Quantifying Liquidity and Default Risks of Corporate Bonds over the Business Cycle

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Motivation

- Default risk only accounts for part of corp bond spread (bond yield minus treasury)
  - Longstaff, Mithal, Neis (2005), default part: Aaa/Aa, 50%; Baa, 70%
  - Structural models with time-varying macroeconomic risks target default component of corporate bonds:
    Chen, Collin-Dufresne, Goldstein (2009); Bhamra, Kuehn, Strebulaev (2010); Chen (2010)

- This paper: structural model to explain total credit spread
  - Introducing time-varying search frictions and macroeconomic risks into a structural corporate bond pricing models

- Match cross-sectionally (ratings) and over the business cycle:
  1. Historical moments of (conditional) total credits spreads
  2. Historical moments of (unconditional) default probabilities
  3. Historical moments of (conditional) non-default risk: bond-CDS spreads and bid-ask spreads

- Model-based decomposition highlights interactions between liquidity and default, allows framework for policy evaluation
Literature Overview

Structural Credit Models:
▶ Leland (1994b): optimal default with “random” maturity
▶ Huang, Huang (2013): “credit risk puzzle” – matching default and recovery, models have trouble matching credit spreads
▶ Chen, Collin-Dufresne, Goldstein (2009); Bhamra, Kuehn, Strebulaev (2010); Chen (2010): Macroeconomic states with countercyclical risk-premium

Liquidity Models:
▶ Amihud, Mendelson (1986): Liquidity shocks & fixed trans. costs
▶ Duffie, Garleanu, Pedersen (2005): Liquidity shocks & fixed holding costs in OTC search market

Empirical liquidity & yield estimations:
▶ Edwards Harris Piwowar (2007): estimates of BA spreads
▶ Bao Pan Wang (2009): Roll’s measure estimates of liquidity of corporate bonds
▶ Very recent liquidity estimations in the crisis:
  ▶ Dick-Nielsen, Feldhuetter, Lando (2011)
  ▶ Friewald, Jankowitsch, Subrahmanym (2012)
Model Setup

Structural credit model in Leland tradition with OTC bond market:

1. Valuation done under **risk-neutral pricing** kernel (physical probabilities recovered via change-of-measure)
2. **Fixed principal** $p$ and **finite maturity** debt leads to continuous debt-rollover
3. Equity holders **optimally default** when option value of keeping firm alive too costly
4. **Liquidity shocks** force bond holders to trade via **OTC search** market

Aggregate shocks to parameters:

1. **2-state Markov chain**: Normal/Good (G) and Recession/Bad (B) periods
2. Jumps in pricing kernel and volatilites (**shocks to real production process, price of risk**)
3. Jumps in secondary market intermediation parameters & borrowing costs (**shocks to financial system**)
Basic Model: Leland-Toft ’96, Leland ’94b

Rollover gain/loss = \[ m \left[ D_H (y_t) - p \right] \]

\( D_H \): bond price; \( p \): face value repayment; \( y = \log(Y) \): log CFs

Firm:
\[ dy = \mu dt + \sigma dZ \]

Default at \( y_b \)
Idiosyncratic Liquidity Shocks

Firm:
\[ dy = \mu dt + \sigma dZ \]

Reissue

\[ D_H: \]
\[ c \ dt \]

Maturity

\[ D_L: \]
\[ (c - hc) dt \]

Default at \( y_b \)

Liq. Shock

Maturity
Secondary Market and Trading Prices: Bid and Ask

Firm: dy = \mu dt + \sigma dZ

Maturity

Default at \( y_b \)

Reissue

\( D_H: \) c dt

\( D_L: (c-hc) dt \)

Liq.
Shock

\( \xi \)

Interdealer
Market

\( A = D_H \)

\( B = D_L + \beta(D_H - D_L) \)

Intermediation

\( \lambda \)
Degrees of Freedom

Firm:
\[ dy = \mu^s dt + \sigma^s dZ \]

Default at \( y_b^s \):
\[ D_H = \alpha_H^{s} v_b^s \]
\[ D_L = \alpha_L^{s} v_b^s \]

\( D_H \):
\[ c \, dt \]

\( D_L \):
\[ (c-hc^s) dt \]

Reissue

Maturity

Liq. Shock

\( \xi^s \)

Interdealer Market

\( \lambda^s + \beta(D_H-D_L) \)

A = D_H

B = D_L

Intermediation
Microfoundations Holding Costs

Interpretation of liquidity shock

- Liquidity shock leads to a higher borrowing rate: $r_s + \chi_s$
- Bond can be used for collateralized borrowing at rate $r_s$
- **Haircut** on bond $h$ (can be function of price, return volatility, ...)

Borrowing costs

- Let $P^s(y) = \frac{A^s(y) + B^s(y)}{2}$ be the mid-point price
- Marginal benefit of bond: $\chi_s [1 - h(y)] P^s(y)$
- Consider following haircut function decreasing in $P^s(y)$

$$h(y) = \frac{\chi N + rN - c}{rP^s(y)} - \frac{\chi}{r}$$

- Plugging in, we have linear price-dependent holding cost

$$hc^s = \chi_s [N - P^s(y)]$$
## Calibration: Secondary Bond Market

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
<th>G</th>
<th>B</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>Liquidity Shock Int.</td>
<td>2</td>
<td></td>
<td>Bond-CDS &amp; Turnover</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Investor’s Bargaining</td>
<td>0.05</td>
<td></td>
<td>Literature</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Intermediation Int.</td>
<td>50</td>
<td>20</td>
<td>Anecdotal</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>Recovery rate of $H$</td>
<td>58.71%</td>
<td>32.56%</td>
<td>Literature</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>Recovery rate of $L$</td>
<td>57.49%</td>
<td>30.50%</td>
<td>Literature</td>
</tr>
<tr>
<td>$N$</td>
<td>Holding Cost Intercept</td>
<td>107</td>
<td></td>
<td>Bond-CDS sprd, Baa in $G$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Holding Cost Slope</td>
<td>0.12</td>
<td>0.17</td>
<td>Investment BA spread</td>
</tr>
</tbody>
</table>
Calibration: Fundamental and Aggregate Shocks

\[
\frac{d\Lambda_t}{\Lambda_t} = -r(s_t) \, dt - \eta(s_t) \, dZ_t^m + \sum_{s_t \neq s_t'} \left( e^{\kappa(s_{t-}, s_t)} - 1 \right) \, dM_t^{(s_{t-}, s_t)}
\]

<table>
<thead>
<tr>
<th>Model Parameters</th>
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<tbody>
<tr>
<td>Symbol</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>( r )</td>
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<td>( \zeta )</td>
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<tr>
<td>( e^\kappa )</td>
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<tr>
<td>( \mu )</td>
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<tr>
<td>( \eta )</td>
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<tr>
<td>( \sigma_m )</td>
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<tr>
<td>( \sigma_i )</td>
</tr>
<tr>
<td>( m )</td>
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<tr>
<td>( \omega )</td>
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</tbody>
</table>
Default Probabilities (5 year bonds)

- Default Prob: 1920-2011 (unconditional)

<table>
<thead>
<tr>
<th></th>
<th>Aaa/Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity = 5 years</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Default probability (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td>0.7</td>
<td>1.3</td>
<td>3.1</td>
<td>9.8</td>
</tr>
<tr>
<td>model</td>
<td>0.5</td>
<td>1.5</td>
<td>3.7</td>
<td>9.9</td>
</tr>
</tbody>
</table>
Credit spreads (5 year bonds)

- Credit spread: 1994-2012, merge FISD and TRACE

<table>
<thead>
<tr>
<th></th>
<th>Aaa/Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State G</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td>55.7</td>
<td>85.7</td>
<td>149</td>
<td>315</td>
</tr>
<tr>
<td></td>
<td>(3.7)</td>
<td>(6.6)</td>
<td>(15.5)</td>
<td>(33.8)</td>
</tr>
<tr>
<td>model</td>
<td>72.9</td>
<td>103</td>
<td>170</td>
<td>341</td>
</tr>
<tr>
<td><strong>State B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td>107</td>
<td>171</td>
<td>275</td>
<td>542</td>
</tr>
<tr>
<td></td>
<td>(5.8)</td>
<td>(10.5)</td>
<td>(23.9)</td>
<td>(29.8)</td>
</tr>
<tr>
<td>model</td>
<td>99</td>
<td>148</td>
<td>243</td>
<td>459</td>
</tr>
</tbody>
</table>
Bid-Ask Spread (5 year bonds)

<table>
<thead>
<tr>
<th>Bid-Ask spreads (bps)</th>
<th>State G</th>
<th>State B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Superior</td>
<td>Investment</td>
</tr>
<tr>
<td>data</td>
<td>40</td>
<td>50</td>
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<tr>
<td>model</td>
<td>38</td>
<td>50</td>
</tr>
</tbody>
</table>

Calibration Results on Bid-Ask Spread (in bps)

- State B bid-ask spread is normal time numbers multiplied by ratio of bid-ask spread implied by Roll’s measure as in Bao et al (2012)

Translating to Haircuts:

- Model implied haircut: Aaa/Aa, 9.0%; A, 10%, Baa, 12%, Ba, 18%.
- BIS (2010): Aaa/Aa 6.7%, Baa 12%, higher yield 23%. 
## Bond-CDS Spread (5 year bonds)

- Sample: 2005-2012, firms with CDS, 5- and 10-year bonds
- Note: Empirical evidence sheds doubt on “CDS perfectly liquid” assumption
- Crisis: 08Q4 to 09Q1

<table>
<thead>
<tr>
<th>Bond-CDS spreads (bps) 5 years</th>
<th>Aaa/Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State G</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>data</td>
<td>27.7</td>
<td>44.4</td>
<td>74.6</td>
<td>104</td>
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<tr>
<td></td>
<td>(6.6)</td>
<td>(5.8)</td>
<td>(8.7)</td>
<td>(11.2)</td>
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<td>model</td>
<td>55.5</td>
<td>62.1</td>
<td>78.6</td>
<td>99.6</td>
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<td><strong>State B</strong></td>
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<tr>
<td>data</td>
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<td>125</td>
<td>182</td>
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<td></td>
<td>(5.1)</td>
<td>(2.1)</td>
<td>(18.0)</td>
<td>(39.2)</td>
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<tr>
<td>model</td>
<td>74.5</td>
<td>90.6</td>
<td>116</td>
<td>160</td>
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</table>
Counterfactual: Perfect Liquidity (5 year bonds)

- Setting holding cost $hc = 0$ or liquidity shock $\xi = 0$

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<tr>
<th>Maturity = 5 years</th>
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<td></td>
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<td>72.9</td>
<td>103</td>
<td>170</td>
<td>341</td>
</tr>
<tr>
<td>$hc = 0$</td>
<td>10.1</td>
<td>26.5</td>
<td>68.8</td>
<td>194</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State B</th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
<td>99</td>
<td>148</td>
<td>243</td>
<td>459</td>
</tr>
<tr>
<td>$hc = 0$</td>
<td>12.9</td>
<td>33.8</td>
<td>84.0</td>
<td>221</td>
</tr>
</tbody>
</table>
Structural Decomposition

**Standard View:**
- CDS spread measures “Default component”
- Bond-CDS spread = Credit spread - CDS spread measures “Non-default” component

**Our view:**
- Split **default component** based on investor w/o liquidity shocks but s.t. same $y_b$ into (i) **pure default** part based illiquidity free model (w. different $y_b^{LT}$) and (ii) residual of **liquidity-driven default**
- Split **liquidity component**, which is yield spread minus default component, into (i) **pure liquidity** part based on same search frictions but risk-free bonds and (ii) residual **default-driven liquidity**

Concentrate on 5 year bonds; pick $y$ so that bonds has avg credit spread of rating class in G state
Structural Decomposition: G State

- Aaa/Aa 72.9 bps
- A 103 bps
- Baa 170 bps
- Ba 341 bps

Components:
- Pure Default
- Liq-driven Default
- Pure Liq
- Default-driven Liq
Structural Decomposition: B State

Aaa/Slash1 Aa
99 bps
A 148 bps
Baa 243 bps
Ba 459 bps

Default/Minus driven Liq
Pure Liq
Liq/Minus driven Default
Pure Default

Default–driven Liq
Liq–driven Default
Pure Liq
Ba 459 bps
Structural Decomposition: Change $G \rightarrow B$

- Aaa/Slash1Aa: 26.2 bps
- A: 45 bps
- Baa: 73.5 bps
- Ba: 118 bps

- Default/Minus driven Liq
- Pure Liq
- Liq-driven Default
- Pure Default
- Default-driven Liq
Injecting liquidity means improving dealer contact intensity $\lambda^s$ and decreasing un-collateralized borrowing premium $\chi^s$. 
Liquidity Provision

- **Policy evaluation:** What are the effects of “injecting liquidity” on lowering “borrowing cost”?

- **Experiment:** Improve secondary market \((\lambda_B, \chi_B)\) to \((\lambda_G, \chi_G)\): Improved liquidity \(\rightarrow\) safer bond \(\rightarrow\) better liquidity

<table>
<thead>
<tr>
<th></th>
<th>Credit Spread w/o policy</th>
<th>Credit Spread w policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa/Aa G</td>
<td>72.9</td>
<td>54.7</td>
</tr>
<tr>
<td>Aaa/Aa B</td>
<td>99.1</td>
<td>59.2</td>
</tr>
<tr>
<td>Ba G</td>
<td>341</td>
<td>295</td>
</tr>
<tr>
<td>Ba B</td>
<td>459</td>
<td>347</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Contributions to Change (%)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>LIQ→DEF</td>
</tr>
<tr>
<td>Aaa/Aa G</td>
<td>30</td>
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<tr>
<td>Aaa/Aa B</td>
<td>30</td>
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<tr>
<td>Ba G</td>
<td>62</td>
</tr>
<tr>
<td>Ba B</td>
<td>67</td>
</tr>
</tbody>
</table>
Conclusion

- Tractable structural model that embeds aggregate liquidity effects in a capital structure model

- Ability to explain total credit spread, i.e., credit risk premium and liquidity premium, cross-sectionally (across rating classes) and over the business cycle (across aggregate states)

- Holding costs motivated by cost of collateralized vs un-collateralized borrowing (haircuts)

- Counterfactual analysis reveals sizable benefits of injecting liquidity in bad times through default-liquidity interaction terms
Calibration

1. Solve the model for “random maturity” bonds to solve for default boundaries \( \mathbf{y}_b = [y_b(G), y_b(B)]^\top \) (system of equations).

2. Calculate prices of fixed maturity (5 years) bonds numerically given \( \mathbf{y}_b \) above. Compute the implied CDS spread assuming CDS contracts are immune to liquidity problems.

3. Within each rating class, map the observed distribution of market leverage into a distribution of initial cash-flow states \( y \) to circumvent Jensen’s inequality problems (David, 2008; Bhamra et al., 2010).

4. Compute conditional aggregate model implied moments of
   - Total credit spread, (unconditional) default probabilities
   - Measures of bond illiquidity that are endogenous to the model (bid-ask spreads, Bond-CDS spreads)

One set of parameters to explain the entire ratings cross-section.
Jensen’s Inequality: Heterogeneity in the Data

- Rating class defined by the *empirical distribution* of market leverages as given by distribution $d\mathbb{P}_{\text{rating}}$ for each quarter.
Jensen’s Inequality: Implementation

- David (2008), Burma et al. (2010): map each firm-quarter to its model counterpart, then aggregate within each rating class
  - Perfectly match *empirical leverage distribution* for any rating class

- Market leverage: $ML = \frac{D}{D+E}$, so if $ML' > 0$, then 1-to-1 mapping between $ML$ and $y$

- Each $ML$ then implies a cash-flow state $y$, which in turn implies a credit-spread, liquidity measure and default probability

- Average credit spread for each rating class:
  \[
  cs_{rating} = \int_0^1 cs (y (ML)) \, d\mathbb{P}_{rating}
  \]

- Given rating, we explain average credit spread across firms, **not** credit spread of an average firm because strong non-linearities
Calibration: Post-Default Bond Market

- For counterfactual analysis, we need bond recovery w/o post-default illiquidity. So need **ultimate recovery**
- Moody's *Default and Recovery Database* covering 1987-2012
- Risk adjust: discounting these return with a public market benchmark (*SP500*) over the same horizon, known as Public Market Equivalent (PME)

<table>
<thead>
<tr>
<th>Default Time</th>
<th># Def. Bonds</th>
<th>Net PME</th>
<th>Emergence (Yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Recession</td>
<td>512</td>
<td>0.3126</td>
<td>1.37</td>
</tr>
<tr>
<td>Recession</td>
<td>130</td>
<td>0.5537</td>
<td>1.31</td>
</tr>
<tr>
<td>Full Sample</td>
<td>642</td>
<td>0.3613</td>
<td>1.35</td>
</tr>
</tbody>
</table>

- Ultimate recovery rate $\hat{\alpha}$: $\hat{\alpha}_G = 87.96\%$, $\hat{\alpha}_B = 64.68\%$. 
CDS Spread

- Assuming CDS contracts are perfectly liquid
- A CDS contract with maturity $T$ requires a flow payment $f$ that solves

$$\mathbb{E}^Q \left[ \int_0^{\min\{\tau, T\}} e^{-rt} f \cdot dt \right] = \mathbb{E}^Q \left[ 1_{\{\tau \leq T\}} e^{-r\tau} LGD (s) \right]$$

where

- $\tau$ is the first time the firm defaults
- loss-given-default $LGD (s)$: face-value $p$ minus recovery value right at default at state $s$

- CDS spread:

$$CDS\_Spread = \frac{f}{p} = \frac{\mathbb{E}^Q \left[ 1_{\{\tau \leq T\}} e^{-r\tau} LGD (s) \right]}{p \cdot \mathbb{E}^Q \left[ \int_0^{\min\{\tau, T\}} e^{-rt} dt \right]}$$

- Bond-CDS spread: Bond credit spread minus CDS spread