Abstract

Many organizations rely on donations of money, time, and effort to function. Here we consider how such organizations motivate donors who are concerned with “making a difference”. Its tension is between a donor’s desire to be marginally important against the firm’s desire to make important objectives less precarious. We show the factors that lead firms to first undertake unimportant objectives, deferring more important ones. We also show how these issues lead mission based organizations to become less focused than is technologically efficient and to potentially benefit from being failing to diagnose the returns to activities.

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Perhaps the most common way to encourage donations of time, effort, or money is to ask the donor to “make a difference”. Such exhortations are ubiquitous, and seen in workplaces, schools, charitable solicitations and a myriad of other settings. The focus of this paper is how organizations can encourage such contributions. Its central conclusion is that they are likely to benefit from inefficiently allocating their resources. For the main body of the paper, these inefficiencies take the form of failing to prioritize valuable opportunities over less valuable ones. The underlying logic is simple. Donors give more if an important endeavor will not be done without their contribution. This requires important initiatives to be precarious. Yet firms do not like such activities to be precarious, because by definition they are sometimes not done. Instead, they would prefer to prioritize high value activities. Our focus is on how organizations manage such tradeoffs.

The results depend on three ingredients. First, donors to organizations care about the impact of their contribution - this marginal calculation is how we interpret “making a difference”. Second, these organizations potentially exhibit diminishing returns. These two assumptions imply that an individual donor may perceive her marginal contribution as being of little value, as the donations of others would fund the most valuable activities. Yet all donors are marginal, which implies aggregate donations can be very sensitive to perceptions of diminishing returns. The final ingredient is a series of inefficient practices used by organizations to mitigate diminishing returns, done to attract more donors.

We model a firm that provides public goods, but cannot fund more public goods than it can raise in donations. These public goods, or projects, vary in their quality (either high or low). The donors care about the marginal impact of their donations on welfare, which depends on their beliefs of the donations of others. There is also randomness in donations. In the benchmark model with which we begin the paper, we assume that the firm can choose a priority rule over projects. So, for example, they could carry out all high value projects first, which we call triage, or they could delay these more valuable opportunities such that they are only done with enough funding. As there is randomness in total donations, carrying out high value projects first is appealing, as adequate funding may not be attained to carry them out. We characterize the optimal priority given to high value projects, and only in

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2 In the most literal interpretation, donors could be offering money and these public goods could be clients seen by a service agency, grant disbursement by a funding agency, food donations to a food pantry, and so on. Alternatively, firms could be attempting to induce more effort from their workers, who care about the benefits that accrue from those efforts.
special cases does it involve triage. Only if donors are sufficiently unresponsive on the margin to “making a difference” will this arise. When we assume that noise is characterized by a Normal distribution, we show that there is a single partition in which all high valuation projects are done, but in general, it does not involve triage. We show in this case how the desire to prioritizes low value opportunities over higher value ones depends only on the marginal generosity of donors and the value of a low value project. As donors become sufficiently generous, the likelihood of high value projects being done converges to 50%.

We provide a number of extensions. The outcomes above arise if donors and the firm have aligned preferences. Yet there are often cases where they are not aligned - for example, donors often like visible manifestations of their donations, whereas organizations might prefer unrestricted giving. In cases where the objectives of the firm are not aligned with those of donors, we show that triage never occurs and that instead endeavors preferred by donors are, on average, done less than half of the time.

These results show the central tradeoff with donors who want to make a difference. Yet this outcome relies on the dual assumptions that a firm can commit to turn down high value projects to do less valuable ones, and that they can communicate this to donors. In reality, this may be difficult. As a result, much of the remainder of the paper is concerned with how such desires to render marginal activities more valuable can be implemented in practice.

The first issue is that commitment may not be possible: instead, once funds are received, the firm carries out projects in descending order of their value. To address this case, we allow the value of projects to be determined by investments made by the firm, but where they cannot commit to a priority rule. Consider a case where there is a technological return to specializing investments on a small number of initiatives. We show that, despite this, the firm follows a lack of focus, by investing in a wide range of projects. Specifically, the outcome involves a partition of projects where returns are equalized. The reason for this is that if the firm focuses its investments narrowly, donors realize that their marginal contributions will be on less valuable opportunities, and hence will not give. The only way in which to overcome this when the firm cannot commit is to invest less in a wider set of projects.

We then show how organizations can temper diminishing returns through ignorance. Consider an extension of the model above where firms vary in their diagnostic efficiency, namely their ability to identify high value opportunities. There is clearly a value to efficiently identifying project quality - firms can commit their resources to the most valuable opportunities. Yet there is a countervailing incentive here, in that efficient firms exhibit greater diminishing returns. This harms donations. As a result, there is a strategic value to
inefficiency. If marginal donor generosity is sufficiently high, the firm always benefits from being less efficient if it cannot commit to a priority rule.

We then extend this logic to consider how mission based organizations offer access to their services. As an example, consider a setting where a client (project) in potential need of the firm’s services can be “undeserving”, “in need”, or “in severe need”. Usual economic logic would stress a value to distinguishing between these cases. Suppose that the organization can choose optimal diagnosis. If it is not too costly to do so, the firm always rules out the undeserving. However, for sufficiently generous donors, they never distinguish between the two other cases. An interpretation of this is that they diagnose clients as “qualified” or “not qualified” but make attempt to distinguish between those in need in those in severe need. Yet the reason for this practice is not ethical, but rather to encourage donors.

In each of these cases, donations arises from inefficiency diagnosis because they flatten the relationship between firm priority and project quality - if the firm cannot identify good projects, some good ones are likely missed early on in the priority ranking. We show that another avenue by which this arises is when there is misalignment in preferences between donors and the firm, where the initiatives favored by the firm need not be those most liked by donors. Once again, we show that for sufficiently generous donors, the firm benefits from misalignment as donors give more.

The central idea of the basic model is that the priority of various activities is upended to encourage donations. We also extend the model in a different direction by identifying the value of firms investing in projects that are of narrower interest than is possible. In particular, we show the value of focusing on projects that no donor likes more than another project that is available, but instead than others like less.

The backdrop to this work is a long and rich literature on the relationship between efficiency and firm growth (Jovanovic, 1982, Hopenhayn, 1992, Syverson, 2011). As Jovanovic (p.649) puts it, “the efficient grow and survive, the inefficient decline and fall”. Empirical evidence for privately consumed goods supports this view.\(^3\) As a contrast, we consider the outcome for privately consumed goods, where we show that efficiency - by prioritizing higher quality projects - maximizes consumer welfare. The key difference arises because for private goods, only one person can consume each good, whereas for public goods, all donors evaluate their contribution at the same set of marginal points.

It is often argued that competition can mitigate firm inefficiencies. We extend the model\(^3\) This occurs both through internal organization of firms, and the fact that efficient, lower cost, firms have larger markets shares.
to consider the impact of competition. Here we show that competition increases donations but reduces allocative efficiency if the market is sufficiently competitive. This is because competition adds a “business stealing” return to distorted project choice. In the limiting case of undifferentiated Bertrand competition, welfare always falls relative to the case of a monopolist public goods provider.

Before describing the model, it is worth discussing relevant literature. First, the central point of the paper is that mission based organizations may benefit from allocating resources inefficiently. There is a small theoretical literature on how agency concerns can change how mission based organizations operate. With the interpretation of donations here being effort by employees, one could interpret this paper as belonging to that vein of work. Yet the employees of mission based organizations often seem to exhibit an altruistic zealotry that suggests that looking outside worker agency issues may be valuable. We do so here by arguing that the contortions of mission based organizations may be generated by their need to attract donors. Second, the model requires two key assumptions: that donors care about marginal impact, and that the recipients of these donations exhibit diminishing returns. We describe a large body of empirical work on the charitable sector that supports both assumptions.

Finally, it is worth addressing the range of potential applications of these ideas. The model is described in terms of a generic donor. Its most literal interpretation is an external donor offering cash to a non-profit. Yet there are other potential applications. By semantics, the donation could be an employee in a mission based organization accepting lower compensation for doing so. Yet the work may also be relevant to organizations without obvious social objectives. Instead, what matters is that workers feel worth from their marginal actions (see Dur and Van Lent, 2018, for evidence on this). From this perspective, any organization could be designing tasks to enhance their marginal significance.

Section 1 describes the basic model, and the desire to delay high value opportunities so that they are precarious. In Section 2, we consider how this can be implemented when the firm cannot commit. We then identify other ways to organize to make donors more marginal in Section 3. Section 4 addresses the impact of competition. Section 5 discusses relevant


5A most natural application here would be how firms design jobs to make workers feel value in their actions (Hackman and Oldham, 1976). A central component in the literature of such job design is “task significance”, where workers can see the impact of their actions for the objectives of the organization that employs them.
1 Model

Consider a firm that carries out projects (or equivalently serves clients) subject to the constraint that it can spend no more than the revenue it receives from small donors. Each project carried out costs $c$, where $c$ is small, and total donations are $D \geq 0$. The projects are of two types, either low value with surplus $v > 0$, or high value with payoff $\overline{v}$, where $\overline{v} > v$. There is an elastic supply of low value projects but an exogenous and known number $p$ high value projects. The firm knows each project’s type.

The firm carries out projects in priority order $v(i), i = 0, 1, 2, \ldots$ until its budget is spent. We assume for now that the firm can commit to a priority rule, where it carries out project type $v(i) \in \{v, \overline{v}\}$ as the $i$th one done. This is observed by donors. Donors are small, in the sense that each donor funds one project.\footnote{Two issues arise with positive discrete project costs. First, there is the familiar public goods problem where more than one donor is needed to implement a project. This can give rise to complementarities across donors, which is not the interest of the paper. Second, with discrete costs and small donors, the points of marginality become the discrete set $f(1), f(2), f(3), \ldots$ and so on. Such integer complications are not the central issue of the paper, as so we ignore it by assuming $c$ small. Finally, note $c$ small does not necessarily imply that $p$ becomes small, as $p$ can be scaled by $\frac{1}{c}$. For example, if instead we assumed that the firm needs donations of $p$ to carry out all $\overline{v}$ projects, then the donations needed to carry out all high value projects is invariant to $c$.} It receives a return of 0 on any project not done. For any realization of $D$, the objective of the firm is to maximize the surplus from all projects carried out

$$S = \sum_{i=0}^{D} v(i).$$

(1)

The expected number of donors $\overline{D}$ depends on marginal impact. Let $\overline{S}$ be expected surplus, and $\Delta \overline{S}$ be the change in expected surplus from one more donor. Donor $j$ gives if $\Delta \overline{S} \geq \lambda^j$, where $\lambda^j \geq 0$ is her personal cost of a donation. This is the sense in which donors want to make a difference. The distribution of $\lambda$ is such that there are $A(\lambda) \geq 0$ donors with costs below $\lambda$, where $A' > 0$.

There is also ex post randomness in donations. We assume that the number of donors $D$ is given by

$$D = \overline{D} + \epsilon$$

(2)

where $D \geq 0$. This randomness could either be through the level donations or the costs of
providing services. Let \( F \) be the distribution of \( D \), with mean \( \overline{D} \) and continuous density \( f \). Donations are non-negative, so that the distribution of \( F \) is truncated at \( f(0) = F(0) \), as in for example a Tobit exercise. We assume that the distribution of \( \epsilon \) is independent of \( \overline{D} \) other than through the truncation. We treat the number of donors as a continuous variable. As a result, for any \( \overline{D} \), the probability that there are exactly \( x \) donors is \( f(x) \).

Below we consider the case where \( F \) is a truncated Normal distribution and the distribution is Tobit. Accordingly, let \( \phi \) be the density of a Normal distribution with mean zero and variance \( \sigma^2 \), with its CDF given by \( \Phi \). In that case, \( D \sim N(\overline{D}, \sigma^2) \) for \( x > 0 \), and \( \phi(0) = \Phi(0) \).

The timing of the model is as follows. The firm first commits to \( v(i), i = 0, 1, 2, \ldots \), which is observed by donors. Donors then give, resulting in a distribution \( F \) with mean \( \overline{D} \), with realization \( D \). Projects are then carried out according to the priority rule, and the game ends.

The expected number of donors is given by

\[
\overline{D} = A(\Delta \overline{S}),
\]

subject to \( \Delta \overline{S} \) being the equilibrium belief from a distribution with mean \( \overline{D} \). Remember that the firm chooses a priority rule \( v(i) \). Given (3), expected funding is therefore given by

\[
\overline{D} = A(\int_0^\infty v(x)f(x)dx),
\]

where \( f \) has mean \( \overline{D} \). This is the \( F \) distribution weighted average of the expected return to a marginal contribution with a priority rule \( v(x) \).

The model has the possibility that total contributions to the firm can be 0. In order to focus on cases where the donor believes that their contribution always has some value, we make Assumption 1.

**Assumption 1:** \( F(0) \) is small.

This allows us to ignore the boundary condition, which is largely used to allow us to restrict attention to a strictly positive number of donors. Below, we show an alternative interpretation where this simplification is not necessary.

With this benchmark, now consider the provision of public goods. Given Assumption 1, the donor believes his donation will either fund a low value or a high value project. For any

\footnote{We can think of this as a limited liability constraint where if the firm is to make losses, it can simply liquidate.}
there is a set of feasible densities \( f_\Gamma \), where for notational simplicity below we do not condition on \( \Gamma \). Then let \( \Gamma \) be the set where the firm places its \( p \) high valuation projects in order to do so. Then if \( \Delta_v = \overline{v} - \underline{v} \),

\[
\overline{D} = A(\overline{v} + \int_\Gamma \Delta_v f(x) dx)
\]

The firm has one choice here - to choose the set \( \Gamma \). It does so to maximize \( E[\int_0^D v(i)di] \) subject to (5), \( f(x) \) is feasible for \( x > 0 \), and Assumption 1. We use the term *triaze* when \( v(i) = \overline{v} \) for all \( x \leq p \). Our interest is in whether the firm will place some \( \overline{v} \) projects at \( x > p \).

The problem is simplified by two observations that we show in the Appendix. First, the firm can locate its high value projects at *any* densities of the truncated Normal. To see this, assume that the firm wishes to place a high value project at density \( f(x) \) instead of at \( f(y) \). It cannot simply switch priorities \( x \) and \( y \) for the reason that the expected number of donations may change, say by \( \delta \). Then the new priorities with those densities are given by \( x + \delta \) and \( y + \delta \) respectively. The firm can implement this by additionally placing \( \delta \) low value projects in priorities 1, 2, ..., \( \delta \).\(^8\) The second simplification is that, because of that use of the low value projects, the marginal value of an additional donor is \( (1 - F(0))\overline{v} \), which given Assumption 1, we approximate by \( \overline{v} \).\(^9\)

Then consider the impact of the firm changing a project from type \( \underline{v} \) to type \( \overline{v} \) at priority \( x > p \). The probability of a project of priority \( x \) being completed is \( 1 - F(x) \), so raising \( v \) has a direct marginal return of \( (1 - F(x))\Delta_v \). However, from (4) it also raises expected funds locally by \( A'f(x)\Delta_v \). Then the return to increasing \( v \) in position \( x \) is approximately \( A'\Delta_v f(x)\underline{v} + (1 - F(x))\Delta_v \). For notational convenience, let \( \alpha = A' \). The firm’s objective is then simple - place all \( \overline{v} \) projects in the \( p \) positions where

\[
\alpha f(x)\underline{v} + (1 - F(x))
\]

is highest. This condition generates the outcomes of this section of the paper.

To show the desire to render high value projects precarious, begin by considering the value of \( x \) where (6) is maximized, which we denote the *optimal priority*. Maximizing \( \alpha f(x)\underline{v} + (1 - F(x)) \) with respect to \( x \) yields

\[
\alpha \underline{v} = \frac{f(x^*)}{f'(x^*)}
\]

\(^8\)It can do this because there is an elastic supply of low value projects: this is the only role for assuming a large number of \( \underline{v} \) projects.

\(^9\)The extra dollar is used - in expectation - on the first low value project carried out, which can be funded with probability \( 1 - F(0) \approx 1 \).
This is clearly not zero in general: instead, it depends on the hazard of the density. Note however that the firm will only avoid triage if \( f'(x^*) > 0 \) over some range of the distribution. Otherwise, giving is maximized by carrying high value projects first.\(^{10}\) However, in cases where the density rises at some point in the distribution, delaying high value projects may be optimal.

**Normal Distribution:** In order to render the problem more tractable, and to show optimal delay, we consider the case where the distribution is Normal. Then (7) becomes

\[
x^* = D - \frac{\sigma}{\alpha v}.
\]

First, this is clearly not zero. Second, the optimal priority is below the mean: this arises because the density of Normal distribution is maximized at the mean. Second, the likelihood of a project at the optimal priority being completed depends on only two parameters, the generosity of donors \( \alpha \), and the value of low return projects \( v \). It is increasing in the marginal willingness to donate, \( \alpha \). As \( \alpha \to \infty \), the probability of the maximal delay project being completed converges to \( \frac{1}{2} \). (This is because the mode is the mean for the Normal distribution.) On the other hand, as \( \alpha \) gets small, \( x^* \) hits the boundary of 0, and there is no value to delaying valuable opportunities.

For the Normal distribution, the percentile rank of \( \Gamma \) is determined by only those two parameters. Specifically, it neither depends on average donations, nor its randomness. That it moves one for one with the mean of the distribution is not surprising, given that the shape of the Normal distribution is invariant to the mean. More notable is that the probability that a project of priority \( x^* \) is completed does not depend on \( \sigma^2 \), despite the presence of \( \sigma \) in (8). This is because \( \Phi(x^*) \) independent of \( \sigma^2 \): specifically, \( \Phi(-\frac{\sigma}{\alpha v}) \) is independent of \( \sigma \), and the optimal priority is located the \( -\frac{1}{\alpha v} \) percentile of a standard Normal.\(^{11}\)

\(^{10}\)Consider two other distributions - defined only over the positive line - that are commonly used, namely the Exponential and Rayleigh. The Exponential has a constant hazard function. As a result, triage is always optimal. By contrast, the hazard for the Rayleigh distribution is increasing in \( x \). For \( \alpha \) large enough, the outcome is above: the firm places its high value projects in an intermediate range of \( x \)'s. These outcomes are shown in the Appendix.

\(^{11}\)\( \Phi(-\frac{\sigma}{\alpha v}) \) is equivalent to \( G(-\frac{1}{\alpha v}) \) for a standard Normal. Note that this shows that the outcome for \( x^* \) is independent of variance. However, the *ordering* of projects is affected by variance. As an example, consider a case with little uncertainty and one with more, and assume that the probability of the first high valuation project occurs at the 25th percentile of the distribution. Then with low variance, the 25th percentile occurs at a higher value of \( x \) than with a high variance. As a result, the ordering of projects has more low valuation ones done first when the variance is low, whereas the list of low valuation ones is shorted with more variance.
This identifies the single point that maximizes the location of a high value project. The firm has, of course, to allocate not one high value project but \( p \). In general, these could be placed in many different locations, and need not necessarily be in a single partition. However, because \( \phi(x) \) is hump shaped and single peaked for a Normal distribution, in that case all \( \overline{v} \) projects are located in a single partition around \( x^* \). Then all high value projects are located between \( x^* - b \) and \( x^* + a \), where \( b = (p - a) \). When \( x - b > 0 \), these are characterized by

\[
\phi(x^* + a) - \phi(x^* - b) = \frac{\Phi(x^* + a) - \Phi(x^* - b)}{\alpha v}.
\]

Furthermore, as \( \alpha \) gets large, \( x^* \) converges to \( D \) and then partition of \( v \) projects is between \( D - \frac{p}{2} \) and \( D + \frac{p}{2} \). Note that in that case, half of all high value projects are more likely to remain undone.

We have so far emphasized how high value projects may be delayed. However, parameter values may be such that all \( v \) cases have higher priority than any \( v \) ones, which we refer to as triage. In the Appendix we show that triage arises only if there is no \( x > p \) that offers a higher return than locating at \( x = 0 \). This arises if

\[
f(p) - f(0) \leq \frac{F(p)}{\alpha v},
\]

where the distribution has a mean of \( A(v + F(p)(\overline{v} - v)) \).

A number of features lead to triage. First, it arises if donors are not sufficiently generous. For one interpretation of this, note that when \( \alpha \) is low, \( p \) becomes high relative to \( D \). As \( f' < 0 \) for \( x > D \), this implies that for \( p \) sufficiently high, \( (10) \) always holds. One possible interpretation of this is that small firms do not need to distort in order to attract funders, as they have not hit significant diminishing returns. We return to this below. Second, triage arises if \( v \) is low, as the value of additional funds is not high enough. Proposition 1 summarizes the results of this section.

**Proposition 1** The firm places its high value projects in the \( p \) values where \( (6) \) is maximized. Triage occurs if \( (10) \) holds. For the Normal distribution, the firm locates its high value activities around \( x^* = D - \frac{\sigma}{\alpha v} \), where the range is from \( x^* - (p - a) \) to \( x^* + a \), where for \( x^* - (p - a) > 0 \), \( \phi(x^* + a) - \phi(x^* - (p - a)) = \frac{\Phi(x^* + a) - \Phi(x^* - (p - a))}{\alpha v} \). As \( \alpha \to \infty \), \( x^* = D \) and the partition converges to \((\frac{D - p}{2}, \frac{D + p}{2})\).

Proposition 1 illustrates the tradeoff between rendering projects marginal to be valuable to donors and the danger that these projects are not funded.\(^{12}\)

\(^{12}\)In some ways, the tradeoff is akin to the canonical monopoly problem. In (6), the firm chooses priorities
General Operating Expenses: Assumption 1 rules out the possibility that donors perceive that their donation has no value. Consider an extension of the model where donors only care about the high value projects, where any residual funds are used for projects that donors do not value - in reality, charitable institutions find it especially hard to fundraise for General Operating Expenses, as it hardly resonates with most donors. As a result, consider the case where the firm values these low value projects at \( v \) but the donor values them at 0. Then, even if we drop Assumption 1, expected donations are given by \( \bar{D} = A(\int \Delta_v f(x) dx) \), and all the qualitative results continue to hold.

Archdiocese of Chicago: Another case of interest is where the interests of donors are inversely related to those of the firm. As an example, every year the Archdiocese of Chicago runs its annual fund raising campaign, where individual parishes raise funds. These donations are allocated to two “projects”. Specifically, some funds are retained by the Archdiocese and used for its own purposes, while some can be retained by individual parishes. This setting differs in one important way from the setting above, in that local parishes prefer money to go to themselves, while the Archdiocese prefers to have the centralized funds. Hence they do not agree on the ranking of projects.

To address this case, consider a case where the donor values the projects at \( \bar{v} \) and \( v \) as before, but where the firm now values the \( v \) project at \( v_f > \bar{v} \). Then the equivalent condition to (6) is given by \( \alpha \Delta_v f(x) v_f + (1 - F(x))(\bar{v} - v_f) \), which is optimized at

\[
\frac{f'(x^*)}{f(x^*)} = \frac{\bar{v} - v_f}{\alpha \Delta_v v_f},
\]

which, as \( \bar{v} - v_f < 0 \) implies that for the Normal distribution the optimal priority is greater than the mean. Hence, the qualitative results apply in this case, but result in longer delays relative to the preferences of the consumer. In the example above, the Archdiocese had to choose how allocate aggregate donations between these two. The outcome chosen by the Archdiocese is to “delay” the projects preferred by the donors, where they mandated that to trade off marginal gains of delaying valuable projects against the danger they do not get done. It is not hard to see the analogy to a monopolist trading off a higher price against a lower probability of a sale. There are two ways in which this differs. First, in the standard monopoly problem, if the buyer does not pay the price, then the good is not sold. We assume here that the firm cannot simply threaten to not carry out any projects. (As one example, a social service agency cannot threaten to not serve any needy clients unless it reaches a specific funding target. That surely would violate an ethical constraint among donors, efficient as it might be.) Instead the most it can do is use the funds for less efficient ends. Second, this setting is conceptually different from the standard monopoly problem in that here, a donor’s valuation of the good depends on our analog of the price, the priority rule.
for each parish, they need to collect a certain amount for the Archdiocese but beyond that point, 100% of extra funds would go to the parish.

**Distributional Assumptions:** In order to provide concrete examples of how projects are prioritized, we have restricted attention to the case where the errors are truncated Normal. This is important for three reasons. First, the density of the Normal rises over some range. Second, for the Normal distribution, all high value projects are optimally placed in a single partition.\(^{13}\) Finally, throughout the paper we have assumed that \( \epsilon \) is independent of \( D \), and that as a result, increasing \( D \) does not exclude any densities \( f(x) \). This arises because the shape of the distribution of \( F \) does not depend on its mean. But for some distributions, densities change with the mean.\(^{14}\) We have assumed that \( \epsilon \) is independent of \( D \) both for technical reasons, but additionally for empirical relevance. Specifically, while it seems intuitive that an institution may render projects precarious to make donors feel needed, it seems harder to imagine that they do so to change higher moments of the distribution.

## 2 Extensions

Here we extend the model in a number of directions.

### 2.1 Investment

The outcomes above arise where the firm is exogenously assigned \( p \) high value projects. An alternative is to allow the firm to invest in projects. These investments are observed by donors. Assume that without investment, all projects have return \( v \) but the firm has total (manpower) resources of \( m \) that are assigned to projects. The return can be increased to \( v + \Delta(x) \) at cost \( C(\Delta) \), where \( C'(\Delta) > 0, C''(\Delta) \geq 0 \), and \( C'(0) = k \), so there is a fixed cost of \( k \) to investing. The manpower budget constraint is that \( \sum x C(\Delta(x)) \leq m \). Given the generality of this cost function, let \( \alpha \) be a constant.

As above, the marginal value of \( \Delta \) at \( x \) is \( \alpha f(x)v + (1 - F(x)) \). As a result, for two points

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\(^{13}\)This is not an outcome that need arise with other distributions. For example, a distribution with many atoms could have many disconnected points with high value projects.

\(^{14}\)For example, consider the Weibull, which is defined over the positive line, which has a scale and a shape parameter. When the hazard rate rises with \( x \), as the shape parameter rises, the distribution flattens. As a result, the set of available densities changes as the mean rises, and more relevantly the height of the mode falls.
Two familiar cases naturally arise. First consider the case where investment has only a fixed cost: \( C(v - \bar{v}) = k \) for \( v \leq \bar{v} \), but \( C(v - \bar{v}) = \infty \), for \( v > \bar{v} \). Then the same results as above arise, where \( p \equiv \frac{m}{k} \). The second case is where \( C''(\Delta) = 0 \), in which case the firm investments \( m \) on a single projects, at \( x^* \). Hence the results naturally apply to settings where the firm chooses project specific investments.\(^{15}\)

### 2.2 Commitment and Lack of Focus

The outcomes above rely on the ability of the firm to commit to carrying out low value projects over higher value ones. However, once the firm has received donations, it is tempted to simply renge and carry out projects via triage.\(^{16}\) Without some commitment mechanism, the only credible outcome is triage, with its potentially lower level of equilibrium funding. Hence, some mechanism of restricting firms from doing this is necessary for the results above. Here we show how strategic investment can render marginal donations more valuable. In order to isolate these effect, we assume here that the distribution \( F \) is degenerate. (As a benchmark, note that the optimal outcome above with degenerate \( F \) is to place a \( \bar{v} \) project at priority \( A(\bar{v}) \), and to place the remaining \( p - 1 \) high value projects at any priorities lower than \( A(\bar{v}) \).)

To address this, we consider a case where the firm has a resource, manpower, that it can assign to projects. Its manpower resource is \( m \). Without investment of manpower, a project has a return \( v \). However, the firm can increase this by \( \Delta \) by assigning manpower of \( \Delta \), at a cost of \( C(\Delta) \), where \( C'(0) = 0 \), \( C'(x) > 0 \) for \( x > 0 \), and \( C'' = 0 \). As a benchmark, note the

\[^{15}\text{There is an additional potential distortion here, in that the firm may choose to invest in technologically dominated investment opportunities. To see this, assume now that there is a fixed cost} \; k_x \text{ in order to increase} \; v \text{ to } \bar{v} \text{ in priority } x. \text{ Here the cost can vary with } x. \text{ Efficiency without the donor responses here implies that a firm should invest in a project } x \text{ over project } y \text{ if } \frac{k_x}{k_y} \leq \frac{1-F(y)}{1-F(x)}. \text{ However, here they will do so only prioritize investment in } x \text{ over } y \text{ if } \frac{k_x}{k_y} < \frac{f'(y)\Sigma x}{f'(x)\Sigma x}. \text{ Once again, outcomes that are marginally more likely will be more likely to be invested in, even though their technological returns are lower.} \]

\[^{16}\text{More generally, it has to be able to commit to burn } \Delta, \text{ in the event that donations fall below some level. Take the example of a matching grant, where fund raising has to reach a particular level to be matched. This only works if the firm can credibly burn that money if those funding levels are not reached. In reality, this can be difficult to do. A concern that arises with matching funds is that even if the funds are not raised, the donor of the matching funds simply will simply donate anyway.} \]
outcome that arises with a degenerate $F$ when the firm can commit to a priority ordering. Let $D(m) = A(v + C^{-1}(m))$ be the maximum investment in a single project. Then the firm assigns that to the marginal project, and invests only in that project.\footnote{This is the only outcome that depends on the assumption that $C'' = 0$ above. When $C'' > 0$ in the case where the firm can commit, it may spread out the investment over more than one project. We make the assumption of $C'' = 0$ to render the comparison below more stark. The constrained outcome is identical even when $C'' > 0$.}

Now consider the case where the firm cannot commit. The strategy above is not credible. Let $\Delta(i)$ be the $i$th highest value of $\Delta$. The inability to commit implies that we must now add the constraint that $\Delta(i)$ is non-increasing in $i$. Then expected donations are given by $D = A(\int_0^\infty (v + \Delta^*(x))f(x)dx)$, where $\Delta^*$ is the equilibrium choice. As $F$ is degenerate, this is $A(v + \Delta^*(D))$. The outcome is that $\Delta(i)$ is constant over some range, as described in Proposition 2.

**Proposition 2** When the firm cannot commit to the order of projects, it invests an equal amount $\Delta^*(i) = \frac{m}{m^*}$ in the first $b^*$ projects, where $b^*$ is uniquely defined by $A(v + C^{-1}(\frac{m}{m^*})) = b^*$. It does not invest in any other projects.

Proposition 2 illustrates the strategic response- it spreads its investments across a wide range of projects, even though technologically there is no reason to do so. By doing so, it can credibly communicate to donors that their contributions will matter more on the margin. By contrast, the firm that can commit specializes in a single project. Two intuitive interpretations arise. First, mission based organizations exhibit a lack of focus. Without the commitment problem here, the firm could invest heavily in single project to encourage donations. However, this offers little reason to give. To overcome this, the firm spreads itself thin, as a way of credibly persuading donors that their contributions are indeed marginal. Yet this is costly to the firm - rather than raise $A(v + C^{-1}(m))$ donations, it can only attain $A(v + C^{-1}(\frac{m}{m^*}))$.

Second, a simple extension offers a view of mission based organizations as excessively “egalitarian”, whereby they over-invest in bad projects relative to good ones to equalize returns. Specifically, assume that in the absence of investment, returns to projects differ, where project $k$’s return is given by $\psi_k$, $k = 1, 2, 3, \ldots$. Order these such that $\psi_k \geq \psi_{k+1}$. Then compare the outcomes when the firm can commit relative to when it cannot. When it can commit, it will invest in only one project, $\psi_1$, and will choose its priority to be the marginal project, $x^*$, where donations are given by $A(\psi_1 + C^{-1}(m)) = x^*$. When the firm cannot commit, this outcome is not feasible, as the firm will simply carry out that project.
first. Instead, the firm has to equalize returns over a range of projects for the same reason as above. However, here this implies that the firm invests more in lower $v_k$ projects than when their return without investment is higher. In this sense, the firm is excessively egalitarian.

3 Other ways of creating marginal benefits to donors

The central premise of the paper is that organizational inefficiency may be a way to render donors more marginally valuable. Above we showed how a failure to target investment opportunities may be one way to do this. Here we consider cases where the firm cannot commit to deferring better projects to do worse ones, nor does it have the capacity to choose investments on individual projects. Here we address three other mechanisms to render donors more marginally valuable - by being less efficient, by failing to diagnose need, and by focusing on projects that are of narrow interest. In each of these case, we generate randomness through a different avenue, so we assume that $F$ is degenerate.

3.1 Inefficiency

Here we identify the return to a measure of firm efficiency, namely its ability to identify $v$. To model strategic inefficiency, begin by assuming that firms vary in their ability to identify $v$. Let there now be a total supply of projects given by $N$, of which $p$ are of value $v$. It receives a signal $s$ on $p$ projects. Here, however, this does not unambiguously identify them as high value projects. Instead, the probability that such a project is of value $v$ is $\gamma$, where $1 \geq \gamma \geq p$. The metric of firm efficiency is $\gamma$, which is observed by donors. However, it cannot commit to the order of projects, and so all projects with a signal $s$ are carried out first.

The timing of the revised model is as follows. First, the firm is endowed with its type $\gamma$, which is observed by the donor. Then donor $j$ gives based on $\lambda_j$ and her equilibrium beliefs of $\Delta S$. The firm then carries out projects.

The equilibrium of the game is described in the Appendix. Remember that the central issue of the paper is that donations fall when marginal activities are of low value. This is manifested here when the marginal project funded does not have the signal $s$. When the marginal project does not have signal $s$, its expected quality is $v + \frac{p(1-\gamma)\Delta}{N-p}$. The marginal project does not have signal $s$ if $\bar{D} = A\left(\bar{v} + \frac{p(1-\gamma)\Delta}{N-p}\right) > p$. Consider that case. Here

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18 More formally, the firm chooses a range $1, 2, \ldots, x^*$ to invest in, where $x^*$ maximizes $A(v_{x^*} + \Delta^*(x^*))$, subject to $\sum_{i=1}^{x^*} \Delta^*(i) = m$ and $v_x + \Delta^*(x) = v_{x+1} + \Delta^*(x+1)$. 

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donations are decreasing in firm efficiency: \( D'_1 = -A' \frac{p \Delta v}{N-p} < 0 \). This is because the firm can better prioritize earlier projects as it can identify them better. Expected surplus is given by
\[
S = p(\bar{v} - (1 - \gamma)\Delta v) + \nu(N - p)(\bar{v} + \frac{p(1 - \gamma)\Delta v}{N-p}),
\]
where \( \nu = \frac{D_1(\gamma) - p}{N-p} \leq 1 \) is the probability that a project without signal \( s \) will be completed. The problem is only interesting if \( \nu < 1 \). Then
\[
\frac{dS}{d\gamma} = p\Delta v(1 - \nu) + A'(\gamma)(\bar{v} + \frac{p(1 - \gamma)\Delta v}{N-p}).
\]
The first part of this is the usual efficiency gain from better allocation - as \( \nu < 1 \), identifying high quality projects has value as they are more likely to be completed. However, this must be traded off against reduced funding, given by the second term. However, the second term dominates for large enough \( A' \). Proposition 3 immediately follows.

**Proposition 3** If \( \bar{D} > p \), donations are decreasing in firm efficiency. Furthermore, if \( \nu < 1 \) and \( A' \) is large enough, firm surplus is decreasing in its efficiency.

### 3.2 Optimal Diagnosis and “Qualified”

The previous section had one potentially important assumption: there were no projects that the firm should turn down. If we had included projects that had negative value, more random completion would have added an additional cost of inefficiency. We turn to this issue here in order to address how mission based organizations provide access to their services. In particular, here we outline how optimal diagnosis can simply be to exclude cases that are below a threshold, but to make no distinctions beyond that. In more casual terms, they simply deem clients “qualified” but make no more attempt to diagnose severity of their needs. (As one example, such institutions could use a “first come, first served” policy among those who pass a qualification standard.)

So far, we have treated the ability to diagnose as a parameter. Now consider a setting where firms can choose how they diagnose “severity” of cases. As an example, in many social service settings, clients must pass a test of some form to determine if they qualify for benefits. Here we allow the firm to decide what that filter should be. We do so in an extended setting where in addition to cases \( \bar{v} \) and \( \bar{v} \), there are cases that are “undeserving”, with return \( v^- < 0 \). Assume that in the population, these cases occur in proportions \( p_{\bar{v}}, p_v, \) and \( 1 - p_{\bar{v}} - p_v \). There are \( N \) cases in total. Randomness here arises through diagnosis choices by the firm, so once again we assume that \( F \) is degenerate.
A natural way to interpret \( v \) is where higher \( v \) cases have more severe characteristics than do lower \( v \) ones. Accordingly, consider a diagnosis technology where at cost \( \kappa_0 \) the firm can verify that rule out cases of type \( v^- \), and at cost \( \kappa_1 \), case \( v \) can additionally be ruled out. Without incurring any costs, we now assume that they cannot distinguish between cases.

The timing of this revised game is as follow. First, the firm commits to its diagnosis technology - whether to incur cost \( \kappa_0 \), \( \kappa_1 \), or to not diagnose at all. After observing this, donors give to the firm, the firm allocates its funds in the ex post efficient way (given whatever information it has), and the game ends.

First consider ruling out undeserving cases. This part is standard, and consists of no more than a comparison of better allocative efficiency (and the higher funding that comes from this) against the cost of diagnosis. Without any diagnosis, the payoff to the firm is

\[ D_0(p_{v \bar{v}} + p_{\bar{v} v} + (1 - p_{v} - p_{\bar{v}})v^-) \]

Donations are given by

\[ D_0 = A(p_{v \bar{v}} + p_{\bar{v} v} + (1 - p_{v} - p_{\bar{v}})v^-) \]

and so surplus is

\[ (p_{v \bar{v}} + p_{\bar{v} v} + (1 - p_{v} - p_{\bar{v}})v^-)D_0 \]

If the firm incurs the cost of \( \kappa_0 \)’s, surplus is

\[ \left( \frac{p_{v \bar{v}} + p_{\bar{v} v}}{p_{v} + p_{\bar{v}}} \right)D_1, \]

where

\[ D_1 = A\left( \frac{p_{v \bar{v}} + p_{\bar{v} v}}{p_{v} + p_{\bar{v}}} \right) \]

Then if

\[ \left( \frac{p_{v \bar{v}} + p_{\bar{v} v}}{p_{v} + p_{\bar{v}}} \right)D_1 - (p_{v \bar{v}} + p_{\bar{v} v} + (1 - p_{v} - p_{\bar{v}})v^-)D_0 \geq \kappa_0, \]

the firm excludes undeserving cases.

Of more conceptual interest is whether to distinguish between \( v \) and \( \bar{v} \) cases. This depends on the return to the marginal case. If \( A’ \) is low, the marginal case is \( \bar{v} \), and the same kind of calculus as above continues to hold. What changes is when funding is sufficiently generous to allow some \( v \) projects to be done - this requires that \( D > Np_{v} \). In that case, there is a tradeoff beyond the cost of diagnosis: with better diagnosis, a larger fraction of cases carried out will be of high value, but as funding is based on the marginal case, fewer cases will be done. Specifically, if \( D_2 \) is the level of donations, surplus with additional diagnosis is given by

\[ Np_{v \bar{v}} + (D_2 - Np_{v})v \]

Hence, all high value cases are done first, and only the residual will be lower value cases. Hence there is better assignment of cases. However, \( D_2 = A(v) \), as donors see their marginal dollars being spent on less important cases. Hence, diagnosis is only optimal if

\[ Np_{v \bar{v}} + (A(v) - Np_{v})v - \kappa_1 \geq A\left( \frac{p_{v \bar{v}} + p_{\bar{v} v}}{p_{v} + p_{\bar{v}}} \right) \left( \frac{p_{v \bar{v}} + p_{\bar{v} v}}{p_{v} + p_{\bar{v}}} \right) \]

Proposition 4 follows.

**Proposition 4** Assume that (15) holds but (16) is violated. Then optimal diagnosis is to distinguish between cases \( v^- \) and the other two, but no more. This can arise even if \( \kappa_1 = 0 \).
The notable part of this outcome is that firms may eschew diagnosing severity, even if it is costless, despite the allocative benefits that triage would allow. Indeed, firms may be willing to incur a cost not to diagnose. When this arises, it offers a particular view of mission based organizations as egalitarian. Specifically, there is a gatekeeper that determines whether a client is “qualified”. Once qualified, no distinctions are made between clients, despite variation in severity. Once possible implementation of this is that clients are treated in a “first come, first served” random way. Yet such egalitarianism is not used here for ethical reasons, but rather as a mechanism to encourage donations, as marginal cases continue to have value.

3.3 Misaligned Preferences

Failure to diagnose $v$ is potentially valuable to the firm because it flattens the relationship between the firm’s no-commitment priority ranking and the quality of the marginal project to the donor. As a result, funding rises. Another mechanism through which such flattening can be achieved is when there is misalignment of preferences between the donor and firm. To see this, consider a case where the firm values projects at $v$ and $v'$ as above, but where the degree of alignment with the donor preferences is given by $\rho < 1$. Specifically, the donor’s valuation is $v_d$, where $v_d = \rho v + (1 - \rho) v'$ when the firm’s valuation is $v$ and $v_d = (1 - \rho) v' + \rho v$ when the firm’s valuation is $v'$.

Consider the return to misaligned preferences for donations. Clearly, if the marginal project carried out is of type $v'$, misalignment harms donations as the marginal project has value $\rho v + (1 - \rho) v' < v$. However, for sufficiently generous donors, the marginal projects without commitment is one of quality $v$. With aligned preferences, donations are then given by $A(v)$. However, with misaligned preferences, relationship between priority and donor value flattens to increase donations, which are now given by $A((1 - \rho) v' + \rho v)$. Once again, donations are increased, though here through misalignment of preferences.

3.4 Promoting Narrow Interests

The central idea of the paper is that project choice becomes distorted in order to render donors marginal. Up to now, this has arisen through the order of activities. Here we consider firms investing in projects that are of narrow interest, in the sense of implementing a project that some people like less, but no one likes more, than some alternative. While this reduces surplus from the project, it may make it more likely to be implemented, as
interested donors will - correctly - realize that they are more marginal.

The distortion is project choice here is its breath. As a result, we ignore priority by assuming that the firm can only carry out a single project from a set of $n$. It requires funding of 1. There are $r$ donors who can potentially fund the project. We assume that each potential funder is capable of funding the project.\footnote{In settings where there is a single project, one way to encourage donations is where it takes more than one person to donate to the project. In that case, if the person is marginal, the discreteness of the project implies that the donors offering is multiplied, as otherwise the project is not done. This is rather like a matching grant.} Project $n$ offers benefits of $v$ to only $n \leq r$.

The firm values donations for two ends. First, it values the project at $V(n)$, where $V'(n) \geq 0$. Second, it carries out activities other than the project. Specifically, if it raises more than $1$, it uses the additional funds on other activities which has surplus $Z > 0$ to the firm. (Think of these as general operating expenses.) However, each donor value these only at $y$, where the problem is characterized by Assumption 2.

**Assumption 2:** $\bar{v} > 1 > y$.

Note that in the absence of the second inequality here, all donors would give.

The timing is as follows. First, the firm chooses $n \leq r$ (say by building some initial infrastructure needed for the project - architectural drawings would be a literal example). Each of the $r$ donors then decides whether to donate 1 to the project or not. If no funds are raised, the project is not done. If at least one donor gives a dollar, then the project is done and any residual donations have marginal value $Z$.

A donor will give if she gets benefits from the project, and sees herself as sufficiently marginal. It should be obvious that there can be no pure strategy equilibrium of this game. Specifically, if no one donates, there is an incentive for one donor to deviate, as she is always marginal. However, if it is believed that this strategy is followed, no one will donate as they are not marginal. Similarly, given Assumption 1, there is no pure strategy equilibrium where no one donates.

Given this, consider a symmetric mixed strategy equilibrium. In that equilibrium, each of the $n$ donors who value the project gives 1 with probability $p$, where $0 < p < 1$, and all other donors give 0. For this to be an equilibrium, it must be that the donor is indifferent about giving. Any donor is marginal only if the other $n - 1$ potential donors have not given.
This implies that \( p(n) \) is defined by
\[
\alpha(\bar{v}(1 - p(n))^{n-1} + (1 - (1 - p(n))^{n-1})v) = 1, \tag{17}
\]
or \( p(n) = 1 - (\frac{1-v}{\bar{v}-v})^{n-1} \). Total expected donations are then given by \( np(n) \). Note the relationship between \( n \) and expected donations:
\[
\frac{dnp(n)}{dn} = p(n) + \frac{n}{(n-1)^2} (1 - p(n)) \log(\frac{1-v}{\bar{v}-v}). \tag{18}
\]
The first term here is positive (there are more potential donors), and the second negative (as they are less likely to be marginal), so its sign depends on parameter values. However, for \( \frac{1-v}{\bar{v}-v} \) high enough, it is declining in \( n \), for all \( n > 1 \). If this holds, donations are maximized by implementing projects that are of value to only only person!

Now consider the firm’s preferences and choice of \( n \). Its expected surplus is given by
\[
np(n)Z + (1 - (1 - p(n))^n)(V(n) - Z). \tag{19}
\]
The first term here is expected donations, \( np(n) \), times the marginal value of excess dollars, \( Z \), while the second term is the extra utility generated by the project, implemented with probability \( 1 - (1 - p(n))^n \).

The firm chooses \( n \) to maximize (19) subject to (17). It is straightforward to provide parameter values where restricting interest is optimal. As one example, consider the case where \( V = 0 \). In that case, the firm maximizes expected donations, and the example above of \( \frac{1-v}{\bar{v}-v} \) high enough applies. Yet again, project choice is distorted to encourage donors interested in making a difference.

## 3.5 A Behavioral Implementation

The basic model above assumed that firms defer high value projects to persuade rational donors that their contributions are marginal. Consider an alternative to this where donors use a much simpler rule of thumb. Specifically, they simply observe the set of projects that is available to the firm when they donate, and give based on its expected value rather than marginal value. Now consider a temporal setting where at the beginning of, say, a month, the firm is endowed with its projects, of which \( p \) are of type \( \bar{v} \). They choose when to do

\[\text{Notice that this implies that an increase in } v \text{ increases the probability of giving, as it must reduce the likelihood of being marginal from the indifference condition. Hence, once again, there is a reason to focus resources to increase giving.}\]
these projects during the month. Half of its expected donors arrive on the 1st of the month, with the remaining half arriving on the 15th. However, there is randomness in how many donors show up on the 15th.

When donors arrive, they observe the set of projects that have not yet been done, and donate based on its expected value. This behavioral rule for donating would generate the same outcomes as above. Specifically, if the firm uses triage over the month, by beginning with high value projects, it can alleviate the problem that later donors may not show up. However, if they do so, the donors who arrive on the 15th will see a low quality set of residual projects and, as a result, they will donate less. In that setting, the same tradeoff as above arises, though here it arises from the behavioral rule that donors give based on the average quality of outstanding projects.

3.6 The Efficiency of Private Consumption

Up to now, we have simply posited funding opportunities for the firm based on marginal impact. Here we provide a micro-foundation by adding an alternative privately consumed good. In this way, we can additionally address how the issues here are handled by a firm providing such private goods. Assume now that potential donors derive utility from a privately consumed good, $R$, and a public good, $S$. Donor $j$’s welfare is given by $U_j = R_j + S + g_j I_j$, where $R_j$ is donor $j$’s consumption of the private good, and $g_j$ if ‘a ‘warm glow” she received from donating to the public good, where $I_j$ is an indicator variable equal to 1 if she donates and 0 otherwise. Units of $S$ are chosen such that the potential donors weights private and public consumption equally. As above, the public good is the aggregation of many projects (or equivalently clients), $S = \sum_{i=0}^{P} v(i)$. Let $S_{-j}$ be the surplus generated by the donations of all agents other than $j$, and $\Delta S = S - S_{-j}$ be the marginal contribution of agent $i$. We can rewrite the consumer’s utility as

$$U_j = R_j + S_{-j} + \Delta S + g_j I_j.$$  

(20)

Each donor has total resources of 1, and chooses whether to allocate $c$ of her budget to the public good. Public goods each cost $c$ to produce, and consumption goods cost $r$. The objective of the public goods firm is to maximize (1). The consumer has no control over $S_{-j}$, and so compares the marginal impact of each expenditure on her welfare. Taking expectations, consumer $j$ gives to the public good if $g_j \geq \Delta S / e$. Our interest is in total donations, so let $A(\Delta S)$ be the number of donors where $g_j$ exceeds $1/e - \Delta S / e$. As a result, total expected donations to the public good are given by $\mathcal{D} = A(\Delta S)$, where $A' > 0$. However,
additionally assume that there is randomness in donations by $D = \mathcal{D} + \epsilon$. Then consider the case where the distribution of donations $F$ is a truncated Normal distribution. Then $D \sim N(\mathcal{D}, \sigma^2)$ for $x > 0$, and $\phi(0) = \Phi(0)$, as required. This offers a simple micro-foundation of the donor’s preferences.

**Privately Consumed Goods:** This structure allows us to contrast the efficiency of the private good with that of the public good. Assume now that another firm provides the private good. To retain symmetry with the provider of the public good, now assume that it too has goods of different types, either low value with surplus $V$, or high value with payoff $\overline{V}$. There is an elastic supply of low value projects but an exogenous and known number $P$ high value projects. The firm knows each project’s type and carries out projects (or equivalently serves consumers) subject to the constraint that it can spend no more than the revenue it receives.\(^{21}\)

To determine outcomes, we need to identify which consumer gets which private good. As all consumers value the good equally, we consider the case where consumers report their demand, with total demand $R$, and each consumer is given a random priority $1, 2, ..., R$, and received good $V(i)$ if their priority is $i$.\(^{22}\) To mimic a competitive market for private goods provision, assume that the objective of the private firm is to choose the priority of its goods $V(i)$ to maximize consumer surplus: $\sum_{i=0}^{R} V(i)$, where $R = \frac{1 - \epsilon c}{r}$. Note now that $\mathcal{D}$ is now given by the number of donors where $g_j$ exceeds $\frac{EV}{r} - \frac{\Delta S}{c}$, as consumers now value the marginal value of a privately consumed good at $EV$. Note (ignoring the truncation of $D$ at 0) that $R \sim N(\frac{1 - \epsilon c D}{r}, \sigma^2)$. Let this distribution be given by $\Phi_R$.

Then consider a consumer matched to a priority $x$. She receives the good with probability $1 - \Phi_R(x)$. Hence, her willingness to pay is $V(x)[1 - \Phi_R(x)]$. As all consumers are identical, this implies that aggregate willingness to pay is

$$\int_{0}^{\infty} V(x)[1 - \Phi_R(x)]dx.$$ \(^{(21)}\)

This has an intuitive maximizer: all high valuation goods are produced before any low

\(^{21}\)The interpretation for the private good is slightly different: consider this as inventory that it produces based on demand, where a buyer of the good chooses among the available inventory.

\(^{22}\)Relaxing this assumption does not change the optimal priority chosen. Consider a case where the firm has $n$ consumers. Given that it has $P$ high valuation projects, it could offer each consumer a $\frac{P}{n}$ change of the high valuation good, or it could give $p_c$ consumers the high valuation good with the highest priority. Whether it wishes to do so depends on whether the probability of buying the good is concave in $v$ or convex. In the case that it is linear or concave, it is optimal to give all consumers the random allocation. However, in either case it is optimal to choose to always place the the high valuation goods earliest in the priority list.
valuation ones. The reason for this is that as one consumer is attached to each good, the firm maximizes the average return to a consumer. (By contrast, with the public good, all consumers are the marginal consumer.) Then as there is a danger that high valuation projects are not done if demand is not high enough, it makes sense to front-load the highest valuation endeavors. Hence privately consumed good are always provided efficiently.

This serves to illustrate the key issue underlying the provision of public goods. Specifically, all donors get the same marginal return even though there are many infra-marginal goods - in the mind of the donor, if they deviate and do not donate, those goods will be provided by others. By contrast, each consumer consumers a single private good, and only one consumer gets the marginal good. As a result, the focus of the organization providing the public good is on the margin, whereas with private goods the firm integrates over the returns to both marginal and infra-marginal goods.

4 Competition

It is often suggested that competition is a remedy to the ailment of firm inefficiency, and many mission-based agencies do compete with each other. Here we address how competition affects outcomes. There are two reasons to introduce competition. The first is simply to understand robustness. A second reason is to address the welfare implications of the model above. Specifically, one interpretation of the results above is that while internal distortions arise to attract donations, it is to a good end, namely, that it allows agencies to expand and meet the needs of a larger set of opportunities. As such, these distortions are socially beneficial as there are few harmful welfare implications. Here we add competition to show that this may no longer the case.

The are now two firms competing for donors, and are indexed by $k = 1, 2$. Each firm has donations $D_k$, where as above, $D_k$ has mean $\bar{D}_k$, with distribution $F_k$, normally distributed for $D_k > 0$ with common variance $\sigma^2$. Noise for each firm is independent. The two firms are differentiated in the minds of donors. Specifically, the value that donor $j$ gets from donating to firm $k$ is now given by

$$u^j_k = \int_0^{\infty} v_k(x)f_k(x)dx - \gamma^j_k,$$  \hspace{1cm} (22)

It is well known that monopolistic competition can give rise to excess entry, and hence potentially harm welfare. This has been illustrated for the charitable sector in important work by Rose Ackerman, 1982. This is not our concern. As a result, to address other issues associated with competition, we ignore entry and instead simply add a competitor to the firm above.
where $\gamma^j_k$ measures the idiosyncratic preference that donor $j$ has for firm $k$ and $v_k$ is the priority ordering of firm $k$. We consider the case where the two firms are horizontally differentiated and normalize the average disutility of donating to zero: $E\gamma_1 = E\gamma_2 = 0$. The distribution of $\gamma_k$ is $A$, and is independent across $k$. We also here need to consider the potential donor base, which we normalize to 1.

The only variation from the basic model above is that now, at the beginning of the period, each firm simultaneously commits to $v_k(x)$. Donors then give. The distributions $F_k$ then generate outcomes for each firm, projects are carried out, and the game ends.\(^{24}\)

Our interest here is in how the addition of another firm affects both total expected donations and the efficiency of the two firms. First consider its effect on total donations. If the firm 1 did not compete with the other, total expected donors are $A(\Delta S_1)$. Holding strategies fixed, expected donations rise from the addition of firm 2 because there are circumstances where donors give to firm 2 even if they do not like firm 1. The number of people who not give to firm 1 is $1 - A(\Delta S_1)$, of which $A(\Delta S_2)$ will give to firm 2. Total expected donations are then given by

$$D_c = A(\Delta S_1) + A(\Delta S_2) - A(\Delta S_1)A(\Delta S_2).$$  \(23\)

This is just the usual benefit of offering choices to horizontally differentiated donors.

Of more interest here is how it affects the strategic choice of firms. To address this, note that expected donations to, say, firm 1 are now given by

$$D_1 = A(\Delta S_1)(1 - \frac{A(\Delta S_2)}{2}).$$  \(24\)

What matters now is that the return to changing $v_1(x)$ now depends on the actions of the other firm. Specifically, consider the exercise carried out in (6), where at priority $x$ a low value project is substituted with a high value project. The marginal return to this is now given by

$$\Delta_v f(x)vA'(\Delta S_1)(1 - \frac{A(\Delta S_2)}{2}) + (1 - F(x))\Delta_v.$$  \(25\)

Three features differentiate this from the case without competition. First, the $A'(\Delta S_1)$ term is being evaluated at a different level of donors than without competitors. Depending on

\(^{24}\)In the spirit of the rest of the literature, we have assumed that the objective of the firm is to maximize the surplus generated by its firm. In a setting where ones competitors are also carrying out missions, this may not be appropriate. So, for example, if two institutions are feeding the poor in a particular neighborhood, donors may seem them as easily substitutable. But if this is the case, then one firm may not worry if its competitor gets the donor, as the poor will still be fed. A simple way to implement this would be to assume that the objective of firm $i$ becomes $D_i + \lambda D_j$. If $\lambda$ is sufficiently high, then while the firm could take its competitors donor by distorting priorities, it would choose not to do so.
the density of donors, this can increase or decrease the marginal return to changing $\Delta S_1$. Second, and economically more significant, the firm now has to share the market with the other firm, so increases in potential donors has to be discounted by the fact that some of these prefer firm 2: this is reflected in the $1 - \frac{A(S_2)}{2}$ term. Finally, the marginal return to the firm’s actions depend on the actions of the other firm, as the $1 - \frac{A(S_2)}{2}$ isn’t simply a constant.

We simplify by considering two cases: one with little differentiation between firms, and a second where they are highly differentiated. First consider the case where $\gamma_j^i$ is close to 0 for all $j$, and the outcome is close to undifferentiated Bertrand competition. With the addition of a competitor in firm 2, the share of the market obtained by firm is close to 0 for $\Delta S_1 < \Delta S_2$, and close to 1 for $\Delta S_1 \geq \Delta S_2$. Bertrand competition then implies that both firms choose the priority rule that maximizes the return to the marginal consumer, where $x^* = \bar{D}_i$. This offers the minimal allocative efficiency, because it implies that high value projects on average are only completed half of the time. Note further that welfare falls here, because as $\gamma^i_j \to 0$, $\bar{D}_e \to A(\Delta S_1)$ as there is no differentiation of firms. Hence, aggregate donations are unchanged yet allocative efficiency falls. Proposition 5 follows.

**Proposition 5** With undifferentiated Bertrand competition, both firms choose $x^* = \bar{D}_i$. In this case, competition reduces welfare.

Proposition 5 shows how competition can exacerbate firm inefficiency. The economics of this should be easy to see - with undifferentiated Bertrand, there is no welfare gain per se from more firms, as donors don’t care which one they engage with. As a result, the only impact is strategic. As is standard, undifferentiated Bertrand competition results in firms orienting their actions to benefit consumers, as they become infinitely elastic. This implies distorting efficiency maximally to attract donors, resulting in greater inefficiency.

It should not be a surprise that with more differentiation between the two firms, this need not be the case, and that the differentiated value of firms offers a direct benefit to donors. We show this in the Appendix.

### 5 Empirical Observations

Remember that the model has two essential ingredients: that donors care about their marginal impact, and institutions suffer from diminishing returns. We argue here that there

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25As a benchmark, note that without competition, this would imply that say firm 1’s choice is given by $\int_0^{\infty} v(x)dF(x) = 0$, as all donors have a normalized cost of 0 of donating.
is considerable evidence for both. First consider the evidence on marginal impact. The most natural link is to the literature on matching gifts for charities, where a gift by a donor is matched by funds from elsewhere.\textsuperscript{26} Empirical evidence (List, 2011) clearly shows that such matches are an effective way of encouraging donations. Beyond matching grants, the U.S. Trust survey described 94% of high net worth donors as motivated by “their gift can make a difference” (U.S. Trust, 2016). Similarly, Grant, 2008, offers experimental evidence to show how a fund raising experiment for student scholarships is enhanced by telling the respondents about the social impact of their donations.\textsuperscript{27}

The model also requires diminishing returns. A test of this is the large literature on whether crowd-out occurs in charitable giving. This relates both to diminishing returns and donors caring about marginal impact. The most common setting to study this is to address whether government support of an organization reduces private donations. Evidence on such crowd out arises in Andreoni, 1993, Andreoni and Payne, 2009, Bolton and Katok, 1998, and Robert, 1984 (though see Riber and Wilhelm, 2002). Also related is work on charitable donations and firm size. The relation between the size of a charitable organization and its donations has been experimentally tested by Borgloh et al, 2013. In their setting, potential donors were given information on the income streams of charities, and could choose between a larger and smaller charity. They find that donors are much more likely to give to smaller charities, their interpretation being that “the higher impact of the own donation, and the neediness of the charity organization are decisive for choosing the small organization” (p.1).

The model most closely resembles a nonprofit institution seeking external donors of money. An alternative setting is employees in mission based organizations taking lower wages based on their perceived contribution to the mission.\textsuperscript{28} Such issues are of economic significance, as the nonprofit sector employs 7% of the U.S. workforce. Furthermore, empirical estimates of the reduction in wages in that sector range from 10% to 15% without

\textsuperscript{26}Note that that the model also can be interpreted as the optimal way to design a matching grant. Assume that the firm can assign a match $\Delta_e$ if fund raising reached some level $x$. Identical analysis would show the value of matching is maximized at $x^*$. \textsuperscript{27}The greatest response in fund raising was found for a setting where the respondents were prompted by being told that their donations “made a difference in the lives of others”, consistent with a necessary ingredient for this paper’s results. \textsuperscript{28}These cases requires one additional component. In order for the same logic to hold with employees, it must be the case that all employees - even those working on low valuation projects - can carry out the counterfactual of imagining what would occur if they were not employed. Specifically they can understand that the musical chairs of reallocated workers would mean that the loss from their departure is the marginal project carried out, not necessarily the project that they are engaged in.
extensive worker demographic controls (Leete, 2001, Ruhm and Borkowski, 2003, and Salamon and Sokolowski, 2005) to closer to 6% with more controls (Hirsch et al., 2017).

An alternative donation could be the effort of employees. Through this lens, its insights may extend beyond the confines of overtly socially oriented organizations. As one example, Dur and van Lent, 2018, report that 77% of all workers report that it is “important or very important” for them to have a job that is socially useful. Economists are used to addressing the motivation of workers through compensation. Yet there is also a large management literature, deriving from Hackman and Oldham, 1976, on how Job Design can encourage worker motivation. A major building block is Task Significance, where workers can see the impact of their actions for the objectives of the organization.\textsuperscript{29}

Next consider the evidence on competition among not for profits. Since Rose Ackerman, 1982, potential distortions from competition arise through the channel of “business stealing”. In this paper, such business stealing arises by firms distorting their priorities in order to become more appealing to donors. In Rose Ackerman, the mechanism is through excessive fund raising.\textsuperscript{30} Empirical evidence that corroborates this relationship, with Feigenbaum, 1987, Castenada et al., 2008, and Bose, 2015, all showing how fund raising expenses rise with competition. While fund raising is clearly not the margin on which inefficiency is modeled here, it does suggest at least the willingness of non-profits to increasingly distort some activities in competitive settings to attract donors.

Finally, there is evidence for the charitable sector on the relationship between donor generosity and one measure of firm efficiency, namely, the proportion of donations that go to the mission rather than to administrative expenses. A striking feature of this literature is its inconclusiveness, with weak relationships generally being the outcome (Yoruk, 2013, Karlan and Wood, 2014, Parson, 2007, and Buchheit and Parsons, 2006). These are not direct tests of this section, of course, as our measure of efficiency is diagnostic. However, the absence of a relationship is consistent with the diminishing returns assumption needed here. With diminishing returns, the impact of changing the fraction of funds going to the mission is indeterminate. Substitution effects result in less donations. However, there is also an income effect, where firms that use more of their funds on mission may not need marginal funds as much. As such, this literature may offer useful additional evidence on diminishing returns.

\textsuperscript{29}See Fried and Ferris, 1987, and Humphrey, et al., 2007, for empirical evidence.
\textsuperscript{30}Also see Bilodeau and Slivinski, 1997, and Aldashev and Verdier, 2010.
6 Conclusion

Economists have been formally studying motivation for four decades, and this is not the first paper to argue that institutions that rely on a sense of mission change their practices to encourage contributions. Yet that literature typically views such practices as ways of overcoming employee agency problems. More generally, most work by economists on motivation has addressed the link between compensation and effort. Despite undoubted progress on this issue, one of the primary lessons of the literature on agency theory has been how often workers have objectives far beyond the “$\beta$” that links their actions to their pay.

Here motivation is generated by a “need to be needed”. Given the zeal that the employees of many non-profits exhibit, such a lens does not seem out of place. We believe this leads to a number of novel outcomes. First, mission based organizations appear to fail to prioritize important objectives. One manifestation of this, when they cannot commit, is a failure to focus, here by investing in a wider range of activities than is technologically sensible. It can also lead donors to penalize more efficient organizations, by them (correctly) inferring that “they don’t need the money”. It can also result in firms failing to diagnose the needs of their clients in the technologically efficient way. Finally, it argues that competition may not be a panacea: while it renders donors happier to donate, it does not imply that its clients are better served. Finally, while the paper is written most closely linked to a non-profit setting, its implications may be broader. Firms of all stripes rely on the willingness to workers to “volunteer”, and this work argues that a myriad of practices may help workers find worth in their actions. As such, these insights may have more general value.

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31 A well known case is Aghion and Tirole’s 1997, on real authority where an agent is more willing to exert effort if, for example, it makes is more likely a policy she gets private benefits from is implemented. Another example is Dewatripont et al., 1999, where mission based firms excessively focus, because broad missions are so diffuse that it make it difficult to identify good performance. Prendergast, 2007, shows how agency concerns had lead to the selection of agents who exhibit hostility to the beneficiaries of public services, as it leads to greater effort. Finally, Glaeser and Shleifer, 2001, examine a setting where firms may be tempted to short-change their consumers, and show that they may take non-profit status to deter themselves from doing so.
References


Appendix

Proof of Proposition 1: The firm chooses \( v(x) \), where \( x \) is the rank of the project carried out, subject to \( v(x) \in \{v, \overline{v}\} \), and the number of slots including \( \overline{v} \) being \( p \). For any \( v(x) \), expected donations are then given by \( \overline{D} = A(\int_0^\infty v(x)f(x)dx) \). The objective of the firm is then to \( \max \sum_{i=0}^D v(i) \) subject to \( D \sim (\overline{D}, \sigma^2) \), \( v(x) \in \{v, \overline{v}\} \), and the number of slots including \( \overline{v} \) being \( p \).

Consider a strategy where \( p-1 \) high valuation projects are in a set \( \Gamma \), and the firm chooses to locate the last high valuation project rather than a low valuation project at a point \( x \). Holding \( f(x) \) and \( F(x) \) constant, this locally increases expected donations by \( A'f(x)\Delta v \). It also raises the expected value of the \( x \) project by \( (1 - F(x))\Delta v \). If \( \mu \) is the marginal return to a dollar raised in donations, then the expected value of increasing \( v \) at point \( x \) is \( A'\Delta v f(x)\mu + (1 - F(x))\Delta v \). What remains to be determined is \( \mu \) which is nailed down by the need to retain \( f(x) \) and \( F(x) \) constant. Specifically, the mean of the distribution of \( F \) rises by \( A'\Delta v f(x) \) by this strategy. As a result, to maintain the distribution of all other high valuation projects, the firm places \( A'\Delta v f(x) \) projects of value \( v \) from slots \( x = 0 \) to \( x = A'\Delta v f(x) \). The value of these additional low valuation projects placed in positions 0 to \( \Delta_H f(x) \) is \( \int_0^{A'\Delta v f(x)} \overline{v}(1 - F(z))dz \), which by the assumption that \( F(0) \) is small implies that this is approximately equal to \( A'\Delta v f(x)\overline{v} \). This then characterizes not only the strategy used by the firm, but also that the marginal value of funds raised is \( \overline{v} \). As a result, the marginal return to increasing project quality at point \( x \) is given by \( A'\Delta v f(x)\overline{v} + (1 - F(x))\Delta v \).

This exercise shows the marginal return conditional on the set of \( f \) and \( F \) being fixed. Yet when total donations rise by \( A'f(x)\Delta v \), the Normal distribution shifts to the right by this much. This implies that all densities between \( f(0) \) and \( f(A'f(x)\Delta v) \) are now available to the firm, where if \( \phi \) and \( \Phi \) are the density and cumulative distribution of a Normal distribution with mean 0 and variance \( \sigma^2 \), then \( f(x) = \phi(-\overline{D}_T + x) \) and \( F(x) = \Phi(-\overline{D}_T + x) \). However, as \( f' > 0 \) for these local changes, the firm cannot prefer these over those located to the right of that, and so they are never chosen if the initial choice is not triage.

The objective of the firm is then to choose the \( \rho \) values of \( x \) where \( \alpha f(x)\overline{v} + 1 - F(x) \) is maximized. Differentiating this with respect to \( x \) yields \( \alpha f'(x)\overline{v} - f(x) \), and so the firm’s welfare from a high value project is maximized at \( f'(x^*) = \frac{1}{\alpha\overline{v}} \). For a Normal distribution with zero mean, \( -\frac{\sigma}{\overline{v}} f'(z) = f(z) \). As a result, \( \frac{f'(x^*)}{f(x^*)} = \frac{1}{\alpha\overline{v}} \) simplifies to
\[
  x^* = \overline{D} - \frac{\sigma}{\alpha\overline{v}}.
\] (26)

Now consider the location of the \( \rho \) high value projects. For the Normal distribution, for
\( x < D, \alpha f'(x)v - f(x) = (\alpha v - \frac{x}{\alpha})f'(x) \), which is increasing until \( x = x^* \) and continuously declining after. As a result, all high value projects are located in a single partition around \( x^* \), between \( x^* - b \) and \( x^* + a \), \((a, b) > 0\), with \( a - b = p \), where

\[
f(x^* + a) - f(x^* - b) = \frac{F(x^* + a) - F(x^* - b)}{\alpha v}
\]  

(27)

We have not, however, shown whether \( x^* - b > 0 \). We use the term triage to refer to the outcome where all \( v \) cases have higher priority than any \( u \) ones, so that \( x^* - b = 0 \). First note that a sufficient condition for triage is that \( x^* \leq 0 \). This arises if \( \alpha \leq \frac{\sigma}{Dv} \). Intuitively, if donations are not sufficiently forthcoming, the desire to allocate efficiently dominates. Yet there remain cases where \( x^* > 0 \) and triage still arises. Specifically, expected funding under triage is given by

\[
D_T = A((F(p + d)v + (1 - F(p + d)))v),
\]

with expected surplus

\[
(1 - F(p))v + \int_0^p vF(x)dx + \int_p^\infty v dF(x)
\]

(28)

where here \( F \) has mean \( D_T \). This implies that if \( \phi \) and \( \Phi \) are the density and cumulative distribution of a Normal distribution with mean 0 and variance \( \sigma^2 \), then \( f(x) = \phi(-D_T + x) \) and \( F(x) = \Phi(-D_T + x) \). Note that \( \Gamma' > 0 \) for \( x \leq x^* \) and is negative beyond that. This implies that we need only compare points 0 and \( p \) to determine whether triage is optimal. This arises if

\[
\alpha v f(0) \geq \alpha u f(p) - F(p),
\]

(29)

or \( f(p) - f(0) \leq \frac{F(p)}{\alpha u} \).

**Proof of Proposition 2:** The firm now chooses its investment in projects, where project \( x \) receives investment \( \Delta(x) \), subject to the budget constraint that total investment cannot exceed \( m \). Given the constraint that projects must be done in the order of their productivity, we can order these such that \( \Delta(i) \geq \Delta(j) \) for \( i > j \). The objective of the firm is to choose \( \Delta(i) \), \( i \in (0, \infty) \), to maximize \( \int_0^D (v + \Delta(i))di \), subject to \( D = A(v + \Delta(D)) \), \( \Delta'(i) \leq 0 \), and \( \int_0^\infty C(\Delta(i))di \leq m \).

First note that it is never the case that \( \Delta(D) < \Delta(i) \), for any \( i < D \), as all projects \( i \leq D \) are completed, and reallocating resources to project \( D \) increases donations. As \( \Delta'(i) \leq 0 \), this implies that there is a constant \( \Delta \) for all \( i \in [0, D] \). Furthermore, there is no value to increasing \( v \) for \( i > D \), and so the firm chooses \( \Delta(i) = \frac{m}{D} \). The equilibrium condition is then determined by the condition that the marginal project is exactly the number of donors, \( A(v + \frac{m}{D}) = b^* \) as required. Note that the left hand side of this expression is decreasing in \( b^* \) and the right hand side increasing, and so this is uniquely defined.
Proof of Proposition 3: Consider the equilibrium of the game. In the equilibrium, the expected number of donors has to be consistent with the marginal project implied by that number of donors. They give based on their belief of the marginal project. The marginal project is either a project with the signal or one without. There are three cases. The first is the low generosity case, where \( A(\gamma \varpi + (1 - \gamma) v) \leq p \). In that case, the marginal project has signal \( s \). In that case, donations are given by \( D_0(\gamma) \), where \( D_0(\gamma) = A(\gamma \varpi + (1 - \gamma) v) \), which is increasing in \( \gamma \). Hence without diminishing returns having an impact, efficient firms attract more funding.

The second case is high generosity. The expected value of projects without signal \( s \) is \( v + p \Delta v / (N - p) \). Then for sufficiently generous donors \( A(v + p \Delta v / (N - p)) > p \). Let \( D_1(\gamma) \) be donations. The expected value of projects without signal \( s \) is \( v + p \Delta v / (N - p) \), and so

\[
D_1(\gamma) = A \left( v + \frac{p(1 - \gamma) \Delta v}{N - p} \right). \tag{30}
\]

The final case is intermediate generosity. This is where \( A(\gamma \varpi + (1 - \gamma) v) > p > A(v + p \Delta v / (N - p)) \). In this case, if the donor believes that \( \varpi \) is the marginal project, she gives enough such that it no longer is, while if she believes that \( v \) is the marginal project, she doesn’t give enough to make that the case. As a result, there is no equilibrium where the donor \( i \) gives if \( \Delta S > \lambda_i \), as such actions are not consistent with the beliefs. Instead the outcome is simple: there are exactly \( p \) donors, and they fund all high value projects. The marginal donor is not indifferent here, unlike the reduced form that we have used above (this is why we endogenize preferences here). If more than \( p \) donors arrive, the marginal project no longer is \( \varpi \) and so fewer than \( p \) donors would want to donate, and if less than \( p \) donate, more would want to. Hence exactly \( p \) donors is the outcome in the intermediate generosity case.

Note that the high generosity case is the only one where the issues of the paper are relevant, as this is where the marginal project is lower than inframarginal ones. As such, consider that case. Here donations are decreasing in firm efficiency: \( D'_1 = -A' \frac{p \Delta v}{N - p} < 0 \). Expected surplus is then given by

\[
S = p(\varpi - (1 - \gamma) \Delta v) + \nu(N - p)(v + \frac{p(1 - \gamma) \Delta v}{N - p}), \tag{31}
\]

where \( \nu = \frac{D_1(\gamma) - p}{N - p} \leq 1 \) is the probability that a project without signal \( s \) will be completed. The problem is only interesting if \( \rho < 1 \): otherwise all \( N \) projects are done. Then

\[
\frac{dS}{d\gamma} = p\Delta v(1 - \nu) + A'(\gamma)(v + \frac{p(1 - \gamma) \Delta v}{N - p}). \tag{32}
\]
The first part of this is the usual efficiency gain from better allocation - as $\nu < 1$, identifying high quality projects has value as they are more likely to be completed. However, this must be traded off against reduced funding, given by the second term. However, the second term dominates for large enough $A'$.

**More general distributions:** Rewrite (6) as $[\alpha \nu \eta(x) + 1](1 - F(x))\Delta_v$, where $\eta(x)$ is the hazard rate evaluated at $x$. First consider the Exponential distribution $f(x) = \lambda e^{-\lambda x}$, where the hazard rate is given by $\frac{f(x)}{S(x)} = \lambda$ where $S(x)$ is the survival rate. The analog to (6) is $[\alpha \nu + 1]e^{-\lambda x} \Delta_v$. This is always decreasing in $x$, and triage is always used.

An alternative is the Rayleigh distribution, $F(x) = 1 - e^{-x^2/\sigma^2}$, for $x > 0$. Then the hazard rate is $\frac{f(x)}{S(x)} = \frac{x}{\sigma}$, which is linearly increasing in $x$. Here the analog to (6) is $[\alpha \nu \frac{x}{\sigma} + 1]e^{-\frac{x^2}{\sigma^2}} \Delta_v$. Triage may now no longer be optimal. Specifically, if $\Omega(x) = [\alpha \nu \frac{x}{\sigma} + 1]e^{-\frac{x^2}{\sigma^2}}$, then $\Omega'(x) > 0$ iff $\alpha \sigma \nu > \frac{x}{\sigma} + \alpha x^2 \nu$. This is positive for small $x$ but decreasing for larger $x$. This implies that the firm places its high value projects in an intermediate range of $x$’s.

**Differentiated Competition** Consider the case where $\gamma_i \sim U[0, a]$ and $a > \Delta \Sigma$. Here the density is flat and sufficiently disperse. In the absence of competition, the benchmark analog to (6) is $f(x)\frac{1}{a} + (1 - F(x))$. With competition, the return to firm 1 increasing $v$ by $\Delta_v$ at point $x$ is given by

$$f(x)v\left(\frac{1}{a} - \frac{\Delta \Sigma^2}{a^2}\right) + (1 - F(x)).$$

(33)

With $a$ large, the marginal role of the other firm’s marginal actions on the firm is low. In equilibrium, both firms follow a similar strategy, implying an optimal delay of

$$x^* = D - \frac{\sigma}{v\left(\frac{1}{a} - \frac{\Delta \Sigma}{a^2}\right)}.$$  

(34)

This has less incentives to distort project priority than without competition. This is for two reasons. First, if preferences are relatively flat ($a$ high), then there is little business stealing from the competitor firm. Second, the firm’s market is smaller (by half) from having a competitor, and so the marginal benefit to being more attractive to donors falls.

A pretty robust implication of increased competition is that it gives more rents to consumers. In a setting where firms are characterized by agency problems, competition can alleviate a variety of “x-inefficiency” problems. However, this is not the case here. Instead, the only distortions come from then desire of donors to matter on the margin. One way

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32 As the mean of the exponential is $\frac{1}{\lambda}$, this implies that $\lambda$ is defined by $\lambda = \frac{1}{D}$.

33 This implies that $D = \sqrt{\pi} \sigma$, as this is the expected value of the Rayleigh.
then to interpret the results above with a single firm is a monopoly-like tradeoff between the firm trying to implement the socially optimal outcome and the willingness of donors to give. The impact of competition is then to make donors more elastic, giving their preferences more weight in the equilibrium allocation. As a result, it should not be surprising that competition can reduce efficiency.