A Theory of Stock Exchange Competition and Innovation:
Will the Market Fix the Market?

[ONLINE APPENDICES]
Eric Budish, Robin S. Lee, and John J. Shim
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A Institutional Background

This appendix provides further details regarding Unlisted Trading Privileges (UTP) and Regulation National Market System (Reg NMS), the two key U.S. stock market regulations which guide our model and which were discussed more briefly in Section 2.

We note that while our discussion focuses on the United States, there are economically similar regulations for stock exchanges in Canada and somewhat similar regulations in Europe. Regulations for futures exchanges, on the other hand, are quite different from those for stock exchanges, both in the U.S. and abroad. In particular, there is no analogue of UTP in futures markets because each contract is proprietary to a particular exchange. Similarly, there are differences between the regulation of stock exchanges and the regulation of financial exchanges for other financial instruments like government bonds, corporate bonds, foreign currency, etc.; in particular, the information dissemination provisions of Reg NMS are often economically different in these asset classes.

A.1 Unlisted Trading Privileges (UTP)

Section 12(f) of the 1934 Exchange Act, passed by Congress, directed the Securities and Exchange Commission to “make a study of trading in unlisted securities upon exchanges and to report the results of its study and its recommendations to Congress.” Since that time, the right of one exchange to facilitate trading in securities that are listed on other exchanges has undergone several evolutions. In its current form, passed by Congress in the Unlisted Trading Privileges Act of 1994 (U.S. Congress, 1994) and clarified by the SEC in a Final Rule effective November 2000 (U.S. Securities and Exchange Commission, 2000), one exchange may extend unlisted trading privileges (UTP) to a security listed on another exchange immediately upon the security’s initial public offering on the listing exchange, without any formal application or approval process through the SEC. Prior to 1994, exchanges had to formally apply to the SEC for the right to extend UTP to a particular security; such

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1In Canada’s version of the Order Protection Rule (which goes by the same name), the key difference is that the rule applies to the full depth of the order book, not just the first level (Canadian Securities Administrators, 2009). In Europe, instead of the (prescriptive) Order Protection Rule there are (principles-based) best execution regulations (Petrella, 2010). Note however that principles-based best execution requirements leave some ambiguity with regard to whether market participants have to “pay attention” to quotes from small exchanges, which could affect innovation incentives; whereas under the Order Protection Rule there is no such ambiguity. This seems a good topic for future research.
approval was “virtually automatic” following a delay of about 30-45 days (Hasbrouck, Sofianos and Sosebee, 1993). Between the passage of the UTP Act of 1994 and the Final Rule in 2000, extension of UTP was automatic but only after an initially two-day, and then one-day, delay period after the security first began trading on its listing exchange. For further historical discussion of UTP, please see the background section of the 2000 Final Rule document, and also Amihud and Mendelson (1996).

A.2 Regulation National Market System (Reg NMS)

Regulation National Market System (“Reg NMS”), was passed in June 2005 and implemented beginning in October 2007 (U.S. Securities and Exchange Commission, 2005). It is a long and complex piece of regulation, with roots tracing to the Securities Exchange Act Amendments of 1975 and the SEC’s “Order Handling Rules” promulgated in 1996. For the purpose of the present paper, however, there are two core features to highlight.

The first is the Order Protection Rule, or Rule 611. The Order Protection Rule prohibits an exchange from executing a trade at a price that is inferior to that of a “protected quote” on another exchange. A quote on a particular exchange is “protected” if it is (i) at that exchange’s current best bid or offer; and (ii) “immediately and automatically accessible” by other exchanges. Reg NMS does not provide a precise definition of “immediately and automatically accessible,” but the phrase certainly included automated electronic continuous limit order book markets and certainly excluded the NYSE floor system with human brokers. A June 2016 rules clarification issued by the SEC indicated that exchanges can use market designs that impose delays on the processing of orders and still qualify as “immediate and automatic” so long as (i) the delay is of a de minimis level of less than 1 millisecond, and (ii) the purpose of the delay is consistent with the efficiency and fairness goals of the 1934 Exchange Act (U.S. Securities and Exchange Commission, 2016c). This rules clarification suggests that quotes on an exchange that adopts an asymmetric delay or frequent batch auctions would be protected under Rule 611 so long as the delay period or batch interval satisfies the de minimis delay standard. However, the SEC has not formally ruled on either proposal, nor has the SEC proactively clarified what market designs are and are not allowed within the de minimis rule Budish (2016c).

2The goal of the National Market System is described by the SEC as follows: “The NMS is premised on promoting fair competition among individual markets, while at the same time assuring that all of these markets are linked together, through facilities and rules, in a unified system that promotes interaction among the orders of buyers and sellers in a particular NMS stock. The NMS thereby incorporates two distinct types of competition—competition among individual markets and competition among individual orders—that together contribute to efficient markets.” See U.S. Securities and Exchange Commission (2005), pg 12.

3For an overview of Reg NMS, a good source is the introductory section of the SEC’s final ruling itself (U.S. Securities and Exchange Commission, 2005). For an overview of the National Market System prior to Reg NMS, good sources are O’Hara and Macey (1997) and the SEC’s “Market 2000” study (U.S. Securities and Exchange Commission, 1994).

4As noted in the main text, sophisticated market participants can take on responsibility for compliance with the Order Protection Rule themselves, absolving exchanges of the responsibility for checking quotes on other exchanges, by using an order type denoted intermarket sweep order (ISO). The relevant aspects of Reg NMS are Rule 600(b)(30) for the definition of ISOs, Rule 611(b)(5) for the exchange’s exemption from ensuring compliance with the Order Protection Rule for ISOs, and Rule 611(c) for this compliance obligation instead residing in the sender of the ISO.

5In 2019, EDGA proposed an asymmetric delay with a delay period of 4 milliseconds, and proposed for the exchange not to have protected quotes (i.e., for the quotes not to count as immediately and automatically accessible). The SEC rejected EDGA’s proposal in part citing that the delay of 4 milliseconds was not adequately justified under the fairness standards of the Exchange Act (“the Commission does not believe that the Exchange has supported its assertions and demonstrated that the LP2 delay mechanism is appropriately tailored to address latency arbitrage and not permit unfair discrimination”, U.S. Securities and Exchange Commission, 2020b, pg 24). In September 2016, three months after the June 2016 rules clarification, CHX proposed an asymmetric delay of 350 microseconds (0.350 milliseconds). However, the proposal was withdrawn after CHX was acquired by the NYSE Group. See the introduction and footnote 7 in the main text for more details.
The second key provision to highlight is the Access Rule, or Rule 610. Intuitively, in order to comply with the Order Protection Rule, exchanges and market participants must be able to efficiently obtain the necessary information about quotes on other exchanges and efficiently trade against them. As the SEC writes (pg. 26), “...protecting the best displayed prices against trade-throughs would be futile if broker-dealers and trading centers were unable to access those prices fairly and efficiently.”

The Access Rule has three sets of provisions that together are aimed at ensuring such efficient “search and access.” First, Rule 610(c) limits the trading fee that any exchange can charge to 0.3 pennies, which, importantly, is less than the minimum tick size of 1 penny. This ensures that if one exchange has a strictly better displayed price than another exchange, the price is economically better after accounting for fees. Second, Rule 610(d) has provisions that together ensure that prices across markets do not become “locked” or “crossed” — specifically, each exchange is required to monitor data from all other exchanges and to ensure that it does not display a quote that creates a market that is locked (i.e., bid on one exchange equal to ask on another exchange) or crossed (i.e., bid on one exchange strictly greater than an ask on another exchange). Together, then, rules 610(c) and 610(d) ensure that there is a well-defined “national best bid and offer” (NBBO) across all exchanges (at least ignoring the complexities that arise due to latency, see Section 4 of Budish, 2016). Third, Rule 610(a) prevents exchanges from charging discriminatory per-share trading fees based on whether the trader in question does or does not have a direct relationship with the exchange. In our model, the notion of a direct relationship with the exchange is captured by the decision of whether to buy exchange-specific speed technology, which represents exchange products like proprietary data feeds, co-location, and connectivity. What Rule 610(a) ensures is that market participants face the same trading fee schedule, whether or not they have such a direct relationship.
B Theory Appendix

B.1 Preliminaries

We first provide notation and definitions used in this Theory Appendix.

Trading Firms. In the model there are $N$ “fast” trading firms (TFs) that possess general-purpose speed technology, and a continuum of “slow” TFs that do not possess such technology. This technology affects the priority with which orders are processed by a continuous limit order book exchange. We index all TFs by $i$ or $k$. In this appendix, we will explicitly note whenever we are referring to a fast or a slow TF; any references to a particular TF $i$ or $k$ without noting its speed will allow for that TF to be either fast or slow.

Investor Strategies. In the equilibria that we construct, when there are multiple exchanges investors employ routing table strategies to break ties when they are indifferent over transacting on different exchanges. Routing table strategies are defined to be a vector of fixed weights $\gamma = (\gamma_1, \ldots, \gamma_M)$ such that, in the event that there are multiple exchanges contained in set $\mathcal{J}$ that offer depth at the same best price net of trading fees, an investor determines how much to purchase on each exchange using the following procedure: the investor “consumes” liquidity at rate $\gamma_j / \sum_{k \in \mathcal{J}} \gamma_k$ on each exchange $j \in \mathcal{J}$ until either (i) its demand is satisfied, or (ii) one or more exchanges in $\mathcal{J}$ no longer has any depth remaining at the best price, in which case the investor updates the set of exchanges that offer depth at the best price, updates its rate of consumption across exchanges, and continues consuming liquidity as before.

For example, consider three exchanges across which $(0.25, 1, 1)$ units of the security are available at the same best price net trading fees. An investor with unit demand and routing table strategies $(1/3, 1/3, 1/3)$ initially consumes liquidity at an equal rate from all three exchanges until the first exchange no longer has any available liquidity at the best price; this occurs after the investor has purchased 0.75 units in total. For the last 0.25 units that the investor demands, the investor consumes equally from the remaining two exchanges. To operationalize this strategy, the investor submits a limit order to purchase $(0.25, 0.375, 0.375)$ units from the three exchanges.\(^6\)

Orders and Liquidity Provision. We denote by $o_{ij} \in \mathcal{O}$ the order for TF $i$ submitted to exchange $j$, where $\mathcal{O}$ is the set of all potential combinations of messages. We allow for three types of messages that TFs can send to a particular exchange $j$: (i) standard limit orders, which take the form $(q_i, p_i)$ and indicate that the TF is willing to buy (if $q_i > 0$) or sell (if $q_i < 0$) up to $|q_i|$ units at price $p_i$; (ii) cancellations of existing limit orders in $\omega_j$, i.e., exchange $j$’s order book; and (iii) immediate-or-cancel orders (IOCs), which are standard limit orders that, if not fully executed in a given period, have any portion that is remaining cancelled by the exchange at the end of the period. An order submitted to a particular exchange may also contain no messages (i.e., $o_{ij} = \emptyset$); such an order maintains the TF’s existing limit orders on that exchange, if any exist. A TF can adjust an existing limit order (e.g., change the price) by cancelling the old limit order and submitting a new one. Denote by $\mathbf{o}_i \equiv \{o_{ij}\}_{j \in \mathcal{M}}$ the set of orders submitted by TF $i$ to all exchanges, where $\mathcal{M}$ represents the set of all exchanges.

\(^6\)Note that routing table strategies allow investors to (essentially) employ lexicographic preferences over exchanges when submitting orders: e.g., if there are three exchanges with depth available at the best price net trading fees, allowing $\gamma = (1 - \varepsilon - \varepsilon^2, \varepsilon, \varepsilon^2)$ for $\varepsilon > 0$ sufficiently small approximates an investor consuming depth from exchange 1 before moving to exchange 2, and then consuming depth from exchange 2 before moving to exchange 3. If $\gamma_j = 0$ for all $j \in \mathcal{J}$, we assume that an investor splits his demand uniformly among exchanges contained in $\mathcal{J}$. It is without loss to assume that routing table strategy weights sum to 1.
We say that a limit order provides liquidity if it offers to buy (or sell) some positive quantity at a price less than (or greater than) the current value of \( y \). Denote by \( \text{LO}_{ij}(o_{ij}; \omega_j) \) the set of TF \( i \)'s liquidity-providing limit orders on exchange \( j \), given the prior state of exchange \( j \)'s order book \( \omega_j \) and the processing of any messages contained in \( o_{ij} \). We say that an order \( o_{ij} \) provides liquidity on exchange \( j \) if \( \text{LO}_{ij}(o_{ij}; \omega_j) \) is non-empty; this can occur if either (i) \( o_{ij} \) contains a limit order that provides liquidity; or (ii) TF \( i \) has outstanding liquidity-providing limit orders on exchange \( j \) and \( o_{ij} \) does not cancel all of these limit orders. As noted in the main text, because we have assumed that investors are equally likely to arrive needing to buy or sell one unit of the security and the distribution of jumps in \( y \) is symmetric about zero, it is convenient to focus on the provision of liquidity via combinations of two limit orders: i.e., for a given quantity \( l \) and fundamental value \( y \), a limit order to buy the security at \( y - s/2 \), and a limit order to sell at \( y + s/2 \) for some (bid-ask) spread \( s \geq 0 \). We say \( o_{ij} \) provides \( l \) units of liquidity at spread \( s \) if \( \text{LO}_{ij}(o_{ij}; \omega_j) \) contains such a combination of limit orders.

There are two sets of relationships between orders that we refer to in this Appendix. We say that \( o'_i \) (weakly) withdraws liquidity relative to \( o_i \) if any limit order providing liquidity (at a given price and quantity on a particular exchange) contained in \( o'_i \) is also contained in \( o_i \), and any messages contained in \( o'_i \) but not in \( o_i \) are cancellations of existing limit orders. This implies that, for every exchange \( j \), any limit order providing liquidity contained in \( \text{LO}_{ij}(o'_i; \omega_j) \) is also contained in \( \text{LO}_{ij}(o_i; \omega_j) \). We say that \( o'_i \) is a (strict) price improvement over \( o_i \) if, for any \( q \in (0, 1] \) and any exchange \( j \in \mathcal{M} \), buying and selling \( q \) units on exchange \( j \) is weakly cheaper trading against limit orders in \( \text{LO}_{ij}(o'_i; \omega_j) \) than against limit orders in \( \text{LO}_{ij}(o_i; \omega_j) \), and there exists some quantity \( q \in (0, 1] \) and exchange \( j \) for which it is strictly cheaper to buy or sell \( q \) units trading against limit orders in \( \text{LO}_{ij}(o'_i; \omega_j) \) than against limit orders in \( \text{LO}_{ij}(o_i; \omega_j) \). (If there is no ability to buy (or sell) \( q \) units trading against limit orders in \( \text{LO}_{ij}(\cdot) \) on exchange \( j \), then the cost of buying (or selling) \( q \) units against limit orders in \( \text{LO}_{ij}(\cdot) \) is considered infinite.) Note that if \( o_i \) does not provide liquidity on any exchange, then any \( o'_i \) providing liquidity at any (finite) price on any exchange represents a price improvement over \( o_i \).

For thinking practically about the order book equilibrium concept, it is important to reiterate that our use of the language “submit orders” includes the possibility that an order contains no messages to one or more exchanges (i.e., \( o_{ij} = \emptyset \) for TF \( i \) and some exchanges \( j \)). Such “empty” orders simply maintain the TF’s outstanding limit orders on those exchanges, if it has any. As noted in the main text, trading games can be interpreted as lasting a sufficiently short amount of time (e.g., one millisecond or potentially even less) so that in most trading games, no exogenous Period-2 events occur (i.e., \( \lambda_{\text{invest}} + \lambda_{\text{private}} + \lambda_{\text{public}} \) is small). In the equilibria that we analyze, in this case that no exogenous Period-2 events occur, then in the next Period 1 all TFs send empty orders to all exchanges, hence simply maintaining their existing limit orders in \( \omega \). It is in this sense that these equilibria will capture the idea that each exchange’s limit order book settles into a rest point between Period-2 events, i.e., between arrivals of an investor, informed trader, or public information.

### B.2 Order Book Equilibrium (OBE)

Let \( E_{\pi_i}(o_i, o_{-i}) \) represent TF \( i \)'s expected profits from a trading game given the Period-1 orders \( o_i \) that it submits and the Period-1 orders submitted by all other trading firms, denoted \( o_{-i} = \{o_{kj}\}_{k \neq i, j \in \mathcal{M}} \), taking as given the state \((y, \omega)\) at the beginning of Period 1 of the given trading game, and assuming that all market participants employ their essentially unique optimal strategies in Period 2 of the trading game (or, in the event of public information when there is a Discrete exchange, play a Nash equilibrium). Expectations are taken

\(^7\)In all proofs and materials contained in this Appendix, any references to liquidity provision, unless explicitly noted, are with respect to liquidity provided at a spread within the support of \( J \) given the current state of \( y \).
over the potentially random sequence in which the orders \((\mathbf{o}_t, \mathbf{o}_{-t})\) are processed by exchanges in Period 1, the random action of nature in Period 2, and, in the event of a sniping race in Period 2 on one or more Continuous exchanges, the random sequence in which TFs’ orders are processed by exchanges.

**Definition B.1.** An order book equilibrium (abbreviated OBE) of our trading game is a set of orders \(\mathbf{o}^* \equiv \{\mathbf{o}^*_i\}\) submitted by all TFs in Period 1 given state \((y, \omega)\) that satisfies the following two conditions:

1. **No safe profitable price improvements.** No TF \(i\) has a strictly profitable price improvement that is safe, defined as remaining strictly profitable even if some other TF profitably withdraws liquidity in response to TF \(i\)’s deviation.

   Formally, for any TF \(i\), if \(\mathbf{o}'_i\) is a price improvement over \(\mathbf{o}^*_i\) and is strictly profitable meaning that \(E_{\pi}(\mathbf{o}'_i, \mathbf{o}^*_{-i}) > E_{\pi}(\mathbf{o}^*_i, \mathbf{o}^*_{-i})\), then there is some other TF \(k\) and profitable reaction \(\mathbf{o}'_k\) that withdraws liquidity relative to \(\mathbf{o}^*_k\) and renders TF \(i\)’s deviation no longer strictly profitable: i.e.,
   \[
   E_{\pi}(\mathbf{o}'_i, (\mathbf{o}'_k, \mathbf{o}^*_k)) \leq E_{\pi}(\mathbf{o}^*_i, (\mathbf{o}^*_k, \mathbf{o}^*_{-ik})) + E_{\pi}(\mathbf{o}^*_k, (\mathbf{o}'_i, \mathbf{o}^*_{-ik})) > E_{\pi}(\mathbf{o}^*_k, (\mathbf{o}'_i, \mathbf{o}^*_{-ik})),
   \]
   where \(\mathbf{o}^*_{-ik}\) denotes orders in \(\mathbf{o}^*\) for all TFs other than TF \(i\) and TF \(k\).

2. **No robust deviations.** No TF \(i\) has any other strictly profitable deviation (i.e., not a price improvement) that is robust, defined as remaining strictly profitable if, in response to TF \(i\)’s deviation, some other TF engages in a profitable reaction that is either: (a) a withdrawal of liquidity; or (b) a safe profitable price improvement (as defined in 1.).

   Formally, for any TF \(i\), if \(E_{\pi}(\mathbf{o}'_i, \mathbf{o}^*_{-i}) > E_{\pi}(\mathbf{o}^*_i, \mathbf{o}^*_{-i})\) for some deviation \(\mathbf{o}'_i\) that is not a price improvement over \(\mathbf{o}^*_i\), then there is some other TF \(k\) and profitable reaction \(\mathbf{o}'_k\) that renders TF \(i\)’s deviation no longer strictly profitable, and either: (a) \(\mathbf{o}'_k\) withdraws liquidity relative to \(\mathbf{o}^*_k\); or (b) \(\mathbf{o}'_k\) is a safe profitable price improvement, and hence is a profitable price improvement that remains strictly profitable for TF \(k\) even if any other TF, including TF \(i\), withdraws liquidity in response.

**Discussion.** As discussed in the main text, OBE strictly weakens Markov perfect equilibrium for our infinitely repeated trading game (and Nash equilibrium for a single play of our trading game) by relaxing the requirement that no strictly profitable unilateral deviations exist for any set of Period-1 orders. OBE instead only requires that there are no strictly profitable unilateral deviations that remain strictly profitable even if another TF were to profitably react in a particular way.

Central to the definition of OBE is the concept of a safe profitable price improvement, which is a strictly profitable price improvement that remains strictly profitable even if some other TF profitably withdraws liquidity in response. Allowing for withdrawals in response to price improvements protects TFs that are providing liquidity in equilibrium from being undercut by the “have your cake and eat it too” deviation discussed in Section 3.2.1, in which some other fast TF attempts to both earn the revenues from liquidity provision and the revenues from sniping the liquidity it just undercut. Hence, by imposing this “safe” requirement, OBE requires that for a strictly profitable price improvement to challenge equilibrium existence, it must remain strictly profitable due to the act of liquidity provision alone, and not from also continuing to snipe any liquidity that is no longer profitable to offer. We believe that the absence of safe profitable price improvements is a necessary condition for an exchange’s order book to be at a “rest point,” whereby no TF wishes to modify or adjust its outstanding
orders given the anticipation of likely reactions by rivals, and captures the spirit of competitive liquidity provision as discussed and assumed in Glosten and Milgrom (1985).

Condition 2 of OBE requires that there are no other strictly profitable deviations (i.e., not price improvements) that remain strictly profitable even if another TF profitably reacted with either (a) a withdrawal, or (b) a safe profitable price improvement. By allowing for safe profitable price improvements as reactions to other strictly profitable deviations (e.g., deviations that worsen liquidity), OBE requires that for such deviations to challenge equilibrium existence, they must not incentivize the provision of new liquidity at more competitive prices. We believe this, too, is a necessary condition for an exchange’s order book to be at a rest point, and is in the spirit of competitive liquidity provision as in Glosten and Milgrom (1985). Additionally, as in Condition 1, we allow for withdrawals as a reaction to other such deviations to protect TFs that are providing liquidity in equilibrium from the “have your cake and eat it too” deviation.

To provide intuition for how Conditions 1 and 2 help ensure both the existence of equilibria and the uniqueness of certain equilibrium features, consider the case analyzed in Section 3.2.1 where there is a single exchange that charges zero trading fees and all $N$ fast TFs have purchased ESST from this exchange. In this setting, consider a single play of the Stage 3 trading game that begins with an empty limit order book where $y = 10$ and $s^*_{\text{continuous}} = 2$. Assume that all market participants play their essentially unique optimal strategies (described in the main text) in Period 2 of the trading game.

First, note that there does not exist an OBE of this trading game where only one unit of liquidity is provided at a spread strictly greater than $s^*_{\text{continuous}}$ in Period 1. To see why, assume that in Period 1, some TF $i$ provides one unit of liquidity at spread $s = 4$ around $y$ (i.e., bid 8, ask 12), and no other liquidity is provided by any other TF. This violates Condition 1, as there exists a safe profitable price improvement for some fast TF $k \neq i$ to provide a unit of liquidity at spread $s' = 4 - 2\varepsilon$ around $y$ (i.e., bid $8 + \varepsilon$, ask $12 - \varepsilon$) for any $\varepsilon \in (0, 1]$; even if TF $i$ reacted to the deviation by withdrawing his liquidity, TF $k$ strictly prefers providing liquidity at spread $s'$ to sniping TF $i$’s liquidity at spread $s > s^*_{\text{continuous}}$.

Next, note that there does not exist an OBE where only one unit of liquidity is provided at a spread strictly narrower than $s^*_{\text{continuous}}$ in Period 1. To see why, assume that in Period 1, some TF $i$ provides one unit of liquidity at spread $s = 1$ around $y$ (i.e., bid 9.5, ask 10.5), and no other liquidity is provided by any other TF. This violates Condition 2, as there exists a robust deviation for TF $i$ to widen its spread to $s^*_{\text{continuous}}$ around $y$ (i.e., bid 9, ask 11) on its one unit of liquidity: there is no other liquidity that could be profitably withdrawn, and there is no safe profitable price improvement for any other TF (as fast TFs earn more from sniping liquidity at $s^*_{\text{continuous}}$ than providing liquidity at a strictly narrower spread, and slow TFs earn strictly negative profits providing liquidity at a strictly narrower spread).

Thus, Conditions 1 and 2 of OBE rule out equilibria in which one unit of liquidity is provided at any spread other than $s^*_{\text{continuous}}$: at strictly wider spreads, there are safe profitable price improvements to offer liquidity at more competitive prices; at strictly narrower spreads, there are robust deviations to increase prices on the offered liquidity.

Now consider a candidate equilibrium where TF $i$ provides one unit of liquidity at exactly $s^*_{\text{continuous}}$ around $y$ (i.e., bid 9, ask 11), and no other liquidity is provided by any other TF. As we will discuss, there exist three types of strictly profitable unilateral deviations, implying that this candidate equilibrium is not a Nash equilibrium. However, none of these deviations remain strictly profitable if other TFs are able to engage in the

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8A deviation could either weakly or strictly improve liquidity on some exchange and not count as a price improvement, e.g., by strictly worsening liquidity on some other exchange.
reactions discussed above, and hence these deviations do not prevent this candidate equilibrium from satisfying
the conditions for OBE (see Proposition 3.1).

First, there are strictly profitable price improvements that involve some fast TF $k \neq i$ adding a unit of
liquidity at bid $9 + \varepsilon$, ask $11 - \varepsilon$ for sufficiently small $\varepsilon > 0$: in doing so, TF $k$ attempts to “have his cake
and eat it too” by earning revenues from liquidity provision at a strictly narrower spread while also continuing
to snipe TF $i$’s orders that it just undercut. However, these deviations are not safe (Condition 1): TF $i$ can
profitably withdraw liquidity in response to TF $k$’s price improvement and render the deviation unprofitable,
as TF $k$ would prefer to snipe $i$’s liquidity at $s^*_{\text{continuous}}$ than provide liquidity at a narrower spread.

Second, consider the deviations by some fast TF $k \neq i$ to provide additional liquidity $l > 0$ at spread
$s^*_{\text{continuous}}$. Such deviations are strictly profitable for TF $k$ for $l \leq 1$ when TF $i$ is slow, in a variation of the
“have your cake and eat it too” deviation: when TF $i$ is slow, TF $k$’s liquidity is added to the order book prior
to any of TF $i$’s liquidity, and TF $k$ can earn both revenues from liquidity provision and revenues from sniping
TF $i$. However, these deviations, which are not price improvements, are not robust as there is a profitable
withdrawal for TF $i$ that renders them no longer strictly profitable (Condition 2a): following a withdrawal by
TF $i$, TF $k$ would earn the same amount from deviating as it would have from not deviating and sniping TF
$i$’s single unit of liquidity at $s^*_{\text{continuous}}$.

Last, there are strictly profitable deviations for TF $i$ to widen its spread — for example, to say $s' = 4$ (i.e.,
bid 8, ask 12) — on its one unit of liquidity. However, these deviations are not robust as there is a safe profitable
price improvement reaction by some fast TF $k \neq i$ that renders them unprofitable (Condition 2b): for example,
if TF $i$ deviates by widening its spread to $s' = 4$, then TF $k$ can profitably react by providing a unit of liquidity
at bid $8 + \varepsilon$, ask $12 - \varepsilon$ for sufficiently small $\varepsilon > 0$. This profitable price improvement remains strictly profitable
for $k$ even if TF $i$ were to withdraw any liquidity, and thus is safe. (This is the same reasoning used above to
show that a single unit of liquidity provided at any spread $s > s^*_{\text{continuous}}$ is not an equilibrium.)

Hence, the deviations that challenge the existence of a Nash equilibrium for a single play of our trading
game, or a Markov perfect equilibrium for our infinitely repeated trading game, no longer do so for an OBE.

B.3 Proofs for Section 3

B.3.1 Proof of Proposition 3.1 (Equilibrium in the BCS Environment)

We focus on the Stage 3 subgame with a single exchange ($M = 1$) charging zero trading fees ($f_1 = 0$), and with
all $N$ fast TFs having purchased exchange specific speed technology (ESST) from this exchange.

Existence. As discussed in the main text, Period 2 behavior for investors, informed traders, and trading
firms is governed by essentially unique optimal strategies and described in the statement of the proposition. We
now prove that there exists an OBE for Period 1 given state $(y, \omega)$ where a single unit of liquidity is provided
at spread $s^*_{\text{continuous}}$ around $y$.

Consider the following set of candidate equilibrium orders in Period 1. A single TF $i$ (fast or slow) submits
an order such that he provides exactly one unit of liquidity at spread $s^*_{\text{continuous}}$ around $y$; if he has liquidity
outstanding from the previous trading game in $\omega$, he maintains, adjusts or withdraws it as necessary so that
he provides exactly one unit at spread $s^*_{\text{continuous}}$ around $y$. All other TFs do not provide any liquidity (which
includes withdrawing any existing liquidity in $\omega$, if present).

To show that these orders comprise an OBE, first consider deviations by TF $i$. It is not profitable for TF
$i$ to adjust the quantity of liquidity that it provides: withdrawing any amount of liquidity offered at spread
\( s_{\text{continuous}}^{*} \) is not strictly profitable; and offering additional liquidity beyond the initial one unit is strictly unprofitable, as doing so only incurs additional adverse selection and sniping costs without any additional benefits. Reducing the spread on any amount of liquidity is also strictly unprofitable. Last, although there is a strictly profitable deviation by TF \( i \) to increase its spread to \( s' > s_{\text{continuous}}^{*} \) for \( l \leq 1 \) units of liquidity that it provides, such a deviation is not robust. To see why, consider as a reaction the profitable price improvement by some fast TF \( k \neq i \) to provide \( l \) units at spread \( s_{\text{continuous}}^{*} \), and an additional \( 1 - l \) units as a stub quote (i.e., liquidity provided at a spread outside the support of \( J \)). This reaction renders TF \( i \)'s deviation unprofitable; furthermore, the reaction is safe since \( k \) would prefer to offer such liquidity even if TF \( i \) were to withdraw any of its liquidity: providing \( l \) units of liquidity at \( s_{\text{continuous}}^{*} \) is strictly preferable to sniping the same amount of liquidity at \( s' > s_{\text{continuous}}^{*} \), and the stub quote ensures that \( k \) prefers to engage in its reaction even if TF \( i \) were to withdraw any of its liquidity. Hence, there are no robust deviations (or safe profitable price improvements) for TF \( i \).

Next, consider potential deviations for other TFs (who do not provide any liquidity given equilibrium strategies):

1. Consider the deviation by some TF \( k \neq i \) to provide \( l > 0 \) additional units of liquidity at some spread \( s' > s_{\text{continuous}}^{*} \). This is strictly unprofitable for both slow and fast TFs, as the additional liquidity incurs only adverse selection and sniping costs without any benefits of being traded against by an investor.

2. Consider the deviation by some TF \( k \neq i \) to provide \( l > 0 \) additional units of liquidity at spread \( s' = s_{\text{continuous}}^{*} - \varepsilon \) for \( \varepsilon > 0 \). If TF \( k \) is slow, this deviation is strictly unprofitable as slow TFs earn negative profits in expectation when offering any liquidity at a spread strictly less than \( s_{\text{continuous}}^{*} \). If TF \( k \) is fast, this “undercutting” of TF \( i \) is strictly profitable for sufficiently small \( \varepsilon > 0 \) as TF \( k \) earns revenues from both liquidity provision (earning priority over \( i \) at a cost of just \( \varepsilon \)) and from sniping TF \( i \)'s liquidity. But this deviation, which is a profitable price improvement for TF \( k \), does not remain strictly profitable (and hence is not safe) if TF \( i \) profitably withdraws \( l \) units of its own liquidity offered at spread \( s_{\text{continuous}}^{*} \) in response: by (3.1), liquidity provision and stale quote sniping are equally profitable at \( s_{\text{continuous}}^{*} \) for a fast TF, implying that TF \( k \) would have preferred to snipe at \( s_{\text{continuous}}^{*} \) than provide liquidity at a strictly narrower spread, \( s' < s_{\text{continuous}}^{*} \).

3. Consider the deviation by some TF \( k \neq i \) to provide \( l > 0 \) additional units of liquidity at \( s_{\text{continuous}}^{*} \). If TF \( k \) is slow, this deviation is not strictly profitable as slow TFs do not earn strictly positive expected profits when offering liquidity at spread \( s_{\text{continuous}}^{*} \). If TF \( k \) is fast, this deviation is only strictly profitable for some \( l > 0 \) if (i) strictly less than one unit of liquidity is resting in the order book from the previous trading game (at spread \( s_{\text{continuous}}^{*} \)), and (ii) TF \( i \) is slow. To see why (i) is required for the deviation to be profitable, note that if one unit (or more) of liquidity is resting in the order book from the previous trading game, then any additional liquidity provided by TF \( k \) in Period 1 would have worse queue priority than this unit, and thus would not be filled by an investor upon arrival in Period 2. TF \( k \)'s liquidity would then only bear sniping and adverse selection costs, and earn negative profits. To see why (ii) is required for the deviation to be profitable, note that if TF \( i \) is fast and less than one unit of liquidity is resting in the order book from the previous trading game, TF \( k \)'s liquidity for sufficiently small \( l > 0 \) will be added to the order book in Period 1 before any of TF \( i \)'s new liquidity with probability 1/2 (due to the random sequence in which orders are processed by the exchange among TFs with the same speed technology) and thus filled by an investor upon arrival in Period 2. TF \( k \)'s deviation would then earn in
expectation:

\[ l \times \left( \frac{1}{2} \cdot \lambda_{\text{invest}} \cdot \frac{s^*_\text{continuous}}{2} - \left( \frac{N - 1}{N} \lambda_{\text{public}} + \lambda_{\text{private}} \right) \cdot L(s^*_\text{continuous}) \right), \]

where the term \( \frac{1}{2} \cdot \lambda_{\text{invest}} \) reflects the probability that TF \( k \)'s liquidity is added before TF \( i \)'s new liquidity and is filled by an investor in Period 2. Substituting in \( \lambda_{\text{invest}} \cdot \frac{s^*_2}{2} = (\lambda_{\text{public}} + \lambda_{\text{private}}) \cdot L(s^*_\text{continuous}) \) from (3.1) into the previous expression and simplifying yields:

\[ l \times \left( 2 - \frac{N}{2N} \lambda_{\text{public}} - \frac{1}{2} \lambda_{\text{private}} \right) \cdot L(s^*_\text{continuous}), \]

which is strictly negative since \( N \geq 3 \). In the case that (i) and (ii) are both satisfied, then the deviation for sufficiently small \( l > 0 \) is strictly profitable for the same reason that the deviation discussed above involving liquidity provision at a strictly narrower spread \( s' = s^*_\text{continuous} - \varepsilon \) (for sufficiently small \( \varepsilon > 0 \)) is profitable: TF \( k \) earns revenues from both liquidity provision and from sniping slow TF \( i \)'s liquidity. However in this case, also as above, TF \( i \) has a profitable reaction to withdraw \( l \) units of its own liquidity, rendering the deviation by TF \( k \) not strictly profitable and hence not robust. Hence, there are no safe profitable price improvements or robust deviations for any TF \( k \neq i \). Thus, these orders comprise an OBE for Period 1 given state \((y, \omega)\).

In this OBE, each fast TF earns the same amount in expectation \((\frac{1}{N} \lambda_{\text{public}} \cdot L(s^*_\text{continuous}))\), whether it provides liquidity or snipes stale quotes, and slow TFs earn zero in expectation, whether they provide liquidity or do nothing, for each iteration of the trading game given the publicly observed state \((y, \omega)\). Hence, repeated play of this OBE comprises an equilibrium of the infinitely repeated Stage 3 trading game.

**Uniqueness.** As discussed in the main text, Period 2 behavior for investors, informed traders and trading firms is governed by essentially unique optimal strategies and described in the statement of the proposition. We now prove that in any OBE for Period 1 given state \((y, \omega)\), a single unit of liquidity is provided at spread \( s^*_\text{continuous} \) around \( y \).

First, we show that there cannot be an OBE with \( l > 1 \) units of liquidity offered at the end of Period 1. Assume by contradiction that such an OBE exists. Focus on liquidity offered at the worst price. If such liquidity would never be filled by an investor in Period 2—which can occur if there is at least one unit of liquidity offered at a strictly better price—then any TF offering such liquidity would have a robust deviation to withdraw this liquidity, as such liquidity only bears adverse selection and sniping costs without liquidity provision benefits; thus, this cannot be an OBE. Hence, if there are \( l > 1 \) units of liquidity offered, all liquidity offered at the worst price must in expectation filled by an investor in Period 2 with some probability that is strictly positive, but less than 1 (since \( l > 1 \)). However, in this case, any TF offering liquidity at the worst price has a profitable price improvement to reduce the spread on its liquidity by some small amount \( \varepsilon > 0 \), thereby ensuring that its liquidity would be filled by an investor with certainty in Period 2; furthermore, this deviation remains profitable even if other TFs withdrew liquidity, and hence is safe. Contradiction.

Next, we show that there cannot be an OBE with \( l < 1 \) units of liquidity offered at the end of Period 1. Assume by contradiction that such an OBE exists. Consider the strictly profitable unilateral deviation by any fast TF to offer \( 1 - l \) additional units of liquidity at spread \( s^*_\text{continuous} \). This is a safe profitable price improvement, as reactions that withdraw offered liquidity do not render this deviation weakly unprofitable. This cannot be an OBE; contradiction. Hence, exactly a single unit of liquidity must be offered at the end of
Period 1 in any OBE.

Last, we show that in any OBE, the single unit of liquidity that is offered must be at bid-ask spread $s^*_\text{continuous}$ around $y$. Assume by contradiction that there exists an OBE where exactly one unit of liquidity is offered at the end of Period 1, but not all of it is offered at spread $s^*_\text{continuous}$. Assume first that in such an equilibrium, $l \leq 1$ units are offered at a spread $s < s^*_\text{continuous}$ by some TF $i$. Consider the strictly profitable unilateral deviation by TF $i$ to increase its spread to $s^*_\text{continuous}$ on its offered liquidity. This deviation is robust: there is no withdrawal or safe profitable price improvement that renders the deviation weakly unprofitable, as any fast TF considering a price improvement that undercuts TF $i$'s liquidity at $s^*_\text{continuous}$, as opposed to providing liquidity at a narrower spread, and any slow TF cannot profitably provide liquidity at any spread $s' \leq s^*_\text{continuous}$. This cannot be an OBE; contradiction. Assume next that in such an equilibrium, $l \leq 1$ units are offered at a spread $s > s^*_\text{continuous}$ by some TF $i$. There is a safe profitable price improvement by some fast TF $k \neq i$ to undercut and provide $l$ units at spread $s^*_\text{continuous}$, as there are no withdrawals of liquidity that render the deviation weakly unprofitable (since TF $k$ prefers to provide liquidity at $s^*_\text{continuous}$ to sniping liquidity provided at $s > s^*_\text{continuous}$). This cannot be an OBE; contradiction.

### B.3.2 Supporting Lemmas for Proposition 3.2

The proof of Proposition 3.2 relies on the following supporting lemmas.

**Lemma B.1.** Consider the Stage 3 trading game in any subgame where: (i) all $N$ fast TFs have purchased ESST from the same set of exchanges; and (ii) trading fees are zero for all exchange contained in the non-empty set $J \subseteq M$ and strictly positive for all exchanges $m \notin J$. Then:

1. Existence: for any vector of market shares $\sigma^* = (\sigma^*_1, \ldots, \sigma^*_M)$ such that $\sum_{j \in J} \sigma^*_j = 1$ and $\sigma^*_m = 0$ if $m \notin J$, there exists an OBE in which TFs in aggregate provide $\sigma^*_j$ units of liquidity on each exchange $j$ at spread $s^*_\text{continuous}$ in Period 1.

2. Uniqueness: any OBE has exactly one unit of liquidity provided in aggregate at spread $s^*_\text{continuous}$ in Period 1, where liquidity is provided across exchanges according to some vector of market shares $\sigma^* = (\sigma^*_1, \ldots, \sigma^*_M)$ such that $\sum_{j \in J} \sigma^*_j = 1$ and $\sigma^*_m = 0$ if $m \notin J$.

(We do not require the uniqueness portion of Lemma B.1 for our main results, but state and prove it here for completeness.)

**Proof.** Condition on state $(y, \omega)$ at the beginning of this trading game.

**Existence.** Consider any vector of exchange market shares $\sigma^* = (\sigma^*_1, \ldots, \sigma^*_M)$ such that $\sum_{j \in J} \sigma^*_j = 1$ and $\sigma^*_m = 0$ if $m \notin J$. Consider the following candidate equilibrium strategies. In Period 1, a single TF $i$ (either fast or slow) submits an order to each exchange $j \in J$ to provide exactly $\sigma^*_j$ units of liquidity at spread $s^*_\text{continuous}$ around $y$ (maintaining, adjusting or withdrawing any outstanding liquidity from the previous trading game as necessary). All other TFs do not provide any liquidity (which includes withdrawing any existing liquidity in $\omega$, if present). In Period 2, an investor sends IOCs to all exchanges that, in aggregate, trade up to one unit in their desired direction, prioritizing their demand across exchanges based on the lowest value of $s_j / 2 + f_j$ (where for each exchange $j$, $s_j$ is the lowest spread at which liquidity is offered and $f_j$ is the trading fee), and breaking ties according to routing table strategies given by $\gamma^* = \sigma^*$; additionally, if there are any remaining orders that are profitable to trade against based on the publicly observed state $y$, the investor trades against those as well. An
informed trader sends IOCs to trade against any orders on any exchange that are profitable to trade against based on their privately observed $y$. If there is a publicly observable jump in $y$, TF $i$ sends messages to cancel all liquidity providing orders; and all fast TFs not providing liquidity send IOCs to each exchange $j$ to try to trade against (snipe) any orders that are profitable to trade against based on the new value of $y$.

We now show that these strategies comprise an equilibrium. As discussed in the main text, Period-2 strategies are essentially unique, and there are no strictly profitable Period-2 deviations. Arguments analogous to those used above in the proof of Proposition 3.1 can then be used to show that any strictly profitable Period-1 deviations are either not safe (if they are price improvements) or robust. First, if TF $i$ engages in a strictly profitable deviation to increase its spread on any exchange, there is a safe profitable price improvement by some fast TF $k \neq i$ to provide liquidity at spread $s^*_{\text{continuous}}$ on any exchange, rendering the deviation not strictly profitable and hence not robust. Second, if some fast TF $k \neq i$ adds additional liquidity on any exchange at any spread weakly less than $s^*_{\text{continuous}}$ and finds it profitable to do so, TF $i$ can profitably withdraw any amount of liquidity on any exchange and render TF $k$’s deviation not strictly profitable. Thus, these strategies comprise an OBE. Note that in this equilibrium, in each trading game each fast TF earns (gross ESST fees) expected profits of $\sigma^*_{j} \times \frac{N^*_{\text{continuous}}}{N}$ on exchange $j$ from either liquidity provision or sniping activity; this implies that each fast TF earns in aggregate $\frac{N^*_{\text{continuous}}}{N}$ per-trading game across all exchanges—the same amount that each fast TF would earn in equilibrium if there was only a single exchange.

**Uniqueness.** Consider any equilibrium where $l = (l_1^*, \ldots, l_M^*)$ units of liquidity are provided across exchanges at the end of Period 1. In any OBE, we now prove that exactly a single unit of liquidity is provided in aggregate among all exchanges with zero trading fees (i.e., $\sum_{j \in J} l_j^* = 1$ and $l_m^* = 0$ if $m \notin J$) at spread $s^*_{\text{continuous}}$ around $y$ following Period 1. This follows from establishing the following three results.

First, in Period 1, exactly one unit of liquidity must be provided in aggregate across all exchanges. Using the same arguments as in the proof of Proposition 3.1, if less than one unit is provided, then any fast TF would have a safe profitable price improvement to add some small amount of liquidity at spread $s^*_{\text{continuous}}$ to some exchange $j \in J$; and if more than one unit is provided, then some TF offering liquidity at the worst price has a robust deviation to either withdraw such liquidity, or reduce the spread by some positive amount $\varepsilon > 0$ to guarantee that it would be transacted against by an investor in Period 2.

Second, all liquidity that is provided on any exchange $j \in J$ must be provided at spread $s^*_{\text{continuous}}$. The same arguments used in the proof of Proposition 3.1 establish that if instead some quantity of liquidity were provided on any exchange $j \in J$ at spread $s' \neq s^*_{\text{continuous}}$ by some TF $i$, then there would then exist either a robust deviation for TF $i$ to increase its spread to $s^*_{\text{continuous}}$ if $s' < s^*_{\text{continuous}}$, or a safe profitable price improvement for some fast TF $k \neq i$ to undercut TF $i$ if $s' > s^*_{\text{continuous}}$.

Third, any positive quantity of liquidity cannot be provided on any exchange $m$ where $f_m > 0$. Assume not, and there exists an equilibrium in which some quantity of liquidity $l > 0$ is provided on exchange $m$ at some spread $s'$ by some (fast or slow) TF $i$. Consider first the case where $\frac{s'}{2} + f_m > s^*_{\text{continuous}}$. In this case, there is a safe profitable price improvement for some fast TF $k \neq i$: TF $k$ can profitably provide the same amount of liquidity $l$ on any exchange $j \in J$ at spread $s^*_{\text{continuous}}$ (since an investor would strictly prefer to transact on exchange $j$ at spread $s^*_{\text{continuous}}$ than on exchange $m$ at $s'$), and this would remain strictly profitable for TF $k$ even if TF $i$ were to withdraw its liquidity in response (as TF $k$ would strictly prefer to provide liquidity on exchange $j$ at $s^*_{\text{continuous}}$ than snipe liquidity on exchange $m$ at $s'$). Contradiction. Consider next the case where $\frac{s'}{2} + f_m \leq s^*_{\text{continuous}}$. In this case there is a strictly profitable deviation for TF $i$, that is also robust, to withdraw all liquidity on $m$ and offer the same amount of liquidity on any exchange $j \in J$ at spread $s^*_{\text{continuous}}$. Note that in this equilibrium, in each trading game each fast TF earns (gross ESST fees) expected profits of $\sigma^*_{j} \times \frac{N^*_{\text{continuous}}}{N}$ on exchange $j$ from either liquidity provision or sniping activity; this implies that each fast TF earns in aggregate $\frac{N^*_{\text{continuous}}}{N}$ per-trading game across all exchanges—the same amount that each fast TF would earn in equilibrium if there was only a single exchange.
s^\ast_{\text{continuous}} (and avoid trading fees). The reason this deviation is robust is that there are no withdrawals (since only i provides liquidity), and there are no safe profitable price improvements by other TFs that render the deviation not strictly profitable: any fast TF \( k \neq i \) prefers sniping liquidity on \( j \) at \( s^\ast_{\text{continuous}} \) than offering it on any exchange at a lower spread, and any slow TF cannot profitably provide liquidity at a lower spread. Contradiction.

Thus, any OBE involves exactly a single unit of liquidity provided in aggregate at spread \( s^\ast_{\text{continuous}} \) around \( y \) following Period 1, and liquidity is only provided on exchanges with zero trading fees. Given essentially unique optimal Period-2 strategies, it then follows that transaction volume upon the arrival of an investor in Period 2 coincides with liquidity provision for all exchanges (i.e., \( \sigma_j^2 = l_j^2 \) for all \( j \in \mathcal{M} \)).

Lemma B.2. (“Lone-Wolf Lemma”) Consider the Stage 3 trading game in any subgame where: (i) trading fees on all exchanges are zero; (ii) fast TF \( i \), referred to as the “lone-wolf,” has purchased exchange-specific speed technology (ESST) only on exchanges contained in the set \( \mathcal{J} \subseteq \mathcal{M} \); and (iii) all other fast TFs have purchased ESST on the same set of exchanges \( \mathcal{J} \), where \( \mathcal{J} \subseteq \mathcal{J'} \subseteq \mathcal{M} \). There exists an OBE for Period 1 of this trading game where exactly one unit of liquidity is provided only on exchanges contained in \( \mathcal{J} \) by TF \( i \) at spread \( \bar{s}_N \) in Period 1, where \( \bar{s}_N \) solves:

\[
\lambda_{\text{invent}} \frac{\bar{s}_N}{2} - \left( \frac{N-2}{N-1} \lambda_{\text{public}} + \lambda_{\text{private}} \right) L(\bar{s}_N) = \frac{\lambda_{\text{public}} L(\bar{s}_N)}{N}, \tag{B.1}
\]

and TF \( i \) earns in expectation at least \( \pi^\ast_{\text{lone-wolf}} = N^{-1} \lambda_{\text{public}} L(\bar{s}_N) \) per-trading game gross of ESST fees, where \( \pi^\ast_{\text{lone-wolf}} \in (N^{-2} \times \Pi^\ast_{\text{continuous}}, \Pi^\ast_{\text{continuous}}) \). Furthermore, among OBE in which TF \( i \) is the sole liquidity provider in Period 1 of each trading game (i.e., all other TFs play strategies in which they provide no liquidity), it is unique that TF \( i \) provides exactly one unit of liquidity at spread \( \bar{s}_N \) solely on exchanges contained in \( \mathcal{J} \).

Proof. In this proof, all references to TF profits are in expectation for each trading game, gross ESST fees.

Preliminaries. Define the spread \( \bar{s}_N \) to be the minimum spread the lone-wolf TF \( i \) must charge on exchange \( j \) for one unit of liquidity so that \( i \) breaks even in expectation when the \( N-1 \) other fast TFs have also purchased ESST from \( j \) and no liquidity is provided on any other exchange; i.e., \( \bar{s}_N \) is the solution to:

\[
\lambda_{\text{invent}} \frac{\bar{s}_N}{2} - \left( \frac{N-1}{N-1} \lambda_{\text{public}} + \lambda_{\text{private}} \right) L(\bar{s}_N) = 0. \tag{B.2}
\]

We refer to \( \bar{s}_N \) as the zero-variable profit spread. The difference between the definition of \( \bar{s}_N \) and the definition of \( s^\ast_{\text{continuous}} \) in (3.1) is that \( \bar{s}_N \) does not incorporate the opportunity cost of sniping, worth \( \frac{1}{N} \lambda_{\text{public}} L(\cdot) \).

Next, we provide intuition for the spread \( \bar{s}_N \) defined in (B.1). Assume that conditions (i)-(iii) in the statement of the Lemma hold, and the lone-wolf TF \( i \) provides one unit of liquidity at spread \( \bar{s}_N \) on some exchange \( j \in \mathcal{J} \). Then any other fast TF \( k \neq i \) would be indifferent between (i) sniping TF \( i \) on exchange \( j \) (earning the right-hand side of (B.1)), and (ii) TF \( i \) not providing any liquidity, and TF \( k \) instead providing one unit of liquidity at \( \bar{s}_N \) on some exchange \( j' \notin \mathcal{J} \) (earning the left-hand side of (B.1), where TF \( k \) only risks being sniped by \( N-2 \) other TFs who have ESST on exchange \( j' \)).

We now prove that \( \bar{s}_N < \bar{s}_N < s^\ast_{\text{continuous}} \). The first inequality, \( \bar{s}_N < \bar{s}_N \), follows from comparing (B.2) to (B.1), which can be re-written as \( \lambda_{\text{invent}} \frac{\bar{s}_N}{2} - ((\frac{1}{N} + \frac{N-2}{N-1}) \lambda_{\text{public}} + \lambda_{\text{private}}) L(\bar{s}_N) = 0 \). It is straightforward to show that the coefficient on \( \lambda_{\text{public}} \) is greater in (B.1) than in (B.2): \( \frac{1}{N} + \frac{N-2}{N-1} = 1 - \frac{1}{N(N-1)} > 1 - \frac{1}{N} = \frac{N-1}{N} \). Hence, it follows that \( \bar{s}_N > \bar{s}_N \). The second inequality, \( \bar{s}_N < s^\ast_{\text{continuous}} \), follows using similar logic: in (3.1),

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which defines \( s^*_{\text{continuous}} \); \( \lambda_{\text{public}} \) enters the equation with a coefficient of 1; however, in (B.1), which defines \( \tilde{s}_N \), \( \lambda_{\text{public}} \) enters with a coefficient strictly less than 1.

The rest of the proof proceeds in three parts. First, we establish that an OBE with the properties outlined in the statement of the Lemma exists (Existence). Second, we establish that any OBE in which TF \( i \) is the sole liquidity provider in Period 1 of each trading game must have these properties (Uniqueness). Last, we prove that \( s^*_{\text{lone-wolf}} \in \left(\frac{(N-2)}{\sum_{N-1}^{N}} \times \Pi^*_{\text{continuous}} \Pi^*_{\text{continuous}}\right) \) (Profit Bound).

**Existence.** We now prove that there is an OBE for Period 1 of each trading game in which the lone-wolf TF \( i \) provides one unit of liquidity at spread \( \tilde{s}_N \) across exchanges according to any arbitrary vector of shares \( \sigma^* = (\sigma^*_1, \ldots, \sigma^*_M) \) s.t. \( \sum_{j \in J} \sigma^*_j = 1 \) and \( \sigma^*_j = 0 \) if \( j \notin J \), and no additional liquidity is provided by any other TF. Consider equilibrium strategies where in Period 1, TF \( i \) submits orders to provide one unit of liquidity at spread \( \tilde{s}_N \) across exchanges in \( J \) according to \( \sigma^* \) (maintaining, adjusting or withdrawing any outstanding liquidity from the previous trading game as necessary), and other TFs do not provide any liquidity (which includes withdrawing any existing liquidity in \( \omega \), if present); and in Period 2, strategies follow those described in the proof of Lemma B.1 (where investors break ties across exchanges using routing table strategies \( \gamma^* = \sigma^* \)). The right-hand-side of (B.1) represents the gross expected payoffs that any fast TF \( k \neq i \) expects to obtain by sniping TF \( i \) across all exchanges; the left-hand-side represents the gross expected payoffs that any fast TF \( k \) would anticipate if TF \( k \) were instead the sole liquidity provider on some other exchange \( m \notin J \) at spread \( \tilde{s}_N \). Hence, no fast TF \( k \neq i \) has a strictly profitable deviation—for example, by undercutting \( i \) or providing additional liquidity at a spread weakly smaller than \( \tilde{s}_N \) on any exchange—that remains profitable if TF \( i \) reacts by withdrawing any liquidity that is no longer profitable to offer. Since \( \tilde{s}_N < s^*_{\text{continuous}} \), any slow TF also has no robust deviations or safe profitable price improvements. Last, by similar arguments used in the proof of Proposition 3.1, there are no robust deviations for TF \( i \): if TF \( i \) widened its spread on any amount of liquidity, the deviation would be rendered unprofitable by another fast TF \( k \)'s safe profitable price improvement to provide that amount of liquidity on some exchange \( m \notin J \) at spread \( \tilde{s}_N \) (which, by (B.1), is more profitable for TF \( k \) than sniping TF \( i \) at any spread strictly greater than \( \tilde{s}_N \)); and TF \( i \) reducing its spread or adjusting the amount of liquidity that it provides would strictly reduce profits. Thus, these strategies comprise an OBE.

**Uniqueness.** We now prove that among OBE in which TF \( i \) is the sole liquidity provider in Period 1 of each trading game, it is unique that TF \( i \) provides exactly one unit of liquidity at spread \( \tilde{s}_N \) solely on exchanges contained in \( J \).

First, note that TF \( i \) must offer exactly one unit of liquidity in aggregate: otherwise, TF \( i \) would find it profitable to withdraw liquidity (if it offered strictly greater than one unit of liquidity) or have a safe profitable price improvement to add liquidity at spread \( \tilde{s}_N \) on some exchange in \( J \) (if it offered strictly less than one unit of liquidity).

Second, note that such liquidity must be offered only on exchanges in \( J \). Assume not, and some positive amount of liquidity \( l_m > 0 \) is offered by TF \( i \) on some exchange \( m \notin J \). Consider first the case where such liquidity is offered by TF \( i \) at spread \( s' > s^*_{\text{continuous}} \). In this case, there would be a safe profitable price improvement by some other fast TF \( k \neq i \) to undercut TF \( i \) and offer this amount of liquidity on exchange \( m \) at spread \( s^*_{\text{continuous}} \); contradiction. Consider next the case where such liquidity is offered by TF \( i \) at spread \( s' \leq s^*_{\text{continuous}} \). In this case, TF \( i \) would then have a robust deviation to withdraw that liquidity from \( m \) and offer instead the same amount of liquidity on some exchange in \( J \) at spread \( \tilde{s}_N \) (as discussed above when establishing Existence, no fast TF \( k \neq i \) would find offering liquidity at any spread less than \( \tilde{s}_N \) on any exchange strictly preferable to sniping TF \( i \)'s liquidity on an exchange in \( J \) at spread \( \tilde{s}_N \)); contradiction.
Last, TF $i$ must offer this single unit of liquidity at spread $\tilde{s}_N$. If any amount of liquidity were offered at a lower spread, TF $i$ would have a robust deviation to increase its spread to $\tilde{s}_N$; and if any amount of liquidity were offered at a strictly greater spread, there would be a safe profitable price improvement by some fast TF $k \neq i$ to provide the same amount of liquidity on some exchange $m \notin \mathcal{J}$ at spread $\tilde{s}_N$.

Profit Bound. Define

$$\pi_N^{\text{lone-wolf}} \equiv \lambda_{\text{invest}} \frac{\tilde{s}_N}{2} - \left( \frac{N-1}{N} \lambda_{\text{public}} + \lambda_{\text{private}} \right) L(\tilde{s}_N)$$

(B.3)

to be the expected profits per trading game (gross ESST fees) that the lone-wolf TF $i$ makes providing a single unit of liquidity at spread $\tilde{s}_N$ across exchanges contained in $\mathcal{J}$ when there are $N$ total fast TFs (including him) that also have purchased ESST on exchanges contained in $\mathcal{J}$ and no other TF provides liquidity.

We now prove the stated bounds on $\pi_N^{\text{lone-wolf}}$. First, the upper bound,

$$\pi_N^{\text{lone-wolf}} < \Pi^{\text{continuous}}_{\text{continuous}} = \lambda_{\text{invest}} \frac{s^*_\text{continuous}}{2} - \left( \frac{N-1}{N} \lambda_{\text{public}} + \lambda_{\text{private}} \right) L(s^*_\text{continuous})$$

follows since, comparing the right-hand side of (B.3) to the right-hand side of the above expression, $\tilde{s}_N < s^*_\text{continuous}$ and $L(\tilde{s}_N) > L(s^*_\text{continuous})$. To obtain the lower bound, first solve for $\lambda_{\text{invest}} \frac{\tilde{s}_N}{2}$ in (B.1) and substitute this expression into the right-hand side of (B.3) to obtain:

$$\pi_N^{\text{lone-wolf}} = \left( \frac{1}{N} + \frac{N-2}{N} - \frac{N-1}{N} \right) \frac{\Pi^{\text{continuous}}_{\text{continuous}}} {N} \lambda_{\text{public}} L(\tilde{s}_N)$$

$$= \frac{N-2}{(N-1)N} \lambda_{\text{public}} L(\tilde{s}_N)$$

$$> \frac{N-2}{(N-1)N} \lambda_{\text{public}} L(s^*_\text{continuous}) = \frac{N-2}{(N-1)N} \Pi^{\text{continuous}}_{\text{continuous}}$$

where the inequality on the last line follows from $L(\tilde{s}_N) > L(s^*_\text{continuous})$.

B.3.3 Money-Pump with Negative Trading Fees

Before proceeding with our results for the full exchange competition game, we make the following remark regarding negative trading fees.

Remark B.1. Suppose some exchange $m \in \mathcal{M}$ sets a trading fee $f_m < 0$. For any arbitrarily large dollar amount $P > 0$, there exist TF strategies in Period 1 of any trading game in Stage 3 where TFs earn profits weakly greater than $P$, at the expense of exchange $m$, without engaging in self-dealing.

To see this, consider the following Period 1 strategies for any two distinct TFs $i$ and $k$: TF $i$ submits an order to exchange $m$ to buy $Q \equiv P/(2 \times |f_m|)$ units at price $y$, and TF $k$ submits an order to exchange $m$ to sell $Q$ units at price $y$. There are two cases to examine. First, if there is no other liquidity provided on exchange $m$ buying (or selling) at prices weakly greater than (or less than) $y$ in Period 1, then TF $i$ and $k$ transact $Q$ units with one another in Period 1, and TF $i$ and $k$ together earn $2 \times (Q \times |f_m|) = P$ in profits at the expense of exchange $m$. Second, if (for some reason) there is other liquidity provided on exchange $m$ buying (or selling) at prices weakly greater than (or less than) $y$ in Period 1, then it still must be the case that either TF $i$ or TF $k$ transacts at least $Q$ units at a price no worse than $y$ in that Period, implying that TFs trading these $Q$ units, in aggregate, earn $P$ in profits at the expense of exchange $m$. 

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Hence, if an exchange charges negative trading fees, it creates a “money-pump” that TFs, even without self-dealing, can take advantage of, and exposes the exchange to arbitrarily large losses. For this reason, we restrict exchanges to charge non-negative trading fees when analyzing our full exchange competition game.

B.3.4 Proof of Proposition 3.2 (Equilibrium of the Full Exchange Competition Game)

For any vector of ESST fees $F^*$ that satisfies (3.2) and market shares $\sigma^*$ such that $\sum_{j \in M} \sigma^*_j = 1$, consider the following candidate equilibrium strategies:

- In Stage 1, each exchange $j$ charges $F_j^*$ for ESST and sets trading fees $f_j = 0$;
- In Stage 2, all $N$ fast TFs buy ESST from exchange $j$ only if (i) its ESST fee $F_j \leq F_j^*$, (ii) $f_j = \min_{k \in M} f_k$, and (iii) $\sigma^*_j > 0$;
- In Stage 3, in Period 1 of each trading game:
  1. On the candidate equilibrium path: If all fast TFs purchase ESST from the same set of exchanges $J \subseteq M$ where $f_j = 0$ for all $j \in J$, then in Period 1 of each trading game, some (fast or slow) TF $i$ submits orders to provide $\sigma^*_j / (\sum_{k \in J} \sigma^*_k)$ amount of liquidity on each exchange $j \in J$ at spread $s^*_{\text{continuous}}$ around $y$ (maintaining, adjusting or withdrawing any outstanding liquidity from the previous trading game as necessary), all other TFs submit orders such that they provide no liquidity on any exchange, and no liquidity is provided elsewhere.
  2. If one fast TF $i$ purchases ESST from a non-empty strict subset of exchanges $J' \subset M$, and all other fast TFs $k \neq i$ purchase ESST from a strictly greater set of exchanges $J$ (so that $J' \subset J \subseteq M$) where $f_j = 0$ for all $j \in J$, then in Period 1 of each trading game, TF $i$ is the “lone-wolf” liquidity provider and submits orders to provide one unit of liquidity on some exchange $j \in J'$ at spread $\tilde{s}_N$ (defined in (B.1)) around $y$ (maintaining, adjusting or withdrawing any outstanding liquidity from the previous trading game as necessary), all other TFs submit orders such that they provide no liquidity on any exchange, and no liquidity is provided elsewhere.
  3. If one fast TF $i$ purchases ESST from a set of exchanges $K \subseteq M$ and all other fast TFs $k \neq i$ purchase ESST from a strict subset of exchanges $J \subset M$, where $f_j = 0$ for all $j \in J$ and $K \nsubseteq J$, then in Period 1 of each trading game:
    (a) If $J \subset K$ (so that TF $i$ purchases from a strictly greater set of exchanges than all other TFs), strategies are as in Case 1 above and liquidity is provided only on exchanges in $J$;
    (b) If $J \cap K = \emptyset$ (so that TF $i$ purchases from no exchanges contained in $J$), strategies are analogous to Case 1 above: some fast TF $k \neq i$ which has purchased ESST on exchanges in $J$ submits orders to provide $\sigma^*_j / (\sum_{k \in J} \sigma^*_k)$ amount of liquidity on each exchange $j \in J$ at spread $s^*_{\text{continuous}}$ around $y$ (maintaining, adjusting or withdrawing any outstanding liquidity from the previous trading game as necessary) and all other TFs submit orders such that they provide no liquidity on any exchange;
    (c) Otherwise (which occurs if $K$ contains a non-empty strict subset of exchanges in $J$ and at least one exchange outside of $J$), strategies are as in Case 2 above where TF $i$ is the lone-wolf liquidity provider, and provides one unit of liquidity at spread $\tilde{s}_N$ on some exchange contained in $J' = J \cap K$ (maintaining, adjusting or withdrawing any outstanding liquidity from the
previous trading game as necessary), all other TFs submit orders such that they provide no liquidity on any exchange, and no liquidity is provided elsewhere.

- In Stage 3, in Period 2 of each trading game, an investor sends IOCs to all exchanges that, in aggregate, trade up to one unit in their desired direction, prioritizing their demand across exchanges based on the lowest value of $s_j/2 + f_j$ (where for each exchange $j$, $s_j$ is the lowest spread at which liquidity is offered and $f_j$ is the trading fee), and breaking ties according to routing table strategies given by $\gamma^* = \sigma^*$; additionally, if there are any remaining orders that are profitable to trade against based on the publicly observed state $y$, the investor trades against those as well. An informed trader sends IOCs to trade against any orders on any exchange that are profitable to trade against based on their privately observed $y$. If there is a publicly observable jump in $y$, TF $i$ sends messages to cancel all liquidity providing orders; and all fast TFs (except the liquidity provider TF $i$ if $i$ is fast) send IOCs to each exchange $j$ to try to trade against (snipe) any orders that are profitable to trade against based on the new value of $y$.

Note that these candidate equilibrium strategies dictate play in all subgames that are reachable via any sequence of unilateral deviations in Stages 1 and 2. Specifically, Stage 2 strategies prescribes play for all fast TFs given any choice of ESST and trading fees chosen by exchanges in Stage 1 (where, as discussed earlier in Section B.3.3, we restrict exchanges to charge non-negative trading fees because negative fees create a money pump). Stage-3-Period-1 strategies prescribe play in any subgame reachable following Stage 2 if no more than one fast TF engages in a deviation. For these Stage-3-Period-1 strategies, Case 1 prescribes play on the equilibrium path when no fast TF deviates, and all fast TFs purchase from the same set of exchanges. Cases 2 and 3(a)-(c) prescribe play when some fast TF purchases from a different set of exchanges than all other fast TFs, and comprehensively covers play depending on whether TF $i$ purchases ESST from a: (Case 2) strict subset, (Case 3(a)) strict superset, (Case 3(b)) non-overlapping set, or (Case 3(c)) partially-overlapping set of exchanges.

We now show that these strategies comprise an equilibrium.

First, consider Stage 1 deviations for exchanges regarding their choice of ESST fees and trading fees. If any exchange lowers its ESST fee, it strictly reduces its profits as it earns less from the sale of ESST but outcomes would otherwise remain the same. If any exchange increased its ESST fee, the exchange would earn zero profits as no fast TF would purchase ESST from it, and liquidity would only be provided on other exchanges in the subsequent OBE outcome of the Stage 3 trading game (see Lemma B.1). If any exchange increased its trading fee, it also would earn zero profits for the same reasons. Hence, exchanges have no strictly profitable unilateral deviations.

Next, we turn to Stage 2 strategies for fast TFs. By following candidate strategies in Stage 2 given exchanges did not deviate in Stage 1, all fast TFs earn $\frac{1}{N} \Pi_{\text{continuous}}^* - \sum_j F_j^*$ which, by condition (3.2), is positive. Potentially profitable unilateral deviations for any fast TF involve the purchase of ESST from a strict subset of exchanges (as purchasing ESST from no exchanges yields no profits, and being the only fast TF to purchase ESST from an exchange yields no benefit due to our fair-access assumption). In subgames following such deviations, prescribed strategies comprise the unique OBE for Period 1 of the subsequent “lone-wolf” Stage 3 trading game given no liquidity is provided by other TFs (Lemma B.2), and the deviating fast TF earns in expectation $\pi_{\text{one-wolf}}^N$ per trading game (gross trading fees). Condition (3.2) ensures that this deviation is not profitable for any fast TF. Similar arguments establish that there are no strictly profitable deviations for fast TFs in Stage 2 given at most one exchange engaged in any deviation in Stage 1.

Finally, given equilibrium play in Stages 1 and 2, Lemma B.1 establishes that Stage-3-Period-1 strategies comprise an OBE. Stage-3-Period-2 strategies are easily seen to be optimal, as discussed in the main text.
B.3.5 Proof of Proposition 3.3 (Bound on ESST Fees)

Consider any vector of ESST fees $\mathbf{F}' = (F'_1, \ldots, F'_M)$ that maximizes $\sum_{j \in M} F'_j$ among all vectors of ESST fees that satisfy condition (3.2). This condition, satisfied by any equilibria described by Proposition 3.2, can be rewritten as:

$$
\sum_{j : \sigma^*_j > 0} F'_j \leq \frac{\Pi^*_{\text{continuous}}}{N} - \max(0, \pi^*_N \text{wolf} - \min_j F'_j).
$$

Since the upper bound on the total sum of ESST fees across exchanges (the left-hand side) is increasing in the minimum ESST fee (on the right-hand side), such a vector $\mathbf{F}'$ must have the same ESST fees for all exchanges: i.e., there must be a constant $\tilde{F}$ such that $F'_j = \tilde{F}$ for all $j \in M$. Hence, the vector of ESST fees that maximizes the sum over all ESST fees and satisfies condition (3.2) is unique and involves each exchange charging the same amount $\tilde{F}$. This implies that, in any equilibrium described by Proposition (3.2), each fast TF pays at most

$$
M \times \tilde{F} \leq \frac{1}{N} \Pi^*_{\text{continuous}} - (\pi^*_N \text{wolf} - \tilde{F})
$$

in ESST fees across all exchanges. Substituting in the lower bound $\pi^*_N \text{wolf} > \frac{N-2}{(N-1)N} \Pi^*_{\text{continuous}}$ from Lemma B.2 into the above equation and re-arranging terms yields:

$$
\tilde{F} < \frac{1}{(M-1)(N-1)N} \Pi^*_{\text{continuous}}
$$

$$
(\Leftrightarrow) \quad M \times N \times \tilde{F} < \frac{M}{(M-1)(N-1)} \Pi^*_{\text{continuous}},
$$

where $M \times N \times \tilde{F}$ is the upper bound on the total amount of ESST fees earned by all exchanges.

B.4 Proofs For Section 5

Preliminaries: Equilibrium Spreads on Discrete. Denote by $\bar{s}_{\text{discrete}}(f)$ the zero-variable profit spread for a liquidity provider on Discrete given Discrete charges a trading fee $f \geq 0$; such a spread solves:

$$
\lambda_{\text{invest}}(\frac{\bar{s}_{\text{discrete}}(f)}{2} - f) - \lambda_{\text{private}} L(\bar{s}_{\text{discrete}}(f), f) = 0,
$$

where $L(s, f) \equiv E(J - \frac{s}{2} + f | J > \frac{s}{2} + f) \Pr(J > \frac{s}{2} + f)$ represents the expected loss to a liquidity provider providing liquidity at spread $s$ on an exchange with trading fee $f$ in the event of being adversely traded against. The first term on the left-hand-side of (B.4) represents the revenues a liquidity provider earns when an investor arrives (i.e., half the spread less the trading fee), and the second term is the expected loss from informed trading. A unique solution $\bar{s}_{\text{discrete}}(f)$ exists for any $f \geq 0$ (and is strictly positive) by the same arguments used to establish the existence and uniqueness of $s^*_{\text{continuous}}$ in the main text (see the discussion following equation (3.1)). Note that when the trading fee on Discrete is zero ($f = 0$), $\bar{s}(f) = s^*_{\text{discrete}}$, where $s^*_{\text{discrete}}$ is defined in (5.1) in the main text.

Define $f^*_{\text{discrete}}$ to be the trading fee so that an investor is indifferent between trading on Discrete at spread $\bar{s}_{\text{discrete}}(f^*_{\text{discrete}})$ with trading fee $f^*_{\text{discrete}}$, and trading on a Continuous exchange with no trading fee at the zero-variable profit spread $\bar{s}_{\text{continuous}} \equiv \bar{s}_N$, defined in equation (B.2). As the following lemma establishes, under a technical assumption, $f^*_{\text{discrete}}$ exists and is unique.

Lemma B.3. Assume that the jump size distribution is continuously differentiable. Then there exists a unique...
solution \( f_{\text{discrete}}^* \) to:
\[
\frac{s_{\text{discrete}}(f_{\text{discrete}}^*)}{2} + f_{\text{discrete}}^* = \frac{s_{\text{continuous}}}{2}.
\]  

Furthermore, if \( f < (>) f_{\text{discrete}}^* \), then \( \frac{s_{\text{discrete}}(f)}{2} + f < (>) \frac{s_{\text{continuous}}}{2} \).

Proof. Let \( H(s, f) = \lambda_{\text{invest}}(\frac{s}{2} - f) - \lambda_{\text{private}}L(s, f) \). Define \( s(f) \) to be the solution to \( H(s(f), f) = 0 \) (hence, \( s_{\text{discrete}}(f) = s(f) \)). Since the jump size distribution is continuously differentiable, \( H(\cdot) \) is as well, and by the implicit function theorem (given \( \frac{dH}{ds} \neq 0 \), which is satisfied), the function \( s(f) \) exists and is continuously differentiable with
\[
s'(f) = -\frac{\partial H}{\partial s} = \frac{\lambda_{\text{invest}} + \lambda_{\text{private}}L_f(s, f)}{\lambda_{\text{invest}}/2 - \lambda_{\text{private}}L_s(s, f)},
\]
where \( L_f(\cdot) \) and \( L_s(\cdot) \) represent partial derivatives of \( L(\cdot) \). We next establish that \( \frac{s_{\text{discrete}}(f)}{2} + f \) is strictly increasing in \( f \); differentiating this expression with respect to \( f \) implies that a sufficient condition for it to be strictly increasing in \( f \) is \( s'(f) > -2 \). Substituting in for \( s'(f) \) and re-arranging terms yields:
\[
\frac{\lambda_{\text{invest}}}{\lambda_{\text{private}}} > (L_s(s, f) - L_f(s, f))/2.
\]
This inequality always holds since the left-hand-side is strictly positive, and the right-hand-side is weakly negative.\(^9\) Since \( \frac{s_{\text{discrete}}(f)}{2} + f \) is thus strictly increasing and continuous in \( f \), and since it is less than \( \frac{s_{\text{continuous}}}{2} \) for \( f = 0 \) but greater than \( \frac{s_{\text{continuous}}}{2} \) when \( f = \frac{s_{\text{continuous}}}{2} \), there exists a unique solution to (B.5). The rest of the statement directly follows.

We maintain the assumption that the jump size distribution is continuously differentiable for the rest of this section.

### B.4.1 Proof of Proposition 5.1 (Equilibrium Stage 3 with a Discrete and a Continuous Exchange)

The statement of Proposition 5.1 assumes that both Continuous and Discrete charge zero trading fees. Here, we prove a more general version of Proposition 5.1, and allow Discrete to charge a weakly positive trading fee.

For the rest of this proof, consider any Stage 3 subgame with a single Continuous and single Discrete exchange, where all fast TFs have purchased ESST from Continuous, and trading fees are zero on Continuous and equal to \( \bar{f} \in [0, f_{\text{discrete}}^*] \) on Discrete (where \( f_{\text{discrete}}^* \) is defined in (B.5)). We will prove that in any equilibrium of this Stage 3 subgame: exactly one unit of liquidity is provided on Discrete at bid-ask spread

\(^9\)Let \( G_{\text{jump}} \) denote the jump size distribution and \( g_{\text{jump}} \) its associated density. To establish that \( (L_s(s, f) - L_f(s, f))/2 \leq 0 \), note that
\[
L(s, f) = E(J - \frac{s}{2} + f | J > \frac{s}{2} + f) Pr(J > \frac{s}{2} + f) = \int_{\frac{s}{2} + f}^{\infty} [t - \frac{s}{2} + f]g_{\text{jump}}(t)dt.
\]
Hence,
\[
L_s(s, f) = -\int_{\frac{s}{2} + f}^{\infty} \frac{g_{\text{jump}}(t)}{2} dt - [(\frac{s}{2} + f) - \frac{s}{2} + f] \times \frac{g_{\text{jump}}(s/2 + f)}{2} = -\frac{(1 - G_{\text{jump}}(s/2 + f))}{2} - f \times g_{\text{jump}}(s/2 + f),
\]
\[
L_f(s, f) = \int_{\frac{s}{2} + f}^{\infty} g_{\text{jump}}(t)dt - [(\frac{s}{2} + f) - \frac{s}{2} + f] \times g_{\text{jump}}(s/2 + f) = (1 - G_{\text{jump}}(s/2 + f)) - 2f \times g_{\text{jump}}(s/2 + f),
\]
and \( (L_s(s, f) - L_f(s, f))/2 = -(1 - G_{\text{jump}}(s/2 + f)) \), which is weakly negative since \( G_{\text{jump}}(x) \leq 1 \) for all \( x \).
\(s_{\text{discrete}}(\sqrt{f})\) around the current value of \(y\) following Period 1, and no liquidity is provided elsewhere; Period 2 behavior is as described in the statement of the Proposition; and that such an equilibrium exists.\(^{10}\)

**Existence.** Consider the following Stage 3 strategies given state \((y, \omega)\). In Period 1, a single TF \(i\) submits an order to Discrete to provide one unit of liquidity at spread \(s_{\text{discrete}}(\sqrt{f})\) around \(y\); if he has liquidity outstanding from the previous trading game, he maintains, adjusts or withdraws it as necessary so that he provides exactly one unit at spread \(s_{\text{discrete}}(\sqrt{f})\) around \(y\) on Discrete, and no liquidity on Continuous. All other TFs do not provide any liquidity on any exchange. In Period 2, an investor sends IOCs to all exchanges that, in aggregate, trade up to one unit in their desired direction, prioritizing their demand across exchanges indexed by \(j\) based on the lowest value of \(s_j/2 + f_j\) (where for each exchange \(j\), \(s_j\) is the lowest spread at which liquidity is offered), and breaking ties in an arbitrary fashion; additionally, if there are any remaining orders that are profitable to trade against based on the publicly observed state \(y\), the investor trades against those as well. An informed trader sends IOCs to trade against any orders on any exchange that are profitable to trade against based on their privately observed \(y\). If there is a publicly observable jump in \(y\), TF \(i\) sends messages to cancel all liquidity providing orders; and all fast TFs not providing liquidity send IOCs to each exchange \(j\) to try to trade against (snipe) any orders that are profitable to trade against based on the new value of \(y\).\(^{11}\)

Using similar arguments used in the proof of Lemma B.1, it is straightforward to show that there are no safe profitable price improvements or robust deviations in Period 1, or profitable unilateral deviations in Period 2, and hence these strategies comprise an equilibrium for the Stage 3 trading game. For example, in Period 1, any increase by TF \(i\) in its spread on Discrete to \(s_{\text{discrete}}(\sqrt{f}) + \varepsilon\) for any \(\varepsilon > 0\) and any amount of liquidity is not a robust deviation, as it is rendered unprofitable by a safe profitable price improvement from another TF to provide the same amount of liquidity at spread \(s_{\text{discrete}}(\sqrt{f}) + \varepsilon/2\) on Discrete.

**Uniqueness.** Consider a Stage 3 trading game where there is some positive amount of liquidity offered on Discrete following Period 1, and in Period 2 there is the arrival of public information. We first prove that in any Nash equilibrium of this subgame, on Discrete either: (i) all stale quotes are canceled; or (ii) not all stale quotes are cancelled and either (a) no trade occurs, or (b) if the auction results in trade, the auction price is the new value of \(y\). (Behavior on the Continuous exchange in the event of public information is still characterized by the essentially unique optimal strategies described in the main text.) First, note that there exist Nash equilibria (described in the Existence portion of this proof) where all liquidity providing orders at stale prices are canceled on Discrete. Now, assume by contradiction that there exists a candidate Nash equilibrium in which some liquidity providing orders are not canceled on Discrete, and trade occurs in Period 2 on Discrete at a price not equal to the new value of \(y\). This implies that either (i) trade occurs at a price at which the liquidity-providing TF earns negative profits, or (ii) trade occurs at a price at which the liquidity-taking TF earns negative profits. In either case, one of these TFs has a robust deviation to cancel its unprofitable orders as a Discrete exchange processes cancellations prior to processing any new orders. Contradiction. Thus, either no trade occurs in Period 2 on Discrete following the arrival of public information, or if it does occur, it must occur at price \(p = y\). Note that these results imply that in any equilibrium of Stage 3, there are no latency arbitrage rents on Discrete.

We now prove that in any Stage 3 equilibrium, exactly one unit of liquidity is provided on Discrete following

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\(^{10}\)It is straightforward to use the same arguments in this proof to establish that this statement holds even if there exist other Discrete exchanges with trading fees that are strictly greater than \(\sqrt{f}\).

\(^{11}\)When \(\sqrt{f} = 0\), there also exist equilibria in which, following a publicly-observed jump in \(y\) that exceeds \(s_{\text{discrete}}/2\), TF \(i\) does not withdraw all of its offered liquidity on Discrete, and multiple TFs submit IOCs to Discrete to purchase one unit of the security at price \(y\). In such equilibria, all trades in the auction occur at the new value of \(y\).
Period 1 at spread $\hat{s}_{\text{discrete}}(\hat{f})$, and no liquidity is provided on Continuous.

First, using similar arguments used in the proof of Lemma B.1, it is straightforward to establish that exactly one unit of liquidity in aggregate must be provided in any Stage 3 equilibrium.

Next, we show that some positive amount of liquidity cannot be provided on Continuous. Assume by contradiction that there exists an equilibrium in which some positive amount of liquidity is provided on Continuous at spread $\hat{s}$. If $\hat{s} < \hat{s}_{\text{continuous}}$ (the zero-variable profit spread on Continuous), then the liquidity provider would have a robust deviation to withdraw it. If $\hat{s} > \hat{s}_{\text{continuous}}$, then there is a safe profitable price improvement by any slow TF to provide the same amount of liquidity on Discrete at some spread $s' \in (\hat{s}_{\text{discrete}}(\hat{f}), \hat{s}_{\text{continuous}} - 2\hat{f})$. Contradiction.12

Last, we show that the one unit of liquidity on Discrete must be provided at spread $\hat{s}_{\text{discrete}}(\hat{f})$. Assume by contradiction that there exists an equilibrium where some positive amount of liquidity on Discrete is provided at spread $\hat{s} \neq \hat{s}_{\text{discrete}}(\hat{f})$. If $\hat{s} < \hat{s}_{\text{discrete}}(\hat{f})$ (the zero-variable profit spread on Discrete given informed trading), then the liquidity provider would have a robust deviation to withdraw it. If $\hat{s} > \hat{s}_{\text{discrete}}(\hat{f})$, then there is a safe profitable price improvement by any slow TF to provide the same amount of liquidity at any spread $s' \in (\hat{s}_{\text{discrete}}(\hat{f}), \hat{s})$. Contradiction.

### B.4.2 Proof of Proposition 5.2 (Equilibrium with a Discrete and a Continuous Exchange)

**Existence.** First, we establish that if Discrete charged any trading fee $f' > f^*_{\text{discrete}}$ (where $f^*_{\text{discrete}}$ is the solution to equation (B.5)) and Continuous had zero trading fees, then in any Stage 3 equilibrium, no liquidity can be provided on Discrete. To see why, assume by contradiction that there exists an equilibrium in which Discrete charges a trading fee $f' > f^*_{\text{discrete}}$, and there is positive liquidity provided on Discrete in Stage 3. The lowest spread at which liquidity could be profitably offered on Discrete is the zero-variable profit spread $\hat{s}_{\text{discrete}}(f')$. At this spread, the total price considered by an investor contemplating trading on Discrete is $\hat{s}_{\text{discrete}}(f')/2 + f' > \hat{s}_{\text{continuous}}/2$ (by Lemma B.3 and the definition of $f^*_{\text{discrete}}$). This implies that there exists a safe profitable price improvement for some fast TF on Continuous to provide liquidity on Continuous at spread $s' \in (\hat{s}_{\text{continuous}}, \hat{s}_{\text{discrete}}(f') + 2f')$, as such liquidity at spread $s'$ on Continuous would be preferred by investors to the liquidity on Discrete and earns strictly positive profits for the deviating TF. Contradiction.

Next, consider the following candidate equilibrium strategies. In Stage 1, Discrete charges positive trading fees $f^*_{\text{discrete}}$. Continuous charges zero trading fees and zero ESST fees. In Stage 2, fast TFs do not purchase ESST from Continuous if ESST fees are weakly positive, and purchase ESST from Continuous if ESST fees are strictly negative. In Stage 3, market participants use strategies described in the Proof of Proposition 5.1, with the modification that investors break ties in favor of Discrete.13

We now show that these strategies comprise an equilibrium. In Stage 1, Continuous has no strictly profitable deviations: charging strictly positive ESST or trading fees do not affect profits; charging strictly negative ESST fees earns strictly negative profits; and, as discussed earlier in Section B.3.3, we restrict exchanges to charge non-negative trading fees because negative fees create a money pump. Discrete also has no strictly profitable deviations: by Proposition 5.1, reducing trading fees yields lower profits for Discrete as it does not affect Stage

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12 As long as the spread on Discrete $s' < \hat{s}_{\text{continuous}} - 2\hat{f}$, an investor would prefer to transact on Discrete (paying $\frac{s' + \hat{f}}{2}$) than transact on Continuous (paying $\frac{\hat{s}_{\text{continuous}}}{2}$).

13 If trading fees are restricted to be in discrete units (e.g., in units of $0.0001$), then there also exist equilibria in which investors always break ties in favor of Continuous: in such equilibria, Discrete charges the greatest trading fee $f$ such that $\frac{\hat{s}_{\text{discrete}(f)}}{2} + f < \hat{s}_{\text{continuous}}$, and liquidity is only offered on Discrete in each trading game.
3 trading game behavior; and any higher trading fees results in all trading activity in Stage 3 occurring on Continuous and zero profits (as established above). In Stage 2, there are no strictly profitable deviations by any fast TF (as purchasing ESST does not affect profits). Last, in Stage 3, similar arguments used in the Existence portion of the proof for Proposition 5.1 establish that these strategies comprise an equilibrium of the Stage 3 trading game.

Uniqueness. We now prove that in any equilibrium, (i) Discrete charges trading fees equal to $f^\ast_{\text{discrete}}$; (ii) in every iteration of the trading game, exactly one unit of liquidity is offered on Discrete at spread $\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}})$ and no liquidity is provided on Continuous; (iii) Continuous earns zero profits; and (iv) Discrete earns expected per-trading-game profits that exceed $\frac{N-1}{N}\Pi^\ast_{\text{continuous}}$.

For claim (i), first consider a candidate equilibrium where Discrete charges trading fee $f < f^\ast_{\text{discrete}}$. Since $\bar{s}_{\text{discrete}}(f) + f < \bar{s}_{\text{continuous}}$, then by continuity of $\bar{s}_{\text{discrete}}(\cdot)$, there exists trading fee $f' = f + \varepsilon$ for sufficiently small $\varepsilon > 0$ that Discrete could charge such that $\bar{s}_{\text{discrete}}(f') + f' < \bar{s}_{\text{continuous}}$ and would yield Discrete strictly higher profits as it would still capture all trading volume but obtain higher trading revenues; contradiction. Next, consider a candidate equilibrium where Discrete charges trading fee $f > f^\ast_{\text{discrete}}$. In this candidate equilibrium, either Discrete has zero trading volume in Stage 3, or positive trading volume. In the case that Discrete has zero trading volume, since there exists some strictly positive trading fee $f' < f^\ast_{\text{discrete}}$ that Discrete could charge such that $\bar{s}_{\text{discrete}}(f') + f' < \bar{s}_{\text{continuous}}$ and yields positive trading volume on Discrete in any Stage 3 equilibrium (by Proposition 5.1), there is a profitable deviation for Discrete to charge $f'$ instead; contradiction. In the case that Discrete has positive trading volume, then using similar arguments as in Lemma B.3 and above, since there exists some strictly positive trading fee $f' > 0$ that Continuous could charge such that $\bar{s}_{\text{continuous}}(f') + f' < \bar{s}_{\text{discrete}}(f)$ and $\bar{s}_{\text{continuous}}(f')$ is the analogous zero-variable profit spread on Continuous given Continuous charges trading fees $f'$ and moves all trading volume to Continuous in any Stage 3 equilibrium, there is a profitable deviation for Continuous to charge $f'$ instead; contradiction. Thus, Discrete cannot charge any trading fee $f > f^\ast_{\text{discrete}}$. Hence, Discrete must charge trading fees equal to $f^\ast_{\text{discrete}}$.

For claim (ii), any equilibria in which strictly less than or strictly greater than one unit of liquidity is provided in aggregate across all exchanges can be ruled out using similar arguments as in Lemma B.1. Now consider a candidate equilibrium in which exactly one unit of liquidity is offered in aggregate, but strictly positive liquidity in aggregate across all exchanges can be ruled out using similar arguments as in Lemma B.1. Next consider a candidate equilibrium where Discrete charges trading fee $f > f^\ast_{\text{discrete}}$. In this candidate equilibrium, either Discrete has zero trading volume in Stage 3, or positive trading volume. In the case that Discrete has zero trading volume, since there exists some strictly positive trading fee $f' < f^\ast_{\text{discrete}}$ that Discrete could charge such that $\bar{s}_{\text{discrete}}(f') + f' < \bar{s}_{\text{continuous}}$ and yields positive trading volume on Discrete in any Stage 3 equilibrium (by Proposition 5.1), there is a profitable deviation for Discrete to charge $f'$ instead; contradiction. In the case that Discrete has positive trading volume, then using similar arguments as in Lemma B.3 and above, since there exists some strictly positive trading fee $f' > 0$ that Continuous could charge such that $\bar{s}_{\text{continuous}}(f') + f' < \bar{s}_{\text{discrete}}(f)$ and $\bar{s}_{\text{continuous}}(f')$ is the analogous zero-variable profit spread on Continuous given Continuous charges trading fees $f'$ and moves all trading volume to Continuous in any Stage 3 equilibrium, there is a profitable deviation for Continuous to charge $f'$ instead; contradiction. Thus, Discrete cannot charge any trading fee $f > f^\ast_{\text{discrete}}$. Hence, Discrete must charge trading fees equal to $f^\ast_{\text{discrete}}$.

Claim (iii) directly follows from (i) and (ii).

We have now proved that in any equilibrium, Discrete charges trading fees equal to $f^\ast_{\text{discrete}}$ and one unit of liquidity is provided in each trading game on Discrete at spread $\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}})$ and no liquidity is provided on Continuous. We now establish claim (iv).

First, substituting in the expression for $\bar{s}_{\text{continuous}}$ yields:

$$\lambda_{\text{invest}}\left(\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}}) + f^\ast_{\text{discrete}}\right) + f^\ast_{\text{discrete}} \text{ given by (B.5) into (B.2) yields:}$$

$$\lambda_{\text{invest}}\left(\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}}) + f^\ast_{\text{discrete}}\right) + f^\ast_{\text{discrete}} \text{ given by (B.5) into (B.2) yields:}$$

$$\lambda_{\text{invest}}\left(\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}}) + f^\ast_{\text{discrete}}\right) + f^\ast_{\text{discrete}} \text{ given by (B.5) into (B.2) yields:}$$

Equation (B.4) provides the expression for the zero-variable profit spread on Discrete at $f^\ast_{\text{discrete}}$:

$$\lambda_{\text{invest}}\left(\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}}) - f^\ast_{\text{discrete}}\right) - \lambda_{\text{private}}L(\bar{s}_{\text{discrete}}(f^\ast_{\text{discrete}}), f^\ast_{\text{discrete}}) = 0.$$
Subtracting this expression from (B.6) and re-arranging yields:

$$2f^*_{\text{discrete}} \times (\lambda_{\text{invest}} + \lambda_{\text{private}}Pr(J > \frac{s^*_\text{discrete}(f^*_{\text{discrete}})}{2} + f^*_{\text{discrete}})) = \frac{N-1}{N} \lambda_{\text{public}}L(\bar{s}_{\text{continuous}}), \quad (B.7)$$

(since $L(\bar{s}_{\text{discrete}}(f^*_{\text{discrete}}), f^*_{\text{discrete}}) - L(\bar{s}_{\text{continuous}}) = 2f^*_{\text{discrete}} \times Pr(J > \frac{s^*_\text{discrete}(f^*_{\text{discrete}})}{2} + f^*_{\text{discrete}})$). The left-hand side of (B.7) represents Discrete’s total expected revenues per-trading game from trading fees $f^*_{\text{discrete}}$: Discrete earns $2f^*_{\text{discrete}}$ in trading fees each time an investor arrives and trades (with probability $\lambda_{\text{invest}}$), or an informed trader arrives and trades (with probability $\lambda_{\text{private}} \times Pr(J > \frac{s^*_\text{discrete}(f^*_{\text{discrete}})}{2} + f^*_{\text{discrete}})$). The right-hand side of (B.7) represents $(N-1)/N$ share of the total “sniping prize” at a spread of $\bar{s}_{\text{continuous}}$: since $\bar{s}_{\text{continuous}} < s^*_\text{continuous}$ and $L(\cdot)$ is decreasing in the spread, the right-hand side is strictly greater than $(N-1)/N$ share of $\Pi^*_\text{continuous}$, and the result follows.

### B.4.3 Proof of Proposition 5.3 (Equilibrium with Multiple Discrete Exchanges)

**Existence.** Consider the following candidate equilibrium strategies. In Stage 1, all Discrete exchanges charge zero trading fees; any Continuous exchange charges zero trading fees and zero ESST fees. In Stage 2, fast TFs do not purchase ESST from any Continuous exchange with weakly positive ESST fees, and purchase ESST from any Continuous exchange with strictly negative ESST fees. In Stage 3 Period 1, a single TF $i$ submits an order to any Discrete exchange with the minimum trading fee (denoted $j$) to provide one unit of liquidity at spread $\bar{s}_{\text{discrete}}(\tilde{f})$ around $y$: if he has liquidity outstanding from the previous trading game, he maintains, adjusts or withdraws it as necessary so that he provides exactly one unit at spread $\bar{s}_{\text{discrete}}(\tilde{f})$ around $y$ on Discrete, and no liquidity on Continuous. All other TFs do not provide any liquidity on any exchange (which includes withdrawing any existing liquidity in $\omega$, if present). In Period 2, investors, informed traders, and TFs use strategies described in the Proof of Proposition 5.1, with the modification that investors break ties in favor of Discrete exchange $j$.

We now check that these strategies comprise an equilibrium. In Stage 1, Continuous exchanges have no profitable deviations: charging strictly positive ESST or trading fees do not affect profits; charging strictly negative ESST fees earns strictly negative profits; and, as discussed earlier in Section B.3.3, we restrict exchanges to charge non-negative trading fees because negative fees create a money pump. Any Discrete exchange also has no strictly profitable deviations: increasing trading fees results in no trading volume and revenues given equilibrium strategies. In Stage 2, there are no strictly profitable deviations by any fast TF (as purchasing ESST does not affect profits). Last, similar arguments used in the Existence portion of the proof for Proposition 5.1 establish that these strategies comprise an equilibrium of the Stage 3 subgame.

**Uniqueness.** We now prove that in any equilibrium, (i) at least one Discrete exchange charges zero trading fees; (ii) in every iteration of the trading game, exactly one unit of liquidity is offered only on Discrete exchanges with zero trading fees at spread $s^*_\text{discrete}$ (equivalent to $\bar{s}_{\text{discrete}}(0)$) around the current value of $y$ following Period 1; (iii) no liquidity is provided on Discrete exchanges with positive trading fees or on Continuous exchanges; and (iv) all exchanges and trading firms earn zero profits. For claim (i), consider a candidate equilibrium where all Discrete exchanges charge strictly positive trading fees, and the minimum trading fee is $f > 0$. The same

\[\text{Denote by } J \text{ the set of all Discrete exchanges with zero trading fees and let } \sigma^* \text{ be any arbitrary vector of market shares. As in Proposition 3.2, there also exist equilibria with multiple Discrete exchanges in which TF } i \text{ provides } \sigma^*_j/(\sum_{k \in J} \sigma^*_k) \text{ amount of liquidity on each exchange } j \in J \text{ (thereby providing one unit of liquidity in aggregate), and investors break ties when indifferent according to routing table strategies given by } \gamma^* = \sigma^*.\]
logic underlying why undifferentiated Bertrand competition results in marginal cost pricing implies that this cannot be an equilibrium: for some Discrete exchange, there exists a profitable Stage 1 deviation to charge a slightly lower trading fee \( f' = f - \varepsilon \) for some \( \varepsilon > 0 \) as this would guarantee that all subsequent trading volume would occur on that exchange (the same arguments used in Proposition 5.1 establish that all Stage 3 equilibria involve all trading volume occurring on the Discrete exchange with the lowest trading fee). Claims (ii) and (iii) follow directly from the arguments used in Proposition 5.1. Claim (iv) directly follows from claims (i) and (ii).

B.4.4 Proof of Proposition 5.4 (Prisoner’s Dilemma Payoffs)

The result follows directly from \( \Pi^D > 0 \) and the following Lemma.

Lemma B.4. Assume that ESST fees \( F^* \) satisfy condition (3.2). Then \( NF_j^* < \Pi^D \) for any exchange \( j \).

Proof. Let \( \bar{F} \) be the most that any exchange \( j \) can charge for ESST fees given (3.2). This maximum is realized for exchange \( j \) when all other exchanges charge 0; condition (3.2) then becomes

\[
\bar{F} \leq \frac{1}{N} \Pi^*_{\text{continuous}} - \frac{\Pi^*_{\text{lone-wolf}}}{N} < \frac{1}{(N - 1)N} \Pi^*_{\text{continuous}},
\]

where the last inequality follows from the lower bound on \( \frac{\Pi^*_{\text{lone-wolf}}}{N} \) given by Lemma B.2. Hence, \( N\bar{F} < \frac{1}{N-1} \Pi^*_{\text{continuous}} \). Since \( \Pi^D > \frac{N-1}{N} \Pi^*_{\text{continuous}} \) by Proposition 5.2 (and since \( N \geq 3 \)), the result follows. \( \square \)
C Additional Empirical Evidence on the Stage 3 Trading Game

In this appendix, we report additional evidence on the Stage 3 Trading Game presented in Section 4.1. Specifically, we present Stylized Facts #1 and #2 for the “Top 8” exchanges in 2015, which include the 3 taker-maker exchanges in addition to the 5 maker-taker exchanges that we study in the main text. We also present a version of Figure 4.3 from Stylized Fact #3 from the beginning of the Reg NMS era. Before going into the additional results, we describe the institutional details of taker-maker exchanges, which help interpret Stylized Facts #1 and #2 with the expanded Top 8 sample.

Taker-Maker Exchanges. In 2015, the top 5 exchanges by market share all used the “maker-taker” pricing model, in which the provider of liquidity (“maker”) gets a rebate and the taker of liquidity pays a fee. The next 3 exchanges by market share all used the “taker-maker”, or “inverted”, pricing model, in which the taker of liquidity gets a rebate, and the provider of liquidity pays a fee. Together, these Top 8 exchanges accounted for 98% of share volume in 2015, of which 83% was the top 5 maker-takers and 15% was the 3 taker-makers.

The difference between taker-maker and maker-taker fee structures means that liquidity at the same displayed quoted price is economically more attractive for the taker of liquidity on the taker-maker exchange than on the maker-taker exchange. Put differently, if a liquidity provider offers liquidity at the same quoted price on both types of exchanges, it is in effect offering a better price on the taker-maker exchange. For example, suppose there is displayed depth on both a taker-maker exchange and a maker-taker exchange at the national best offer price, say $10.00. Suppose that on both exchanges, the rebate is $0.0029, the fee is $0.0030, and hence the net fee collected by the exchange (i.e., the fee minus the rebate) is $0.0001. However, on the taker-maker exchange, the taker gets the rebate of $0.0029 and the liquidity provider pays the fee of $0.0030, whereas on the maker-taker exchange the liquidity provider gets the rebate and the taker pays the fee. This means that if the taker transacts on the maker-taker exchange, they in effect pay $10.00 + $0.0030 = $10.0030, whereas on the taker-maker exchange, the liquidity provider gets the rebate and the taker pays the fee. This means that if the taker transacts on the maker-taker exchange, they in effect pay $10.00 - $0.0029 = $9.9971. That is, they save $0.0059 by transacting on the taker-maker exchange. This example illustrates that depth on a taker-maker exchange, when offered, is economically more attractive than depth on a maker-taker exchange, and will be consumed first by optimizing market participants. This effect is observable in the additional evidence for Stylized Facts #1 and #2 below.

Stylized Fact #1: Many Exchanges Simultaneously at the Best Bid and Best Offer (Additional Evidence) Figure C.1 presents results for the Top 8 exchanges, i.e., all exchanges with meaningful market share. We present the results separately for NYSE-listed symbols and non-NYSE listed symbols because, as mentioned in the main text, non-NYSE listed symbols do not trade on NYSE (but do trade everywhere else) and NYSE listed symbols trade everywhere. Hence, for NYSE listed symbols the maximum number of exchanges out of the Top 8 that could be at the best bid or offer is 8, whereas for non-NYSE listed symbols the maximum is 7.

If we look at the Top 8, all exchanges are at the best bid (similarly, best offer) in about 50% of milliseconds. There is a small peak at 5 exchanges for NYSE-listed stocks and 4 exchanges for non-NYSE-listed stocks. As mentioned above, if a liquidity provider quotes the same price on a taker-maker exchange as on a maker-taker exchange, it is in effect offering a price that is roughly half a penny (the sum of the fee and rebate) better for the taker of liquidity and worse for itself as the provider of liquidity (see Table 4.1 in Section 4.2 for exact numbers). Therefore it makes economic sense that it will often be the case that the best price is found on all of
Notes: The data is from NYSE TAQ. Percent of time indicates the percent of symbol-side-millisecond (e.g. SPY-Bid-10:00:00.001) for which the number of exchanges at the best price was equal to N. The figure considers the Top 8 exchanges; for discussion of Top 8 see the text. An exchange was at the best price for a symbol-side-millisecond if the best displayed quote on that exchange was equal to the best displayed quote on any of the eight exchanges, all measured at the end of the millisecond. Sample is 100 highest volume symbols that satisfy data-cleaning filters (see text for description) on all dates in 2015.

Stylized Fact #2: Depth Equals Volume. (Additional Evidence) Figure C.2 presents a scatter-plot of $\text{VolumeShare}_{ijt}$ against $\text{DepthShare}_{ijt}$ for the Top 8 exchanges (see the main text in Section 4.1 Stylized Fact #2 for details on how we calculate shares). Each dot in the figure represents a symbol-exchange-date tuple. Dots are color coded by exchange and are labeled in the figure.

The figure shows that most of the depth-volume data falls along the 45 degree line. Formally, the slope of a regression of volume share on depth share is 0.991 (s.e. 0.020), and the $R^2$ of the relationship is 0.865. Just as the figure in the main text, the Top 5 exchanges – NYSE, Nasdaq, NYSE Arca, BZX (sometimes referred to simply as BATS), and EDGX – are tightly scattered along the 45 degree line. The taker-maker exchanges – EDGA, BYX (sometimes referred to as BATS Y), Nasdaq BX – are clustered at the bottom left of the figure and have a steeper slope than the maker-taker exchanges. That is, the taker-maker exchanges have volume shares that are typically greater than depth shares. The reason for this, as mentioned in the example above, is that the taker-maker exchanges pay a rebate to the taker of liquidity. Thus, while depth on taker-maker exchanges is comparatively rare, when there is depth on taker-maker exchanges it is more economically attractive, after accounting for fees, than depth at the same pre-fee price on the larger maker-taker exchanges. For this reason, it makes sense that taker-maker exchanges have volume shares that are larger than their depth shares.

Stylized Fact #3: Exchange Market Shares are Interior and Relatively Stable. (Additional Evidence) In the main text for Stylized Fact #3, we presented aggregate weekly exchange market shares from January 2011 to December 2015 for the Top 8 exchanges. Figure C.3 presents exchange market shares for the Top 8 exchanges from October 2007, the start of the Reg NMS era, through the end of 2015. There are several “jumps” in the data, in particular for BZX in late 2008 and EDGA and EDGX in 2010. Al-
Figure C.2: 2015 Daily Volume Share vs. Depth Share: Top 8 Exchanges

Notes: The data is from NYSE TAQ for the Top 8 exchanges. The dark line depicts the 45-degree line which is the depth share to volume share relationship predicted by the theory. Observations are symbol-date-exchange shares, with shares calculated among the Top 8. For details of share calculations see the text. Sample is 100 highest volume symbols that satisfy data-cleaning filters (see text for description) on all dates in 2015.

Figure C.3: Exchange Market Shares: 2007 – 2015

Notes: The data is from NYSE TAQ and covers October 2007 to December 2015 for the Top 8 exchanges. The market shares are based on all on-exchange trading volume in shares.
Figure C.4: 2015 Exchange Market Shares: Per Stock

Notes: The data is from NYSE TAQ. Observations are symbol-exchange averages of symbol-exchange-date market shares in 2015. In a given box, the middle line is the median and the edges of the box are the 25th and 75th percentiles. The lines on top of and below the box (whiskers) go out to the interquartile range multiplied by +/-1.5. The dots are symbol-exchange outliers that fall outside of that range.

though these exchanges were officially approved at the time of the jump, they operated as off-exchange venues, or ATS’s, and had significant market shares before they were officially approved. Thus, although the data show jumps in market share when these exchanges were approved, the market share change in going from an ATS to an exchange was likely much more smooth.

Figure C.4 explores how market shares vary across stocks. For each stock in the top 100, we compute its average market share per exchange over all dates in 2015. We then present this data as a box plot. Each box represents the 25th-75th percentile range for symbol market shares on that exchange, with the solid horizontal line in the middle of the box representing the median. The lines above and below the box represent the full range, with dots for outliers. As can be seen, while there is of course variation across symbols, most of the variation in the data is driven by the exchange.
D Supporting Details for Evidence on Trading Fees

In this appendix, we provide supporting details for the evidence in Stylized Facts #4-#5 presented in Section 4.2.

D.1 Expanded Table of Range of U.S. Equity Exchange Trading Fees

Table D.1 shows the range of regular-hours trading fees for the top 8 exchanges as reported in historical fee schedules. This table extends Table 4.1 in Section 4.2 by adding the fee ranges for special fee programs, as well as the fee ranges by listing exchange (referred to as the “Tape” a stock is included on: Tape “A” represents stocks listed on NYSE, “B” on NYSE Arca, and “C” on Nasdaq). As discussed in the main text, 7 of the 8 exchanges have total fees that are negative for at least one fee program. The only exception is EDGA, which has a minimum total fee per share per side of $0.00005 of 0.5 mills.

D.2 Details for Average Trading Fee Calculations in Table 4.1

In this appendix, we provide supporting details for the calculation of average per-share per-side trading fees for the three major exchange families as reported in Table 4.1. The calculations themselves are available in a supporting spreadsheet available in the online appendix.

**BATS.** For BATS, the April 2016 S-1 provides a net trading revenue figure of $81.0M, as we reported in the text of Section 4.3 and a matched share volume figure of 1.5 billion shares per day which corresponds to 378 billion shares per year (252 trading days). We cross-checked the volume figure with the NYSE TAQ data and found 367.9 billion shares in that data set, which is within rounding error. Using the S-1 figures for consistency with what follows, we obtain net revenue per share of $81M / 378 billion shares = $0.000214 which corresponds to $0.000107 per-share per side. This figure includes revenue from regular-hours trading, which is what we want, but it also includes revenue from opening and closing auctions and routing, which we want to strip out.

For BATS, the auction volume is minimal (0.13 billion shares per NYSE TAQ), so even under the assumption that all auction volume pays the maximum auction fee (which, depending on the order type utilized, ranges from zero to 5 mills for the opening auction and 10 mills for the closing auction), auction revenue does not move the needle. Routing volume on the other hand is significant, at approximately 25.2 billion shares per the S-1 (0.1 billion per day times 252 trading days). BATS reports routing and clearing costs of $43.7M in their S-1, which is 17.3 mills per share. We use a variety of data regarding routing fees based on the ultimate destination of the trade (e.g., a directed ISO versus a take on another exchange versus a take on a dark venue) to obtain a back-of-envelope estimate for BATS’s routing revenue per share of 22.8 mills per share and hence net routing revenue of 5.5 mills per share. This in turn implies net routing revenue for BATS overall of 5.5 mills * 25.2 billion shares = $13.8 million. Subtracting this revenue from BATS’s total revenue as reported above yields $67.2 million of regular-hours trading revenue, or $0.000178 per share and $0.000089 per-share per-side, as reported in Table 4.1. We caveat that the routing estimate is particularly back-of-envelope, so the reader may prefer to utilize the $0.000107 figure reported above or to adjust for routing in some other way.

**Nasdaq.** Nasdaq last reported U.S. cash equity net trading revenues in 2013; in 2015, they only report equity net trading revenues globally, not for the U.S. Our first step therefore is to take the 2015 number for
<table>
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<th>Taker Fee</th>
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**Notes:** This table summarizes the fee schedules for the top 8 exchanges retrieved from Internet Archive (Wayback Machine) dated from February 28, 2015 to September 1, 2015 (BATS Global Markets, Inc., 2015a,b,c,d; Nasdaq, Inc., 2015a,b; NYSE, 2015a; NYSE Arca Equities, Inc., 2015). In general, we determine the max rebates based on what a trading firm that satisfies the exchange’s highest volume tier would pay or receive, and the min rebates and fees tend to be the baseline for adding or taking liquidity. We consider all volume-based incentives for regular-hours liquidity provision, but we do not include additional incentives for trading off hours, trading at the open or close, creating non-displayed midpoint liquidity, sending retail orders, routing, or for trading securities with a share price below $1. The “Regular” program corresponds to the fees and rebates a firm would receive if it does not qualify for additional incentive programs detailed below, which often either involve an additional volume threshold, a National Best Bid and Offer quoting requirement, or an off-hours trading requirement. The Designated Liquidity Provider (Nasdaq DLP) program rewards market participants who maintain a one or two-sided quote on specified Nasdaq-listed ETFs for at least 15% of the trading day. The National Best Bid or Offer Setter (BZX/BYX NBBO Setter) program rewards participants who send orders that set the new national best bid or offer, as well as fulfill an additional volume requirement. The Supplemental Liquidity Provider (NYSE SLP) program rewards participants who quote at the NBBO at least 10% of the trading day, as well as fulfill an additional volume requirement. The Designated Market Maker (NYSE DMM) program rewards participants who make commitments to satisfy a wide variety of requirements involving market depth, volume, NBBO quoting, capital, and others every month. The Qualified Market Maker (Nasdaq BX QMM) program grants a discount on making liquidity for participants who actively quote at the NBBO. We also separately report fees by “Tape” or listing exchange. Tape “A” represents stocks listed on NYSE, “B” on NYSE Arca, and “C” on Nasdaq. In the table, NA indicates that an exchange does not charge different fees by listing exchange.
global cash equity trading revenues less transaction-based expenses (i.e., rebates), of $253 million, and multiply it by the 2013 ratio of U.S.:Global equity trading revenues, which was $107M/$193M = 55%. This yields $140.3M for 2015 U.S. cash equity net trading revenues. Nasdaq reports matched U.S. equity share volume of 327.7 billion; this is close to the figure we obtain in the TAQ as a cross-check (329.4 billion shares). We thus obtain net revenue per share of $140.3M / 327.7 billion shares = $0.000428, or $0.000214 per-share per-side. We caveat that this figure will be incorrect if the 2015 U.S.:Global ratio is meaningfully different from the 2013 ratio.

The next step is to deduct auction volume and revenue, which are both significant. We obtain auction volume from TAQ, of 5.3 billion shares annually for the opening auction and 20.2 billion shares for the closing auction. For the opening auction, we use a fee of 15 mills per-share per-side, which is the fee for regular market-on-open and limit-on-open orders that participate in the auction. This ignores fees for some other less-common order types as well as a fee cap for high-volume users of $20,000 per month. For the closing auction, Nasdaq has a fee schedule with 6 tiers based on volume levels. The fee ranges from 8 mills for the highest-volume tier to 15 mills for the lowest. We assume an equal six-way split across the six tiers to obtain 12.1 mills. Together, the opening and closing auction account for 25.5 billion shares traded and $64.6 million of revenue.

Last, we deduct routing revenue. Routing is prominently discussed in Nasdaq financial statements but they do not report any specific numbers. Since the Nasdaq routing business appears to be at least somewhat similar to the BATS routing business, we utilize the BATS net routing revenue per share number computed above (5.5 mills) and the BATS routed volume as a % of total volume (6.7%), to obtain net routing revenue of $12.0 million on 21.8 billion shares.

When we subtract auction revenue and volume, and subtract routing revenue, we obtain 302.2 billion regular-hours shares traded and $63.6 million of regular-hours net trading revenue, for $0.000211 per share and $0.000105 per-share per-side, as reported in Table 4.1. As a sensitivity analysis, we assume that we have overestimated auction revenues by 25%, for example, due to the monthly fee caps. This would change the figure to $0.000132 per-share per-side.

**NYSE** NYSE’s parent company, Intercontinental Exchange (ICE), reports in its 2015 10-K that NYSE’s U.S. cash equities revenues, net of transaction based expenses (i.e., rebates), were $220 million in 2015. The ICE 10-K reports average daily matched volume of 1,187M shares for Tape A, 296M shares for Tape B, and 206M for Tape C. Multiplied by 252 trading days this yields annual volume of 425.6 billion shares, which is close to the TAQ number. This yields revenue per share of $220M / 425.6 billion shares = $0.000517, or $0.000258 per-share per-side.

Next, we deduct auction revenue and volume. We get opening and closing auction volume for NYSE, NYSE Arca, and NYSE Mkt from the TAQ data. These volumes are significant for both NYSE and NYSE Arca, with 11.1 billion and 1.9 billion shares of volume for the open, and 48.4 billion and 9.7 billion shares of volume for the close, respectively. For the opening auction, we use a fee of 10 mills for NYSE and NYSE Arca and 15 mills for NYSE Mkt and 15 mills for NYSE Arca, based on their fee schedules. As with Nasdaq, there are some discounts (in particular for NYSE designated market makers) and monthly caps, which we do not attempt to account for here, but rather do so below in a sensitivity analysis. For the closing auction, NYSE has a range of fees from 6 mills to 10 mills depending on volume tier; we use an equal-weighted average of the tiers to obtain 7.7 mills. NYSE Arca’s closing auction fee is 10 mills and NYSE Mkt’s is 8.5 mills. Combined across these three venues and combining both the open and close, we obtain $123.3M of total auction revenue.

For routed volume, we utilize that the ICE 10-K reports both matched volume and handled volume; the
difference is what is routed. This comes to 10.8 billion shares annualized across the 3 tapes. We utilize the same 5.5 mills net routing fee number from BATS, lacking any better source. This comes to $5.9M of total routing revenue.

When we subtract auction revenue and volume, and subtract routing revenue, we obtain 353.5 billion regular-hours shares traded and $90.7 million of regular-hours net trading revenue, for $0.000257 per share and $0.000128 per-share per-side, as reported in the table. As a sensitivity analysis, we assume that we have overestimated auction revenues by 25%, for example, due to the monthly fee caps. This would change the figure to $0.000172 per-share per-side.
E Supporting Details for Evidence on Exchange-Specific Speed Technology Revenue (F)

In this appendix, we provide supporting details for the evidence presented in Section 4.3. Appendix E.1 provides supporting material for the magnitude of ESST revenue documented in Stylized Fact #6. Appendix E.2 provides supporting material for the magnitude of ESST revenue documented in Stylized Fact #7.

E.1 Details for Data and Co-Location Revenue Estimates for Nasdaq and NYSE

In this appendix, we provide supporting details for our calculations of market data and co-location/connectivity revenues for Nasdaq and NYSE in 2015, which we reported in Section 4.3.

Nasdaq’s fiscal year 2015 10-K reports market data and co-location/connectivity revenue only at the global level – $399M and $239M, respectively.\textsuperscript{15} To get from global to the U.S., for market data, we utilize information in its 2013 10-K filing that breaks out its market data business geographically: U.S. is 72% of the total in 2013, and we assume this ratio holds in 2015. For co-location/connectivity, we use Nasdaq’s overall 2015 U.S.:global revenue ratio, of 71%. Last, we need to separate out Nasdaq’s U.S. Equities business from its U.S. Options business. We take two approaches. First, we assume that Nasdaq’s market data and co-location revenue from U.S. Equities vs. U.S. Options is proportional to its trading volume in U.S. Equities vs. U.S. Options. Second, we assume that Nasdaq’s U.S. Options business generates the same market data and co-location revenue as BATS’s U.S. Options business, scaled up for Nasdaq’s larger U.S. Options volume than BATS. The first approach assumes that every 1 option traded on Nasdaq generates the same market data and co-location revenue as 100 shares of stock; the second approach assumes that 1 option traded on Nasdaq generates the same market data and co-location revenue as 1 option traded on BATS. These two approaches yield a range for Nasdaq’s U.S. Equities revenue of $222.4M-$267.3M for Market Data, $121.0M-$139.0M for Co-Location/Connectivity, and $343.3M-$406.4M combined.

NYSE was acquired by ICE, a large futures exchange conglomerate, in Nov 2013. ICE’s 2014 10-K filing therefore gives significant detail on the contribution of the NYSE business to the overall ICE business, for 2014, the first full year of integration (and also for the Nov-Dec 2013 period). The filing reports that NYSE’s U.S. businesses (not including Euronext, which ICE divested) contributed $430M to its data services business in 2014; this includes both market data and co-location/connectivity, for both U.S. equities and U.S. options. The filing also reports that $202M of this was for co-location/connectivity, implying $228M for market data. ICE’s 2015 10-K filing reports that it reclassified an additional $60M of revenue, for 2014, from its “other” category to its data services business, and that this revenue corresponds to “NYSE connectivity fees and colocation service revenues”\textsuperscript{16} Therefore the adjusted 2014 totals are $262M for co-location/connectivity and $228M for market data. Comparison of ICE’s 2014 and 2015 10-K filings suggest a growth rate of its overall data services business from 2014 to 2015, of which the NYSE business was by far the largest component, of 12.3%.\textsuperscript{17} For comparison,

\textsuperscript{15} The $239M for global co-location/connectivity also contains revenues from a small Nordic region Broker Services business, which when last reported separately was $19M; we subtract out this $19M from Nasdaq’s global “Access and Broker Services” business in the analysis that follows.

\textsuperscript{16} It is hard to know but we guess that this adjustment reflects post-merger alignment of accounting practices between NYSE and ICE, that in principle should have been reflected in the 2014 10-K but that was not completed until the 2015 10-K.

\textsuperscript{17} This growth figure accounts for several other ICE acquisitions in this time period. 2014 ICE data services revenue was $691M but includes just 12 weeks of the SuperDerivatives business, which contributed $12M in those 12 weeks; therefore 2014 revenue pro forma for the SuperDerivatives business was $731M. 2015 data services revenue was $871M.
BATS’s U.S. equities growth rate for the 2014 to 2015 period was 19.2% for co-location/connectivity and 12.4% for market data,\textsuperscript{18} which suggests that the 12.3% growth rate computed from ICE data is reasonable for NYSE. We use this growth rate to obtain estimates for 2015 for NYSE’s overall U.S. business, and then utilize the same two methods described above for Nasdaq to obtain estimates for NYSE’s U.S. Equities business. This yields a range of $218.9-$241.5M for U.S. equities market data, $251.6-$281.5M for U.S. equities co-lo/connectivity, and $470.5-$523.0M combined.

E.2 Details for Data and Co-Location Revenue Growth Estimates for Nasdaq and BATS

In this appendix we provide supporting details for the data on Nasdaq and BATS exchange-specific speed technology (ESST) revenue growth discussed in Stylized Fact #7. As discussed in the main text, our goal is to get a sense of magnitudes for ESST growth over time by looking at revenue growth in the financial reporting categories that contain U.S. equities ESST revenues.

From 2006 to 2012, Nasdaq reported co-location/connectivity revenues in the category “Access services revenues.” In 2013, Nasdaq changed the reporting category to “Access and Broker Services Revenues,” which incorporated Nasdaq’s small Nordic broker services business ($19M in 2012) into the category. In 2015, Nasdaq appears to have adjusted its accounting practices to reclassify some revenue in “Access and Broker Services Revenues” to “Technology Solutions,” which led to a downward revision of $18M for the 2014 revenue figures based on the 2015 reporting method as reported in Nasdaq’s fiscal year 2015 10-K, so it is of a similar magnitude as the upward revision in 2013. In 2016, Nasdaq changed the reporting category again to “Trade Management Services Revenues,” but this appears to be a change in the category name only with no revision to revenue figures reported in previous 10-Ks. As noted in the main text, we view the periods 2006-2012 and 2015-2017 as yielding reliable apples-to-apples growth rates, and the period 2012-2015 seems relatively flat with the caveat that there were multiple reporting changes.

From 2006 to 2012, Nasdaq reported its U.S. equities proprietary market data revenue in the category “U.S. market data products.” Over this period, Nasdaq also separately reports tape revenues as “Net U.S. tape plans.” Starting in 2013, Nasdaq reports only combined U.S. data revenue (i.e., including both proprietary and tape) in the segment “U.S. market data products,” and also made some segment reporting changes, moving $27M of revenue from “Index data products” out of U.S. market data products into its own reporting category. Starting in 2014, Nasdaq reports only total market data revenue instead of separating out U.S., international, and index market data separately. To get a roughly apples-to-apples time series, we make the following two adjustments. First, from 2014 onwards, we use the 2013 ratio of U.S. to total market data revenue (72%) to get a U.S. market data revenue estimate. Second, from 2013 onwards, we subtract out 2012 tape revenue of $117M to get to U.S. market data revenue excluding tape plan revenue.\textsuperscript{19} These two assumptions together imply that any revenue

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\textsuperscript{18}BATS’s 2014 numbers include just 11 months of Direct Edge revenue versus 12 months in 2015. If we conservatively assume that the Direct Edge business is 50% of BATS’s overall business, then we can take the unadjusted 2014-to-2015 growth rates, of 23.7% for co-location/connectivity and 16.9% for market data, and reduce them by 50% \cdot \frac{11}{12} \approx 4.5 \text{ percentage points}, to obtain 2014 to 2015 growth rates that are apples-to-apples.

\textsuperscript{19}We feel comfortable treating tape revenues as flat since 2012 since tape revenues are based on depth and market shares, and we show in Stylized Fact #3 that market shares are roughly flat and show in Stylized Fact #2 that depth shares line up one-for-one with market shares. Nasdaq’s market shares by share volume from 2011 to 2017 were: 29.1%, 29.5%, 28.9%, 31.3%, 28.0%, 26.0%, 28.0%. Moreover, in the last three years that Nasdaq did report tape revenues but includes $50M of revenues from 2015 acquisitions of Interactive Data and Trayport; therefore a like-for-like 2015 revenue number is $821M, or 12.3% more than the adjusted 2014 figure.
growth in Nasdaq’s total market data category since 2014 is attributed 72% to Nasdaq’s U.S. proprietary market
data segment with the remaining 28% to international and index market data revenues. Our sense is that this
convention is conservative since Nasdaq reports that both international market data revenue and index data
revenue were relatively flat in the years leading up to 2013 (International: $83M in 2011 to $77M in 2013; Index:
$24M in 2011 to $27M in 2013).

We use four data sources for BATS co-location/connectivity revenue: BATS’s 2012 S-1 statement (revenue
from 2009-2011), BATS’s 2016 S-1 statement (revenue from 2010-2015), the CBOE/BATS 2016 proxy statement
(revenue for 9 months of 2016, which we annualize), and CBOE’s 2017 annual report (which reports BATS’s
contribution to CBOE revenues for 10 months, which we annualize; CBOE’s acquisition of BATS was finalized on
Feb 28 2017). BATS states in its 2012 S-1 that it began charging for co-location/connectivity revenue, described
as “port fees,” in Q4 2009 (pg. 64) so we report numbers starting in 2010. Before 2012, the reporting segment
was called “Other Revenues” in BATS’s 2012 S-1; BATS describes the category by stating “Other revenues
consist of port fees, which represent fees paid for connectivity to our markets, and, more recently, additional
value-added products revenues.” The reporting segment changed to “Port Fees and Other” in BATS’s 2016 S-1;
the revenue reported for 2011 in BATS’s 2016 S-1 is within 1% of the 2011 revenue reported in BATS’s 2012
S-1 so we conclude that the change was almost entirely a renaming of the reporting category rather than a
substantive change. The reporting segment changed again to “Connectivity Fees and Other” for 2016 in the
CBOE/BATS proxy statement, which was a change in name only (revenue reported from previous years are
consistent with the 2016 S-1). In 2017, as a part of CBOE, BATS’s revenue from co-location/connectivity is split
across two CBOE segments, “Access fees” and “Exchange services and other fees.” CBOE separately reports
BATS’s contributions to these categories and we report their sum, annualized to twelve months.

BATS reports in its 2016 S-1 that its two original BATS exchanges, BZY and BYX, only began charging
for proprietary market data in Q3 2014 (pg. 94). We thus use numbers starting in 2015 (this is also the first
full year that revenue associated with Direct Edge is included in BATS’s filings). BATS market data revenue
for U.S. equities is included in the category “Market Data Fees” in BATS’s 2016 S-1 (2015 revenue) and in the
2016 CBOE/BATS proxy statement (9 months of revenue from 2016, which we annualize). BATS’s market data
revenue for 2017 comes from CBOE’s 2017 annual report, which provides BATS’s contribution to the category
“Market Data Fees” for 10 months in 2017 (which we annualize). We know that tape revenue is a significant
fraction of market data revenue, and utilize percentages provided in BATS’s filings to estimate tape revenue,
which we can subtract from overall market data revenue as reported in BATS’s filings. BATS reports in its 2016
S-1 that 84% of market data revenue in 2015 comes from tape revenue (pg. 21), and reports in the CBOE/BATS
merger proxy that 79% comes from tape revenue in 2016 (pg. 56). Using these percentages, we estimate that
2015 tape revenue is $110M and 2016 tape revenue is $116M. We also assume that 2017 tape revenue is flat
from 2016 levels. If we subtract these tape revenue estimates from the overall market data revenue reported
in BATS’s filings, we get $21.0M in 2015, $30.8M in 2016 and $38.3M in 2017 (growth of 35.3% per year).
These numbers include data revenues related to BATS’S European equities and U.S. options business, so they
overstate the level but likely understate the growth rate of BATS’S U.S. proprietary market data since 2015.

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20BATS’s combined market share for its four exchanges by share volume declined slightly from 36.3% in 2016 to 34.5%
in 2017, so if anything 2017 tape revenues would be slightly smaller than 2016.

21The CBOE/BATS proxy statement reports on pg. 311-312 that, of the growth in market data revenue of $10.7M
for the first 9 months of 2016 versus the same period in 2015, $7.4M came from U.S. proprietary market data (“pricing
changes in proprietary market data that were implemented in the third quarter of 2015 and the first quarter of 2016”) versus
$0.7M from U.S. options (pg. 312) and $0.7M from European equities (pg. 318). Unfortunately, we are not able

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F Discussion of Discrete vs. Continuous with Tick-Size Constraints and Agency Frictions

In this Appendix we discuss competition between a Discrete exchange and one or more Continuous exchanges when there are tick-size constraints and agency frictions.

For the purpose of this discussion we make the following assumptions:

- **Tick-size constraints.** We assume that stocks trade in increments of $\text{ticksize} = \$0.01$. This reflects tick-size regulations in U.S. stock markets for stocks with a nominal share price greater than $\$1$ (Reg NMS Rule 612).

- **Agency frictions.** We assume that whenever there is liquidity at the same quoted price on Continuous and Discrete, investors always break ties in favor of Continuous. This tie-breaking in favor of Continuous is independent of any fee differences (discussed below). Investors route orders to Discrete only if Discrete has a quoted price that is at least a full tick better for the investor, i.e., when they are mandated to do so under the Reg NMS order protection rule. This assumption is meant to capture, in a simple and worst-case way, any agency frictions that might favor trading on Continuous markets over Discrete markets (in the spirit of Battalio, Corwin and Jennings, 2016).

- **Maker-Taker fees.** We assume that the Continuous market uses a maker-taker fee schedule with a take fee equal to the regulatory maximum under the Reg NMS Access Rule of $+30$ mills ($0.0030$) per share, and a make fee equal to $-30$ mills per share (i.e., make rebate of 30 mills), for a net fee of 0. This approximates current practice as discussed in Section 4.2. Additionally, in conjunction with the tie-breaking assumption in the previous bullet point, this assumption about fees is a worst-case for Discrete. Since we have assumed that investors only route to Discrete when liquidity is a full tick more attractively priced, Discrete would like to charge investors a higher fee than they pay on Continuous — investors would be willing to pay a higher fee conditional on trading, since whenever they trade on Discrete they are saving a full tick. However, the Continuous market is already charging investors the regulatory maximum. Therefore, if Discrete is to charge a positive fee, it does so by charging a take fee of 30 mills, just as on Continuous, and a make fee of $-(30 - f_D)$ mills (i.e., rebate of $30 - f_D$ mills), where $f_D$ denotes the net fee to Discrete per share.

- **Distribution of fundamental values.** For the purpose of discussion, we assume that the fundamental value $y$ of the security is uniformly distributed such that all values between any two relevant ticks are equally likely.

- **Magnitude of latency arbitrage.** For the purpose of discussion, we utilize the estimate from Aquilina, Budish and O’Neill (2020) that the latency arbitrage tax on liquidity is 0.42 basis points (0.0042%) of traded volume.$^{22}$

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$^{22}$As shown in Section 5.5 of Aquilina, Budish and O’Neill (2020), for the purpose of computing the amount by which eliminating latency arbitrage would reduce the cost of liquidity, it is technically more accurate to use latency-arbitrage profits as a proportion of non-race traded volume as opposed to as a proportion of all traded volume. This non-race volume latency arbitrage tax is 0.53 basis points as opposed to 0.42 basis points. To err on the side of conservatism we use the 0.42 basis points figure.
Given our assumptions about tick-sizes and fees, if there were a single Discrete exchange operating in isolation, the equilibrium best offer would be 
\[
\left\lceil y + \frac{s^*_{\text{discrete}} \cdot \text{discrete}}{2} - (0.0030 - f_D) \right\rceil ,
\]
where the notation \( \left\lceil x \right\rceil \) denotes rounding up to the nearest whole-penny increment. (For simplicity, we focus discussion on just the offer, the bid being symmetric.) If there were a single Continuous exchange operating in isolation, the equilibrium best offer would be 
\[
\left\lceil y + \frac{s^*_{\text{continuous}} \cdot \text{continuous}}{2} - (0.0030) \right\rceil .
\]
TFs would only be able to offer liquidity on Discrete at a strictly better whole-penny increment than on continuous if

\[
\left\lceil y + \frac{s^*_{\text{discrete}} \cdot \text{discrete}}{2} - (0.0030 - f_D) \right\rceil < \left\lceil y + \frac{s^*_{\text{continuous}} \cdot \text{continuous}}{2} - (0.0030) \right\rceil ,
\]
that is, if the magnitude of latency arbitrage, represented by \(\frac{s^*_{\text{continuous}} - s^*_{\text{discrete}} \cdot \text{discrete}}{2} - f_D\), is large enough to “cross a tick” given the current fundamental value \(y\), and accounting for any difference in fees and rebates.

Given the assumption that fundamental values are uniformly distributed between any two relevant ticks, the probability that the fundamental value \(y\) satisfies condition (F.1) is:

\[
\frac{s^*_{\text{continuous}} - s^*_{\text{discrete}} \cdot \text{discrete}}{\text{ticksize}} - f_D,
\]
truncated below at 0 (in case \(f_D\) is too large) and above at 1 (in case \(\frac{s^*_{\text{continuous}} - s^*_{\text{discrete}} \cdot \text{discrete}}{2} - f_D\) exceeds the tick size). For example, if \(\frac{s^*_{\text{continuous}} - s^*_{\text{discrete}} \cdot \text{discrete}}{2} - f_D = 0.0020\), then the probability that \(y\) is such that condition (F.1) holds is 20%. Our assumption about the magnitude of latency arbitrage enables us to compute Discrete’s market share for symbol \(i\) from (F.2) as:

\[
FBA_{\text{share}}_i = \frac{(0.0042\% \cdot \text{shareprice}_i - f_D)}{\$0.01}
\]
again truncating below at 0 and above at 1.

We use TAQ data in 2015 to compute (F.3) for all symbols with \(\text{shareprice}_i > \$5\) that traded continuously throughout the year under the same ticker, with \(\text{shareprice}_i\) calculated as the volume-weighted average trade price for symbol \(i\) and net fees per share \(f_D\) ranging from 0 mills to 30 mills. A net fee \(f_D\) of 0 mills corresponds to a take fee of +30 mills and a make fee of -30 mills (i.e., a rebate) as on the Continuous market, whereas a net fee \(f_D\) of 30 mills corresponds to a take fee of +30 mills and a make fee of 0. We then use \(FBA_{\text{share}}_i\) to compute overall FBA market shares by share volume and dollar volume (relative to the symbol universe). We also use \(FBA_{\text{share}}_i\) to compute annual FBA revenue, which is \(FBA_{\text{share}}_i\) multiplied by \(f_D\) and by overall share volume for symbol \(i\), then summed over all symbols. The results are summarized in Table F.1.

At a net fee of \(f_D = 0\), i.e., the same maker-taker fee structure as the Continuous market, the Discrete exchange’s share is computed as 18.7% in share volume and 37.2% in dollar volume. The reason why dollar volume is meaningfully higher than share volume is that tick-size constraints are less binding for high nominal share price stocks.

At a net fee of \(f_D = 10\) mills, the Discrete exchange’s share is computed as 10.4% in share volume and 27.9% in dollar volume. Fee revenues are $92.1 million per year. Fee revenues are maximized at about $106 million per year, using a net fee of about 20 mills.

Clearly, this exercise is very back-of-the-envelope. In particular, it utilizes a magnitude for latency arbitrage taken from UK equity markets, whereas if better data were available in U.S. equity markets it would be possible to directly calculate both the average level and the cross-sectional heterogeneity across stocks and ETFs.
Table F.1: FBA Market Share and Revenue with Tick-Size Constraints and Agency Frictions

<table>
<thead>
<tr>
<th>FBA Net Fee Per Share (Mills) (*)</th>
<th>FBA Share (% of Share Volume)</th>
<th>FBA Share (% of Dollar Volume)</th>
<th>FBA Annual Revenue ($ Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18.7%</td>
<td>37.2%</td>
<td>0.9</td>
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<td>1</td>
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<td>15.7</td>
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<td>35.2%</td>
<td>29.6</td>
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<tr>
<td>3</td>
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<td>34.3%</td>
<td>41.8</td>
</tr>
<tr>
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<td>14.8%</td>
<td>33.3%</td>
<td>52.5</td>
</tr>
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<td>32.4%</td>
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<td>13.2%</td>
<td>31.4%</td>
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<td>30.5%</td>
<td>76.8</td>
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<td>29.6%</td>
<td>82.8</td>
</tr>
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<td>11.0%</td>
<td>28.8%</td>
<td>87.8</td>
</tr>
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<td>10.4%</td>
<td>27.9%</td>
<td>92.1</td>
</tr>
<tr>
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<td>7.8%</td>
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<td>17.6%</td>
<td>103.0</td>
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<tr>
<td>30</td>
<td>3.7%</td>
<td>15.2%</td>
<td>97.2</td>
</tr>
</tbody>
</table>

(*) FBA Net Fee = Take Fee + Make Fee. As discussed in the text, we hold fixed the Take Fee at 30 mills ($0.0030) per share, and vary the Make Fee from -30 mills (i.e., rebate of $0.0030 per share) to 0.

**Notes:** Data from NYSE TAQ in 2015. The symbol universe is all stocks with \( \text{shareprice}_i \geq 5 \) that traded continuously throughout the year under the same ticker. We first calculate \( \text{FBAshare}_i \) for each symbol \( i \) according to equation (F.3), with \( \text{shareprice}_i \) calculated as the volume-weighted average trade price of the symbol over all trading days in 2015, and \( f_D \) set to the values in the “FBA Net Fee Per Share” column. We then use the FBA market shares for each symbol to compute overall FBA market shares by share volume and dollar volume, with overall market shares expressed relative to the symbol universe. FBA annual revenue is computed as overall FBA share volume times the net fee \( f_D \).

of latency arbitrage in the U.S.. It also incorporates complicated market frictions in a very stylized manner. Nevertheless, we hope that this exercise provides the reader with a useful sense of magnitudes both for interpreting the Discrete vs. Continuous theoretical results in Section 5.2 and for thinking about the market design exclusivity period idea in Section 6.2.
References


