Low Inflation:
High Default Risk AND High Equity Valuations

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ABSTRACT

We develop an asset-pricing model with endogenous corporate policies that explains how inflation jointly impacts real asset prices and corporate default risk. Our model includes two empirically founded nominal rigidities: fixed nominal debt coupons (sticky leverage) and sticky cash flows. Taken together, these two frictions result in higher real equity prices and credit spreads when expected inflation falls. An increase in expected inflation has opposite effects, but with smaller magnitudes. In the cross section, the model predicts that the negative impact of expected inflation on real equity values is stronger for low leverage firms. We find empirical support for the model’s predictions.

JEL Classification Numbers: E44, G12, G32, G33

Keywords: low inflation, default risk, equity, leverage, credit spreads
1 Introduction

Corporate default risk spikes during times of low expected inflation. But so do firms’ equity valuations, despite increased bankruptcy risk. Figure 1 documents these two empirical facts for the U.S. over the period 1970Q2–2016Q4. Panel A illustrates the strong negative relationship between expected inflation and the number of quarterly defaults in the U.S., whereas Panel B shows a similar negative relationship between expected inflation and the price-dividend ratio. In this paper, we attempt to explain how shareholders can rationally value equity more favorably during periods of low expected inflation, despite facing greater bankruptcy risk. We propose a theory that reconciles this apparent contradiction and provide novel empirical evidence that these relationships are robust features of the data.¹

Existing theories have overlooked the connection between these two empirical relationships and only examined them separately. One branch of the literature focuses on the link between expected inflation and default risk, but yields counterfactual implications for equity valuation. See, for example, Bhamra, Fisher, and Kuehn (2011), Kang and Pflueger (2015), or Gomes, Jermann, and Schmid (2016), in which lower expected inflation reduces both the nominal risk-free rate and the expected growth rate of a firm’s nominal cash flows. Both effects increase firms’ indebtedness and default risk, but reduce equity prices. Another branch of the literature investigates the link between expected inflation and equity values, but remains silent on implications for default risk. See, for example, Modigliani and Cohn (1979), Feldstein et al. (1980), Ritter and Warr (2002), Sharpe (2002), and Campbell and Vuolteenaho (2004).² A common explanation for the link between inflation and equity prices is money illusion: Investors discount real cash flows with nominal discount rates.³

In contrast to the existing literature, we propose a unified treatment of the empirical facts we illustrate in Figure 1. We build on Bhamra, Kuehn, and Strebulaev (2010a,b) and Chen (2010) and construct an asset-pricing model with fluctuating levels of expected inflation to explain these apparently contradictory observations.⁴ Our framework provides predictions on default risk and equity valuation from a corporate finance perspective, whereby firms’ financing and default policies are endogenous. Firms issue

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¹We show the relationships displayed in Figure 1 are not an artifact of firm aggregation. Specifically, we find that the same sets of firms experience an increase in default risk and equity valuation when expected inflation falls. Furthermore, firm-level regressions reveal that equity valuation and default risk jointly decrease with expected inflation after controlling for firm characteristics and variations in aggregate financial/economic conditions.


³Alternative explanations are the non-neutrality of inflation and the existence of an inflation risk premium. We describe the relevant literature in more detail below.

⁴Bhamra et al. (2010a,b) and Chen (2010) analyze firms’ capital-structure and default decisions, as well as levered asset prices, in a consumption-based model with changing macroeconomic conditions. Our key modelling contribution relative to the above papers is the introduction of a new state variable (stochastic expected inflation) and two specific nominal rigidities (sticky leverage and sticky cash flows). Understanding how these two nominal rigidities impact the
nominal debt and equity, which are priced by a representative agent with Epstein-Zin-Weil preferences. The economy switches randomly between expansion and recession, creating intertemporal macroeconomic risk. A two-state Markov regime-switching model with parameter estimates based on quarterly U.S. consumption data over the period 1970Q2-2016Q4 determines the switches between real states. We introduce three expected inflation states (low, moderate, and high) via a second, independent Markov regime-switching process that matches the one-year mean inflation forecast from the Survey of Professional Forecasters. We refer to fluctuations in the expected inflation rate as inflation risk, which is distinct from real macroeconomic risk.

We consider two key frictions in the model, both of which act as nominal rigidities. First, firms keep their nominal debt coupons fixed. This stickiness of leverage means changes in expected inflation impact real asset prices via shifts in the real values of debt coupons. Second, price rigidity in the goods market implies sticky nominal cash flow growth in the short run, and so expected nominal cash flow growth changes less than one-for-one with changes in expected inflation. We denote this friction as sticky cash flows and find strong empirical support for this nominal rigidity in U.S. data. Our assumption is consistent with the evidence on the stickiness of output prices (see, e.g., Nakamura and Steinsson (2008), Gorodnichenko and Weber (2016)), Pasten, Schoenle, and Weber (2019) and way expected inflation affects endogenous corporate financial policies and real asset prices forms the novel theoretical core of this paper.
D’Acunto, Liu, Pflueger, and Weber (2018) and nominal rigidities are also central to explain the real effects of large-scale asset purchase programs (Elenev (2019)).

We show that these two empirically motivated nominal frictions are sufficient to explain the stylized facts in Figure 1. Our model predicts that an individual firm experiences both higher credit risk and higher equity valuation during periods of lower expected inflation. In addition, we extend our analysis to a simulated economy of firms, whose distribution of leverage ratios is structurally similar to that in the data. Hence, we show that the negative impact of expected inflation on equity valuation and default risk continues to persist—and thus does not cancel out—when aggregating over a cross section of firms.

The sticky leverage nominal rigidity is the key driver of our theoretical result that a fall in expected inflation increases credit risk and credit spreads. In the model, a fall in expected inflation reduces a firm’s performance in nominal terms, which increases its default probability. Hence, the stickiness of leverage drives the negative association between credit risk and expected inflation. Changes in expected inflation affect equity valuation through two distinct channels. First, a lower nominal cash flow growth rate caused by a fall in expected inflation decreases equity value. Second, firm cash flows are discounted at a lower nominal risk-free rate, which increases equity value. The latter effect dominates the former because the nominal risk-free rate varies one-for-one with expected inflation, whereas the nominal cash flow growth rate varies less than one-for-one with expected inflation. Hence, equity valuation decreases with expected inflation, and this effect arises because of sticky cash flows. We obtain a similar theoretical prediction when replacing cash-flow stickiness by a rule for nominal interest rates in which the Taylor principle holds, which implies a greater sensitivity of the nominal risk-free rate to expected inflation.

The model also generates the convexity in the relations depicted in Figure 1. A decrease in expected inflation increases the value of equity, and it appears natural to assume an increase in expected inflation of the same size will result in an equal-sized decrease of equity values. But such an analysis is incomplete, because it ignores how the present value of firm cash flows depends non-linearly on the nominal risk-free rate, and thus on the level of expected inflation, via nominal discounting. The relation between equity valuation and expected inflation is thus asymmetric, and so low expected inflation is not the mirror image of high expected inflation. Lower expected inflation increases equity

\[\text{On the theoretical side, menu-cost models generate a band of inaction, rationalizing price non-adjustment to shocks (see, e.g., Mankiw (1985) and Ball and Mankiw (1994)). Nominal price rigidities are the leading explanation of the real effects of monetary policy.}

\[\text{Under this alternative setting, the nominal risk-free rate varies more than one-for-one with expected inflation, while the nominal cash flow growth rate varies one-for-one with expected inflation. The nominal risk-free rate continues to be more sensitive than the nominal cash flow growth rate to changes in expected inflation, which induces a similarly negative relation between equity valuation and expected inflation.}

\[\text{This prediction arises although default probabilities are convex in the distance-to-default, which implies that an increase in default risk depresses the value of equity more than a decrease in default risk of the same size. But we show this effect is not sufficient to offset the asymmetry arising from nominal discounting.}
prices more than higher expected inflation depresses them, which implies that fluctuations in expected inflation increase equity valuation on average.\textsuperscript{8} Hence, inflation risk has a positive—and not a negative—effect on real asset values. Our paper thus contributes to understanding the impact of inflation fluctuations on asset pricing and in showing that inflation risk can be economically beneficial to investors.

At first glance, it may appear that the sticky leverage assumption drives the model’s results for credit risk while the sticky cash flow assumption drives the equity valuation results. If the two assumptions operated independently, it could call into question jointly studying how expected inflation affects both equity valuation and credit risk. However, the sticky leverage assumption does impact equity valuation: in the cross section, the impact of expected inflation on equity values varies with firm’s leverage. A decrease in expected inflation reduces the nominal risk-free rate, which increases the present value of nominal coupons paid to debtholders. The higher value of debt partially offsets the initial increase in equity value. Therefore, a fall in expected inflation increases the value of levered equity, but this increase becomes smaller for firms with more debt outstanding. The model thus predicts the increase in equity prices during times of low expected inflation should be less pronounced for high-leverage firms.

We provide a detailed empirical investigation of the impact of expected inflation on both equity valuation and default risk. Our empirical analysis has two aims. First, we test the cross-sectional prediction that financial leverage reduced the sensitivity of equity values to expected inflation. Second, we verify that the negative and asymmetric relations are robust at the firm level. We use CRSP-Compustat merged data from April 1972 to December 2016 and exploit two firm-level measures of equity valuation: the market-to-book (M/B) ratio and the price-dividend ratio.\textsuperscript{9} We compute a firm’s financial distress risk and its implied physical default probability, following Campbell, Hilscher, and Szilagyi (2008). A portfolio analysis with firms sorted on their financial leverage ratios shows that default risk and equity valuation decrease with the level of expected inflation, for both low and high-leverage firms. The reduction in equity valuation is, however, stronger for less-levered firms, which means that a high level of debt reduces, rather than exacerbates, the sensitivity of equity prices to changes in nominal conditions. The validation of this cross-sectional prediction provides support for our model.

A potential concern is that variations in expected inflation reflect changes in economic or financial conditions, and that the resulting changes in default risk and equity valuation at the portfolio level are due to grouping heterogeneous firms. We address these issues by running firm-level regressions with 743,536 firm-quarter observations and a rich set of financial, macroeconomic, and firm-level controls.

\textsuperscript{8}To reach this conclusion, we compare the model’s prediction with that of an hypothetical economy with expected inflation set at its unconditional mean.

\textsuperscript{9}The availability of forecasts for inflation determines the starting point of the sample.
We find that the negative and asymmetric impact of expected inflation on a firm’s equity valuation and default risk are highly statistically significant, and that the results remain robust to different samples of firms and subperiods. The results continue to hold when we condition on firms that remain in our sample throughout the period, which ensures a firm-selection effect does not explain our findings. Our empirical analysis thus provides robust support that U.S. firms indeed display higher equity values, despite higher default risk, when expected inflation decreases.

Our paper makes several contributions. First, we build a model of multiple firms issuing debt and equity with the option to default, where expected inflation impacts firms’ asset prices. We explain the negative relation between equity valuation and expected inflation with sticky cash flows. Our model generates the negative impact of expected inflation on default risk through sticky leverage, which induces variations in real leverage. Second, we find that equity prices and default risk are more sensitive to a change in expected inflation when expected inflation is currently low than when it is high, which suggests a fundamental asymmetry in the effects of inflation risk. This asymmetry is important in light of the extremely low inflation levels we have observed during and after the Great Recession. Third, in the cross section, we show equity prices vary more with expected inflation for less-levered firms. Finally, we empirically validate all these predictions at the firm level, which provides new evidence regarding the joint response of equity valuation and default risk to variations in expected inflation.

Existing studies going back to Fama (1981) provide explanations for the negative relation between equity valuation and expected inflation, based on the idea that inflation is non-neutral because it has a negative effect on real growth.10 Agents demand a positive inflation risk premium, which reduces equity prices (e.g., Eraker, Shaliastovich, and Wang (2015)). Our theory generates a negative relation without requiring an inflation risk premium. In fact, we intentionally have a real stochastic discount factor, which is completely independent of the nominal state, in line with the view that periods of low/high inflation can be associated with either good or bad economic conditions.11 Our model shows that sticky leverage combined with sticky cash flows is sufficient to generate the relations between inflation, equity valuation, and default risk we observe in the data.

This paper also contributes to the literature exploring empirically the relation between inflation and equity returns. Chen, Roll, and Ross (1986) and Ang, Briere, and Signori (2012) find inflation risk is priced in the cross section of U.S. equity returns, whereas Boons et al. (2019) show the inflation

10Other studies exploring theoretically the interaction between inflation and equity returns include Day (1984), Stulz (1986), Wachtler (2006), Gabaix (2008), Hess and Lee (1999), Chen (2010), Bansal and Shaliastovich (2013), and Gomes et al. (2016).
11There is no consensus that agents should like higher or lower inflation. For example, Piazzesi and Schneider (2006) show inflation predicts consumption growth negatively, whereas Boons, Duarte, de Roon, and Szymanowska (2019) suggest the relation is time-varying. This finding is consistent with the evidence inflation periods do not always reflect a bad state of the economy. See, for example, Bekaert and Wang (2010), Campbell, Sunderam, and Viceira (2017), and David and Veronesi (2013).
risk premium varies over time conditional on the relation between inflation and the real economy. We depart from this literature by explaining the relation between inflation and equity valuation without linking inflation to consumption. Our work is closely related to Weber (2015) who shows how inflation risk impacts equity returns via a sticky-price channel. We combine the idea of sticky cash flows with sticky leverage. Finally, Kang and Pflueger (2015), which studies how inflation risk impacts corporate bond prices is another closely related paper. Our paper complements this study by jointly studying expected inflation, default risk, and equity prices in a unified framework. Furthermore, we provide empirical evidence and a theoretical explanation for the asymmetric relation between asset prices and expected inflation.

In Section 2 we present a simple model to clarify the qualitative relationships between equity valuation, credit risk, and expected inflation, whereas in Section 3 we describe the fully fledged consumption-based asset-pricing model with inflation risk. In Section 4 we derive asset prices together with optimal default and capital-structure decisions. We discuss the model’s theoretical predictions in Section 5. In Section 6 we provide novel empirical evidence for the relationship between expected inflation, equity values and debt values. We also confront our model with the data. In Section 7 we conclude.

2 Intuition from a Simple Model

In this section, we consider a simple, static model with exogenous financing and default policies. We develop intuition for the negative impact of expected inflation on equity valuation and credit risk. We also discuss why equity prices and credit risk are more sensitive to a decrease in expected inflation than to an increase in expected inflation, that is, why the relations are asymmetric. Appendix OA.A provides details on derivations and proofs.

2.1 Economy

To value nominal asset prices, we specify a price index $P_t$ that satisfies

$$\frac{dP_t}{P_t} = \mu_P dt,$$

where $\mu_P$ is expected inflation, which is constant. We assume the price index is locally risk free. The nominal risk-free rate is equal to the real interest rate $r$ plus expected inflation $\mu_P$, i.e. $r^\$ = r + \mu_P$.

Consider a firm with time $t$ nominal cash flow $X_t$. Under the risk-neutral probability measure $Q$, the dynamics of $X_t$ is given by

$$\frac{dX_t}{X_t} = \hat{\mu}_X dt + \sigma_X dW_t^Q,$$

where $\hat{\mu}_X$ and $\sigma_X$ are the risk-neutral drift and volatility of $X_t$, respectively. This model allows us to derive asset prices and optimal capital-structure decisions in the presence of inflation risk.
where $W_t^Q$ is a standard Brownian motion. The nominal cash-flow growth volatility $\sigma_X$ equals real cash-flow growth volatility, as the price index is locally risk free. The expected nominal cash-flow growth is the sum of real expected cash-flow growth $\tilde{\mu}_Y$ and a multiple $\varphi$ of expected inflation $\mu_P$, that is, $\tilde{\mu}_X = \tilde{\mu}_Y + \varphi \mu_P$, where $\varphi$ captures the sensitivity of nominal cash-flow growth to expected inflation. Cash flows are sticky when $\varphi < 1$.

Strong empirical evidence exists supporting the notion of sticky cash flows (see, e.g., Nakamura and Steinsson (2008) and Gorodnichenko and Weber (2016)), which we confirm by estimating the parameter $\varphi$ with U.S. data. We regress the consensus forecast for the growth rate of corporate profits over the next 12 months on the consensus forecast for inflation over the same period, using data from the Survey of Professional Forecasters. The estimate of $\varphi$ is 0.415 in column (4) of Table 1, which controls for variations in real macroeconomic conditions. This estimate is significantly lower than 1 ($t$-stat of 3.5), which indicates that the relation between expected nominal cash flow growth and expected inflation is less than one-for-one. Hence, cash flows are sticky with respect to changes in nominal conditions. Consistent with this evidence, we hereafter assume $\varphi = 0.5$, which lies in the confidence interval of our estimate.

Table 1 [about here]

The firm issues equity and a bond. The corporate bond pays out a fixed nominal coupon of $c$ dollars per unit of time until default which occurs at the first passage time $\tau_D = \inf_{t>0} \{X_t \leq X_D\}$, for some fixed default threshold $X_D$. The debt coupon is constant in nominal terms, that is, leverage is sticky. The firm has no residual value when default occurs and there are no taxes.\(^{12}\)

Consider first the case of a bond with no default risk. The nominal price of this bond is given by

$$B_{f,t}^\$ = \frac{c}{r + \mu_P}, \quad (3)$$

from which we can immediately obtain the real bond price $B_{f,t} = B_{f,t}^\$/P_t$. Sticky leverage implies that both the real and nominal prices of a bond without default risk are decreasing in expected inflation, simply because the real value of the nominal coupon decreases with expected inflation. We report this relation in Panel A of Figure 2, using three levels of expected inflation: low ($\mu_P = 1\%$), moderate ($\mu_P = 3\%$), and high ($\mu_P = 5\%$).

Figure 2 [about here]

\(^{12}\)We relax these assumptions in the full model (Section 3).
A corporate bond subject to default risk has a different exposure to expected inflation. The nominal price of such bond is equal to

$$B_t^S = B_{f,t}^S \left[ 1 - q_{D,t}^S(\mu_P) \right],$$

(4)

where the term $0 < q_{D,t}^S(\mu_P) < 1$ is the Arrow-Debreu default claim, that is, the date $t$ price of the security that pays out one dollar at the time of default. A fall in expected inflation reduces a firm’s performance in nominal terms, which increases the Arrow-Debreu default claim (for any value of $\varphi > 0$). This effect originates from the nominal debt coupons that are constant and, thus, not adjusted with expected inflation. This is a direct consequence of the stickiness of leverage, as in Bhamra et al. (2011), Kang and Pflueger (2015), and Gomes et al. (2016).

A change in expected inflation has now two opposing effects on corporate debt valuation: the present value of the nominal coupons, $B_{f,t}^S$, decreases with expected inflation, while the Arrow-Debreu default claim, $q_{D,t}^S(\mu_P)$, also decreases with expected inflation. The presence of default risk dampens the price sensitivity of a risky corporate bond to expected inflation, relative to that of a risk-free bond (Panel A). The yield of a default-risky corporate bond ($y^S = c/B_t^S$) then moves less than one-for-one with expected inflation, whereas the nominal risk-free rate ($r^S = c/B_{f,t}^S$) moves one-for-one with expected inflation (Panel B). The credit spread, which is defined as the nominal yield of the corporate bond minus the nominal risk-free rate

$$s_t = y^S - r^S = \frac{c}{B_t^S} - \frac{c}{B_{f,t}^S},$$

(5)

decreases with expected inflation (Panel C). Without leverage stickiness, the corporate bond subject to default risk displays a similar sensitivity to expected inflation as the risk-free counterpart, thereby turning off the negative exposure of credit risk to expected inflation. With sticky leverage, the relation between the credit spread and expected inflation is not only negative but also convex, as expected inflation impacts default risk non-linearly via the Arrow-Debreu default claim (Panel D). Intuitively, a decrease in expected inflation increases default risk more when default risk is currently high, which is when expected inflation is low.

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13 Without leverage stickiness, the firm would optimally reduce the level of the nominal debt coupon to offset the increase in default risk when expected inflation decreases.

14 We can directly interpret the predictions on the credit spread as predictions on default risk, because the bond has no value in default.

15 The Arrow-Debreu default claim satisfies $q_{D,t}^S(\mu_P) = e^{-a(\mu_P)(x_t - x_D)}$, with $x_t = \ln X_t$ and $x_D = \ln X_D$. $x_t - x_D$ is the distance-to-default in logarithms and so $a(\mu_P)$ acts as a discount rate, which is increasing in expected inflation. For a given distance-to-default, the relation between the Arrow-Debreu default claim $q_{D,t}^S(\mu_P)$ and expected inflation $\mu_P$ is negative and convex, as verified in Appendix OA.A.
We turn now to equity valuation. The nominal value of equity is given by the nominal firm value less the nominal bond value

\[
S_t^S = V_t^S - B_t^S, \tag{6}
\]

with

\[
V_t^S = \frac{X_t - X_{Dq,D,t}}{r^S - \bar{\mu}_X} = \frac{X_t - X_{Dq,D,t}}{r - \bar{\mu}_Y + (1 - \varphi)\mu_P}, \tag{7}
\]

where \(V_t^S\) is the present value of the firm’s cash flows up until default. Observe that \(V_t^S\) is decreasing with expected inflation when \(\varphi < 1\) (Panel E), and so the equity value, \(S_t^S\), is also decreasing with expected inflation (Panel F).

Expected inflation affects equity valuation through two distinct channels. First, a fall in expected inflation reduces the nominal cash flow growth rate, which decreases equity valuation. Second, firm cash flows are discounted at a lower nominal risk-free rate, which increases equity valuation. The latter effect dominates the former because the nominal risk-free rate varies one-for-one with expected inflation, whereas the nominal cash flow growth rate varies less than one-for-one with expected inflation when \(\varphi < 1\). Hence, equity valuation decreases with expected inflation, and this relation arises from sticky cash flows. A change in the nominal risk-free rate impacts equity values non-linearly via nominal discounting, which implies that the impact of changes in expected inflation on equity value is stronger when expected inflation is lower. Hence, the relation between equity valuation and expected inflation is negative and asymmetric.

To sum up, Figure 2 shows that equity valuation and credit risk are both negatively related to expected inflation. Hence, a firm displays higher equity prices and, at the same time, faces higher credit spreads (or default risk) when expected inflation decreases. Furthermore, a change from moderate to low expected inflation has a greater impact than a change from moderate to high expected inflation, although we consider symmetric variations in expected inflation. Hence, low expected inflation is not the mirror image of high expected inflation.

This analysis demonstrates that firms can have higher levered equity valuations and higher default risk when expected inflation decreases. This simple model assumes no arbitrage and does not make specific assumptions about preferences. The two critical features driving both relations are sticky cash flows, for which we find strong support in the data, and sticky leverage, which is an empirically grounded friction in the corporate debt market. Based on these insights, we now consider a dynamic version of the model with endogenous corporate policies to study how fluctuations in expected inflation jointly impact equity valuation and credit risk in a richer environment.
3 Model

This section presents a dynamic asset-pricing model with firms facing real and nominal risk. We first define aggregate consumption and inflation and derive the real and nominal stochastic discount factors, using an Epstein-Zin-Weil representative agent. We then derive the asset values of firms, which issue nominal debt and equity, and describe their optimal financing and default decisions.

3.1 Aggregate economic variables

Aggregate consumption at date $t$ is denoted by $C_t$ and its dynamics are given exogenously by

$$\frac{dC_t}{C_t} = \mu_{C,t} dt + \sigma_{C,t} dZ_t,$$

(8)

where $Z_t$ is a standard Brownian motion under the physical probability measure $\mathbb{P}$. The conditional first and second moments of aggregate consumption growth, $\mu_{C,t}$ and $\sigma_{C,t}$, respectively, can take different values, depending on the current state of the real economy, denoted by $\nu_t$. The real economy is risky and transitions between a recession state, $\nu_t = R$, and an expansion state, $\nu_t = E$, according to a two-state Markov chain. The probability under the physical probability measure of moving from the expansion state to the recession state within an instant $dt$ is $\lambda^{\text{real}}_{ER} dt$. Similarly, the probability under the physical measure of moving from the recession state to the expansion state within an instant $dt$ is $\lambda^{\text{real}}_{RE} dt$. We have $\mu_{C,R} < \mu_{C,E}$ and $\sigma_{C,R} > \sigma_{C,E}$ to ensure the mean and volatility of consumption growth are procyclical and countercyclical, respectively.

Inflation dynamics are specified exogenously. The date $t$ level of the price index is denoted by $P_t$ and satisfies

$$\frac{dP_t}{P_t} = \mu_{P,t} dt,$$

(9)

where we neglect inflation volatility stemming from small Brownian shocks, and assume that date $t$ conditional expected inflation, $\mu_{P,t}$, depends on the nominal state $\epsilon_t$. We assume three nominal states: a low expected inflation state, $\epsilon_t = L$; a moderate expected inflation state, $\epsilon_t = M$; and a state of high expected inflation, $\epsilon_t = H$. From the definition of the nominal state, $\mu_{P,L} < \mu_{P,M} < \mu_{P,H}$. The physical probability of moving from the nominal state $l$ to $k$, within the instant $dt$, is $\lambda^{\text{real}}_{lk} dt$, and the probability of moving back within a later instant is $\lambda^{\text{real}}_{kl} dt$, where $k, l \in \{L, M, H\}$ and $l \neq k$. 

10
For ease of notation, we combine the real and nominal states into six distinct states, where the current combined state is denoted by $s_t = (\nu_t, \epsilon_t)$. In summary, the different states are

<table>
<thead>
<tr>
<th>State Description</th>
<th>State</th>
<th>Coordination</th>
<th>$\sigma_{C,t}$</th>
<th>$\mu_{P,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession &amp; Low Expected Inflation (RL)</td>
<td>1</td>
<td>$(R, L)$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,L}$</td>
</tr>
<tr>
<td>Recession &amp; Moderate Expected Inflation (RM)</td>
<td>2</td>
<td>$(R, M)$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,M}$</td>
</tr>
<tr>
<td>Recession &amp; High Expected Inflation (RH)</td>
<td>3</td>
<td>$(R, H)$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,H}$</td>
</tr>
<tr>
<td>Expansion &amp; Low Expected Inflation (EL)</td>
<td>4</td>
<td>$(E, L)$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,L}$</td>
</tr>
<tr>
<td>Expansion &amp; Moderate Expected Inflation (EM)</td>
<td>5</td>
<td>$(E, M)$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,M}$</td>
</tr>
<tr>
<td>Expansion &amp; High Expected Inflation (EH)</td>
<td>6</td>
<td>$(E, H)$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,H}$</td>
</tr>
</tbody>
</table>

The transitions between combined real and nominal states are given exogenously by a six-state Markov chain. Real and nominal regimes switch independently over period $dt$, and so the physical probability of the combined state switching from $s_{t-} = (\nu_{t-}, \epsilon_{t-})$ to $s_t = (\nu_t, \epsilon_t)$, where $s_t \neq s_{t-}$ within a time interval of length $dt$ is given by

$$
\lambda_{s_{t-}, s_t} dt = \lambda_{\nu_{t-}, \nu_t}^{\text{real}} \lambda_{\epsilon_{t-}, \epsilon_t}^S dt,
$$

where $\lambda_{\nu_{t-}, \nu_t}^{\text{real}} = 1$ if $\nu_{t-} = \nu_t$ (i.e. the real state does not change) and $\lambda_{\epsilon_{t-}, \epsilon_t}^S = 1$ if $\epsilon_{t-} = \epsilon_t$. (that is, the nominal state does not change).

### 3.2 Representative agent and stochastic discount factors

The representative agent has the continuous-time analog of Epstein-Zin-Weil preferences. The real stochastic discount factor (SDF) at time $t$, $\pi_t$, depends on the state of the real economy and is given by (see Appendix OA.B for the derivation)

$$
\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\psi}} C_t^{-\gamma} \left(p_{C,t} \int_0^t p_{C,u}^{-1} du\right)^{-\frac{\gamma-1}{1-\psi}},
$$

where $\beta$ is the rate of time preference, $\gamma$ is the coefficient of relative risk aversion (RRA), and $\psi$ is the elasticity of intertemporal substitution under certainty (EIS). The date $t$ value of the claim to aggregate consumption per unit of time is denoted by $p_{C,t}$. This price-consumption ratio depends only

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16The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1989) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness for the finite-horizon case.
on the real state of the economy, denoted by \( \nu \):

\[
p_{C,t} = \begin{cases} 
  p_{C,R}, & \text{if } \nu_t = R \\
  p_{C,E}, & \text{if } \nu_t = E.
\end{cases}
\]  \(13\)

When \( \psi > 1 \), \( p_{C,t} \) is procyclical and the real SDF increases in the recession state, \( R \).\(^{17}\)

The real stochastic discount factor at date-\( t \), \( \pi_t \), evolves as follows

\[
\frac{d\pi_t}{\pi_t \mid_{\nu_{t-},\nu_t}} = -r_i dt - \gamma \sigma_{C,i} dZ_t + (\omega_{ij} - 1) dN^P_{ij,t}, \ i, j \in \{R, E\}, \ j \neq i,
\]  \(14\)

where \( r_i \) is the equilibrium real risk-free interest rate in state \( i \in \{R, E\} \), given by

\[
r_i = \begin{cases} 
  r_R, & i = R, \\
  r_E, & i = E,
\end{cases}
\]  \(15\)

with \( r_E > r_R \) so that the real interest rate is procyclical with respect to the real economy. The real interest rates are identical to those of Bhamra et al. (2010a) and Bhamra et al. (2010b), and given in Appendix OA.B. Two types of risk are priced. The increment in the standard Brownian motion \( dZ_t \) represents small but frequent changes in unexpected consumption growth, and \( \gamma \sigma_{C,i} \) is the associated price of risk. The compensated Poisson process \( N^P_{\nu_{t-},\nu_t,t} \) is given by

\[
N^P_{\nu_{t-},\nu_t,t} = dN_{\nu_{t-},\nu_t,t} - \lambda_{\nu_{t-},\nu_t}^{\text{real}} dt, \ \nu_{t-},\nu_t \in \{R, E\}, \ \nu_t \neq \nu_{t-}
\]  \(16\)

where \( N_{\nu_{t-},\nu_t,t} \) is a Poisson process that jumps up by 1 when the real state of the economy switches; that is, \( N_{\nu_{t-},\nu_t,t} = 1 \) if \( \nu_t \neq \nu_{t-} \). The increment in the compensated Poisson process, \( dN^P_{\nu_{t-},\nu_t,t} \), is a martingale (under the physical measure \( P \)) that represents the risk that the real state of the economy changes with associated price of risk \( \omega_{ij} \). The representative agent only prices real risks. If the real state of the economy moves from expansion to recession, that is, \( \nu_{t-} = E \) and \( \nu_t = R \), then \( \omega_{ER} = \omega \), where \( \omega = \left( \frac{p_{C,R}}{p_{C,E}} \right)^{\frac{\gamma - 1}{\psi}} \). Similarly, if the real state of the economy moves from recession to expansion, that is, \( \nu_{t-} = R \) and \( \nu_t = E \), then \( \omega_{RE} = \omega^{-1} \). Intuitively, a negative shock to the economy results in an increase in the real SDF, and so \( \omega > 1 \).

\(^{17}\)The price-consumption ratios for each real state are derived from a coupled system of nonlinear algebraic equations given in (OA.41) of the Online Appendix.
The pricing of securities is based on the risk-neutral switching probabilities per unit of time (that is, transition intensities), \( \hat{\lambda}_{s_{t-}, s_t} \), which are related to the physical switching probabilities, \( \lambda_{s_{t-}, s_t} \), via

\[
\hat{\lambda}_{s_{t-}, s_t} = \omega_{\nu_{t-}, \nu_t} \lambda_{\nu_{t-}, \nu_t}^{real} \lambda_{s_{t-}, s_t}, \quad s_{t-} \neq s_t,
\]

where \( \omega_{\nu_{t-}, \nu_t} = 1 \) if \( \nu_{t-} = \nu_t \) (that is, the real state does not change). Hence, under Epstein-Zin-Weil preferences, \( \omega_{ER} = \omega = \omega_{RE}^{-1} \) acts as a distortion factor, distorting physical transition intensities. The representative agent cares about future consumption growth and prefers early resolution of intertemporal risk (\( \gamma > 1/\psi \)), and so \( \omega > 1 \), which implies the risk-neutral probability per unit of time of switching from expansion to recession is higher than the physical probability. The agent prices only real risks, so the risk-neutral and the physical transition intensities coincide when only the nominal state changes (\( \omega_{\nu_{t-}, \nu_t} = 1 \) when \( \nu_{t-} = \nu_t \)).

Financial securities have nominal prices, which requires us to consider a nominal stochastic discount factor for asset pricing. The date-\( t \) nominal SDF, denoted by \( \pi^s_t \), is defined as

\[
\pi^s_t = \frac{\pi_t}{P_t},
\]

whose dynamics satisfy

\[
\frac{d\pi^s_t}{\pi^s_t} \bigg|_{s_{t-}=i, s_t=j} = -r^s_i dt - \gamma \sigma_{C,i} dZ_t + \sum_{j \neq i} (\omega_{ij} - 1) dN_{ij,t}^P.
\]

\( r^s_i \) is the nominal interest rate in state \( i \), given by

\[
r^s_i = r_i + \mu_{P,i}.
\]

The nominal interest rate depends on both real and nominal states and can thus takes six different values; it changes when the conditional moments of consumption growth change and also when expected inflation changes. The nominal risk-free rate is lowest during the recession/low-inflation state and highest during the expansion/high-inflation state.

### 3.3 Firm cash flows

The date-\( t \) level of the real cash flow of an individual firm is denoted by \( Y_t \) and evolves under the physical probability measure \( \mathbb{P} \) according to the process

\[
\frac{dY_t}{Y_t} = \mu_{Y,t} dt + \sigma_{Y,t} dW_t.
\]
Real cash flows have a conditional expected growth rate $\mu_{Y,t}$ and a conditional volatility $\sigma_{Y,t}$. Both moments are identical across firms. Increments in the standard Brownian motion $W$ (under $P$) represent frequent but small shocks to the firm’s cash-flow growth. We assume cash-flow shocks are independent across firms and from shocks to consumption growth. Consequently, systematic risk in real cash flows is exclusively associated with low-frequency but severe changes in economic conditions. The expected growth rate is higher in expansions than in recessions, whereas the conditional volatility is lower in expansions than in recessions. In sum, for all firms, we have $\mu_{Y,t} = \mu_{Y,R}$, $\sigma_{Y,t} = \sigma_{Y,R}$ in recessions and $\mu_{Y,t} = \mu_{Y,E}$, $\sigma_{Y,t} = \sigma_{Y,E}$ in expansions, where $\mu_{Y,R} < \mu_{Y,E}$ and $\sigma_{Y,R} > \sigma_{Y,E}$.

Because firms issue nominal securities and pay nominal taxes, investors care about the dynamics of nominal cash flows. The firm’s nominal date $t$ cash-flow level is then given by $X_t$, where

$$X_t \equiv Y_t P_t^\varphi,$$

which satisfies

$$\frac{dX_t}{X_t} = \mu_{X,t} dt + \sigma_{X,t} dW_t,$$

with $\mu_{X,t} = \mu_{Y,t} + \varphi \mu_{P,t}$. Because we ignore instantaneous Brownian shocks to the price index, the volatility of the nominal cash flows is given by $\sigma_{X,t} = \sigma_{Y,t}$. The sticky-cash-flow parameter, $\varphi$, captures the extent to which changes in inflation expectations are reflected in the firm’s cash-flow growth rate.

Overall, firms exhibit heterogeneity in their cash flows due to firm-specific shocks but, at the same time, all firms have identical conditional moments for the expected cash-flow growth rate.

4 Asset Prices and Corporate Financing Decisions

In this section, we derive asset prices together with optimal default and capital-structure decisions.

4.1 Nominal debt and leverage stickiness

Firms pay taxes on nominal cash flows $X_t$ and issue debt to shield profits from taxes. Each firm has a debt contract that is characterized by a constant and perpetual nominal debt coupon $c$. Leverage is sticky because the coupon is fixed in nominal terms. Hence, when the nominal state changes, the real coupon changes, which affects asset valuations. Consequently, sticky leverage acts as a nominal rigidity. In other words, firms cannot adjust the nominal quantity of debt to news about the inflation state.

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\[18\] We ignore a non-zero correlation between real cash flows and consumption, because the asset-pricing and corporate financing implications are negligible. See, for example, Bhamra et al. (2010a, 2010b).
4.2 Liquidation value

A firm is liquidated when its nominal cash flows reach a state-dependent boundary \( X_{D,i} \), which equityholders select to maximize equity value.

The nominal asset value at the time of liquidation, denoted by \( A_{i,t}^\$ \) in state \( i \in \{1,...,6\} \), corresponds to the present value of the after-tax nominal unlevered cash flows:

\[
A_{i,t}^\$ = (1 - \eta)X_t \frac{1}{r_{A,i}},
\]

where \( \eta \) is the corporate tax rate and \( \frac{1}{r_{A,i}} \) is defined by

\[
\frac{1}{r_{A,i}} = E_t \left[ \int_t^\infty \frac{\pi_u^S X_u}{\pi_t^S X_t} du \left| s_t = i \right. \right].
\]

The value of \( r_{A,i} = v_{A,i}^{-1} \) is given by the reciprocal of the \( i \)'th element of the vector \( V_A = [v_{A,1}, \ldots, v_{A,6}]^\top \) where

\[
V_A = (R_A - \hat{\Lambda})^{-1}1_{6 \times 1}.
\]

\( 1_{6 \times 1} \) is a \( 6 \times 1 \) vector of ones, \( R_A \) is the following \( 6 \times 6 \) diagonal matrix

\[
R_A = \text{diag}(r_1^S - \mu_{X,1}, \ldots, r_6^S - \mu_{X,6}),
\]

and \( \hat{\Lambda} \) is the \( 6 \times 6 \) risk-neutral generator matrix of the Markov chain characterizing the real and nominal states of the economy, defined by

\[
[\hat{\Lambda}]_{ij} = \hat{\lambda}_{ij}, \ i, j \in \{1,...,6\}, \ j \neq i,
\]

\[
[\hat{\Lambda}]_{ii} = -\sum_{j \neq i} \hat{\lambda}_{ij}, \ i, j \in \{1,...,6\}, \ j \neq i.
\]

We can interpret \( r_{A,i} \) as the discount rate for a perpetuity with stochastic expected growth rate \( \mu_{X,i} \), which is currently equal to \( \mu_{X,i} \). If the economy stays in state \( i \) forever, the discount rate reduces to the standard expression \( r_{A,i} = r_i^S - \mu_{X,i} \). In general, however, the economy can change states, and so the discount rate depends on the risk-neutral generator matrix of the Markov chain governing the economy’s transitions. The presence of the risk-neutral generator matrix as opposed to the physical generator matrix incorporates the pricing of risk. However, as previously discussed, our SDF does not give rise to an inflation risk premium.
4.3 Arrow-Debreu default claims

Default risk is central to firm valuation. We now express the value of a firm’s assets as a function of a set of Arrow-Debreu default claims. We define an Arrow-Debreu default claim as an asset that pays out $1 if default occurs in state $j$ and the current state is $i$. We denote the nominal price of such a security by $q_{D,ij,t}^S$, which satisfies (see Appendix OA.F)

$$q_{D,ij,t}^S = E_t \left[ \frac{\pi_{\tau_D}^S}{\pi_t^S} I_{\{s_{\tau_D} = j\}} \left| s_t = i \right. \right],$$

(30)

where $\tau_D$ is the date at which default occurs and $I_{\{s_{\tau_D} = j\}}$ is an indicator function that equals 1, if default occurs in state $j$, and zero otherwise.

When valuing assets that depend on the level of cash flows at time of default, $X_{\tau_D}$, we have to consider additional Arrow-Debreu securities, because our economy features “deep defaults.” These defaults can occur when the state of the economy jumps from its current state to a worse state. Default boundaries are countercyclical and can suddenly move upward when the economy deteriorates, for example, when a low expected inflation period starts. In such a situation, a fraction of firms may immediately default upon a change in state. Consider a firm that has a nominal cash-flow level of $10 while the default boundary is $8$. If the economy suddenly deteriorates by moving into a new state where the default boundary is $11$, the firm will immediately default. In fact, all firms with a nominal cash-flow level below $11$ would default, thereby creating a default cluster. More formally, we can consider a firm with a nominal cash-flow level $X_\tau_D$, at time $\tau_D$, which is the time just before default, where $X_\tau_D$ is below the new state’s default boundary, $X_{D,j}$. This firm will default as soon as the economy enters the new state, and so $X_\tau_D = X_{\tau_D} < X_{D,j}$ ($X_\tau_D = X_{\tau_D}$ because $X$ is a continuous process). Hence, it is not necessarily the case that at default, a firm’s cash-flow level is at the default boundary. Consequently, to value securities that depend on a firm’s cash flows, we need a modified set of Arrow-Debreu default claims. We derive them in Appendix OA.G.

This second type of Arrow-Debreu default claims pay out $\frac{X_{\tau_D}}{X_{D,j}}$ at default if default occurs in state $j$ and the current state is $i$. The date-$t$ nominal price of this security is denoted by $\hat{q}_{D,ij,t}^S$, where

$$\hat{q}_{D,ij,t}^S = E_t \left[ \frac{\pi_{\tau_D}^S}{\pi_t^S} \frac{X_{\tau_D}}{X_{D,j}} I_{\{s_{\tau_D} = j\}} \left| s_t = i \right. \right].$$

(31)

Overall, thirty six Arrow-Debreu default prices exist for each type, because six states characterize the aggregate economy.
4.4 Corporate bond value

A firm that issues debt promises to pay the nominal coupon \( c \) per unit time. If the firm defaults, debtholders recover a fraction of the after-tax unlevered asset value of the firm, whereas the remaining fraction is lost due to liquidation costs. We denote the constant recovery rate by \( \alpha \). Hence, the date \( t \) nominal value of corporate debt, conditional on the current state being \( i \), is given by

\[
B_{i,t} = c E_t \left[ \int_t^{\tau_D} \frac{\pi^u}{\pi^s_t} du \right] + \alpha E_t \left[ \frac{\pi^s_{\tau_D}}{\pi^s_t} A^s_{\tau_D}(X_{\tau_D}) du \right]. \tag{32}
\]

The above expression is simply the present value of future coupon flows up until some random default time, \( \tau_D \), plus the present value of the unlevered firm assets net of liquidation costs. We can rewrite the above expression as

\[
B_{i,t} = c \left( \frac{1}{r^s_{P,i}} - \sum_{j=1}^{6} q^s_{D,ij,t} \frac{1}{r^s_{P,j}} \right) + \sum_{j=1}^{6} \alpha A^s_j(X_{D,j}) q^s_{D,ij,t}, \tag{33}
\]

where \( r^s_{P,i} \) is the nominal discount rate for a perpetuity paying a flow of \( 1 \) dollar in perpetuity. Observe that \( \frac{1}{r^s_{P,i}} = E_t \left[ \int_t^{\infty} \frac{\pi^u}{\pi^s_t} du | s_t = i \right] \).

To gain intuition for corporate bond price (33), note \( c \frac{1}{r^s_{P,i}} \) is the present value in nominal terms of a default-free bond paying a coupon flow of \( c \) dollars in perpetuity. The expression \( c \sum_{j=1}^{6} q^s_{D,1,ij,t} \frac{1}{r^s_{P,j}} \) is the present value of coupons lost because of the possibility of default, and \( \sum_{j=1}^{6} \alpha A^s_j(X_{D,j}) q^s_{D,ij,t} \) is the present value of the assets recovered.

The nominal discount rate for a constant nominal perpetuity, \( r^s_{P,i} \), is given by \( r^s_{P,i} = v_{B,i}^{-1} \), where \( v_{B,i} \) is the \( i \)-th element of the vector \( V_B = [v_{B,1}, ..., v_{B,6}]' \)

\[
V_B = (R^s - \tilde{\Lambda})^{-1} 1_{6 \times 1}, \tag{35}
\]

and \( R^s \) represents the \( 6 \times 6 \) diagonal matrix such that \( R^s_{ii} = r^s_i \). Therefore, \( r^s_{P,i} \) accounts for the possibility that the nominal risk-free rate takes different future values as macroeconomic fundamentals and expected inflation fluctuate over time.
4.5 Equity value

Shareholders are entitled to the firm’s cash flows net of taxes and debt servicing as long as the firm does not default. When the firm is in default, which occurs at some random time $\tau_D$, shareholders recover nothing and lose their rights to any future cash flows. The nominal value of equity at date $t$, conditional on the current state $i$, is then given by

$$S^S_{i,t} = (1 - \eta) E_t \left[ \int_t^{\tau_D} \pi^S_u (X_u - c) du \mid s_t = i \right]$$  \hspace{1cm} (36)

$$= A^S_t (X_t) - (1 - \eta) \frac{C}{T_P} \sum_{j=1}^{6} \left( A^S_j (X_{D,j}) q^S_{D,ij,t} - (1 - \eta) q^S_{D,ij,t} \frac{C}{T_P} \right).$$  \hspace{1cm} (37)

The first two terms of (37) represent the present value of cash flows net of coupon payments in the absence of default, whereas the summation term captures the present value of the net cash flows that shareholders lose in the case of default.

4.6 Default and capital structure decisions

Shareholders maximize the value of their default option by choosing when to default. The state-contingent endogenous default boundary $X_{D,s,t}$ depends on the current real and nominal state of the economy, that is, $s_t \in \{1, \ldots, 6\}$. Expected inflation matters for default decisions because a change in the nominal cash flow growth is not offset by a change in the nominal coupon rate; that is, leverage is sticky. Hence, equityholders are entitled to smaller expected future cash flows when expected inflation is low than when expected inflation is high.

The default boundaries satisfy the following six standard smooth-pasting conditions

$$\frac{\partial S^S_{s,t} (X)}{\partial X} \bigg|_{X = X_{D,s,t}} = 0, \ s_t \in \{1, \ldots, 6\}. \hspace{1cm} (38)$$

Shareholders also choose the optimal nominal coupon to maximize firm value at date 0 by balancing marginal tax benefits from debt against marginal expected distress costs. Two features are noteworthy. First, as is standard in the capital-structure literature (Leland, 1994), by maximizing firm value, shareholders internalize debtholders’ value at date 0. However, in choosing default times, they ignore the considerations of debtholders. This feature creates the usual conflict of interest between equity- and debtholders. Second, the optimal coupon depends on the state of the economy at date 0. We denote the date 0 coupon by $c_{s_0}$, where, to emphasize this dependence, $s_0$ is the date 0 state of the
Shareholders choose the coupon to maximize date-0 firm value $F^S_{s_0,0} = B^S_{s_0,0} + S^S_{s_0,0}$

$$c_{s_0} = \arg \max_c F^S_{s_0,0}(c). \quad (39)$$

We obtain the optimal default and capital-structure decisions numerically by maximizing equation (39) subject to the conditions in equation (38). As a result, the optimal default boundaries depend on the debt policy, which the initial financing state determines. Hence, if the economy is in state $i$, the default boundary for nominal earnings is given by $X_{D,i}(c_{s_0})$, where $i$ denotes the dependence on the current state and $c_{s_0}$.

## 5 Theoretical Predictions

This section discusses how changes in expected inflation affect corporate asset prices and default risk.

### 5.1 Calibration

We calibrate the model to the U.S. economy over the period 1970Q1-2016Q4.\footnote{The availability of the data on expected inflation determines our start date.} The real states are characterized by the conditional moments of aggregate consumption growth. We obtain the transition probabilities $\lambda^\text{real}_{\nu_{i-1} \to \nu_t}$ by estimating a two-state Markov regime-switching model on quarterly U.S. consumption, jointly with real aggregate earnings data. We proxy for aggregate consumption with real non-durable goods plus service-consumption expenditures from the Bureau of Economic Analysis. We compute real cash flows using aggregate nominal earnings from S&P, as provided by Robert J. Shiller’s website, that we deflate with the consumer price index.\footnote{We use the Consumer Price Index for All Urban Consumers: All Items, which we retrieve from the Federal Reserve Bank of St. Louis. Real earning shocks are winsorized at the 99\textsuperscript{th} percentile.} The conditional moments of real consumption and cash-flow growth rates are reported in Table 2.\footnote{Following Bhamra et al. (2010a,b), we account for an additional 22.58% of firm-specific volatility. The total cash-flow volatility is thus approximately 25% for our benchmark firm, which is the average volatility of firms with outstanding rated corporate debt.} The nominal growth rate of cash flows ranges between -9.91% in recession/low inflation (RL) to 7.50% in the expansion/high inflation (EH). The estimates of the long-run physical probabilities of being in a recession and expansion are $f_R = f_1 + f_2 + f_3 = 18.2\%$ and $f_E = f_4 + f_5 + f_6 = 81.8\%$, respectively.

| Table 2 [about here] |

We determine the nominal states based on quarterly expected inflation data. We use the mean, one-year-ahead inflation forecasts from the Survey of Professional Forecasters, as reported by the Federal Reserve Bank of Philadelphia. We estimate a three-state Markov regime-switching model and
set expected inflation to the unconditional mean in the medium nominal state. We discipline the chain such that the stationary probabilities of being in low-, moderate- or high-expected inflation regimes are $f_L = f_1 + f_4 = 25\%$, $f_M = f_2 + f_5 = 50\%$, and $f_H = f_3 + f_6 = 25\%$, respectively. This calibration ensures symmetry in the probabilities of being in the $L$ or $H$ expected-inflation regimes. The expected inflation rate is 1.96\% in low expected inflation (L), 3.54\% with moderate expected inflation (M), and 5.13\% during times of high expected inflation (H).

Regarding the firm parameters, the corporate tax rate is set to $\eta = 15\%$ and the liquidation value in default is $\alpha = 50\%$. We normalize the initial value of the cash flow to $X_0 = 1$. Preferences involve a risk aversion of $\gamma = 10$, an elasticity of intertemporal substitution (EIS) of $\psi = 2$, and a subjective discount factor of $\beta = 0.03$. Finally, the sensitivity of nominal cash-flow growth to expected inflation is set to $\varphi = 0.5$, following the empirical evidence discussed in Section 2.

Table 3 reports the firm-level predictions. Unconditionally, the firm services a debt coupon $c$ that is equal to 80.22\% of the initial cash flow and defaults optimally when the firm’s cash flows fall to a boundary that is approximately equal to 26\% of the initial level. With such policies, financial leverage is 44.89\% and the credit spread is 156 bps. These model-implied moments are consistent with the empirical counterpart of an average Baa firm, which displays an average leverage ratio of 43.28\% and a bond spread of 158 bps (Huang and Huang, 2012). Similarly, Kang and Pflueger (2015) report a leverage ratio of 41\% and a credit spread of 153 bps. The model generates a price-dividend (P/D) ratio of 67.8, which we compute as the value of equity $S_t^\$ divided by the nominal dividend paid to equityholders $D_t^\$.\footnote{Corporate income after debt servicing is taxed at the rate $\eta$ and the after-tax distribution to equityholders (dividend) is thus given by $D_t^\$ = (1 - $\eta$)(X_t - c).} This value is comparable to what we observe empirically for an average U.S. firm (see Table 7).

Table 3 [about here]

5.2 Expected inflation and default risk

The model predicts that default risk decreases with the level of expected inflation. Nominal cash flows grow less rapidly in times of lower expected inflation, which increases a firm’s likelihood of default and thus its credit spread (Table 3).

When the economy moves into a state of lower expected inflation, shareholders would benefit from reducing the level of nominal debt to attenuate the increase in credit risk. However, firms are stuck with their outstanding debt issues, as the debt level (coupon) remains fixed in nominal terms despite a change in expected inflation. Hence, the nominal rigidity in the firm’s capital structure induces default risk to vary with nominal conditions. The same effect of sticky leverage can be seen in Bhamra et al. (2011), Kang and Pflueger (2015), and Gomes et al. (2016).
A change in expected inflation also impacts the valuation of corporate debt, but there are now two opposing effects. On the one hand, lower expected inflation reduces the nominal risk-free rate, which increases the present value of the coupons and thus the value of corporate debt. On the other hand, firms face greater default risk when expected inflation falls, which reduces the value of debt. Overall, the first effect dominates, as reported in Table 3. The model thus predicts that periods of lower expected inflation are jointly associated with higher credit risk and higher debt valuation. The variation in default risk across nominal conditions arises from the sticky leverage channel, while the effect of nominal discounting drives the variation in debt valuation. Observe that the impact of expected inflation on default risk and debt valuation remain negative with and without cash flow stickiness, as illustrated in Figure 3.

Figure 3 [about here]

5.3 Expected inflation and equity valuation

We now explore the impact of expected inflation on equity valuation. A fall in expected inflation results into higher default risk, and one might thus expect lower equity valuation. However, another mechanism, which works in the opposite direction, generates an increase in equity prices when expected inflation falls. The nominal risk-free rate changes one-for-one with lower expected inflation, but the expected nominal cash flow growth rate changes less than one-for-one with expected inflation. Because expected inflation affects the nominal discount rate more than the growth rate of nominal cash flows, the present value of cash flows decreases with expected inflation. This effect, which arises directly from the stickiness of cash flows, dominates the default risk channel. Equity valuation thus decreases with expected inflation (see Table 3). As a counterfactual, Figure 3 shows that turning off cash flow stickiness in the model reverts the relation between equity prices and expected inflation, as the only effect that prevails in this case is the default risk channel.

5.4 Low versus high expected inflation: Asymmetry

Figure 3 shows that the relation between equity valuation and expected inflation is non-linear, which implies that lower expected inflation is not the mirror image of higher expected inflation. As reported in Table 3, equity valuation increases by 9.6% (from 11.77 to 12.90) when the economy switches from moderate to low expected inflation. By contrast, equity valuation decreases by only 3.1% (from 11.77 to 11.41) when expected inflation switches from moderate to high expected inflation. Similarly, the P/D ratio increases by 6.72 but decreases by 2.15. The relation is thus asymmetric.

\[^{23}\text{We consider a firm being in the expansion state, but the message is qualitatively similar when considering the recession state.}\]
The impact of a decrease in expected inflation on equity prices is stronger than the impact of an increase in expected inflation, although both states are equally likely. The reason is that a shift in the nominal risk-free rate, upon a change in expected inflation, impacts equity values non-linearly via the discounting of nominal cash flows. This prediction arises although default probabilities are convex in the distance-to-default, which implies that an increase in default risk depresses the value of equity more than a decrease in default risk of the same size. But we find this effect is not sufficient to offset the asymmetry arising from nominal discounting. Figure 3 shows the same asymmetric effect applies to the value of debt and to the credit spread.

A direct implication of this asymmetry is that the presence of inflation risk increases unconditional asset valuation. Given the convex relation between equity value and expected inflation, the average equity value across the low and high expected inflation states is higher than the equity value during an average expected inflation state. Following the same reasoning, inflation risk also increases debt and firm valuation, on average.

To quantify the role of inflation risk, we compare the results of the full model (see Table 3) with the case in which we switch off variations in the nominal state (Table 4). In this latter specification, the expected inflation rate is set at its unconditional mean, which corresponds to the “moderate inflation” regime. Table 5 indicates that inflation risk increases asset valuations, on average, adding up to 0.66% of firm value. This prediction translates, using a simple back-of-the-envelope calculation, into an increase in firm value of approximately US$425 billion, given a market capitalization of listed companies at the NYSE of US$19.3 trillion (as of June 2016) and an average leverage ratio of 0.3. Inflation risk has thus economically important asset-pricing implications for investors.

Tables 4 and 5 [about here]

5.5 Cross-sectional predictions

Our theory suggests firms are differently exposed to variations in expected inflation. In particular, we predict equity prices of firms with higher leverage are less sensitive to expected inflation. This cross-sectional variation arises because changes in expected inflation affect equity values through two opposite channels. First, lower expected inflation increases the value of unlevered equity through sticky cash flows. Second, the value of debt also increases, through sticky leverage, which partially offsets the increase in unlevered equity valuation. The latter channel is expected to be naturally stronger for high-leverage firms. By contrast, no such opposing forces should drive the relation between credit risk and expected inflation conditional on leverage, which implies weaker cross-sectional variations in credit spreads than in equity prices.

Table 6 verifies these predictions for firms with low versus high financial leverage, which are firms with high and low cash flow levels (relative to an identical debt coupon), respectively. In this analysis,
the leverage ratios are 35% and 55% for the two sets of firms, respectively. The results confirms that a fall in expected inflation (from H to L) generates a greater increase in equity valuation (2.1 vs 0.77) for the less-levered firms than for the more-levered firms.\textsuperscript{24} Hence, higher leverage reduces–rather than exacerbates–the sensitivity of equity valuation to changes in nominal conditions.

Table 6 [about here]

5.6 Alternative channels and aggregation of firms

We show a rational model with specific frictions can explain why shareholders value stocks more favorably when default risk increases, that is, in times of low expected inflation. We find these relations hold when accounting for macroeconomic risk and in the case of endogenous corporate policies. Hence, the joint impact of expected inflation on equity valuation and default risk is not due to changes in real economic conditions. In addition, the asset-pricing implications of expected inflation do not vanish when shareholders optimally adjust the firm’s capital structure and the timing of default to the presence of inflation risk.

5.6.1 Alternative channels

Several complementary channels can amplify the impact of expected inflation on asset prices and credit risk. First, a rule for nominal interest rates which satisfies the Taylor principle can generate a similar negative relation between equity valuation and expected inflation. To see that, consider the nominal interest rate in state $i$ satisfies

$$r_i^S = \phi_0 + \phi_\mu \mu_{C,i} - \phi_\sigma \sigma_{C,i}^2 + \phi_P \mu_{P,i}.$$  

(40)

If the price index is locally risk-free, then no arbitrage implies that

$$r_i^S - r_i = \mu_{P,i},$$  

(41)

and so the real risk-free rate depends on expected inflation:

$$r_i = \phi_0 + \phi_\mu \mu_{C,i} - \phi_\sigma \sigma_{C,i}^2 + (\phi_P - 1)\mu_{P,i}.$$  

(42)

\textsuperscript{24}In comparison, the difference in the relation between credit risk and expected inflation for high- versus low-leverage firms is negligible. Observe that the difference is slightly positive due to a convexity effect, as the sensitivity of credit spreads to news increases with the level of credit risk (David, 2008).
The cash flow discount rate $r_{A,i}$ in state $i$, in absence of cash flow stickiness (i.e. $\mu_{X,i} = \mu_{Y,i} + \mu_{P,i}$), is given by

$$r_{i} - \mu_{X,i} = \phi_{0} + \phi_{C}\mu_{C,i} - \phi_{\sigma}\sigma_{C,i} + (\phi_{P} - 1)\mu_{P,i} - \mu_{Y,i}. \quad (43)$$

For $\phi_{P} > 1$, $r_{i} - \mu_{X,i}$ is increasing in $\mu_{P,i}$, so we get the same predictions in the current setup for asset valuations as with sticky cash flows.\(^{25}\) As a result, a model in which a monetary authority sets short-term nominal interest rates endogenously and in which the Taylor principle is satisfied could deliver similar predictions without relying on sticky cash flows.

Second, an inflation risk premium might be present. Expected inflation can be non-neutral in the model by having a negative effect on real consumption growth. Investors would then demand a positive inflation risk premium in times of high expected inflation, which reduces equity valuation. Rather than following this route, we demonstrate that higher expected inflation can negatively affect equity prices although inflation risk remains unpriced. Indeed, we intentionally consider a model in which inflation risk is absent from the real stochastic discount factor, such that the inflation risk premium does not drive any of our predictions. We show that nominal conditions, although they have no effect on the representative agent’s pricing kernel, nonetheless have a non-trivial influence on asset prices.

A third channel is behavioral. Investors may discount real cash flows with nominal discount rates, which induces real equity valuations to decrease with expected inflation. In this paper, we assume that the agent is fully rational and thus does not suffer from any type of money illusion.

A combination of these alternative mechanisms would reinforce the quantitative predictions of this paper, especially regarding the impact of expected inflation on equity valuation. While analyzing the role of these different channels is beyond the scope of this paper, it is certainly a relevant avenue for further research.

Rather than proposing a theory that embeds all these features, our paper highlights the minimum set of frictions that are necessary to explain the seemingly conflicting relations between default risk, equity valuation, and expected inflation in a standard corporate finance model. The key channel for the relation between default risk and expected inflation is the presence of sticky leverage, whereas sticky cash flows drive the negative relation between equity valuation and expected inflation. Therefore, we find that both sticky cash flows and sticky leverage, which are plausible channels, help us understand how expected inflation jointly affects equity valuation and default risk.

\(^{25}\) In fact, we have $\phi_{P} - 1 = 1 - \varphi$, that is, $\phi_{P} = 2 - \varphi = 2 - 0.5 = 1.5$, which is a standard calibration target in New Keynesian models (Gorodnichenko and Weber (2016)).
5.6.2 Aggregation of firms

We obtain the results so far for a single firm that remains at its optimal capital structure. In the real world, firms’ leverage ratios frequently deviate from their optimal levels. These deviations are not symmetric and do not cancel each other in the cross-section. We now verify the role of inflation risk also holds in the case of an economy of firms, with a cross-section of leverage ratios that is structurally similar to that in the data.

For this exercise, we simulate an aggregate dynamics of firms, in the spirit of Bhamra et al. (2010a,b). At date zero, the economy is in the expansion and moderate inflation regime (EM), and 10,000 firms are “born” and choose their optimal capital structure. In each period, firm cash flows are hit by idiosyncratic shocks and the state of the economy may also randomly switch. Since all firms are initially identical and the real/nominal regimes are common to all firms, the cross-sectional differences in leverage are attributable only to different evolutions in their real cash flows. We specify a birth rate of new firms to generate a (stationary) distribution of leverage ratios that matches the estimates of their 25th, median, and 75th counterparts in the U.S. economy (Table 7). To minimize the impact of initial conditions, we use the cross-section at the last quarter (after 1,000 years) to study the aggregate dynamic implications of inflation risk. That is, we compute each firm’s asset valuation and credit risk measures, which we then aggregate for each nominal state.

Figure 4 shows the impact of expected inflation on equity valuation and credit spreads is similar whether we consider an individual firm or a simulated economy of firms. Our findings regarding the joint relations between equity valuation, default risk, and expected inflation are thus robust, as these relations do not vanish when aggregating a large cross-section of firms.

Figure 4 [about here]

6 Empirical Analysis

This section has two aims. First, we show that the empirical relations which we seek to explain in our theoretical model are indeed present in the data. Second, we test our theoretical cross-sectional prediction that the relation between equity valuation and expected inflation is stronger for less levered firms.

6.1 Data

Our empirical analysis is based on the following data. Expected inflation is the year-on-year expected GDP-deflator inflation from the Federal Reserve Bank of Philadelphia’s Survey of Professional Fore-
casters. We consider two measures of equity valuation: the firm’s market-to-book (M/B) equity ratio and the price-dividend ratio. Default risk is measured by a firm’s financial-distress risk, following Campbell, Hilscher, and Szilagyi (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Appendix A provides details on the computation of these measures. Accounting variables are from Compustat Fundamental Quarterly data, whereas stock price variables are from CRSP. The dataset spans from April 1972 to December 2016. Table 7 displays the summary statistics.

Table 7 [about here]

6.2 Relations between equity valuation, default risk, and expected inflation

We first analyze the relations between equity valuations and default risk with expected inflation. Figure 5 displays the results for the price-dividend ratio (top panels), the market-to-book ratio (middle panels), and the physical bankruptcy probability (bottom panels). The panels plot the (value-weighted) averages of the firm characteristics against the level of expected inflation observed in the corresponding quarter. We disentangle the relations by level of financial leverage, which we define as long-term debt and debt in current liabilities over the sum of the numerator and stockholders’ equity. The left panels report portfolios of firms with below-median leverage, whereas the right panels report firms with above-median leverage. Each panel uses a quadratic regression to fit the data.

Figure 5 [about here]

This graphical analysis suggests the price-dividend ratio, the market-to-book ratio, and the bankruptcy probability are all negatively related to the level of expected inflation. Importantly, each portfolio contains the same set of firms, thereby indicating a decrease in expected inflation simultaneously increases both a firm’s equity valuation and its default risk. Furthermore, as our model predicts, the relations based on equity valuation appear to be stronger for low-leverage firms.

6.3 Portfolio sorts

As a formal test of these cross-sectional relations, we now exploit double sorts. We first sort all firms into two portfolios based on their financial leverage. We then create three equal-sized portfolios depending on the level of expected inflation.

Table 8 reports the results. Panel A shows for conditional double sorts that both equity valuation and default risk decrease in expected inflation. The high expected inflation-minus-low expected inflation estimates are all negative and statistically significant within each leverage sort. In terms of magnitude, firms with low (high) leverage display an average price-dividend ratio of 102.8 (66.4) when
expected inflation is low and 47.0 (25.4) when expected inflation is high. The market-to-book ratios are 4.24 (2.25) and 1.95 (0.98), respectively. These differences are economically large. Further, the double difference by leverage ratios (that is, the difference between estimates of the high expected inflation minus low expected inflation estimates across high- and low-leverage firms) are also highly statistically significant. These tests show that the relation between equity valuation and expected inflation is negative and stronger for firms with lower levels of financial leverage, consistent with our theory.

The conditional double sorts also indicate that the negative relation between default probability and expected inflation is weakly stronger for high-leverage firms (H-L=3.59-4.98=-1.39) than for low-leverage firms (H-L=2.15-3.22=-1.07). Both the sign and the (low) magnitude of this difference are consistent with the cross-sectional prediction of our model. Panel B of Table 8 shows that all these results remain similar when we perform unconditional double sorts.

Table 8 [about here]

6.4 Firm-level regressions

We now show that the negative relations are robust features of the data and, in particular, hold for individual firms. It is important to ensure that the empirical relations between expected inflation, equity valuation and default risk are not an artifact of the partial aggregation of firms. To this end, we examine how valuation ratios and default risk at the firm level vary with expected inflation, while keeping constant other firm characteristics and aggregate economic conditions.

Our main regression specification is as follows:

\[ E_{i,j,t} = \delta_{P} \mu_{P,t} + \mathbf{X}'_{i,j,t} \delta C_1 + \mathbf{Y}'_t \delta C_2 + \gamma_j + \epsilon_{i,j,t}, \]  \hspace{1cm} (44)

where \( E_{i,j,t} \) denotes the equity valuation for firm \( i \) in industry \( j \) at quarter \( t \), measured as the price-dividend ratio or the market-to-book ratio. In the analysis of default risk, \( E_{i,j,t} \) captures firm \( i \)'s default probability computed in quarter \( t \). Keeping the same notation as in the model, \( \mu_{P,t} \) reflects expected inflation in quarter \( t \). We denote by \( \mathbf{X}_{i,j,t} \) and \( \mathbf{Y}_t \) the vectors of firm and global characteristics that we use as control variables. We include industry fixed effects (\( \gamma_j \)) to control for time-invariant differences across industry groups and cluster standard errors \( \epsilon_{i,j,t} \) at the quarter level to allow for correlations in error terms of unknown form across firms in a given quarter.

Equity valuations and default probabilities vary with firm characteristics; therefore, accounting for such drivers is critical. Following Fama and French (2015), we consider the level of investment, profitability, and firm size as firm-level controls (see Appendix A for details of the variable definitions).

\footnote{We bootstrap the double difference to calculate standard errors.}
We also include the year-on-year growth rate of U.S. industrial production, a recession indicator based on the NBER business-cycle dates, the trailing one-year return of the S&P 500 index, and the slope of the yield curve measured by the yield spread between the 10-year Treasury note and the three-month Treasury bill, because these factors predict U.S. defaults.\footnote{See, for example, Das, Duffie, Kapadia, and Saita (2007), Duffie, Saita, and Wang (2007), Campbell et al. (2008), Duffie, Eckner, Horel, and Saita (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2011), and Azizpour, Giesecke, and Schwenkler (2018).} We also control for the recent period of unconventional monetary policies by including a dummy variable that is equal to 1 over the 2008Q1–2016Q4 period, and zero otherwise. These data are from the Federal Reserve Bank of St. Louis.

Table 9 reports the regression results. We see in Columns (1)–(2) that expected inflation is a strong driver of the price-dividend ratio and the market-to-book ratio, beyond the information contained in firm fundamentals and economic/financial conditions. A one-standard-deviation decrease in the expected inflation rate increases the price-dividend ratio by 16.7, which is economically sizable. Column (3) reports similar results for the level of distress risk.

Table 9 [about here]

We now turn to another central prediction of the model: A decrease in expected inflation has a stronger impact on equity valuation and default risk than an increase in expected inflation. The following analysis tests for such asymmetry in the data. To investigate a potential non-linearity in the relation between the valuation ratios (or default risk) and expected inflation, we interact expected inflation with a dummy variable, \( D_{L,M} \), that takes the value of 1 when expected inflation is below the 75\textsuperscript{th} percentile. This choice follows from our calibration, in which high expected inflation corresponds to the top quartile. Columns (1)–(3) of Table 10 show the relation between equity valuations and expected inflation is stronger when expected inflation is lower. The difference in the sensitivity to expected inflation is economically and statistically significant. The same result holds for distress risk. The empirical support for such asymmetry confirms than an increase in expected inflation is not the mirror image of a decrease in expected inflation.

Table 10 [about here]

6.5 Robustness analysis

In this section, we report several alternative tests to probe the robustness of our empirical findings. We first address the potential concern that variations in expected inflation reflect changes in economic or financial conditions, in particular given the low inflation levels observed during and after the Great Recession. It is therefore critical to exploit a measure of expected inflation that is independent of the
business cycle. To this end, we first orthogonalize the level of expected inflation with respect to the NBER recession indicator and reproduce our portfolio analysis of Table 8 with this orthogonalized measure. Table 11 displays the results. Alternatively, Table 12 focuses on the 1972Q2–2007Q4 period to ensure that observations during the Great Recession and onwards do not drive the relation between expected inflation with equity valuation and default risk. In both analyses, the results continue to hold with the same economic magnitude and statistical significance.

We go through the same exercise for the firm-level regressions, which all control for indicators of NBER recession and post-2007 years. Columns (4)–(6) of Tables 9 and 10 repeat the baseline analysis of Columns (1)–(3) but for the 1972Q2–2007Q4 period, thereby excluding observations during which equity valuation and default risk are most sensitive to expected inflation. The relations continue to be negative and asymmetric. Furthermore, our rich set of financial, macroeconomic, and firm-level controls allows us to disentangle the impact of nominal and real conditions. Hence, we can rule out the concern that a high or a low inflation environment reflects a bad state of the economy, thereby driving equity valuation and default risk.

Our robustness analysis also considers alternative samples. First, we compare the findings with and without financial firms and utilities in Table 13, because they operate in regulated markets or have special capital structures. Second, Columns (1)–(3) Table 14 exclude all tech firms, which tend to display relatively high equity valuations. The results remain similar in all of these cases.

Finally, we address the concern that the high levels of expected inflation in the 1970s may be a primary driver of our results. We thus exclude the pre-1980 period and report the results in Columns (4)–(6) of Table 14. This analysis shows that our findings are not driven by the changes in equity valuation and default risk as a consequence of the large variations in expected inflation during that period.

6.6 Summary

Our empirical investigation of the impact of expected inflation on equity valuation and default risk highlights three main findings. First, we document that the relations are robust feature of the data and, in particular, hold for individual firms. Firm-level regressions reveal that equity valuation and default risk jointly decrease with expected inflation, even after controlling for firm characteristics or for variations in aggregate financial, economic, and monetary conditions. Second, the relations are asymmetric, that is, a decrease in expected inflation has a stronger impact on a firm’s default risk and
equity valuation when expected inflation is low than when it is high. Third, we validate the cross-sectional prediction of our theory that the relation between equity valuation and expected inflation is stronger for less levered firms. Corporate indebtedness thus reduces, rather than exacerbates, the sensitivity of equity valuation to changes in nominal conditions. Hence, our analysis provides novel empirical evidence that the relations are negative and asymmetric at the firm-level, and vary across financial leverage ratios.

7 Conclusion

Default risk increases in times of low expected inflation but so does equity valuation. Our empirical contribution is to provide new evidence that these relations are robust features of the data, not only at the market level but also for individual firms. In particular, we show that these relations are asymmetric and strongest for less financially-levered firms. Our theoretical contribution is to develop a model which jointly rationalizes these stylized patterns in the data. In the model, inflation risk impacts real asset prices via two empirically grounded nominal frictions: sticky leverage and sticky cash flows. There are two key mechanisms at play. First, long-term nominal debt coupons are fixed, but expected inflation varies. This stickiness in leverage makes expected future real debt coupons dependent on future expected inflation, ensuring that inflation risk impacts real corporate bond values and hence default risk. Second, the expected cash flow growth rate is less sensitive to variations in expected inflation than the nominal risk-free rate. This stickiness in cash flows makes equity prices decreasing in the nominal risk-free rate and hence in expected inflation.

Our model thus implies that lower expected inflation simultaneously increases real asset values and default risk. Importantly, the relations are asymmetric, as a decrease in expected inflation increases real equity values by more than an increase in expected inflation of equal size. The effect on equity prices is also stronger for firms with less leverage. Hence, leverage dampens rather than exacerbates the sensitivity of equity valuation to inflation expectations. We find support for the model predictions in the data, lending credence to the idea that sticky leverage and sticky cash flows are important channels for understanding the impact of inflation risk on real asset values and corporate default risk.
References


Table 1: Estimation of cash flow stickiness

This table reports estimates of the degree of cash flow stickiness, as determined by the sensitivity of expected cash flow growth to expected inflation. Expected cash flow growth is measured as the mean forecast for the one-year-ahead corporate profit growth rate, while expected inflation is measured as the mean forecast for one-year-ahead inflation. All growth rates are annualized. We report standard errors corrected for heteroskedasticity and serial correlation in parentheses. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively. Forecast data are obtained from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. The control variables are retrieved from the Federal Reserve of St-Louis. The sample period is 1970Q2–2016Q4.

<table>
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<th>Dependent Variable: Expected Profit Growth</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Inflation</td>
<td>0.382**</td>
<td>0.397**</td>
<td>0.425**</td>
<td>0.415**</td>
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<tr>
<td></td>
<td>(0.180)</td>
<td>(0.176)</td>
<td>(0.174)</td>
<td>(0.168)</td>
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<tr>
<td>Expected GDP Growth</td>
<td>3.773***</td>
<td>4.068***</td>
<td>4.265***</td>
<td>4.278***</td>
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<tr>
<td></td>
<td>(0.285)</td>
<td>(0.327)</td>
<td>(0.318)</td>
<td>(0.317)</td>
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<td>Consumption Growth</td>
<td>-0.259</td>
<td>-0.024</td>
<td>-0.014</td>
<td></td>
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<tr>
<td></td>
<td>(0.167)</td>
<td>(0.171)</td>
<td>(0.182)</td>
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<tr>
<td>Industrial Production Growth</td>
<td>-0.196***</td>
<td>-0.189***</td>
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<tr>
<td></td>
<td>(0.061)</td>
<td>(0.064)</td>
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<td>NBER Recession</td>
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<td>0.377</td>
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<td></td>
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<td>(1.227)</td>
</tr>
<tr>
<td>Constant</td>
<td>-4.280***</td>
<td>-4.349***</td>
<td>-5.278***</td>
<td>-5.376***</td>
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<td>(1.011)</td>
<td>(0.999)</td>
<td>(0.982)</td>
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<td>185</td>
<td>185</td>
<td>185</td>
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<tr>
<td>$R^2$</td>
<td>0.575</td>
<td>0.581</td>
<td>0.605</td>
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Table 2: Model calibration

This table presents the parameter values of the model. Panel A reports the conditional moments of the economic environment. Panel B reports the conditional firm characteristics. We calibrate the model to the aggregate U.S. economy using real consumption data (non-durable goods plus service consumption expenditures). Expected inflation is measured as the mean forecast for one-year-ahead inflation. The moments of cash flows are estimated using Robert J. Shiller’s aggregate earnings data. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. Each column displays the predictions for a specific state of the economy: the expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. We retrieve the consumption data from the Bureau of Economic Analysis, while the forecast data are obtained from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. All estimates are in percentage points and annualized when applicable. The sample period is 1970Q2-2016Q4. The calibration is detailed in Section 5.1.

<table>
<thead>
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<th>Unconditional</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>State 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R &amp; L</td>
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<tr>
<td>Panel A: Economic Environment</td>
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<td></td>
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<tr>
<td>Stationary Probability</td>
<td>4.54</td>
<td>9.09</td>
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<tr>
<td>Consumption Growth Rate</td>
<td>1.76</td>
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<td>Consumption Growth Volatility</td>
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<td>Expected Inflation</td>
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<td>Nominal Interest Rate</td>
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<td>Risk-Free Discount Rate</td>
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<td>Cash Flow Discount Rate</td>
<td>4.32</td>
<td>4.55</td>
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<td>Panel B: Firm Characteristics</td>
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<tr>
<td>Real Cash Flow Growth Rate</td>
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<td>-10.89</td>
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<td>Cash Flow Stickiness</td>
<td>0.50</td>
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<td>Tax Rate</td>
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<td>15.00</td>
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Table 3: Firm policies and asset prices

This table presents the predictions of the model regarding endogenous firm policies and asset valuation. Panel A reports the coupon and the conditional default boundaries. The capital structure is chosen optimally in the state of expansion with moderate inflation. Panel B reports the conditional asset pricing quantities for the economy. Each column displays the predictions for a specific state of the economy: the expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

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<thead>
<tr>
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<td></td>
<td>State 1</td>
<td>State 2</td>
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<tr>
<td>Stationary Probability</td>
<td>0.0454</td>
<td>0.0909</td>
</tr>
</tbody>
</table>

Panel A: Corporate Policies

| Default Boundaries (Coupon: 0.8022) | 0.2655 | 0.2689 | 0.2679 | 0.2598 | 0.2625 | 0.2611 |

Panel B: Asset Pricing Quantities

<table>
<thead>
<tr>
<th>P/D Ratio</th>
<th>Equity Value</th>
<th>Debt Value</th>
<th>Market Leverage (%)</th>
<th>Credit Spreads (bps)</th>
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<tr>
<td>67.75</td>
<td>11.39</td>
<td>9.22</td>
<td>44.89</td>
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<td>56.24</td>
<td>9.45</td>
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<td>8.58</td>
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<td>8.41</td>
<td>8.10</td>
<td>49.06</td>
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<td>9.16</td>
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<td>67.87</td>
<td>11.41</td>
<td>8.61</td>
<td>43.76</td>
<td>139.08</td>
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</table>
Table 4: Firm policies and asset prices – constant inflation

This table presents the predictions of the model without fluctuating nominal conditions. Expected inflation is constant and set to its unconditional mean over the sample period, which corresponds to the “moderate inflation” state (M). Panel A reports the coupon and the conditional default boundaries. The capital structure is chosen optimally in the state of expansion. Panel B reports the conditional asset pricing quantities for the economy. Each column displays the predictions for a specific state of the economy, which can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>State 2</td>
</tr>
<tr>
<td>R &amp; M</td>
<td>0.1817</td>
</tr>
</tbody>
</table>

**Panel A: Corporate Policies**

Default Boundaries (Coupon: 0.7952)

<table>
<thead>
<tr>
<th>Unconditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 2</td>
</tr>
<tr>
<td>R &amp; M</td>
<td>0.2659</td>
</tr>
</tbody>
</table>

**Panel B: Asset Pricing Quantities**

<table>
<thead>
<tr>
<th>Unconditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 2</td>
</tr>
<tr>
<td>R &amp; M</td>
<td>65.39</td>
</tr>
<tr>
<td>Equity Value</td>
<td>11.38</td>
</tr>
<tr>
<td>Debt Value</td>
<td>9.09</td>
</tr>
<tr>
<td>Market Leverage (%)</td>
<td>44.58</td>
</tr>
<tr>
<td>Credit Spreads (bps)</td>
<td>152.52</td>
</tr>
</tbody>
</table>

Table 5: Asset pricing implications of nominal risk

This table presents the impact of nominal risk on asset prices. It reports differences in asset pricing predictions between a model with fluctuating expected inflation and a model with constant expected inflation. In the latter case, the expected inflation rate is constant and set to its unconditional mean (i.e. moderate inflation state), and the model predictions are those of Table 4. The differences in asset values are in relative terms (%). The differences in leverage are in percentage points, while the difference in credit spreads are in basis points. Each column reports model predictions for a different current state of the economy. The expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

<table>
<thead>
<tr>
<th>Unconditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
</tr>
<tr>
<td>R &amp; L</td>
<td>0.0454</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Change in Stationary Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>R &amp; L</td>
</tr>
<tr>
<td>G &amp; M</td>
</tr>
<tr>
<td>E &amp; R</td>
</tr>
<tr>
<td>E &amp; M</td>
</tr>
<tr>
<td>E &amp; H</td>
</tr>
</tbody>
</table>

| Change in P/D Ratio | 3.60 | 11.27 | 1.86 | -1.02 | 11.71 | 1.98 | -1.18 |
| Change in Equity Value | 0.05 | 7.46 | -1.63 | -4.41 | 7.88 | -1.51 | -4.57 |
| Change in Debt Value | 1.43 | 12.11 | -0.43 | -5.99 | 13.00 | -0.41 | -6.37 |
| Change in Firm Value | 0.66 | 9.76 | -1.04 | -5.20 | 10.11 | -1.03 | -5.35 |
| Change in Market Leverage (%) | 0.31 | 1.06 | 0.30 | -0.42 | 1.14 | 0.27 | -0.47 |
| Change in Credit Spreads (bps) | 3.49 | 9.72 | 2.83 | 0.64 | 10.88 | 1.70 | -0.80 |
Table 6: Cross-sectional predictions

This table presents the cross-sectional impact of nominal risk by market leverage. The table reports asset pricing predictions for firms that differ in their levels of cash flow, which generates cross-sectional differences in market leverage. Predictions are reported across nominal conditions for a firm with low (35%) and high (55%) leverage. The expected inflation rate can be low (L), moderate (M), or high (H), while the real economy is set at its unconditional state. The table also displays the difference in results between the high (H) and the low (L) expected inflation state, as well as the double difference. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

<table>
<thead>
<tr>
<th>Expected Inflation</th>
<th>P/D Ratio</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Leverage</td>
<td>High Leverage</td>
</tr>
<tr>
<td>L</td>
<td>117.16</td>
<td>45.32</td>
</tr>
<tr>
<td>M</td>
<td>106.53</td>
<td>41.56</td>
</tr>
<tr>
<td>H</td>
<td>102.84</td>
<td>40.50</td>
</tr>
<tr>
<td>H-L</td>
<td>−14.32</td>
<td>−4.82</td>
</tr>
<tr>
<td>Double Difference</td>
<td>9.49</td>
<td>−0.93</td>
</tr>
</tbody>
</table>
Table 7: Descriptive statistics

This table reports the summary statistics of the main variables. Financial variables at the firm level are value-weighted. Expected inflation is from the Survey of Professional Forecasts from the Federal Reserve Bank of Philadelphia. The default probability is the marginal probability of bankruptcy or failure over the next quarter, which is computed as in Campbell et al. (2008). Section 6.1 provides details on the computation of the firm variables. N is the number of observations. The sample period is 1972Q2–2016Q4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>25% Perc</th>
<th>Median</th>
<th>75% Perc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Inflation (%)</td>
<td>3.773</td>
<td>1.896</td>
<td>2.292</td>
<td>3.308</td>
<td>4.765</td>
</tr>
<tr>
<td>Price-Dividend Ratio</td>
<td>75.816</td>
<td>61.153</td>
<td>23.827</td>
<td>39.634</td>
<td>70.625</td>
</tr>
<tr>
<td>Market-Book Ratio</td>
<td>3.173</td>
<td>2.076</td>
<td>0.955</td>
<td>1.554</td>
<td>2.673</td>
</tr>
<tr>
<td>Default Probability (bps)</td>
<td>3.831</td>
<td>0.008</td>
<td>2.883</td>
<td>4.800</td>
<td>9.028</td>
</tr>
<tr>
<td>Market Leverage</td>
<td>0.233</td>
<td>0.242</td>
<td>0.041</td>
<td>0.209</td>
<td>0.458</td>
</tr>
<tr>
<td>Net Income to Total Assets (%)</td>
<td>0.612</td>
<td>2.116</td>
<td>-0.230</td>
<td>0.532</td>
<td>1.209</td>
</tr>
<tr>
<td>Excess Return (%)</td>
<td>4.061</td>
<td>47.223</td>
<td>-30.277</td>
<td>-2.068</td>
<td>24.405</td>
</tr>
<tr>
<td>Return Volatility (%)</td>
<td>33.153</td>
<td>31.795</td>
<td>28.508</td>
<td>42.815</td>
<td>66.603</td>
</tr>
<tr>
<td>Size to Market</td>
<td>-8.963</td>
<td>2.593</td>
<td>-12.595</td>
<td>-10.837</td>
<td>-8.376</td>
</tr>
<tr>
<td>Short Term Assets to Total</td>
<td>0.063</td>
<td>0.100</td>
<td>0.018</td>
<td>0.050</td>
<td>0.114</td>
</tr>
<tr>
<td>Log Share Price</td>
<td>0.579</td>
<td>2.370</td>
<td>0.257</td>
<td>0.797</td>
<td>2.476</td>
</tr>
<tr>
<td>Change in Total Assets (%)</td>
<td>2.496</td>
<td>8.001</td>
<td>-1.870</td>
<td>1.307</td>
<td>5.430</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.099</td>
<td>9.496</td>
<td>-0.020</td>
<td>0.040</td>
<td>0.080</td>
</tr>
<tr>
<td>Size</td>
<td>6.140</td>
<td>2.048</td>
<td>4.613</td>
<td>6.067</td>
<td>7.572</td>
</tr>
<tr>
<td>IP Growth (%)</td>
<td>1.709</td>
<td>4.218</td>
<td>0.585</td>
<td>2.742</td>
<td>5.149</td>
</tr>
<tr>
<td>Slope (%)</td>
<td>1.450</td>
<td>1.421</td>
<td>0.270</td>
<td>1.430</td>
<td>2.270</td>
</tr>
<tr>
<td>N</td>
<td>743,536</td>
<td>743,536</td>
<td>743,536</td>
<td>743,536</td>
<td>743,536</td>
</tr>
</tbody>
</table>
Table 8: Equity valuation and default risk by expected inflation and leverage

This table reports double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4.

<table>
<thead>
<tr>
<th>Expected Inflation</th>
<th>Nobs</th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Distress Risk</th>
<th>Default Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low Leverage</td>
<td>High Leverage</td>
<td>Low Leverage</td>
<td>High Leverage</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>L</td>
<td>172</td>
<td>102.80</td>
<td>66.39</td>
<td>4.24</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.05)</td>
<td>(1.55)</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>M</td>
<td>169</td>
<td>97.13</td>
<td>54.94</td>
<td>4.03</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.82)</td>
<td>(1.35)</td>
<td>(0.11)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>H</td>
<td>169</td>
<td>46.97</td>
<td>25.39</td>
<td>1.95</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.77)</td>
<td>(0.50)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>H-L</td>
<td></td>
<td>-55.83</td>
<td>-41.00</td>
<td>-2.29</td>
<td>-1.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.37)</td>
<td>(0.93)</td>
<td>(0.07)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Double Difference</td>
<td></td>
<td>16.26</td>
<td>1.14</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.00)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B. Unconditional Double Sorts

<table>
<thead>
<tr>
<th>Expected Inflation</th>
<th>Nobs</th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Distress Risk</th>
<th>Default Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low Leverage</td>
<td>High Leverage</td>
<td>Low Leverage</td>
<td>High Leverage</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>L</td>
<td>172</td>
<td>98.74</td>
<td>64.34</td>
<td>4.05</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.91)</td>
<td>(1.58)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>M</td>
<td>169</td>
<td>94.75</td>
<td>53.59</td>
<td>3.92</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.69)</td>
<td>(1.34)</td>
<td>(0.11)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>H</td>
<td>169</td>
<td>51.43</td>
<td>28.59</td>
<td>2.18</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.85)</td>
<td>(0.57)</td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>H-L</td>
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<td>-47.31</td>
<td>-35.75</td>
<td>-1.86</td>
<td>-1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.25)</td>
<td>(0.96)</td>
<td>(0.07)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Double Difference</td>
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<td>0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.00)</td>
<td></td>
<td></td>
</tr>
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</table>
Table 9: Regressions on expected inflation

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, firm characteristics, and macro aggregates. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

<table>
<thead>
<tr>
<th></th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Expected Inflation ($\mu_P$)</td>
<td>−8.15</td>
<td>−0.15</td>
<td>−0.19</td>
<td>−9.90</td>
<td>−0.14</td>
<td>−0.18</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.51)</td>
<td>(0.00)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Investment</td>
<td>104.24</td>
<td>1.64</td>
<td>−0.80</td>
<td>107.26</td>
<td>1.69</td>
<td>−0.62</td>
</tr>
<tr>
<td></td>
<td>(2.69)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(3.21)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Profitability</td>
<td>−0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>−0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>log Size</td>
<td>3.56</td>
<td>0.24</td>
<td>−0.05</td>
<td>3.67</td>
<td>0.23</td>
<td>−0.04</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.22)</td>
<td>(0.01)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>IP Growth</td>
<td>31.63</td>
<td>−0.02</td>
<td>−0.90</td>
<td>8.85</td>
<td>0.16</td>
<td>−0.51</td>
</tr>
<tr>
<td></td>
<td>(8.55)</td>
<td>(0.15)</td>
<td>(0.23)</td>
<td>(10.53)</td>
<td>(0.16)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>S&amp;P Return</td>
<td>15.89</td>
<td>0.39</td>
<td>−0.02</td>
<td>12.18</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(2.86)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(3.18)</td>
<td>(0.05)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Yield Curve</td>
<td>−1.61</td>
<td>−0.06</td>
<td>−0.07</td>
<td>−2.03</td>
<td>−0.05</td>
<td>−0.07</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.30)</td>
<td>(0.00)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Leverage</td>
<td>−33.77</td>
<td>−1.83</td>
<td>1.56</td>
<td>−35.55</td>
<td>−1.80</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.83)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Recession</td>
<td>1.57</td>
<td>0.02</td>
<td>0.20</td>
<td>1.18</td>
<td>0.05</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(1.77)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(2.23)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Dummy$_{post\ 2008}$</td>
<td>−15.63</td>
<td>−0.23</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Industry FE</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nobs</td>
<td>743,536</td>
<td>743,536</td>
<td>743,536</td>
<td>592,966</td>
<td>592,966</td>
<td>592,966</td>
</tr>
<tr>
<td>R$^2$</td>
<td>17.22%</td>
<td>33.65%</td>
<td>49.21%</td>
<td>19.78%</td>
<td>35.95%</td>
<td>46.50%</td>
</tr>
</tbody>
</table>
This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, firm characteristics, and macro aggregates. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $D_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

<table>
<thead>
<tr>
<th></th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Inflation ($\mu_P$)</td>
<td>$-3.95$</td>
<td>$-0.09$</td>
<td>$-0.01$</td>
<td>$-4.18$</td>
<td>$-0.09$</td>
<td>$-0.01$</td>
</tr>
<tr>
<td></td>
<td>$(0.38)$</td>
<td>$(0.01)$</td>
<td>$(0.02)$</td>
<td>$(0.38)$</td>
<td>$(0.01)$</td>
<td>$(0.02)$</td>
</tr>
<tr>
<td>$\mu_P \times D_{L,M}$</td>
<td>$-12.65$</td>
<td>$-0.10$</td>
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Table 11: Equity valuation and default risk by orthogonolized expected inflation

This table reproduces Table 8 when the level of expected inflation is orthogonolized with respect to NBER recessions. We present double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia and orthogonolized with respect to NBER recessions. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4.

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Panel B. Unconditional Double Sorts

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Table 12: Equity valuation and default risk by expected inflation and leverage (1970–2007)

This table reports double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2007Q4.

### Panel A. Conditional Double Sorts

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### Panel B. Unconditional Double Sorts

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Table 13: Regressions on expected inflation – Convexity: excl Finance and Utilities

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, firm characteristics, and macro aggregates. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $D_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

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<td>(0.02)</td>
<td>(1.02)</td>
<td>(0.04)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Recession</td>
<td>3.04</td>
<td>0.00</td>
<td>0.23</td>
<td>-0.34</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(1.61)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(1.43)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Dummy_{post 2008}</td>
<td>-33.63</td>
<td>-0.16</td>
<td>-0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Industry FE</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
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<tr>
<td>Nobs</td>
<td>437,012</td>
<td>437,012</td>
<td>437,012</td>
<td>356,938</td>
<td>356,938</td>
<td>356,938</td>
</tr>
<tr>
<td>$R^2$</td>
<td>16.37%</td>
<td>35.50%</td>
<td>48.31%</td>
<td>19.01%</td>
<td>38.40%</td>
<td>48.31%</td>
</tr>
</tbody>
</table>
Table 14: Regressions on expected inflation – Convexity: no Tech or pre-1980

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, firm characteristics, and macro aggregates. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $D_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4 in columns (1)–(3) but excludes all tech firms and 1980Q1–2016Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

<table>
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<tr>
<th></th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
<th>P/D Ratio</th>
<th>M/B Ratio</th>
<th>Default Risk</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Expected Inflation ($\mu_P$)</td>
<td>$-3.95$</td>
<td>$-0.09$</td>
<td>$-0.01$</td>
<td>$-2.55$</td>
<td>$-0.08$</td>
<td>$-0.03$</td>
</tr>
<tr>
<td></td>
<td>$(0.38)$</td>
<td>$(0.01)$</td>
<td>$(0.02)$</td>
<td>$(0.29)$</td>
<td>$(0.01)$</td>
<td>$(0.02)$</td>
</tr>
<tr>
<td>$\mu_P \times D_{L,M}$</td>
<td>$-12.58$</td>
<td>$-0.10$</td>
<td>$-0.14$</td>
<td>$-13.95$</td>
<td>$-0.11$</td>
<td>$-0.12$</td>
</tr>
<tr>
<td></td>
<td>$(0.69)$</td>
<td>$(0.01)$</td>
<td>$(0.01)$</td>
<td>$(0.65)$</td>
<td>$(0.02)$</td>
<td>$(0.01)$</td>
</tr>
<tr>
<td>Investment</td>
<td>$102.68$</td>
<td>$1.63$</td>
<td>$-0.80$</td>
<td>$115.65$</td>
<td>$1.79$</td>
<td>$-0.87$</td>
</tr>
<tr>
<td></td>
<td>$(2.67)$</td>
<td>$(0.04)$</td>
<td>$(0.05)$</td>
<td>$(2.30)$</td>
<td>$(0.04)$</td>
<td>$(0.04)$</td>
</tr>
<tr>
<td>Profitability</td>
<td>$0.11$</td>
<td>$0.03$</td>
<td>$0.00$</td>
<td>$0.13$</td>
<td>$0.01$</td>
<td>$0.00$</td>
</tr>
<tr>
<td></td>
<td>$(0.09)$</td>
<td>$(0.01)$</td>
<td>$(0.00)$</td>
<td>$(0.02)$</td>
<td>$(0.00)$</td>
<td>$(0.00)$</td>
</tr>
<tr>
<td>log Size</td>
<td>$3.30$</td>
<td>$0.24$</td>
<td>$-0.06$</td>
<td>$3.45$</td>
<td>$0.25$</td>
<td>$-0.06$</td>
</tr>
<tr>
<td></td>
<td>$(1.17)$</td>
<td>$(0.00)$</td>
<td>$(0.00)$</td>
<td>$(1.19)$</td>
<td>$(0.00)$</td>
<td>$(0.00)$</td>
</tr>
<tr>
<td>IP Growth</td>
<td>$48.06$</td>
<td>$0.15$</td>
<td>$-0.72$</td>
<td>$69.22$</td>
<td>$0.25$</td>
<td>$-0.20$</td>
</tr>
<tr>
<td></td>
<td>$(6.00)$</td>
<td>$(0.13)$</td>
<td>$(0.23)$</td>
<td>$(7.46)$</td>
<td>$(0.17)$</td>
<td>$(0.27)$</td>
</tr>
<tr>
<td>S&amp;P Return</td>
<td>$17.82$</td>
<td>$0.39$</td>
<td>$-0.05$</td>
<td>$14.41$</td>
<td>$0.37$</td>
<td>$-0.16$</td>
</tr>
<tr>
<td></td>
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<td>$(0.04)$</td>
<td>$(0.06)$</td>
<td>$(1.84)$</td>
<td>$(0.05)$</td>
<td>$(0.06)$</td>
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<tr>
<td>Yield Curve</td>
<td>$-0.34$</td>
<td>$-0.04$</td>
<td>$-0.06$</td>
<td>$0.16$</td>
<td>$-0.04$</td>
<td>$-0.06$</td>
</tr>
<tr>
<td></td>
<td>$(0.21)$</td>
<td>$(0.00)$</td>
<td>$(0.01)$</td>
<td>$(0.20)$</td>
<td>$(0.00)$</td>
<td>$(0.01)$</td>
</tr>
<tr>
<td>Leverage</td>
<td>$-33.46$</td>
<td>$-1.82$</td>
<td>$1.57$</td>
<td>$-36.85$</td>
<td>$-1.94$</td>
<td>$1.59$</td>
</tr>
<tr>
<td></td>
<td>$(0.75)$</td>
<td>$(0.02)$</td>
<td>$(0.02)$</td>
<td>$(0.76)$</td>
<td>$(0.02)$</td>
<td>$(0.02)$</td>
</tr>
<tr>
<td>Recession</td>
<td>$1.82$</td>
<td>$0.03$</td>
<td>$0.20$</td>
<td>$1.73$</td>
<td>$0.03$</td>
<td>$0.19$</td>
</tr>
<tr>
<td></td>
<td>$(1.22)$</td>
<td>$(0.02)$</td>
<td>$(0.03)$</td>
<td>$(1.27)$</td>
<td>$(0.02)$</td>
<td>$(0.03)$</td>
</tr>
<tr>
<td>Dummy\text{post} 2008</td>
<td>$-22.89$</td>
<td>$-0.30$</td>
<td>$0.03$</td>
<td>$-23.30$</td>
<td>$-0.30$</td>
<td>$0.04$</td>
</tr>
<tr>
<td></td>
<td>$(0.94)$</td>
<td>$(0.02)$</td>
<td>$(0.03)$</td>
<td>$(0.95)$</td>
<td>$(0.02)$</td>
<td>$(0.03)$</td>
</tr>
</tbody>
</table>

| Industry FE                  | X         | X         | X            | X         | X         | X            |
| Nobs                         | 733,532   | 733,532   | 733,532      | 656,631   | 656,631   | 656,631      |
| R²                           | 17.89%    | 33.82%    | 49.68%       | 15.79%    | 31.68%    | 47.96%       |
Figure 2: Expected inflation and asset prices – Simple model

The figure illustrates the impact of inflation on equity value (upper-left panel), credit spread (upper-right panel), debt value (lower-left panel), and market leverage (lower-right panel). The expected inflation rate is either low (1%), moderate (3%), or high (5%). Predictions are obtained with the static corporate finance model with exogenous capital structure and default policies discussed in Section 2. We set the parameter values to $\mu_Y = 2\%$, $\sigma_Y = 15\%$, $X_D = 0.5$, $X_0 = 1$, and $r = 4\%$. 

(A) Debt Value

(B) Bond Yield (bps)

(C) Credit Spread (bps)

(D) Arrow Debreu Default Claim

(E) Firm Value

(F) Equity Value
Figure 3: Expected inflation and asset prices – Full model

The figure illustrates the impact of expected inflation on equity value (upper-left panel), credit spread (upper-right panel), debt value (lower-left panel), and market leverage (lower-right panel). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. Predictions for the full model (sticky cash flows) are compared to the predictions of a model without sticky cash flows ($\varphi = 1$). Both sets of predictions are normalized to unity in the moderate expected inflation state. Firms have the corporate policies presented in Table 3. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.
Figure 4: Expected inflation and asset prices – Economy of firms

The figure illustrates the impact of expected inflation on equity value (upper-left panel), credit spread (upper-right panel), debt value (lower-left panel), and market leverage (lower-right panel). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. Predictions for a representative firm are compared to the predictions for a cross-section of firms, which differ in their leverage ratios. Both sets of predictions are normalized to unity in the moderate expected inflation state. All firms have initially the corporate policies presented in Table 3. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.
Figure 5: Equity valuation, default risk, and expected inflation

This figure plots the relations between expected inflation and the price-dividend ratios (top panels), the market-to-book ratios (middle panels), and default risk (bottom panels). We report the relations by levels of market leverage. The left panels show portfolios of firms with below-median leverage, whereas the right panels report firms with above-median leverage. Each observation represents the value-weighted average of the valuation metric across firms for a given level of expected inflation. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), which is computed as in Campbell et al. (2008). Section 6.1 provides additional details on the data. The sample period is 1972Q2–2016Q4.
APPENDIX

A Data description

This section describes the data used in our empirical analysis (section 6).

A.1 Valuation ratios

Market-to-book ratio ($MB$) is computed as $ME/BE$. Book equity ($BE$) is shareholders’ equity ($SEQQ + CEQQ + PSTKQ$ or $ATQ - LTQ$), plus balance sheet deferred taxes and investment tax credit ($TXDITCQ$) if available, minus the book value of preferred stock ($PSTKQ$) as in Weber (2018). Market capitalization ($ME$) is the product of quarter-end price ($PRC$) and share outstanding ($Shrot$).

The price-dividend ratio is computed as the share price divided by the sum of dividend payments over the last 12 months. We construct dividend payments using cum-dividend return and ex-dividend returns, as in Beeler and Campbell (2012).

A.2 Default risk

We follow Campbell, Hilscher, and Szilagyi (2008) to calculate financial distress risk ($FR$) as the logit transformed bankruptcy probability, while excluding leverage in the measurement. $FR$ is then calculated as

$$FR = -9.16 - 20.26 * NIMTAVG - 7.13 * EXRETAVG + 1.41 * SIGMA$$
$$- 0.045 * RSIZE - 2.13 * CASHMTA + 0.075 * MB - 0.058 * PRICE,$$

where

$$NIMTAVG_t = \sum_{i=0}^{3} \frac{1 - \phi^3}{1 - \phi^3} (\phi^{3(i-1)} NITMA_{t-3i})$$

$$EXRETAVG_t = \sum_{i=0}^{11} \frac{1 - \phi}{1 - \phi} (\phi^{i-1} EXRET_{t-i})$$

$NIMTA$ and $EXRET$ are net income over total assets ($NIQ/ATQ$) and the log of gross excess returns over the value-weighted S&P500 returns, respectively. $SIGMA$ is the square root of the annualized sum of squared stock returns over a 3-month period. $RSIZE$ is the log of firm’s market equity over the total valuation of all firms in the S&P500. $CASHMTA$ is cash and short-term investments over total assets ($CHEQ/ATQ$). $MB$ is the market-to-book value of equity. $PRICE$ is the log of price.
per share. The associated 1-quarter bankruptcy probability for firm $i$ at time $t$ is then

$$P_{t-1}(Y_{i,t} = 1) = \frac{1}{1 + exp(-FR_{i,t-1})}.$$ 

### A.3 Leverage, investment and profitability

Market leverage is the sum of long term debt and debt in current liabilities over the sum of debt and market capitalization $((DLCQ + DLTTQ)/(DLCQ + DLTTQ + ME))$ as in Freyberger, Neuhierl, and Weber (2017).

Investment and profitability are calculated following Fama and French (2015) as revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity $(IBQ - COGSQ - XSGAQ - XINTQ)/BE$ and the percentage change in total asset.
ONLINE APPENDIX
OA.A Simple Model

Under the physical probability measure $\mathbb{P}$, real cashflow growth is given by

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dW_t,$$  \hspace{1cm} (OA.1)

where $W$ is a standard Brownian motion under $\mathbb{P}$. The date $t$ nominal cashflow is given by $X_t = Y_t P_t^\phi$, and so nominal cashflow growth is given by

$$\frac{dX_t}{X_t} = (\mu_Y + \phi \mu_P) dt + \sigma_Y dW_t.$$  \hspace{1cm} (OA.2)

The real SDF is given by

$$\frac{d\pi_t}{\pi_t} = -r dt - \Theta dZ_t,$$  \hspace{1cm} (OA.3)

where $Z$ is a standard Brownian motion under $\mathbb{P}$ such that $dZ_t dW_t = \rho dt$. In this section, there is no risk premium associated with sudden shifts in the state of the economy. In contrast with the main model, we therefore assume $\rho > 0$ to ensure that the risk premium is not zero. Consequently, conditional on date $t$ information, the risk-neutral probability of event $A$ occurring at date $T$ is given by

$$E_t^Q[1_A] = E_t \left[ \frac{M_T}{M_t} 1_A \right],$$  \hspace{1cm} (OA.4)

where $M$ is an exponential martingale under $\mathbb{P}$, defined by

$$\frac{dM_t}{M_t} = -\Theta dZ_t, \, M_0 = 1.$$  \hspace{1cm} (OA.5)

The exogenous price index is given by

$$P_t = P_0 e^{\mu_P t},$$  \hspace{1cm} (OA.6)

where $\mu_P$ is the constant inflation rate.

The nominal SDF is given by $\pi^\$ = $\pi_t / P_t$, and so

$$\frac{d\pi^\$}{\pi^\$} = -r^\$ dt - \Theta dZ_t,$$  \hspace{1cm} (OA.7)

where

$$r^\$ = r + \mu_P.$$  \hspace{1cm} (OA.8)

The price-index is not stochastic, so there is no inflation risk premium. We can therefore price risk under the risk-neutral measure $\mathbb{Q}$ with no additional adjustment for an inflation risk premium. If
there were a risk premium, we would have to define a different probability measure in order to discount nominal cashflows with the nominal interest rate. Using Girsanov’s Theorem, we obtain the evolution of $X$ under $Q$:

$$\frac{dX_t}{X_t} = (\hat{\mu}_X + \varphi \mu_P)dt + \sigma_X dW^Q_t.$$  

(OA.9)

where $W^Q$ is a standard Brownian motion under the risk-neutral measure $Q$ and $\hat{\mu}_X = \mu_X - \rho \sigma_X \Theta$ is the risk-neutral expected nominal cash flow growth rate.

The date-$t$ nominal after-tax abandonment value of the firm is given by

$$A^S_t = A^S(\xi_t) = (1 - \eta)\xi_t E_t \left[ \int_t^\infty \frac{\pi^S_u}{\pi^S_t} \xi_u \right] = (1 - \eta)\xi_t E_t \left[ \int_t^\infty e^{-r^S(u-t)} \frac{\xi_u}{\xi_t} du \right].$$  

(OA.10)

The date-$t$ nominal price of the corporate bond is given by

$$B^S_t = c E_t \left[ \int_t^{\tau_D} \frac{\pi^S_u}{\pi^S_t} du \right] + \alpha E_t \left[ \pi^S_{\tau_D} (\xi_{\tau_D}) \right]$$

$$= c E_t \left[ \int_t^{\tau_D} e^{-r^S(u-t)} du \right] + \alpha E_t \left[ e^{-r^S(\tau_D-t)} A^S(\xi_{\tau_D}) du \right]$$

(OA.11)

which we can rewrite as

$$B^S_t = c E_t^Q \left[ \int_t^\infty e^{-r^S(u-t)} du - e^{-r^S(\tau_D-t)} E^Q_{\tau_D} \int_t^{\tau_D} e^{-r^S(u-\tau_D)} du \right] + \alpha E_t^Q \left[ e^{-r^S(\tau_D-t)} A^S(\xi_{\tau_D}) du \right]$$

$$= c E_t^Q \left[ \int_t^\infty e^{-r^S(u-t)} du - e^{-r^S(\tau_D-t)} E^Q_{\tau_D} \int_t^{\tau_D} e^{-r^S(u-\tau_D)} du \right] + \alpha E_t^Q \left[ e^{-r^S(\tau_D-t)} A^S(\xi_{\tau_D}) \right]$$

$$= \frac{c}{r^S} E_t^Q \left[ 1 - e^{-r^S(\tau_D-t)} \right] + \alpha E_t^Q \left[ e^{-r^S(\tau_D-t)} A^S(\xi_{\tau_D}) \right]$$

(OA.12)

$$B^S_t = \frac{c}{r^S} (1 - q^S_{D,t}) + \alpha A^S(\xi_{D}) q^S_{D,t},$$

where

$$q^S_{D,t} = E_t^Q \left[ e^{-r^S(\tau_D-t)} \right]$$

(OA.17)

is the date-$t$ price of the Arrow-Debreu default claim, which pays off 1 unit of the numeraire (1 dollar) at the time of default $\tau_D$. 

OA-2
From the principle of no arbitrage, the price of the Arrow-Debreu default claim, \( q_D^S(x) \), (where \( x = \ln X \)) satisfies

\[
E_t^Q[dq_D^S(x) - q_D^S(x)r^S dt] = 0.
\] (OA.18)

Applying Ito’s Lemma gives the ordinary differential equation

\[
\frac{1}{2} \sigma_Y^2 q_D''^S(x) + \left( \mu_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2 \right) q_D'^S(x) - r^S q_D(x) = 0.
\] (OA.19)

The general solution of the above ordinary differential equation is given by

\[
q_D^S(x) = k - e^{a_- x} + k + e^{a_+ x},
\]
where \( a_- \) and \( a_+ \) are the roots of the following quadratic in \( a \)

\[
\frac{1}{2} \sigma_Y^2 a^2 + \left( \mu_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2 \right) a - r^S = 0.
\] (OA.20)

It follows that

\[
a_\pm = -\mu_Y + \mu_P - \frac{1}{2} \sigma_Y^2 \pm \sqrt{\left( \mu_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2 \right)^2 + \frac{r + \mu_P}{2 \sigma_Y^2}}.
\] (OA.21)

We know that \( a_- a_+ = -r^S \), so \( a_- \) and \( a_+ \) are of opposite sign if \( r^S > 0 \). We can also see that \( a_+ > 0 \) if \( r^S > 0 \). Therefore, \( a_- < 0 \) if \( r^S > 0 \). From the no-bubble condition \( \lim_{x \to \infty} |q_D(x)| < \infty \), we see that \( c_+ = 0 \) and so \( q_D^S(x) = k_- e^{a_- x} + k_+ e^{a_+ x} \). The boundary condition \( q_D(x_D) = 1 \) (where \( x_D = \ln X_D \)) implies that \( q_D^S(x) = e^{a_-(x-x_D)} \), and so

\[
q_D^S(x) = e^{-a(\mu_P)(x_1-x_D)},
\] (OA.22)

where

\[
a(\mu_P) = \frac{\hat{\mu}_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2}{\sigma_Y^2} - \sqrt{\left( \frac{\hat{\mu}_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2}{\sigma_Y^2} \right)^2 + \frac{r + \mu_P}{2 \sigma_Y^2}},
\] (OA.23)

and of course \( a(\mu_P) > 0 \), if \( r^S > 0 \). Now

\[
\frac{\partial \ln q_D^S}{\partial \mu_P} = -\left( x_1 - x_D \right) \frac{\partial a(\mu_P)}{\partial \mu_P} - a(\mu_P) \frac{\partial (x_1 - x_D)}{\partial \mu_P}.
\] (OA.24)

We now show that \( \frac{\partial a(\mu_P)}{\partial \mu_P} > 0 \) if \( \varphi > -1/a(\mu_P) \) (provided \( r^S > 0 \)). Observe that

\[
\frac{1}{2} \sigma_Y^2 a(\mu_P)^2 - \left( \hat{\mu}_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2 \right) a(\mu_P) - r^S = 0.
\] (OA.25)
Differentiating with respect to $\mu_P$ gives
\[
\sigma_Y^2 \left[ a(\mu_P) - \left( \mu_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2 \right) \right] \frac{\partial a(\mu_P)}{\partial \mu_P} = 1 + \varphi a(\mu_P). \tag{OA.26}
\]
Hence
\[
\sigma_Y^2 \left( \frac{\mu_Y + \varphi \mu_P - \frac{1}{2} \sigma_Y^2}{\sigma_Y^2} \right)^2 + \frac{r + \mu_P}{2} \frac{\partial a(\mu_P)}{\partial \mu_P} = 1 + \varphi a(\mu_P), \tag{OA.27}
\]
and so $\frac{\partial a(\mu_P)}{\partial \mu_P} > 0$ if $\varphi > -1/a(\mu_P)$ (provided $r^S > 0$). Hence, holding the distance to default, $x_t - x_D$, fixed, the price of the Arrow-Debreu default claim, $q^s_{D,t}$, decreases with expected inflation.

The date-$t$ value of levered equity (in nominal units) after taxes is given by
\[
S^s_t = (1 - \eta) E^Q_t \left[ \int_t^{\tau_D} e^{-r^S(u-t)} (X_u - c) du \right]. \tag{OA.28}
\]
Therefore, we obtain
\[
S^s_t = (1 - \eta) \left( X_t E^Q_t \left[ \int_t^{\tau_D} e^{-r^S(u-t)} \frac{X_u}{X_t} du - e^{-r^S(\tau_D-t)} \frac{X_D}{X_t} E^Q_{\tau_D} \int_{\tau_D}^{\infty} e^{-r^S(u-\tau_D)} \frac{X_u}{X_D} du \right] \right)
\]
\[
- c E^Q_t \left[ \int_t^{\tau_D} e^{-r^S(u-t)} du \right] \tag{OA.29}
\]
\[
= (1 - \eta) X_t \left( \frac{1}{r^S - \hat{\mu}_X} - E^Q_{\tau_D} \left[ e^{-r^S(\tau_D-t)} \frac{X_D}{X_t} \right] \frac{1}{r^S - \hat{\mu}_X} - \frac{c}{r^S} (1 - q^s_{D,t}) \right) \tag{OA.30}
\]
\[
= (1 - \eta) \left( \frac{X_t - X_D q^s_{D,t}}{r^S - \hat{\mu}_X} - \frac{c}{r^S} (1 - q^s_{D,t}) \right) \tag{OA.31}
\]

With an endogenous default policy, the default time $\tau_D$ is chosen to maximize the value of levered equity. The smooth pasting condition $\frac{\partial S^s_t}{\partial X_t} \bigg|_{X_t = X_D} = 0$ determines the default boundary $X_D$, i.e.
\[
\frac{1 - X_D}{r^S - \hat{\mu}_X} \frac{\partial q^s_{D,t}}{\partial X_t} \bigg|_{X_t = X_D} + \frac{c}{r^S} \frac{\partial q^s_{D,t}}{\partial X_t} \bigg|_{X_t = X_D} = 0, \tag{OA.33}
\]
and so
\[
X_D = \frac{c + (1 - \varphi) \mu_P - \hat{\mu}_Y}{r + \mu_P} \frac{a(\mu_P)}{1 + a(\mu_P)}. \tag{OA.34}
\]
It follows that, for a fixed nominal coupon, $c$, we have
\[
\frac{\partial x_D}{\partial \mu_P} = \frac{(1 - \varphi) r - (r - \hat{\mu}_Y)}{(r + (1 - \varphi) \mu_P - \hat{\mu}_Y)(r + \mu_P)} + \frac{1}{a(\mu_P)(1 + a(\mu_P))} \frac{\partial a(\mu_P)}{\partial \mu_P}. \tag{OA.35}
\]
With an endogenous default policy, the distance to default is impacted by inflation. A priori, it is possible that equity holders will choose to default later when inflation is higher, that is the distance to default will increase. However, for the calibration we have chosen, equityholders default earlier when inflation is higher, because the present value of the coupons they have to pay to bondholders is increased. Even if this were not the case, \( \frac{\partial}{\partial \mu} \) is much larger than \( \frac{\partial (x_t - x_D)}{\partial \mu} \), so any increase in distance to default would not change the overall sign of \( \frac{\partial \ln q_{D,t}}{\partial \mu} \).

**OA.B The Economy**

First, we introduce some notation related to jumps in the state of the economy. Suppose that during the small time-interval \( [t - \Delta t, t) \) the economy is in state \( i \) and that at time \( t \) the state changes, so that during the next small time interval \( [t, t + \Delta t) \) the economy is in state \( j \neq i \). We then define the left-limit of \( s \) at time \( t \) as

\[
s_{t-} = \lim_{\Delta t \to 0} s_{t-\Delta t}, \quad (OA.36)
\]

and the right-limit as

\[
s_t = \lim_{\Delta t \to 0} s_{t+\Delta t}. \quad (OA.37)
\]

Therefore \( s_{t-} = i \), whereas \( s_t = j \), so the left- and right limits are not equal. If some function \( E \) depends on the current state of the economy i.e. \( E_t = E(s_t) \), then \( E \) is a jump process which is right continuous with left limits, i.e. RCLL. If a jump from state \( i \) to \( j \neq i \) occurs at date \( t \), then we abuse notation slightly and denote the left limit of \( E \) at time \( t \) by \( E_i \), where \( i \) is the index for the state, i.e. \( E_{t-} = \lim_{s \uparrow t} E_s = E_i \). Similarly \( E_t = \lim_{s \downarrow t} E_s = E_j \). We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

\[
f(c, v) = \beta \left( h^{-1}(v) \right)^{1-\gamma} u \left( c/h^{-1}(v) \right), \quad (OA.38)
\]

where

\[
u(x) = \frac{x^{1-\gamma} - 1}{1 - \frac{1}{\psi}}, \psi > 0,
\]

\[
h(x) = \begin{cases} 
\frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1, \\
\ln x, & \gamma = 1.
\end{cases}
\]

The representative agent’s value function is given by

\[
J_t = E_t \int_t^\infty f(C_t, J_t) \, dt. \quad (OA.39)
\]
Proposition OA.1 The SDF of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by

\[
\pi_t = \begin{cases} 
(\beta e^{-\beta t})^{\frac{1}{1-\psi}} C_t^{-\gamma} \left( p_{C,t} e^{\int_0^t \rho_{C,s} ds} \right)^{-\frac{\gamma - \frac{1}{\psi}}{1-\psi}}, & \psi \neq 1 \\
\beta e^{-\beta \int_0^t [1+(\gamma-1)\ln(V_s^{-1})] ds} C_t^{-\gamma} V_t^{-(\gamma-1)}, & \psi = 1 
\end{cases}
\] (OA.40)

When \( \psi \neq 1 \), the price-consumption ratio in state \( i \), \( p_{C,i} \), satisfies the nonlinear equation system:

\[
p_{C,1}^{-1} = \tau_i + \gamma \sigma_{C,i}^2 - \mu_{C,i} - \left( 1 - \frac{1}{\psi} \right) \lambda_{ij} \left( \frac{p_{C,j}/p_{C,i}}{1 - \gamma} - 1 \right), \quad i, j \in \{ R, E \}, j \neq i.
\] (OA.41)

where

\[
\tau_i = \beta + \frac{1}{\psi} \mu_{C,i} - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_{C,i}^2, \quad i \in \{ R, E \}.
\] (OA.42)

When \( \psi = 1 \), define \( V_i \) via

\[ J = \ln(C V) \].
(OA.43)

Then \( V_i \) satisfies the nonlinear equation system:

\[
\beta \ln V_i = \mu_{C,i} - \frac{\gamma}{2} \sigma_{C,i}^2 + \lambda_i \frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma} i \in \{ R, E \}, j \neq i.
\] (OA.44)

**OA.C Derivation of the real SDF**

In this section, we derive the real SDF shown in Proposition OA.1. When we refer to states of the economy within this proof we mean only the real states, \( L, H \).

Duffie and Skiadas (1994) show that the SDF for a general normalized aggregator \( f \) is given by

\[
\pi_t = e^{\int_0^t f_v(C_s,J_s) \, dt} f_c(C_t, J_t),
\] (OA.45)

where \( f_c(\cdot, \cdot) \) and \( f_v(\cdot, \cdot) \) are the partial derivatives of \( f \) with respect to its first and second arguments, respectively, and \( J \) is the value function given in (OA.39). The Feynman-Kac Theorem implies

\[
f(C_t, J_t) \big|_{\nu_t = i} \, dt + E_t [ dJ_t \big|_{\nu_t = i} ] = 0, \quad i \in \{ R, E \}.
\]

Using Ito’s Lemma we rewrite the above equation as

\[
0 = f(C_t, J_t) + CJ_t \mu_{C,i} + \frac{1}{2} C^2 J_t \sigma_{C,i}^2 + \lambda_i (J_j - J_i),
\] (OA.46)
for \( i, j \in \{ R, E \} \), \( j \neq i \). We guess and verify that \( J = h(CV) \), where \( V_i \) satisfies the nonlinear equation system

\[
0 = \beta u \left( V_i^{-1} \right) + \mu_{C,i} - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_i \left( \frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma} \right), \quad i, j \in \{ R, E \}, \ j \neq i. \tag{OA.47}
\]

Substituting (OA.38) into (OA.45) and simplifying gives

\[
\pi_t = \beta e^{-\beta \int_0^t \left[ 1 + (\gamma - \frac{1}{\psi}) u(V_i^{-1}) \right] dt} C_t^{-\gamma} V_t^{-\left( \frac{1}{\psi} \right)}. \tag{OA.48}
\]

When \( \psi = 1 \), the above equation gives the second expression in (OA.40). We rewrite (OA.47) as

\[
\beta \left[ 1 + \left( \gamma - \frac{1}{\psi} \right) u \left( V_i^{-1} \right) \right] = \tau_i - \left( \gamma - \frac{1}{\psi} \right) \lambda_i \left( \frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma} \right) = \left[ \gamma \mu_{C,i} - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], \quad i, j \in \{ R, E \}, \ j \neq i,
\]

where \( \tau_i \) is given in (OA.42). Setting \( \psi = 1 \) in (OA.49) gives (OA.44). To derive the first expression in (OA.40) from (OA.48) we prove that

\[
V_i = (\beta p_{C,i})^{1 - \frac{1}{\psi}}, \ \psi \neq 1. \tag{OA.50}
\]

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption, \( C^* \), and risky asset portfolio, \( \varphi \), to maximize her expected utility

\[
J^*_t = \sup_{C^* \varphi} E_t \int_t^{\infty} f \left( C^*_t, J^*_t \right) dt.
\]

Observe that \( J^* \) depends on optimal consumption-portfolio choice, whereas the \( J \) defined previously in (OA.43) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, \( C^* \), invests a proportion, \( \varphi \), of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, \( W \), evolves according to the dynamic budget constraint:

\[
\frac{dW_t}{W_t} = \varphi_t \left( dR_{C,t} - r_t dt \right) + r_t dt - \frac{C^*_t}{W_t} dt,
\]

where \( dR_{C,t} \) is the cumulative return on the claim to aggregate consumption. We define \( N_{i,t} \) as the Poisson process which jumps upward by one whenever the real state of the economy switches from \( i \) to \( j \neq i \). The compensated version of this process is the Poisson martingale

\[
N_{i,t}^P = N_{i,t} - \lambda_i t, \ i \in \{ R, E \}
\]
It follows from applying Ito’s Lemma to \( P = p_C C \) that the cumulative return on the claim to aggregate consumption is

\[
dR_{C,t} = \frac{dP_t + C_t dt}{P_t^{-}} = \mu_{RC,t} dt + \sigma_{RC,t} dB_{C,t} + \sigma_{RC,t}^P dN_{t-}^P,
\]

where

\[
\mu_{RC,t} = \mu_{C,i} + \frac{1}{2} \sigma_{C,i}^2 + \lambda_i \left( \frac{p_{C,j}}{p_{C,i}} - 1 \right) + \frac{1}{p_{C,i}},
\]

\[
\sigma_{C,t} = \sigma_{C,i},
\]

\[
\sigma_{RC,t}^P = \frac{p_{C,i}}{p_{C,i}} - 1,
\]

for \( i \in \{R, E\}, j \neq i \). The total volatility of the return to holding the consumption claim, when the current state is \( i \), is given by

\[
\sigma_{RC,i} = \sqrt{\sigma_{C,i}^2 + \lambda_i \left( \sigma_{RC,i}^P \right)^2}.
\]

Note that \( C^* \) is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, \( C \) is aggregate consumption, and since it is continuous, its left and right limits are equal, i.e. \( C_{t-} = C_t \).

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent’s optimization problem is

\[
\sup_{C^*, \phi} f \left( C^*_{t-}, J^*_{t-} \right) |_{u_{t-}=i} dt + E_t \left[ dJ^*_{t} | u_{t-}=i \right] = 0, \ i \in \{R, E\}.
\]

Applying Ito’s Lemma to \( J^*_{t} = J^* (W_t, u_t) \) allows us to write the above equation as

\[
0 = \sup_{C^*, \phi} f \left( C^*_t, J^*_t \right) + W_t J^*_t W \left( \phi_i (\mu_{RC,i} - r_i) + r_i - \frac{C^*_i}{W_i} \right) + \frac{1}{2} W_t^2 J^*_t W W \phi_i^2 \sigma_{RC,i}^2 + \lambda_i \left( J^*_j - J^*_i \right), \ i \in \{R, E\}, j \neq i.
\]

We guess and verify that \( J^*_t = h_t (W_t F_t) \), where \( F_t \) satisfies the nonlinear equation system

\[
0 = \sup_{C^*, \phi} \beta u \left( \frac{C^*_i}{W_i F_i} \right) + \left( \phi_i (\mu_{RC,i} - r_i) + r_i - \frac{C^*_i}{W_i} \right) - \frac{1}{2} \gamma \phi_i^2 \sigma_{RC,i}^2 + \lambda_i \left( \frac{(F_j/F_i)^{1-\gamma} - 1}{1-\gamma} \right), \ i \in \{R, E\}, j \neq i.
\]

From the first order conditions of the above equations, we obtain the optimal consumption and portfolio policies:

\[
C^*_i = \beta \psi F_i^{-\psi-1} W_t, \ i \in \{R, E\},
\]

\[
\phi_i = \frac{1}{\gamma} \frac{\mu_{RC,i} - r_i}{\sigma_{RC,i}^2}, \ i \in \{R, E\}.
\]
The market for the consumption good must clear, so \( \varphi_i = 1, \ W_i = P_i, \ C_i^* = C \) (and thus \( J = J^* \)). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence

\[
\mu_{RC,i} - r_i = \gamma \sigma_{RC,i}^2,
\]

and

\[
p_{C,i} = \beta^{-\psi} F_{i}^{1-\psi}.
\]

The above equation implies that for \( \psi = 1, \ p_{C,i} = 1/\beta \). The equality, \( J = J^* \), implies that \( CV_i = WF_i \). Hence, \( F_i = p_{C,i}^{-1} V_i \). Using this equation to eliminate \( F_i \) from (OA.51) gives (OA.50). Substituting (OA.50) into (OA.48) and (OA.49) gives the expression in (OA.40) for \( \psi \neq 1 \) and (OA.41).

**OA.D The evolution of the real SDF**

In this section we derive the evolution of the real SDF, as given in (14).

We define \( N_{\nu_- \nu_-,t} \) as the Poisson process which jumps upward by one whenever the real state of the economy switches from \( \nu_- \) to \( \nu_\neq \nu_- \). The compensated version of this process is the Poisson martingale (under the physical probability measure \( \mathbb{P} \)), \( N_{\nu_- \nu_-,t}^{P} = N_{\nu_- \nu_-,t} - \lambda_{\nu_- \nu_-,t} \). We start by proving that the real SDF satisfies the stochastic differential equation

\[
\frac{d\pi_t}{\pi_{\nu_-}} = -r_i dt + \frac{dM_t}{M_{\nu_-}}|_{\nu_- = i},
\]

where \( M \) is a martingale under \( \mathbb{P} \) such that

\[
\frac{dM_t}{M_{\nu_-}}|_{\nu_- = i} = -\Theta_i^B dZ_t + \Theta_i^P dN_{ij,t}^P, \ j \in \{E, R\}, \ j \neq i,
\]

\( r_i \) is the risk-free rate in state \( i \) given by

\[
r_i = \begin{cases} \tau_R + \lambda_{RE}^\text{real} \left[ \frac{\gamma - \frac{1}{\gamma^2}}{\gamma - 1} \left( \omega^{\frac{\gamma - 1}{\gamma^2}} - 1 \right) - (\omega - 1) \right] , & i = R; \\ \tau_E + \lambda_{ER}^\text{real} \left[ \frac{\gamma - \frac{1}{\gamma^2}}{\gamma - 1} \left( \omega^{\frac{\gamma - 1}{\gamma^2}} - 1 \right) - (\omega - 1) \right] , & i = E, \end{cases}
\]

where \( \omega \) is the solution of

\[
G(\omega) = 0,
\]

\( \omega \) is the solution of

\[
G(\omega) = 0,
\]

\( OA-9 \)
and

\[ G(x) = \begin{cases} 
-\frac{1}{\gamma-1} \left( \frac{x^{\gamma-1}}{\gamma-1} - 1 \right), & \psi \neq 1; \\
\frac{\tau_R + \gamma \sigma_{C,E}^2 - g_{E}}{\gamma-1} \left( \frac{x^{\gamma-1}}{\gamma-1} - 1 \right), & \psi = 1.
\end{cases} \] (OA.56)

\( \Theta^B_i \) is the market price of risk due to Brownian shocks in real state \( i \), given by

\[ \Theta^B_i = \gamma \sigma_{C,i}, \quad i \in \{ R, E \}, \] (OA.57)

and \( \Theta^P_i \) is the market price of risk due to Poisson shocks when the economy switches out of state \( i \):

\[ \Theta^P_{ij} = \omega_{ij} - 1, \quad i, j \in \{ R, E \}, \quad j \neq i. \] (OA.58)

We begin the proof by noting that if we define

\[ \omega_{ij} = \frac{\pi_t}{\pi_{t-i}} \left| \nu_t = i, \nu_t = j \right|, \quad i, j \in \{ R, E \}, \quad j \neq i, \] (OA.59)

then (OA.40) implies that

\[ \omega_{ij} = \begin{cases} 
\frac{p_{C,j}}{p_{C,i}} \left( \frac{x^{\gamma-1}}{\gamma-1} - 1 \right), & \psi \neq 1; \\
\frac{V_j}{V_i} \left( \omega_{ij}^{\gamma-1} - 1 \right), & \psi = 1.
\end{cases} \] (OA.60)

The above equation implies that \( \omega_{ER} = \omega_{RE}^{-1} \), so we can set \( \omega_{ER} = \omega_{RE}^{-1} = \omega \).

We now show how to determine \( \omega \). Using (OA.60) we rewrite (OA.41) and (OA.44) as

\[ p_{C,i} = \frac{1}{\rho_i + \gamma \sigma_{C,i}^2 - \mu_{C,i} + \lambda_{ij}^{\text{real}} \left( \omega_{ij}^{\gamma-1/\gamma} - 1 \right)}, \quad i, j \in \{ R, E \}, \quad j \neq i, \] (OA.61)

and

\[ \beta \ln V_i = \mu_{C,i} - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_{ij}^{\text{real}} \omega_{ij} - 1 \left( 1 - \gamma \right), \quad i, j \in \{ R, E \}, \quad j \neq i, \] (OA.62)

respectively. Therefore, from (OA.60) and the above two equations it follows that \( \omega \) is the solution of Equation (OA.55).
We now derive expressions for the risk-free rate and risk prices. Ito’s Lemma implies that the state-price density evolves according to

\[
\frac{d\pi_t}{\pi_t} = -\frac{1}{\pi_t} \frac{\partial \pi_t}{\partial t} dt + \frac{1}{\pi_t} C_t \frac{\partial \pi_t}{\partial C_t} dC_t + \frac{1}{2} \frac{1}{\pi_t} C_t^2 \frac{\partial^2 \pi_t}{\partial C_t^2} \left( \frac{dC_t}{C_t} \right)^2 \\
+ \lambda_{\nu_{-\nu}} \frac{\Delta \pi_t}{\pi_t} dt + \Delta \pi_t \frac{dN_P}{\nu_{-\nu},t},
\]

where \( \Delta \pi_t = \pi_t - \pi_{t-} \). The definition (OA.59) implies

\[
\frac{\Delta \pi_t}{\pi_t} = \omega_{\nu_{-\nu}} - 1.
\]

Together with some standard algebra that allows us to rewrite (OA.63) as

\[
\frac{d\pi_t}{\pi_t} = -\left( \kappa_{\nu_{-\nu}} + \gamma \mu_{C,\nu_{-\nu}} - \frac{1}{2} \gamma (1 + \gamma) \sigma^2_{C,i} + \lambda_{\nu_{-\nu}} \left( 1 - \omega_{\nu_{-\nu}} \right) \right) dt - \gamma \sigma_{C,\nu_{-\nu}} dZ_t + (\omega_{\nu_{-\nu}} - 1) dN^P_{\nu_{-\nu},t}.
\]

Comparing the above equation with (OA.52), which is standard in an economy with jumps, gives (OA.57) and (OA.58), in addition to

\[
\pi_{t-} = \kappa_i + \gamma \mu_{C,i} - \frac{1}{2} \gamma (1 + \gamma) \sigma^2_{C,i} + \lambda_{ij} \left( 1 - \omega_{ij} \right), \quad i, j \in \{ R, E \}, \quad j \neq i,
\]

where

\[
\kappa_i = \begin{cases} 
\beta \left[ 1 + \left( \gamma - \frac{1}{\psi} \right) \left( \frac{\beta \mu_{C,i}}{1 - \psi} \right)^{-1} \right], & \psi \neq 1, \quad i, j \in \{ R, E \}, \quad j \neq i; \\
\beta \left[ 1 + (\gamma - 1) \ln(V_{ij}^{-1}) \right], & \psi = 1, \quad i, j \in \{ R, E \}, \quad j \neq i,
\end{cases}
\]

We use Equations (OA.61) and (OA.62) to eliminate \( p_{C,i} \) and \( V_i \) from (OA.64) to obtain

\[
\kappa_i = \begin{cases} 
\tau_i - \left( \gamma - \frac{1}{\psi} \right) \lambda_{ij} \left( \frac{\omega_{ij}}{1 - \gamma} \right)^{-1} - \left[ \gamma \mu_{C,i} - \frac{1}{2} \gamma (1 + \gamma) \sigma^2_{C,i} \right], & \psi \neq 1, \quad i, j \in \{ R, E \}, \quad j \neq i; \\
\tau_i + \lambda_{ij} \left( \omega_{ij} - 1 \right) - \left[ \gamma \mu_{C,i} - \frac{1}{2} \gamma (1 + \gamma) \sigma^2_{C,i} \right], & \psi = 1, \quad i, j \in \{ R, E \}, \quad j \neq i,
\end{cases}
\]

so

\[
\tau_i = \begin{cases} 
\tau_i - \left( \gamma - \frac{1}{\psi} \right) \lambda_{ij} \left( \frac{\omega_{ij}}{1 - \gamma} \right)^{-1} + \lambda_{ij} \left( 1 - \omega_{ij} \right), & \psi \neq 1, \quad i, j \in \{ R, E \}, \quad j \neq i; \\
\tau_i, & \psi = 1, \quad i \in \{ R, E \}.
\end{cases}
\]

Taking the limit of the upper expression in the above equation gives the lower expression, so (OA.54) follows. The total market price of consumption risk in real state \( i \) accounts for both Brownian and
Poisson shocks, and is thus given by

\[ \Theta_i = \sqrt{(\Theta_i^B)^2 + \lambda_{i,j} (\Theta_i^P)^2}, \quad i, j \in \{R, E\}, \quad j \neq i. \]  

(OA.67)

Because the Poisson and Brownian shocks in (OA.53) are independent and their respective prices of risk are bounded, \(M\) is a martingale under the actual measure \(\mathbb{P}\). Thus, \(M\) defines the Radon-Nikodym derivative \(\frac{d\mathbb{Q}}{d\mathbb{P}}\) via \(M_t = \mathbb{E}_t \left[ \frac{d\mathbb{Q}}{d\mathbb{P}} \right]\). It is a standard result (see Elliott (1982)) that the risk-neutral switching probabilities per unit time are given by

\[ \hat{\lambda}_{ij}^{\text{real}} = \lambda_{ij}^{\text{real}} E_t \left[ \frac{M_t}{M_{t-}} \mid \nu_t = i, \nu_t = j \right], \quad j \neq i. \]

The jump component in \(d\pi\) comes purely from \(dM\). Thus, using (OA.59), we can simplify the above expression to obtain \(\hat{\lambda}_{ij}^{\text{real}} = \lambda_{ij}^{\text{real}} \omega_{ij}\), from which we can derive (17).

We deduce the properties of the risk distortion factor, \(\omega\), from the properties of the function \(G\) defined in (OA.56). We restrict the domain of \(G\) to \(x > 0\). First we consider the case where \(\psi \neq 1\). We assume that the price-consumption ratios, \(p_{C,i}\), \(i \in \{R, E\}\), are strictly positive. Therefore, \(G\) is continuous. We observe that if \(G\) is monotonic, then by continuity, \(G(1)\) and \(G'(1)\) are the same (opposite) in sign iff \(\omega < 1\) (\(\omega > 1\)). Clearly, in both cases, \(\omega\) is unique. To establish monotonicity note that

\[ G'(x) = \frac{1}{\gamma - \frac{1}{\psi}} \left[ \frac{1}{\tau_1 + \gamma \sigma_{C,R}^2 - \mu_{C,R}} \right] \left( \frac{1}{\gamma - \frac{1}{\psi}} - 1 \right) \left( \frac{1}{\gamma - \frac{1}{\psi}} \right) ^{\frac{1}{\gamma - 1}} x^\gamma - \frac{1}{\gamma - 1} \]

The above equation implies that for \(x > 0\), if \(p_{C,R}\) and \(p_{C,E}\) are strictly positive, then \(G'(x)\) does not change sign. Therefore, \(G\) must be monotonic. Now we use the following expressions:

\[ G(1) = 1 - \frac{\tau E + \gamma \sigma_{C,E}^2 - \mu_{C,E}}{\tau R + \gamma \sigma_{C,R}^2 - \mu_{C,R}}, \]

and

\[ G'(1) = \frac{1}{\gamma - \frac{1}{\psi}} \left[ \frac{(\tau R + \gamma \sigma_{C,R}^2 - \mu_{C,R}) \lambda_{RE}^{\text{real}} + (\tau E + \gamma \sigma_{C,E}^2 - \mu_{C,E}) \lambda_{RE}^{\text{real}}}{(\tau R + \gamma \sigma_{C,R}^2 - \mu_{C,R})^2} \right], \]

to relate the signs of \(G(1)\) and \(G'(1)\) to the properties of the agent’s preferences. Note that \(G'(1) < 0\), \((G'(1) > 0)\) iff \(\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} > 0\), \(\left(\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} < 0\right)\). We assume that \(\tau_i + \gamma \sigma_{C,i}^2 - \mu_{C,i} > 0\) for \(i \in \{R, E\}\), which
is equivalent to assuming that if the economy were always in real state $i$, then the price-consumption ratio would be positive. Simple algebra tells us that $\pi_i + \gamma \sigma_{C,i}^2 - \mu_{C,i} = \beta + \left( \frac{1}{\psi} - 1 \right) \left( \mu_{C,i} - \frac{1}{2} \gamma \sigma_{C,i}^2 \right)$.

We know that $\mu_{C,R} - \frac{1}{2} \gamma \sigma_{C,R}^2 < \mu_{C,R} - \frac{1}{2} \gamma \sigma_{C,E}^2$. Therefore $G(1) < 0$, $(G(1) > 0)$ iff $\psi > 1$, $(\psi < 1)$. Consequently, $G(1)$ and $G^*(1)$ are the same (opposite) in sign iff $\gamma < 1/\psi$ ($\gamma > 1/\psi$). It then follows that $\omega > 1$ ($\omega < 1$) iff $\gamma > 1/\psi$ ($\gamma < 1/\psi$), assuming that $\psi \neq 1$.

Similarly, when $\psi = 1$, if we assume that $V_i > 0$ for $i \in \{R, E\}$, then we can prove that: $\omega > 1$ if $\gamma > 1$ ($\gamma < 1$) and $g_i - \frac{1}{2} \gamma \sigma_{C,i}^2$, $i \in \{R, E\}$ are of the same (opposite) sign. Now, if $\gamma < 1$, then $\pi_R + \gamma \sigma_{C,R}^2 - \mu_{C,R} > 0$ implies $\mu_{C,R} - \frac{1}{2} \gamma \sigma_{C,R}^2 > 0$, which means $\mu_{C,i} - \frac{1}{2} \gamma \sigma_{C,i}^2$, $i \in \{R, E\}$ cannot be of opposite sign. Therefore, $\omega > 1$ iff $\gamma > 1$.

So, for $\psi > 0$, $\omega > 1$ ($\omega < 1$) iff $\gamma > 1/\psi$ ($\gamma < 1/\psi$). It follows that $\omega = 1$ iff $\gamma = 1/\psi$.

**OA.E Liquidation Value**

The abandonment or liquidation value of a firm is just its unlevered value, i.e. the present value of future cashflows, ignoring coupon payments to debtholders and default risk. Small, but frequent shocks to a firm’s real cashflow growth are modelled by changes in the standard Brownian motion $W_t$. Small, but frequent shocks to the real SDF are modelled by changes in the standard Brownian motion $Z_t$. The assumption that $dZ_t dW_t = 0$ means that small, but frequent shocks to cashflow growth are not priced. However, changes in the expected real cashflow growth rate are driven by the same Markov chain as those driving jumps in the SDF. Hence, changes in unlevered firm value driven by changes in the expected real cashflow growth rate will be priced.

Suppose the economy is currently in state $i$. Then, the risk-neutral probability of the economy switching into a different state $j \neq i$ during a small time interval of length $\Delta t$ is $\lambda_{ij} \Delta t$ and the risk-neutral probability of not switching is $1 - \lambda_{ij} \Delta t$. We can therefore write the unlevered nominal firm value in state $i$ as

$$A^S_{i,t} = (1 - \eta) X_i \Delta t + e^{-r^S_{i,t} \mu_{X,i} \Delta t} \left[ (1 - \lambda_{ij} \Delta t) A^S_i + \sum_{j \neq i} \lambda_{ij} \Delta t A^S_j \right], \quad i, j \in \{1, \ldots, N\}, \quad j \neq i, \quad (OA.68)$$

where $N = 6$ is the number of states in the economy.

The first term in (OA.68) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The discount rate is just the standard discount rate for a perpetuity. Observe that the volatility of cashflow growth does not appear in the discount rate, because $dZ_t dW_t = 0$. The continuation value is the average of $A^S_{i,t}$ and $A^S_{j,t}$, weighted by the risk-neutral probabilities of being in states $i$ and $j \neq i$ a small instant $\Delta t$ from now. For example, with risk-neutral probability $\lambda_{ij} \Delta t$ the economy will be in state $j \neq i$ and the unlevered nominal firm value will be value will be $A^S_{j,t}$. The continuation value is discounted back at a rate reflecting the nominal interest rate rate $r^S_{i,t}$ and the expected nominal earnings growth rate over that instant which is $\mu_{X,i}$.
observe that there is no difference between the physical and risk-neutral nominal earnings growth rates, because \(dZ_t dW_t = 0\).

We take the limit of (OA.68) as \(\Delta t \to 0\), to obtain

\[
0 = (1 - \eta) X - (r_i^s - \mu_{X,i}) A_i^s + \sum_{j \neq i} \lambda_{ij} \left( A_j^s - A_i^s \right), \quad i \in \{1, \ldots, N\}, j \neq i.
\]

To obtain the solution of the above linear equation system, we define

\[
v_{A,i} = \frac{1}{1 - \eta} A_i^s X,
\]

the before-tax nominal price-earnings ratio in state \(i\). Therefore

\[
\begin{pmatrix}
\text{diag} \left( r_1^s - \mu_{X,1}, \ldots, r_N^s - \mu_{X,N} \right) - \hat{\Lambda} \end{pmatrix} \begin{pmatrix}
v_{A,1} \\
v_{A,2} \\
\vdots \\
v_{A,N}
\end{pmatrix} = 1_{N \times 1},
\]

where \(1_{N \times 1}\) is a \(6 \times 1\) vector of ones, \(\text{diag} \left( r_1^s - \mu_{X,1}, \ldots, r_N^s - \mu_{X,N} \right)\) is a \(N \times N\) diagonal matrix, with the quantities \(r_1^s - \mu_{X,1}, \ldots, r_N^s - \mu_{X,N}\) along the diagonal and \(\hat{\Lambda}\), defined by \([\hat{\Lambda}]_{ij} = \hat{\lambda}_{ij}, i, j \in \{1, \ldots, N\}\), where

\[
\hat{\lambda}_{ij} = \omega_{ij} \lambda_{ij}, j \neq i \quad (OA.70)
\]

\[
\hat{\lambda}_{ii} = - \sum_{j \neq i} \omega_{ij} \lambda_{ij}, j \neq i \quad (OA.71)
\]

is the generator matrix of the Markov chain for the combined state of the economy under the risk-neutral measure. Solving (OA.69) gives (26), if \(\det \left( \text{diag} \left( r_1^s - \mu_{X,1}, \ldots, r_N^s - \mu_{X,N} \right) - \hat{\Lambda} \right) \neq 0\).

Similarly, we can show that the before-tax value of the claim to the real earnings stream \(Y\), when the current state is \(i\) is given by \(P_{i,t}^Y = p_i Y_t\), where

\[
\begin{pmatrix}
p_1 \\
p_2 \\
\vdots \\
p_N
\end{pmatrix}^\top = \left( \text{diag} \left( r_1^s - \mu_{Y,1}, \ldots, r_N^s - \mu_{Y,N} \right) - \hat{\Lambda} \right)^{-1} 1_{6 \times 1}
\]

Hence, from the basic asset pricing equation

\[
E_t \left[ \frac{dP_t^Y}{P_t^Y} + Y dt \right] - r_{\nu_\tau} dt \left| \nu_\tau = i \right. = -E_t \left[ \frac{d\pi_t}{\pi_t} \frac{dP_t^Y}{P_t^Y} \right] \left| \nu_\tau = i \right.,
\]

we obtain the unlevered risk premium:

\[
E_t \left[ \frac{dP_t^Y}{P_t^Y} + Y dt \right] - r_{\nu_\tau} dt \left| \nu_\tau = i \right. = -\sum_{j \neq i} \left( \hat{\lambda}_{ij} - \lambda_{ij} \right) \left( \frac{p_j}{p_i} - 1 \right) dt, i, j \in \{1, \ldots, M\},
\]

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where $M = 2$ is the number of real states in the economy. Applying Ito’s Lemma,

$$dP^X_{i,t} = p_i dX_t + \sum_{j \neq i} \lambda_{ij} (p_j - p_i) dt + \sum_{j \neq i} (p_j - p_i) dN^P_{ij,t}, \quad i, j \in \{1, \ldots, M\},$$

Thus, the volatility of returns on unlevered equity in state $i$ is given by

$$\sigma_{R,i} = \sqrt{\sum_{j \neq i} \lambda_{ij} \left(\frac{p_j}{p_i} - 1\right)^2}, \quad i, j \in \{1, \ldots, M\}.$$ 

### OA.F Arrow-Debreu Securities – Default

The Arrow-Debreu default claim denoted by $q_{D,ij}$ is the value of a unit of consumption paid if default occurs in state $j$ and the current state is $i$. In a static capital structure model, these are the only Arrow-Debreu claims needed. Given the initial state (in which the firm selected its capital structure, there are $N^2$ such claims: \{q_{D,ij}\}_{i,j \in \{1,\ldots,N\}}. We assume, without loss of generality, that the regimes are labelled so that the default boundaries respect a monotonic ordering $X_{D,1} > \ldots > X_{D,N}$.

We say that a firm’s earnings are in the default region $D_k, k = 0, \ldots, N - 1$, when they fall in the interval $(X_{D,k+1}, X_{D,k}]$, assuming that $X_{D,0} \rightarrow \infty$. Region $D_N$ is $(-\infty, X_{D,N}]$.

**Proposition OA.2** Let $A_k$ be

$$A_k = \begin{pmatrix} 0_{N-k \times N-k} & -I_{N-k \times N-k} \\ 2S^{-1}_{x,k}(\Lambda_k - R_k) & 2S^{-1}_{x,k}M_{x,k} \end{pmatrix},$$

where $0_{n \times m} \in \mathbb{R}^{n \times m}$ denotes a matrix of zeros, $I_{n \times n} \in \mathbb{R}^{n \times n}$ denotes the $n$-dimensional identity matrix, $\Lambda_k, R_k, M_{x,k}$, and $S_{x,k}$ are the $N-k$ by $N-k$ matrices obtained by removing the first $k$ rows and columns of $\Lambda$,

$$R = \text{diag}(r_1, \ldots, r_N), \quad M_x = \text{diag}(\mu_{x,1}, \ldots, \mu_{x,N}), \quad \text{and} \quad S_x = \text{diag}(\sigma^2_{x,1}, \ldots, \sigma^2_{x,N}),$$

with $\tilde{\mu}_{x,i} = \tilde{\mu}_x - \frac{1}{2} \sigma^2_{X,i}$ and $\sigma_{x,i} = \sigma_{X,i}$ the drift and diffusion coefficient of $x = \log X$ under the risk-neutral measure.

Given the integration constants $h_{i,j}(\omega)$, the default Arrow-Debreu in region $D_k$ are given by

**Region $D_0$:**

$$q_{D,ij}(x) = \sum_{l=1}^{N} h_{ij}(\omega_{0,l}) e^{-\omega_{0,l}x}, \quad \text{(OA.73)}$$

where $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the $N$ positive eigenvalues of $A_0$. 

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Region $\mathcal{D}_k, k \in \{1, \ldots, N - 1\}$:
\[
q_{D,i,j}(x) = \delta_{ij}, \quad i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\},
\]
\[
q_{D,i,j}(x) = \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l)e^{-\omega_l x} - [A^{-1}_k B_k]_{i-k,j}, \quad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.
\]
\]
\[
\text{(OA.74)}
\]
where $\omega_{k,l}$ are the $2(N-k)$ eigenvalues of $A_k$ and
\[
B_k = \begin{pmatrix}
\begin{array}{cc}
0_{N-k \times k} & 0_{N-k \times N-k}
\end{array}
\end{pmatrix}, \quad B_k = \begin{pmatrix}
\begin{array}{ccc}
\frac{2}{\sigma_{k+1}} & \frac{2}{\sigma_{k+1}} & \frac{2}{\sigma_{k+1}}
\end{array}
\end{pmatrix}.
\]
Region $\mathcal{D}_N$:
\[
q_{D,i,j}(x) = \delta_{ij}, \quad \forall i, j.
\]
\]
\[
\text{(OA.75)}
\]
In each default region, for each $\omega$, the integration constants $h_{k+1,1}(\omega) \equiv [h_{k+1,j}(\omega)]_{j=1,\ldots,N} \in \mathbb{R}^{1 \times N}$, are identified by the boundary conditions (Section OA.F.3), and the remaining integration constants
\[
H_k(\omega) = \begin{pmatrix}
\begin{array}{ccc}
h_{k+2,1}(\omega) & \cdots & h_{k+2,N}(\omega)
\end{array}
\end{pmatrix},
\]
\]
\[
\text{(OA.76)}
\]
are given by
\[
H_k(\omega) = G_k^{-1}(\omega) \ g_{k+1,1}(\omega) \ h_{k+1,1}(\omega)
\]
\]
\[
\text{(OA.77)}
\]
where $g_{k+1,k+1}(\omega) \equiv [g_{i,k+1}(\omega)]_{i=k+2,\ldots,N} \in \mathbb{R}^{(N-k-1) \times 1}$ comprises the last $N-k-1$ elements of the first column of
\[
G(\omega) = 2S^{-1}_x(\Lambda - R) - \omega(2S^{-1}_x M_x - \omega I_{N \times N}).
\]
\]
\[
\text{(OA.78)}
\]
and $G_k(\omega)$ is the $N-k-1$ by $N-k-1$ matrix obtained by removing the first $k+1$ rows and columns of $G(\omega)$.

The next two subsections outline the proof of Proposition OA.2.
OA.F.1 Region $D_0$: $X_t > X_{D,1}$

We start by analyzing the case where earnings at the current date $t$ are above the highest default boundary, i.e. $X_t > X_{D,1}$. Hence, if earnings hit the boundary $X_{D,j}$ from above for the first time in state $j$, \( \{q_{D,ij}\}_{i,j\in\{1,...,N\}} \) will pay one unit of consumption; otherwise, the security expires worthless.

Since each Arrow-Debreu default claim is effectively a perpetual digital put, their values can be derived by solving a system of ordinary differential equations, derived from the standard equations:

\[
E_t^Q[dq_{D,ij} - r_i q_{D,ij} dt] = 0, \ i,j \in \{1,\ldots,N\}.
\] (OA.79)

Using Ito’s Lemma, the above equation can be rewritten as the following second-order ordinary differential-equation system:

\[
\frac{1}{2} \sigma_{x,i}^2 \frac{d^2 q_{D,ij}}{dx^2} + \mu_{x,i} \frac{dq_{D,ij}}{dx} + \sum_{k \neq i} \lambda_{ik} (q_{D,kj} - q_{D,ij}) = r_i q_{D,ij}, \ i,j \in \{1,\ldots,N\},
\] (OA.80)

where $\mu_{x,i} = \tilde{\mu}_{X,i} - \frac{1}{2} \sigma_{X,i}^2$ and $\sigma_{x,i} = \sigma_{X,i}$ are the drift and diffusion coefficient of $x = \log X$ under the risk-adjusted measure.

In order to solve this system of ODEs, define

\[
 z_{ij} = q_{D,ij}, \ i,j \in \{1,\ldots,N\}
\] (OA.81)

\[
 z_{N+i,j} = \frac{dq_{D,ij}}{dx}, \ i,j \in \{1,\ldots,N\}.
\] (OA.82)

Then, we obtain the following first order linear system

\[
 \frac{dz_{ij}}{dx} - z_{N+i,j} = 0, \ i,j \in \{1,\ldots,N\},
\] (OA.83)

\[
 \frac{dz_{N+i,j}}{dx} + 2\frac{\tilde{\mu}_{x,i}}{\sigma_{x,i}^2} z_{N+i,j} + \sum_{k \neq i} 2\frac{\tilde{\lambda}_{ik}}{\sigma_{x,i}^2} (z_{kj} - z_{ij}) - 2r_i \frac{1}{\sigma_{x,i}^2} z_{ij} = 0, \ i,j \in \{1,\ldots,N\}.
\]

Expressing the above equation system in matrix form gives

\[
 Z' + A_0 Z = 0_{2N \times N},
\] (OA.84)

where the $ij$’th element of the $2N$ by $N$ matrix, $Z$, is

\[
 [Z]_{ij} = z_{ij}, \ i \in \{1,\ldots,2N\}, j \in \{1,\ldots,N\},
\] (OA.85)

and $Z' = \frac{dZ}{dx}$.

\[29\) Note that since the puts are perpetual, $\frac{\partial q}{\partial t} = 0$. Hence, $q$ is solely a function of the stochastic process $x = \log X$.\]
To solve eq. (OA.84), one first finds the eigenvectors and eigenvalues of $A_0$. Their defining equation is

$$A_0 \varepsilon_i = \omega_i \varepsilon_i, \ i \in \{1, \ldots, 2N\}, \tag{OA.86}$$

where $\omega_i$ is the $i$'th eigenvalue and $\varepsilon_i$ is the corresponding eigenvector. Note that $A_0$ has $N$ positive and $N$ negative eigenvalues (Jobert and Rogers (2006)).

It follows from (OA.86) that the eigenvalues of $A_0$ are the roots of its characteristic polynomial; that is, any eigenvalue $\omega$ is a solution to the following $2N$'th-order polynomial:

$$\det(A_0 - \omega I) = 0.$$ 

To simplify the above expression for the characteristic polynomial, we then use the following identity from Silvester: If $F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$, where $F_{ij}, i, j \in \{1, 2\}$ are $N$ by $N$ matrices, any two of which commute with each other, then

$$\det F = \det(F_{11}F_{22} - F_{12}F_{21}). \tag{OA.87}$$

Since

$$A_0 - \omega I = \begin{pmatrix} -\omega I & -I \\ 2S_x^{-1}(\hat{\Lambda} - R) & 2S_x^{-1}M_x - \omega I \end{pmatrix}$$

and diagonal matrices commute with all other matrices of the same size, any pair of the $N$ submatrices in $A_0$ commute. Therefore, one can apply (OA.87) and

$$0 = \det(A_0 - \omega I) = \det(\omega(\omega I - 2S_x^{-1}M_x) - 2S_x^{-1}(\hat{\Lambda} - R)). \tag{OA.88}$$

When $N \leq 2$ the above polynomial is of order 4 or less and can be solved exactly in closed-form. When $N \geq 3$, it must be solved numerically. Once the eigenvalues have been obtained, the eigenvectors are obtained by solving (OA.86). We then define the $2N$ by $N$ matrix of eigenvectors, $E$, by stacking the eigenvectors as follows

$$E = (\varepsilon_1, \ldots, \varepsilon_{2N}).$$

Hence, the $ij$'th component of $E$ is the $i$'th element of the $j$'th eigenvector, i.e.

$$E_{ij} = (\varepsilon_j)_i.$$ 

Given the $E$ matrix, we can define the $2N$ by $N$ matrix $W$ via

$$EW = Z. \tag{OA.89}$$
We can then rewrite eq. (OA.84) as

\[ EW' + A_0 EW = 0_{2N \times N}, \]  

\[ \iff E^{-1}(EW' + A_0 EW) = W' + E^{-1}A_0 EW = 0_{2N \times N}, \tag{OA.90} \]

\[ \iff W' + DW = 0_{2N \times N}, \tag{OA.91} \]

where

\[ D = E^{-1}A_0 E = \text{diag}(\omega_1, \ldots, \omega_{2N}). \tag{OA.93} \]

The first order differential equation system of eq. (OA.92) is similar to that of eq. (OA.84), with the notable difference that \( D \) is a diagonal matrix while \( A_0 \) isn’t. Making use of this, we premultiply both sides of eq. (OA.92) by the integrating factor \( e^{Dx} \), which yields

\[ e^{Dx}W' + e^{Dx}DW = (e^{Dx}W)' = 0_{2N \times N}. \]

Integrating the above equation gives

\[ e^{Dx}W = K, \]

where \( K \) is a \( 2N \) by \( N \) matrix of constants of integration. Therefore, the general solution of eq. (OA.92) is

\[ W = e^{-Dx}K, \]

which, given eq. (OA.89), implies

\[ Z = Ee^{-Dx}K = e^{-Dx}EK, \tag{OA.94} \]

given that \( D \) is \( 2N \times 2N \) and diagonal, and that \( E \) is \( 2N \times 2N \).

Thus,

\[ q_{D,ij}(x) = \sum_{l=1}^{2N} h_{ij}(\omega_l)e^{-\omega_l x}, \tag{OA.95} \]

where the \( h_{ij}(\omega_l) \) are constants of integration that depend on the eigenvalues.

Note that, for any eigenvalue \( \omega \) of \( A_0 \), the solution \( q_{D,ij} = h_{ij}(\omega)e^{-\omega x} \), with \( h_{ij}(\omega_l) = 0, \forall \omega_l \neq \omega \), solves (OA.95). Indeed, we then have

\[ Z = \begin{bmatrix} H(\omega) \\ -\omega H(\omega) \end{bmatrix} e^{-\omega x}, \text{ where } H(\omega) = \begin{bmatrix} h_{1,1}(\omega) \\ h_{1,2}(\omega) \end{bmatrix} \]

and

\[ Z' = -\omega Z. \]

Hence, (OA.84) implies that

\[ (-\omega I_{2N \times 2N} + A_0)Z = 0_{2N \times N}, \]
or, equivalently,
\[
\begin{pmatrix}
-\omega I_{N \times N} & -I_{N \times N} \\
2S_x^{-1}(\hat{A} - R) & 2S_x^{-1}M_x - \omega I_{N \times N}
\end{pmatrix}
\begin{pmatrix}
H(\omega) \\
-\omega H(\omega)
\end{pmatrix}
= 0_{2N \times N}.
\tag{OA.96}
\]

This particular solution is important since it allows us to express \(N(N-1)\) of the \(N^2\) integration constants in terms of the first \(N\) ones. Indeed, simplifying (OA.96) gives
\[
-\omega I_{N \times N}H(\omega) + I_{N \times N}H(\omega) = 0_{N \times N},
\]
\[
(2S_x^{-1}(\hat{A} - R) - \omega(2S_x^{-1}M_x - \omega I_{N \times N}))H(\omega) = 0_{N \times N},
\]
where the first equation is trivial. To solve the second equation, we first consider
\[
G(\omega)(h_{1j}(\omega),...,h_{Nj}(\omega))^T = 0_{N \times 1},
\tag{OA.97}
\]
where
\[
G(\omega) = 2S_x^{-1}(\hat{A} - R) - \omega(2S_x^{-1}M_x - \omega I_{N \times N}).
\]
We denote the \(ij\)'th element of \(G(\omega)\) by \(g_{ij}(\omega)\). We know from (OA.88) that \(\det G(\omega) = 0\). Thus, the equations
\[
\sum_{k=1}^{N} g_{ik}(\omega)h_{kj}(\omega), \ i \in \{1,...,N\}
\tag{OA.98}
\]
are linearly dependent. However, the system
\[
\sum_{k=1}^{N} g_{ik}(\omega)h_{kj}(\omega), \ i \in \{2,...,N\}
\tag{OA.99}
\]
is linearly independent, allowing us to solve for \(h_{kj}(\omega), k \in \{2,...,N\}\) in terms of \(h_{1j}(\omega)\), for \(j \in \{1,...,N\}\), that is
\[
H_k(\omega) = G_k^{-1}(\omega) g_{k+1,1}(\omega) h_{k+1,1}(\omega).
\]
Solving the above linear equation system gives us \(h_{ij}(\omega), i \in \{2,...,N\}, j \in \{1,...,N\}\) in terms of \(h_{1j}(\omega), j \in \{1,...,N\}\). Hence, for each eigenvalue \(\omega\) of \(A_0\), this leaves us with \(N\) free integration constants. Then, to ensure the finiteness of the Arrow-Debreu prices as \(x \to \infty\), focus on the \(N\) positive eigenvalues of \(A_0\). That is, we obtain \(q_{D,ij}\), the Arrow-Debreu prices in region \(\mathcal{D}_0\), where \(X > X_{D,1}\):
\[
q_{D,ij}(x) = \sum_{l=1}^{N} h_{ij}(\omega_{0,l})e^{-\omega_{0,l}x},
\tag{OA.100}
\]

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where, without loss of generality, $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the $N$ positive eigenvalues of $A_0$. Note that (OA.100) contains $N^2$ free integrations constants $h_{ij}(\omega_{0,l})$, which will be identified by the value and smooth pasting conditions (Section OA.F.3).

**OA.F.2 Region $D_k$:** $X_{D,k+1} < X \leq X_{D,k}$

We now turn to the analysis of Arrow-Debreu securities in region $D_k$, i.e. when current earnings are above default boundary $k+1$, but below default boundary $k$. Note that the above analysis in default region $k=0$ can be seen as a special case of the analysis below, with $X_{D,0} \to \infty$.

First, note that $q_{D,ij} = \delta_{ij}, \forall i \leq k$. Indeed, if earnings are currently lower than $X_{D,k} < X_{D,k-1} < \ldots$, and if the current state is $i \leq k$, then the firm is in default and the present value of a dollar when the firm defaults in state $j$ is 1 if $i = j$, and 0 otherwise. In particular, this means that, in region $D_N$, where $X \leq X_{D,N}$, we have

$$q_{D,ij} = \delta_{ij}.$$

Applying (OA.79) to the unknown $q_{D,ij}, i > k$, yields the following system of ODEs

$$\frac{dz_{k+1,j}}{dx} - z_{N+k+1,j} = 0,$$

$$\frac{dz_{k+2,j}}{dx} - z_{N+k+2,j} = 0,$$

$$\vdots$$

$$\frac{dz_{N,j}}{dx} - z_{2N,j} = 0,$$

$$\frac{dz_{N+k+1,j}}{dx} + \frac{2\mu_{x,k+1}}{\sigma^2_{x,k+1}} z_{N+k+1,j} + \sum_{l=1}^{k} \frac{2\lambda_{k+1,l}}{\sigma^2_{x,k+1}} (\delta_{lj} - z_{k+1,j})$$

$$+ \sum_{l=k+2}^{N} \frac{2\lambda_{k+1,l}}{\sigma^2_{x,k+1}} (z_{lj} - z_{k+1,j}) - \frac{2r_{k+1}}{\sigma^2_{x,k+1}} z_{k+1,j} = 0,$$

$$\frac{dz_{N+k+2,j}}{dx} + \frac{2\mu_{x,k+2}}{\sigma^2_{x,k+2}} z_{N+k+2,j} + \sum_{l=1}^{k} \frac{2\lambda_{k+2,l}}{\sigma^2_{x,k+2}} (\delta_{lj} - z_{k+2,j})$$

$$+ \sum_{l=k+1,l\neq k+2}^{N} \frac{2\lambda_{k+2,l}}{\sigma^2_{x,k+2}} (z_{lj} - z_{k+2,j}) - \frac{2r_{k+2}}{\sigma^2_{x,k+2}} z_{k+2,j} = 0,$$

$$\vdots$$

$$\frac{dz_{2N,j}}{dx} + \frac{2\mu_{x,N}}{\sigma^2_{x,N}} z_{2N,j} + \sum_{l=1}^{k} \frac{2\lambda_{N,l}}{\sigma^2_{x,N}} (\delta_{lj} - z_{N,j}) + \sum_{l=k+1}^{N-1} \frac{2\lambda_{N,l}}{\sigma^2_{x,N}} (z_{lj} - z_{N,j}) - \frac{2r_{N}}{\sigma^2_{x,N}} z_{N,j} = 0,$$
for $j = \{1, \ldots, N\}$. Rewriting the above equation system in matrix form, we obtain

$$Z_k' + A_k Z_k + B_k = Z_k' + A_k (Z_k + A_k^{-1}B_k) = \tilde{Z}_k' + A_k \tilde{Z}_k = 0,$$  (OA.101)

where $\tilde{Z}_k = (Z_k + A_k^{-1}B_k)$, $Z_k$ is the following $2(N-k)$ by $N$ matrix

$$Z_k = \begin{pmatrix}
z_{k+1,1} & z_{k+1,2} & \cdots & z_{k+1,N} 
z_{k+2,1} & z_{k+2,2} & \cdots & z_{k+2,N} 
\vdots & \vdots & \ddots & \vdots 
z_{N,1} & z_{N,2} & \cdots & z_{N,N} 
z_{N+k+1,1} & z_{N+k+1,2} & \cdots & z_{N+k+1,N} 
z_{N+k+2,1} & z_{N+k+2,2} & \cdots & z_{N+k+2,N} 
\vdots & \vdots & \ddots & \vdots 
z_{2N,1} & z_{2N,2} & \cdots & z_{2N,N}
\end{pmatrix}.$$

Note that the $B_k$ matrix of constants arises from the $\delta_{lj}$’s appearing in the above differential equations:

(i) These appear only in the last $N-k$ equations. Hence, the first $N-k$ rows of the $B_k$ matrix will comprise of zeros.

(ii) Since the sum in which the $\delta_{lj}$’s appear are from 1 to $k$, $\delta_{lj}$ will be zero for all $l$ whenever $j > k$. Hence, the last $N-k$ columns of the $B_k$ matrix will comprise of zeros.

Thereafter, the development made, in region $\mathcal{D}_0$, between equations (OA.86) and (OA.94) can be applied to $\tilde{Z}_k$ in (OA.101) to yield

$$\tilde{Z}_k = e^{-D_k x} E_k K_k,$$  (OA.102)

or, equivalently,

$$Z_k = e^{-D_k x} E_k K_k - A_k^{-1} B_k.$$  (OA.103)

Therefore,

$$q_{D,ij}(x) = \delta_{ij}, i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\}$$

$$q_{D,i,j}(x) = \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l) e^{-\omega_l x} - [A_k^{-1}B_k]_{i-k,j}, i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.$$  

Note that the $-k$ offset on the rows of the $2(N-k) \times N$ matrix $A_k^{-1}B_k$ simply accounts for the fact the first row of this matrix corresponds to the $(k+1)^{th}$ Arrow-Debreu security. Once more, for each
eigenvalue $\omega$ of $A_k$, the particular solution

$$Z_k = \left( \frac{H_k(\omega)}{-\omega H_k(\omega)} \right) e^{-\omega y} - A_k^{-1} B_k,$$

where

$$H_k(\omega) = \left( \frac{h_{k+1}(\omega)}{H_k(\omega)} \right),$$

can be used to express the constants in all lines of $H_k$ but the first, as functions of the $h_{k+1,1}, \ldots, h_{k+1,N}$ constants. This leaves us with $N$ free constants for each of the $2(N-k)$ eigenvalues.

**OA.F.3 Boundary Conditions for Arrow-Debreu Default Claims**

Given the above development, we are left with $N^2$ free integration constants in region $D_0$, and $2(N-k)N$ free constants in region $D_k, k \in \{1, \ldots, N-1\}$. Hence, we still have to solve for the $N^3$ constants,

$$N^2 + \sum_{k=1}^{N-1} 2(N-k)N = N^2 + 2N \sum_{k=1}^{N-1} (N-k) = N^3,$$

that satisfy the $N^3$ boundary conditions (value matching & smooth pasting) of the problem at hand. For each default boundary $X_{D,k}, k \in \{1, \ldots, N\}$, we have that:

(VM) The value of the $N + (N-k)N$ Arrow-Debreu securities with $i \geq k$, must be the same on both sides of the default boundary, i.e.

$$q_{D_{k-1},ij}(x_{D,k}) = q_{D_k,ij}(x_{D,k}), \text{ where } i \in \{k, \ldots, N\}, j \in \{1, \ldots, N\}; \quad (OA.105)$$

(SP) The dynamics of the $(N-k)N$ Arrow-Debreu securities with $i > k$, must be the same on both side of the default boundary, i.e.

$$\left. \frac{dq_{D_{k-1},ij}}{dx} \right|_{x_{D,k}} = \left. \frac{dq_{D_k,ij}}{dx} \right|_{x_{D,k}}, \text{ where } i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}; \quad (OA.106)$$

where the $q_{D_k}$ notation highlights that the Arrow-Debreu claim is computed in the $D_k$ region.

**OA.G Modified Arrow-Debreu Default Claims**

Arrow-Debreu securities provide the expected value of a 1$ cash flow conditional on the state of the world in which they occur. In particular, in region $D_k$ at time $t$, Arrow-Debreu prices

$$q_{D,ij,t}(x) = E_t \left[ \frac{\pi_{\nu_D}}{\pi_t} 1_{\nu_D = j} | \nu_t = i \right] \quad (OA.107)$$

$$= E_t^{Q} \left[ 1_{\nu_D = j} | \nu_t = i \right] \quad (OA.108)$$

$$= Q \left[ \nu_D = j | \nu_t = i \right], \quad (OA.109)$$

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can be interpreted risk-adjusted probability that default, occurring at unknown time $\tau_D$, will occur in state $j$ conditional on the current state of the economy, $\nu_t$, being $i$. For cash flows that do not depend on the level of earnings when the firm defaults, $X_{\tau_D} = e^{x_{\tau_D}}$, these Arrow-Debreu securities yield a straightforward approach to derive the cash flows’ expected values. If a cash flow does depend on $X_{\tau_D}$, the Arrow-Debreu securities may not be as useful.

In a continuous model, the earnings always approach the default boundary from above and default occur when $X_{\tau_D} = X_D$; that is, there is no uncertainty with respect to the level of earnings upon default and Arrow-Debreu securities can readily be used to compute expected cash flows. In our economy, however, “deep defaults” can occur when the state of the economy jumps from its current state to a worse state.

Recall that we ordered the default boundaries such that $X_{D,1} > \ldots > X_{D,N}$; hence, state $N$ is the best state of the economy, state 1 is the worst. When the state of the economy jumps toward a better state, the default boundary decreases as growth opportunities improve; hence, if the firm was not in default, it is even further away from default after the jump. However, if the level of earnings is $X_{D,j+1} < X_{\tau_D} \leq X_{D,j}$ prior to a jump to state $j$ at time $\tau_D$, the firm automatically defaults. The level of earnings $X_{\tau_D}$ is then only a fraction of the default boundary $X_{D,j}$, and the firm will thus be able to honor its obligations to the debtholders, for instance, only partially.

We thus introduce “modified” Arrow-Debreu securities

$$
\tilde{q}_{D,ij,t}(x) = E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} X_{\tau_D}^1_{\nu_{\tau_D}=i | \nu_t = j} \right],
$$

$$
= E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} e^{x_{\tau_D}-x_{D,j}^1_{\nu_{\tau_D}=i | \nu_t = j}} \right],
$$

$$
= E_t^Q \left[ e^{x_{\tau_D}-x_{D,j}^1_{\nu_{\tau_D}=i | \nu_t = j}} \right],
$$

to account for the uncertainty surrounding the recovery rate. Note that, as long as shareholders have no bargaining power, they shouldn’t care about the depth of deep defaults.

Technically, the (standard) Arrow-Debreu securities are special cases of their modified counterparts, with $\frac{x_{\tau_D}}{x_{D,j}} = 1$. Moreover, when in region $D_0$, deep defaults are not a direct concern as the firm would survive even to a jump to the worse state, state 1. Hence, the general solution in (OA.100) holds. However, in region $D_k, k > 0$, applying (OA.79) to the unknown $\tilde{q}_{D,ij}, i > k$, accounting for deep defaults, yields the following system of ODEs

$$
\frac{dz_{i,j}}{dz_{N+i,j}} - z_{N+i,j} = 0,
$$

$$
\frac{dz_{N+i,j}}{dx} + \frac{2\hat{\mu}_{x,i}}{\sigma_{x,i}^2} z_{N+i,j} + \sum_{l=1}^{k} \frac{2\hat{\lambda}_{i,l}}{\sigma_{x,i}^2} (e^{x-x_{D,j}^l} \delta_{lj} - z_{i,j})
$$

$$
+ \sum_{l=k+1, l \neq i}^{N} \frac{2\hat{\lambda}_{i,l}}{\sigma_{x,i}^2} (z_{l,j} - z_{i,j}) - \frac{2r_j}{\sigma_{x,i}^2} z_{i,j} = 0,
$$

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with \( i \in \{k+1, \ldots, N\} \) and \( j \in \{1, \ldots, N\} \). This can be written equivalently in matrix form as

\[
Z'_k + A_k Z_k + \widetilde{B}_k = 0, \quad \text{(OA.113)}
\]

where

\[
\widetilde{B}_k = \begin{pmatrix}
0_{N-k \times k} & 0_{N-k \times N-k} \\
\overline{B}_k^0 & 0_{N-k \times N-k}
\end{pmatrix},
\]

and

\[
\overline{B}_k^0 = \begin{pmatrix}
2\frac{\hat{\lambda}_{k+1,1}}{\sigma_{k+1}} e^{x-x_{D,1}} & 2\frac{\hat{\lambda}_{k+1,2}}{\sigma_{k+1}} e^{x-x_{D,2}} & \cdots & 2\frac{\hat{\lambda}_{k+1,k}}{\sigma_{k+1}} e^{x-x_{D,k}} \\
2\frac{\hat{\lambda}_{k+2,1}}{\sigma_{k+2}} e^{x-x_{D,1}} & 2\frac{\hat{\lambda}_{k+2,2}}{\sigma_{k+2}} e^{x-x_{D,2}} & \cdots & 2\frac{\hat{\lambda}_{k+2,k}}{\sigma_{k+2}} e^{x-x_{D,k}} \\
\vdots & \vdots & \ddots & \vdots \\
2\frac{\hat{\lambda}_{N,1}}{\sigma_N} e^{x-x_{D,1}} & 2\frac{\hat{\lambda}_{N,2}}{\sigma_N} e^{x-x_{D,2}} & \cdots & 2\frac{\hat{\lambda}_{N,N}}{\sigma_N} e^{x-x_{D,N}}
\end{pmatrix}.
\]

Now, \( \overline{B}_k \) is not constant with respect to \( x \) anymore, but \( \overline{B}_k' = \overline{B}_k \). Hence, letting \( \overline{Z}_k = Z_k + (A_k + I)^{-1}\overline{B}_k \), we have

\[
\overline{Z}_k' + A_k \overline{Z}_k = Z_k' + (A_k + I)^{-1}\overline{B}_k + A_k Z_k + A_k (A_k + I)^{-1}\overline{B}_k
\]

\[
= Z_k' + A_k Z_k + \overline{B}_k = 0. \quad \text{(OA.115)}
\]

Once more, the development made between equations (OA.86) and (OA.94) can be applied to \( \overline{Z}_k \) in (OA.115) to yield

\[
\overline{Z}_k = e^{-D_k x} E_k K_k, \quad \text{(OA.116)}
\]

or, equivalently,

\[
Z_k = e^{-D_k x} E_k K_k - (A_k + I)^{-1}\overline{B}_k. \quad \text{(OA.117)}
\]

Therefore,

\[
\overline{q}_{D,ij}(x) = \delta_{ij} e^{x-x_{D,j}} = \delta_{ij} \frac{X}{X_{D_i}}, \quad i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\},
\]

\[
\overline{q}_{D, i, j}(x) = \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l) e^{-\omega_l x} - [(A_k + I)^{-1}\overline{B}_k]_{i-k,j}, \quad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.
\]

For \( 1 \leq i \leq k \), if the earnings at current time \( t \) are \( X_t < X_{D,i} \) while the current state is \( i \), it must be that the state just jumped to state \( i \) at time \( t^- \) and the firm is now in (deep) default, hence the first equation in the above system.
**OA.H    Bond Prices**

In this proof it is not necessary to distinguish between the state of the economy at dates \( t - \) and \( t \). The central part of our proof consists of proving that

\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^g}{\pi_t^g} ds \left| s_t = i \right. \right] = \frac{1}{r_{P,i}^g} - \sum_{j=1}^{N} \frac{q_{D,ij}^g}{r_{P,j}^g}, \tag{OA.118}
\]

where \( r_{P,i}^g \), the discount rate for a fixed nominal perpetuity, when the economy is in state \( i \), is given by

\[
r_{P,i}^g = \left( E_t \left[ \int_t^{\infty} \frac{\pi_s^g}{\pi_t^g} ds \left| s_t = i \right. \right] \right)^{-1}, \tag{OA.119}
\]

and

\[
E_t \left[ \frac{\tau_D}{\tau_D} A_{T_D}^g (X_{\tau_D}) \left| s_t = i \right. \right] = \sum_{j=1}^{N} A_{j}^g (X_{D,j}) q_{D,ij}^g. \tag{OA.120}
\]

To prove (OA.118), we note that

\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^g}{\pi_t^g} ds \left| s_t = i \right. \right] = E_t \left[ \int_t^{\infty} \frac{\pi_s^g}{\pi_t^g} ds \left| \nu_t = i \right. \right] - E_t \left[ \frac{\tau_D}{\pi_t^g} \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_t = i \right. \right],
\]

and conditioning on the event \( \{\nu_{\tau_D} = j\} \), we obtain

\[
E_t \left[ \frac{\pi_{\tau_D}^g}{\pi_t^g} \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_t = i \right. \right] = \sum_{j=1}^{N} E_t \left[ \Pr (\nu_{\tau_D} = j \left| s_t = i \right. ) \frac{\pi_{\tau_D}^g}{\pi_t^g} \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_{\tau_D} = j \right. \right].
\]

Since consumption is Markovian, so is the state-price density, which implies that

\[
E_t \left[ \Pr (s_{\tau_D} = j \left| s_t = i \right. ) \frac{\pi_{\tau_D}^g}{\pi_t^g} \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_t = i \right. \right] = E_t \left[ \Pr (s_{\tau_D} = j \left| s_t = i \right. ) \frac{\pi_{\tau_D}^g}{\pi_t^g} \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_{\tau_D} = j \right. \right].
\]

Therefore

\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^g}{\pi_t^g} ds \left| s_t = i \right. \right] = E_t \left[ \int_t^{\infty} \frac{\pi_s^g}{\pi_t^g} ds \left| s_t = i \right. \right] \tag{OA.121}
\]

\[
- \sum_{j=1}^{N} E_t \left[ \Pr (\nu_{\tau_D} = j \left| \nu_t = i \right. ) \frac{\pi_{\tau_D}^g}{\pi_t^g} \nu_t = i \right. \right] E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s^g}{\pi_{\tau_D}^g} ds \left| s_{\tau_D} = j \right. \right].
\]

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Conditional on being in state \(i\), the value of a claim which pays one risk-free unit of consumption in perpetuity is 
\[
E_t \left[ \int_t^\infty \frac{\pi_s^t}{\pi_t^t} ds \bigg| s_t = i \right],
\]
so the discount rate for this perpetuity, \(r_P^i\), is given by (OA.119). Consequently, (OA.121) implies
\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^t}{\pi_t^t} ds \bigg| s_t = i \right] = \frac{1}{r_P^i} - \sum_{j=1}^N E_t \left[ \Pr(s_t = i|s_{\tau_D} = j) \frac{\pi_s^t}{\pi_t^t} \bigg| s_t = i \right].
\]

To obtain (OA.118) from the above expression, we note that
\[
q_{D,ij,t}^s = E_t \left[ \Pr(s_{\tau_D} = j|s_t = i) \frac{\pi_{\tau_D}^t}{\pi_t^t} \bigg| s_t = i \right].
\]

To prove (OA.120), we condition on the event \(\{s_{\tau_D} = j\}\) to obtain
\[
\alpha E_t \left[ \frac{\pi_{\tau_D}^t}{\pi_t^t} A_{\tau_D}^s (X_{\tau_D}) \bigg| s_t = i \right] = \alpha \sum_{j=1}^N A_j^s (X_{D,j}) E_t \left[ \frac{X_{\tau_D}}{X_{D,j}} \Pr(s_{\tau_D} = s_j|s_t = i) \frac{\pi_{\tau_D}^t}{\pi_t^t} \bigg| s_t = i \right].
\]

Using (OA.123) to simplify the above expression we obtain (33).

**OA.I Equity risk premium and equity volatility**

Applying Ito’s Lemma to \(S_{i,t}^s\) gives
\[
\frac{dS_{i,t}^s}{S_{i,t}^s} + (X_t - c)dt = \frac{S_{i,t}^s}{X_t} \frac{\partial S_{i,t}^s}{\partial X_t} dX_t + \frac{1}{2} \frac{S_{i,t}^s}{X_t^2} \left( \frac{dX_t}{X_t} \right)^2 + \sum_{j=1}^N \frac{S_{j,t}^s - S_{i,t}^s}{S_{i,t}^s} dN_{ij,t} + \frac{(X_t - c)dt}{S_{i,t}^s}, \quad i, j \in \{1, \ldots, N\}.
\]

Observe that
\[
\frac{\partial S_{i,t}}{\partial X_t} = (1 - \eta) \frac{1}{r_{A,i}} - \sum_{j=1}^N \left( A_j^s (X_{D,j}) \frac{\partial q_{D,ij,t}^s}{\partial X_t} - (1 - \eta) \frac{\partial q_{D,ij,t}^s}{\partial X_t} \frac{c}{r_{P,j}} \right), \quad i, j \in \{1, \ldots, N\}.
\]

Define the date-\(t\) conditional nominal expected return
\[
\mu_{R,i,t}^s = \frac{1}{dt} E_t \left[ \frac{dS_{s_{t-},t}^s}{S_{s_{t-},t}^s} \right. \left. \bigg| s_{t-} = i \right], \quad i \in \{1, \ldots, N\}, \quad \text{(OA.127)}
\]
and the date-$t$ conditional real expected return
\[
\mu_{R,i,t} = \frac{1}{dt} E_t \left[ \frac{dS_{s_{t-},t} + (Y_t - c/P_t)dt}{S_{s_{t-},t}} \right] s_{t-} = i, i \in \{1, \ldots, N\}. \tag{OA.128}
\]
Observe that $\mu_{R,i,t}$ depends on both real and nominal states, because of sticky leverage and sticky cash flows.

The basic asset pricing equation is
\[
\mu_{R,i,t} - \mu_{P,i} = -\frac{1}{dt} E_t \left[ \frac{d\pi_{s_t}^S dS_{s_{t-},t}}{\pi_{s_{t-}}^S S_{s_{t-},t}} \right] s_{t-} = i, i \in \{1, \ldots, N\}. \tag{OA.129}
\]
Hence
\[
\mu_{R,i,t} - \mu_{P,i} = \sum_{j \neq i} (1 - \omega_{ij}) \frac{S_j^S - S_i^S}{S_i^S} \lambda_{ij}, i,j \in \{1, \ldots, N\}. \tag{OA.130}
\]
Observe that because
\[
\mu_{R,i,t}^S = \mu_{R,i,t} + \mu_{P,i}, i \in \{1, \ldots, N\}, \tag{OA.131}
\]
and
\[
\mu_{P,i}^S = \mu_{P,i}, i \in \{1, \ldots, N\}, \tag{OA.132}
\]
we have
\[
\mu_{R,i,t} - \mu_{R,i,t}^S - \mu_{P,i} = \mu_{P,i}^S - \mu_{i,t}^S, i \in \{1, \ldots, N\}. \tag{OA.133}
\]
The unexpected stock return in state $i$ is given by
\[
\sum_{j \neq i} \sigma_{R,ij}^P dN_{ij,t}, i,j \in \{1, \ldots, N\}, \tag{OA.134}
\]
where
\[
\sigma_{R,ij}^P = \frac{S_j}{S_i} - 1, i,j \in \{1, \ldots, N\}. \tag{OA.135}
\]
Now the risk premium in state $i$ is
\[
\mu_{R,i} - \mu_i = \sum_{j \neq i} (\omega_{ij} - 1) \sigma_{R,ij}^P \lambda_{ij}, i,j \in \{1, \ldots, N\}. \tag{OA.136}
\]
First, note that if $\log S = \log f(X)$ and $x \equiv \log X$, then
\[
\frac{\partial \log S}{\partial \log X} = \frac{\partial f(e^x)}{\partial x} = \frac{1}{f(e^x)} f'(e^x) e^x = \frac{X}{S} f'(X). \tag{OA.137}
\]
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Second, recall that

\[ S_{i,t}^s = A_i^s(X_t) - (1 - \eta)v_{B,i}c - \sum_{j=1}^N \left( A_j^s(X_{D,j})q_{D,ij,t}^s(X_t) - (1 - \eta)q_{D,ij,t}^s(X_t)v_{B,i}c \right) \quad (OA.138) \]

\[ = (1 - \eta)X_t v_{A,i} - (1 - \eta)v_{B,i}c - \sum_{j=1}^N \left( A_j^s(X_{D,j})q_{D,ij,t}^s(X_t) - (1 - \eta)q_{D,ij,t}^s(X_t)v_{B,i}c \right), \quad (OA.139) \]

where we made explicit that the Arrow-Debreu prices, \( q_{D,ij,t}^s \), depend on the current value of \( X_t \). Hence,

\[ \frac{\partial \log S_{i,t}}{\partial \log X_t} = \frac{X_t}{S_{i,t}} \left[ \frac{A_i^s(X_t)}{X_t} - \sum_{j=1}^N \left( A_j^s(X_{D,j}) \frac{\partial q_{D,ij,t}^s}{\partial X_t} - (1 - \eta) \frac{\partial q_{D,ij,t}^s}{\partial X_t} v_{B,i}c \right) \right]. \quad (OA.140) \]