This paper analyzes debt maturity structure for borrowers with private information about their future credit rating. Borrowers' projects provide them with rents that they cannot assign to lenders. The optimal maturity structure trades off a preference for short maturity due to expecting their credit rating to improve, against liquidity risk. Liquidity risk is the risk that a borrower will lose the nonassignable rents due to excessive liquidation incentives of lenders. Borrowers with high credit ratings prefer short-term debt, and those with somewhat lower ratings prefer long-term debt. Still lower rated borrowers can issue only short-term debt.

I. INTRODUCTION AND OVERVIEW

This paper analyzes the choice of debt maturity by firms. Short-term debt matures before the cash flows arrive from a firm's investments and must be refinanced at terms that depend on its future credit rating. Long-term debt, in contrast, has maturity matching the timing of the cash flows. Maturity is thus measured relative to the timing of arrival of cash flows, rather than in calendar time. In this paper I ask how borrowers choose maturity structure and how their choice depends on their credit rating. As a stylized fact, firms with high credit ratings issue short-term debt (commercial paper) directly to investors. Firms with lower credit ratings issue long-term bonds or borrow through financial intermediaries such as banks. The lower rated borrowers' bank loans are of relatively short term. Borrowers who rely heavily on short-term debt are then a mix of the very high and low rated borrowers, with the middle rated borrowers using more long-term debt. This paper develops a model that explains this behavior.

Debt maturity choice is analyzed as a trade-off between a borrower's preference for short-term debt due to private information about the future credit rating, and liquidity risk. Liquidity risk from short-term debt arises from the borrower's loss of control rents in the event that lenders are unwilling to refinance when bad
news arrives. Liquidity risk is the risk that a solvent but illiquid borrower is unable to obtain refinancing. The model fits this characterization because, even without using their private information, some borrowers who are liquidated would invest their own capital (if it were available) to avoid default. The model characterizes borrowers who are averse to bearing this risk.

The main result is that there is a credit rating such that those with higher ratings prefer short-term debt as a type of "bridge financing" that allows them to choose to refinance when good news arrives, while lower rated borrowers prefer long-term debt. In addition, some very low rated borrowers have no choice but to use short-term debt, despite the control that it gives to lenders. The model thus identifies two categories of short-term borrowers: the low rated who have no choice, and the high rated who use short-term debt to time their borrowing to take advantage of the arrival of information. Borrowers with ratings in between these rely more heavily on long-term debt.

The model measures the borrower's credit rating for a given amount of leverage, and this might alternatively be termed its credit reputation. Another interpretation of the model is of the choice of debt maturity for a leveraged buyout (a recapitalization without outside equity) conducted by incumbent management who have private information about the firm's prospects. Liquidity risk arises from debt that is of shorter maturity than assets. A key determinant of the actual calendar maturity is the timing of cash flows. This paper abstracts from cash flow timing differences and provides a theory of the gap between debt maturity and asset maturity.¹

Short maturity debt creates liquidity risk because sometimes the borrower is unable to refinance and the lender liquidates when the borrower would not choose to if he or she were the sole owner of the firm. Short-term lenders liquidate (are unwilling to refinance current management) too often from the borrower's point of view because there are constraints on pledging future rents to lenders: the amount that can be pledged to lenders may be less than the value they receive from liquidation, yet the total future rents exceed the liquidation value. These constraints arise from many possible sources: private information by borrowers, moral hazard, and monitoring costs. These contracting problems imply that

¹ The only empirical study I am aware of that looks at this gap is Morris (1989), but it does not use credit rating data.
difficulties in obtaining initial financing persist at refinancing time and can lead to excessive liquidation.

This approach to modeling liquidity risk differs from that in Diamond and Dybvig [1983], but the two approaches are complementary. In Diamond and Dybvig lenders refuse to refinance an illiquid borrower (a bank) who would otherwise remain solvent because of strategic uncertainty about other lenders’ actions. The approach in this paper builds on Diamond [1987], which illustrates the liquidity risk of attempting a refinancing for borrowers subject to moral hazard. My analysis, especially that in Section V, also relates to Hart and Moore [1989], who show that some types of debt renegotiation provide a disadvantage to short-term debt. Hart and Moore also analyze several more general renegotiation issues unrelated to maturity structure, and that paper and Scharfstein and Bolton [1990] examine effects of inefficient liquidation under symmetric information. Also related is Sharpe [1989], who concludes that all debt should be long term, in a model where liquidation is never efficient and all information is public. 2

The second element of this paper’s model is private information of the borrowers. A borrower’s willingness to choose short-term debt and thus subject his or her financing costs to new information might depend on the private information. Flannery [1986] presents a model of this influence on maturity choice when there is no liquidity risk. 3 His model implies that all borrowers select short-term debt, unless short-term debt causes higher transaction costs, because no borrowers have reason to prematurely reveal bad news about themselves. I incorporate liquidity risk, and this leads to a trade-off for borrowers because short-term debt can lead to lost control rents. In my model even borrowers with information that their credit rating is likely to improve can prefer long-term debt.

Section II outlines the basic model. Section III describes the information that will arrive about borrowers and shows how lenders respond to the information given short- and long-term debt. Section IV presents some simple examples to illustrate a trade-off between the liquidity risk of lost control rents, and the incorporation of new information about a borrower’s credit rating


3. Also related is Boyd and Prescott [1986] on the signaling value of agreeing to have one’s project evaluated.
into the cost of financing. Section V develops a model in which to compare several types of debt contracts. Section VI shows how the borrower’s credit rating influences maturity structure choice. Section VII generalizes this to an optimal mix of short- and long-term debt. Section VIII discusses firms whose credit rating is so poor that no one will lend long term. Section IX provides some more general motivation for the existence and magnitude of nonassignable control rents. Section X uses a game-theoretic equilibrium refinement to show that the equilibrium is reasonable. Section XI concludes the paper.

II. THE MODEL

Borrowers have no capital, and each needs to fund an indivisible investment project. The project’s ex ante prospects and the ex post cash flows are private information observed only by the borrower. The borrower’s project can be liquidated. If it is liquidated, the borrower can be prevented from consuming its current or future cash flows and control rents. In this case, an optimal financial contract is a debt contract enforced by the right to liquidate if the debt is not repaid in full. The only reason I use the assumption of unobservable cash flows is in restricting attention to debt contracts.

Borrowers and lenders are risk neutral. Lenders consume at date 2 and have a constant returns-to-scale investment technology that returns $R$ per unit invested per period. One unit invested at date 0 returns $R$ units at date 1; and if this is invested until date 2, the terminal value is $R^2$. A borrower with a given credit rating can borrow as long as lenders receive an expected return of $R$ per period, per unit loaned. The model abstracts from unexpected changes in riskless interest rates. One interpretation of this is that the borrower hedges these changes using interest rate futures, options, or swaps. Because borrowers have no private information about future riskless interest rates, they can hedge these risks without revealing any information about themselves.

There are two types of projects, and each borrower has one type of project. The borrower’s type is his private information. Both types require $1 in (outside) capital at date 0 and produce cash flows only on date 2 (none on date 1). Each project yields a cash flow of $X > 0$ when successful, and each project also produces a nonassignable control rent of $C$ if the management has control at
All projects can be liquidated at date 1 for \( L \). A successful project yields a higher return when not liquidated, because \( L < X/R \). Liquidation should be broadly interpreted to include other situations where the borrower loses control of the project at date 1 and lenders attempt to capture maximum repayment: see Section IV where moral hazard causes the loss without liquidation. Projects have no liquidating value at date 2. The two types of project differ only in the probability that the return \( X \) is received. The two types of borrower are as follows.

**Type G**  
The project returns a cash flow of \( X > R^2 \) for sure at date 2. This is a positive net present value project in terms of cash flows.

**Type B**  
The project returns a cash flow of \( X \), with probability \( \Pi \), and returns zero with probability \( 1 - \Pi \). If \( X < R^2 \), and the project has a negative net present value in terms of cash flows.

Lenders’ information about a borrower is the credit rating. As of date 0, it is as follows. A borrower has a credit rating \( f \) if lenders assign the borrower a probability \( f \) to being type G and \( 1 - f \) to being type B. Recall that the borrower knows his or her own type. The lower a borrower’s credit rating, the higher is the promised interest rate, due to the higher default rate of type B’s. As of date 0, the probability of repayment of a loan maturing on date 2 is: \( \Pi + f(1 - \Pi) \).

Additional information about borrowers arrives at date 1. This information is described in the next section.

### III. MATURITY STRUCTURE CHOICE: INFORMATION ARRIVAL

Information arrives at date 1 about each borrower. This information arrives privately to lenders and is not verifiable, so contracts written contingent on the information cannot be enforced. If borrowers refinance at date 1, the terms of refinancing will depend on the information, since the credit rating will be either upgraded or downgraded. I assume that there are only two realizations of information at date 1. Denote the credit rating (conditional probability that a borrower is of type G) given an

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4. The following generalizations can be made without changing our results. The control rent can differ by type of borrower, it can be risky, and it can be related to the realized cash flow. The control rent can be per period (the rent in the first period is a “sunk benefit” as of date 1).
upgrade by \( f^u \), and rating given a downgrade by \( f^d \). I assume that \( f^u > f > f^d \): the probability of being of type G is greater given an upgrade than given a downgrade.

All borrowers receive either an upgrade or a downgrade. For simplicity, I assume that all type B borrowers receive a downgrade, implying that \( f^u = 1 \) because only type G’s ever receive an upgrade. All of my results hold when there is a positive probability of an upgrade of a type B, as long as the probability differs from that of a type G, but this extra parameter is not needed. Let \( e \) denote the probability that a type G borrower receives an “erroneous” downgrade (\( 1 - e \) is the probability of an upgrade for a type G). Because all type B’s are downgraded, Bayer’s Law implies that \( e \), the probability that a type G borrower receives a downgrade is \( e = \frac{f^d(1 - f)}{f(1 - f^d)} \), where \( f^d \) is the credit rating given a downgrade and \( f \) is the initial (date 0) credit rating of the borrower.

A. Maturity Choice

Long-term debt is debt floated at date 0 that matures at date 2, with no refinancing at date 1. The face value \( \rho \) of this debt is set so that lenders who lend $1 get an expected return of \( R^2 \), realizing that debt is repaid with probability \( \Pi + f(1 - \Pi) \). The face value of a long-term bond is given by \( \rho = R^2(\Pi + f(1 - \Pi))^{-1} \), if \( \rho \leq X \). If \( \rho > X \), then borrowers with credit rating \( f \) cannot borrow long term, because they cannot provide lenders with an expected return of \( R^2 \).

On date 1 each borrower receives either an upgrade or a downgrade. The information does not influence the face value of long-term debt and does not lead to liquidation because long-term lenders have no such rights, but the information does influence the secondary market value of that debt. The date 1 market value is useful for comparison with the case of short-term debt because it measures the value received by lenders from borrowers who receive upgrades and downgrades. Let \( V^u_i \) and \( V^d_i \) denote the date 1 market value of the long-term debt given, respectively, an upgrade and a downgrade. Given an \textit{upgrade}, the long-term debt is repaid for sure, and its date 1 market value is \( V^u_i = \rho/R \). Given a \textit{downgrade}, the long-term debt is repaid with probability \( q^d = \Pi + f^d(1 - \Pi) \), and its date 1 value is \( V^d_i = q^d\rho/R \).

Short-term debt is debt financed at date 0, maturing on date 1 with face value \( r_1 \). The date 1 repayment comes either from refinancing at date 1 at interest rates contingent on the realization of date 1 information or from the proceeds of liquidation at that
The face value of short-term debt issued at date 1 is set so that given the information about a borrower at that date, lenders at date 1 get an expected return of $R$ per unit invested (e.g., if they invest $R$, they receive an expected return of $R^2$). The amount that must be raised at date 1 depends (among other things) on $r_1$, the face value of short-term debt issued at $t = 0$.

When the borrower cannot repay in full at date 1, the decisions made at that time serve the interests of date 0 short-term lenders, because they have the control rights to force liquidation when they are not paid in full. Suppose that the face value of the refinanced debt at date 1, given a downgrade, is $r^d$. The debt issued by downgraded borrowers at date 1 will be repaid at date 2 with probability $q^d = \Pi + (1 - \Pi)f^d$ (and repays zero with probability $1 - q^d$). Debt with face value $r^d$ raises $q^d r^d/R$ on date 1. The debt maturing at date 1 can be repaid in full by a new debt issue if $q^d X/R > r_1$, because $X$ is the most a borrower can pay. If the maturing debt cannot be repaid in full, lenders choose the resolution that maximizes their repayment. Making a concession to “work out” a default by accepting the maximum that a new debt issue can raise yields $q^d X/R$, while liquidation yields $L$ (and destroys the borrower’s control rent). Let the value of date 1 repayment received by date 0 short-term lenders given a downgrade be denoted by $V^d = \min[r_1, \max\{L, q^d X/R\}]$.

Short-term borrowers who receive an upgrade refinance to pay the full face value of their date 0 debt: they pay $r_1$ on date 1, and the value on date 1 of date 0 short-term debt to an upgraded borrower is $V^u = r_1$. This debt maturing at date 2 is repaid with probability 1; the face value of debt issued at date 1 with good news is then $r^u = r_1 R$. Let $p_j$ be the probability that news $j$ arrives for a borrower of unknown type. Note that $p^u = (1 - e)f$ and $p^d = 1 - p^u$. The expected return of a $t = 0$ short-term lender is $p^u r_1 + p^d V^d$. Equating this to the expected return $R$, required by one-period lenders, implies that $r_1 = [R - p^d V^d]/p^u$.

Section V nests the various short- and long-term alternatives in a single model, by viewing the maturity structure choice as

5. The old lenders can be the same people as the new lenders.
affecting the date 1 information contingent payoffs received by lenders. Debt maturity structure influences the date 1 liquidation decision and the face value of refinanced debt at date 1. Before presenting this analysis, the next section illustrates the important issues with a series of numerical examples that show some determinants of the maturity structure preferred by type G borrowers. The maturity preferred by type G's turns out to be offered by all borrowers, because type B's would reveal themselves if they offered another maturity, and they would not be able to borrow.

IV. EXAMPLES OF MATURITY CHOICE WHEN BOTH MATURITIES ARE FEASIBLE

This section presents examples where any short-term debt to be refinanced at date 1 is riskless because liquidation by itself can provide date 0 lenders with a normal rate of return, or \( L = R \). To keep units simple, the examples assume that interest rates are zero, or \( R = 1 \). This gives lenders a simple liquidation rule: liquidate whenever the debt is not repaid in full. It implies that the face value of date 0 short-term debt is \( r_1 = 1 \).

In all examples, I assume the following. The probability of repayment by type B borrowers is \( \Pi = \frac{1}{2} \). The sum of cash flows from successful projects and nonassignable control benefits is \( X + C = \frac{13}{4} \) (\( X \) and \( C \) will vary across examples), except in example 1, where \( X = \frac{13}{4} \) and \( C \) is any nonnegative value. At date 1 information about the type of a borrower arrives, and the information has two realizations: conditional on an upgrade in rating, a borrower is known to be type G for sure; \( f_u = 1 \), where \( f_u \) is the probability of type G given an upgrade. All type B and some type G borrowers get a downgrade. Conditional on a downgrade in rating, the probability of being a type G borrower is \( f_d = \frac{1}{3} \).

**EXAMPLE 1. An example without liquidation.**

Suppose that the date 0 credit rating is \( f = \frac{1}{2} \) (half the borrowers are type G) and the projects return cash of \( X = \frac{13}{4} \) when successful. In this example there will be no liquidation, implying that the control rent \( C \) is not lost on a downgrade. As a result, the

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6. This example illustrates the informational role of short-term debt, similar to Flannery [1986]. Taken by itself, this example suggests that all debt is short term if borrowers have private information about their future prospects.
example applies to any \( C \geq 0 \). The basic example assumes that \( C = 0 \).

**Long-term debt** is repaid by all type G’s and by a fraction \( \Pi = \frac{1}{2} \) of type B’s: the probability of repaying long-term debt is then \( \frac{3}{4} \), and the face value of long-term debt to provide an expected return of 1 is \( \rho = 4/3 \). The payoff of a type G borrower with long-term debt, \( \text{Long} \), is then \( \text{Long} = X + C - \rho = \frac{5}{12} + C \).

**Short-term debt.** Because all type B’s are downgraded, and \( f^d \), the probability of being type G given a downgrade, is \( \frac{1}{3} \), the probability that a type G receives a downgrade is \( e = f^d (1 - f) / f (1 - f^d) = \frac{1}{2} \). Given a downgrade, the repayment probability is \( q^d = \Pi + (1 - \Pi) f^d = \frac{2}{3} \), implying that issuing debt at date 1 with face \( r^d = (q^d)^{-1} = 3/2 \) will raise 1 to repay the initial short-term debt without liquidation. Borrowers who receive an upgrade refinance with short-term debt at date 1 with face value \( r^u = 1 \). The payoff of a type G borrower with short-term debt (facing a probability \( e = \frac{1}{2} \) of a downgrade, \( 1 - e = \frac{1}{2} \) of an upgrade), \( \text{Short} \), is then

\[
\text{Short} = e(X + C - r^d) + (1 - e)(X + C - r^u) \\
= X + C - [e(r^d) + (1 - e)r^u] \\
= 1\frac{1}{3} + C - [\frac{1}{2}(3/2) + \frac{1}{2}(1)] = 1\frac{1}{2} + C > \text{Long} = \frac{5}{12} + C.
\]

When there is no chance of liquidation destroying control rents, short-term debt is preferred by type G borrowers because it lowers their expected financing costs. Short-term debt lowers a good borrower’s expected financing cost because a downgrade that increases a borrower’s repayment is less likely for him or her than for an average borrower.

The type G’s receive lower financing costs because those who receive downgrades provide lenders with a higher date 1 return given short-term debt, \( V^d_i = L = 1 \), than given long-term debt, \( V^d_i = q^d \rho / R = \frac{8}{9} \), and type G’s have a lower than average probability of receiving a downgrade. With short-term debt, type G’s provide a smaller fraction of the payments to lenders (and type B’s a larger fraction) than with long-term debt.

**Example 2. Loss of control.**

Keep everything from example 1, but let \( C = 0.35 \) and \( X = 1.4 \). Example 1 with \( C = 0 \) is our point of comparison. The type G borrower’s payoff from issuing long-term debt with face \( \rho = 4/3 \) remains equal to \( X + C - \rho = \frac{5}{12} \).

With short-term debt lenders liquidate given a downgrade to
receive $L = 1$, because without liquidation, the maximum that they receive at date 2 is $X = 1.4$, with probability $q^d = \frac{2}{3}$, an expected return of $\frac{14}{15}$. Given an upgrade, lenders refinance with $r^u = 1$. A type G borrower’s payoff from issuing short-term debt is $e(0) + (1 - e)(X + C - r^u) = \frac{1}{2} (1.75 - 1) = \frac{5}{12}$. This is less than $\frac{5}{12}$, the payoff from long-term debt. Type G borrowers prefer long-term debt that allows them to always retain the control rents, $C = 0.35$, even though this eliminates sensitivity of their financing costs to favorable new information.

With short-term debt the project is liquidated given a downgrade, even though it is inefficient (taking account of the nonassignable control rent $C$) given date 1 public information. A borrower who had only public information would pay to retain control if the funds were available, but the funds are not available. This is the liquidity risk of loss of control. Example 1 showed that if liquidity risk is absent (no liquidation when $q^dX + C > L$), then short-term debt is preferred. If liquidity risk is present, then long-term debt can be preferred by type G borrowers. It is still true that short-term debt provides lenders with higher date 1 returns than long-term for those who receive a downgrade ($V^g_d = 1 > V^l_d = \frac{5}{12}$), and that type G’s are equivalently less likely than average to receive a downgrade, but now providing this higher repayment by those who receive a downgrade destroys the control rent of $C = 0.35$. The next example illustrates the trade-off between these two effects of short-term debt.

**Example 3.** A good borrower with a higher credit rating prefers short debt.

Keep everything from example 2 (cash return $X = 1.4$, managerial control rent $C = 0.35$, liquidation value $L = 1$, and credit ratings given upgrade, $f^u = 1$, and downgrade, $f^d = \frac{1}{2}$). Holding the credit rating given downgrade constant keeps the ex post costs of financial distress constant. Increase the initial credit rating $f$ to $f = \frac{3}{4}$ (instead of $\frac{1}{2}$). The probability of repayment of long-term debt is then $\Pi + (1 - \Pi)f = \frac{7}{8}$, and the long-term face value is $\frac{8}{7}$. The payoff to a type G from issuing long-term debt is then $X + C - \rho = 1.75 - \frac{5}{7} = \frac{17}{28}$.

Given a date 0 credit rating of $f = \frac{3}{4}$ and a rating given downgrade of $f^d = \frac{1}{3}$, the probability of a type G receiving a downgrade is $e = f^d(1 - f)/(f(1 - f^d)) = \frac{1}{6}$. A higher rated type G borrower (with $f = \frac{3}{4}$) has a lower probability of a given drop in credit rating than a type G borrower with a lower rating (of $f = \frac{1}{2}$,
in example 2). There is still liquidation given a downgrade. The payoff from issuing short debt is 
\[ e(0) + (1 - e)(X + C - r^\mu) = \frac{5}{6}(1.75 - 1) = \frac{5}{6} \]
which exceeds the \( \frac{17}{28} \) payoff from issuing long-term debt. Therefore, the higher rated borrower prefers short debt.

As in example 2 the value of short-term lenders' claim on date 1 given a downgrade is greater than long-term lenders' claim given a downgrade, and the control rent of \( C = 0.35 \) is destroyed given a downgrade. The higher credit rating as of date 0 \( f = \frac{3}{4} \) instead of \( \frac{1}{2} \) in example 2) implies that the probability that a type G borrower receives a downgrade \( (e = \frac{1}{6}) \) relative to an average borrower’s probability of a downgrade \( p^d = ef + (1 - f) = \frac{9}{16}, \) or \( e/p^d = \frac{1}{6}/\frac{9}{16} = \frac{4}{15}, \) which exceeds equivalent ratio in examples 1 and 2 was \( \frac{5}{15}. \)

Type G borrowers in example 2 have a credit rating just above that where pledging all cash flows will lead them to lose their financing. These borrowers are the most averse to bearing the liquidity risk of short-term debt because their probability of getting the small downgrade that leads to liquidation need not be much lower than that of type B borrowers. The extra repayment from type B’s who receive a downgrade is not sufficient to make the type G’s risk loss of their control rents. Conversely, the higher rated type G borrower in example 3 is much less likely than the average high rated borrower to get a very large downgrade. The higher rated type G borrower prefers short-term debt because high debt payments are extracted from liquidating many type B’s for each type G that is downgraded and loses control rents in liquidation.

V. COMPARING DEBT STRUCTURES THAT DIFFER BOTH IN INFORMATION AND LIQUIDATION

The maturity of debt that is preferred by type G borrowers is chosen by all borrowers.\(^7\) Choosing a maturity that only type B’s would prefer would reveal that a borrower was of type B, and then no loan would be made. Section X discusses the borrower beliefs that support this outcome. I analyze the preferred maturity structure of type G borrowers, but this should be interpreted as the

\(^7\) Type B borrowers would always prefer long-term debt, if the choice did not reveal information. However, when type G borrowers prefer short-term debt, the choice of long-term debt would reveal that a borrower was type B, with negative net present value projects, and no one would lend to a revealed type B.
maturity offered by all borrowers with the same date 0 credit rating as the type G borrower analyzed.

The examples in the last section illustrate how debt maturity influences the impact of date 1 information on financing costs, liquidation decisions, and the value of lenders' claims. To provide a framework to develop these ideas, this section examines the preferences of type G borrowers for arbitrary contracts that differ in the date 1 value of lenders' claims \( V^d \) and \( V^u \), and possibly differing date 1 liquidation decisions contingent on downgrade. This result is then applied to the analysis of several classes of debt contracts. Recall that the date 1 value of lenders' claims given a downgrade, \( V^d \), is always higher for short-term debt than for long-term, because either the face value of the debt is increased, or there is liquidation that yields more than increasing the face value.

Table I provides a summary of the paper's notation.

Lemma 1 extends example 1, where liquidation never occurs and where type G borrowers prefer short-term debt.

**Lemma 1.** Comparing two debt contracts issued on date 0, neither of which involves liquidation on a downgrade at date 1, the one with the higher value date 1 market value given a downgrade, \( V^d \), is preferred by type G borrowers. This implies that short-term debt is preferred when there is not liquidation on a downgrade.

**Proof.** See Appendix.

The idea behind Lemma 1 is simple. When the liquidation decision is not affected, increasing the payment by those who get a downgrade lowers the expected financing costs of type G borrowers who are less likely than average to get a downgrade. Lemma 2 characterizes the trade-off between extracting larger payments from those who are downgraded versus losing control rents in liquidation. The lemma ranks debt structures from the point of view of type G borrowers.

**Lemma 2.** Comparing a debt structure that does not imply liquidation and that implies a value of debt claim given a downgrade of \( V^d \), and an alternative debt structure that implies liquidation given a downgrade, type G borrowers prefer the debt structure that implies liquidation if and only if \( C < C^* \): the control rent is less than or equal to a positive bound \( C > 0 \),
TABLE I
SUMMARY OF NOTATION

\begin{align*}
f & = \text{Probability that a borrower is of type G, given date 0 public information, sometimes referred to as the credit rating.} \\
f^d & = \text{Probability that a borrower is of type G, given a downgrade at date 1.} \\
f^u & = \text{Probability that a borrower is of type G, given an upgrade at date 1 (} f^u = 1). \\
\Pi & = \text{Probability that a type B borrower repays at date 2 (probability that the project is successful).} \\
q^d & = \text{Probability that a borrower who receives a downgrade repays at date 2} \\
& \quad (q^d = \Pi + f^d [1 - \Pi]). \\
X & = \text{Return of a successful project (type G project succeeds with certainty).} \\
C & = \text{Control rent of a project not liquidated before date 2.} \\
e & = \text{Probability that a type G borrower receives a downgrade} \\
& \quad \left( e = f^d \frac{[1 - f]}{f [1 - f^d]} \right). \\
p^u & = \text{Probability that a borrower of unknown type receives an upgrade} \\
& \quad (p^u = [1 - e] f). \\
p^d & = \text{Probability that a borrower of unknown type receives a downgrade} \\
& \quad (p^d = 1 - p^u). \\
R & = \text{Expected return required by lenders on a one-unit investment, per period.} \\
\rho & = \text{Face value of long-term debt, maturing at date 2.} \\
r^d & = \text{Face value of short-term debt maturing at date 2, issued after a downgrade at date 1.} \\
r^u & = \text{Face value of short-term debt maturing at date 2, issued after an upgrade at date 1.} \\
L & = \text{Date 1 proceeds of liquidation.} \\
V_d & = \text{Date 1 market value of a debt contract given a downgrade.} \\
V_{d}^s & = \text{Date 1 market value of a short-term debt contract given a downgrade.} \\
V_{d}^l & = \text{Date 1 market value of a long-term debt contract given a downgrade.} \\
V_{u}^s & = \text{Date 1 market value of a short-term debt contract given an upgrade.} \\
V_{u}^l & = \text{Date 1 market value of a long-term debt contract given an upgrade.} \\
\end{align*}

given by

\[
\hat{C} = \left[ \frac{LR}{f^d} \right] - X - R \frac{1}{f^d} \left[ \frac{1}{q^d} - 1 \right].
\]

Sufficient conditions for type G borrowers to prefer liquidation are the absence of control rents, \( C = 0 \), and liquidation yielding sufficiently high proceeds: \( L > [q^d X + f^d C]/R \).

Proof. See Appendix.

The next section applies this to short-term versus long-term debt, where short-term debt is the debt contract that could result in liquidation. If borrowers prefer that lenders liquidate on a downgrade, this is sufficient for them to prefer short-term debt.
The sufficient condition for borrowers to prefer liquidation, \( L > \left[ q^dX + f^dC \right]/R \), has the following interpretation. If \( L > \left[ q^dX + C \right]/R \), there would be liquidation even if the control rent were assignable to lenders, because \( q^d \) is the probability that a loan is repaid given a downgrade and the control rent \( C \) comes to the borrower for sure. In this case, liquidation is efficient given public information. The increased repayment that lenders receive on a downgrade sufficiently reduces borrowing costs given an upgrade to compensate the borrower for the lost control rents. Even if liquidation is slightly inefficient, and \( L \) is less than \( \left[ q^dX + C \right]/R \) but greater than \( \left[ q^dX + f^dC \right]/R \), type G borrowers prefer liquidation because the control rents lost on a downgrade belong to type G borrowers with probability \( f^d \), and the control rents lost by type B borrowers are not considered in the choice made by type G’s.

VI. MATURITY STRUCTURE PREFERENCE BY CREDIT RATING

One can use Lemmas 1 and 2 to show how the choice between short- and long-term debt depends on the credit rating of the borrower. Long-term debt never leads to liquidation on date 1 because lenders do not have control rights on that date. It commits lenders to continue to lend at date 1 with a face value of \( \rho \). All long-term debt implies that the value of the date 1 claim of date 0 lenders given a downgrade is \( V_{i^d} = q^d\rho/R \), where \( \rho = R^2[\Pi + f(1 - \Pi)]^{-1} \).

With short-term debt the date 1 value of a debt claim given a downgrade is \( V^d = \min[R, \max[L, q^dX/R]] > V^d_i \). If the short-term debt is repaid in full given a downgrade, lenders have no liquidation rights. Payments in full given a downgrade implies that lenders are always repaid in full and that date 0 short-term debt is riskless; this leads to \( r_i = R \). If the debt cannot be repaid in full, then lenders have the control right to liquidate: lenders receive the larger of \( L \) and \( q^dX/R \).

Proposition 1 characterizes the minimum cost form of financing for a type G borrower.

**Proposition 1.** If the control rent is zero, the liquidation value is sufficiently close to efficient (it covers lost control rents of type G’s): \( L \geq \left[ q^dX + f^dC \right]/R \), or there is not liquidation on a downgrade: \( q^dX/R > L \), then type G borrowers of all credit ratings prefer short-term debt. If \( C > 0 \), and there is liquidation that is not close enough to efficient: \( L \in (q^dX/R, \)
then for sufficiently low credit ratings, \( f \in (0, S) \), type G borrowers prefer long-term to short-term debt, and the lowest rating that prefers short-term debt is \( S > 0 \), given by

\[
S = \frac{\Pi[R^2 - LR + f^d(X + C + X)]}{(1 - \Pi)(LR - f^d(C + X))}.
\]

The rating \( S \) is increasing in \( L \) and decreasing in \( C \). All ratings prefer long-term debt if \( S > 1 \), or \( f^d > (LR - R^2\Pi)/(X + C - R^2\Pi) \).

**Proof.** See Appendix.

Consider the conditions where low rated type G borrowers prefer long-term debt. These require liquidation and liquidity risk: \( L < [q^dX + f^dC]/R \) implying that liquidation occurs only because the control rent cannot be pledged. The control rent is lost in liquidation, and liquidation yields lenders smaller proceeds than the cost to type G borrowers. A date 0 credit rating \( f \), only a small amount above the downgrade credit rating \( f^d \), implies that a type G is only slightly less likely than an average borrower to receive a downgrade. If a type G borrower is only slightly less likely than an average borrower to receive a downgrade, the loss of the control rent on downgrade makes the borrower unwilling to borrow short term. A type G borrower with a date 0 rating \( f \) well above \( f^d \) must have a much smaller than average chance of a downgrade. If the credit rating given a downgrade, \( f^d \), is low, and if the control rent \( C \) is not too large, then a higher rated type G borrower will prefer short-term debt despite some expected loss of control rents.

These ideas are fairly general and do not depend on there being two realizations of new information. For a large enough downgrade the probability of a good borrower receiving the downgrade must be much less than the average borrower (otherwise the large drop in conditional probability of being good could not occur). Good borrowers are not averse to liquidation given large downgrades, even if they lose control rents in these liquidations because most of those being liquidated will be other types of borrower. For a sufficiently high rated borrower, it will require a large downgrade for liquidation (or other control loss) to occur. Such high rated borrowers are not averse to liquidity risk: a small downgrade will not lead to liquidation, and liquidation will be desired given a large downgrade. Lower rated borrowers, who would lose control given a
comparatively small downgrade, would be averse to the large liquidity risk of short-term debt.

VII. The Optimal Mix of Short- and Long-Term Debt

The model can be generalized to accommodate simultaneous issue of short- and long-term debt. In cases where the optimal choice between the two maturities is long-term debt, the choice of an appropriate maturity mix allows good borrowers to receive some of the benefit from the good news they anticipate, without incurring the liquidity risk. The optimal maturity mix will trade off reduced liquidity risk against increased sensitivity of financing cost to new information.

The choice of optimal maturity mix is analyzed in two steps. The first determines the action of short-term lenders that is preferred as of date 0 by type G borrowers. The second determines a maturity mix that provides lenders an incentive to take the preferred action. The lender action in question is at date 1 given bad news. Lenders either leave management in control, or they liquidate. With all long-term debt management retains control. With all short-term debt a lender’s action maximizes lender wealth, which may not be the action that maximizes the borrower’s ex ante welfare. Type G borrowers desire the largest possible value of \( V_d \), the value of the amount received by lenders from borrowers who get a downgrade, when there is not liquidation. This requires that the face value of date 2 debt given downgrade be at its maximum value \( X \), implying that \( V_d = q^d X/R \). A type G’s preferred liquidation decision as of date 0 is determined by a trade-off between the loss of control rents when the borrower is liquidated given a downgrade, versus the extraction of higher repayment from borrowers who are liquidated given a downgrade. Lemma 3 provides the conditions for type G ex ante value to be maximized by liquidation given a downgrade, as opposed to a refinancing with total face value of debt equal to \( X \).

Lemma 3. Type G borrowers prefer liquidation given a downgrade to refinancing with face value \( X \), iff \( f^d \), the rating given a

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8. More generally, priority as well as maturity would need to be studied. In our model, however, the maximum amount that can be repaid is \( X \) whenever it is not 0. This leaves very little role for differing priority. Priority turns out not to matter for the problem posed here. See Diamond [1990] for a generalization that analyzes optimal debt priority.
downgrade, is less than \((LR - PIx)/(X + C - PIx) = \beta\). Lenders with short-term debt liquidate iff \(f^d < (LR - PIx)/(X - PIx) = \lambda > \beta\). For downgrade credit ratings \(f^d \in (\beta, \lambda)\), lenders liquidate too often.

Proof. See Appendix.

Short-term lenders liquidate whenever G’s would desire it, and at other times as well, because lenders cannot benefit from the control rent \(C\). The liquidity risk of all short-term debt is this excessive liquidation. If the credit rating given a downgrade, \(f^d\), exceeds \(\beta\) but is less than \(\lambda\), then G’s want to avoid liquidation, yet retain some of the benefits from the favorable news that they anticipate.\(^9\)

The equilibrium mix of maturities is the one preferred by type G’s, with a fraction \(\alpha^*\) of short term and \(1 - \alpha^*\) of long term. Proposition 2 provides the largest value of \(\alpha \in [0, 1]\) (the shortest maturity structure) that will lead to short-term lenders leaving the borrower in control given a downgrade, by making sure that the short-term debt can be refinanced by issuing new short-term debt at date 1.

PROPOSITION 2. If the rating given a downgrade, \(f^d\), is between \(\beta\) and \(\lambda\) (from Lemma 3), then an optimal maturity mix is a fraction \(\alpha^* = \min \{\alpha, 1\}\) of senior short-term debt, and \(1 - \alpha^*\) of junior long-term debt. The value of \(\alpha\) is

\[
\alpha = \frac{[X(\Pi + f^d(1 - \Pi)) - 1](\Pi + f^d(1 - \Pi))}{(1 - \Pi)(f^d - f^d)} > 0;
\]

which is increasing in the rating \(f\). If the rating given a downgrade is sufficiently low (\(f^d < \beta\)) or liquidity risk is absent (\(f^d > \lambda\)), then borrowers prefer all short-term debt.

Proof. See Appendix.

Lemma 3 and Proposition 2 describe a maturity mix that

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\(^9\) The downgrade credit rating \(f^d\), (the probability of being type G given a downgrade), is the ratio of a type G probability of a downgrade to the probability of downgrade by an average borrower. The numerator of both \(\beta\) and \(\lambda\), \(LR - PIx\), is the increase in repayment that type B downgraded borrowers make when liquidated versus refinancing and repaying face value \(X\), with probability \(\Pi\). The denominator of \(\beta\) is the net lost rents of a liquidated type G; the total rents \((X + C)\) from a type G project, minus what a downgraded type B borrower pays when not liquidated. The denominator of \(\lambda\) is identical, except it neglects the lost control rents \(C\), which are not available to lenders.
assures that short-term lenders’ actions maximize the ex ante value of type G borrowers. This mix will be offered by all borrowers, supported by the reasonable beliefs that a higher fraction of long-term debt would be offered only by type B borrowers, and that no new information is revealed by a lower fraction. The results are similar to Proposition 1: borrowers with better credit ratings (higher \( f \)) choose a higher fraction of short-term debt. This shorter average maturity is desired because it yields higher sensitivity of financing cost to new information, without leading to liquidation because higher rated borrowers must have a larger fall in credit rating for control to be lost with a given maturity structure.

Another contract similar to the mix of long and short debt described in Proposition 2 is a *resettable rate bond* that is sometimes used in leveraged buyouts. The interest rate is reset (often in an auction) after the bonds have been outstanding for some period, to attempt to make the bonds trade at par in the secondary market. If the credit rating falls, the interest rate must be increased, and vice versa. If there is no rate that will make the bonds trade at par (or if a binding ceiling rate is specified), this is equivalent to short-term bonds that cannot be repaid in full. Some resettable bonds do not put the firm into bankruptcy or give other control rights to lenders if this occurs, implying sensitivity of the cost of capital to new information, but without the risk of lost control rents.

Maturity (and more generally priority) of debt needs to be designed to take account of the differing workout incentives of borrowers and lenders, and possibly, the differing qualities of information of various groups of lenders. The long-term lenders do not need the information possessed by short-term lenders. Because the initiation of workout decisions is the domain of informed short-term lenders, choosing the proper contract structure can provide them with the proper incentives to determine the type of resolution of potential defaults. This argument suggests that commercial bank loans (loans from informed lenders) would optimally be relatively short maturity, and that public debt (uninformed) would be long-term financing. The long-term debt would not play an active role in workout decisions except near its maturity.

**VIII. INABILITY TO BORROW LONG TERM**

Most of our analysis focuses on credit ratings good enough so both long- and short-term debt are possible. Long-term debt is not
possible given credit rating \( f \) if and only if \( \rho > X \), which is equivalent to \( f < [R^2 - \Pi X]/(1 - \Pi) \). Short-term debt is always possible when long-term debt is possible, because when there is a default, lenders always choose a debt restructuring at date 1 that maximizes their proceeds, and setting the face value of date 0 short-term debt equal to \( r_1 = X/R \) would yield at least as high a return as long-term debt. Short-term debt is feasible for some credit ratings when long-term is not only if lenders can obtain sufficiently high returns from liquidation given bad news. This requires that the liquidation value \( L \) exceed the maximum date 1 value without liquidation, \( q^d X/R \). Lemma 4 provides the minimum credit ratings needed to issue both short- and long-term debt.

**Lemma 4.** If the credit rating of a borrower at date 0, \( f \), exceeds \([R^2 - \Pi X]/(1 - \Pi)\), then both short- and long-term debt are feasible. If and only if liquidation yields sufficiently high proceeds, \( L > q^d X/R \), there are credit ratings where only short-term debt is feasible, and the condition to be able to borrow short term is \( f > [X f^d + R^2(1 - f^d) - LR]/(X - LR) \).

The low rated borrowers who are forced to use short-term debt have a substantial probability of loss of control rents in liquidation, but the higher payment from this liquidation is needed to provide lenders a large enough return for them to lend.

The prediction of short-term debt issued directly to the public by those with bad credit ratings has little empirical support. The low rated borrowers choose short-term debt because the returns in liquidation are a large part of the return received by borrowers. If one takes account of the lower costs of arranging a total or partial liquidation when a loan is a private placement or bank loan, then the model suggests that the low rated borrowers choose short-term privately placed debt, and the high rated borrowers choose short-term commercial paper, with the intermediate ratings issuing long-term publicly traded bonds. This is certainly not a well-developed study of the role of financial intermediaries and corporate debt maturity structure, but it simply suggests that the low rated borrower who is forced to choose short-term debt can be matched with the data by adding some workout cost advantages of intermediaries, as in Diamond [1984], [1990], or [1991].

**IX. Other Limits to Pledging Future Rents**

Borrowers' private information about their cash flows and actions, by itself, limits the ability of future rents to serve as
backing for financial claims. The limits are caused by the difficulty of verifying that a low cash flow actually occurred, and the implied incentive problems if there is no unpleasant consequence to the borrower of paying a small amount; see, for example, Townsend [1979], Diamond [1984], Gale and Hellwig [1985], or Hart and Moore [1989]. Either costly monitoring or some punishment such as the possible inefficient liquidation of the borrower's project must follow from a low repayment, or all repayments will be low. For a sufficiently high repayment the monitoring or liquidation is avoided. The higher the probability that realized cash flows in a period fall below this level, the higher are the expected costs or ex post inefficiencies associated with financing the project. High enough costs can prevent the financing of a project that would be worth undertaking if returns were fully observable. These returns in the upper tail of the distribution cannot be assigned to lenders, and play a role identical to the control rent in the model analyzed in the text.

Moral hazard imposes an important limit on rent pledging, because it gives rise to a notion of "debt capacity," with a cost of reduced productive efficiency (lost rents) if the capacity is exceeded.10 This effect is worth illustrating. Assume that control rents are zero, but that type G borrowers have a privately observed choice at date 1 between a safe project that returns G at date 2 for sure, and a risky negative net present value one that returns $B > G$ with probability $\Pi < R^2/B < 1$, and returns zero with probability $1 - \Pi$. The type G borrower's choice depends on the face value of debt at date 2, which depends on the information at dates 0 and 1. Type B borrowers have only the risky project. Let $p$ be the date 2 face value of debt. The type G's date 2 return from selecting the safe project is $G - p$, the return from the risky project is $\Pi(B - p)$, implying that safe projects are selected if and only if $p \leq (G - \Pi B)/(1 - \Pi) \equiv \chi$. The number $\chi$ is the "debt capacity"; if this face value is exceeded, then type G borrowers will select the risky project with lower net present value, destroying rents of $G - \Pi B$. This moral hazard model is identical to the situation in the text, except that the debt capacity $\chi$ is now the limit on assignable cash flow (instead of $X$), the rent lost on a downgrade is now $G - \Pi B$ (instead of $C$), and the probability of debt repayment is $q^d$ if face

value does not exceed $\chi$ (and $\Pi$ if it exceeds $\chi$). Note that the lost rents in this example do not occur in liquidation, but in a “workout” (outside of bankruptcy) that assigns more cash rents to lenders. Therefore, the liquidity risk of refinancing short-term debt is not just risk of liquidation, but more generally risk of lost control rents.

X. Lenders’ Beliefs About Borrower Type by Contract Chosen

This (somewhat technical) section describes the inferences that lenders make from off-equilibrium path actions that support the equilibrium analyzed above, and presents another (uninteresting) equilibrium where no one ever borrows long term. My goal in this section is to demonstrate that the equilibrium studied above is reasonable.

The equilibrium analyzed has all borrowers choosing the least cost form of debt for a good borrower (a type G), where the good borrower makes the choice using a calculation assuming that both long- and short-term debt are available, with both types of debt evaluated using the credit rating $f$ of a borrower, rather than some complicated inference about the type as a function of the choice made. I refer to this equilibrium as the cost-minimizing equilibrium. This is supported by the following beliefs of lenders. If the calculation indicates that short-term debt is cheaper for G’s, then offering long-term is interpreted as indicating that a borrower is of type B, to whom no one would lend. If the calculation indicates that G’s prefer long term, then lenders place no stigma on offering long-term debt. The inference drawn from the off-equilibrium action of offering short-term debt is that the borrower is of type G with probability $f$ (note that any lower probability of being of type G would also assure that short term is off-equilibrium in this case).

An off-equilibrium action is the choice of a maturity that no one offers. It is important to make sure that a given maturity is not off-equilibrium only because lenders are anticipated to make an unreasonable inference from its selection. The criterion I use to rule out unreasonable inferences of lenders from off-equilibrium

11. Moral hazard also interacts with control rent. If the safe project has a control rent $c_s$, and the risky project $c_r$, then the debt capacity is $X = (G - \Pi B + c_g - c_g)/(1 - \Pi)$, and lost rent is $G - \Pi B + c_g$. With $c_g > c_r$, $B > G$ no longer need be assumed: if $X$ is exceeded, project selection is based primarily on the control rent.
actions is the equilibrium dominance criterion of Cho and Kreps [1987], that rules out inferences from off-equilibrium actions that imply some type took an action that makes it worse off than its equilibrium action.

**Lemma 5.** The cost-minimizing equilibrium analyzed in the text satisfies the Cho and Kreps equilibrium domination criterion. Another equilibrium that satisfies the criterion is for borrowers of all credit ratings to issue only short-term debt. No separating equilibrium exists.

**Proof.** See Appendix.

The equilibrium where all borrowers borrow with an identical maturity, even though it is not cost minimizing, is less interesting because it has no cross-sectional predictions about maturity structure. It is supported by the belief that only type B borrowers would borrow at the off-equilibrium maturity, and is analogous to an autarchy equilibrium in a model of trade, where no one produces goods for other than their own consumption because all expect others to do the same, and all anticipate no trade possibilities.

In the cost-minimizing equilibrium, if liquidity risk is sufficiently large, type G borrowers (and all others) choose long-term debt because the saving of interest in the initial round of short-term borrowing is insufficient to offset the risk of lost future rents.

XI. Conclusion

Financing with large amounts of debt of shorter maturity than the cash flows from investments gives substantial control to lenders because the borrower can pay off the old debt only by issuing new debt. If the old debt cannot be repaid in full, lenders have the right to liquidate or take control in some other fashion. Limited ability to assign future cash flows and control rents to lenders implies that they liquidate in too many circumstances. If there is no limit to assigning future rents to outside lenders, short-term debt is preferred by borrowers whose private information is favorable, and this forces other borrowers to choose short-term debt to avoid appearing to have bad news. When it is impossible to contractually assign all future rents, then there is a liquidity risk that limits the use of short-term debt by borrowers who have private good news. The risk of loss of control rents that cannot be assigned to lenders makes good borrowers choose
long-term debt, even though they expect the average news about them to be more favorable than average at the time of refinancing. For a borrower with a sufficiently good credit rating, this liquidity risk is outweighed by the effect of expecting future news to be favorable. For borrowers with lower ratings, the liquidity risk outweighs the information effect. There is a minimum rating such that the benefits of short-term debt are worth the liquidity risk. However, very low rated borrowers may have no choice but to choose short-term debt, despite the incentives for inefficient liquidation that it gives to lenders.

The two types of borrowers that choose primarily short-term debt imply that the chosen debt maturity is not a monotonic function of the borrower’s credit rating. If the model of this paper captures an important part of the debt maturity decision, empirical studies of maturity will measure a mixture of two effects. This could make inferences complicated. One way around this would be to use short-term bank debt as a proxy for the low rated borrowers who must choose short-term debt, and directly placed commercial paper as the proxy for the borrowers who choose short-term debt given the alternative of long-term debt. It is also important to remember that the model holds the amount of leverage constant by ruling out equity. A firm with little total debt would be unlikely to lose control due to inability to repay the debt, independent of the maturity. This suggests that highly leveraged firms would choose longer term debt. Heterogeneous amounts of leverage introduce another complication: the credit rating reported by the rating agencies is the rating on a firm’s bonds, and the rating is a decreasing function of leverage. Similarly, the rating on a firm’s debt of a given maturity is a decreasing function of the amount of debt at that maturity.

A general implication of borrowers’ having private information is that there is a bias toward short-term debt because there can be a stigma associated with long-term debt. This implies that the expected lost control rents from excessive liquidation are not minimized. In addition, the cost of short-term debt is not a fixed cost similar to a transaction cost. Adding a fixed cost to short-term debt (instead of liquidity risk) would imply that those borrowers who were most underrated in the market would be the most willing to select short-term debt and incur the cost. The most underrated good borrowers are those with the lowest rating, implying that a fixed cost of short-term debt would lead the low rated to choose short-term and the high rated to choose long-term. The cost
imposed by liquidity risk is largest for liquidation due to small defaults, and is small for large defaults. In addition, the cost is only incurred if there is a default and liquidation. This implies that it is the high rated borrowers who are willing to bear the liquidity risk of refinancing short maturity debt.

**APPENDIX**

**Proof of Lemma 1**

For the date 0 lenders to receive an expected return of $R$ on their investment of 1 requires that $R = (1 - p^d) r_1 + p^d V^d$, or the face value of their debt is $r_1 = [R - p d V^d] / p^u$.

The payoff $\text{NoLiq}$ from debt not yielding liquidation is

$$\text{NoLiq} = e [X + C - R V^d/q^d] + (1 - e)[X + C - r_1 R]$$

$$= X + C - R \left( \frac{V^d}{q^d} + \frac{(1 - e)}{p^u} [R - p^d V^d] \right)$$

$$= X + C - \frac{R^2}{p^u} (1 - e) - R V^d \left\{ \frac{1}{q^d} - \frac{p^d(1 - e)}{p^u} \right\}$$

$$= X + C - \frac{R^2}{p^u} (1 - e) - R V^d e \left\{ \frac{1}{q^d} - \frac{p^d(1 - e)}{p^u} \right\}$$

$$= X + C - \frac{R^2}{p^u} (1 - e) - R V^d e \left\{ \frac{1}{q^d} - \frac{1}{f^d} \right\}$$

because $f^d = \frac{e p^u}{(1 - e)p^d}$.

$$\frac{d\text{NoLiq}}{d V^d} = e R \left\{ \frac{1}{f^d} - \frac{1}{q^d} \right\} > 0$$

because $q^d = f^d + \Pi (1 - f^d) > f^d$.

Q.E.D.

**Proof of Lemma 2**

Compare a debt structure that yields liquidation with one that does not.

The increase in payoff from choosing the structure that leads to liquidation yielding $L$, versus one that avoids liquidation and has
V^d. The payoff from debt that yields liquidation on a downgrade is
\[ \text{Liq} = (1 - e)[X + C - R \frac{R}{p^d}] = (1 - e)[X + C - (R/p^u)[R - p^dL]]. \]
The Payoff without liquidation is \( N_{\text{NoLiq}} = X + C - R[e(V^d/q^d) + (1 - e)/p^u[R - p^dV^d]] \)

\[
\text{Liq} - N_{\text{NoLiq}} = \frac{p^d(1 - e)}{p^u} R \left[L - V^d\right] - e \left[X + C - R V^d/q^d\right]
\]

\[
= e \left[\frac{p^d(1 - e)}{p^u e} R \left[L - V^d\right] - X + C - R V^d/q^d\right]
\]

\[
= e \left[\frac{\left[L - V^d\right]}{f^d} R - \left[X + C - R V^d/q^d\right]\right]
\]

\[
= e \left[\frac{LR}{f^d} - X + C - RV^d\left[1 - \frac{1}{f^d} - \frac{1}{q^d}\right]\right].
\]

This is a decreasing function of \( C \) and of \( V^d \). It is negative for sufficiently large \( C \). \( \text{Liq} - N_{\text{NoLiq}} \) is nonnegative iff \( C \leq \hat{C} = \left\{ LR \right\} - X - RV^d\left[1 - \frac{1}{f^d} - \frac{1}{q^d}\right]\)

\[
= \left\{ LR \right\} q^d - X - RV^d\left[1 - \frac{1}{f^d} - \frac{1}{q^d}\right].
\]

Because \( q^d > f^d \), and \( LR/q^d > X \), \( \hat{C} > 0 \), and liquidation is desired by type G borrowers for sufficiently small \( C > 0 \). Because \( V^d \leq q^dX/R \), liquidation is preferred if

\[
LR \geq f^d \left[C + X + \left\{\frac{q^d}{f^d} - 1\right\}X\right] = q^dX + f^dC.
\]

Q.E.D.

Proof of Proposition 1

Apply Lemma 1 with \( V^d = q^d \rho/R \), where \( \rho = R^2/(\Pi + (1 - \Pi)f) \).
This yields

\[
C \leq \frac{LR}{f^d} - X - \left(1 - \frac{q^d}{f^d}\right) \frac{R^2}{\Pi + (1 - \Pi)f}
\]
or

\[
R^2(q^d/f^d - 1) - \Pi [C + X - (LR/f^d)] \\
(1 - \Pi) (C + X - (LR/f^d))
\]

\[
= \frac{\Pi (R^2 - LR + f^d (C + X - R^2))}{(1 - \Pi) (LR - f^d (C + X))} \equiv S; \quad \frac{dS}{dL} < 0; \quad \frac{dS}{dC} = \frac{dS}{dX} > 0.
\]

Solving \( S > 1 \) yields \( f^d > (LR - R^2\Pi)/(X + C - R^2\Pi) \).

Q.E.D.

**Proof of Lemma 3**

If type G borrowers could instruct lenders as of date 0 on their preferred resolution of debt given a downgrade, they would make a choice to maximize their own value, realizing that they receive bad news with probability \( e \). If lenders do not liquidate given a downgrade, then by Lemma 1, type G borrowers prefer the maximum possible repayment, because this maximizes \( V^d \). The maximum possible repayment is \( X \), which implies that \( V^d = q^dX/R \). From Lemma 2, type G's prefer liquidation iff

\[
C \leq \frac{LR}{f^d} - \frac{Xq^d}{f^d} = \frac{LR}{f^d} - X(2 - \left[ \frac{f^d + (1 - f^d)\Pi}{f^d} \right])
\]

\[
= \frac{LR}{f^d} - \frac{X(1 - \Pi + \Pi f^d)}{f^d}.
\]

This is equivalent to \( f^d \leq (LR - \Pi X)/(C + X - \Pi X) \equiv \beta \).

The condition for lenders to liquidate is \( LR \geq q^dX = (\Pi + (1 - \Pi)f^d)\) \( X \). This is equivalent to \( f^d \leq (LR - \Pi X)/(X - \Pi X) \equiv \lambda \).

For \( C > 0, \lambda > \beta \).

Q.E.D.

**Proof of Proposition 2**

Type G borrowers are not in agreement with lenders over liquidation decisions given all short-term debt if \( f^d \) is between \( \beta \) and \( \lambda \). A debt structure that implies that lenders do not have the control right to liquidate given a downgrade with \( f^d \in (\beta, \lambda) \) will prevent liquidation. To avoid giving lenders this control right, the total face value of date 2 debt needed to refinance the date 0 short-term debt given a downgrade must not exceed \( X \).

Let the face value of long debt be \( \rho \), date 0 short debt \( r_1 \), and the face value of date 1 refinanced short-term debt given downgrade be
The constraint that implies that control is retained is \( r^d + \rho \leq X \); i.e., the date 0 short debt is riskless.

If \( r_1 \leq q^d(X - \rho) \), the short-term debt is riskless, and the short-term lender cannot remove control. If a fraction \( \alpha \) of the initial debt is (riskless) short term, then its face value is \( r_1 = \alpha/R \), and the face value of short-term debt issued at date 1 (to raise \( r_1 \) and fully repay the initial short-term debt) when there is a downgrade is \( r^d = \alpha R^2/(\Pi + (1 - \Pi)f^d) \). The fraction \( 1 - \alpha \) of long-term debt has face value \( \rho = (1 - \alpha)R^2/(\Pi + (1 - \Pi)f) \). As a result, the constraint that total date 2 maturing debt not exceed \( X \) is \( X \geq r^d + \rho \) yields \( r^* \leq \alpha R^2/(\Pi + (1 - \Pi)f^d) + (1 - \alpha)R^2/(\Pi + (1 - \Pi)f) \). Solving for \( \alpha^* \), the largest value of \( \alpha \) that satisfies this constraint yields

\[
\alpha^* = \frac{[X(\Pi + f(1 - \Pi)) - R^2(\Pi + f^d(1 - \Pi))]}{R^2(1 - \Pi)(f - f^d)} > 0.
\]

If \( \alpha^* > 1 \), the optimal maturity structure is 100 percent short term. Differentiating \( \alpha^* \) with respect to \( f \) yields

\[
\frac{d\alpha^*}{df} = \frac{(\Pi + (1 - \Pi)f^d)[R^2 - X(\Pi + (1 - \Pi)f^d)]}{R^2(1 - \Pi)(f - k)^2} > 0.
\]

An improved credit rating implies a greater fraction of short-term debt. A sufficiently good credit rating implies all short-term maturity structure.

**Proof of Lemma 5**

The Cho-Kreps criterion rules out predictions of optimal lender actions that would require lender inferences of the following form: the inclusion of a type in the group of borrowers believed to deviate to the off-equilibrium action when that type would be worse off than in equilibrium if lenders select a best response to a type distribution of borrowers that excluded that type. It rules out situations where a given type of borrower could explain a deviation to another maturity by noting that the other type would have no incentive to deviate if the lender interpreted the deviation as indicating the given type, but the other type could make no such claim. For example, if short-term debt is off-equilibrium and this must be supported by beliefs that offering short-term debt indicates some possibility of being type B, then this is unreasonable if type B's would not select the deviation (would prefer the equilib-
rium where all offer long term) if deviation to short were interpreted as indicating type G for sure.

Whenever short-term debt is off the equilibrium path, it is supported by the belief that the implication of short term is a probability of type G of \( f \), thus both types are included. For a type to deviate to short term, a higher conditional probability of being of type G would be required, thus there can be no deviations interpreted as indicating type B for sure. If a deviation is interpreted as indicating type G for sure, then the information that will arrive at time 1 is superfluous, and for both long- and short-term debt the final date 2 face value will be \( R \). In this case, both types would want to deviate, implying that the belief that probability of being type G of \( f \) passes the Cho-Kreps test.

If only a subset of borrowers prefer long-term debt, it must be the type B's who prefer it, because the news on date 1 is unfavorable for type B's, and favorable for G's (and the liquidity risk of short-term debt is weakly larger for B's). This implies that there are no separating sequential equilibria.

The only other type of equilibrium is one where a single maturity is always offered, independent of credit rating. This passes the Cho-Kreps test, supported by the belief that the off-equilibrium maturity implies that type is certainly B. This passes the Cho-Kreps test because both types would prefer to deviate to the off-equilibrium action if interpreted as type G for sure.

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REFERENCES


