

FOUNDATIONS
OF
FINANCE

*PORTFOLIO DECISIONS
AND
SECURITIES PRICES*

Eugene F. Fama

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Preface

Among the various fields of economics, finance is somewhat unique in terms of the correspondence between theory and evidence. The purpose of this book is to introduce the theory of finance and the empirical tests of the theory. I concentrate on that part of finance which is concerned with portfolio decisions by investors and the pricing of securities in capital markets.

My view is that the student's motivation to master a theory is enhanced when evidence is presented to show that the theory has some power to explain real world phenomena. Moreover, my classroom experience is that pointless squabbles about the realism of a theory or the assumptions from which it is drawn can be avoided if relevant empirical evidence is presented along with the theory. This is the approach taken in this book.

The first four chapters of the book provide the background statistical material. The goals are (a) to review the statistical tools that are necessary for any nonsuperficial study of finance and (b) to familiarize the reader with the descriptive evidence on the behavior of securities prices that forms the empirical foundation for the theory of finance and the formal tests of that theory. The approach in these chapters is to introduce statistical concepts first and then to use them to describe the behavior of returns on securities. Thus, Chapter 1 studies probability distributions and the properties of samples and then uses the concepts to examine distributions of common stock returns. Chapters 2 and 3 take up the statistical tools that are needed to study the relationships between returns on securities and portfolios. To motivate the study of these tools, some of the rudiments of portfolio theory are introduced in Chapter 2. Chapter 4 uses the statistical concepts presented in Chapters 2 and 3 to study empirically the "market sensitivity" of New York

Stock Exchange common stocks, examining evidence on the extent to which the returns on individual securities are related to market returns.

The core of the book is in Chapters 5 to 9. Three related topics are treated: (a) theory and evidence on capital market efficiency, (b) portfolio theory, and (c) theory and evidence on the relationship between expected return and risk. In an efficient capital market, prices of securities "fully reflect" available information. Chapters 5 and 6 discuss theory and empirical work on capital market efficiency; the former is concerned with the stock market, the latter with the bond market. Chapter 7 develops in detail the portfolio model introduced in Chapter 2 and presents empirical evidence on the effects of diversification in reducing risk. Chapter 8 then considers the characteristics of equilibrium security prices when investors make portfolio decisions according to the model of Chapter 7. The relationship between expected return and risk that comes out of the model of capital market equilibrium in Chapter 8 is put to the test in Chapter 9.

Problems for the reader are scattered through the text. The word "scattered" is used advisedly. The problems are not tucked neatly at the ends of sections, but rather appear whenever I want to reinforce a point or give pause for thought. The problems are an integral part of the text; results contained in them are often referred to in later parts of the text. In light of this, fairness and convenience argue that answers to problems follow the problems in the text. This raises the temptation—and thus the probability—that the problems will not be treated as such, but it is in the student's interest to resist this temptation. The problems allow the reader to keep tabs on his understanding of the material and so to avoid unwarranted euphoria.

The technical prerequisites for reading this book are minimal. Mathematics beyond elementary algebra appears only briefly in two chapters and is not critical to understanding the important material in either chapter. Moreover, I try always to supplement even elementary mathematical arguments with verbal discussions; in cases where the details of a mathematical argument can be skipped, this is so indicated. The book is, however, heavy with formal notation, and the reader is well advised to master the notation as quickly as possible.

Although finance is properly regarded as a branch of economics, the ambitious reader could understand this book without previous formal exposure to economics. Financial economics is, however, easier to grasp if one has some familiarity with habits of economic analysis. Thus, although no specific material is needed, some prior exposure to economics is helpful. Likewise, the book reviews the statistical concepts that it uses, but the presentation is more effective if the reader has had some previous exposure to statistics, though

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not necessarily to the specific statistical concepts that are most useful in finance.

This book is meant to be an introduction to finance, with approximately equal emphasis on theory and evidence. As with any introduction, some picking and choosing of topics is necessary. I have chosen to focus on topics where there is sufficient empirical evidence to draw coherent conclusions about the descriptive power of a theory. I do not claim to cover all the topics that meet this criterion, and one can argue that my choices reflect much personal prejudice. The goal of the book is met, however, if I familiarize the reader with the common methods of analysis in finance sufficiently to tackle original works, both those already available and those yet to come, on his own.

Finally, I am pleased to acknowledge the help of Linda Huegel, who typed several versions of the manuscript; proofreading was provided by Agnes Farris, Vicky Longawa, and Jane Miller. Nicholas Gonedes and Harry Roberts, my colleagues at the University of Chicago, made many valuable comments on the manuscript. My debt to the pioneers of modern finance, who did the original work on which this book is based, is obvious.