A Theory of Bank Capital

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ABSTRACT
Banks can create liquidity precisely because deposits are fragile and prone to runs. Increased uncertainty makes deposits excessively fragile, creating a role for outside bank capital. Greater bank capital reduces the probability of financial distress but also reduces liquidity creation. The quantity of capital influences the amount that banks can induce borrowers to pay. Optimal bank capital structure trades off effects on liquidity creation, costs of bank distress, and the ability to force borrower repayment. The model explains the decline in bank capital over the last two centuries. It identifies overlooked consequences of having regulatory capital requirements and deposit insurance.

Does bank capital structure matter, and if so, how should it be set? Most work on the subject extrapolates an answer from prior work on the capital structure of industrial firms. But bank assets and functions are not the same as those of industrial firms. In fact, one strand of the banking literature suggests banks have a role precisely because they do not suffer the asymmetric information costs of issuance faced by industrial firms (see Gorton and Pennacchi (1990)). Therefore, to really understand the determinants of bank capital structure, we should start by modeling the essential functions banks perform, and then ask what role capital plays. Using this approach, we can see that a bank's capital structure affects its liquidity-creation and credit-creation functions in addition to its stability. The consequent trade-offs imply an optimal bank capital structure. Because customers rely to different extents on liquidity and credit, bank capital structure also determines the nature of the bank's clientele. Our approach will help us better understand the impact of regulations such as minimum capital requirements, and also help suggest the consequences of different recapitalization policies in a banking crisis.

We start by describing the functions a bank performs. Consider a world where a number of entrepreneurs each has a project in need of funding. Each entrepreneur has specific abilities vis-à-vis his project so that the cash flows he can generate exceed what anyone else can generate from it. An entrepreneur cannot commit his human capital to the project, except on a

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spot basis. An outside financier can extract future repayment only by threatening to liquidate the project (taking away the project from the entrepreneur and selling it to the next best user). But, because the entrepreneur can always threaten to withhold his specific skills in the future and thus capture a rent, the financier can extract only a fraction of the cash flows generated. Thus projects are illiquid in that they cannot be financed to the full extent of the cash flows they generate.

An outside financier who lends at an early stage of a project knows how the project is set up, and thus learns how best to redeploy the project’s assets. Such a relationship lender has the specific abilities to lend more to the firm—because the lender has a better liquidation threat than anyone else. However, the lender may not be able to raise much money against the financial asset, that is, the loan that he holds. The amount he raises from outsiders will typically be less than the present value of the payments he can extract from the entrepreneur, precisely because he cannot commit to using his specific abilities on behalf of the less capable outsiders. Thus the source of illiquidity of the real asset (the project) and the financial asset (the loan to it) are the same: an agent’s specific abilities, which lead to non-pledgeable rents. In the case of the project, it is the entrepreneur’s greater ability to run it relative to a second best operator. In the case of the loan, it is the relationship lender’s better ability to recover payments relative to someone who lends against it.

Since an asset is illiquid because specialized human capital cannot easily be committed to it, devices that tie human capital to assets create liquidity. We show in Diamond and Rajan (2001) that a bank, which is a lender financed with demand deposits, is such a device. When the relationship lender issues demand deposits—which are fixed claims with a sequential service constraint where depositors get their money back in the order in which they approach the relationship lender until he runs out of money or assets to sell—the relationship lender (henceforth “bank”) cannot hold up depositors and, instead, has to pay them the promised amount. Intuitively, the sequential service constraint creates a collective action problem among depositors, which makes them run on the bank whenever they think their claim is in danger. Because they run immediately, rather than enter into negotiation, this commits them not to make concessions. When the bank has the right quantity of deposits outstanding, any attempt by the banker to extort a rent from depositors by threatening to withdraw his specific abilities will be met by a run, which disintermediates the banker and drives his rents to zero. Thus the banker will not attempt to extort rents and will pass through all collections directly to depositors. In a world of certainty, the bank maximizes the amount of credit it can offer by financing with a rigid and fragile all-deposit capital structure.1

1 Interestingly, the bank is a source of liquidity both for the depositor and the entrepreneur. When some (or even all) initial depositors want their money back in the ordinary course of business (in contrast to a run), the bank does not need to liquidate the entrepreneur. It simply
However, with uncertainty that is observable but not verifiable (and thus cannot be used in contracting), we introduce the other side of the trade-off. The rigid capital structure could lead to runs when real asset values fall, even without opportunistic behavior by the banker. The banker now has to trade off credit and liquidity creation against the cost of bank runs. It may be optimal for the bank to partially finance itself with a softer claim that can be renegotiated in bad times.

We call such a claim “capital.” It is a long-term claim without a first-come-first-served right to cash flows. It is most easily interpreted as equity in our model because the holders always have the right to liquidate (replace the banker), but it could also be interpreted as long-term debt where this right accrues to holders only if there is a default. Capital holders, unlike depositors, are not subject to an immediate collective action problem. As a result, they cannot commit not to renegotiate. Although this allows the banker to capture some rents in the future, thus reducing his ability to raise money today, it also buffers the bank better against shocks to asset values. The single period optimal bank capital structure is obtained by trading off these costs and benefits of capital. Our model explains why bank capital can be costly, not just in the traditional Myers and Majluf (1984) sense of the asymmetric information cost of issuing new capital, but in the more recurring cost of reducing bank liquidity creation, and the flow of credit.

In a multiperiod setting, however, a bank’s capital structure also influences the amount that the bank can extract from a liquidity-constrained entrepreneur by altering the bank’s horizon when it bargains with its borrowers. This effect is reminiscent of Perotti and Spier (1993), who argue that a more levered capital structure enables equity holders to extract more from workers, but the rationale is quite different. The bank’s ability to extract repayment does not change monotonically in its deposit leverage and also depends in a nonmonotonic way on the characteristics of the entrepreneur’s project, such as the interim cash flows it generates.

In summary, the optimal capital structure for a bank trades off three effects of capital—more capital increases the rent absorbed by the banker, increases the buffer against shocks, and changes the amount that can be extracted from borrowers.

Our framework can be applied to understand a variety of phenomena. For example, by characterizing the kinds of firms that benefit most from bank finance, it can explain the pattern of disintermediation as a financial system develops. As another example, because financial fragility is essential for banks to create liquidity, our model highlights some of the costs (in terms of lower credit and liquidity creation) of regulations such as capital require-
ments that attempt to make the banking system safe. As a third example, because the extent to which a borrower is squeezed by its bank depends on the borrower’s liquidity, the bank’s capital structure, and the liquidity position of other borrowers, a bank capital crisis could lead to large transfers between the banking system and the industrial system, and between various segments of industry. These transfers are a hitherto unexamined cost of banking crises.

The rest of the paper is organized as follows. In Section I, we investigate a one-period model. In Section II, we examine decisions at the interim date and initial dates in a two-period model. In Section III, we examine implications of the model, and in Section IV, the robustness of the model to alternative assumptions. Conclusions follow.

I. Framework

A. Agents, Projects, and Endowments

Consider an economy with entrepreneurs and investors. The economy lasts for two periods and three dates—date 0 to date 2. All agents are risk neutral and the discount rate is zero. Each entrepreneur has a project idea. Let us start by assuming projects last only for one period, requiring an initial investment of $1 at date 1 and ending at date 2. At date 2, the project returns a cash flow \( C^H \) in state \( H \) with probability \( q^H \) and \( C^L \) in state \( L \) with probability \( 1 - q^H \). Cash flows are generated only if the entrepreneur contributes his human capital. The assets created through the initial investment also have value without the entrepreneur’s human capital. This best alternative use is also termed “liquidation value,” and has a random value \( X^s \) with realization \( X^s \) in state \( s \) until \( C^s \) is due to be produced. After that, the value of the assets collapses to zero. Funds can also be invested at any date in a storage technology that returns $1 at the next date for every dollar invested.

Entrepreneurs do not have money to finance their projects. There are a large number of investors, each with less than one unit of endowment at date 0, who can finance entrepreneurs. We will assume for now that \( C^s > X^s \). This will ensure that illiquidity never prevents an entrepreneur from paying at date 2. We also assume that the aggregate endowment exceeds the number of projects by a sufficient amount so that storage is in use at each date. This implies that there is no aggregate shortage of capital or liquidity. As a result, at any date a claim on one unit of consumption at date \( t + 1 \) sells in the market for one unit at date \( t \). The distribution of investors’ endowments is not critical.

B. Contracting

We consider financial contracts that specify that the entrepreneur owns the asset and has to make a payment to the financier, failing which the financier will get possession of the asset and the right to dispose of it as he
pleases. The realized values of cash and liquidation are not verifiable and, therefore, not contractible. A contract can only specify the repayments, $P_t$, the entrepreneur is required to make at date $t$, as well as the assets the financier gets if the entrepreneur defaults. If $P_t < \infty$, this is a debt contract with promised payment $P_t$. If $P_t = \infty$, this is an equity contract where the outside investor is free to liquidate or replace the entrepreneur (as in Hart and Moore (1994)).

For simplicity, we assume that partial liquidation is not possible, and on liquidation (replacement of the entrepreneur), the lender gets all the proceeds. The entrepreneur can liquidate himself for as much as the relationship lender.

C. Relationship-Specific Collection Ability

The initial financier of a project acquires the specific skills to put assets to their best alternative use and obtains $X^s$ from liquidation, whereas everyone else who does not have access from the beginning can generate only $\beta X^s$, where $\beta < 1$. What we have in mind is that the initial financier is with the project from the beginning and sees how the entrepreneur puts it together. This financier will build relationships with the entrepreneur’s employees, suppliers, and customers, as well as have a good understanding of the competition. Therefore, he has a better understanding of how and where to find a replacement for the entrepreneur, as well as how best to dispose of the assets. Later financiers come in when the project is already put together, and have much less ability to understand the details of its working or the key participants.

For example, the initial financier comes into contact with the entrepreneur’s key lieutenants on a daily basis when the firm is young. This detailed knowledge helps him when the firm is more mature because he has a better ability to choose an appropriate replacement for the entrepreneur. Later financiers only deal regularly with the Assistant Treasurer of the more mature firm, and have little knowledge of the competence of the more senior second tier of management.

We also assume, $q^H X^H + (1 - q^H) X^L > 1 > \beta[q^H X^H + (1 - q^H) X^L]$. This assumption ensures that the financier will have to be able to use specific skills in at least one of the states for the loan to be worth making.

Because educating the initial relationship financier takes time and effort, we assume that there can be just one financier for each entrepreneur. We assume that the relationship financier needs constant close contact with the entrepreneur to maintain his advantage so that if he sells the financial claim or it is seized from him, he loses his specific skills next period. This assumption simplifies the analysis but is not necessary. In Diamond and Rajan (2001) we get similar results when the intermediary retains relationship lending skills no matter what happens to the ownership of the financial claim.
D. Bargaining with the Entrepreneur

Any agent can commit to contributing his human capital to a specific venture only in the spot market. In particular, because he cannot commit future human capital in the initial contract, at date 2 the entrepreneur may attempt to renegotiate the terms of the contract (henceforth the loan) that he agreed to in the past, using the threat of withholding human capital. We assume bargaining at date 2 takes the following form: the entrepreneur offers an alternative payment from the one contracted in the past and commits to contribute his human capital if the offer is accepted. The lender can (1) reject the offer and liquidate the asset immediately, (2) accept the offer, or (3) reject the offer and sell the assets to a third party. The game gives all the bargaining power to the entrepreneur, apart from the lender’s option to liquidate. This is for simplicity only, and modified versions of our results hold when there is more equal bargaining power. If the entrepreneur’s offer is accepted, the entrepreneur contributes his human capital, and the offered payment is made. The sequence is summarized in full generality in Figure 1.
Example 1: Suppose that it is date 2, state \( s \), and the entrepreneur has promised to pay \( P_2 = X^s \). The entrepreneur knows the relationship lender can obtain \( X^s \) through liquidation. As a result, he pays \( X^s \) because, by assumption, he generates enough cash flow to do so.

E. Intermediation

With the assumptions that one individual’s endowment is not enough to fund the project, that there be at most one initial lender acquiring specific skills, and that specific skills are necessary, at least in some state, for investors to break even, investors have no option but to delegate the acquisition of specific collection skills to an intermediary at date 0. In this “large scale” case, it will turn out the intermediary must use demand deposits to commit not to renegotiate with investors. Another (equivalent) motivation when lending over two periods for the use of demand deposits is if all investors have an uncertain need for liquidity at date 1. Diamond and Rajan (2001) show that even when one individual can fully finance the project and become the relationship lender, financing through a bank will dominate financing directly by an individual so long as the individual has a high enough probability of a need for liquidity (and will need to raise funds from others) at an intermediate date. The implications of these two motivations for banks with demand deposits (large scale of borrowing versus an uncertain need for liquidity) are the same. For reasons of space only, we will use large scale as the motivation in what follows.

F. Holdup by an Intermediary

The relationship lender is an intermediary who has borrowed from other investors. In the same way as the entrepreneur can negotiate his repayment obligations down by threatening not to contribute his human capital, the intermediary can threaten not to contribute his specific collection skills and thereby capture a rent from investors. The intermediary, by virtue of his position in the middle, can choose with whom to negotiate first. The intermediary will negotiate first with outside investors before concluding any deal with the entrepreneur (else his threat to withhold his collection skills is without bite). So he will open negotiations with investors by offering a different schedule of repayments. The negotiations between an intermediary and investor(s) take much the same form as the negotiations between the entrepreneur and a lender (see Figure 2). The investor can (1) reject the proposed schedule and bargain directly with the entrepreneur as in Figure 1 (this is equivalent to the investor seizing the “asset”—the loan to the entrepreneur—from the intermediary), (2) accept the proposed schedule, or (3) bargain with the intermediary over who will bargain with the entrepreneur. It is best to see the effect of this potential holdup by the intermediary in our example.

\footnote{We show in Diamond and Rajan (2001) that what is crucial is not the sequence of bargaining but that the intermediary have contingent ownership of the loan.}
Example 1, continued: Suppose the intermediary funds his loan to the entrepreneur by borrowing from several investors. Assume for now that there are no problems of collective action among the investors (we will later call such investors “capital”). Suppose that the promised payment by the entrepreneur, $P_2$, is high. No matter what the initial contract with investors, at date 2 the intermediary can threaten not to collect on the loan to the entrepreneur and instead let investors collect it. The investors, because of their poorer liquidation skills, can expect to extract at most $\beta X^*$ from the entrepreneur. The intermediary’s threat not to collect can thus allow him to capture some of the extra amount that only he can collect. If the intermediary and investors split the additional amount extracted equally, the investors will get $[(1 + \beta)/2]X^*$ and the intermediary will get the remainder, or $[(1 - \beta)/2]X^*$. Thus, at date 1, the intermediary’s inability to commit to employ his specific collection skills at date 2 prevents him from pledging to repay more than a fraction, $(1 + \beta)/2$, of what he collects from the entrepreneur.
G. Depositors as Investors

The difference between a generic intermediary financed by “capital” as described above and a bank financed by demand deposits is that the sequential service nature of demand deposits creates a collective action problem that prevents the banker from negotiating depositors down. As a result (for a detailed proof, see Diamond and Rajan (2001)), with the appropriate level of outstanding deposits, the bank can commit to pass on whatever it collects to depositors without extracting rents on the way.

To sketch the reason, we have to first specify the terms of the deposit contract. The deposit contract allows the investor to withdraw at any time. He forms a line with other depositors who decide to withdraw at that time. If the banker does not pay him the full promised nominal repayment $d$, the depositor has the right to seize bank assets (cash and loans) equal in market value (as determined by what an ordinary investor would pay for the assets—see above) to $d$. Depositors get paid or seize assets based on their place in line. Therefore if bank assets are insufficient to pay all depositors, the first one in line gets paid in full whereas the last one gets nothing.

Suppose the banker announces that he intends to renegotiate and makes an initial offer. Depositors can (1) accept the new terms, (2) join a line, with positions allocated randomly, to seize the bank’s assets of loans and cash based on what is due to them in the original contract, which we call a run, or (3) refuse the offer, but negotiate without seizing bank assets (see Figure 3). All depositors choose between these alternatives simultaneously.

At the end of this stage, either the banker or the depositor will be in possession of the loan to the entrepreneur. If depositors have seized the loan, the banker is disintermediated, and the entrepreneur can directly initiate negotiations with depositors by making an offer. The subsequent steps follow the sequence that we have already documented above and in Figure 1.

There is an essential difference between an intermediary bargaining with investors who simply have ordinary debt or equity claims on the intermediary and the banker bargaining with demand depositors. If the banker attempts to renegotiate or takes any other action that would impair the value of deposits, depositors will choose to run in an attempt to grab a share of the bank’s assets and come out whole. As we will argue shortly, the run, by disintermediating the banker, will destroy his rents even though he will continue to have specific skills in the short run. Fearing disintermediation, the banker will not attempt to renegotiate and will pass through the entire amount collected from the entrepreneur to depositors.

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3 An equivalent assumption to depositors seizing loans is that they demand cash and the bank is forced to sell loans at their market value to third parties to meet cash demands. The net effect is the same—unskilled parties are in possession of the loans after the run.
Banker threatens to withdraw his human capital from the bargaining unless a depositor (D) make concessions. If agreement reached, banker will make agreed payments after negotiating with entrepreneur.

**Example 1, continued:** How much can the banker commit to pay in state $s$ from the loan with face value $P_2 = X^s$? Let the banker issue demand deposits at date 1 with face value $d = X^r$ in total, raising the money from many depositors. A depositor with claim $ad$ is permitted to take cash or portions of the loan with market value equal to $ad$ (or to force this amount of loan to be sold to finance the payment of the deposit). The market value of the loan is $\{(1 + \beta)X^s)/2\} < X^s$, so not all the depositors will be paid in full if they run. If the banker should offer depositors less than $d = X^r$, then

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**Figure 3. Bargaining with depositors.** *If this part of the tree is entered after $D$ rejects the entrepreneur's offer, $D$ now has the option of liquidating or negotiating with the entrepreneur but not of accepting the initial offer.*
each depositor has the unilateral incentive to run to the bank to get paid in full whenever other depositors have not done so first. Therefore, when other depositors have not run on the bank, a given depositor will not make any concessions, preferring to run instead. Finally, once a run has fully disintermediated the bank’s assets, the entrepreneur and depositors, who now hold the loan to the entrepreneur, can negotiate. Depositors can hire the banker to collect the full $X^*$ for a fee of $[(1 - \beta)/2]X^*$. Knowing this, the entrepreneur will offer to pay $[(1 + \beta)/2]X^*$ directly to the depositors who now hold the loan, and the banker will receive zero. Consequently, a bank run drives the banker’s rents to zero. Disciplined by the threat of a complete loss in rents, the banker can commit at date 1 to pay the depositors at date 2 the entire amount $P_2 = X^*$ extracted from the firm.

Demand deposits thus allow the bank to create liquidity, allowing it to borrow more (i.e., $X^*$) from depositors than the market value of the loan to the entrepreneur (i.e., $[(1 + \beta)/2]X^*$). They work by creating a collective action problem. Depositors are individually better off refusing to renegotiate with the banker (even though collectively depositors are weakly better off renegotiating with the banker than seizing assets). Therefore, depositors grab assets first and negotiate later, but the later negotiations cut out the banker.\footnote{Another way of seeing this is that once the loan is made, the banker’s skills are useful only in effecting transfers, not in creating value. Once depositors are in possession of the loan, the banker adds nothing to the coalition of the entrepreneur and the depositors, and hence his rents are driven to zero.} Despite the possibility of efficient bargaining after a run, the banker is disciplined by a run. Hence the banker does not attempt to renegotiate, and pays out the full amount collected, taking only an infinitesimal rent for his specific skills.

H. Financing Through a Mix of Deposits and Other Claims

We have seen that investors holding nondeposit claims are negotiated down by the intermediary, whereas depositors are not. What if both kinds of investors simultaneously hold claims on the intermediary? Let investors (capital) hold a claim that gives them the residual value after deposits, $d$, are paid out. Capital can seize the intermediary’s assets (cash and loans) if the intermediary does not make an acceptable offer, but it then becomes responsible for paying depositors. In effect, this assumption that capital can always seize assets is tantamount to assuming that capital is outside equity, or represents so high a level of subordinate debt that the intermediary is always in default at date 1. We show that a rent typically goes to the banker so that the bank is no longer a complete pass-through.

Let the banker threaten not to collect the loan at date 2. We have already argued that he will be unsuccessful in negotiating depositors down. Hence this threat must be directed at capital.
Example 1, continued: Without the banker, capital will be able to collect only $\beta X^s$ from the entrepreneur. If the face value of deposits exceed $\beta X^s$, capital will not be able to avoid a run if the banker quits, and will get zero. The net amount available to capital and the banker if the banker does use his skills in collecting the loan is $X^s - d$. Because neither can get any of the surplus without the other’s cooperation, they split the surplus, and each gets $\frac{1}{2}(X^s - d)$. Similarly, if the face value of deposits is lower than $\beta X^s$, the banker gets $[(1 - \beta)/2]X^s$, and capital gets $[(1 + \beta)/2]X^s - d$.

Thus far, we have assumed that enough can be extracted from the entrepreneur to pay off depositors. If that is not true, there will be a bank run. Rather than proceeding case by case, let us present the general result.

**Lemma 1:**

1. If $\min[P_2, X^s] < d$, depositors will run on the bank and will be paid

   $\frac{1}{2}\min[P_2, \beta X^s] + \frac{1}{2}\min[P_2, X^s]$.

2. If $\min[P_2, X^s] \geq d$, and
   
   (2.a) if $\min[P_2, \beta X^s] < d$, then there is no run, depositors get paid $d$, the banker gets $\frac{1}{2}[\min[P_2, X^s] - d]$, and capital gets $\frac{1}{2}[\min[P_2, X^s] - d]$.
   
   (2.b) if $\min[P_2, \beta X^s] > d$, then there is no run, depositors get paid $d$ and
   
   (2.b.1) if $P_2 \leq \beta X^s$, the banker gets $0$ whereas capital gets $P_2 - d$.
   
   (2.b.2) if $X^s > P_2 \geq \beta X^s$, the banker gets $\frac{1}{2}[P_2 - \beta X^s]$ whereas capital gets $\frac{1}{2}(P_2 + \beta X^s) - d$.
   
   (2.b.3) if $P_2 \geq X^s$, the banker gets $[(1 - \beta)/2]X^s$ whereas capital gets $[(1 + \beta)/2]X^s - d$.

I. Optimal Bank Capital Structure with One-Period Projects

From the lemma, it is clear that if deposits are set very high the banker’s rents will be driven to zero, but the amount going to the depositors will also be reduced because the inevitable run will lead to a loss of the banker’s valuable services. By contrast, if deposits are set low, the banker absorbs substantial rents. The trade-off between these two effects results in the optimal bank capital structure at date 1. Let us examine this trade-off more closely.

Given the two possible states at date 2, there are two levels of deposits to consider, low and high: $d = X^L$ and $d = X^H$. If the banker issues a low level of deposits, he will capture a rent when $X^H$ is realized (of either $\frac{1}{2}[X^H - X^L]$)

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5 In terms of the amounts that can be raised at date 1, it does not make sense to set deposits lower than $d = X^L$ and a level of deposits between $d = X^L$ and $d = X^H$ is dominated by $d = X^H$. Hence the focus on these two levels.
or \([1 - \beta/2]X^H\) depending on whether \(X^L > \beta X^H\). For now, assume that \(X^L < \beta X^H\). Then the expected total date 2 payment the banker can commit to make to depositors plus other claimants when date 2 deposits are low enough to be risk-free, \((d_2 = X^L)\), is \(q^H[(1 + \beta)/2]X^H + (1 - q^H)X^L = D_{\text{Safe}}\).

Alternatively, to avoid absorbing a rent when the realized liquidation value is \(X^H\), the banker could operate with a high level of deposits, \(d = X^H\). However, a bank run would occur if \(X^L\) is the realization. Once the run occurs, the sum of the value to depositors, the banker, and any other claimants on the bank falls to the market value of the loan, or \([(1 + \beta)/2]X^L\). So the expected total payment the banker makes to outsiders, that is, to depositors plus other claimants, when deposits are high is given by \(q^H X^H + (1 - q^H) \times [(1 + \beta)/2]X^L = D_{\text{Risky}}\).

At date 1, the most that the bank can commit to pay to outsiders at date 2 is \(\max\{D_{\text{Safe}}, D_{\text{Risky}}\}\). This is strictly less than the total value the banker can collect from the borrower, \(E[X]\), whenever the value of the asset is uncertain. We can also calculate \(D_{\text{Safe}}\) and \(D_{\text{Risky}}\) if \(X^L \geq \beta X^H\). \(D_{\text{Risky}}\) is unchanged, and \(D_{\text{Safe}}\) is given by

\[
q^H \frac{(X^H - X^L)}{2} + X^L.
\]  

The following lemma follows.

**LEMMA 2:**

1. If \(q^H X^H < (1 - q^H) X^L\), then \(D_{\text{Safe}}\) is greater than \(D_{\text{Risky}}\).
2. If \(X^L < q^H X^H\), then \(D_{\text{Risky}}\) is greater than \(D_{\text{Safe}}\).
3. If \(X^L > q^H X^H \geq (1 - q^H) X^L\), there is a \(\beta^*\) such that \(D_{\text{Safe}} > D_{\text{Risky}}\) iff \(\beta < \beta^*\).

**Proof:** See the appendix.

\(D_{\text{Safe}} > D_{\text{Risky}}\) implies a capital structure with safe deposits raises more external financing than a capital structure with risky deposits. This is true if the expected costs of distress because of a run outweigh the expected rent that goes to the banker if deposits are too low. Because rent absorption takes place in “high” states whereas distress takes place in “low” states, the bank capital structure that raises the most value ex ante is one with relatively fewer deposits when bad times are anticipated and more deposits when good times are anticipated—the level of deposits should be a leading indicator.

Perhaps less obvious, when the intrinsic liquidity of project assets, \(\beta\), falls, the bank can again raise more by issuing more capital. The intuition here is that the banker’s rent in the high state is relatively unaffected by the illiquidity of bank assets once they are sufficiently illiquid—capital has to share half the collections over the value of deposits with the banker because it
cannot pay depositors on its own. However, the cost to investors of a bank run increases with illiquidity. Therefore, the bank raises relatively more when assets become more illiquid by adopting a safer capital structure.

**Corollary 1:** If the conditions of Lemma 2(3) hold, $\bar{D}^{\text{Risky}} - \bar{D}^{\text{Safe}}$ increases with a mean preserving spread in the distribution of $\bar{X}$.

Finally, the risk of the loan repayments is proportional to the risk of the underlying collateral. So the corollary suggests that the capital structure that raises the most at date 1 contains more deposits as the distribution of loan repayments shift to the tails. The intuition is that as value shifts to the tails, it becomes more important for the banker to commit to pay out the repayments extracted in the high state, while the costs incurred through financial distress in the low state become relatively unimportant. Note that this is observationally equivalent to "risk-shifting" behavior (riskier bank loans are correlated with higher leverage), though the direction of causality is reversed and bank management maximizes the amount raised, not the value of equity.

**J. Implication: The Decline in Bank Capitalization**

Berger, Herring, and Szego (1995) present evidence that book capital-to-assets ratios for banks have been falling steadily in the United States, from about 55 percent in 1840 to the low teens today. Although the passage of regulations providing greater implicit government capital to the banks could explain some of the decline, bank capital also declined over periods with little or no regulatory change. Our model suggests that as the underlying liquidity of projects, $\beta$, increases, the capital structure that raises the most value up front contains more deposits. Thus as the advantage of banks relative to arm's length lenders has decreased with financial development, perhaps because of improvements in information availability, the size of market, and the legal environment, our model suggests bank capital structures will become more levered. This could explain the historic decline in capital ratios.

**K. The Future as Collateral**

A number of recent papers (see, for example, Holmstrom and Tirole (1997, 1998); Krishnamurthy (1999)) have suggested that collateral constraints—the inadequacy of individual or aggregate wealth—prevent value-enhancing transactions from taking place. In their work, current wealth serves to bond promises (also see Jensen and Meckling (1976)). In the context of our model, outside lenders can only get at $\beta E[\bar{X}]$ of the entrepreneur's assets, and this serves as the outside collateral the entrepreneur can use to borrow. How-

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6 Our view that bank capital structure can allow for risky deposits contrasts with the view of Merton and Perold (1993) in which capital structure is always maintained such that deposits are completely safe because depositors want safe deposits.
ever, in our work, the bank is not just useful because it uses existing collateral more efficiently, but because it enhances the value of existing collateral more than other lenders. The banker has no direct collateral of his own but he has specific collection skills, which can be a future source of rents. By creating an institutional structure that kills these rents if he misbehaves, the banker bonds his behavior. Thus, the bank in our model can pledge, in addition to existing collateral, $\beta E[X]$, an additional amount $\max\{\tilde{D}^{\text{Safe}}, \tilde{D}^{\text{Risky}}\} - \beta E[X]$. So banks are especially valuable when the collateral value of assets $\beta E[X]$ is low and specific collection skills important.

II. Multiperiod Projects

Thus far, we have examined single-period projects financed by one-period banks. Two factors simplified our analysis: the entrepreneur had enough cash to repay what could be extracted, and the date being analyzed was the last date. Let us now examine lending to two-period projects, with projects starting at date 0 and ending at date 2. Let subscripts denote the date and let the assumptions we have made about cash flows and liquidation values hold looking forward from date 1 to date 2. We assume further that

$$\min \left\{ E[\tilde{C}_1 + \tilde{C}_2, \frac{\tilde{C}_1 + \tilde{C}_2}{X_1^{s_1}}] \right\} > 1$$

for all realizations of date 1 state $s_1$, \(2\)

so that the entrepreneur’s initial project produces greater total cash flow returns—viewed from both the date 0 investment and the date 1 opportunity cost of $X_1^{s_1}$—than storage. Because we are interested in knowing the maximum that can be financed, let us assume the entrepreneur contracts up front to pay $P_1 = \infty$.

From the previous section, we know the maximum the entrepreneur can commit to pay at date 2 is $E[X_2|s_1]$, and the maximum the bank can raise against this at date 1 is $\max\{\tilde{D}^{\text{Safe}}, \tilde{D}^{\text{Risky}}\}$, where for notational simplicity, we have suppressed the dependence of these terms on the date 1 state. Let us now determine how much the entrepreneur will pay at date 1.

A. The Most Interesting Case

Consider the most illuminating case. Let $\tilde{D}^{\text{Safe}} > \tilde{D}^{\text{Risky}}$ so that the banker can raise the most funds at date 1 by maintaining a safe capital structure at date 2 with deposits low enough to avoid runs. Also let

$$C_1^{s_1} + \tilde{D}^{\text{Safe}} > X_1^{s_1}, \tilde{D}^{\text{Safe}} > \frac{1 + \beta}{2} X_1^{s_1}.$$ \(3\)
The first inequality implies the cash the entrepreneur can generate together with the amount the bank can raise against the entrepreneur’s best date 2 promises are greater than the amount obtained from liquidation at date 1. The second inequality implies that the amount the banker can collect by liquidating at date 1 exceeds the amount the bank can raise against the entrepreneur’s best date 2 promises, which, in turn, exceeds the value of selling the loan (by the third inequality).

A.1. Solution Strategy

Because depositors can demand payment at any time and capital can replace the banker, we assume without loss of generality that the banker pays off all financial claimants every period. So we first determine how much the banker needs to pay out at date 1, which is a function of his capital structure coming into that date. The banker’s ability to pay all date 1 claimants depends on (1) how much he can raise by issuing claims against anticipated date 2 payments—which we know from the previous section and (2) how much cash the entrepreneur pays at date 1. So the second step is to determine the entrepreneur’s offer when he knows that the banker will respond, keeping in mind his (the banker’s) ability to meet his obligations.

The sequence of moves is as follows. The entrepreneur opens negotiations at date 1 by making a take-it-or-leave-it offer to the banker. Before concluding these negotiations, the banker then negotiates with capital, at the end of which the banker accepts the entrepreneur’s offer, the banker liquidates the entrepreneur, or capital takes over and negotiates with the entrepreneur. The entrepreneur’s opening offer is only available for the banker to accept, and if capital takes over, the entrepreneur will open with a new offer.

A.2. How Much Does the Bank Have to Pay Out at Date 1?

Let us start by determining how much the banker has to pay claimants at date 1 and how this varies with the level of maturing deposits $d_1$ contracted at date 0 (i.e., with capital structure). Because maturing deposits cannot be renegotiated, all we need to do is determine how much the banker pays capital.

The banker will first make an offer to capital (as in Figure 2). Capital can accept, or reject the offer and enter the equal probability take-it-or-leave-it-offer game, after which it can still take over the bank if it finds the offer unsatisfactory. As this is capital’s best response, we now determine how much the banker has to offer to avoid loss of control.

Suppose capital rejects the banker’s take-it-or-leave-it offer. Capital can then “liquidate” the bank and negotiate directly with the entrepreneur. If capital liquidates immediately, it can obtain $\beta X_1^*$. If capital were to wait until date 2 to liquidate, it would get $\beta E[X_2|s_1]$. Therefore, after rejecting a final offer from the banker, capital expects $\max\left\{\beta \max\{X_1^*, E[X_2|s_1]\} - d_1, 0\right\}$.

By contrast, if capital makes the take-it-or-leave-it offer (and gets all the surplus), it does not have to give the banker anything for his services (because it owns the loan to the entrepreneur and the banker has no right to
collect without the legal authority embedded in the loan). Therefore, it asks the banker to collect from the entrepreneur, and capital gets the ensuing loan repayment net of deposit payments of Max[$X_{1t}^i - d_1, 0$].

Anticipating the outcome of the equal probability take-it-or-leave-it-offer game, the banker will have to offer capital

$$\pi_{1t}^i = \frac{1}{2} \text{Max}[\beta \text{Max}[X_{1t}^{i\prime}, E(X_{2t}|s_1)] - d_1, 0] + \frac{1}{2} \text{Max}[X_{1t}^i - d_1, 0].$$  (4)

Note that the payment is state-contingent, not because capital has an explicit state-contingent contract, but because capital’s bargaining power depends on the date 1 state. By contrast, the payment to deposits is fixed so long as the bank is not run or liquidated. On inspection, the total payment, $\pi_{1t}^i + d_1$, that has to go to date 1 claimants is increasing (though sometimes only weakly) in the level of deposits. Capital structure coming into date 1 therefore affects the total amount the banker has to pay out. Now let us see how this amount and the amount the banker can raise against the entrepreneur’s future payments affects negotiations between the entrepreneur and the banker.

### A.3. Negotiations Between Banker and Entrepreneur

Given that he has the cash, the entrepreneur is indifferent about paying it at date 1 or paying it at date 2, and the banker is either indifferent or prefers an earlier payment (if he is undercapitalized). We can thus focus without loss of generality on payment offers by the entrepreneur such that $P_{2t}^{i\prime} > 0$ only if $P_{1t}^{i\prime} = C_1$, that is, the entrepreneur promises a positive date 2 payment only if he has no more cash to make date 1 payments. For the banker to accept an offer, two conditions must hold. First, the amount paid by the entrepreneur at date 1 together with any date 1 amounts the banker raises by issuing new claims against future recoveries from the entrepreneur have to be enough for the banker to pay the depositors and capital coming into date 1. So if $\text{Pledgeable}(P_{2t}^{i\prime})$ is the amount the bank can raise today against a date 2 promise of $P_{2t}^{i\prime}$ by the entrepreneur, we require

$$P_{1t}^{i\prime} + \text{Pledgeable}(P_{2t}^{i\prime}) \geq \pi_{1t}^{i\prime} + d_1.$$  (5)

7 Alternatively, capital could ask the banker to do nothing at date 1, and pay everything he can commit to pay out of date 2 collections. We have seen that the banker can commit to pay the capital and deposits withdrawn at date 2 at most $D^{safe}$. However, in the current case, capital prefers immediate liquidation because $X_{1t}^i > D^{safe}$ from equation (3).

8 More specifically, $\text{Pledgeable}(P_{2t}^{i\prime}) = q_{2t}^{H}\left(P_{2t}^{i\prime} + (1 - q_{2t}^{H})\min[P_{2t}^{i\prime}, X_{2t}^l]\right)$ if $P_{2t}^{i\prime} \leq \beta X_{2t}^U$ and the capital structure can be set so that the bank does not collect a rent. It is $q_{2t}^{H}(1 + \beta/2)P_{2t}^{i\prime} + (1 - q_{2t}^{H})X_{2t}^l$ if $P_{2t}^{i\prime} > \beta X_{2t}^U$ and the bank does collect a rent at date 2. If $P_{2t}^{i\prime} > X_{2t}^l > \beta X_{2t}^U$, the expression is $q_{2t}^{H}((P_{2t}^{i\prime} + X_{2t}^l)/2) + (1 - q_{2t}^{H})X_{2t}^l$. 
Second, the banker should get more over the two dates after paying out all claimants than if he liquidates and pays claimants. Because the required payment to claimants does not depend on whether he liquidates or not, this implies

\[ P_1^{s_1} + q_2^H P_2^{s_1} + (1 - q_2^H) \text{Min}[P_2^{s_1}, X_2^L] \geq X_1^{s_1} \] (6)

where we have suppressed the dependence of date 2 values on the date 1 state for notational simplicity. We will now show that, depending on how much cash the entrepreneur has and the bank’s capital structure coming into date 1, the entrepreneur’s total payments to the bank may exceed \( \text{Max}[X_1^{s_1}, E(X_2|s)] \), which is the total payment a long horizon lender could extract, even though the date 1 liquidation threat is what enables the banker to extract repayment. It may be useful to first outline the intuition with the numerical example.

A.4. Numerical Example

Let \( \beta = 0, X_1^{s_1} = 0.99, X_2^L = 0.8, X_2^H = 1.4, \) and \( q_2^H = 0.5. \) \( DSafe \) is given by equation (1) and equals 0.95, and \( DRisky = 0.9. \)

Because \( E[X_2|s_1] = 1.1 > X_1^{s_1} = 0.99, \) the banker will get 1.1 in expectation at date 2 from the entrepreneur if he turns down the entrepreneur’s offer. The bank can raise \( DSafe = 0.95 \) at date 1 against these payments by the entrepreneur.

Let the bank’s outstanding deposits coming into date 1, \( d_1, \) be 0. Then the total payments the bank has to make date 1 claimants is \( \pi_1^{s_1} + d_1 = \pi_1^{s_1} = 0.99 \) (substituting values in equation (4)). Because the bank can raise more even after rejecting the entrepreneur’s offer, capital structure at date 1 does not constrain the banker’s response, and he will reject any offer that pays less than 1.1. Moreover, the difference between the expected inflow of 1.1 and the outflow of 0.99/2 (to pay off date 1 claimants) will be a rent to the banker.

As the level of deposits increases, the total payout to date 1 claimants increases. When \( d_1 \) exceeds 0.91, the bank’s total payout to date 1 claimants exceeds \( DSafe = 0.95. \) Because \( DSafe \) is the maximum the bank can raise at date 1 against future promises, its horizon shortens and it will liquidate at date 1 if not paid enough by the entrepreneur. It turns out that the entrepreneur’s cash position now matters in determining his payment. To see this, let \( d_1 = 0.99 \) so that from equation (4), \( \pi_1^{s_1} + d_1 = 0.99. \)

An entrepreneur with \( C_1^{s_1} < 0.04 \) will always be liquidated. This is because the most the entrepreneur can offer without being liquidated is \( C_1^{s_1} + DSafe < 0.99, \) and the bank needs 0.99 to avoid a run by depositors. But at \( C_1^{s_1} = 0.04, \) the entrepreneur can offer an immediate payment of \( P_1^{s_1} = C_1^{s_1} = 0.04 \) and a future payment of \( P_2^{s_1} = 1.4. \) This will be accepted because the banker gets \( C_1^{s_1} + DSafe = 0.99 \) to pay off maturing date 1 deposits. The total amount the banker will collect from the entrepreneur over date 1 and date 2 is \( C_1^{s_1} + E[X_2|s] = 0.04 + 1.1 = 1.14, \) which exceeds \( \text{Max}[X_1^{s_1}, E(X_2|s_1)] = 1.1, \) the maximum amount the banker could collect if he was patient and not constrained by capital structure.
Why does the banker collect more? Intuitively, the banker’s need to pay claimants at date 1 shortens his horizons and makes his date 1 liquidation threat credible even though it is inferior to the date 2 liquidation threat. To avoid liquidation by the banker, the entrepreneur pays everything he can today, and the bank raises the rest against future promises by the entrepreneur. Because only a fraction of the future payments by the entrepreneur translate into current cash raised by the banker (the entrepreneur pays 1.1 in expectation but the banker can raise only 0.95 against it) the entrepreneur overpays to avoid liquidation. Of the total of 1.14 the entrepreneur pays, 0.99 will be paid to outside investors and the banker will keep the rest as rent.

As the entrepreneur’s date 1 cash inflows increase further beyond 0.04, he can make more of his payments in cash and less in inefficient date 2 promises that involve paying an additional rent to the bank. Eventually, the date 2 promise falls to such a level that it no longer requires the bank’s special skills to collect (the loan to the entrepreneur becomes liquid), and the bank’s rent falls to zero. Therefore, the total payment made by the entrepreneur falls as he generates more cash, and when \( C_1 \approx 0.19 \), his payment bottoms out at 0.99. Note that the entrepreneur now pays less than Max\([X_1, E(X_2 s_1)] = 1.1\), and the banker’s short horizon clearly hurt his ability to collect.

A.5. More Formally

More formally, as equation (5) indicates, if deposits due at date 1 are high so that the bank has to pay much out at date 1 while the entrepreneur generates little cash at date 1 so that \( P_1 \) is small, he may have to promise to pay \( P_2 \) at date 2 for the bank to raise enough to pay off date 1 claimants (we assume as in the example that \( X_2 > \beta X_2 \)). But such a high promised payment implies that the banker will get a date 2 rent of \( (q_2^H/2)(P_2 - X_2^H) \). So an entrepreneur with little date 1 cash has to use an inefficient means of payment—date 2 promises that have an element of leakage in that some of it goes as a rent to the bank.

Just because a rent goes to the bank at date 2 does not imply the entrepreneur will overpay. The bank also has to be highly levered. To see why, if the amount owed by the bank to date 1 claimants is less than \( X_1 \), so that the banker gets some rents at date 1, the entrepreneur could ask to offset the rent the bank collects at date 2 by paying less at date 1. But the highly levered bank pays out everything it gets at date 1 to claimants, so the rent at date 2 cannot be offset and becomes entirely excess payment by the entrepreneur. In particular, the total amount the entrepreneur pays is

\[
X_1 + \text{Max} \left\{ \left[ \frac{q_2^H}{2} (P_2 - X_2^H) - (X_1 - d_1 - \pi_1) \right], 0 \right\} 
\]

which is the sum of the bank’s liquidation threat and the net uncompensated rent the bank gets (the term in square brackets in equation (7)). The higher \( d_1 \) is, the lower the date 1 rent going to the bank is and the greater the
uncompensated date 2 rent. When $d_1 = X_1$, the bank’s date 1 rent is zero and all the rent paid at date 2 is entirely excess payment to the bank. A
cash-poor entrepreneur can thus pledge up to $P_2 = X_2$. The highly levered bank can extract up to $X_1 + q_2((1 - \beta)/2)X_2$ from this entrepreneur, which, using equation (3), is greater than $E[X_2|s_1]$. So it is the combination of an illiquid borrower and a highly levered bank that enables the latter to extract more from the former.

Of course, a deposit-intensive date 1 capital structure that shortens the bank’s horizons can also hurt its ability to extract repayment if the entrepreneur’s project generates a lot of cash at date 1. To see this, if the entrepreneur generates enough cash at date 1 so that $P_2 \leq X_2$, the total payment given by equation (7) is only $X_1$. If $E[X_2|s_1] > X_1$, the entrepreneur will pay less to the bank than a patient bank can extract, and the shortening of horizon makes the bank “weak.” Thus the amount that can be extracted from the entrepreneur depends in a nonmonotonic way on the bank’s leverage and the entrepreneur’s liquidity.

There is some empirical evidence supporting our model. Hubbard, Kuttner, and Palia (1999) find that bank dependent borrowers (but not the most credit-worthy among them) tend to pay higher rates when their bank is highly levered. More work, of course, needs to be done to test the detailed implications of our model.

B. Related Literature

Although others (Berglof and Von Thadden, 1994), Bolton and Scharfstein (1996), and Dewatripont and Tirole (1994) have analyzed the role of multiple creditors in “toughening” up a borrower’s capital structure, we do not know of any other work that examines the effect of a tough capital structure on an intermediary’s behavior towards borrowers. The closest work to ours is that of Perotti and Spier (1993), who examine the role of senior debt claims on management’s ability to extract concessions from unions. In their model, management can credibly threaten to underinvest by taking on senior debt. Of course, this is simply a ploy to extract concessions from unions. In our model, a deposit-intensive capital structure allows the bank to credibly threaten to liquidate. The threat of a run commits the bank to liquidate if the present and future payments offered are too small, and the larger payments imply a rent to the banker (because their collection requires the banker’s skills).

C. General Characterization of Date 1

Thus far, we have only examined a special case, albeit one that contains the most interesting implications. More generally, at date 1, the banker will try to maximize the sum of his date 1 and date 2 rent, conditional on being able to pay claimants at date 1. So we have the following proposition.
Proposition 1: If the entrepreneur has to renegotiate his payment at date 1, the outcomes are as follows.

1. If \( d_1 > \max \{ D_{\text{Safe}}, D_{\text{Risky}}, X_1^{*}\} \), the entrepreneur offers nothing at date 1 and there is a bank run. The run reduces the amount collected by depositors to \( \max[(1 + \beta)/2]X_1^{*}, \beta E[X_2|s_1] \), and drives the payoff of capital and the banker to zero. In the rest of the proposition, the level of \( d_1 \) is assumed less than or equal to \( \max \{ D_{\text{Safe}}, D_{\text{Risky}}, X_1^{*}\} \).

2. If \( \max \{ D_{\text{Safe}}, D_{\text{Risky}} \} \geq X_1^{*} \), then the bank cannot use its date 1 liquidation threat. If \( D_{\text{Risky}} > D_{\text{Safe}} \), there is a level of date 1 net deposits beyond which the amount collected from the entrepreneur falls from \( E[X_2|s_1] \) to \( D_{\text{Risky}} \). If \( D_{\text{Risky}} \leq D_{\text{Safe}} \), the level of date 1 deposits has no effect on total collections which are always \( E[X_2|s_1] \).

3a. If \( D_{\text{Risky}} < D_{\text{Safe}} < X_1^{*} \), there is a \( d^* \) such that for every \( d_1 > d^* \), we can find a \( C_1^{\text{Liq}}(d_1) \) such that the entrepreneur will be liquidated with some probability if he defaults at date 1 when \( C_1^{*} < C_1^{\text{Liq}} \).

3b. If, further, \( X_1^{*} > E[X_2|s_1] \), there is a \( d^{**} \) such that for every \( d_1 > d^{**} \), there is a range \( [C_1^{*}, C_1^{**}] \) such that the bank extracts more than \( X_1^{*} \) from the entrepreneur if \( C_1^{*} \in [C_1^{*}, C_1^{**}] \). For any given \( d_1 > d^{**} \), the amount extracted by the bank increases until \( C_1^{*} = C_1^{**} \) and then decreases monotonically as \( C_1^{**} \) increases.

3c. If \( E[X_2|s_1] > X_1^{*} \), there is a \( d^{***} \) such that for every \( d_1 > d^{***} \), there is a range \( [C_1^{*}, C_1^{***}] \) such that the bank extracts more than \( E[X_2|s_1] \) from the entrepreneur iff \( C_1^{*} \in [C_1^{*}, C_1^{***}] \). There is \( \bar{C}_1 > C_1^{***} \) such that the bank extracts only \( X_1^{*} \) iff \( C_1^{***} \geq \bar{C}_1 \).

   For any given \( d_1 > d^{***} \), the amount extracted by the bank increases until \( C_1^{***} = C_1^{***} \) and then decreases monotonically as \( C_1^{***} \) increases until \( C_1^{***} = \bar{C}_1 \).

4. If \( D_{\text{Safe}} < D_{\text{Risky}} < X_1^{**} \), then capital structure has no effect on the expected amount the bank extracts if \( E[X_2|s_1] \leq X_1^{*} \). When \( E[X_2|s_1] > X_1^{*} \), there is a \( d \) such that the bank extracts less than \( E[X_2|s_1] \) iff \( d_1 = d \).

Proof: See the appendix.

Proposition 1 shows how a preexisting bank capital structure and the entrepreneur’s liquidity will affect the payments that will be made by the entrepreneur. Let us now move back to date 0.

D. Capital Structure at Date 0

At date 0, the banker simply aggregates the effects across date 1 states and chooses a capital structure that maximizes the amount of surplus he captures over the two periods. If banks are competitive, all projects for which the banker can raise sufficient funds by pledging payments to outside depositors and capital are funded.\(^9\) First, let bankers start with no personal funds.

\(^9\) For a paper that obtains equilibrium pricing and quantities of bank capital, see Gorton and Winton (1995).
D.1. Trade-offs When Banker Has No Funds of His Own

When the banker has no personal funds, the level of deposits going into date 1 will be set such that it minimizes the rent that flows to the banker, provided the project can be fully funded. Let there be two states at date 1 also, \( H \) and \( L \). The maximum the banker can raise from outsiders at date 1 in state \( s_1 \) is \( D^1_{s_1} = \max\{X^1_{s_1}, D^\text{Risky}_{s_1}, D^\text{Safe}_{s_1}\} \). Without loss of generality, let the amount that can be raised at date 1 in state \( H \) exceed the amount in state \( L \). The banker can finance with safe deposits at date 0 if \( d_1 < D^L_{s_1} \). This implies a date 1 rent to the banker when state \( H \) occurs. The total amount that can be raised through deposits and capital at date 0 is then

\[
\tilde{D}^\text{Safe}_0 = D^H_1 + q_H^1 \left[ \max\left( \frac{1}{2} \beta \max\{X^H_1, E[X^2_2|H]\} - D^L_1, 0 \right) + \frac{1}{2}(D^H_1 - D^L_1) \right],
\]

where the second term is the rent that accrues to capital at date 1 in state \( H \).

If deposits, \( d_1 \), exceed \( D^L_1 \), there will be a run at date 1 in state \( L \). This reduces the payment to outsiders in that state to \( \max\left(\frac{1+\beta}{2}X^L_1, \beta E[X^2_2|L]\right) \). The maximum that can be raised at date 0, given a run in the low state at date 1, is then obtained by setting \( d_1 = D^H_1 \). The date 0 amount raised is

\[
\tilde{D}^\text{Risky}_0 = q^H_1 \tilde{D}^H_1 + (1-q^H_1) \max \left\{ \frac{1+\beta}{2}X^L_1, \beta E[X^2_2|L] \right\}.
\]

It is now easy to see the date 0 capital structure under competition. For example, if \$1 has to be raised and \( \tilde{D}^\text{Risky}_0 \geq 1 > \tilde{D}^\text{Safe}_0 \), and \( \tilde{D}^\text{Risky,H} \geq \tilde{D}^\text{Safe,H} \), no rents need be given to the banker and the firm is best off borrowing from a risky bank. By contrast, if \( \tilde{D}^\text{Safe}_0 \geq 1 > \tilde{D}^\text{Risky}_0 \), the project cannot be financed with risky deposits. The bank will issue a level of deposits at date 0 that will be safe in all states at date 1. It will issue capital to fund the rest of the project. So even under competition, rents will accrue to the banker, simply because he is liquidity constrained (in the sense of having no inside capital) and cannot pay for the rents up front.

D.2. Trade-offs When Banker Has Funds of His Own

Now let the banker have the endowment to pay up front for the rents he extracts. The level of deposits going into date 1 is determined by trading off the total amount collected from the entrepreneur (which varies with the level of deposits as seen in the previous section) against the risk of runs (which increases with deposits). Under the conditions of Proposition 1, 3b and 3c, a highly levered bank may now have a comparative advantage in funding an entrepreneur who expects to generate only modest amounts of cash at interim dates—the bank can extract more from such an entrepreneur and thus can lend more money up front. By contrast, as proposition 1 suggests, if \( E(X^2_2|s_1) > X^1_{s_1} \), an entrepreneur with high anticipated date 1 cash inflows may prefer a well-
capitalized bank because such a bank can wait to liquidate, and will collect $E(\hat{X}_2|s_1)$. Thus our theory predicts a matching between banks with different capital structures and particular entrepreneurs.

E. Multiple Borrowers

We argued earlier that the bank's ability to extract more than $\max[X_1^+, E(X_2|s_1)]$ does not change qualitatively if it has multiple borrowers instead of one. Although this is obvious when the bank has to threaten all borrowers with immediate liquidation to raise enough to pay claimants, consider the case where the bank's capital structure allows it some slack so that it can treat borrowers asymmetrically. It turns out there is a unique equilibrium that mirrors the single borrower case.

At date 1, borrowers will, if possible, simultaneously make the lowest offer that ensures they are not liquidated. Let borrower $j$ offer a payment schedule $(\hat{P}_1^j, \hat{P}_2^j)$. To avoid integer constraints, we assume that each borrower is small (alternatively we can allow partial liquidation of a borrower). For simplicity, we assume that all borrowers' projects have identical liquidation values, but allow the cash generated, $C_1^j$, to differ. It is straightforward, but notationally messy, to generalize to allow borrowers with different liquidation values.

In choosing between offers, the banker has two concerns. He wants to maximize how much he collects from borrowers with the constraint that he has to pay off maturing claims (of value $d_1$). Clearly, when each borrower $j$ makes an offer with an expected total payment of $X_1$, and the banker can raise enough to pay off $d_1$, there will be no overpayment.

If, however, the banker cannot raise $d_1$, then borrowers will have to make offers that involve overpayment and/or some will be liquidated. Let us now determine how the banker will respond to each offer. If the banker decides against liquidating, he can reject the offer and thus collect at date 2 or he can accept the offered payments. So the cost of liquidating borrower $j$ is the foregone continuation value less the receipts from immediate liquidation, $\max[\hat{P}_1^j + E(\hat{P}_2^j), E(X_2)] - X_1$ where $E(\hat{P}_2^j)$ is the expected date 2 payment by the borrower if his offer is accepted, and all variables are contingent on the date 1 state. The benefits from liquidating borrower $j$ are that the bank may get more today to meet claimants' needs. This is the liquidation value $X_1$ less the amount that can be raised immediately if the borrower were continued, $\max[\hat{P}_1^j + \text{Pledgeable}(\hat{P}_2^j), \text{Pledgeable}(X_2)]$. Thus the cost to benefit ratio of liquidating is

$$\Gamma_j = \frac{\max[\hat{P}_1^j + E(\hat{P}_2^j), E(X_2)] - X_1}{X_1 - \max[\hat{P}_1^j + \text{Pledgeable}(\hat{P}_2^j), \text{Pledgeable}(X_2)]}.$$ 

Now suppose the banker receives the borrowers' offers and knows he has to liquidate some to repay claimants. Of the borrowers for whom the denominator in $\Gamma$ is positive (i.e., the banker gets more immediately by liquidating
than by continuing the borrower), the banker will prefer liquidating the borrower with the lowest \( g_j \). Intuitively, the banker gets more current bang for the sacrifice of a future buck when he liquidates such a borrower. Of course, the banker will never liquidate a borrower for whom the denominator is zero or negative, because he can raise (weakly) more by continuing such a borrower.

Anticipating such a response, borrowers will first attempt to minimize the denominator by making offers that have the maximum pledgeable content. For instance, they will offer to pay out all current cash. But if even after setting \( P^1_j = C^1_j \), borrower \( j \)'s \( g_j \) is too low, he will have to raise \( P^2_j \) to increase \( g_j \). Let \( \hat{g}_j \) be the maximum \( g_j \) borrower \( j \) can offer. Now consider a candidate equilibrium \( \hat{g}^* \) such that borrowers with \( \hat{g}_j < \hat{g}^* \) are liquidated, whereas borrowers with \( \hat{g}_j > \hat{g}^* \) lower their offer until \( \hat{g}_j = \hat{g}^* \) or their total expected payment equals \( X_1 \) (no one can pay less than this). The unique equilibrium is the lowest \( \hat{g}^* \) that allows the banker to raise \( d_1 \).

It is easily confirmed that all our earlier intuition carries through. *Ceteris paribus*, borrowers with more cash flow \( C^1_j \) have a higher \( \hat{g}_j \) and will have to make a lower total payment than a borrower with less cash because the former offer more value up front. The really cash poor will be liquidated because they cannot make up sufficiently through future promises for the low cash value of their offers. Moreover, the more levered the bank (higher \( d_1 \)), the greater is the equilibrium \( \hat{g}^* \), and the greater the payments extracted from borrowers who are not liquidated.

It is not the case that the banker singles out particular borrowers to make them pivotal to his continuation, and thus extracts more. Rather, knowledge of the pressure the banker is under to meet his claims forces borrowers to self-select in their offers as they attempt to avoid liquidation.

What is particularly interesting is that there are now spillover effects among borrowers. A borrower is worse off as his fellow borrowers become more cash rich—because he will have to pay more to make his offer as attractive as their front-loaded offers—whereas he is better off if their liquidation value increases because they are more likely candidates for liquidation. These interborrower effects deserve further study.

### III. Policy Implications

Our framework allows us to comment on the effects of policies such as capital requirements. We describe the trade-offs highlighted in our model that may throw additional light on the policy debate.

#### A. The Effects of Minimum Capital Requirements

Minimum capital requirements specify a minimum capital-to-asset ratio required to enter banking or to continue to operate as a bank (see Benston et al. (1986), Berger et al. (1995), and Kane (1995)). Because there is a level of deposits (and thus a level of capital) that maximizes the amount that the banker can pledge to outside investors, requiring more capital will
make the bank safer but also increase the banker’s rents, reduce the amount
the banker can pledge to outsiders, and raise the bank’s effective cost of
capital.

Now consider the effect of a binding current capital requirement on a bank’s
interaction with borrowers. If a very strict capital requirement is imposed, such
as allowing no deposits at all in the future, the most that a banker can commit
to pay outsiders at date 2 is the market value of its loans (pledgeability goes
down from \(\max(D_{\text{Safe}}, D_{\text{Risky}})\) to \([(1 + \beta)/2]E[X_2|s_1]\) as more capital is re-
quired). By contrast, the banker can collect his full liquidation threat from bor-
rowers immediately, and this threat is unchanged by the requirements. As a
result, given a preexisting set of claimants that have to be paid, an increase in
future capital requirements will shorten the banker’s horizons and make it more
likely that the banker will use the immediate liquidation threat.

The shortening of the banker’s horizons has different effects on borrowers.
A borrower with very little cash will be liquidated. A borrower with moderate
cash will pay more because future promises from the borrower have less
value under the stricter capital requirements. Finally, a borrower with lots
of cash and for whom the future liquidation threat is more valuable than the
immediate one will pay less. Thus an increase in capital requirements can
cause a “credit crunch” for the cash poor and potentially alleviate the debt
burden of the cash rich; greater safety has adverse distributional conse-
quences. Finally, and paradoxically, by reducing the bank’s future ability to
pledge, an abrupt transition to higher capital requirements can lead to a
bank run because maturing deposits may exceed what the bank can pledge.

A.1. Long Run Effects of Capital Requirements

The amount that a bank can raise at date 0 depends on what it can
commit to pay out at date 1. If the bank relies on liquidation threats at
date 1, capital requirements do not reduce pledgeability. But if \(X_1^1 <
\max(D_{\text{Safe}}, D_{\text{Risky}})\), higher capital requirements reduce the amount that can
be pledged to those outside the bank. This can prevent the funding of
entrepreneurs with projects with payoffs in the more distant future.

In summary, capital requirements have subtle effects, affecting the flow of
credit and even making the bank riskier. These effects emerge only when the
capital requirements are seen in the context of the functions the bank per-
forms rather than in isolation.

B. The Effects of Deposit Insurance

Thus far, we have not considered the effect of deposit insurance. In prac-
tice, bank deposits below a certain amount have explicit insurance, whereas
bank deposits above that may enjoy some implicit insurance if the bank is
too big to fail.

At one extreme, when all depositors are insured, the insurer intervenes early
to back deposits, and enjoys no special powers in negotiating, deposits will
have no disciplinary effect. In such a situation, deposits are essentially no dif-
ferent from capital, and banks are safe but do not create liquidity (implying that if banks raise deposits in excess of the market value of loans, the excess is the result of a subsidy provided by the deposit insurer). Moreover, even if deposit insurance is fairly priced, it interferes with private contracting and weakly reduces aggregate welfare. On the other hand, when some deposits are uninsured (or when there is a positive probability that some deposits will not be bailed out) and the insurer takes his own time coming to the banker’s aid, we could get very similar effects to those in the model. Runs by uninsured depositors would still lead to some disintermediation, and this would provide some discipline. Furthermore, if the deposit insurer has a committed policy of closing a bank when its capital is too low (and somehow enforces this commitment), then our results again follow. See Diamond (1999) for an analysis of Japanese banking in recent times using this approach.

We have not considered the possibility of panic-based runs (perhaps resulting from depositor fears that refinancing will not be available) or aggregate liquidity shortages that are central to the rationale for deposit insurance according to Diamond and Dybvig (1983) or Holmstrom and Tirole (1997). If these events have a positive probability of occurring, deposit insurance can have benefits that have to be traded off against its costs of reduced commitment. Firm conclusions await further research.

C. Intervention in a Crisis

Consider a financial crisis where a number of firms are short of cash and are threatened with liquidation by banks, while banks themselves are insolvent and face imminent runs by depositors. If the regulatory authorities want to minimize failure, how best should they target resources?

Although a complete answer is not possible without parameterizing the problem more fully, our model points to some issues that are often overlooked. It is usually thought that the infusion of cash (i.e., capital) into either the industrial sector or the banking sector should make both sectors better off. This is not the case because the infusion of cash can increase (and sometimes decrease) the targeted sector’s bargaining power vis à vis the other sector.

If the industrial sector gets cash, some firms will be able to avoid liquidation and repay their loans. This may not improve the health of the banking sector because it would have recovered the money anyway by liquidating. Other firms may be able to survive by committing to pay the banks more in the long run, with little effect on the banks’ current state. And still other firms will be able to take advantage of the banking sector’s weakened state and short horizon and negotiate their repayments down in return for immediate payment. This will hurt the long-run viability of the banking sector. Finally, the cash infusion that goes to firms that are already cash rich will have no effect on repayments or on failures. On net, the industrial sector will definitely be made better off by the infusion, though some of it may go to waste (see Holmstrom and Tirole (1998)). The banking sector may be made worse off depending on the distribution of borrowers in the economy.
Consider now a cash (i.e., capital) infusion to the banking sector. An infusion only large enough to prevent bank runs from taking place may simply lead to the industrial sector being squeezed harder. If the infusion did not take place, banks would have to sell loans to stave off a run (or actually be run), and firms would be able to negotiate their debts down with new creditors. The infusion helps banks just survive without selling loans, but forces them to be tough with their borrowers. Some firms will be liquidated, and others may just survive by mortgaging their futures to the banks. The industrial sector could be made worse off by such an infusion. Of course, a large cash infusion will extend bank horizons, enabling banks to use long-run liquidation threats, and help the industrial sector escape liquidation without transferring excessive value to the banking sector. Thus the recapitalization of the banking sector may have to be really large to have a positive influence on the industrial sector. More work is needed to quantify these effects.

IV. Robustness

Before we conclude, let us examine how robust our model is to changes in assumptions. This will also help highlight what is really critical to the model.

A. Actions Other Than Threats to Quit

Because the financial asset requires the banker’s collection skills, the threat of dismissal is not always a credible sanction. However, a run serves to discipline and thus control adverse banker actions that can be observed by outsiders. Actions that can be controlled by the threat of disintermediation include the bank operating inefficiently, making poor credit decisions, incurring excessive labor costs, or even substituting assets. The threat to quit should be viewed as a metaphor for such actions.

B. Can the Intermediary Do Without Demandable Debt?

Three characteristics of demand deposits are important in controlling these actions. First, depositors can ask for repayment at any time. Second, they have priority over any other claim if they ask for repayment. Third, if there are multiple depositors, each one can establish priority with respect to the other only by seizing cash and forcing disintermediation. Could other claims have similar properties?\(^\text{10}\)

\(^\text{10}\) We assume the information possessed by depositors is obtained freely and depositors do not spend money to monitor the bank. A generalization of our approach is to consider either low cost monitoring of information about banker actions, or situations where a subset of depositors learn banker actions and incipient runs reveal information to other depositors. This would allow us to incorporate the insights of Calomiris and Kahn (1991), where the fact that the first few depositors get paid in full provides incentives for the monitoring of malfeasance, and of Rajan and Winton (1995) or Park (1999) where potentially impaired senior creditors have the strongest incentive to monitor a borrower.
It turns out that any such claim looks very much like a deposit. For example, suppose the bank finances at date 1 by issuing a single class of short-term debt maturing in one period. If, at date 2, the banker attempts to renegotiate payments, the short-term creditors will have no option but to give in; because they are treated identically, they are better off accepting the banker’s terms. The important difference between short-term debt and demand deposits here is that there is no collective action problem with the former.

To induce such a problem, it is necessary that some creditors should be able to achieve priority only by demanding payment. This requires some classes of debt to mature before the banker’s threat can be carried out, and also requires that it be impossible for the banker to promise maturing creditors a future claim equal in worth to what they can get immediately. These claimants will then demand immediate payment. The amount of disintermediation that will occur is then equal to the amount of demand deposits plus the amount of debt maturing before the banker’s adverse action is carried out. The banker’s rents will be restricted to a function of the extra value that he can collect on the assets that remain in the bank.

Finally, we have ignored throughout the paper any rationale for investors themselves to want demandable claims. If, as in Diamond and Rajan (1999a), investors have random liquidity or payment needs, then demandable deposits would be preferable to short maturity debt even if they have similar disciplinary effects.

C. Could the Entrepreneur Issue Deposits?

One could also ask why the entrepreneur does not reduce his cost of capital by directly issuing demand deposits. It turns out that because the entrepreneur’s human capital is still essential ex post to the generation of cash flows, he cannot commit to extracting lower rents by issuing demand deposits. Intuitively, depositors in the firm will seize the firm’s assets after a run. But because the entrepreneur is still the best user of the assets, they will rehire the entrepreneur after they take the assets and thus will be forced to pay him his rents. Unlike the banker, the entrepreneur is not redundant ex post and hence demand deposits that induce depositors to grab assets do not discipline him. So long as there is some time after the holder of the claim on the entrepreneur takes his claim before he must irrevocably liquidate, the entrepreneur will be retained. This is unlike the situation with a banker, where a brief time to be rehired does not reverse the discipline of a run. As a result, demand deposits will be much less effective in the capital structure of industrial firms, and firm capital structure will tend towards irrelevance.

D. Cash and Collateral

Thus far, we have not examined what happens if either the entrepreneur or the bank store cash. If cash is simply treated as an asset with $\beta = 1$, it turns out that the storage of cash has no effect on our results. For example,
at date 1 only net debt, \( P_1 - C_f \), or \( d_1 - c_b \) matter, where \( C_f \) is cash stored by the firm at date 0 and \( c_b \) is cash stored by the bank. So everything that is achieved by holding cash is achieved by taking on less debt.

Stored cash does have use if it cannot be seized by the lender but can be used at the borrower’s discretion. Essentially, as in Hart and Moore (1998), it is one way to make simple contracts more contingent. To see this, let \( C_1^L + E[\tilde{X}_2|L] < X_1^L \) so that the bank will liquidate in the low date 1 state if the entrepreneur defaults. To avoid liquidation in the low state, the entrepreneur must not be required to pay more than \( P_1 \leq C_1^L \). But this will limit what he can pay in the high state to \( C_1^L + E[\tilde{X}_2|H] \). If the date 1 liquidation threat in the high state allows the bank to collect more than this (i.e., \( X_1^H > C_1^L + E[\tilde{X}_2|H] \)) then liquidation in the low state may be averted only at the cost of drastically reducing the total amount the entrepreneur can commit to pay. Stored cash that the entrepreneur has complete discretion over can help in this situation.\(^{11}\) The bank could lend the entrepreneur \( 1 + x \) at date 0, set \( P_1 = \infty \), and have him hold cash of \( C_f = x \) where \( x = X_1^L - C_1^L - E[\tilde{X}_2|L] \). In the low state at date 1, the entrepreneur can now avoid liquidation by paying \( X_1^L \) in cash and promises. In the high state, the entrepreneur will pay \( X_1^H \). So the collateral value in the high state at date 1 can be fully utilized without incurring liquidation.\(^{12}\)

Our model therefore predicts entrepreneurs will hold extra cash to keep control either when a cash shortage could lead to liquidation or when a cash shortage could increase the amount that the lender can extract. These roles for cash can be part of the original implicit deal and anticipated by the bank.\(^{13}\)

E. Uncertainty and Incomplete Contracts

We have assumed that the uncertainty is noncontractible so the bank cannot write state-contingent deposit contracts that would allow the promised payment to depositors to fluctuate with the state.\(^{14}\) Alternatively, if the state were contractible, the bank could purchase insurance against poor borrower

\(^{11}\) In other words, the cash is held in such a form that it is not available to the bank when the bank liquidates—either because the borrower has transformed it (see Myers and Rajan (1998)) or because the borrower has stored it in a form only he can access.

\(^{12}\) It turns out that the role cash plays is identical to that played by a clause giving the borrower an inviolable claim to a fixed quantity of the assets on liquidation (see Diamond and Rajan (1999)).

\(^{13}\) This suggests a role for cash balances different from the traditional one. Instead of giving the bank greater comfort or collateral, fungible cash balances that can be drawn down at the discretion of the entrepreneur offer him a way to limit the bank’s power in a way that enhances overall efficiency.

\(^{14}\) While deposits cannot be contingent on the state, we do allow loans seized from the bank to be sold at a market price that is state contingent. More plausibly, the bank sells loans, and realizes cash to repay depositors. If loans are heterogeneous, and the bank can choose what to sell, it may be hard to infer from a few loan sales what the state is. But many loans are sold only if the bank is largely disintermediated. Therefore loan sale prices will reflect the state only if the bank is run. In general, therefore, loan sale prices cannot be used to make normal deposit payments contingent on the state, even if sale prices were verifiable.
repayment outcomes, rather than using capital as an indirect hedge against uncertainty.\footnote{15 We do not explicitly model the constraints that prevent contingent contracting. Previous work has motivated these limits in settings very similar to ours by private information (Townsend (1979), Diamond (1984)), unobservable renegotiation possibilities (Hart and Moore (1999)), coalition formation (Bond (1999)), or collateral constraints (Holmström and Tirole (1998), Krishnamurthy (1999)).} We intend in future work to examine the relative roles of capital and risk management (see Stulz (1996)) in settings where some limited contingent contracts would be feasible.

In both our model and the model in Diamond (1984), a borrower’s uncertain ability to repay leads useful commitment devices to be ex post costly for some realizations. In Diamond (1984), it is shown that deposit contracts should be contingent on observable aggregate shocks (or risk management contracts should be conditioned on these shocks), but uncertainty remains because idiosyncratic shocks cannot be written into contracts. If one believes that there are some easily contractible aggregate shocks, or easily diversifiable idiosyncratic ones, one should interpret the uncertainty in our model as conditional on the realization of these shocks. Diversification and risk management are substitutes for capital. Without a theory of the effects of bank capital, it has not been possible to analyze the trade-offs between these responses to uncertainty. We hope that our approach will provide a foundation for this analysis.

\section*{V. Conclusion}

We have presented a theory of bank capital in a model where the bank’s asset side and liability side are intimately tied together. We have identified at least three areas affected by bank capital: bank safety, the bank’s ability to refinance at low cost, and the bank’s ability to extract repayment from borrowers or its willingness to liquidate them. A large number of avenues for future research have only been sketched and deserve much more detailed exploration.

\section*{Appendix}

Proof of Lemma 2: When \(X^L_2 < \beta X^H_2\), it is easily checked that \(\tilde{D}^{Safe} > \tilde{D}^{Risky}\) iff

\[
q^H_2 X^H_2 < (1 - q^H_2) X^L_2.
\]

(A1)

When \(X^L_2 \geq \beta X^H_2\), \(\tilde{D}^{Safe} > \tilde{D}^{Risky}\) iff

\[
(1 - q^H_2) \left(1 - \frac{\beta}{2}\right) X^L_2 > q^H_2 \left(\frac{X^H_2 - X^L_2}{2}\right).
\]

(A2)
(1) If $q_2^H X_2^H < (1 - q_2^H) X_2^L$, then $\tilde{D}_{\text{Safe}}>\tilde{D}_{\text{Risky}}$. By inequality (A1), this is certainly true when $X_2^L < \beta X_2^H$. Now consider $X_2^L \geq \beta X_2^H$. We know

$$(1 - q_2^H)X_2^L > q_2^H X_2^H \Rightarrow (1 - q_2^H) \frac{(1 - \beta)}{2} X_2^L > q_2^H \frac{(1 - \beta)}{2} X_2^H.$$  

But $X_2^L \geq \beta X_2^H$. So

$$q_2^H \frac{(1 - \beta)}{2} X_2^H \geq q_2^H \left(\frac{X_2^H - X_2^L}{2}\right).$$

It follows that

$$(1 - q_2^H) \frac{(1 - \beta)}{2} X_2^L > q_2^H \left(\frac{X_2^H - X_2^L}{2}\right),$$

hence $\tilde{D}_{\text{Safe}}>\tilde{D}_{\text{Risky}}$.

(2) If $X_2^L \leq q_2^H X_2^H$, then $\tilde{D}_{\text{Risky}}>\tilde{D}_{\text{Safe}}$. By inspection, inequalities (A1) and (A2) are reversed when $X_2^L \leq q_2^H X_2^H$. Hence $\tilde{D}_{\text{Risky}}>\tilde{D}_{\text{Safe}}$.

(3) If $X_2^L > q_2^H X_2^H \geq (1 - q_2^H) X_2^L$, there is a $\beta^*$ such that $\tilde{D}_{\text{Safe}} > \tilde{D}_{\text{Risky}}$ iff $\beta < \beta^*$. The relative size of $\tilde{D}_{\text{Safe}}$ and $\tilde{D}_{\text{Risky}}$ is unaffected by $\beta$ when $X_2^L < \beta X_2^H$. When $X_2^L \geq \beta X_2^H$, by inspection, there is a $\beta'$ such that inequality (A2) holds for $\beta < \beta'$. Also, $X_2^L \geq \beta X_2^H \Rightarrow \beta \leq X_2^L/X_2^H$. Therefore, $\tilde{D}_{\text{Safe}} > \tilde{D}_{\text{Risky}}$ when $\beta < \min[\beta', (X_2^L/X_2^H)]$. Q.E.D.

**Proof of Proposition 1 (sketch):**

(1) If $d_1 > \text{Max}\{\tilde{D}_{\text{Safe}}, \tilde{D}_{\text{Risky}}, X_1^s\}$, pledgeable bank assets are less than the face value of deposits. Depositors know that not enough cash can be raised to pay off all the maturing deposits; therefore, they will run to seize loans (or force the bank to sell them to third parties for cash). After the run, the loans will be in the hands of depositors. The entrepreneur can make them a direct offer. Whether they accept or reject depends on how much they can get by rejecting. Because the banker has been disintermediated, he has valuable collection skills this period at date 1, but they will dissipate by date 2. So if the depositors negotiate with the banker to act as their liquidating agent at date 1, the banker will collect $X_1^s$ from the entrepreneur and pocket a fee of $[(1 - \beta)/2]X_1^s$, leaving $[(1 + \beta)/2]X_1^s$ for depositors. Alternatively, the depositors may prefer to wait until date 2 and exercise their expected liquidation threat of $b E[X_2|s_1]$. Thus depositors can expect to get $\text{Max}\{[(1 + \beta)/2]X_1^s, b E[X_2|s_1]\}$ by rejecting the offer. This is the offer the entrepreneur will make to them directly, and the banker will get cut out.

The entrepreneur has the ability to make this offer; even if he has no cash, he can liquidate himself and pay depositors $[(1 + \beta)/2]X_1^s$. 


and he can always promise $\beta E[\tilde{X}_2|s_1]$. Would he prefer to not liquidate himself and take his chance with the banker? The answer is no. To avoid liquidation, he would need to offer the bank cash and future promises totaling $X_{1t}^s$. But because the bank has no better ability to enforce future payments by the entrepreneur, when the entrepreneur can avoid liquidation by the banker, he also can promise depositors $[(1 + \beta)/2]X_{1t}^s$ without liquidating himself.

(2) \[ \text{Max}\{D_{\text{Safe}}, D_{\text{Risky}}\} \geq X_{1t}^s \text{ only if } E[\tilde{X}_2|s_1] > X_{1t}^s. \] If the banker rejects an offer from the entrepreneur, he will simply wait until date 2 to threaten liquidation and extract cash. Liquidation at date 1 is dominated. As a result, the entrepreneur will offer $P_1^t = 0$, $P_2^t = X_{2t}^s$ and the offer will be accepted. The banker will want to use a safe level of date 2 deposits as far as possible because this ensures he gets to extract $E[\tilde{X}_2|s]$ from the entrepreneur. Let $d_1^s$ be such that $\pi_{1t}^s + d_1^s = D_{\text{Safe}}$ (if there are many $d_1^s$ that satisfy this equality, let $d_1^s$ be the highest). If $D_{\text{Safe}} < D_{\text{Risky}}$, then for $d_1^s > d_1^s$, the banker will not be able to issue safe date 2 deposits and still meet the needs of investors at date 1 (because, by inspection, $\pi_{1t}^s + d_1$ strictly increases in $d_1$ after an initial region where it does not change with $d_1$). Once he has to issue risky date 2 deposits, funds raised at date 1 go up to meet investor claims, but funds extracted at date 2 from the entrepreneur go down from $E[\tilde{X}_2|s]$ to $D_{\text{Risky}}$. If $D_{\text{Safe}} \geq D_{\text{Risky}}$, there is no need to switch to issuing risky deposits.

(3a) If $D_{\text{Risky}} < D_{\text{Safe}} < X_{1t}^s$, then the banker will liquidate when he gets more from doing so than continuing, or when he simply cannot continue even though he wants to. For a borrower not to be able to stave off liquidation, either of the following conditions must be true:

\[ E(\tilde{X}_2|s_1) + C_{1t}^s < X_{1t}^s \quad (A3) \]

or

\[ D_{\text{Safe}} + C_{1t}^s < \pi_{1t}^s + d_1. \quad (A4) \]

The first condition is that the entrepreneur not have enough resources to bribe the banker. The second is that the banker cannot raise enough resources even with the entrepreneur’s full cooperation to be able to commit enough to investors without liquidating. It then follows that $C_{1t}^{LQ} = \text{Max}[X_{1t}^s - E(\tilde{X}_2|s_1), \pi_{1t}^s + d_1 - D_{\text{Safe}}]$ where it should be noted that $\pi_{1t}^s$ is also dependent on $d_1$.

(3b) We claim that $d^{**}$ solves $d_1^s + \pi_{1t}^s = X_{1t}^t - b_2$ where $b_2 = q_2^H[(1 - \beta)/2]X_{2t}^H$ is the maximum rent the bank can extract at date 2 when deposits are set at the safe level.\footnote{If multiple $d^{**}$ solve the equation $d_1^s + \pi_{1t}^s = X_{1t}^t - b_2$ (because the LHS is weakly increasing in $d_1$ initially), we pick the highest such value.} To see why this is true, note that the bank can
extract more than $X_1^s$ only when the entrepreneur, to placate outside investors, makes date 2 promises whose value to the bank exceeds their current pledgeable value. Were it not for this pressure, that is, if the entrepreneur only had to satisfy the bank, he would simply pay the bank a total of $X_1^s$ over the two periods and the bank would be satisfied. Therefore the question we have to ask is what is the minimum level of deposits at which the entrepreneur, in placating the outside investors, ends up overpaying. Consider an entrepreneur who has just enough cash, $C_1^s$, to enable the bank to meet the demands of outside investors through current cash and future promises. So $C_1^s = \pi_1^s + d_1 - D_{Safe}$. Such an entrepreneur has to make the maximum date 2 promise of $E[\bar{X}_2|s_1]$ to avoid liquidation at date 1, and hence will end up paying more than an entrepreneur with more current cash (of course, an entrepreneur with less cash will be liquidated half the time, and the entrepreneur will be liquidated half the time. We assume here that liquidation is very costly to the entrepreneur so he always makes an offer to drive the probability of liquidation to zero if he can.

It is easily shown that when $d_1 > d^{**}$, the amount the entrepreneur will overpay is $X_1^s$. Hence we obtain $d^{**}$ as the solution to $d_1 + \pi_1^s = X_1^s - b_2$.

For $d_1 > d^{**}$, the entrepreneur who will overpay the most is one who can just avoid liquidation. So $C_1^s(d_1) = \pi_1^s + d_1 - D_{Safe}$. Obviously, one with less cash will not be able to meet the needs of investors and will not overpay.\(^{17}\) As entrepreneurs have more cash, the date 2 promise necessary to escape liquidation falls, and the rent embedded in the date 2 promise, as well as overpayment, fall. When $C_1^s$ is high enough that the rent to the bank embedded in the date 2 promise falls below the date 1 rent the bank extracts, the entrepreneur will not overpay because he will simply offset one rent against the other. If $P_2^L$ is the promised date 2 payment that avoids liquidation, and assuming $\beta P_2^L > X_2^s$ (only the expression for the rent changes if this is not true, and that case is easily handled), we require

$$q_2^H \frac{(1 - \beta)}{2} P_1^2 = X_1^s - \pi_1^s - d_1 \quad \text{(A5)}$$

\(^{17}\) The banker need not always liquidate such entrepreneurs with probability 1. If the date 1 cash payment and the future promise they offer is sufficiently attractive, the bank may want to continue if it can. So it may enter the alternating offer game with capital. With probability 1/2, capital will get to make the offer and demand the bank liquidate. With probability 1/2, the bank will make the offer and offer capital its small outside option. Anticipating this, the entrepreneur will offer to pay the banker a little over $X_1$ in current and future promises, the offer will be accepted half the time, and the entrepreneur will be liquidated half the time. We assume here that liquidation is very costly to the entrepreneur so he always makes an offer to drive the probability of liquidation to zero if he can.
The right hand side of equation (A5) is the date 1 rent the bank expects and the left hand side is the date 2 rent. Now \( P_1^2 \) is the minimum date 2 promise that will enable the entrepreneur to commit enough to the bank’s investors to avoid liquidation, so that it solves

\[
q_2^H \frac{(1 + \beta)}{2} P_1^2 + (1 - q_2^H) X_2^L + C_1^{**} = \pi_1^{s_1} + d_1.
\]  

(A6)

Substituting for \( P_1^2 \) from equation (A6) in equation (A5), we get \( C_1^{**} \).

(3c) The first part is similar to (3b) and is omitted. Again, when the date 2 rent the entrepreneur has to give the bank exactly equals the date 1 rent the bank gets from liquidation, the entrepreneur does not pay more than \( X_1^{s_1} \). So \( C_1 = C_1^{**} \) calculated above.

(4) If \( D_{\text{Safe}} < D_{\text{Risky}} X_1^{s_1} \), the bank will threaten liquidation if \( E[\hat{X}_2|s_1] \leq X_1^{s_1} \). But the promises a cash-constrained entrepreneur will make do not embed a date 2 rent to the bank because \( D_{\text{Risky}} \) (which is the most valuable promise) does not contain a rent. So the banker will not be able to extract more than \( X_1^{s_1} \) regardless of \( d_1 \). When \( E[\hat{X}_2|s_1] > X_1^{s_1} \), a low level of \( d_1 \) such that \( d_1 + \pi_1^{s_1} \leq D_{\text{Safe}} \) will enable the bank to continue until date 2 regardless of the entrepreneur’s offer and extract \( E[\hat{X}_2|s_1] \). But once deposits exceed this level, the bank will have to liquidate at date 1 if it turns down the entrepreneur’s offer (because financing using the higher level of deposits inherent in \( D_{\text{Risky}} \) gives it no extra rent, and it is better off liquidating). As a result, entrepreneurs will offer the bank cash and date 2 promises that give the bank no rent, and the bank will extract no more than \( X_1^{s_1} \). Q.E.D.

REFERENCES


A Theory of Bank Capital


Kashyap, Anil, Raghuram Rajan, and Jeremy Stein, 1999, Banks as liquidity providers: An explanation for the co-existence of lending and deposit taking, Working paper, University of Chicago.

Krishnamurthy, Arvind, 1999, Collateral constraints and the credit channel, Working paper, Northwestern University.


Myers, Stewart C., and Nicholas S. Majluf, 1984, Corporate financing and investment decisions when firms have information that investors do not have, Journal of Financial Economics 13, 187–221.


