Delegated asset management, investment mandates, and capital immobility

Zhiguo He\textsuperscript{a,b}, Wei Xiong\textsuperscript{c,b,*}

\textsuperscript{a} University of Chicago, Booth School of Business, United States
\textsuperscript{b} NBER, United States
\textsuperscript{c} Princeton University, United States

\textbf{A B S T R A C T}

This paper develops a model to explain the widely used investment mandates in the institutional asset management industry based on two insights: first, giving a manager more investment flexibility weakens the link between fund performance and his effort in the designated market, and thus increases agency cost. Second, the presence of outside assets with negatively skewed returns can further increase the agency cost if the manager is incentivized to pursue outside opportunities. These effects motivate narrow mandates and tight tracking error constraints to most fund managers except those with exceptional talents. Our model sheds light on capital immobility and market segmentation that are widely observed in financial markets, and highlights important effects of negatively skewed risk on institutional incentive structures.

\section{1. Introduction}

The institutional asset management industry has experienced rapid growth in the last two decades. In 2008, the global asset management industry managed a total of around $90 trillion, through various types of funds, such as pension funds, mutual funds, insurance funds, hedge funds, private equity funds, and exchange traded funds. This sheer size already makes institutional investors a key player in the financial markets. Their distinctive institutional incentive structures also make their preferences and investment characteristics very different from those of individual investors. The Bank for International Settlements (BIS) initiated a working group on incentive structures in the institutional asset management industry. By interviewing more than one hundred industry practitioners from 14 countries, the working group identified several general trends in this industry. One of them is the use of more stringent investment mandates, i.e., “a tiering and narrowing of investment mandates, enhanced by an increasing emphasis on relative performance measurement, narrowing tracking errors and more pervasive use of other investment constraints, such as limits on investing in specific securities or...
been widely adopted to analyze the implicit incentives, explicit incentives. The career-concern framework has approaches to analyze the effects of these implicit and explicit incentives from investors to fund families, and two layers of implicit and explicit incentives: one layer of incentive structures and investment strategies. Financial service companies, such as Fidelity and TIAA-CREF, offer families of investment funds for investors to choose from and typically charge a fixed fee based on assets under management. New fund inflows after superb fund performance provide implicit incentives for the fund family to hire a capable fund manager (e.g., Brown, Harlow, and Starks, 2006; Chevalier and Ellison, 1997). An equally important aspect which has not received much attention in the empirical literature is that the majority of individual fund managers’ compensation is a relative-performance-based bonus (Bank for International Settlements, 2003). These observations suggest that the asset management industry builds on two layers of implicit and explicit incentives: one layer of implicit incentives from investors to fund families, and another layer of explicit incentives from fund families to fund managers. The literature has developed distinctive approaches to analyze the effects of these implicit and explicit incentives. The career-concern framework has been widely adopted to analyze the implicit incentives, while the optimal contracting approach has been used to study the explicit incentives.4

Our model adopts the optimal contracting approach to focus on explicit incentives of individual fund managers. This approach allows us to derive investment mandates, which are usually explicitly stated in fund prospectuses and enforced by managers’ compensation contracts. Specifically, we analyze a model with a risk-neutral principal delegating capital to a risk-averse fund manager. In light of the two-layered incentive structure discussed above, one can interpret the principal as a fund family, which hires the manager to manage one of its funds. Different from the aforementioned models, our model allows the manager to face investment opportunities in several markets instead of one. The manager has a primary market. While his expertise endows him some private information about the asset return in this market, he can improve the precision of his information by exerting unobservable costly effort. Thus, the principal needs to incentivize the manager to work. In addition, the manager’s expertise also gives him a free signal about the asset return in another market, whose identity is known only to the manager. This free signal is not as precise as his signal about the primary market, but is nevertheless useful.

We deviate from a commonly used framework (e.g., Bhattacharya and Pfleiderer, 1985; Dynvig, Farnsworth, and Carpenter, 2010), in which the principal incentivizes the manager to acquire information and truthfully report the information to the principal, who then makes investment decisions based on the reported information. Our model builds on the premise that the principal is unable to execute the investment decisions. Instead, the manager is responsible for not only acquiring information but also making investment decisions, consistent with the common practice in the money management industry. And, as only the manager can implement the investment decisions, we assume further that the incentive contract cannot be contingent upon the manager’s investment positions. Under this setting, the principal needs to motivate the manager not only to exert effort in acquiring information about the primary market, but also to make efficient investment choices to take advantage of his information about the primary market as well as the secondary market. We are particularly interested in analyzing the joint implications of these two dimensions on

---

1 Jame (2010) finds evidence that trades made by pension funds in non-Standard & Poor’s (S&P) 500 stocks significantly outperform their trades in S&P 500 stocks, and that tracking error constraints imposed on pension funds weaken the performance of their trades by roughly 30 basis points per year.

2 An argument based on investors’ demands for specialized funds can explain the existence of investment mandates that accompany specialized funds, but cannot explain the varying degree of tracking error constraints across funds.

3 The size of the bonus component in individual asset managers’ compensation varies considerably across countries. However, at least in some countries, there seems to be a general trend towards a higher share of variable compensation in total pay over recent years...US managers can earn average bonuses of 100% and higher. In the United Kingdom, where the median fund manager will get a bonus of about 100%, exceptional asset managers can earn as much as six times their base salary in the form of bonuses” (Bank for International Settlements, 2003, p. 23).


5 In practice, a fund manager can obscure his positions through complex financial contracts and thus can game any compensation scheme that is based upon his positions. The scandals of rogue traders, such as Nick Leeson of the bankrupted British bank Barings and Jerome Kerviel of French bank Societe Generale, vividly demonstrated that they were able to hide their positions from their supervisors for prolonged periods. These observations motivate us to consider incentive contracts based only on the fund performance and the primary market return.
the optimal incentive contract and the resulting investment efficiency.

A key insight of our model is that there is a trade off between ex post investment efficiency and ex ante incentive provision efficiency. Allowing the manager to take advantage of outside opportunities when he fails to find a good opportunity in the primary market is ex post efficient, but implementing this efficient strategy weakens the link between the fund performance and his unobservable effort in the primary market. This is because the manager can generate good performance either by effort in the primary market or by random luck (an opportunity unrelated to effort) from outside. In the language of Holmstrom (1979), implementing the efficient investment strategy reduces the ex ante incentive provision efficiency by making benchmarking more difficult.

Building on this trade off, our model shows that it can be optimal to confine the manager in his primary market depending on his cost of effort and outside investment opportunities. Intuitively, this holds true if his cost of effort is sufficiently high or if his free outside opportunities are only modest. Although the principal cannot directly observe the manager's investment position, he can implement such a strategy by imposing a tight limit on the manager's tracking error of the primary market return. More precisely, he can prevent the manager from seeking any outside opportunity by penalizing the manager's good performance if it deviates substantially away from the benchmark primary market return. On the other hand, if the manager's cost effort is sufficiently low or if his outside opportunities are abundant, it is optimal to incentivize him to pursue opportunities both inside and outside the primary market by granting a sufficient tolerance on tracking error and by rewarding him for beating the primary market.

The incentive to pursue outside opportunities can also induce the manager to seek unwarranted negatively skewed risk even when he finds no good opportunity. As widely recognized by academia and policy makers, active risk-seeking is a severe problem in designing institutional risk management systems and incentive structures. Lowenstein (2000) attributes the financial crisis of the hedge fund Long Term Capital Management to its negatively skewed risk in the increasingly complex financial markets, our model also explains the aforementioned trend of narrowing investment mandates in the delegated asset management industry.

The widely used narrow investment mandates can have important implications for asset market dynamics. Duffie (2010) highlights capital immobility, i.e., capital often fails to flow to liquidity distressed markets that offer profitable opportunities, as an important factor in understanding asset market liquidity. According to our model, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations constrain most fund managers from moving capital into other liquidity distressed markets. Instead, the strategic decisions of allocating capital across different market segments are often left to the less informed investors themselves. As a result, the flow of capital is likely to be delayed. This explanation of

---

7 Malliaris and Yan (2009) and Makarov and Plantin (2010) analyze fund managers' risk-seeking incentives driven by convex fund flow by using career-concern models. The career-concern models are not convenient for analyzing investors' active deterrence of managers' risk-seeking because investors are typically passive in these models. In contrast, our model shows that the incentive to seek negatively skewed risk is an inherent by-product of incentivizing fund managers to pursue profitable opportunities, and deterring such risk-seeking may require inducing them to use suboptimal investment strategies.

8 The mechanism of a negatively skewed risk differ from that of the standard volatility risk in delegated asset management (e.g., Ou-Yang, 2003; Palmolino and Prat, 2003, Cadenillas, Cvitanic, and Zapatero, 2007). These papers typically find that pay for performance is still useful even when fund managers can choose return volatility. In our model, incentive compensation on the primary market itself triggers active seeking of negatively skewed risk outside the primary market, which motivates the use of narrow investment mandates.
capital immobility based on institutional constraints at the originating end of capital flow is distinct from the other explanations based on information barriers about asset fundamentals at the receiving end.

Narrow investment mandates can also help explain the de facto segmentation of various asset markets from the broad financial markets in the absence of explicit regulatory and physical constraints on investment. Collin-Dufresne, Goldstein, and Martin (2001), Gabaix, Krishnamurthy, and Vigneron (2007), Garleanu, Pedersen, and Poteshman (2009), and Bekoort, Harvey, Lundblad, and Siegel (2008) provide evidence of risk premiums for market-specific risk factors in the corporate bond market, mortgage-backed securities market, S&P 500 index option market, and emerging stock markets. These findings are broadly consistent with our model in the sense that investors heavily rely on professional fund managers to invest in these markets and agency considerations can motivate narrow mandates on the fund managers. As a result, they are exposed to market specific risk. With fund managers likely being the marginal investor, these markets can exhibit premia for market specific risks and thus de facto segmentation.

The literature has recognized the importance of restrictions on investment strategies in motivating fund managers’ efforts. Admati and Pfleiderer (1997) point out that in the absence of portfolio restrictions, a manager can use portfolio choice to offset the incentive intended by his compensation contract. By using a setting whereby the principal can directly observe and contract on the fund manager’s investment positions, Dybvig, Farnsworth, and Carpenter (2010) show that imposing portfolio restrictions can improve the efficiency of incentive provisions. Our model adopts a more general setting in which the principal cannot observe the manager’s investment positions, and derives the optimal incentive contract that improves the signal precision by focusing on agency frictions only. See Massa (2003) and Mamaysky and Spiegel (2002) for studies of how heterogeneity among individual investors in terms of investment horizon and risk preferences can motivate mutual fund families to offer funds specializing in different markets or strategies.

2. The basic model

2.1. Setup

We consider a single-period principal-agent model where a risk-neutral principal delegates capital to a risk-averse agent. As we discussed before, the asset management industry has a two-layered incentive structure with fund families charging investors fixed management fees while compensating individual fund managers based on fund performance. We focus on the explicit incentives of individual fund managers who directly make investment decisions. Thus, we interpret the principal—agent relationship as a fund family (the principal) hiring a fund manager (the agent) to manage one of its funds.

The manager’s utility function over consumption $U(\cdot)$ satisfies $U(0) = 0$, $U'(\cdot) > 0$, and $U''(\cdot) < 0$. Throughout, we focus on the specification that

$$U(c) = c^{1-\alpha}, \quad \alpha \in (0,1).$$

The principal hires the manager to actively invest his money in a primary market, which we denote by market $A$. We can broadly interpret this market as a specific market sector, such as the Treasury bond market, the mortgage bond market, the U.S. stock market, or a regional stock market. We assume for simplicity that the return from this market can only take two possible values, a positive value $r$ or a negative one $-r$, with equal probability:

$$\tilde{r}_A = \begin{cases} r & \text{with probability 0.5,} \\ -r & \text{with probability 0.5,} \end{cases}$$

The manager – who possesses certain expertise that normal investors lack – obtains a private signal $s_A$ regarding the likelihood of the market going up or down. The signal takes two possible values, 1 or $-1$. If the return is positive (or negative), the signal is more likely to take the value 1 (or $-1$):

$$\Pr(s_A = 1 | \tilde{r}_A = r) = \Pr(s_A = -1 | \tilde{r}_A = -r) = 0.5 + \Delta_A + \theta.$$  \hfill (3)

The term $\Delta_A + \theta > 0$ measures the precision of the signal $s_A$ in revealing the return in market $A$. There are two components in the signal precision: the first part $\Delta_A$ captures the manager’s knowledge about the market without any effort on the job, while the second part $\theta$ represents his effort in acquiring additional information. The effort $\theta$ takes binary values, zero and $e$, corresponding to “shirking” and “working,” respectively. By working hard (e.g., conducting a thorough analysis), the manager improves the signal precision by $e$. We impose $\Delta_A + e \leq 0.5$ to make the probability meaningful. To differentiate the precision of the signal with and without the manager’s signal, risk-neutral principal is interested in maximizing expected fund return, and this assumption, by ruling out various hedging and diversification needs, allows us to focus on agency frictions only. See Massa (2003) and Mamaysky and Spiegel (2002) for studies of how heterogeneity among individual investors in terms of investment horizon and risk preferences can motivate mutual fund families to offer funds specializing in different markets or strategies.
effort, we denote $s^e_A$ as the signal with effort and $s^0_A$ as the signal without effort.

The effort incurs a private (utility) cost $k$ to the manager and is unobservable to the principal. For simplicity, we also assume that the manager exerts effort before he receives any signal.\(^1\) We assume that the manager has an additive utility function over consumption and effort:

$$U(c, \theta) = U(c) - \frac{k}{e} \theta, \text{ where } \theta \in [0, e].$$

The fund has one unit of initial capital. To deliver the key insight without getting into unnecessary complications, we simplify the manager’s investment choices. First, the manager cannot short sell any asset and cannot borrow either.\(^2\) Second, he always invests all of the fund in one asset: either in market $A$, the risk-free asset, or something else.\(^3\) We normalize the return of the risk-free asset to be zero. Then, if he observes a positive signal on market $A$, the expected return is positive and he will invest the fund in market $A$; if he observes a negative signal, then he should stay out of market $A$. But, should he then invest the fund in the risk-free asset or something else? In reality, a fund manager often has expertise beyond his primary market. An important question faced by every fund is whether the manager should be incentivized to pursue outside opportunities when the primary market lacks a good one.

We capture this idea by assuming that the manager can access a set of outside markets. These markets have independently and identically distributed returns with the same binomial distribution as market $A$. Before the manager makes his investment decision, he also receives a free signal about one of these markets, which we denote by market $B$. This market is randomly drawn from the pool of outside markets. Neither can the principal observe the signal, nor which market the signal is about. The precision of the manager’s free signal on market $B$ is $A_B \in \{0, 1/2\}$, i.e.,

$$\Pr(s_B = 1 | \hat{p}_B = r) = \Pr(s_B = -1 | \hat{p}_B = -r) = 0.5 + A_B. \quad (4)$$

We also assume that $A_B \leq A_A$, i.e., the manager is better informed about his primary market.

Denote the manager’s investment choice by $x \equiv \{x_A, x_B, x_0\}$, where $x_i \in \{0, 1\}$ indicates the manager’s investment position in market $i$ with $i \in \{A, B\}$, and $x_0 \in \{0, 1\}$ is his position in the risk-free asset. The borrowing constraint requires that $x_A + x_B + x_0 = 1$. We denote the set of all feasible investment choices by $X = \{x\}$. The fund’s return $\hat{r}_F$ can take three possible values, i.e., $\hat{r}_F \in \{r, 0, -r\}$.\(^4\)

\(^1\) In an earlier version of this paper, we allowed the principal to observe the identity of market $B$ and therefore to write the incentive contract based on the return of market $B$ as well. The key results of our paper remain similar.

\(^2\) Besides, we can further show that peer evaluation, i.e., basing one manager’s compensation on his relative performance to other funds trading in the same market, cannot help in optimal contracting. The reason is simple: in our model, conditional on the true state of the primary market return, the signals are independent across managers. Thus, if the contract incorporates the realized market returns, it has already used the best information for relative performance evaluation (one can formally show this result using the sufficient statistics argument in Holmström, 1979).

\(^3\) While many mutual funds explicitly impose various restrictions on investment positions such as short sales, use of leverages, and investing in derivatives, the enforcement of these restrictions often rely on random auditing and ex post penalties of the funds’ advisors and the Security Exchange Commission, e.g., Almazan, Brown, Carlson, and Chapman, 2004, footnote 2. This confirms the difficulty for fund investors to observe fund positions in practice and the potential challenge in enforcing investment restrictions. Under the premise that investment positions are not observable to the principal, our model not only justifies investment mandates but also derives penalties against tracking errors as an enforcement mechanism for investment mandates.
Thus, an incentive contract $\Pi$ is a mapping from the information set $\Omega$ generated by $f_A$ and $f_B$ to non-negative payments to the manager: $\Pi : \Omega \equiv \{u, d\} \times \{u, d\} \to \mathbb{R}_+$.

We rule out negative wages to the manager due to limited liability.\footnote{We assume that the manager is both risk-averse and protected by limited liability. If we were assuming a risk-neutral manager, then limited liability would have generated a non-zero agency cost but the agency cost would not be affected by efficiency of benchmarking in implementing different investment strategies. The reason is that with risk-neutral managers, it is optimal for the principal to shift all positive rewards to one good outcome (across two investment strategies, say, $\Omega$) where the manager’s investment strategy is identifiable. This gives rise to the same agency cost across different investment strategies.} The fund return can take three possible values: $u$ (up with a return of $r$), zero, or $d$ (down with a return of $-r$). The return of market $A$ can take two possible values, $u$ (up) or $d$ (down). There are six possible outcomes. Therefore, the contract only needs to specify six contingent payments. Denote $\omega = (f_A, f_B) \in \Omega$ as a possible outcome. It is easier to work with the payment in terms of the manager’s utility ($\pi_\omega$) than in terms of dollars ($c_\omega$), where $\pi_\omega = U(c_\omega)$. We write the contract as

$$\Pi = \{p_\omega = (u, u, \ldots, u, d, d, \ldots, d) \}.$$ \hspace{1cm} (5)

For instance, $p_\omega$ is the manager’s utility when the primary market is down but the fund return is up. Then, $c_\omega = U^{-1}(\pi_\omega)$ is the cost of compensating the manager for this outcome.

### 2.2.2. Contracting problem

For a given contract $\Pi = \{\pi_\omega\}$, the fund manager maximizes his expected utility by first making his optimal effort and then investment choices based on the signals he receives:

$$\max_{\theta \in \{0, \varepsilon\}, x \in \mathbb{X}} \sum_{\omega \in \Omega} p_\omega(\theta, x) \pi_\omega - \frac{k}{\varepsilon} \theta,$$

where $p_\omega$ is the probability of outcome $\omega$. The manager’s effort and investment choices $\theta$ and $x$ determine the outcome probability $p_\omega$. We write $\theta^*(\Pi)$ and $x^*(\Pi)$ as the manager’s optimal effort and investment choices, respectively, in response to a given contract $\Pi$.

By using different incentive contracts, the principal can induce different investment choices from the manager. When the manager’s effort cost is sufficiently low, the first-best combination of effort and investment strategies is that the manager exerts effort in the primary market and then follows a so-called “two-tiered” investment strategy, as shown in Table 1: if the manager receives a positive signal about the primary market $A$, he will invest the fund capital in it; if not, he will invest in market $B$ if his free signal about market $B$ indicates a good opportunity; finally, he will invest in the risk-free asset if his signals about both markets $A$ and $B$ are negative. This strategy instructs the manager to take advantage of opportunities outside his primary market. Alternatively, the fund can also implement a “single-market” strategy, as shown in Table 1: the manager will invest in the primary market if his signal on the market is favorable and otherwise put the fund capital in the risk-free asset. Relative to the two-tiered strategy, this strategy requires the same effort cost but forgoes a valuable investment opportunity outside the primary market. As we will show later, this seemingly inferior strategy dominates the two-tiered strategy under certain conditions because of its more efficient incentive provision.\footnote{In an earlier draft of the paper, we have considered a symmetric setting, in which markets $A$ and $B$ are ex ante symmetric to the manager in the sense that his endowed signals about the two markets have the same precision and both can be improved by his personal effort. This symmetric setting allows us to evaluate a so-called combined-market strategy, according to which the manager seeks to find the best opportunity in these markets by exerting effort to improve his signals about both markets. We show that this strategy is desirable if the manager’s cost of effort is sufficiently low—lower than the level derived for the optimality of the two-tiered strategy. As this analysis substantially complicates the presentation but adds little additional insight, we choose to leave it out of the paper and instead focus on the comparison between the single-market and two-tiered strategies.}

As the principal cannot directly observe the manager’s investment position, he has to rely on the incentive contract to induce the manager to implement any intended strategy. Put differently, when the principal finds it optimal to use the single-market strategy, he cannot just tell the manager not to invest outside the primary market. In order to enforce this narrow investment mandate, the principal needs a set of detection and penalty mechanisms for preventing potential violation. In our model, the principal can use the fund’s tracking error relative to the primary market return to imperfectly detect the manager’s deviation from investing in the primary market. By penalizing the manager for a large tracking error, the principal can prevent such a deviation.

The manager has a reservation utility of $U$, which represents his forgone outside opportunity cost by managing this fund. The participation constraint requires that

$$\sum_{\omega \in \Omega} p_\omega(\theta^*(\Pi), x^*(\Pi)) \pi_\omega - \frac{k}{\varepsilon} \theta^*(\Pi) \geq U.$$

Because of limited liability, the manager earns some positive rent in our model, and for simplicity throughout this paper, we assume that $U$ is sufficiently small so that the manager’s participation constraint is not binding.

The principal’s payoff from outcome $\omega$ is the portfolio return minus the compensation cost:

$$W_\omega = 1 + \bar{f}_A(\omega) - U^{-1}(\pi_\omega).$$ \hspace{1cm} (6)

The principal maximizes the expected payoff from the fund by choosing an optimal incentive contract, i.e.,

$$V = \max_{\Pi} \sum_{\omega \in \Omega} p_\omega(\theta^*(\Pi), x^*(\Pi)) W_\omega,$$

subject to the manager’s participation and incentive compatibility constraints.

We can further decompose the principal’s expected payoff into two components:

$$V = \sum_{\omega \in \Omega} p_\omega(1 + \bar{f}_A(\omega)) - \sum_{\omega \in \Omega} p_\omega U^{-1}(\pi_\omega),$$ \hspace{1cm} (7)

where the first part is the expected fund return, which is determined by the manager’s effort and investment
strategy; and the second part is the expected cost of compensating the fund manager. This decomposition suggests the following two-step method to solve for the optimal contract: first, find the least costly contract to implement each of the two effort and investment strategies; then, compare these least costly contracts to determine the optimal contract that offers the highest expected net payoff to the principal.

2.3. Single-market strategy

We start with analyzing the least costly contract for implementing a single-market strategy in market \( A \), as indicated in Table 1. The contract induces the following effort and investment choices from the fund manager: the manager exerts effort only in market \( A \); after receiving the signal \( s_A^e \), he invests all the fund capital in market \( A \) if the signal is positive, and invests in the risk-free asset otherwise, regardless of his signal \( s_B \) about opportunities outside the primary market. Note that there is an opportunity loss when the manager’s signals suggest that the primary market lacks a good investment opportunity while another market, market \( B \), offers a good one (\( s_A^e = -1, s_B = 1 \)).

\[ s_A^e = 1, s_B^e = -1 \]

Table 1

<table>
<thead>
<tr>
<th>Signal realizations</th>
<th>Single-market</th>
<th>Two-tiered</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_A^e = 1, s_B^e = -1 )</td>
<td>Market ( A )</td>
<td>Risk-free asset</td>
</tr>
<tr>
<td>( s_A^e = -1, s_B^e = 1 )</td>
<td>Market ( A )</td>
<td>Market ( B )</td>
</tr>
</tbody>
</table>

2.3.1. Incentive compatibility

The fund manager has two unobservable actions: exerting effort to obtain a precise signal and making the investment choice. In contrast to the costly effort on information acquisition, the investment choice per se does not involve any personal cost, and the incentive compatibility constraint regarding the investment choice is slack (which we will verify later) when implementing the single-market strategy. Here, we discuss the manager’s incentive compatibility constraint regarding his effort choice. Taking the manager’s investment choice as given, his expected utility from exerting effort on acquiring a more precise signal about market \( A \) is

\[ U(c, \theta) \]

\[ = 0.5[(0.5 + A_A + e)\pi_u^e + (0.5 - A_A - e)\pi_d^e] \\
+ (0.5 + A_A + e)\pi_u^d + (0.5 - A_A - e)\pi_d^d] - k. \]  

(8)

Take \( \pi_u^e \) for example. The probability of the outcome \( (\tau_a = a) \) is the probability of state \( \tau_A = u \) (which is 0.5), multiplied by the probability of the manager receiving a positive signal \( s_A^e = 1 \) conditional on \( \tau_A = u \) and the manager exerting effort (which is 0.5 + \( A_A + e \)). Similarly, the manager’s expected utility from shirking is

\[ E[U(c, \theta) \mid \text{shirking with } s_A^e] \]

\[ = 0.5[(0.5 + A_A)\pi_u^e + (0.5 - A_A)\pi_d^e] \\
+ (0.5 + A_A)\pi_u^d + (0.5 - A_A)\pi_d^d]. \]  

(9)

Therefore, the manager’s incentive compatibility constraint regarding exerting effort requires that the value of (8) is no less than that of (9), i.e.,

\[ 0.5(\pi_u^e - \pi_d^e + e\pi_u^d - e\pi_d^d) \geq k. \]  

(10)

In (10), the coefficient of each utility term in the bracket gives the manager’s incentive differential between “shirking” and “working” for a particular outcome \( \omega \). For instance, consider \( \pi_u^e \). By working, the probability of getting \( \pi_u^e \) is \( (0.5 + A_A + e) / 2 \), while by shirking, the probability becomes \( (0.5 + A_A) / 2 \). The difference between these two probabilities is exactly the coefficient 0.5\( e \) in front of \( \pi_u^d \) in condition (10). The higher this coefficient, the more effective the payment \( \pi_u^d \) in motivating the manager to exert effort. We also call this coefficient the incentive leverage of the payment.

2.3.2. The least costly contract

The least costly contract for implementing the single market strategy is determined by

\[ \min_{\pi_u^e, \pi_d^e, \pi_u^d, \pi_d^d \in \mathbb{R}^t} \sum p_\omega U^{-1}(\pi_u^e) = 0.5[(0.5 + A_A + e)U^{-1}(\pi_u^e) \\
+ (0.5 - A_A - e)U^{-1}(\pi_d^e) + (0.5 + A_A + e)U^{-1}(\pi_u^d) \\
+ (0.5 - A_A - e)U^{-1}(\pi_d^d)], \]

subject to the incentive compatibility constraint in (10), which is binding in the solution.

In (10), two outcomes \( (\tau_A = u) \) and \( (\tau_A = d) \), which represent poor performance relative to market \( A \), have negative incentive leverages. Any payment to the manager for these outcomes is a reward for failure and thus should be minimized to zero (i.e., \( \pi_u^d = \pi_d^d = 0 \)). On the other hand, \( \pi_u^e \) and \( \pi_d^e \) represent rewards for good performance in outcomes \( (\tau_A = u) \) and \( (\tau_A = d) \). Using the standard Lagrange method, the first-order conditions provide that

\[ U'[U^{-1}(\pi_u^e)] = U'[U^{-1}(\pi_u^d)] = \frac{(0.5 + A_A + e)}{2e}, \]

(11)

where \( \lambda \) is the Lagrange multiplier for the incentive compatibility constraint in (10). Combining this result with (10), we have \( \pi_u^e = \pi_d^d = k/e \).

We also need to specify payments for two off-equilibrium outcomes \( (\tau_A = u) \) and \( (\tau_A = d) \) to prevent the manager from investing in the secondary market \( B \):

\[ \pi_u^d = \pi_d^d = 0. \]

We verify in Appendix A that under these terms, the manager will never deviate to invest in market \( B \).
The following proposition summarizes the contract derived above.

**Proposition 1. The least costly contract for implementing the single-market strategy uses the following payments:**
\[
\begin{align*}
\pi_u &= \pi_d = k/e, \\
\pi_u^0 &= \pi_d^0 = \pi_u^0 = 0.
\end{align*}
\] (12)

The principal’s expected payoff from implementing this strategy is
\[
V^{SM} = 1 + (\Delta_A + e)u - (0.5 + \Delta_A + e)U^{-1}(k/e).
\] (13)

This contract benchmarks the manager’s performance to the return of his designated market. The manager receives a positive reward if he secures the positive return of the market or avoids its negative return. Otherwise, he receives nothing. Consistent with the benchmarking idea, the same fund performance (zero), could lead to two different compensations (zero or k/e) depending on whether the market return is positive or negative.

Another notable point is that the contract gives a zero payment for both \(\pi_u^0\) and \(\pi_u^0\), the outcomes in which the manager delivers maximum deviation from the primary market return. These terms represent penalties against the manager’s tracking errors, which are often used in practice according to the Bank for International Settlements (2003) report. These penalties discourage the manager from investing outside the primary market and serve the role of implementing a narrow investment mandate.

### 2.4. Two-tiered strategy

Implementing the single-market strategy imposes an efficiency loss by restricting the manager from taking advantage of opportunities outside the primary market. This subsection studies a two-tiered strategy (see Table 1) which improves on this dimension: the manager exerts effort on acquiring a precise signal \(S_A\) about market \(A\); if this signal is favorable, he invests in market \(A\); if \(S_A\) is unfavorable but his free signal \(S_B\) indicates a good outside opportunity in market \(B\), he invests in market \(B\); otherwise, he invests in the risk-free asset. Similar to the single-market strategy, the two-tiered strategy also induces the manager’s effort in market \(A\). However, in contrast to the single-market strategy, the two-tiered strategy instructs the manager to pursue outside opportunities if necessary.

#### 2.4.1. The least costly contract

We derive the least costly contract for implementing the two-tiered strategy in a way similar to the single-market strategy. By exerting effort and following the intended investment strategy, the manager’s expected utility is

\[
\mathbb{E}[U(c, \theta)] \text{exerting effort and following the two-tiered investment strategy} = 0.25(1 + 2\Delta_A + 2e) + (0.5 + \Delta_A - e)(0.5 + \Delta_B)\pi_u^0 + 0.25(1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e)(0.5 + \Delta_B)\pi_u^0 + 0.25(0.5 - \Delta_A - e)(0.5 - \Delta_B)\pi_d^0 + 0.25(0.5 + \Delta_A + e)\pi_d^0 + 0.25(0.5 - \Delta_A - e)\pi_u^0 - k.
\] (14)

The manager can also adopt a deviation strategy by shirking and then following the two-tiered investment strategy based on his free signals \(S_A^0\) and \(S_B^0\):

\[
\mathbb{E}[U(c, \theta)] \text{shirking and following the two-tiered investment strategy} = 0.25[(1 + 2\Delta_A + (0.5 - \Delta_A)(0.5 + \Delta_B)\pi_u^0 + 0.25(0.5 + \Delta_A)(0.5 + \Delta_B)\pi_d^0 + 0.25(0.5 - \Delta_A)(0.5 - \Delta_B)\pi_u^0 + 0.25(0.5 + \Delta_A)(0.5 + \Delta_B)\pi_d^0.
\] (15)

Incentive compatibility requires that (14) dominates (15), which is equivalent to

\[
0.25[(1 - 2\Delta_A + (0.5 + \Delta_A)(0.5 + \Delta_B)\pi_u^0 + 0.25(0.5 - \Delta_A)(0.5 - \Delta_B)\pi_u^0 + 0.25(0.5 + \Delta_A)(0.5 + \Delta_B)\pi_d^0 + 0.25(0.5 - \Delta_A)(0.5 - \Delta_B)\pi_d^0.
\] (16)

This is an important constraint in implementing the two-tiered strategy.

The principal also needs to ensure that after receiving a negative signal about market \(A\) and a positive signal about market \(B\), the manager is willing to invest in market \(B\) instead of the risk-free asset. Investing in market \(B\) exposes the manager to the risk that the realized return might be negative, while investing in the risk-free asset allows the manager to lock in the sure return zero. The comparison of the two depends on the structure of the manager’s incentive contract. Specifically, given \(S_A^0 = -1\) and \(S_B^0 = 1\), the manager’s expected utility from investing in market \(B\) is

\[
(0.5 - \Delta_A - e)(0.5 + \Delta_B)\pi_u^0 + (0.5 - \Delta_A - e)(0.5 - \Delta_B)\pi_u^0 + (0.5 + \Delta_A + e)(0.5 + \Delta_B)\pi_d^0 + (0.5 + \Delta_A + e)(0.5 - \Delta_B)\pi_u^0.
\] (17)

while his expected utility from investing in the risk-free asset is

\[
(0.5 - \Delta_A - e)\pi_u^0 + (0.5 + \Delta_A + e)\pi_d^0.
\] (18)

Implementing the two-tiered strategy thus requires (17) dominate (18):

\[
(0.5 - \Delta_A - e)(0.5 + \Delta_B)\pi_u^0 + (0.5 - \Delta_A - e)(0.5 - \Delta_B)\pi_u^0 + (0.5 + \Delta_A + e)(0.5 + \Delta_B)\pi_d^0 + (0.5 + \Delta_A + e)(0.5 - \Delta_B)\pi_u^0 - (0.5 - \Delta_A - e)\pi_u^0 - (0.5 + \Delta_A + e)\pi_d^0 \geq 0.
\] (19)

This constraint also holds in the least costly contract.

The least costly contract minimizes the expected compensation cost \(E[U(\pi_u^0)]\):

\[
0.25[(1 + 2\Delta_A + 2e) + (0.5 - \Delta_A - e)(0.5 + \Delta_B)U^{-1}(\pi_u^0) + 0.25(0.5 - \Delta_A - e)U^{-1}(\pi_u^0) + 0.25(1 - 2\Delta_A - 2e) + (0.5 + \Delta_A + e)(0.5 - \Delta_B)U^{-1}(\pi_d^0) + 0.25(0.5 - \Delta_A - e)(0.5 - \Delta_B)U^{-1}(\pi_d^0) + 0.25(0.5 + \Delta_A + e)(0.5 + \Delta_B)U^{-1}(\pi_d^0).
\] (20)
subject to the incentive constraints in (16) and (19) and
that all payments are non-negative. We denote the
Lagrange multipliers associated with the two incentive
constraints as \( \lambda_1 \geq 0 \) and \( \lambda_2 \geq 0 \), respectively. The follow-
ing proposition characterizes the least costly contract. We also verify other deviation strategies based on this incen-
tive contract in Appendix A.

Proposition 2. The least costly contract for implementing the
two-tiered strategy gives zero payments for the following outcomes:
\[
\pi^0_u = \pi^0_d = \pi^2_d = 0,
\]
and positive payments for \( \pi^u_u, \pi^u_d, \) and \( \pi^2_d \), which are given in
Appendix A and satisfy \( \pi^u_u < \pi^u_d \) and \( \pi^2_d < \pi^u_d \). Under the
sufficient conditions (35) and (36) in Appendix A, this contract also deters the use of other deviation strategies.

To implement the two-tiered strategy, the expected compen-
sation cost is
\[
K^{TT} = 0.25((1+2.A_A+2 \varepsilon)+(0.5-\lambda A_A-\varepsilon)(0.5+\lambda A_B))U^{-1}(\pi^u_u) \\
+(0.5+\lambda A_A+\varepsilon)(0.5+\lambda A_B)U^{-1}(\pi^u_d) + (0.5+\lambda A_A+\varepsilon)U^{-1}(\pi^2_d)),
\]
and the principal's expected payoff is
\[
1+(e+\lambda A_A+0.5 \lambda A_B)r-K^{TT}.
\]

Proposition 2 shows that to encourage the manager to
pursue potential investment opportunities outside his
primary market, the least costly incentive contract toler-
ates greater tracking errors than the one for implement-
ing the single-market strategy (Proposition 1). This difference is reflected by the positive payment for the out-
come \( (\varepsilon) \), in which the fund return beats the primary
market return by two notches. Because of the large
tracking error, this seemingly good performance is not
rewarded by the contract derived in Proposition 1. Fur-
thermore, the contract derived in Proposition 2 also
provides a greater incentive slope, i.e., a larger payment
for better performance as reflected by \( \pi^u_u > \pi^u_d \) and
\( \pi^2_d > \pi^u_d \). In contrast, the contract derived in
Proposition 1 provides the same payment for the two good outcomes: \( \pi^u_u = \pi^2_d \).

In practice, hedge funds tend to be more tolerant of
tracking errors and provide greater incentive slopes, whereas mutual funds tend to be more restrictive on
tracking errors and give smaller incentive slopes. Thus,
the contract derived in Proposition 2 is closer to the hedge
fund contracts, while the contract derived in Proposition 1
is closer to the mutual fund contracts.

2.4.2. Higher agency cost due to worse benchmarking

Interestingly, the seemingly superior two-tiered stra-
tegy may be suboptimal because it exacerbates the agency
cost to incentivize the manager to exert effort in his
primary market. This negative impact originates from
two channels. First, the additional investment flexibility
makes “benchmarking” more difficult because it intro-
duces luck from market B into the fund performance. This
weakens the link between the fund performance and the
manager’s effort in market A, and leads to less efficient
incentive provision. Second, implementing the two-tiered
investment strategy requires an additional constraint (19)
on the incentive contract, which further reduces its incen-
tive-provision efficiency. The intuition for the sec-
ond channel is obvious. Thus, we focus on illustrating
the first channel, which is also the key economic insight of
our model.

The negative impact of investment flexibility on incen-
tive provision manifests itself in the payment for the
outcome \( \omega = (\varepsilon) \). In implementing the two-tiered strategy, the probability of this outcome is
\[
P_{\omega} = 0.5(0.5+\lambda A_A+\varepsilon) + 0.25(0.5-\lambda A_A-\varepsilon)(0.5+\lambda A_B).
\]

The first term represents the situation that the primary
market return is high (with probability 0.5) and the
manager spots this opportunity (with probability
0.5+\lambda A_A+\varepsilon). The second term represents an additional
possibility that the primary market return is high (with
probability 0.5) but the manager fails to spot it (with
probability 0.5-\lambda A_A-\varepsilon); instead, the return in market B is
also high (with probability 0.5) and the manager spots
this one (with probability 0.5+\lambda A_B). The second term
represents luck from market B. Such luck increases the
probability for the principal to make the positive payment
and thus adds to the compensation cost.

More interestingly, the luck also reduces the incentive
leverage of \( \pi^u_u \). If the manager shirks, the probability of
this outcome becomes
\[
0.5(0.5+\lambda A_A)+0.25(0.5-\lambda A_A)(0.5+\lambda A_B).
\]

Thus, the difference between (21) and (22) gives the
incentive leverage of \( \pi^u_u \)
\[
Dp_{\omega} = 0.5e-0.25(0.5+\lambda A_B)e.
\]

Comparing to (10) when implementing the single-market
strategy, where \( Dp_{\omega} = 0.5e \), the above incentive leverage is reduced by the possible luck from market B. Intuitively,
the free luck from market B crowds out the need to exert
effort to spot the good opportunity in market A (if there is
one). This crowding-out effect, which is at work only
when implementing the two-tiered strategy, reduces the
manager's gain from exerting his effort in market A and
therefore his ex ante working incentives.

We call the ratio \( p_{\omega}/Dp_{\omega} \), the cost-to-incentive ratio of
the payment, which is first derived in Holmstrom
(1979). The numerator \( p_{\omega} \) captures a cost effect, i.e.,
the larger the probability of the outcome \( \omega \), the higher
the expected cost of each dollar promised to this outcome.
The denominator \( Dp_{\omega} \) captures an incentive effect: the
larger the incentive leverage \( Dp_{\omega} \), the greater the man-
ager’s incentive to exert effort. In implementing the
two-tiered strategy, we have
\[
\frac{p}{Dp}_{\omega} = \frac{0.5(0.5+\lambda A_A+\varepsilon)+0.25(0.5-\lambda A_A-\varepsilon)(0.5+\lambda A_B)}{0.5e-0.25(0.5+\lambda A_B)e}
\]

18 The ratio \( Dp_{\omega}/p_{\omega} \) corresponds to the ratio \( f_{\omega}/f \) in Holmstrom
(1979), where \( f \) is the probability density function of the performance,
and \( f_{\omega} \) is the marginal impact of action \( a \) on the density function.
Holmstrom points out that \( f_{\omega}/f \) is the derivative of log likelihood, and
interprets this measure as how strongly one is inclined to infer from the
performance that the agent did not take the assumed action.
We have decomposed each term in the fraction relative to 
\((0.5 + A_k + \epsilon)/\epsilon\), the corresponding cost-to-incentive ratio in implementing the single-market strategy. It is clear that the investment flexibility unambiguously increases the cost-to-incentive ratio by raising the expected payment and lowering the incentive leverage of the payment.

Overall, implementing the two-tiered strategy requires a higher expected compensation cost, which we formally prove in the following proposition.

**Proposition 3.** The expected compensation cost of implementing the two-tiered strategy is higher than that of the single-market strategy, i.e., \(K^{T} > K^{SM}\). Furthermore, the difference monotonically increases with \(k\).

This proposition shows that in implementing the two-tiered strategy, the additional investment benefit \(0.5r_A A_k\) from encouraging the manager to pursue the opportunity outside the primary market comes with an increased agency cost of \(K^{T} - K^{SM}\). As the increased agency cost monotonically increases with \(k\), the principal will prefer the narrowly mandated single-market strategy if \(k\) is higher than a certain threshold \(k^*\).

We can intuitively relate the model parameter \(k\) to the manager’s ability and the information opacity of the primary market. As the effort cost of the more talented managers is lower, the additional agency cost from encouraging them to pursue opportunities outside their primary markets is also smaller. As a result, we have the following implication:

**Implication** Fund managers with lower ability are more likely to be confined in trading a specific market sector or asset class; on the other hand, managers with higher ability tend to face less stringent investment mandates.

This implication is consistent with a casual observation that hedge fund managers tend to be more talented than mutual fund managers,\(^{19}\) and they also face less stringent investment mandates.

Furthermore, the effort cost is likely to be higher for managers whose primary markets are more informationally opaque. This in turn leads to another testable implication:

**Implication** Fund managers whose primary markets are more informationally opaque face more stringent investment mandates.

### 2.5. Model robustness

Our model makes several simplifying assumptions to make the analysis tractable. These assumptions include the restriction preventing the manager from taking multiple positions in markets \(A\) and \(B\) at the same time, the short-sale constraints in both markets \(A\) and \(B\), and the restriction on the manager’s position to be either one or zero units in each market. These assumptions are not essential to the key economic insight of our model.

**Appendix B** presents an alternative model setting, which relaxes the restrictions on multiple positions and short sales. Specifically, the manager can take either a long or short position of one unit in the primary market \(A\), and at the same time he can also independently take a long or short position of one unit in the secondary market \(B\). We show that the key result derived from our main model prevails—when the manager’s cost of effort in the primary market is sufficiently high or when the precision of the manager’s free signal about market \(B\) is sufficiently low, motivating the manager to pursue the single-market strategy that invests exclusively in the primary market is optimal even though the free signal about the secondary market is nevertheless valuable. The driver of this result is the same as in Section 2.4.2: because the principal cannot directly observe the manager’s investment positions and the fund performance aggregates the manager’s returns from both markets, investment flexibility across both markets impedes the principal’s inference problem of the manager’s effort. As a result, incentive provision becomes inefficient and more costly.

The restriction on the manager’s position of one or zero units serves to prevent the portfolio return from fully revealing the manager’s investment position. With both markets \(A\) and \(B\) having binomial returns, allowing the manager to choose investment position in a continuous range such as \([0, 1]\) would allow the principal to perfectly infer the manager’s position based on his portfolio return and the primary market return—for example, if the principal can design a contract to induce the manager to either invest 100% in market \(A\) or 99% in market \(B\). However, the continuously distributed asset returns in reality render such a revelation mechanism unrealistic. Thus, we do not believe that this assumption is essential to our key economic insight.

### 3. An extended model with negatively skewed risk

When incentivized to pursue investment opportunities outside his primary market, the manager may also seek unwarranted negatively skewed risk even if his signals do not indicate any good opportunity. In light of the recent financial crisis, many observers (e.g., \(\text{Rajan, 2010, Acharya, Cooley, and Richardson, 2010}\) point out that excessive risk-taking (by AIG, Lehman Brothers, and other financial firms), and in many cases active seeking of negatively skewed risk, was a key contributing factor of the crisis. As highlighted by \(\text{Rajan (2010)},\) such tail risk presents a great challenge to the ongoing reform of the financial industry’s risk management system and incentive structure: “We have to find ways to reduce the incentive to take tail risk even while rewarding bankers for performance so that they continue to offer innovative products that meet customer needs and lend to the risky but potentially very successful start-up.”
3.1. Negative skewness and analysis

We suppose that one of the markets outside the manager's primary market, denoted by market C, has a zero expected return and a negative skewness, in addition to market B specified earlier. 20 Again, only the manager knows the identity of this market. Specifically, market C offers the following return:

\[
\hat{r}_C = \begin{cases} 
  r & \text{with probability } \eta_C \in (0.5, 1), \\
  \frac{\eta_C}{1-\eta_C} - r & \text{with probability } 1 - \eta_C \in (0, 0.5).
\end{cases}
\]

A higher value of \( \eta_C \) leads to a more negatively skewed return, i.e., this market gives a positive return \( r \) with a high probability \( \eta_C \) but a large negative return \( -((1-\eta_C)/r) \) with a small probability \( 1 - \eta_C \). The manager does not observe any signal about this market—market C represents a pure gamble. 21

The presence of such a market further complicates the delegation problem between the principal and the manager. Suppose that the manager is compensated by the incentive contract derived in Proposition 2 which aims to induce effort and implement the two-tiered investment strategy in markets A and B. If the manager invests in market C, this can be detected only after the realization of loss \(-((1-\eta_C)/r)\) that is more severe than those from other regular investments. When this occurs, limited liability implies that the principal can only penalize the manager by paying him zero. On the other hand, if the outcome is positive, the principal cannot identify the source of the good performance and has to compensate the manager accordingly to the contract.

The following scenario demonstrates this risk-seeking behavior clearly. Consider the investment problem faced by the manager when he observes a negative signal in market A and a positive signal in market B (i.e., \( s^A = -1 \) and \( s^B = 1 \)). His expected utility from investing in market B is

\[
(0.5 - A - e)((0.5 + A + e)\pi^u_B + (0.5 - A)\pi^d_B) + (0.5 + A + e)((0.5 + A + e)\pi^d_B + (0.5 - A)\pi^u_B),
\]

which, under the contract in Proposition 2, is equal to

\[
(0.5 + A + e)(0.5 - A - e)\pi^u_B + (0.5 + A + e)\pi^d_B.
\]

His expected utility from investing in market C is \( \eta_C((0.5 - A - e)\pi^u_B + (0.5 + A + e)\pi^d_B) \) as he will get compensated after market C gives a positive return and zero otherwise. Thus, the manager will choose to invest in market C if and only if \( \eta_C > 0.5 + A_B \).

That is to say, the manager will ignore a good opportunity in market B and instead seek the unwarranted risk in market C if the negative skewness \( \eta_C \) is sufficiently large. This exactly captures the concern that compensation for positive performance can also motivate the manager to seek negatively skewed risk. 22

This illustration suggests that additional constraints are necessary to prevent the manager from seeking the negatively skewed risk in market C. It turns out that if the manager deviates from the intended investment strategy, he prefers a double-deviation strategy to first shirk and then seek risk in market C regardless of his signals. Deterring such a double-deviation provides the most-binding constraint on the incentive contract. 23 If the manager chooses to shirk in the primary market and then to always invest in market C, his expected utility is (the manager gets zero if market C has negative outcome)

\[
\eta_C(0.5(\pi^u_B + \pi^d_B)).
\]

His expected utility from exerting effort and following the two-tiered investment strategy is given in (14). Thus, the additional constraint is

\[
0.25((1 + 2A + 2B) + (0.5 - A - e)(0.5 + A_B)\pi^u_B + 0.25(0.5 + A + e(0.5 + A_B)\pi^d_B
+ 0.25((1 - 2A - 2B) + (0.5 + A + e)(0.5 - A_B)\pi^d_B + 0.25(0.5 - A - e(0.5 - A_B)\pi^u_B + 0.25(0.5 + A + e)\pi^d_B + 0.25(0.5 - A - e)\pi^u_B) \geq 0.5\eta_C(\pi^u_B + \pi^d_B). \tag{23}
\]

The rewards for positive performance stimulate the risk-seeking behavior because the right-hand side of this inequality increases with \( \pi^u_B \) and \( \pi^d_B \). If these payments are high, the contract has to raise payments for other outcomes (possibly for bad performance), to increase the opportunity cost of seeking the negatively skewed risk. As a result, adding this constraint can further increase the agency cost of implementing the two-tiered investment strategy.

Recall that Proposition 2 shows that the least costly contract involves only three non-zero payments, in the absence of the negatively skewed risk. With the negatively skewed risk in market C, we need to minimize the expected compensation cost in (20) subject to constraints

20 While in practice a fund manager can seek risk in either his primary market or any outside market, this concern is perhaps more severe for outside markets. This is because his primary market is usually tightly defined and thus offers limited flexibility to seek risk. On the other hand, once incentivized to invest outside, the manager's flexibility to seek any type of payoff or risk in the complex financial universe is greatly increased. Thus, our analysis focuses on the manager's risk-seeking incentive that accompanies his incentive to pursue outside investment opportunities.

21 To highlight the damage caused by market C to the manager's incentive, we intentionally make its expected return zero. This expected return per se is not a concern to the risk-neutral principal. Instead, as we will highlight, the presence of this market not only erodes the manager's effort incentive but also makes it more difficult to implement an efficient investment strategy. It should be clear that making the expected return of market C negative by further reducing its negative return will only strengthen our result.

22 The incentive to seek negatively skewed risk will arise as long as the manager faces a sufficiently large reward for good performance and is protected by limited liability. By endogenizing the manager's compensation contract through the agency problem, our model allows us to analyze the interaction between effort-motivating incentive and risk-seeking incentive in determining the optimal incentive structure for fund managers.

23 This situation is similar to the optimality of double-deviation in the dynamic moral hazard problem with private (hidden) saving, where the agent usually finds it optimal to shirk and save concurrently, e.g., He (forthcoming). As investment positions are not observable in our model, investing in market C plays the same role as private saving in that context.
in (16), (19), and (23). The next proposition shows that when \( \eta_C \) is sufficiently large, the constraint in (23) is binding. As a result, the previously zero payments \((\pi_n^d, \pi_u^d, \pi_u^d)\) can now turn positive, and \(\pi_u^d\) turns positive before the other two.

**Proposition 4.** When \( \eta_C \) is sufficiently large, the constraint in (23) is binding. Furthermore, \(\pi_u^d\) turns positive before \(\pi_n^d\) and \(\pi_u^d\) under the sufficient condition that

\[
0.5 + A_B < 2A_A + 2e.
\]

A positive payment \(\pi_u^d\) arises because it increases the opportunity cost for the manager to seek the unwarranted negatively skewed risk, i.e., the left-hand side of (23). Although we only prove that \(\pi_u^d\) turns positive before \(\pi_n^d\) and \(\pi_u^d\) under the given sufficient condition, numerically we have verified that \(\pi_u^d\) and \(\pi_u^d\) always remain zero in a large set of parameter values outside the sufficient condition.24

To further illustrate the effects of negatively skewed risk on the least costly incentive contract for implementing the two-tiered strategy, we adopt the following baseline parameters:

\[
\begin{align*}
r &= 0.25, & A_A &= 0.25, & A_B &= 0.2, & k &= 0.02, \\
e &= 0.2, & \alpha &= 0.6.
\end{align*}
\]

Fig. 1 plots four payments \(\pi_n^u, \pi_u^d, \pi_u^d, \pi_u^d\) as \(\eta_C\) increases from 0.5 to 0.8. The risk-seeking constraint in (23) starts to bind when \(\eta_C\) passes 0.65, a level below 0.5 + \(A_B\) = 0.7. Consistent with our earlier discussion, this suggests that the double-deviation of shirking and risk-seeking is more desirable to the manager than the single deviation of risk-seeking only, and thus the constraint in (23) binds earlier than \(\eta_C > 0.5 + A_B\).

When \(\eta_C\) is between 0.65 and 0.67, the least costly contract offsets the risk-seeking incentive by increasing \(\pi_n^u\), reducing \(\pi_u^d\) and \(\pi_u^d\), and keeping \(\pi_u^d\) at zero. As we have discussed, \(\pi_n^u\) is useful for deterring risk-seeking because its coefficient on the left-hand side of (23) is greater than that on the right-hand side. As a result, the expected compensation cost increases with \(\eta_C\).

When \(\eta_C\) rises above 0.67, simply increasing \(\pi_n^u\) is not enough. Instead, the contract gives a positive payment \(\pi_u^d\) even though it has a negative effect on inducing the manager’s effort in the primary market. To counter the negative incentive effect brought on by \(\pi_u^d\), the contract has to simultaneously increase the other three payments \((\pi_n^u, \pi_u^d, \pi_u^d)\) which have positive incentive effects. Because of the intricate interaction between the manager’s incentive-provision constraint and risk-seeking constraint, the expected compensation cost increases dramatically with \(\eta_C\) once it passes above 0.67.

Furthermore, Fig. 1 shows that the incentive slope \(\pi_u^d - \pi_u^d\) decreases with \(\eta_C\). This is because \(\pi_u^d\) is particularly strong in motivating the risk-seeking behavior—when \(\eta_C\) is large, the coefficient of \(\pi_u^d\) on the right-hand side of (23) exceeds that on the left-hand side. As a result, the contract has to reduce \(\pi_u^d\) to mitigate such an incentive.

The increased agency cost makes the single-market strategy more desirable. Fig. 2 plots the upper threshold \(k^*\) of the manager’s effort cost for the optimality of implementing the two-tiered strategy. When \(\eta_C\) is below 0.65, \(k^*\) is insensitive to \(\eta_C\). As \(\eta_C\) rises above 0.65, \(k^*\) decreases with \(\eta_C\). This plot suggests that in the presence of the negatively skewed risk, only managers with sufficiently high talents (and thus low effort cost) are encouraged to pursue investment opportunities outside their designated markets. The next proposition formally proves this result.

**Proposition 5.** In the presence of the negatively skewed risk in market \(C\), the upper threshold \(k^*\) on the manager’s effort cost for the optimality of implementing the two-tiered strategy decreases with the skewness parameter \(\eta_C\).

### 3.2. Implications

Taken together, our analysis demonstrates that the presence of negatively skewed risk increases the agency cost of encouraging the manager to pursue opportunities outside the primary market. As a result, narrow investment mandates become even more desirable. It is especially useful to interpret this result in light of the increasingly complex financial markets. The rapid development of financial markets in recent years has greatly expanded the space of financial securities and thus made it much easier to access negatively skewed risk, either by buying a structured finance product with the intended risk profile or by selling an out-of-money option like security. Together with this change in the investment environment, our model explains the recent trend of the growing popularity of stringent investment mandates and narrow tracking errors highlighted by the Bank for International Settlements (2003) report.

Our model also shows that for those managers with exceptional talents, the optimal incentive contract not only encourages them to pursue flexible investment strategies but also rewards them generously. Interestingly, the reward covers not just their good performances but also their well-intentioned failures. The reward for failures might appear counter-intuitive because of its seemingly negative incentive effect. But it helps deter risk-seeking because the managers stand to lose such a reward if they choose to seek outside risk. In other words, since managers get paid generously for pursuing the intended strategies, they will find seeking outside risk too costly as it jeopardizes the generous payments guaranteed to them.

Philippon and Reshef (2008) find that wages for financial jobs were excessively high around 1930 and from the mid-1990s to 2006. They attribute the high

---

24 What makes \(\pi_u^d\) different from \(\pi_n^d\) and \(\pi_n^d\)? Although their cost-to-incentive ratios in motivating the manager’s effort are all negative, the cost-to-incentive ratio of \(\pi_u^d\) is \(-(1 - 2A_A - 2e + (0.5 + A_B + e)0.5 - A_B))/(1.5 + A_B)/e, which is lower than (so the absolute value is higher than) that of \(\pi_n^d\) and \(\pi_n^d\) with \((p/Dp_u^d, p/Dp_n^d) = -0.5 - A_A - e)/e. Section 2.4.2 then tells us that it is more difficult for the principal to use the outcome \(\pi_u^d\) to distinguish (bad) performance from (bad) luck. As a result, if the principal has to pay the manager that comes with a negative impact on effort incentives, a payment for \(\pi_u^d\) causes the least damage.
wages to financial deregulation during these periods, which made financial jobs more skill-intensive and complex and thus attracted better talents to the financial industry. In light of our analysis, financial deregulation not only made financial jobs more demanding, but also created more room for traders and fund managers to take on creative negatively skewed risk. As a result, higher wages are necessary not only because the financial workers’ reservation wages were higher, but also because the damages they could do to the firms were also higher.

4. Discussions

The wide usage of narrow investment mandates in the asset management industry have important implications for asset market dynamics. In this section, we discuss such implications on capital immobility and market segmentation.

4.1. Capital immobility

The stringent investment mandates imposed on fund managers can lead to “capital immobility,” i.e., capital often fails to flow to distressed markets that offer profitable opportunities. Duffie (2010) highlights this phenomenon as an important factor in understanding market liquidity. For example, many pundits observe that capital immobility was a key factor leading to the 1998 financial market crisis—margin calls forced the hedge fund Long Term Capital Management to liquidate its large leveraged positions in fixed income securities while not enough capital came in to absorb its liquidation. Froot and O’Connell (1999) show that the supply of capital in the
catastrophe insurance market is inelastic because there are times during which the price of catastrophe insurance seems to be high and the capital of catastrophe insurers is low. Other examples include the depressed convertible bond market after convertible hedge funds faced large redemption of capital from investors in 2005 (e.g., Mitchell, Pedersen, and Pulvino, 2008), the temporary price discount for stocks after fire sales by mutual funds (e.g., Coval and Stafford, 2007), and the distressed market for newly downgraded junk bonds (e.g., Da and Gao, 2008).

Our model provides a new hypothesis for capital immobility during liquidity crises based on agency frictions at the originating end of capital flow. The economy could well have adequate capital. However, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations can motivate stringent investment mandates on the fund managers, which in turn confine the capital in its initial market segments. Even if one segment runs out of capital later and ends up in a liquidity crisis, fund managers in other market segments may be unwilling to move in because of the potential tracking errors. Instead, the strategic decisions of moving capital across different segments are largely left to the less informed investors themselves. As a result, the capital flow is likely to be delayed. Only as the crisis deteriorates will the distressed segment gradually attract capital from other segments, starting from funds that face broader investment mandates and greater tolerance for tracking errors. Eventually, investors will also recognize profit opportunities created by the crisis and move capital from other segments to the distressed segment.

Our agency-based hypothesis of capital immobility complements the growing literature that studies the impact of financial intermediaries’ capital inside the crisis market under the premise that outside capital would not flow in (e.g., Kyle and Xiong, 2001; Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009; He and Krishnamurthy, forthcoming-a,b; Bolton, Santos, and Scheinkman, 2011). These studies typically motivate this premise based on various information barrier arguments about the distressed market at the receiving end of capital flow, e.g., outside investors hesitate to invest in the crisis market because they cannot distinguish whether the price drop is driven by liquidity reasons or worsened fundamentals. Our hypothesis is also different from those based on search frictions (e.g., Duffie, 2010), who suggest that the speed of capital flow depends on the rate of random matching between buyers and sellers.

4.2. Market segmentation

There is growing evidence of de facto segmentation of various asset markets from the broad financial markets, even in the absence of explicit regulatory and physical constraints on investment to these markets. For example, Bekaaert, Harvey, Lundblad, and Siegel (2008) show that many emerging markets are still segmented from the global financial markets even though the regulatory constraints on foreign investment had been largely lifted over the past few decades. In particular, they find that after controlling for financial leverage and earnings volatility, emerging markets display a significantly higher industrial earnings yield (the inverse of price-to-earnings ratio) than that of developed countries. A common argument is that information barriers may prevent investors from fully integrating assets of emerging markets into their portfolios (e.g., Merton, 1987). However, investors can hire professional managers to overcome the information barriers. Then, it remains puzzling that the rapid growth of funds specializing in emerging markets in the recent years has not eliminated the segmentation of these markets.

Several other markets also exhibit similar de facto segmentation. In the corporate bond market, Collin-Dufresne, Goldstein, and Martin (2001) find that proxies for both changes in the probability of future default based on standard fundamental-driven credit risk models and for changes in the recovery rate can explain only a small fraction of the observed credit spread changes. Instead, a market-specific latent factor can explain a large fraction of the residuals. In the mortgage-backed securities market, Gabaix, Krishnamurthy, and Vigneron (2007) find that idiosyncratic prepayment risk carries a risk premium. In the S&P 500 index option market, Garleanu, Pedersen, and Poteshman (2009) find that demand pressure in one option contract increases its price as well as other correlated contracts. The de facto segmentation of these markets is even more puzzling as they are mostly traded by financial institutions and professional traders.

The narrow investment mandates derived in our model provide an explanation of the de facto segmentation of the aforementioned markets. When (uninformed) investors delegate their capital to a professional manager to invest in one of these markets, information barriers in these markets make it necessary to impose a stringent investment mandate on the manager in order to reduce agency cost in the delegation process. In other words, the manager has to invest primarily in this particular market, say, Russia, and his compensation is closely tied to his fund performance. Thus, despite that the manager might work for well-diversified investors, his own pricing kernel is exposed to the idiosyncratic risk of the market. The market will exhibit de facto segmentation if the manager is the marginal investor. To sum up, our model suggests that agency frictions can lead to market segmentation despite the fact that investors can hire professional managers to overcome information barriers in informally opaque markets.

5. Conclusion

We analyze a realistic delegated asset management problem in which a principal hires a fund manager to invest his money in a multi-market environment. This implies that the principal needs to motivate not only the manager’s effort in acquiring information, but also an investment strategy across the markets. Our model highlights a tradeoff between encouraging the manager to pursue the efficient investment strategy and the agency cost of incentivizing him. This tradeoff becomes especially severe when the manager can access negatively skewed
risk outside his primary market. Building on this tradeoff, our model explains the increasingly stringent investment mandates faced by fund managers. Our analysis sheds light on capital immobility and market segmentation that are widely observed in financial markets and highlights important effects of negatively skewed risk on institutional incentive structures.

Appendix A. Proofs of propositions

Proof of Proposition 1. We need to verify that the manager has no incentive to deviate and invest in market \( B \). First, consider a deviation strategy where he exerts effort on market \( A \) and then follows the two-tiered investment strategy discussed in Section 2.4 (i.e., invest in market \( B \) when \( s^0_A = 0 \) and \( s^0_B = 1 \)). The relevant situation is when he observes a negative signal in market \( A \) and a positive signal in market \( B \). Then, his expected utility from investing in market \( B \) is

\[
0.25(0.5 - D_A - e)(0.5 + D_B)\pi_u^A + 0.25(0.5 - D_A - e)(0.5 - D_B)\pi_d^A
+ 0.25(0.5 + D_A + e)(0.5 + D_B)\pi_u^B
+ 0.25(0.5 + D_A + e)(0.5 - D_B)\pi_d^B,
\]

which, under the contract specified in Proposition 1, is equal to

\[
0.25(0.5 - D_A - e)(0.5 + D_B)\pi_u^A.
\]

On the other hand, his expected return from investing in the risk-free asset is

\[
(0.5 + D_A + e)\pi_u^B + (0.5 - D_A - e)\pi_d^A,
\]

which is equal to \( (0.5 + D_A + e)\pi_u^B \) under the contract specified in Proposition 1. As \( \pi_u^A = \pi_u^B \), the manager prefers to invest in the risk-free asset.

Next, we consider the deviation strategy where he exerts no effort and follows a two-tiered investment strategy. Then, his expected utility is equal to

\[
0.25((1 + 2D_A) + (0.5 - D_A)(0.5 + D_B))\pi_u^A
+ 0.25((1 + 2D_A) + (0.5 + D_A)(0.5 + D_B))\pi_u^B
+ 0.25((1 - 2D_A) + (0.5 + D_A)(0.5 - D_B))\pi_d^B
+ 0.25((1 - 2D_A) + (0.5 - D_A)(0.5 - D_B))\pi_d^A
+ 0.25(0.5 + D_A + e)(0.5 - D_B)\pi_u^A
+ 0.25(0.5 + D_A + e)(0.5 - D_B)\pi_d^B,
\]

which is modified from Eq. (14) by removing the manager’s effort. Under the contract given in Proposition 1, the manager’s expected utility is equal to

\[
0.25(1 + 2D_A) + (0.5 - D_A)(0.5 + D_B)\pi_u^A
+ 0.25(0.5 + D_A)(0.5 + D_B)\pi_u^B
+ 0.25(0.5 - D_A)(0.5 + D_B)\pi_d^B
+ 0.25(0.5 - D_A)(0.5 - D_B)\pi_d^A
+ 0.25(0.5 + D_A + e)(0.5 - D_B)\pi_u^A
+ 0.25(0.5 + D_A + e)(0.5 - D_B)\pi_d^B,
\]

which is modified from Eq. (14) by removing the manager’s effort. Under the contract given in Proposition 1, the manager’s expected utility is equal to

\[
0.25(1 + 2D_A) + (0.5 - D_A)(0.5 + D_B)\pi_u^A
+ 0.25(0.5 + D_A)(0.5 + D_B)\pi_u^B
- 0.25((0.5 - D_A)(0.5 + D_B) + 1.5 + 3D_A)^k
\]

\[
< 0.25(0.5 - D_A)(0.5 + D_B)\frac{k}{c} = 0.5(1 + D_A)\frac{k}{c}.
\]

By substituting the equilibrium contract into Eq. (8), the manager’s expected utility from exerting effort and following the single-market strategy is \( 0.5(1 + 2D_A)k/c \), which is strictly higher than that from the deviation strategy. □

Proof of Proposition 2. We need to minimize the expected compensation cost in (20) subject to the two incentive constraints in (16) and (19). For the six incentive payments \( \pi_u^A, \pi_u^B, \pi_d^A, \pi_d^B, \pi_u^B, \) and \( \pi_d^B \), the first-order conditions subject to the two constraints are

\[
\frac{U'(U^{-1}(\pi_u^A))}{U'(U^{-1}(\pi_d^A))} \geq \lambda_1 e(1.5 - D_A) + \lambda_2 (0.5 - D_A - e)(0.5 + D_B),
\]

with equality if \( \pi_u^A > 0; \)

\[
\frac{U'(U^{-1}(\pi_u^B))}{U'(U^{-1}(\pi_d^B))} \geq \lambda_1 e(0.5 + D_B) + \lambda_2 (0.5 + D_A + e)(0.5 + D_B),
\]

with equality if \( \pi_u^B > 0; \)

\[
\frac{U'(U^{-1}(\pi_d^A))}{U'(U^{-1}(\pi_d^B))} \geq \lambda_1 e(1.5 - D_A) + \lambda_2 (0.5 - D_B)(0.5 - D_A - e)(0.5 + D_B),
\]

with equality if \( \pi_d^A > 0; \)

\[
\frac{U'(U^{-1}(\pi_d^B))}{U'(U^{-1}(\pi_d^B))} \geq \lambda_1 e(0.5 + D_B) + \lambda_2 (0.5 + D_A + e)(0.5 - D_A - e)(0.5 + D_B),
\]

with equality if \( \pi_d^B > 0. \)

The following lemma verifies that both \( \lambda_1 \) and \( \lambda_2 \) are positive. □

Lemma 1. \( \lambda_1 > 0 \) and \( \lambda_2 > 0. \)

Proof. First, the incentive constraint in (16) must be binding. This is because if this constraint is slack, the solution to minimize the compensation cost would be to set all payments to be zero. This solution, however, violates the constraint in (16). Thus, \( \lambda_1 > 0. \) Now suppose that \( \lambda_1 > 0 \) but \( \lambda_2 = 0 \), i.e., the constraint in (19) is slack. By minimizing the compensation cost subject to (16), we have

\[
\pi^0_u = \pi^d_u = \pi^d_d = 0, \quad \text{and} \quad \pi^0_d > \pi^u_u > 0.
\]

To see this, setting \( \lambda_2 = 0 \), then

\[
U'(U^{-1}(\pi_u^A)) = \frac{0.5 + D_A + e}{\lambda_1 e(1.5 - D_B)} < \frac{(1 + 2D_A + 2e) + (0.5 - D_B)(0.5 + D_A)}{\lambda_1 e(1.5 - D_B)} = U'[U'^{-1}(\pi_u^A)].
\]

Since \( U'[U'^{-1}(\pi)] \) is strictly decreasing in \( \pi \), we have our claim. Now given this, it is direct to verify that this solution violates the constraint in (19). Therefore, both constraints must be binding, i.e., \( \lambda_1 > 0 \) and \( \lambda_2 > 0. \) □

As \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), the right-hand side of Eq. (30) is negative. Thus, \( \pi^0_u = 0 \). Intuitively, this is because \( \pi^0_u \) has a negative incentive differential \( Dp_{u0} < 0 \) in both of the incentive constraints (16) and (19).

The following lemma further determines \( \pi^d_u \) and \( \pi^d_d \) to be zero.

Lemma 2. If \( \pi^0_u > 0 \), then \( \pi^d_u = \pi^d_d = 0. \)
Proof. Based on (29), \( \pi^0_u > 0 \) requires that \( \lambda_1 e > \lambda_2 (0.5 + A_B + e) \). Therefore, \( \pi^0_u = 0 \) and \( \pi^0_d = 0 \) because the right-hand sides of (27) and (28) are negative, while the left-hand side is always positive. □

As a result, there are only three positive payments: \( \pi^0_u, \pi^0_d, \) and \( \pi^0_{\theta^2} \) in the least costly contract. These three payments, together with \( \lambda_1 \) and \( \lambda_2 \), satisfy the binding incentive constraints in (16) and (19):

\[
(1.5 - A_B)\pi^0_u + (0.5 + A_B)\pi^0_d + e\pi^0_{\theta^2} = 4k, \tag{31}
\]

\[
(0.5 - A_B - e)(0.5 + A_B)\pi^0_u + (0.5 + A_B + e)(0.5 + A_B)\pi^0_d - (0.5 + A_B + e)\pi^0_{\theta^2} = 0, \tag{32}
\]

and the first-order-conditions in (25), (26), and (29).

The following lemma provides the ranks of the three positive payments.

Lemma 3. \( \pi^0_u > \pi^0_d \) and \( \pi^0_u > \pi^0_{\theta^2} \).

Proof. Since \( \lambda_2 > 0 \), Eqs. (26) and (29) directly imply that \( \pi^0_u > \pi^0_d \). To show \( \pi^0_u > \pi^0_{\theta^2} \), note that

\[
\frac{1}{U'(U^{-1}(\pi^0_u))} = \frac{e(1.5 - A_B)\lambda_1 + (0.5 - A_B - e)(0.5 + A_B)\lambda_2}{(1 + 2A_B + e) + (0.5 - A_B - e)(0.5 + A_B)} < \frac{e(1.5 - A_B)\lambda_1}{(1 + 2A_B + e)}.
\]

Because \( \frac{1}{U'(U^{-1}(\pi^0_u))} = \lambda_1 e / (0.5 + A_B + e) + \lambda_2 \), it suffices to show that

\[
(1.5 - A_B) > \frac{1}{0.5 + A_B + e},
\]

which holds because \( 1.5 - A_B < 2 \). □

We need to verify that the manager will not pursue any deviation strategy. Two of these strategies have been considered in the main text. Consider the following deviation strategy: the manager shirks; he invests in market \( B \) if \( s_B^0 = 1 \), otherwise he gambles in market \( B \) regardless of \( s_B \). To prevent the use of this strategy, we require that his expected utility from using it

\[
0.25(1.5 + A_B)\pi^0_u + 0.25(0.5 + A_B^2)\pi^0_d + 0.25(1.5 - A_B)\pi^0_{\theta^2} + 0.25(0.5 - A_B)\pi^0_{\theta^2}, \tag{33}
\]

to be dominated by his expected utility given in (14). Another deviation strategy is shirking, investing in market \( B \) if \( s_B = 1 \), and otherwise gambling in market \( A \). To prevent the use of this strategy we also require that the manager’s expected utility from using this strategy

\[
0.25(1.5 + A_B)\pi^0_u + 0.25(0.5 + A_B)\pi^0_d + 0.25(1.5 - A_B)\pi^0_{\theta^2} + 0.25(0.5 - A_B)\pi^0_{\theta^2}, \tag{34}
\]

to be dominated by that in (14). Note that \( \pi^0_u > \pi^0_d \) and \( \pi^0_d = 0 \) in the derived optimal contract. As \( A_B \geq A_B \), the manager’s expected utility from using the first deviation strategy in (33) dominates that from using the second one in (34). Therefore, to verify that (33) is dominated by (14) it suffices to show that

\[
[1 + 2A_B + (0.5 - A_B)(0.5 + A_B)]\pi^0_u + (0.5 + A_B)\pi^0_d + (0.5 + A_B)\pi^0_{\theta^2} \geq (1.5 + A_B)\pi^0_u + (0.5 + A_B)\pi^0_d,
\]

which is equivalent to

\[
(0.5 - A_B)\pi^0_u + (0.5 + A_B)\pi^0_d(0.5 - A_B) \leq (0.5 + A_B)\pi^0_d.
\]

Since \( \pi^0_u < \pi^0_d \), we have

\[
(0.5 - A_B)\pi^0_u + (0.5 + A_B)\pi^0_d(0.5 - A_B) < (0.5 - A_B - e)\pi^0_u + (0.5 + A_B + e)\pi^0_d(0.5 - A_B)
\]

\[
= 0.5 - A_B (0.5 + A_B + e)\pi^0_d,
\]

where the second equality is derived from the binding constraint in (19). Therefore, the following condition is sufficient to ensure that the two aforementioned deviation strategies do not hold:

\[
0.5 - A_B (0.5 + A_B + e) < 0.5 + A_B, \tag{35}
\]

which requires that \( e \) is relatively small.

Finally, the manager could also shirk and always invest in market \( B \). To prevent the use of this deviation strategy, we require that (14) dominates \( 0.25(\pi^0_u + \pi^0_d + \pi^0_{\theta^2}) \). By using the binding constraint in (19), it suffices to show the following condition:

\[
\begin{align*}
&1 + 2A_B + (0.5 - A_B)(0.5 + A_B) \\
&+ (0.5 + A_B)(0.5 + A_B)(0.5 - A_B - e) \\
&+ 2(0.5 + A_B)(0.5 + A_B)\pi^0_u \geq \pi^0_u, \pi^0_d,
\end{align*}
\]

which holds under the following sufficient condition

\[
2(0.5 + A_B)(0.5 + A_B) > 1. \tag{36}
\]

Proof of Proposition 3. Based on the least costly contract derived in Proposition 2, the expected compensation cost of implementing the two-tiered strategy is

\[
K_{TT} = 0.25[(1 + 2A_B + 2e) + (0.5 - A_B - e)(0.5 + A_B)]U^{-1}(\pi^0_u) + 0.25(0.5 + A_B + e)(0.5 + A_B)U^{-1}(\pi^0_d)^2 + 0.25(0.5 + A_B + e)U^{-1}(\pi^0_{\theta^2}),
\]

which is greater than

\[
0.25(0.5 + A_B + e)(1.5 - A_B)U^{-1}(\pi^0_u) + 0.25(0.5 + A_B + e)(0.5 + A_B)U^{-1}(\pi^0_d)^2 + 0.25(0.5 + A_B + e)U^{-1}(\pi^0_{\theta^2}). \tag{37}
\]

Suppose we minimize (37) by using nonnegative \( \pi^0_u, \pi^0_d, \) and \( \pi^0_{\theta^2} \) subject to (16). It should be clear that the minimum is lower than \( K_{TT} \). The minimum is \( (1/2 + A_B + e)U^{-1}(4k/e) \), which is obtained by letting

\[
\pi^0_u = \pi^0_d = \pi^0_{\theta^2} = \frac{4k}{3e},
\]

Because \( U^{-1}(0) = 0 \) and \( U^{-1} \) is convex

\[
\left(\frac{1}{2} + A_B + e\right)U^{-1}(4k/e) > \left(\frac{1}{2} + A_B + e\right)U^{-1}(k/e) = K_{SM}.
\]

This in turn implies that \( K_{TT} > (1/2 + A_B + e)U^{-1}(4k/e) > K_{SM} \).

We now show that \( K_{TT} - K_{SM} \) is increasing in \( k \). Note that in solving for the least costly contract in implementing the two-tiered strategy, (25), (26), (29), (31), and (32) have the feature that the solution \( \{\pi^0_u, \pi^0_d, \pi^0_{\theta^2}\} \) are proportional to \( k \) and \( \{\lambda_1, \lambda_2\} \).
are proportional to \(k^{2/(1-\alpha)}\) (note that when \(U(c)=c^{1-\alpha}\), \(1/U[U^{-1}(\pi)] \propto \pi^{(1-\alpha)/\alpha} k^{2/(1-\alpha)}\)). As a result, the expected cost \(K^T\) is proportional to \(k^{1/(1-\alpha)}\). Proposition 1 implies that the same statement also holds for \(K^M\). As a result, \(K^T-K^M\) is proportional to \(k^{1/(1-\alpha)}\). As \(K^T-K^M\) is positive, it must be increasing with \(k\). □

**Proof of Proposition 4.** We first show that (23) is binding when \(\eta_c\) is sufficiently large. Since the left-hand side of (23) is independent of \(\eta_c\) while the right-hand side increases with \(\eta_c\), we only need to show that there exists one value of \(\eta_c\) so that the least costly contract derived in Proposition 2 (which does not incorporate the constraint in (23)) violates (23). Because of the binding constraint in (16), we only need to show

\[
[(1+2A_A) + (0.5-A_A)(0.5+\Delta_A)]\pi_u^u + (0.5+A_A)(0.5+\Delta_A)\pi_d^u + (0.5+A_A)\eta_c \leq 2\eta_c (\pi_u^u + \pi_d^u).
\]

Let \(\eta_c = 0.5+\Delta_A\). Then, we need to show that

\[
(0.5-A_A)(0.5+\Delta_A)\pi_u^u + (0.5+A_A)\eta_c^2 < (0.5+A_A)(1.5-\Delta_A)\eta_c^2.
\]

Because \(\pi_u^u < \pi_d^u\) and \(\pi_u^u < \pi_d^u\) in the contract, it suffices to show

\[
(0.5-A_A)(0.5+\Delta_A) < (0.5+A_A)(0.5-\Delta_A),
\]

which holds since \(A_A > \Delta_A\).

To verify the second part of the proposition, we need to derive the first-order conditions for deriving the least costly contract. We repeat the minimization of the total compensation cost in (20) subject to constraints in (16), (19), and (23). We denote the Lagrange multiplier of the new constraint by \(\lambda_3 \geq 0\). The first-order conditions for the six payments are now given below:

\[
(1+2A_A + 2e)(0.5-A_A-e)(0.5+\Delta_A) U'[U^{-1}(\pi_u^{v})] \geq \lambda_1 e(1.5-\Delta_A) + \lambda_3 (0.5-A_A-e)(0.5+\Delta_A) + \lambda_3 [2(2+A_A + 2e - 2\eta_c + (0.5-A_A-e)(0.5+\Delta_A))].
\]

(38)

with equality if \(\pi_u^v > 0\);

\[
(0.5+A_A+e)(0.5+\Delta_A) U'[U^{-1}(\pi_d^{v})] \geq \lambda_1 e(0.5+\Delta_A) + \lambda_2 (0.5+\Delta_A+e)(0.5+\Delta_A) - \lambda_3 [2(0.5+\Delta_A+e)(0.5+\Delta_A)],
\]

(39)

with equality if \(\pi_d^v > 0\);

\[
(1-2A_A-2e)(0.5+\Delta_A+e)(0.5-\Delta_A) U'[U^{-1}(\pi_u^{x})] \geq -\lambda_1 e(1.5+\Delta_A) + \lambda_2 (0.5+\Delta_A+e)(0.5-\Delta_A) + \lambda_3 [1-2A_A-2e + (0.5+\Delta_A+e)(0.5-\Delta_A)],
\]

(40)

with equality if \(\pi_u^x > 0\);

\[
(0.5-A_A-e)(0.5-\Delta_A) U'[U^{-1}(\pi_d^{x})] \geq -\lambda_1 e(0.5-\Delta_A) + \lambda_2 (0.5-A_A-e)(0.5-\Delta_A) + \lambda_3 (0.5-A_A-e)(0.5-\Delta_A),
\]

(41)

with equality if \(\pi_d^x > 0\);

\[
(0.5+\Delta_A+e) U'[U^{-1}(\pi_u^{y})] \geq \lambda_1 e - \lambda_2 (0.5+A_A+e) + \lambda_3 (0.5+A_A+e),
\]

(42)

with equality if \(\pi_u^y > 0\);

\[
(0.5-A_A-e) U'[U^{-1}(\pi_d^{y})] \geq -\lambda_1 e - \lambda_2 (0.5-A_A-e) + \lambda_3 (0.5-A_A-e),
\]

(43)

with equality if \(\pi_d^y > 0\);

By comparing (41) and (43), it is easy to show that \(\pi_u^y = 0\) implies that \(\pi_d^y = 0\). This implies that we only need to compare \(\pi_u^y\) and \(\pi_d^y\). From (40), \(\pi_d^y\) is positive if

\[
-\lambda_1 e + \lambda_2 (0.5+\Delta_A) + \lambda_3 (0.5-\Delta_A) + \lambda_3 1-2A_A-2e
\]

(44)

is positive and zero otherwise; while \(\pi_u^y\) is positive if

\[
-\lambda_1 e - \lambda_2 (0.5-A_A) + \lambda_3 (0.5-\Delta_A) + \lambda_3 1-2A_A-2e
\]

(45)

is positive and zero otherwise. Now consider the following sufficient condition that

\[
0.5+\Delta_A < 2A_A+2e.
\]

Under this sufficient condition, we have

\[
0.5+\Delta_A+e < 0.5-A_A-\varepsilon.
\]

This implies that (44) is greater than (45), i.e., \(\pi_d^y\) becomes positive before \(\pi_u^y\) turns positive. □

**Proof of Proposition 5.** The argument for the agency cost to be increasing with \(k\) follows the same argument in Proposition 3, which implies that the expected compensation cost is of order \(k^{1/(1-\alpha)}\). Note that the derivative of the expected compensation cost with respect to \(\eta_c\) is

\[
0.5+\lambda_3 (\pi_u^v + \pi_d^v) \geq 0,
\]

which is strictly positive when the constraint in (23) is binding. Therefore, the expected compensation cost in the presence of tail risk increases with \(\eta_c\), and as a result, \(k^a\) decreases with \(\eta_c\). □

**Appendix B. An alternative model setting**

In this appendix, we adopt an alternative model setting, in which the manager can invest in both markets A and B and face no short-sales constraints. We show that the key result of our main paper remains robust in this alternative setting.

The primary market A and the outside market B are specified in the same way as in the main model, except that the manager can short sell. That is to say, the manager can take either a long or short position in both markets simultaneously based on his signals \(s_A\) and \(s_B\). We still restrict the size of the position to be one unit. There is no need for the manager to take a zero position as such a position is always dominated by either a long or short position depending on the manager’s signal. This alternative setting relaxes two simplifying assumptions used in the main model—the restrictions on the manager’s positions in multiple markets and short sales.

**B.1. Single-market strategy**

We first analyze the single-market strategy. Suppose that the principal implements the single market strategy, i.e., the manager takes a long (short) position of one unit in market A if \(s_A = 1\) (\(s_A = -1\)), and always ignores his free
signal on market $B$ and takes no position outside market $A$. As a result, the fund performance can take two possible values $\tilde{r}_f \in \{r, -r\}$ and the benchmark return of market $A$ can be $\tilde{r}_A \in \{r, -r\}$. Hence, the incentive constraint specifies four possible payments:

$$\{\pi^f_i\} = \{\pi^u_a, \pi^u_d, \pi^d_u, \pi^d_d\}.$$ 

The manager's expected utility from working (i.e., exerting effort to improve his signal about market $A$) is

$$E[\mathcal{U}(c, \theta) \mid \text{exerting effort and obtain } s^\lambda_A] = 0.5[(0.5 + A + e)\pi^u_a + (0.5 - A - e)\pi^u_d + (0.5 + A)\pi^d_u + (0.5 - A)\pi^d_d] - k.$$ 

For instance, the manager receives $\pi^u_d$ when $r_A = -r$ (which has a probability of 0.5) and when he receives $s^\lambda_A = 1$ (which has a conditional probability of 0.5). Similarly, the manager's utility from shirking is

$$E[\mathcal{U}(c, \theta) \mid \text{shirk and obtain } s^0_A] = 0.5[(0.5 + A + e)\pi^u_a + (0.5 - A - e)\pi^u_d + (0.5 + A)\pi^d_u + (0.5 - A)\pi^d_d].$$

Thus, the incentive constraint is $0.5(\pi^u_a - \pi^u_d + \pi^d_u - \pi^d_d) \geq k/e$. Following the same argument used in the main model, the least costly contract to implement this strategy is

$$\pi^u_a = \pi^u_d = k/e, \pi^d_u = \pi^d_d = 0.$$ 

Furthermore, to prevent deviation, the contract pays zero if the fund delivers $2r$ which immediately reveals that the manager has invested in $B$. Hence, the principal's expected payoff from implementing this single-market strategy is

$$V^{SM} = 2(A + e)r - (0.5 + A + e)U^{-1}(k/e).$$

B.2. Double-market strategy

We now consider implementing a so-called double-market strategy—the first-best investment strategy of taking a long position of one unit in market $i$ ($i \in \{A, B\}$) whenever the signal $s_i = 1$ or a short position of one unit if $s_i = -1$. That is, the manager invests in both markets and independently determines a long or short position in each market based on his signal about the market.

Aggregating the manager's positions in the two markets leads to three possible fund returns:

$$\tilde{r}_f \in \{2r, 0, -2r\}.$$ 

When combined with the two possible returns in the primary market (which the principal observes), the incentive contract can be represented by the following six payments:

$$\{\pi^f_i\} = \{\pi^u_a, \pi^u_d, \pi^d_u, \pi^d_d\}.$$ 

With slight abuse of notation, in the superscript $u$ refers to $\tilde{r}_f = 2r$ and $d$ refers to $\tilde{r}_f = -2r$; while in the subscript $u$ refers to $\tilde{r}_A = r$ and $d$ refers to $\tilde{r}_A = -r$.

We derive the manager's expected utility from working based on the following six impossible payments:

1. To receive payment $\pi^u_a$, the fund return $\tilde{r}_f = 2r$ and the primary market return $\tilde{r}_A = r$. This is possible only if the manager takes long positions in both markets, and both markets have positive returns. With probability 0.5 market $A$ has positive return, and the manager takes a long position there with probability 0.5. There are two scenarios to yield a positive return from market $B$: either market $B$ is positive (with probability 0.5) and the manager takes a long position (with probability 0.5) or market $B$ is negative (with probability 0.5) but the manager takes a short position (with probability 0.5). Taken together, the probability of receiving payment $\pi^u_a$ is $0.5(0.5 + A + e)(0.5 + A + e)$. Similar to Case 1, the probability of receiving payment $\pi^u_d$ is $0.5(0.5 - A + e)(0.5 - A + e)$.

2. To receive payment $\pi^u_d$, the fund performance return $\tilde{r}_f = 2r$ and the primary market return $\tilde{r}_A = -r$. This is possible only if the manager takes long positions in both markets, and both markets have positive returns. Similar to Case 2, the probability of receiving payment $\pi^u_d$ is $0.5(0.5 - A + e)(0.5 + A)$.

3. To receive payment $\pi^d_u$, the fund performance return $\tilde{r}_f = 0$ and the primary market return $\tilde{r}_A = r$ (which has probability 0.5). There are two scenarios to reach this outcome: either because the manager takes a long position in market $A$ (with probability 0.5) and a losing position in market $B$ (with probability 0.5), or because the manager takes a short position in market $A$ (with probability 0.5) and a winning position in market $B$ (with probability 0.5). Taken together, the probability for the manager to receive this payment is

$$0.5(0.5 + A + e)(0.5 - A + e) + (0.5 - A + e)(0.5 + A).$$

4. Similarly, the probability for the manager to receive payment $\pi^d_d$ is

$$0.5(0.5 - A - e)(0.5 + A) + (0.5 + A + e)(0.5 - A).$$

5. Similarly, the probability to receive payment $\pi^d_u$ is $0.5(0.5 + A - e)(0.5 + A)$.

6. Similarly, the probability to receive payment $\pi^d_d$ is $0.5(0.5 - A - e)(0.5 - A)$.

Hence, the manager's expected utility from working is

$$0.5(0.5 + A + e)(0.5 + A + e) \pi^u_a + (0.5 + A + e)(0.5 - A + e) \pi^u_d + (0.5 + A + e)(0.5 - A - e) \pi^d_u + (0.5 - A - e)(0.5 - A) \pi^d_d + (0.5 - A - e)(0.5 + A) \pi^d_d.$$ 

His expected utility from shirking is setting $e = 0$ in the above expression and deleting $k$. Thus, the incentive constraint is

$$0.5(0.5 + A) \pi^u_a + 0.5(0.5 + A) \pi^u_d.$$
\[ \geq \frac{k}{e} + A_0 \pi_u^0 + A_0 \pi_d^0 + 0.5(0.5 - A_0) \pi_u^d + 0.5(0.5 - A_0) \pi_d^d. \]  

(49)

Note that for some of the outcomes, investing in market B makes the manager’s good performance in the primary market unclear to the principal, and hence reduces the efficiency of incentive provision. For instance, consider payment \( \pi_u^0 \). The manager may have received a positive signal about the primary market and thus a good performance in his primary market position, together with a bad performance from his position in market B (due to ex post inaccurate signal \( s_B \)). Alternatively, the manager may have received a negative signal about the primary market and thus a poor performance in his primary market position, together with a good performance from his position in market B (due to ex post accurate signal \( s_B \)). Because of the offsetting performance of the manager’s positions in these two markets and because of the inability for the principal to directly observe the manager’s position in the primary market, the cost-to-incentive ratio \( (p/Dp)_u \) of payment \( \pi_u \) is negative in (49). As a result, the least costly contract gives the manager zero payment for this outcome. Like in the main model, the imperfect inference of the manager’s position in the primary market by his position in market B is the key driver of our result.

### B.3. Optimal contract

To show that the single-market strategy may dominate the double-market strategy, we only need to identify an upper bound of the principal’s expected payoff from the double-market strategy. The upper bound is identified by assuming that the incentive constraint in (49), which only considers the deviation of shirking but still following the intended investment strategy, is the only binding constraint in solving the contract. In the main model, another deviation provides an additional binding constraint, which always lowers the principal’s expected payoff.

With the incentive constraint in (49) as the only binding constraint, the least costly contract in implementing the double-market strategy is

\[ \pi_u^0 = \pi_d^0 = \frac{k}{e(0.5 + A_0)}, \quad \pi_u^d = \pi_d^d = \pi_u = 0, \]

and the principal’s expected payoff is

\[ V^{DM} = [2(A_0 + e) + 2A_0]r \]

\[ - (0.5 + A_0 + e)(0.5 + A_0)U^{-1} \left( \frac{k}{e(0.5 + A_0)} \right). \]

(50)

That \( V^{SM} \) in (47) exceeds \( V^{DM} \) in (50) is equivalent to

\[ (0.5 + A_0 + e)(0.5 + A_0)U^{-1} \left( \frac{k}{e(0.5 + A_0)} \right) > 2A_0r. \]

Take \( x = 0.5 \) in Eq. (1), then \( U^{-1}(x) = x^2 \). Then, the above condition is equivalent to

\[ \frac{k^2}{e^2} \frac{0.5 - A_0}{0.5 + A_0 + e} > 2A_0r. \]

which easily holds when \( A_0 < 0 \) or when \( k \) is sufficiently large. Based on the analysis above, we obtain the following proposition.

**Proposition 6.** When the manager’s cost of effort in his primary market is sufficiently high or when the manager’s free signal about market B is sufficiently imprecise, the single-market strategy of only investing in the primary market dominates the double-market strategy of investing in both markets based on the manager’s signals, even though the manager’s signal about market B is nevertheless useful.

The driver of this result works in the same way as the main model. As the principal cannot directly observe the manager’s investment positions and the fund performance aggregates the manager’s returns from both markets, investing in both markets impedes the principal’s inference problem of the manager’s effort and thus makes incentive provision more costly.

### References


