing fund managers to pursue profitable opportunities, and deterring tail-risk seeking may require inducing them to use suboptimal investment strategies.

The paper is organized as follows. We present a basic model in Section 2. Section 3 extends the model to incorporate tail risk, and Section 4 discusses the model implications. Finally, we conclude in Section 5. All technical proofs are in the appendix.

2 The Basic Model

2.1 Setup

We consider a single-period principal-agent model where a risk-neutral principal delegates capital to a risk-averse agent.\(^3\) As we discussed before, the asset management industry has a two-layered incentive structure with fund families charging investors fixed management fees while compensating individual fund managers based on fund performance. We focus on the explicit incentives of individual fund managers who directly make investment decisions. Thus, we interpret the principal-agent relationship as a fund family (the principal) hiring a fund manager (the agent) to manage one of its funds.

The manager’s utility function over consumption \(U(\cdot)\) satisfies \(U(0) = 0\), \(U'(\cdot) > 0\), and \(U''(\cdot) < 0\). Throughout we focus on the specification that

\[
U(c) = c^{1-\alpha}, \quad \alpha \in (0, 1).
\]

The principal hires the manager to actively invest his money in a primary market, which we denote by market \(A\). We can broadly interpret this market as a specific market sector, such as the treasury bond market, the mortgage bond market, the U.S. stock market, or a regional stock market. We assume for simplicity that the return from this market can only take two possible values, a positive value \(r\) or a negative one \(-r\), with equal probability:

\[
\tilde{r}_A = \begin{cases} 
  r & \text{with probability 0.5} \\
  -r & \text{with probability 0.5} 
\end{cases}
\]

\(^3\)The risk neutrality of the principal implies that he is only interested in maximizing expected fund return. This assumption rules out various hedging and diversification needs of the principal, and allows us to focus on effects of agency frictions on the delegation process. See Massa (2003) and Mamaysky and Spiegel (2002) for studies of how heterogeneity among individual investors in terms of investment horizon and risk preferences can motivate mutual fund families to offer funds specializing in different markets or strategies.
The manager—who possesses certain expertise that normal investors lack—obtains a private signal $s_A$ regarding the likelihood of the market going up or down. The signal takes two possible values 1 or $-1$. If the return is positive (or negative), the signal is more likely to take the value 1 (or $-1$):

$$\Pr(s_A = 1 | \tilde{r}_A = r) = \Pr(s_A = -1 | \tilde{r}_A = -r) = 0.5 + \Delta_A + \theta.$$ 

The term $\Delta_A + \theta > 0$ measures the precision of the signal $s_A$ in revealing the return in market $A$. There are two components in the signal precision: the first part $\Delta_A$ captures the manager’s knowledge about the market without any effort on the job, while the second part $\theta$ represents his effort in acquiring additional information. It can take two values, 0 and $e$, corresponding to “shirking” and “working” respectively. By working hard (e.g., conducting a thorough analysis), the manager improves the signal precision by $e$. We impose $\Delta_A + e \leq 0.5$ to make the probability meaningful. To differentiate the precision of the signal with and without the manager’s effort, we denote $s_A'$ as the signal with effort and $s_A^0$ as the signal without effort.

The effort incurs a private (utility) cost $k$ to the manager and is unobservable to the principal. For simplicity, we also assume that the manager exerts effort before he receives any signal. A central issue of the delegated asset management literature is about how to motivate the manager to exert the costly and unobservable effort and thus improving the fund performance. We assume that the manager has an additive utility function over consumption and effort:

$$U(c, \theta) = U(c) - \frac{k}{e} \theta, \text{ where } \theta \in \{0, e\}.$$ 

The fund has one unit of initial capital. To deliver the key insight without getting into unnecessary complications, we simplify the manager’s investment choices. First, the manager cannot short sell any asset and cannot borrow either. Second, he always invests all of the fund in one asset: either in market $A$, the risk-free asset, or something else. We normalize

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4 We rule out the possibility that the manager makes his effort choice after he observes a free signal about the market. Such a sequential setup complicates the analysis, but does not add much to the economic insight.
6 These two simplifying assumptions are not essential to our model. Allowing fractional positions in individual markets can affect our model in two ways. First, it allows the manager to diversify investment across different markets. However, the fund return is borne by the risk-neutral principal, who does not value diversification and instead prefers the manager to invest in the market with the highest expected return. Furthermore, the risk borne by the manager is specified by his compensation contract and does not need to increase with the fund return risk. Second, allowing fractional positions expands the fund return space.
the return of the risk-free asset to be zero. Then if he observes a positive signal on market A, the expected return is positive and he will invest the fund in market A; if he observes a negative signal, then he should stay out of market A. But, should he then invest the fund in the risk-free asset or something else? In reality, a fund manager often has expertise beyond his primary market. An important question faced by every fund is whether the manager should be incentivized to pursue outside opportunities when the primary market lacks a good one.

We capture this problem by assuming that the manager can access a set of outside markets. These markets have independently and identically distributed returns with the same binomial distribution as market A. Before the manager makes his investment decision, he also receives a free signal about one of these markets, which we denote by market B. This market is randomly drawn from the pool. Neither can the principal observe the signal, nor which market the signal is about. The precision of the manager’s free signal on market B is \( \Delta_B \in (0, 1/2) \), i.e.,

\[
\Pr(s_B = 1 | \tilde{r}_B = r) = \Pr(s_B = -1 | \tilde{r}_B = -r) = 0.5 + \Delta_B.
\]

We also assume that \( \Delta_B \leq \Delta_A \), i.e., the manager is better informed about his primary market. Nevertheless, given that his signal about market B is informative, it is profitable for him to take advantage of the possible investment opportunity in market B if there is not any profitable one in market A. However, as we will show, doing so will interfere with the ex ante efficiency of incentive provision.

We denote the manager’s investment choice by

\[
x = \{ x_A, x_B, x_0 \},
\]

where \( x_i \in \{0, 1\} \) indicates the manager’s investment position in market \( i \) with \( i \in \{ A, B \} \), and \( x_0 \in \{0, 1\} \) is his position in the risk free asset. The borrowing constraint requires that \( x_A + x_B + x_0 = 1 \). We denote the set of all feasible investment choices by \( X = \{ x \} \). The fund’s return \( \tilde{r}_F \) can take three possible values, i.e., \( \tilde{r}_F \in \{ r, 0, -r \} \).

and therefore the contract space for the manager’s compensation. In particular, one might argue that this makes it possible to separate the fund’s investment in the primary market and other markets. For example, if the principal can design a contract to induce the manager to either invest 100% in market A or 99% in market B, then the fund return directly reveals the fund’s investment position. However, the continuously distributed market returns in reality render such a revelation mechanism unrealistic. Allowing the manager to short sell assets can also affect the fund return space. In particular, short-selling allows the manager to profit from negative market returns and thus changes the mapping of the fund’s relative performance (i.e., the fund performance needs to compare to the absolute return of the primary market.) However, we do not expect such changes to alter the key intuition of our model.
2.2 Optimal Contracting

2.2.1 Incentive Contract

The principal writes a compensation contract to induce effort and a certain investment strategy from the manager. For efficient incentive provision, benchmarking the manager’s performance \( \hat{r}_F \) to his primary market return \( \hat{r}_A \) (i.e., using relative performance evaluation) is beneficial.\(^7\) It would be useful to incorporate the return of market \( B \). But this is not feasible because market \( B \) is randomly drawn from a set of outside markets and the principal does not observe its identity. Thus, we focus on incentive contracts based on the fund performance and the primary market return.\(^8\)

Furthermore, we make a realistic assumption that the incentive contract cannot be contingent upon the fund’s investment position. It is unrealistic to contract on fund positions for several reasons. First, it is difficult to find a single measure to summarize the investment positions taken by a real-life fund, which typically holds many positions with different characteristics in many dimensions. Second, while fund families are better monitors of fund managers than investors (e.g., Gervais, Lynch, and Musto (2005)), it is still infeasible for a fund family to continuously monitor each individual investment position of its funds. If reporting of fund positions can only take place at discrete intervals, it will induce window dressing by fund managers to game the reporting system, invalidating the incentive intended by the compensation contract that bases on the reporting. Finally, it is possible for a fund manager to obscure the origin of his performance through complex financial contracts with others, in order to game any compensation scheme that bases on his positions. In fact, the scandals of rogue traders such as Nick Leeson of the bankrupted British bank Barings and Jerome Kerviel of French bank Societe Generale vividly demonstrated that they were able to hide their true positions from their supervisors for prolonged periods.

Thus, an incentive contract \( \Pi \) is a mapping from the information set \( \Omega \) generated by \( \hat{r}_F \)

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\(^7\)Our analysis focuses on incentive contracts based on the performance of a fund and the primary market return. We can further show that peer evaluation, i.e., basing one manager’s compensation on his relative performance to other funds trading in the same market, cannot help in optimal contracting. The reason is simple: In our model, conditional on the true state of the primary market return, the signals are independent across managers. Thus, if the contract incorporates the realized market returns, it has already used the best information for relative performance evaluation (one can formally show this result using the sufficient statistics argument in Holmstrom (1979)). Of course, the principal can improve benchmarking by identifying the realized return in the secondary market \( B \), but the anonymity of market \( B \) rules out this possibility.

\(^8\)In an earlier version of this paper, we have allowed the principal to observe the identity of market \( B \) and therefore to write the incentive contract based on the return of market \( B \) as well. The key results of our paper remain similar.
and \( \pi_A \) to non-negative payments to the manager:

\[
\Pi : \Omega \equiv \{u, 0, d\} \times \{u, d\} \to \mathbb{R}_+.
\]

We rule out negative wages to the manager due to limited liability.\(^9\) The fund return can take three possible values: \( u \) (up with a return of \( r \)), \( 0 \), or \( d \) (down with a return of \( -r \)). The return of market \( A \) can take two possible values, \( u \) (up) or \( d \) (down). There are 6 possible outcomes. Therefore, the contract only needs to specify 6 contingent payments. Denote \( \omega = (\tilde{r}_K, \tilde{r}_A) \in \Omega \) as a possible outcome. It is easier to work with the payment in terms of the manager’s utility \( (\pi_\omega) \) than in terms of dollars \( (c_\omega) \). These terms are equivalent because of the monotone relation \( \pi_\omega = U(c_\omega) \). We write the contract as

\[
\Pi = \{\pi^u_u, \pi^0_u, \pi^d_u, \pi^u_d, \pi^0_d, \pi^d_d\}.
\] (1)

For instance, \( \pi^u_d \) is the manager’s utility when the primary market is down but the fund return is up. Then, \( c^u_d = U^{-1}(\pi^u_d) \) is the cost of compensating the manager for this outcome.

2.2.2 Contracting Problem

For a given contract \( \Pi = \{\pi_\omega\} \), the fund manager maximizes his expected utility by first making his optimal effort and then investment choices based on the signals he receives:

\[
\max_{\theta \in \{0, e\}, x \in X} \sum_{\omega \in \Omega} p_\omega (\theta, x) \pi_\omega - \frac{k}{c} \theta,
\]

where \( p_\omega \) is the probability of outcome \( \omega \). The manager’s effort and investment choices \( \theta \) and \( x \) determine the outcome probability \( \{p_\omega\} \). We write \( \theta^* (\Pi) \) and \( x^* (\Pi) \) as the manager’s optimal effort and investment choices, respectively, in response to a given contract \( \Pi \).

Thus, by using different incentive contracts, the principal can induce different effort and investment choices from the manager. When the manager’s effort cost is sufficiently low, the first best combination of effort and investment strategies is that the manager exerts effort in the primary market and then follows a so called “two-tiered” investment strategy: If the manager receives a positive signal about the primary market, he will invest the fund capital in it; if not, he will invest in market B if his free signal about market B indicates a good opportunity; finally, he will invest in the risk-free asset if his signal about both

\(^9\)We assume that the manager is both risk averse and protected by limited liability. Limited liability is sufficient for generating a non-zero agency cost even if the manager is risk neutral. However, as will become clear later, the agency cost in this case is not affected by efficiency of benchmarking in implementing different effort and investment strategies. The reason is that for a risk neutral manager, the principal can shift all positive rewards to one good outcome, say \((0_d)\), where the manager’s investment strategy is identifiable.
markets A and B are negative. This strategy instructs the manager to take advantage of opportunities outside his primary market. Alternatively, the fund can also implement a “single-market” strategy: the manager will invest in the primary market if his signal on the market is favorable and otherwise put the fund capital in the risk-free asset. Relative to the two-tiered strategy, this strategy requires the same effort cost but forgoes a valuable investment opportunity outside the primary market. As we will show later, this seemingly inferior strategy dominates the two-tiered strategy under certain conditions because of its more efficient incentive provision.

The manager has a reservation utility of $U$, which represents his forgone outside opportunity cost by managing this fund. The participation constraint requires that:

$$\sum_{\omega \in \Omega} p_\omega (\theta^* (\Pi), x^* (\Pi)) \pi_\omega - \frac{k}{e} \theta^* (\Pi) \geq U.$$

For simplicity, throughout this paper we assume that $U$ is sufficiently small so that the manager’s participation constraint is not binding.

The principal’s payoff from outcome $\omega$ is the portfolio return minus the compensation cost:

$$W_\omega = 1 + \tilde{r}_F (\omega) - U^{-1} (\pi_\omega). \quad (2)$$

The principal maximizes the expected payoff from the fund by choosing an optimal incentive contract, i.e.,

$$V = \max_{\Pi} \sum_{\omega \in \Omega} p_\omega (\theta^* (\Pi), x^* (\Pi)) W_\omega,$$

subject to the manager’s participation and incentive compatibility constraints.

We can further decompose the principal’s expected payoff into two components:

$$V = \sum_{\omega \in \Omega} p_\omega (1 + \tilde{r}_F (\omega)) - \sum_{\omega \in \Omega} p_\omega U^{-1} (\pi_\omega), \quad (3)$$

where the first part is the expected fund return, which is determined by the manager’s effort and investment strategy, and the second part is the expected cost of compensating the fund manager. This decomposition suggests the following two-step method to solve for the optimal contract: First, find the least costly contract to implement each of the two effort and investment strategies; then, compare these least costly contracts to determine the optimal contract that offers the highest expected net payoff to the principal.

2.3 Single-market Strategy

We start with analyzing the least costly contract for implementing a single market strategy in market $A$. The contract induces the following effort and investment choices from the
fund manager: the manager exerts effort only in market $A$; after receiving the signal $s^e_A$, he invests all the fund capital in market $A$ if the signal is positive, and invests in the risk free asset otherwise, regardless of his signal $s_B$ about opportunities outside the primary market. Note that there is an opportunity loss when the manager’s signals suggest that the primary market lacks a good investment opportunity while another market, market $B$, offers a good one ($s^e_A = -1, s_B = 1$).

2.3.1 Incentive Compatibility

The fund manager has two unobservable actions: exerting effort to obtain a precise signal and making the investment choice. In contrast to the costly effort on information acquisition, the investment choice per se does not involve any personal cost, and the incentive compatibility constraint regarding the investment choice is slack (which we will verify later). Here, we discuss the manager’s incentive compatibility constraint regarding his effort choice.

Taking the manager’s investment choice as given, his expected utility from exerting effort on acquiring a precise signal about market $A$ is:

$$E\left[U(c, \theta) \mid \text{exerting effort and obtain } s^e_A \right] = 0.5 \left[ (0.5 + \Delta_A + e) \pi_u^0 + (0.5 - \Delta_A - e) \pi_u^0 + (0.5 + \Delta_A + e) \pi_d^0 + (0.5 - \Delta_A - e) \pi_d^0 \right] - k. \quad (4)$$

Take $\pi^u_u$ for example. The probability of the outcome $\left( \tilde{r}_A = u \right)$ is the probability of state $\tilde{r}_A = u$ (which is 0.5), multiplied by the probability of the manager receiving a positive signal $s^e_A = 1$ conditional on $\tilde{r}_A = u$ and the manager exerting effort (which is $0.5 + \Delta_A + e$).

Note that in implementing this strategy, the two outcomes $\left( \pi_d^u \right)$ and $\left( \pi_d^d \right)$ are off equilibrium. Similarly, the manager’s expected utility from shirking is:

$$E\left[U(c, \theta) \mid \text{shirking with } s^0_A \right] = 0.5 \left[ (0.5 + \Delta_A) \pi_u^u + (0.5 - \Delta_A) \pi_u^0 + (0.5 + \Delta_A) \pi_d^0 + (0.5 - \Delta_A) \pi_d^d \right]. \quad (5)$$

Therefore, the manager’s incentive compatibility constraint regarding exerting effort requires that the value of (4) is no less than that of (5), which is equivalent to

$$0.5 \left( e\pi^u_u - e\pi^0_u + e\pi^0_d - e\pi^d_d \right) \geq k. \quad (6)$$

In condition (6), the coefficient of each utility term in the bracket gives the manager’s incentive differential between “shirking” and “working” for a particular outcome $\omega$. For instance, consider $\pi^u_u$. By working, the probability of getting $\pi^u_u$ is $(0.5 + \Delta_A + e)/2$, while by shirking, the probability becomes $(0.5 + \Delta_A)/2$. The difference between these two probabilities
is exactly the coefficient $0.5e$ in front of $\pi_u$ in condition (6). The higher this coefficient, the more effective the payment $\pi_u$ in motivating the manager to exert effort. We also call this coefficient the incentive leverage of the payment.

### 2.3.2 The Least Costly Contract

The least costly contract for implementing the single market strategy is determined by

$$
\min_{\{\pi_u, \pi_0^u, \pi_d^u, \pi_d^0\} \in \mathbb{R}_+^4} \sum p_{\omega} U^{-1}(\pi_\omega) = 0.5 \left[ (0.5 + \Delta_A + e) U^{-1}(\pi_u^0) + (0.5 - \Delta_A - e) U^{-1}(\pi_u^0) + (0.5 + \Delta_A + e) U^{-1}(\pi_d^0) + (0.5 - \Delta_A - e) U^{-1}(\pi_d^0) \right],
$$

subject to the incentive compatibility constraint in (6), which is binding in the solution. Note that $U^{-1}$ is the inverse function of the manager’s utility function $U$.

Two outcomes $(u)$ and $(d)$, which represent poor performance relative to market $A$, have negative incentive leverages. Any payment to the manager for these outcomes is a reward for failure and thus should be minimized to zero (i.e., $\pi_u^0 = \pi_d^0 = 0$). On the other hand, $\pi_u$ and $\pi_d$ represent rewards for good performance in outcomes $(u)$ and $(d)$. Using the standard Lagrange method, the first order conditions provide that

$$
U' [U^{-1}(\pi_u^0)] = U' [U^{-1}(\pi_d^0)] = \frac{(0.5 + \Delta_A + e)}{\lambda e},
$$

where $\lambda$ is the Lagrange multiplier for the incentive compatibility constraint in (6). Combining this result with (6), we have $\pi_u^0 = \pi_d^0 = \frac{k}{e}$.

We also need to specify payments for two off-equilibrium outcomes $(d)$ and $(u)$ to prevent the manager from investing in the secondary market $B$:

$$
\pi_u = \pi_d = 0.
$$

We verify in Appendix A.1 that under these terms, the manager will never deviate to invest in market $B$. The following proposition summarizes the contract derived above.

**Proposition 1** The least costly contract for implementing the single market strategy uses the following payments:

$$
\begin{cases}
\pi_u = \pi_0^u = \frac{k}{e}, \\
\pi_u^0 = \pi_d^d = \pi_d^u = 0.
\end{cases}
$$

The principal’s expected payoff from implementing this strategy is

$$
V^{SM} = 1 + (\Delta_A + e) r - (0.5 + \Delta_A + e) U^{-1}(\frac{k}{e}).
$$
This contract benchmarks the manager’s performance to the return of his designated market. The manager receives a positive reward if he secures the positive return of the market or avoids its negative return. Otherwise, he receives nothing. Consistent with the benchmarking idea, the same fund performance \((0)\), could lead to two different compensations \((0 \text{ or } \frac{h}{e})\) depending on whether the market return is positive or negative.

Another notable point is that the contract gives a zero payment for \((u_d)\), the outcome in which the manager delivers good performance but the performance is traced to outside the primary market. This term represents a penalty for the manager’s tracking error, which is often used in practice according to the BIS report (2003). This penalty discourages the manager from investing outside the primary market and serves the role of implementing a narrow investment mandate.

### 2.4 Two-tiered Strategy

Implementing the single-market strategy imposes an efficiency loss by restricting the manager from taking advantage of opportunities outside the primary market. This subsection studies a two-tiered strategy which improves on this dimension: the manager exerts effort on acquiring a precise signal \(s_A^e\) about market \(A\); if this signal is favorable, he invests in market \(A\); if \(s_A^e\) is unfavorable but his free signal \(s_B\) indicates a good outside opportunity in market \(B\), he invests in market \(B\); otherwise, he invests in the risk free asset. Similar to the single-market strategy, the two-tiered strategy also induces the manager’s effort in market \(A\). However, in contrast to the single-market strategy, the two-tiered strategy instructs the manager to pursue outside opportunities if necessary.

#### 2.4.1 The Least Costly Contract

We derive the least costly contract for implementing the two-tiered strategy in a way similar to the single-market strategy. By exerting effort and following the intended investment strategy, the manager’s expected utility is

\[
\mathbb{E} \left[ U(c, \theta) \mid \text{exerting effort and following the two-tiered investment strategy} \right] = 0.25 \left[ (1 + 2\Delta_A + 2e) + (0.5 - \Delta_A - e)(0.5 + \Delta_B) \right] \pi_u^u + 0.25 (0.5 + \Delta_A + e)(0.5 + \Delta_B) \pi_u^d
\]

\[
+ 0.25 \left[ (1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e)(0.5 - \Delta_B) \right] \pi_d^u
\]

\[
+ 0.25 (0.5 - \Delta_A - e)(0.5 - \Delta_B) \pi_d^d + 0.25 (0.5 + \Delta_A + e) \pi_d^0 + 0.25 (0.5 - \Delta_A - e) \pi_u^0 - k.
\]

The manager can also adopt a deviation strategy by shirking and then following the two-tiered investment strategy based on his free signals \(s_A^0\) and \(s_B\). Then, his expected utility
The manager exerts effort if (10) dominates (11), which is equivalent to:

\[
\mathbb{E} [\bar{U} (c, \theta) | \text{shirking and following the two-tiered investment strategy}] = 0.25 [(1 + 2\Delta_A) + (0.5 - \Delta_A) (0.5 + \Delta_B)] \pi^u_u + 0.25 (0.5 + \Delta_A) (0.5 + \Delta_B) \pi^u_d + 0.25 [(1 - 2\Delta_A) + (0.5 + \Delta_A) (0.5 - \Delta_B)] \pi^d_d + 0.25 (0.5 - \Delta_A) \pi^0_u
\]

The manager exerts effort if (10) dominates (11), which is equivalent to:

\[
0.25 \mathbb{E} [(1.5 - \Delta_B) \pi^u_u + (0.5 + \Delta_B) \pi^0_u - (0.5 + \Delta_B) \pi^d_u - (0.5 - \Delta_B) \pi^d_d] \geq k.
\]

This is an important constraint in implementing the two-tiered strategy.

The equilibrium contract also needs to ensure that after receiving a negative signal about market A and a positive signal about market B, the manager will find it optimal to invest in market B rather than invest in the risk-free asset. Investing in market B exposes the manager to the risk that the realized return might be negative, while investing in the risk-free asset allows the manager to lock in the sure return 0. The comparison of the two depends on the structure of the manager’s incentive contract. Specifically, given \( s_A^* = -1 \) and \( s_B = 1 \), the manager’s expected utility from investing in market B is

\[
(0.5 - \Delta_A - e) (0.5 + \Delta_B) \pi^u_u + (0.5 - \Delta_A - e) (0.5 - \Delta_B) \pi^d_u + (0.5 + \Delta_A + e) (0.5 + \Delta_B) \pi^0_u + (0.5 + \Delta_A + e) (0.5 + \Delta_B) \pi^0_u
\]

while his expected utility from investing in the risk-free asset is

\[
(0.5 - \Delta_A - e) \pi^u_u + (0.5 + \Delta_A + e) \pi^0_u.
\]

Implementing the two-tiered strategy thus requires (13) dominate (14):

\[
(0.5 - \Delta_A - e) (0.5 + \Delta_B) \pi^u_u + (0.5 - \Delta_A - e) (0.5 - \Delta_B) \pi^d_u + (0.5 + \Delta_A + e) (0.5 + \Delta_B) \pi^u_u + (0.5 + \Delta_A + e) (0.5 + \Delta_B) \pi^0_u - (0.5 - \Delta_A - e) \pi^0_u - (0.5 + \Delta_A + e) \pi^d_u \geq 0.
\]

This constraint also binds in the least costly contract.

To implement the two-tiered strategy, the expected compensation cost to the manager

\[
\mathbb{E} [U^{-1} (\pi_u)] \text{ is:}
\]

\[
0.25 [(1 + 2\Delta_A + 2e) + (0.5 - \Delta_A - e) (0.5 + \Delta_B)] U^{-1} (\pi_u^u) + 0.25 (0.5 - \Delta_A - e) U^{-1} (\pi_u^0) + 0.25 [(1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e) (0.5 - \Delta_B)] U^{-1} (\pi_d^u) + 0.25 (0.5 + \Delta_A + e) U^{-1} (\pi_d^0) + 0.25 (0.5 - \Delta_A - e) (0.5 - \Delta_B) U^{-1} (\pi_u^d) + 0.25 (0.5 + \Delta_A + e) (0.5 + \Delta_B) U^{-1} (\pi_d^0).
\]

\(15\)
The least costly contract minimizes the expected compensation cost, subject to the incentive constraints in (12) and (15) and that all payments are non-negative. We denote the lagrange multipliers associated with the two incentive constraints as \( \lambda_1 \geq 0 \) and \( \lambda_2 \geq 0 \) respectively. The following proposition characterizes the least costly contract. We also verify other deviation strategies based on this incentive contract in Appendix A.2.

**Proposition 2** The least costly contract for implementing the two-tiered strategy gives zero payments for the following outcomes:

\[
\pi_u^0 = \pi_u^d = \pi_d^d = 0,
\]

and positive payments for \( \pi_u^u, \pi_d^u, \) and \( \pi_d^0 \), which can be solved through a set of equations given in Appendix A.2. Among these payments, the following is true: \( \pi_u^u < \pi_d^d \) and \( \pi_d^0 < \pi_u^u \). Under the sufficient conditions (31) and (32) in Appendix A.2, this contract also deters the use of other deviation strategies.

To implement the two-tiered strategy, the expected compensation cost is

\[
K^{TT} = 0.25 \left\{ [(1 + 2\Delta_A + 2\varepsilon) + (0.5 - \Delta_A - \varepsilon) (0.5 + \Delta_B)] U^{-1} (\pi_u^u) \\
+ (0.5 + \Delta_A + \varepsilon) (0.5 + \Delta_B) U^{-1} (\pi_d^u) + (0.5 + \Delta_A + \varepsilon) U^{-1} (\pi_d^0) \right\},
\]

and the principal’s expected payoff is

\[
1 + (e + \Delta_A + 0.5\Delta_B)r - K^{TT}.
\]

Proposition 2 shows that to encourage the manager to pursue potential investment opportunities outside his primary market, the least costly incentive contract tolerates greater tracking errors than the one for implementing the single-market strategy (Proposition 1). This difference is reflected by the positive payment for the outcome \( (u_d) \), in which the fund return beats the primary market return by two notches. Because of the large tracking error, this seemingly good performance is not rewarded by the contract derived in Proposition 1. Furthermore, the contract derived in Proposition 2 also provides a greater incentive slope, i.e., a larger payment for better performance as reflected by \( \pi_d^u > \pi_u^u \) and \( \pi_d^0 > \pi_u^0 \). In contrast, the contract derived in Proposition 1 provides the same payment for the two good outcomes: \( \pi_u^u = \pi_d^0 \).

In practice, hedge funds tend to be more tolerant of tracking errors and provide greater incentive slopes, whereas mutual funds tend to be more restrictive on tracking errors and give smaller incentive slopes. Thus, the contract derived in Proposition 2 is closer to the hedge fund contracts, while the contract derived in Proposition 1 is closer to the mutual fund contracts.
2.4.2 Higher Agency Cost

Interestingly, the seemingly superior two-tiered strategy may not be optimal because it exacerbates the agency cost to incentivize the manager to exert effort in his primary market. This negative impact originates from two channels. First, the additional investment flexibility makes “benchmarking” more difficult because it introduces luck from market $B$ into the fund performance. This weakens the link between the fund performance and the manager’s effort in market $A$, and leads to less efficient incentive provision. Second, implementing the two-tiered investment strategy requires an additional constraint (15) on the incentive contract, which further reduces its incentive provision efficiency. The intuition for the second channel is obvious. Thus we focus on illustrating the first channel here.

The negative impact of investment flexibility on the incentive-provision efficiency manifests itself in the payment for the outcome $\omega = \binom{u}{u}$. In implementing the two-tiered strategy, the probability of this outcome is

$$p_\omega = 0.5 \left(0.5 + \Delta_A + e\right) + 0.25 \left(0.5 - \Delta_A - e\right) \left(0.5 + \Delta_B\right). \tag{17}$$

The first term represents the situation that the primary market return is high (with probability 0.5) and the manager spots this opportunity (with probability $0.5 + \Delta_A + e$). The second term represents an additional possibility that the primary market return is high (with probability 0.5) but the manager fails to spot it (with probability $0.5 - \Delta_A - e$); instead, the return in market $B$ is also high (with probability 0.5) and the manager spots this one (with probability $0.5 + \Delta_B$). The second term represents luck from market $B$. Such luck increases the probability for the principal to make the positive payment and thus adds to the compensation cost.

The luck also reduces the incentive leverage of $\pi^u_u$. If the manager shirks, the probability of this outcome becomes

$$0.5 \left(0.5 + \Delta_A\right) + 0.25 \left(0.5 - \Delta_A\right) \left(0.5 + \Delta_B\right). \tag{18}$$

Thus, the difference between equations (17) and (18) gives the incentive leverage of $\pi^u_u$:

$$Dp_\omega = 0.5 e - 0.25 \left(0.5 + \Delta_B\right) e,$$

which is reduced by the possible luck from market $B$. Intuitively, the free luck from market $B$ crowds out the need to exert effort to spot the good opportunity in market $A$ (if there is one). This crowding out effect, which is at work only when implementing the two-tiered
strategy, reduces the manager’s gain from exerting his effort in market $A$ and therefore his ex ante working incentives.

We call the ratio $\frac{p_\omega}{Dp_\omega}$ the cost to incentive ratio of the payment, which is first derived in Holmstrom (1979).\footnote{\(\frac{p_\omega}{Dp_\omega}\) corresponds to \(\frac{f}{f_a}\) in Holmstrom (1979), where \(f\) is the probability density function of the performance, and \(f_a\) is the marginal impact of action \(a\) on the density function. Holmstrom points out that \(\frac{f}{f_a}\) is the derivative of log likelihood, and interprets this measure as how strongly one is inclined to infer from the performance that the agent did not take the assumed action.} The numerator $p_\omega$ captures a cost effect, i.e., the larger the probability of the outcome $\omega$, the higher the expected cost of each dollar promised to this outcome. The denominator $Dp_\omega$ captures an incentive effect: the larger the incentive leverage $Dp_\omega$, the greater the manager’s incentive to exert effort. This ratio thus inversely measures the payment’s incentive-provision efficiency. In implementing the two-tiered strategy, we have

\[
\left( \frac{p}{Dp} \right)_\omega = \frac{0.5 (0.5 + \Delta_A + e) + 0.25 (0.5 - \Delta_A - e) (0.5 + \Delta_B)}{0.5e - 0.25 (0.5 + \Delta_B) e} \]

\[
= \frac{0.5 (0.5 + \Delta_A + e) + 0.25 (0.5 - \Delta_A - e) (0.5 + \Delta_B)}{0.5e - 0.25 (0.5 + \Delta_B) e} \]  

We have decomposed each term in the fraction relative to $\frac{0.5+\Delta_A+e}{e}$, the corresponding cost to incentive ratio in implementing the single market strategy. It is clear that the investment flexibility unambiguously increases the cost to incentive ratio by raising the expected payment and lowering the incentive leverage of the payment.

Overall, implementing the two-tiered strategy requires a higher expected compensation cost, which we formally prove in the following proposition.

**Proposition 3** The expected compensation cost of implementing the two-tiered strategy is higher than that of the single-market strategy, i.e., $K^{TT} > K^{SM}$. Furthermore, the difference monotonically increases with $k$.

This proposition shows that in implementing the two-tiered strategy, the additional investment benefit $0.5\Delta_B$ from encouraging the manager to pursue the opportunity outside the primary market comes with an increased agency cost of $K^{TT} - K^{SM}$. As the increased agency cost monotonically increases with $k$, the principal will prefer the narrowly mandated single-market strategy if $k$ is higher than a certain threshold $k^*$.

We can intuitively relate the model parameter $k$ to the manager’s ability and the information opacity of the primary market. As the effort cost of the more talented managers
is lower, the additional agency cost from encouraging them to pursue opportunities outside their primary markets is also smaller. As a result, we have the following implication:

**Implication 1:** Fund managers with lower ability are more likely to be confined in trading a specific market sector or asset class; on the other hand, managers with higher ability tend to face less stringent investment mandates.

This implication is consistent with a casual observation that hedge fund managers tend to be more talented than mutual fund managers, and they also face less stringent investment mandates.

Furthermore, the effort cost is likely to be higher for managers whose primary markets are more informationally opaque. This in turn leads to another testable implication:

**Implication 2:** Fund managers whose primary markets are more informationally opaque face more stringent investment mandates.

### 3 An Extended Model with Tail Risk

When incentivized to pursue investment opportunities outside his primary market, the manager may seek unwarranted tail risk even if his signals do not indicate any good opportunity. In light of the recent financial crisis, many observers had pointed out that excessive risk taking (by AIG, Lehman Brothers, and other financial firms), and in many cases active seeking of tail-risk, was a key contributing factor of the crisis. As highlighted by Rajan (2010), tail risk presents a great challenge to the ongoing reform of the financial industry’s risk management system and incentive structure: “we have to find ways to reduce the incentive to take tail risk even while rewarding bankers for performance so that they continue to offer innovative products that meet customer needs and lend to the risky but potentially very successful start-up.”

To account for the important effects of tail risk, we suppose that one of the markets outside the manager’s primary market has a zero expected return and a negative skewness. Only the manager knows the identity of this market, which we denote by market $C$. Specifically, this market offers the following return:

$$
\tilde{r}_C = \begin{cases} 
    r & \text{with probability } \eta_C \in (0.5, 1) \\
    -\frac{\eta_C}{1-\eta_C}r & \text{with probability } 1 - \eta_C \in (0, 0.5)
\end{cases}
$$

\[^{11}\text{See Kostovetsky (2009) for evidence of a drop in mutual fund returns as a result of a flight of top-performing young managers from mutual funds to hedge funds.}\]
A higher value of $\eta_C$ leads to a more negatively skewed return, i.e., this market gives a positive return $r$ with a high probability $\eta_C$ but a large negative return $-\frac{\eta_C}{1-\eta_C}r$ with a small probability $1 - \eta_C$. Since the manager has limited liability, he is not as concerned about the large negative return as the principal. Thus, the presence of such a market further complicates the delegation problem between the principal and the manager.

To illustrate this complication, suppose that the manager is compensated by the incentive contract derived in Proposition 2 which aims to induce effort and implement the two-tiered investment strategy in markets $A$ and $B$. Note that if the manager chooses to invest in market $C$, the principal can detect this only after the realization of a large loss $-\frac{\eta_C}{1-\eta_C}r$, which is more severe than those from other regular investments. When this occurs, limited liability implies that the principal can only penalize the manager by paying him zero. On the other hand, if the outcome is positive, the principal cannot identify the source of the good performance and has to compensate the manager according to the contract.

The following scenario clearly demonstrates this risk-seeking behavior. Consider the investment problem faced by the manager when he observes a negative signal in market $A$ and a positive signal in market $B$ (i.e., $s^e_A = -1$ and $s_B = 1$). His expected utility from investing in market $B$ is

$$(0.5 - \Delta_A - e) \left[ (0.5 + \Delta_B) \pi_u^u + (0.5 - \Delta_B) \pi_d^u \right] + (0.5 + \Delta_A + e) \left[ (0.5 + \Delta_B) \pi_u^d + (0.5 - \Delta_B) \pi_d^d \right],$$

which, under the contract in Proposition 1, is equal to $(0.5 + \Delta_B) \left[ (0.5 - \Delta_A - e) \pi_u^u + (0.5 + \Delta_A + e) \pi_d^u \right]$. His expected utility from investing in market $C$ is $\eta_C \left[ (0.5 - \Delta_A - e) \pi_u^u + (0.5 + \Delta_A + e) \pi_d^u \right]$ as he will get compensated after market $C$ gives a positive return and zero otherwise. Thus, the manager will choose to invest in market $C$ if

$$\eta_C > 0.5 + \Delta_B.$$ 

This implies that the manager will ignore a good opportunity in market $B$ and instead seek unwarranted tail risk in market $C$ if the negative skewness of the market is sufficiently large. This outcome is alarming as it exactly captures the concern that compensation for positive performance can also motivate the manager to seek tail risk.$^{12}$

This illustration suggests that additional constraints are necessary to prevent the manager from seeking the tail risk in market $C$. It turns out that if the manager chooses to deviate

$^{12}$The incentive to seek tail risk will arise as long as the manager faces a sufficiently large reward for good performance and is protected by limited liability. By endogenizing the manager’s compensation contract through the agency problem, our model allows us to analyze the interaction between effort-motivating incentive and risk-seeking incentive in determining the optimal incentive structure for fund managers.
from the intended investment strategy, he prefers a double-deviation strategy to first shirk and then seek risk in market \( C \) regardless of his signals. Deterring such a double-deviation provides the most-binding constraint on the incentive contract.\(^{13}\) If the manager chooses to shirk in the primary market and then to always invest in market \( C \), his expected utility is

\[
0.5\eta_C \left( \pi^u_u + \pi^u_d \right).
\]

Note that his bad performance in market \( C \) is self-revealing and the principal can penalize him by giving him a zero payment regardless of the return in the primary market. His expected utility from exerting effort and following the two-tiered investment strategy is given in (10). Thus, the additional constraint is

\[
0.25 \left[ (1 + 2\Delta_a + 2e) + (0.5 - \Delta_a - e) (0.5 + \Delta_b) \right] \pi^u_u + 0.25 (0.5 + \Delta_a + e) (0.5 + \Delta_b) \pi^u_d
\]

\[
+ 0.25 \left[ (1 - 2\Delta_a - 2e) + (0.5 + \Delta_a + e) (0.5 - \Delta_b) \right] \pi^d_d + 0.25 (0.5 - \Delta_a - e) (0.5 - \Delta_b) \pi^u_d
\]

\[
+ 0.25 (0.5 + \Delta_a + e) \pi^0_d + 0.25 (0.5 - \Delta_a - e) \pi^0_u - k \geq 0.5\eta_C \left( \pi^u_u + \pi^u_d \right).
\]

(19)

Note that rewards for positive performance stimulate the risk-seeking behavior because the right-hand side of this inequality increases with \( \pi^u_u \) and \( \pi^u_d \). If these payments are high, the contract has to raise payments for other outcomes (possibly for bad performance), to increase the opportunity cost of seeking the tail risk. As a result, adding this constraint can further increase the agency cost of implementing the two-tiered investment strategy.

In Proposition 2, we have shown that in the absence of tail risk the least costly contract involves only three non-zero payments. In the presence of the tail risk in market \( C \), we now need to minimize the expected compensation cost in (16) subject to constraints in (12), (15) and (19). The next proposition shows that when \( \eta_C \) is sufficiently large, the constraint in (19) is binding. As a result, the previously zero payments \( \{ \pi^d_d, \pi^0_u \} \) can now turn positive. The next proposition also shows that \( \pi^d_d \) turns positive before the other two.

**Proposition 4** When \( \eta_C \) is sufficiently large, the constraint in (19) is binding. Furthermore, \( \pi^d_d \) turns positive before \( \pi^d_u \) and \( \pi^0_u \) under the sufficient condition that

\[
0.5 + \Delta_B < 2\Delta_A + 2e.
\]

\(^{13}\) This situation is similar to the optimality of double-deviation in the dynamic moral hazard problem with private (hidden) saving, where the agent usually finds it optimal to shirk and save concurrently, e.g., Kocherlakota (2004) and He (2010). As investment positions are not observable in our model, investing in market \( C \) plays the same role as private saving in that context.
The reason to give a positive payment $\pi^d_d$ is that it increases the left-hand side of (19), i.e., the opportunity cost for the manager to seek the unwarranted tail risk. Although we will only prove that $\pi^d_d$ turns positive before $\pi^d_u$ and $\pi^0_u$ under the given sufficient condition, we have further verified by using numerical solutions that $\pi^d_d$ and $\pi^0_u$ always remain zero in a large set of parameter values even outside the sufficient condition.

Note that $\pi^d_d$ is zero in the least costly incentive contract derived in Proposition 2, just like $\pi^d_u$ and $\pi^0_u$, when the tail risk is absent. What makes $\pi^d_d$ different from $\pi^d_u$ and $\pi^0_u$ when the tail risk is present? Their cost to incentive ratios in motivating the manager’s effort are all negative. But, the cost to incentive ratio of $\pi^d_d$

\[
\left( \frac{p}{Dp} \right)^d_d = -\frac{(1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e) (0.5 - \Delta_B)}{(1.5 + \Delta_B) e}
\]

is lower than (so the absolute value is higher than) that of $\pi^d_u$ and $\pi^0_u$:

\[
\left( \frac{p}{Dp} \right)^d_u = \left( \frac{p}{Dp} \right)^0_u = -\frac{0.5 - \Delta_A - e}{e}.
\]

This result is due to the same argument in Section 2.4.2: the outcome $\left( \frac{d}{d} \right)$ makes it difficult for the principal to distinguish (bad) performance from (bad) luck. As a result, if the principal has to give a payment that comes with a negative incentive in order to deter risk-seeking, a payment for $\left( \frac{d}{d} \right)$ causes the least damage on the ex ante incentive.

To further illustrate the effects of tail risk on the least costly incentive contract for implementing the two-tiered strategy, we adopt the following baseline parameters:

\[
\begin{align*}
    r &= 0.25, \quad \Delta_A = 0.25, \quad \Delta_B = 0.2, \quad k = 0.02, \quad e = 0.2, \quad \alpha = 0.6.
\end{align*}
\]

Figure 1 plots four payments $\pi^u_u$, $\pi^u_d$, $\pi^0_d$, and $\pi^d_d$ as $\eta_C$ increases from 0.5 to 0.8. The tail-risk constraint in (19) already starts to bind when $\eta_C$ passes 0.65, a level below $0.5 + \Delta_B = 0.7$. Consistent with our earlier discussion, this suggests that the double-deviation of shirking and risk-seeking is more desirable to the manager than the single deviation of risk-seeking only, and thus the constraint in (19) binds earlier than $\eta_C > 0.5 + \Delta_B$.

When $\eta_C$ is between 0.65 and 0.67, the least costly contract offsets the risk-seeking incentive by increasing $\pi^u_u$, reducing $\pi^u_d$ and $\pi^0_d$, and keeping $\pi^d_d$ at zero. As we have discussed in the last section, $\pi^u_u$ is not as efficient as $\pi^d_d$ and $\pi^0_d$ in incentivizing the manager’s effort but it is useful for deterring risk seeking because its coefficient on the left-hand side of (19) is greater than that on the right-hand side. As a result, the expected compensation cost increases with $\eta_C$. 

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When $\eta_C$ rises above 0.67, simply increasing $\pi_u^u$ is not enough. Instead, the optimal contract gives a positive payment $\pi_d^d$ even though it has a negative effect on inducing the manager’s effort in the primary market. To counter the negative incentive effect provided by $\pi_d^d$, the contract has to simultaneously increase the other three payments ($\pi_u^u$, $\pi_d^u$, and $\pi_d^0$) which have positive incentive effects. Because of the intricate interaction between the manager’s incentive-provision constraint and risk-seeking constraint, the expected compensation cost increases dramatically with $\eta_C$ once it passes above 0.67.

Furthermore, Figure 1 shows that the incentive slope $\pi_d^d - \pi_u^u$ decreases with $\eta_C$. This is because $\pi_d^u$ is particularly strong in motivating the risk-seeking behavior—when $\eta_C$ is large, the coefficient of $\pi_d^u$ on the right-hand side of (19) exceeds that on the left-hand side. As a result, the contract has to reduce $\pi_d^u$ to mitigate such an incentive.

The increased agency cost makes the single-market strategy more desirable. Figure 2 plots the upper threshold $k^*$ of the manager’s effort cost for the optimality of implementing the two-tiered strategy. When $\eta_C$ is below 0.65, $k^*$ is insensitive to $\eta_C$. As $\eta_C$ rises above 0.65, $k^*$ decreases with $\eta_C$. This plot suggests that in the presence of tail risk, only managers with sufficiently high talents (and thus low effort cost) are encouraged to pursue investment
opportunities outside their designated markets. The next proposition formally proves this result.

**Proposition 5** *In the presence of the tail risk in market C, the upper threshold $k^*$ on the manager’s effort cost for the optimality of implementing the two-tiered strategy decreases with the tail risk parameter $\eta_C$.***

Taken together, our analysis demonstrates that the presence of tail risk increases the agency cost of encouraging the manager to pursue opportunities outside the primary market. As a result, narrow investment mandates become desirable for more fund managers, whose effort cost is higher than a lowered threshold. In light of the easier accessibility of tail risk in the increasingly complex financial markets in recent years, this result thus explains the recent trend of the growing popularity of stringent investment mandates and narrow tracking errors highlighted by the BIS report (2003).

Our model also shows that for those managers with exceptional talents, the optimal incentive contract not only encourages them to pursue flexible investment strategies but also rewards them generously. Interestingly, the reward covers not just their good performances but also their well-intentioned failures. The reward for failures might appear counter-intuitive
because of its seemingly negative incentive effect. But it helps deter tail-risk seeking because the managers stand to lose such a reward if they choose to seek risk. In other words, since managers are getting paid generously for pursuing the intended strategies, they will find seeking tail risk too costly as it risks the generous payments that they already have.

Philippon and Reshef (2008) find that wages for financial jobs were excessively high around 1930 and from the mid 1990s to 2006. They attribute the high wages to financial deregulation during these periods, which made financial jobs more skill intensive and complex and thus attracted better talents to the financial industry. In light of our analysis, financial deregulation not only makes financial jobs more demanding, but also creates more room for traders and fund managers to take on creative tail risk. As a result, higher wages are necessary not only because the financial workers’ reservation wages were higher, but also because the damages they could do to the firms were also higher.

4 Further Discussions

The wide usage of narrow investment mandates in the asset management industry have important implications for asset market dynamics. In this section, we discuss such implications on capital immobility and market segmentation.

4.1 Capital Immobility

The stringent investment mandates imposed on fund managers can lead to “capital immobility,” i.e., capital often fails to flow to distressed markets that offer profitable opportunities. Duffie (2010) highlights this phenomenon as an important factor in understanding market liquidity. For example, many pundits observe that capital immobility was a key factor leading to the 1998 financial market crisis - margin calls forced the hedge fund Long Term Capital Management to liquidate its large leveraged positions in fixed income securities while not enough capital came to absorb its liquidation. Froot and O’Connell (1999) show that the supply of capital in the catastrophe insurance market is inelastic because there are times during which the price of catastrophe insurance seems to be high and the capital of catastrophe insurers is low. Other examples include the depressed convertible bond market after convertible hedge funds faced large redemption of capital from investors in 2005 (e.g., Mitchell, Pedersen, and Pulvino, 2008), the temporary price discount for stocks after fire sales by mutual funds (e.g., Coval and Stafford, 2007), and the distressed market for newly down-graded junk bonds (e.g., Da and Gao, 2008).
Our model provides a new hypothesis for capital immobility during liquidity crises based on agency frictions at the originating end of capital flow. The economy could well have adequate capital. However, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations can motivate stringent investment mandates on the fund managers, which in turn confine the capital in its initial market segments. Even if one segment runs out of capital later and ends up in a liquidity crisis, fund managers in other market segments may be unwilling to move in because of the potential tracking errors. Instead, the strategic decisions of moving capital across different segments are largely left to the less informed investors themselves. As a result, the capital flow is likely to be delayed. Only as the crisis deteriorates will the distressed segment gradually attract capital from other segments, starting from funds that face broader investment mandates and greater tolerance for tracking errors. Eventually, investors will also recognize profit opportunities created by the crisis and move capital from other segments to the distressed segment.

Our agency based hypothesis of capital immobility complements the growing literature that studies the impact of financial intermediaries’ capital inside the crisis market under the premise that outside capital would not flow in (e.g., Kyle and Xiong (2001), Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), He and Krishnamurthy (2008), and Bolton, Santos, and Scheinkman (2008)). These studies typically motivate this premise based on various information barrier arguments about the distressed market at the receiving end of capital flow, e.g., outside investors hesitate to invest in the crisis market because they cannot distinguish whether the price drop is driven by liquidity reasons or worsened fundamentals. Our hypothesis is also different from those based on search frictions (e.g., Duffie, Garleanu, and Pedersen (2005) and Duffie (2010)), who suggest that the speed of capital flow depends on the rate of random matching between buyers and sellers.

4.2 Market Segmentation

There is growing evidence of de facto segmentation of various asset markets from the broad financial markets, even in the absence of explicit regulatory and physical constraints on investment to these markets. For example, Bekaert, Harvey, Lundblad, and Siegel (2008) show that many emerging markets are still segmented from the global financial markets even though the regulatory constraints on foreign investment had been largely lifted over the past few decades. In particular, they find that after controlling for financial leverage and earnings volatility, emerging markets display a significantly higher industrial earnings yield
(the inverse of price to earnings ratio) than that of developed countries. A common argument is that information barriers may prevent investors from fully integrating assets of emerging markets into their portfolios, e.g., Merton (1987). However, investors can hire professional managers to overcome the information barriers. Then, it remains puzzling that the rapid growth of funds specializing in emerging markets in the recent years has not eliminated the segmentation of these markets.

Several other markets also exhibit similar de facto segmentation. In the corporate bond market, Collin-Dufresne, Goldstein, and Martin (2001) find that proxies for both changes in the probability of future default based on standard fundamental-driven credit risk models and for changes in the recovery rate can explain only a small fraction of the observed credit spread changes. Instead, a market-specific latent factor can explain a large fraction of the residuals. In the mortgage-backed securities market, Gabaix, Krishnamurthy, and Vigneron (2007) find that idiosyncratic prepayment risk carries a risk premium. In the S&P 500 index option market, Garleanu, Pedersen, and Poteshman (2009) find that demand pressure in one option contract increases its price as well as other correlated contracts. The de facto segmentation of these markets is even more puzzling as they are mostly traded by financial institutions and professional traders.

The narrow investment mandates derived in our model provide an explanation of the de facto segmentation of the aforementioned markets. When (uninformed) investors delegate their capital to a professional manager to invest in one of these markets, information barriers in these markets make it necessary to impose a stringent investment mandate on the manager in order to reduce agency cost in the delegation process. In other words, the manager has to invest primarily in this particular market, say Russia, and his compensation is closely tied to his fund performance. Thus, despite that the manager might work for well-diversified investors, his own pricing kernel is exposed to the idiosyncratic risk of the market. The market will exhibit de facto segmentation if the manager is the marginal investor. To sum up, our model suggests that agency frictions can lead to market segmentation despite that investors can hire professional managers to overcome information barriers in informationally opaque markets.

5 Conclusion

We analyze a realistic delegated asset management problem in which a principal hires a fund manager to invest his money in a multi-market environment. This implies that the
principal needs to motivate not only the manager’s effort in acquiring information, but also an investment strategy across the markets. Our model highlights a tradeoff between encouraging the manager to pursue the efficient investment strategy and the agency cost of incentivizing him. This tradeoff becomes especially severe when the manager can access tail risk outside his primary market. Building on this tradeoff, our model explains the increasingly stringent investment mandates faced by fund managers. Our analysis sheds light on capital immobility and market segmentation that are widely observed in financial markets and highlights important effects of tail risk on institutional incentive structures.

A Appendix

A.1 Proof of Proposition 1

We need to verify that the manager has no incentive to deviate and invest in market $B$. First, consider a deviation strategy that he exerts effort on market $A$ and then follows the two-tiered investment strategy discussed in Section 2.4 (i.e., invest in market $B$ when $s_A^e = 0$ and $s_B^0 = 1$). The relevant situation is when he observes a negative signal in market $A$ and a positive signal in market $B$. Then, his expected utility from investing in market $B$ is

$$
0.25 (0.5 - \Delta_A - e) (0.5 + \Delta_B) \pi_u^u + 0.25 (0.5 - \Delta_A - e) (0.5 - \Delta_B) \pi_d^d
$$

$$
+ 0.25 (0.5 + \Delta_A + e) (0.5 + \Delta_B) \pi_u^d + 0.25 (0.5 + \Delta_A + e) (0.5 - \Delta_B) \pi_d^u
$$

which, under the contract specified in Proposition 1, is equal to $0.25 (0.5 - \Delta_A - e) (0.5 + \Delta_B) \pi_u^u$; if his expected return from investing in the risk-free asset is

$$(0.5 + \Delta_A + e) \pi_u^0 + (0.5 - \Delta_A - e) \pi_u^0$$

which is equal to $(0.5 + \Delta_A + e) \pi_u^0$ under the contract specified in Proposition 1. As $\pi_u^u = \pi_u^0$, the manager prefers to invest in the risk-free asset.

Next, we consider the deviation strategy that he exerts no effort and follows a two-tiered investment strategy. Then, his expected utility is

$$
0.25 [(1 + 2\Delta_A) + (0.5 - \Delta_A)(0.5 + \Delta_B)] \pi_u^u + 0.25 (0.5 + \Delta_A)(0.5 + \Delta_B) \pi_u^d
$$

$$
+ 0.25 [(1 - 2\Delta_A) + (0.5 + \Delta_A)(0.5 - \Delta_B)] \pi_d^d + 0.25 (0.5 - \Delta_A)(0.5 - \Delta_B) \pi_u^d
$$

$$
+ 0.25 (0.5 + \Delta_A) \pi_d^0 + 0.25 (0.5 - \Delta_A) \pi_u^0
$$

which is modified from equation (10) by removing the manager’s effort. Under the contract
given in Proposition 1, the manager’s expected utility is equal to

\[
0.25 \left[ (1 + 2A) + (0.5 - D_A) (0.5 + D_B) \right] \pi_u^u + 0.25 (0.5 + D_A) \pi_d^0 \\
= 0.25 \left[ (0.5 - D_A) (0.5 + D_B) + 1.5 + 3D_A \right] \frac{k}{e} \\
< 0.25 \left[ (0.5 - D_A) + 1.5 + 3D_A \right] \frac{k}{e} = 0.5 \left( 1 + D_A \right) \frac{k}{e}.
\]

By substituting the equilibrium contract into equation 4, the manager’s expected utility from exerting effort and following the single-market strategy is \(0.5 \left( 1 + 2D_A \right) \frac{k}{e}\), which is strictly higher than that from the deviation strategy.

**A.2 Proof of Proposition 2**

We need to minimize the expected compensation cost in (16) subject to the two incentive constraints in (12) and (15). For the six incentive payments \(\pi_d^0, \pi_u^d, \pi_d^u, \pi_u^u, \pi_d^u\), and \(\pi_u^0\), the first-order conditions subject to the two constraints are

\[
\frac{(1 + 2D_A + 2e) + (0.5 - D_A - e) (0.5 + D_B)}{U' [U^{-1} (\pi_u^u)]} \geq \lambda_1 e \left( 1.5 - D_B \right) + \lambda_2 (0.5 - D_A - e) (0.5 + D_B) \\
\]

with equality if \(\pi_u^u > 0\);

\[
\frac{(0.5 + D_A + e) (0.5 + D_B)}{U' [U^{-1} (\pi_u^u)]} \geq \lambda_1 e (0.5 + D_B) + \lambda_2 (0.5 + D_A + e) (0.5 + D_B) \\
\]

with equality if \(\pi_u^u > 0\);

\[
\frac{(1 - 2D_A - 2e) + (0.5 + D_A + e) (0.5 - D_B)}{U' [U^{-1} (\pi_d^d)]} \geq -\lambda_1 e \left( 1.5 + D_B \right) + \lambda_2 (0.5 + D_A + e) (0.5 - D_B) \\
\]

with equality if \(\pi_d^d > 0\);

\[
\frac{(0.5 - D_A - e) (0.5 - D_B)}{U' [U^{-1} (\pi_d^d)]} \geq -\lambda_1 e (0.5 - D_B) + \lambda_2 (0.5 - D_A - e) (0.5 - D_B) \\
\]

with equality if \(\pi_d^d > 0\);

\[
\frac{(0.5 + D_A + e)}{U' [U^{-1} (\pi_u^u)]} \geq \lambda_1 e - \lambda_2 (0.5 + D_A + e) \\
\]

with equality if \(\pi_u^d > 0\);

\[
\frac{(0.5 - D_A - e)}{U' [U^{-1} (\pi_d^d)]} \geq -\lambda_1 e - \lambda_2 (0.5 - D_A - e), \\
\]

with equality if \(\pi_d^0 > 0\).

The following lemma verifies that both \(\lambda_1\) and \(\lambda_2\) are positive.
Lemma 6 \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \).

Proof. First, the incentive constraint in (12) must be binding. This is because if this constraint is slack, the solution to minimize the compensation cost would be to set all payments to be zero. This solution, however, violates the constraint in (12). Thus, \( \lambda_1 > 0 \). Now suppose that \( \lambda_1 > 0 \) but \( \lambda_2 = 0 \), i.e., the constraint in (15) is slack. By minimizing the compensation cost subject to (12) we have

\[
\pi^0_u = \pi^d = \pi^d = 0, \text{ and } \pi^u = \pi^0_d > \pi^u_0 > 0.
\]

To see this, setting \( \lambda_2 = 0 \), then

\[
U'[U^{-1}(\pi^u_d)] = U'[U^{-1}(\pi^0_d)] = \frac{0.5 + \Delta_A + \epsilon}{\lambda_1 \epsilon} < \frac{(1 + 2\Delta_A + 2\epsilon) + (0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B)}{\lambda_1 \epsilon (1.5 - \Delta_B)} = U'[U^{-1}(\pi^0_u)]
\]

Since \( U'[U^{-1}(\pi)] \) is strictly decreasing in \( \pi \), we have our claim. Now given this, it is direct to verify that this solution violates the constraint in (15). Therefore, both constraints must be binding, i.e., \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \).

As \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), the right-hand side of equation (26) is negative. Thus, \( \pi^0_u = 0 \). Intuitively, this is because \( \pi^0_u \) has a negative incentive differential \( Dp_{\omega} < 0 \) in both of the incentive constraints (12) and (15).

The following lemma further determines \( \pi^u_d \) and \( \pi^d_d \) to be zero.

Lemma 7 If \( \pi^0_d > 0 \), then \( \pi^u_d = \pi^d_d = 0 \).

Proof. Based on (25), \( \pi^0_d > 0 \) requires that \( \lambda_1 \epsilon > \lambda_2 (0.5 + \Delta_A + \epsilon) \). Therefore, \( \pi^u_d = 0 \) and \( \pi^d_d = 0 \) because the right-hand side of (23) and (24) are negative, while the left-hand side is always positive.

As a result, there are only three positive payments: \( \pi^u_d, \pi^0_d, \) and \( \pi^u_u \) in the least costly contract. These three payments, together with \( \lambda_1 \) and \( \lambda_2 \), satisfy the binding incentive constraints in (12) and (15):

\[
(1.5 - \Delta_B) e \pi^u_u + (0.5 + \Delta_B) e \pi^u_d + e \pi^0_d = 4k \tag{27}
\]

\[
(0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B) \pi^u_u + (0.5 + \Delta_A + \epsilon)(0.5 + \Delta_B) \pi^u_d - (0.5 + \Delta_A + \epsilon) \pi^0_d = 0 \tag{28}
\]

and the first-order-conditions in (21), (22), (25).

The following lemma provides the ranks of the three positive payments.
Lemma 8 \( \pi_u > \pi_d^0 \) and \( \pi_u^0 > \pi_u^d \).

**Proof.** Since \( \lambda_2 > 0 \), equations (22) and (25) directly imply that \( \pi_u^d > \pi_d^0 \). To show \( \pi_u^d > \pi_u^d \), note that

\[
\frac{1}{U'[U^{-1}(\pi_u^d)]} = \frac{e(1.5 - \Delta_B) \lambda_1 + (0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B) \lambda_2}{(1 + 2\Delta_A + 2\epsilon) + (0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B)} < \frac{e(1.5 - \Delta_B) \lambda_1}{(1 + 2\Delta_A + 2\epsilon) + (0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B) + \lambda_2}.
\]

Because \( \frac{1}{U'[U^{-1}(\pi_u^d)]} = \frac{\lambda_1 e}{0.5 + \Delta_A + \epsilon} + \lambda_2 \), it suffices to show that

\[
\frac{1.5 - \Delta_B}{(1 + 2\Delta_A + 2\epsilon) + (0.5 - \Delta_A - \epsilon)(0.5 + \Delta_B)} < \frac{1}{0.5 + \Delta_A + \epsilon}
\]

which holds because \( 1.5 - \Delta_B < 2 \). \( \blacksquare \)

We need to verify that the manager will not pursue any deviation strategy. Two of these strategies have been considered in the main text. Consider the following deviation strategy: the manager shirks; he invests in market \( A \) if \( s_A^0 = 1 \), otherwise he gambles in market \( B \) regardless of \( s_B \). To prevent the use of this strategy, we require that his expected utility from using it

\[
0.25(1.5 + \Delta_A)\pi_u^u + 0.25(0.5 + \Delta_A)\pi_d^u + 0.25[1.5 - \Delta_A]\pi_d^d + 0.25(0.5 - \Delta_A)\pi_u^d
\]

(29)

to be dominated by his expected utility given in (10). Another deviation strategy is shirking, investing in market \( B \) if \( s_B^0 = 1 \), and otherwise gambling in market \( A \). To prevent the use of this strategy we also require that the manager’s expected utility from this strategy

\[
0.25(1.5 + \Delta_B)\pi_u^u + 0.25(0.5 + \Delta_B)\pi_d^u + 0.25[1.5 - \Delta_B]\pi_d^d + 0.25(0.5 - \Delta_B)\pi_u^d
\]

(30)

to be dominated by that in (10). Note that \( \pi_u^u > \pi_d^d \) and \( \pi_u^d = 0 \) in the derived optimal contract. As \( \Delta_A \geq \Delta_B \), the manager’s expected utility from using the first deviation strategy in (29) dominates that from using the second one in (30). Therefore to verify that (29) is dominated by (10) it suffices to show that

\[
[1 + 2\Delta_A + (0.5 - \Delta_A)(0.5 + \Delta_B)]\pi_u^u + (0.5 + \Delta_A)(0.5 + \Delta_B)\pi_d^u + (0.5 + \Delta_A)\pi_d^d
\]

\[
\geq (1.5 + \Delta_A)\pi_u^u + (0.5 + \Delta_A)\pi_d^d,
\]

which is equivalent to

\[
[(0.5 - \Delta_A)\pi_u^u + (0.5 + \Delta_A)\pi_d^u] \leq (0.5 + \Delta_A)\pi_d^d.
\]
Since $\pi_u < \pi_d^0$, we have

$$[(0.5 - \Delta_A) \pi_u + (0.5 + \Delta_A) \pi_d^0] (0.5 - \Delta_B) < [(0.5 - \Delta_A - e) \pi_u + (0.5 + \Delta_A + e) \pi_d^0] (0.5 - \Delta_B)$$

$$= \frac{0.5 - \Delta_B}{0.5 + \Delta_B} (0.5 + \Delta_A + e) \pi_d^0$$

where the second equality is derived from the binding constraint in (15). Therefore, the following condition is sufficient to ensure that the two aforementioned deviation strategies do not bind:

$$\frac{0.5 - \Delta_B}{0.5 + \Delta_B} (0.5 + \Delta_A + e) < 0.5 + \Delta_A,$$

which requires that $e$ is relatively small.

Finally, the manager could also shirk and always invest in market $B$. To prevent the use of this deviation strategy we require that (10) dominates $0.25 (\pi_u + \pi_u^d + \pi_d^d)$. By using the binding constraint in (15), it suffices to show the following condition:

$$\left[1 + 2\Delta_A + (0.5 - \Delta_A) (0.5 + \Delta_B) + \frac{(0.5 + \Delta_A) (0.5 + \Delta_B) (0.5 - \Delta_A - e)}{0.5 + \Delta_A + e}\right] \pi_u^u + 2 (0.5 + \Delta_A) (0.5 + \Delta_B) \pi_d^u \geq \pi_u^u + \pi_d^u,$$

which holds under the following sufficient condition

$$2 (0.5 + \Delta_A) (0.5 + \Delta_B) > 1.$$  (32)

### A.3 Proof of Proposition 3

Based on the least costly contract derived in Proposition 2, the expected compensation cost of implementing the two-tiered strategy is

$$K^{TT} = 0.25 [(1 + 2\Delta_A + 2e) + (0.5 - \Delta_A - e) (0.5 + \Delta_B)] U^{-1} (\pi_u^u)$$

$$+ 0.25 (0.5 + \Delta_A + e) (0.5 + \Delta_B) U^{-1} (\pi_d^u) + 0.25 (0.5 + \Delta_A + e) U^{-1} (\pi_d^0),$$

which is greater than

$$0.25 [(0.5 + \Delta_A + e) (1.5 - \Delta_B)] U^{-1} (\pi_u^u) + 0.25 (0.5 + \Delta_A + e) (0.5 + \Delta_B) U^{-1} (\pi_d^u)$$

$$+ 0.25 (0.5 + \Delta_A + e) U^{-1} (\pi_d^0).$$  (33)

Suppose we minimize (33) by using nonegative $\pi_u^u$, $\pi_d^u$, and $\pi_d^0$ subject to (12). It should be clear that the minimum is lower than $K^{TT}$. The minimum is $(\frac{1}{2} + \Delta_A + e) \frac{3}{4} U^{-1} (\frac{4k}{3e}),$

which is obtained by letting

$$\pi_u^u = \pi_d^u = \pi_d^0 = \frac{4k}{3e}.$$
Because $U^{-1}(0) = 0$ and $U^{-1}$ is convex,

$$\left(\frac{1}{2} + \Delta_A + e\right) \frac{3}{4} U^{-1}\left(\frac{4k}{3e}\right) > \left(\frac{1}{2} + \Delta_A + e\right) U^{-1}\left(\frac{k}{e}\right) = K^{SM}.$$  

This in turn implies that $K^{TT} > \left(\frac{1}{2} + \Delta_A + e\right) \frac{3}{4} U^{-1}\left(\frac{4k}{3e}\right) > K^{SM}$.

We now show that $K^{TT} - K^{SM}$ is increasing in $k$. Note that in solving for the least costly contract in implementing the two-tiered strategy, (21), (22), (25), (27) and (28) have the feature that the solution $\pi_u^u, \pi_d^u, \pi_w^0$ are proportional to $k^{\frac{\alpha}{\gamma-b}}$ (note that when $U(c) = c^{1-\alpha}$, $\frac{1}{U'[U^{-1}(c)]} \propto \pi \frac{\alpha}{\gamma-b} \propto k^{\frac{\alpha}{\gamma-b}}$). As a result, the expected cost $K^{TT}$ is proportional to $k^{\frac{1}{\gamma-b}}$. Proposition 1 implies that the same statement also holds for $K^{SM}$. As a result, $K^{TT} - K^{SM}$ is proportional to $k^{\frac{1}{\gamma-b}}$. As $K^{TT} - K^{SM}$ is positive, it must be increasing with $k$.

### A.4 Proof of Proposition 4

We first show that (19) is binding when $\eta_C$ is sufficiently large. Since the left-hand side of (19) is independent of $\eta_C$ while the right-hand side increases with $\eta_C$, we only need to show that there exists one value of $\eta_C$ so that the least costly contract derived in Proposition 2 (which does not incorporate the constraint in (19)) violates (19). Because of the binding constraint in (12), we only need to show

$$[(1 + 2\Delta_A) + (0.5 - \Delta_A) (0.5 + \Delta_B)] \pi_u^u + (0.5 + \Delta_A) (0.5 + \Delta_B) \pi_d^0 < 2\eta_C (\pi_u^u + \pi_d^u).$$

Let $\eta_C = 0.5 + \Delta_A$. Then, we need to show that

$$(0.5 - \Delta_A) (0.5 + \Delta_B) \pi_u^u + (0.5 + \Delta_A) \pi_d^0 < (0.5 + \Delta_A) (1.5 - \Delta_B) \pi_d^u.$$  

Because $\pi_u^u < \pi_d^u$ and $\pi_d^0 < \pi_d^u$ in the contract, it suffices to show that

$$(0.5 - \Delta_A) (0.5 + \Delta_B) < (0.5 + \Delta_A) (0.5 - \Delta_B),$$

which holds since $\Delta_A > \Delta_B$.

To verify the second part of the proposition, we need to derive the first order conditions for deriving the least costly contract. We repeat the minimization of the total compensation cost in (16) subject to constraints in (12), (15) and (19). We denote the Lagrange multiplier of the new constraint by $\lambda_3 \geq 0$. The first order conditions for the 6 payments are now given below:

$$\frac{(1 + 2\Delta_A + 2e) + (0.5 - \Delta_A - e) (0.5 + \Delta_B)}{U'[U^{-1}(\pi_u^u)]} \geq \lambda_1 e (1.5 - \Delta_B) + \lambda_2 (0.5 - \Delta_A - e) (0.5 + \Delta_B)$$

$$+ \lambda_3 [1 + 2\Delta_A + 2e - 2\eta_C + (0.5 - \Delta_A - e) (0.5 + \Delta_B)]$$  

(34)
with equality if $\pi_u^d > 0$;

$$
\frac{(0.5 + \Delta_A + e) (0.5 + \Delta_B)}{U' [U^{-1} (\pi_u^d)]} \geq \lambda_1 e (0.5 + \Delta_B) + \lambda_2 (0.5 + \Delta_A + e) (0.5 + \Delta_B) - \lambda_3 [2\eta_C - (0.5 + \Delta_A + e) (0.5 + \Delta_B)]
$$

(35)

with equality if $\pi_u^d > 0$;

$$
\frac{(1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e) (0.5 - \Delta_B)}{U' [U^{-1} (\pi_d^u)]} \geq -\lambda_1 e (1.5 + \Delta_B) + \lambda_2 (0.5 + \Delta_A + e) (0.5 - \Delta_B) + \lambda_3 [1 - 2\Delta_A - 2e + (0.5 + \Delta_A + e) (0.5 - \Delta_B)]
$$

(36)

with equality if $\pi_d^u > 0$;

$$
\frac{(0.5 - \Delta_A - e) (0.5 - \Delta_B)}{U' [U^{-1} (\pi_u^d)]} \geq -\lambda_1 e (0.5 - \Delta_B) + \lambda_2 (0.5 - \Delta_A - e) (0.5 - \Delta_B) + \lambda_3 (0.5 - \Delta_A - e) (0.5 - \Delta_B)
$$

(37)

with equality if $\pi_u^d > 0$;

$$
\frac{(0.5 + \Delta_A + e)}{U' [U^{-1} (\pi_d^u)]} \geq \lambda_1 e - \lambda_2 (0.5 + \Delta_A + e) + \lambda_3 (0.5 + \Delta_A + e)
$$

(38)

with equality if $\pi_d^u > 0$;

$$
\frac{(0.5 - \Delta_A - e)}{U' [U^{-1} (\pi_u^d)]} \geq -\lambda_1 e - \lambda_2 (0.5 - \Delta_A - e) + \lambda_3 (0.5 - \Delta_A - e)
$$

(39)

with equality if $\pi_u^d > 0$.

By comparing (37) and (39), it is easy to show that $\pi_u^d = 0$ implies that $\pi_d^u = 0$. This implies that we only need to compare $\pi_d^u$ and $\pi_u^d$. From (36), $\pi_d^u$ is positive

$$
-\lambda_1 e \frac{1.5 + \Delta_B}{0.5 + \Delta_A + e} + \lambda_2 (0.5 - \Delta_B) + \lambda_3 (0.5 - \Delta_B) + \lambda_3 \frac{1 - 2\Delta_A - 2e}{0.5 + \Delta_A + e}
$$

(40)

is positive and zero otherwise; while $\pi_u^d$ is positive if

$$
-\lambda_1 e \frac{0.5 - \Delta_B}{0.5 - \Delta_A - e} + \lambda_2 (0.5 - \Delta_B) + \lambda_3 (0.5 - \Delta_B)
$$

(41)

is positive and zero otherwise. Now consider the following sufficient condition that

$$
0.5 + \Delta_B < 2\Delta_A + 2e.
$$

Under this sufficient condition, we have

$$
\frac{1.5 + \Delta_B}{0.5 + \Delta_A + e} < \frac{0.5 - \Delta_B}{0.5 - \Delta_A - e}.
$$

This implies that (40) is greater than (41), i.e., $\pi_d^u$ becomes positive before $\pi_u^d$ turns positive.
A.5 Proof of Proposition 5

The argument for the agency cost to be increasing with $k$ follows the same argument in Proposition 3, which implies that the expected compensation cost is of order $k^{\frac{1}{1+\alpha}}$. Note that the derivative of the expected compensation cost with respect to $\eta_C$ is $0.5 \lambda_3 (\pi_u^1 + \pi_d^1) \geq 0$, which is strictly positive when the constraint in (19) is binding. Therefore the expected compensation cost in the presence of tail risk increases with $\eta_C$, and as a result $k^*$ is decreases with $\eta_C$.

References


Da, Zhi and Pengjie Gao (2008), Clientele change, persistent liquidity shock, and bond return reversal after rating downgrades, Working paper, University of Notre Dame.


He, Zhiguo, and Arvind Krishnamurthy (2008), A model of capital and crisis, NBER working paper #14366.


Jame, Russell (2010), Organizational structure and fund performance: pension funds vs. mutual funds, Job market paper, Emory University.


Kostovetsky, Leonard (2009), Human capital flows and the financial industry, Working paper, University of Rochester.


Malliaris, Steven and Hongjun Yan (2009), Reputation concerns and slow-moving capital, Working paper, Yale SOM.


Makarov, Igor and Guillaume Plantin (2010), Rewarding trading skills without inducing gambling, Working paper, LBS.


Palomino, Frédéric and Andrea Prat (2003), Risk taking and optimal contracts for money managers, *Rand Journal of Economics* 34, 113-137.


Vayanos, Dimitri and Paul Woolley (2008), An institutional theory of momentum and reversal, Working paper, LSE.