15.1 Stationary vs. Non-stationary time series.

- One important aspect of a time series model is whether it is mean reverting.
- That is, can we expect that over the long run the process will tend to an average value?
- Stationary processes will tend to revert to their mean value and a non-stationary process will not.
- Obviously, the models have very different implications about the long run behavior of a process.
Here are 4 simulations from the model AR(1) model with $\beta_1=1$. Each series begins at the value 5 and has 100 observations. $Y_t = 0 + Y_{t-1} + \epsilon_t$

- Sometimes the series wander up, sometimes down, sometimes up then down.

- Where do you think the process will be in period 200?

- There is no force driving the series back to a “mean” value.
Here are 4 simulations from the AR(1) model with $\beta=.5$. Each series begins at the value 5.

\[ Y_t = 2 + .5Y_{t-1} + \varepsilon_t \]

- Contrary to the case when $\beta_1=1$, when $\beta_1=.5$ the series is attracted back to its mean value of

\[ \mu = \frac{\beta_0}{1-\beta_1} = \frac{2}{1-.5} = 5 \]

- Clearly, in period 200, we would expect the process to be somewhere around 5. A little above, or a little below.
• Let’s look more closely at where we should expect an AR(1) model to be in the future and how sure we are.

• To keep the notation simple, our discussion will use time 0 as the point that we start looking at future values of $Y$.

• Later I’ll show you the formula for the general case where we forecast out $k$ periods into the future, starting at an arbitrary time $t$.

\[
Y_1 = \beta_0 + \beta_1 Y_0 + \varepsilon_1 \\
Y_2 = \beta_0 + \beta_1 Y_1 + \varepsilon_2 \\
= \beta_0 + \beta_1 \left( \beta_0 + \beta_1 Y_0 + \varepsilon_1 \right) + \varepsilon_2 \\
= \beta_1^2 Y_0 + (1 + \beta_1) \beta_0 + \beta_1 \varepsilon_1 + \varepsilon_2 \\
Y_3 = \beta_0 + \beta_1 Y_3 + \varepsilon_3 \\
= \beta_0 + \beta_1 \left( (1 + \beta_1) \beta_0 + \beta_1^2 Y_0 + \beta_1 \varepsilon_1 + \varepsilon_2 \right) + \varepsilon_3 \\
= \beta_1^2 Y_0 + (1 + \beta_1 + \beta_1^2) \beta_0 + \beta_1^2 \varepsilon_1 + \beta_1 \varepsilon_2 + \varepsilon_3
\]
\[ Y_t = \beta_1' Y_0 + \frac{1 - \beta_1'}{1 - \beta_1} \beta_0 + \sum_{j=0}^{t-1} \beta_1' \varepsilon_{t-j} \]

This stuff is known at time 0

\[ = \beta_1' Y_0 + \left( 1 - \beta_1' \right) \mu + \sum_{j=0}^{t-1} \beta_1' \varepsilon_{t-j} \]

\[ \mu = \frac{\beta_0}{1 - \beta_1} \]

(The last line comes from \( \mu = \frac{\beta_0}{1 - \beta_1} \))

• This is a useful express that we can use to understand forecasts.
Let’s find the $t$-step ahead forecast that we make at time zero.

$$E(Y_t | Y_0) = E \left( \beta_1' Y_0 + (1 - \beta_1') \mu + \sum_{j=0}^{t-1} \beta_1^j \varepsilon_{t-j} | Y_0 \right)$$

$$= \beta_1' Y_0 + (1 - \beta_1') \mu + E \left( \sum_{j=0}^{t-1} \beta_1^j \varepsilon_{t-j} | Y_0 \right)$$

$$= \beta_1' Y_0 + (1 - \beta_1') \mu + \sum_{j=0}^{t-1} \beta_1^j E(\varepsilon_{t-j} | Y_0)$$

$$= \beta_1' Y_0 + (1 - \beta_1') \mu$$

• This expression tells us how to forecast out multiple periods given an initial value of $Y=Y_0$.
  
  $$\beta_1' Y_0 + (1 - \beta_1') \mu$$

• Clearly there is nothing special about starting the forecast at time period 0. Here’s what the formula looks like for a $k$-period ahead forecast made at an arbitrary time $t$. Let $Y_t^k = E(Y_{t+k} | Y_t)$ denote the forecast of $Y_{t+k}$ given $Y_t$. Then

$$Y_t^k = \beta_1^k Y_t + (1 - \beta_1^k) \mu$$
Case 1: $|\beta_1|<1$

If $|\beta_1|<1$ then the $k$-period ahead forecast is given by:

$$Y_t^{\hat{k}} = \beta_1^k Y_t + \left(1 - \beta_1^k\right) \mu$$

- This makes the nature of the forecast perfectly clear. It says that the forecast is a weighted average of the last value $Y_t$ and the mean of $Y$.

- The further out we forecast the more weight we put on $\mu$ and the less weight on $Y_t$.

Now let's see what happens when $\beta_1=1$

- From before, we have:

$$Y_t = \beta_1^t Y_0 + \left(1 + \beta_1 + \cdots + \beta_1^{t-1}\right) \beta_0 + \beta_1^{t-1} \epsilon_1 + \cdots + \beta_1 \epsilon_{t-1} + \epsilon_t$$

so with $\beta_1=1$ we get:

$$Y_t = Y_0 + (t) \beta_0 + \epsilon_1 + \cdots + \epsilon_{t-1} + \epsilon_t$$

$$= Y_0 + \beta_0 t + \left(\text{known at time 0}\right) \sum_{j=0}^{t-1} \epsilon_j$$

Intervening (future) values of $\epsilon$
So, now the forecast is given by:

\[
E(Y_t | Y_0) = E\left(Y_0 + \beta_0 t + \sum_{j=0}^{t-1} \varepsilon_j \right)
\]

\[
= Y_0 + \beta_0 t + \sum_{j=0}^{t-1} E(\varepsilon_j | Y_0)
\]

\[
= Y_0 + \beta_0 t
\]

More generally, when $|\beta_1|=1$, for an arbitrary starting point $t$, and forecast horizon $k$

\[
Y_t^k = Y_t + k \beta_0
\]

- So as $k$ gets large here, the forecast doesn’t converge to a “mean” $\mu$.
- If $\beta_0<0$ the forecast becomes very negative.
- If $\beta_0>0$ the forecast becomes very large.
- If $\beta_0=0$ the forecast is simply $Y_t$. 
15.2 Forecast errors.

- How big will our forecast errors be?
- To answer this question we want to know the difference between the actual value of $Y_{t+k}$ and the forecasted value.
- We again use $Y_t^k = E(Y_{t+k} | Y_t)$ denote the forecast of $Y_{t+k}$ given $Y_t$.
- Using this notation, we are interested in the error associated with the $k$-step ahead forecast error $e_t^k = (Y_{t+k} - Y_t^k)$

- In this section we find the variance of for $e_t^k$.
- Just like in the one-step ahead forecast, this tells us about our uncertainty.
• Let’s start with case 1 i.e. $|\beta_t|<1$.

• We can write $Y_{t+k}$ as

$$Y_{t+k} = \beta_1^k Y_t + (1 - \beta_1^k) \mu + \sum_{j=0}^{k-1} \beta_1^j \varepsilon_{t+k-j} = Y_t^f + \sum_{j=0}^{k-1} \beta_1^j \varepsilon_{t+k-j}$$

Again, this is (weighted) sum of intervening values of $\varepsilon$.

$$e_t^k = (Y_{t+k} - Y_t^f) = \sum_{j=0}^{k-1} \beta_1^j \varepsilon_{t+k-j}$$

• Let’s find the forecast error variance:

$$\text{Var}(e_t^k) = \text{Var}\left( \sum_{j=0}^{k-1} \beta_1^j \varepsilon_{t+k-j} \right)$$

but the $\varepsilon_t$ are iid. So…

$$\text{Var}(e_t^k) = \sum_{j=0}^{k-1} \beta_1^{2j} \text{Var}(\varepsilon_{t+k-j}) = \sigma^2 \sum_{j=0}^{k-1} \beta_1^{2j} = \frac{(1 - \beta_1^{2k})}{(1 - \beta_1^2)} \sigma^2$$

• For large $k$ this becomes

$$\text{Var}(e_t^k) = \frac{\sigma^2}{(1 - \beta_1^2)}$$
Does this make sense?

The size of the forecast error variance is determined by $\sigma^2$ and $\beta_1$ and the forecast horizon $k$.

- The size of the “surprise” in each period is determined by $\sigma^2$.
- The larger $\beta_1$ is, the larger the swings are that $Y_t$ can take away from the mean.
- The forecast error variance is increasing as we forecast further into the future.
- The forecast error variance converges to a fixed number as $t$ becomes large. The fixed number is just the variance of $Y$.

Case 2 i.e. $|\beta_1|=1$

- Here $Y_t = Y_t + k\beta_0 + \sum_{j=0}^{k-1} \epsilon_{t+j} = Y_t^k + \sum_{j=0}^{k-1} \epsilon_{t+j}$

  so $\epsilon_t^k = Y_{t+k} - Y_t^k = \sum_{j=0}^{k-1} \epsilon_{t+j}$

- The variance of $\epsilon_t^k$ is then:

  $$Var(\epsilon_t^k) = Var\left(\sum_{j=0}^{k-1} \epsilon_{t+j}\right) = k\sigma^2$$

  - This is completely different from the case where $|\beta|<1$.
  - The forecast error variance is proportional to the forecast horizon, $k$.
  - The variance only depends on $k$, not $t$. 


• Of course, in practice we don’t know the true parameters so we plug in our best guesses:
• The \( k \)-step ahead forecast when \(|\beta_1|<1\) is given by:
\[
\hat{Y}_t^k = b_1^k Y_t + \frac{(1-b_1^k)b_0}{(1-b_1)} = b_1^k Y_t + (1-b_1^k)\bar{y}
\]
where \( \bar{y} \) is the sample average of \( Y \).

• The best guess for the \( k \)-step ahead forecast error variance when \(|\beta_1|<1\) is given by:
\[
E(Y_{t+k} - \hat{Y}_t^k)^2 \approx \frac{(1-b_1^{2k})}{(1-b_1^2)} s^2
\]

• Hence when \(|\beta_1|<1\) the 95% prediction interval for \( Y_{t+k} \) is given by:
\[
\hat{Y}_t^k \pm 2 \sqrt{\frac{(1-b_1^{2k})}{(1-b_1^2)} s^2}
\]
• For the case where $\beta_1 = 1$ we have

$$Var(e_i^k) = Var\left(\sum_{j=0}^{k-1} \varepsilon_{t+j}\right) = k\sigma^2$$

• So the 95% prediction interval for the k step ahead forecast is given by:

$$Y_i \pm 2\sqrt{k\sigma^2}$$

(remember that $Y_i^k = Y_i$)

---

**Forecast for $\beta_1 = .8$ and $\beta_0 = 2.5$ $\sigma = 1$**
Forecast for $\beta_1=1$ and $\beta_0=0$, $\sigma=1$

Stationary vs. non-stationary summary

|                      | Stationary models $|\beta|<1$ | Non-stationary models $|\beta|=1$ |
|----------------------|-----------------|----------------------------------|
| **Forecasts**        |                 |                                  |
| Mean revert          | $Y^k = \beta^k Y + (1 - \beta^k) \mu$ | $Y^k = Y_t + k \beta_0$ |
| **Forecast errors**  |                 |                                  |
| Initially increase   | $Var(e^k) = \frac{(1 - \beta^2)}{(1 - \beta^k)} \sigma^2$ | $Var(e^k) = k \sigma^2$ |
15.3 Trending series: the trend stationary models

- Sometimes time series can trend up or down, but not in the same way as a random walk.
- If a process is stationary after removing a trend then it is called a trend stationary process.
- Hence $Y_t$ is a trend stationary process if:

$$Y_t - \left( \beta_0 + \delta t \right) = \tilde{Y}_t$$

- Where $\tilde{Y}_t$ is a mean zero stationary process.

Hence the trend stationary model says that the deviations of $Y_t$ from the trend line have mean zero and, mean revert.
I drew this with $\delta > 0$, but $\delta$ could also be less than zero which means the series trends down.

Here’s a simulation from the model

$$Y_t = (2.5 + .5t) + \tilde{Y}_t$$
Here’s the trend line

\[ trend = (2.5 + .5t) \]

Here’s \((Y_t - \text{trend})\), a stationary AR(1)

\[ (Y_t - \text{trend}) = \tilde{Y}_t \]
Forecasting a trend stationary model

- The trend stationary model says that the deviations of $Y$ from a trend line follow an stationary model. Let $\tilde{Y}_t^k$ denote the $k$-period ahead forecast of $\tilde{Y}_t$

- If $\tilde{Y}_t$ follows an AR(1) we know the $k$-step ahead forecast is given by:

$$\tilde{Y}_t^k = \beta_1^k \tilde{Y}_t + \left(1 - \beta_1^k\right) \mu = \beta_1^k \tilde{Y}_t$$

($\mu=0$, since $\beta_1=0$ right?)
• $\tilde{Y}_t^k$ is the k-step ahead forecast of the deviation of $Y_t$ from the trend line. That is, it's the expectation of $Y_t$-trend.

• Hence, to get the forecast of $Y_t$ we simply add back in the trend line at time period $t+k$ to our forecast of the deviation $\tilde{Y}_t^k$.

$$E(Y_{t+k} | Y_t) = \underbrace{\tilde{Y}_t^k}_{\text{Expected deviation}} + \underbrace{\beta_0 + \delta(t+k)}_{\text{trend line}}$$

• For the trend stationary AR(1) model:

$$Y_t^k = \beta_1^k \tilde{Y}_t + \beta_0 + \delta(t+k)$$

• The forecast of a trend stationary model is composed of the forecast of the deviation of $Y$ from the trend plus the trend line.

• The long run forecast of $Y$ does not converge to $\mu$, but rather to the trend line (so this is a non-stationary model).

• Hence in the long run our forecast just reverts to the trend line!
15.4 What are the forecast errors?

\[ e^k_t = (Y_{t+k} - \hat{Y}_{t+k}) \]

\[ = \left( (\hat{Y}_{t+k} + \beta_0 + \delta(t+k)) - (\hat{Y}_t + \beta_0 + \delta(t+k)) \right) \]

\[ = \hat{Y}_{t+k} - \hat{Y}_t \]

- This is just the forecast error associated with the deviation of \( Y_{t+k} \) from the trend, i.e. the forecast error associated with the AR part! We already know what these look like:

\[ Var(e^k_t) = \frac{(1 - \beta_1^2k)}{(1 - \beta_1^2)} \sigma^2 \]

\[ Y_t - (2.5 + .5t) = \tilde{Y}_t \text{ where } \tilde{Y}_t = .8\tilde{Y}_{t-1} + \epsilon_t \]
Trend stationary summary

- Forecasts converge to the trend line.
- Forecast errors do not diverge, but behave like a stationary model with a variance that increases initially then reaches a fixed value.
- Like the random walk model, the trend stationary model does not mean revert to a fixed number (mean).
- Unlike the random walk model, the trend stationary model forecast errors do not increase without bound.

Estimation

- In a first step, estimate the trend line by running the regression:
  \[ Y_t = \beta_0 + \delta t + \epsilon_t \]

- Next, create the series \( \hat{Y}_t = Y_t - b_0 - dt \)
  where \( b_0 \) and \( d \) are the estimates of the intercept and the trend slope respectively.
- Next, model the de-trended series with an ARMA model.
15.5 Tests for a random walk

• We saw that there is a big difference between the properties of a stationary $|\beta_1|<1$ and non-stationary $\beta_1=1$ model.

• Sometimes it is difficult to tell the difference in a given sample between the two.

• One plot is $\beta_1=1$ and the other $\beta_1=.9$. 

![Graphs comparing plots for different values of beta]
• Clearly, we are interested in running the regression \( Y_t = \beta_0 + \beta_1 Y_{t-1} \) and testing the null that \( \beta_1 = 1 \).

• Why can’t we test the null of stationarity?

• The “t-stat” from this regression doesn’t follow a t-distribution.

• Neither the mean or variance of \( Y \) exists!

• Fortunately some smart guys in the 70’s and 80’s figured out how to do the test correctly.

• They figured out the correct p-values for the usual t-test.

• The critical value is not 2 and depends on the sample size. The actual distribution is called the Dickey-Fuller distribution.

• Fortunately, most software does this test for us.
Here is the test for series 1

Null Hypothesis: AR(1) has a unit root
Exogenous: None
Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

<table>
<thead>
<tr>
<th>Test</th>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-1.869970 0.0560</td>
<td></td>
</tr>
<tr>
<td>Test critical values: 5% level</td>
<td>-2.560330</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% level</td>
<td>-1.944105</td>
</tr>
<tr>
<td></td>
<td>10% level</td>
<td>-1.614590</td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(AR(0))
Method: Least Squares
Date: 07/14/07 Time: 15:46
Sample (adjusted): 2,100
Included observations: 99 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)(-1)</td>
<td>-0.076730</td>
<td>0.037889</td>
<td>-1.969702</td>
<td>0.0560</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.034063</td>
<td>0.017961</td>
<td>0.017961</td>
<td>0.0560</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.034930</td>
<td>0.003767</td>
<td>0.003767</td>
<td>0.0560</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>1.07507</td>
<td>0.017961</td>
<td>0.017961</td>
<td>0.0560</td>
</tr>
<tr>
<td>Durbin-Watson Stat</td>
<td>1.921742</td>
<td>2.349955</td>
<td>2.349955</td>
<td>0.0560</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-1.431262</td>
<td>2.229120</td>
<td>2.229120</td>
<td>0.0560</td>
</tr>
</tbody>
</table>

Since the p-value is small we can reject the null of $\beta_1=1$ at the 6% level

Unit root test for series 2

Null Hypothesis: AR(1) has a unit root
Exogenous: None
Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

<table>
<thead>
<tr>
<th>Test</th>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>0.198417 0.7418</td>
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</tr>
<tr>
<td>Test critical values: 5% level</td>
<td>-2.585330</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% level</td>
<td>-1.944195</td>
</tr>
<tr>
<td></td>
<td>10% level</td>
<td>-1.614590</td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(AR(1))
Method: Least Squares
Date: 07/14/07 Time: 15:50
Sample (adjusted): 2,100
Included observations: 99 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)(-1)</td>
<td>0.007032</td>
<td>0.038311</td>
<td>0.198417</td>
<td>0.7418</td>
</tr>
<tr>
<td>R-squared</td>
<td>-0.018988</td>
<td>0.017961</td>
<td>0.017961</td>
<td>0.0560</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.007032</td>
<td>0.038311</td>
<td>0.017961</td>
<td>0.0560</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.067777</td>
<td>0.017961</td>
<td>0.017961</td>
<td>0.0560</td>
</tr>
<tr>
<td>Durbin-Watson Stat</td>
<td>2.782358</td>
<td>2.888527</td>
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<td>0.0560</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-1.387267</td>
<td>2.050232</td>
<td>2.050232</td>
<td>0.0560</td>
</tr>
</tbody>
</table>

Since the p-value is large we don’t reject the null of $\beta_1=1$
15.6 Seasonal Models.

Many time-series data exhibit some sort of seasonality (e.g., the beer production data, January effect in stock returns).

Include indicator variables in the model to control for seasonality.

Example: Domestic Beer Production

Regression of beer production on month indicator variables:

```
MTB > regr 'Beer' 11 'Jan'-'Nov'
```

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>StDev</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>13.2417</td>
<td>0.2403</td>
<td>55.10</td>
<td>0.000</td>
</tr>
<tr>
<td>Jan</td>
<td>1.9117</td>
<td>0.3399</td>
<td>5.62</td>
<td>0.000</td>
</tr>
<tr>
<td>Feb</td>
<td>1.6933</td>
<td>0.3399</td>
<td>4.98</td>
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</tr>
<tr>
<td>Mar</td>
<td>3.9367</td>
<td>0.3399</td>
<td>11.58</td>
<td>0.000</td>
</tr>
<tr>
<td>Apr</td>
<td>3.9833</td>
<td>0.3399</td>
<td>11.72</td>
<td>0.000</td>
</tr>
<tr>
<td>May</td>
<td>5.0833</td>
<td>0.3399</td>
<td>14.96</td>
<td>0.000</td>
</tr>
<tr>
<td>Jun</td>
<td>5.1900</td>
<td>0.3399</td>
<td>15.27</td>
<td>0.000</td>
</tr>
<tr>
<td>Jul</td>
<td>4.9783</td>
<td>0.3399</td>
<td>14.65</td>
<td>0.000</td>
</tr>
<tr>
<td>Aug</td>
<td>4.5817</td>
<td>0.3399</td>
<td>13.48</td>
<td>0.000</td>
</tr>
<tr>
<td>Sep</td>
<td>2.0167</td>
<td>0.3399</td>
<td>5.93</td>
<td>0.000</td>
</tr>
<tr>
<td>Oct</td>
<td>1.9233</td>
<td>0.3399</td>
<td>5.66</td>
<td>0.000</td>
</tr>
<tr>
<td>Nov</td>
<td>0.1183</td>
<td>0.3399</td>
<td>0.35</td>
<td>0.729</td>
</tr>
</tbody>
</table>

\( S = 0.5887 \quad R-Sq = 92.0\% \quad R-Sq(adj) = 90.5\% \)

December is the excluded category.
MTB > let k1 = 2/sqrt(72)
MTB > print k1
K1    0.235702

MTB > acf 'RESID'

ACF of RESID

-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
+----+----+----+----+----+----+----+----+----+----+
1   0.112                          XXXX
2  -0.066                        XXX
3   0.062                          XXX
4  -0.223                    XXXXXXX
5  -0.218                     XXXXXX
6  -0.091                        XXX
7  -0.180                        XXX
8  -0.015                          X
9   0.323                          XXXXXXXXX
10  0.159                          XXXXX
11  0.095                          XXX
12  0.124                          XXX
13 -0.109                          XXX
14 -0.044                          XX
15 -0.174                      XXXXX
16 -0.297                   XXXXXXXX
17   0.026                          XX
18  0.090                          XXX

This acf looks a lot better than the one we got from the AR(1) model residuals.

MTB > regr 'Beer' 12 'Beer(t-1)' 'Jan'-'Nov'

Predictor       Coef StDev  T        P
Constant      11.724       1.770       6.62    0.000
Beer(t-1      0.1136      0.1312       0.87    0.390
Jan           1.9280      0.3612       5.34    0.000
Feb           1.4896      0.4163       3.58    0.001
Mar           3.7577      0.4008       9.37    0.000
Apr           3.5495      0.6075       5.84    0.000
May           3.6442      0.6125       5.58    0.000
Jun           4.6259      0.7365       6.28    0.000
Jul           4.4021      0.7489       5.88    0.000
Aug           4.0295      0.7244       5.56    0.000
Sep           1.5095      0.6790       2.22    0.030
Oct           1.7076      0.4243       4.02    0.000
Nov          -0.0868      0.4172      -0.21    0.836

S = 0.5949      R-Sq = 92.0%     R-Sq(adj) = 90.4%

Past production doesn’t seem to be significant once seasonality is controlled for.
The acf looks pretty much the same:

MTB > acf 'RESID2'

ACF of RESID2

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