The Fiscal Roots of Inflation

John H. Cochrane*

December 9, 2019

Abstract

Unexpected inflation devalues nominal government bonds. It must therefore correspond to a decline in expected future surpluses, or a rise in their discount rates, so that the real value of debt equals the present value of surpluses. I measure each component via a vector autoregression, in response to inflation, recession, monetary and fiscal policy shocks. Discount rates, rather than deficits, account for much inflation and deflation. Monetary policy smooths the inflationary response to fiscal shocks. I interpret the results through a fiscal theory of monetary policy.

*Hoover Institution, Stanford University and NBER. I thank seminar participants, reviewers, and Harald Uhlig for helpful comments. Data, code, and updates are at http://faculty.chicagobooth.edu/john.cochrane.
1. Introduction

This paper measures the fiscal roots of inflation. I start with a linearized version of the government debt flow identity,

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - s_{t+1}. \quad (1)$$

The log debt to GDP ratio at the end of period $t+1$, $v_{t+1}$, is equal to its value at the end of period $t$, $v_t$, increased by the log nominal return on the portfolio of government bonds $r_{t+1}^n$ less inflation $\pi_{t+1}$, less log GDP growth $g_{t+1}$, and less the primary surplus. The parameter $\rho = e^{-(r-g)}$ is a constant describing the linearization point. I derive this identity in the Appendix.

Iterating forward, we have a present value identity,

$$v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j}^n - \pi_{t+j} - g_{t+j} \right). \quad (2)$$

The log value of government debt, divided by GDP, is the present value of future surpluses, discounted at the ex-post real return, adjusted by GDP growth.

Taking time $t+1$ innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ and rearranging, we have the unexpected inflation identity,

$$\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} \left( r_{t+1}^n - g_{t+1} \right) = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left( r_{t+1+j}^n - \pi_{t+1+j} - g_{t+1+j} \right). \quad (3)$$

A decline in the present value of surpluses, coming either from a decline in surpluses or a rise in their discount rates, must correspond to a lower real value of the debt. This reduction can come about by unexpected inflation, or by a decline in nominal long-term bond prices.

I use a vector autoregression (VAR) to measure each component of the unexpected inflation identity (3) accompanying a variety of other shocks. The first main message is that discount rates matter. Unexpected inflation corresponds more to a rise in the discount rate for government debt than to a decline in primary surpluses.

I interpret the results through the lens of the fiscal theory of monetary policy: models with interest rate targets, fiscal theory of the price level, and, potentially, sticky prices, as described

---

1 More precisely, $s_{t+1}$ is $\rho$ times the real primary surplus divided by GDP, and scaled by the steady state debt-to-GDP ratio, so its units are real surplus divided by real value. With $\rho = 1$, it can also represent the real primary surplus divided by the previous period’s real value of debt — either definition leads to the same linearization. For brevity, I refer to $s_{t+1}$ as simply the “surplus.” I impute the surplus from the other terms of (1), so its identity really only matters when one wishes to assess the accuracy of approximation, which I do below, or to assess an independent data source on surpluses that does not conform to the identity.
in Cochrane (2019a), Cochrane (2019b). (More literature below.) In this interpretation, changes in expected surpluses and discount rates cause unexpected inflation. We study the fiscal roots rather than the fiscal consequences of inflation. With this view, the point of the paper is to establish a set of facts for constructing such models, as atheoretical VARs guided the construction of conventional monetary models. Causal language below refers to this interpretation.

But the above identities hold in almost all macroeconomic models used to quantitatively address inflation. (They assume that the present value is finite – loosely that \( r > g \). I presume this case without further comment.) Therefore, the results can also be interpreted as measures of the passive-fiscal adjustments to an active-money regime, and the standard new-Keynesian model in particular. A well-specified active-money regime must spell out a realistic passive-fiscal policy. The fact that discount rates do much of the adjusting, rather than the ex-post lump-sum taxes alluded to in many theoretical footnotes, changes the fiscal underpinnings of such models substantially.

Moreover, since the analysis is based on identities, ex-post as well as ex-ante, for any conditioning information set or model of discount rate formation, I do not test anything, and the empirical results do nothing to establish one or another causal story. But which element in an identity moves is still an interesting measurement.

The finding that discount rates account for much inflation variation is key for making a fiscal analysis reasonable – or for understanding fiscal underpinnings of standard models. Why, for example, did inflation fall in the great recession of 2008, when deficits skyrocketed? Yes, the same equation holds in both active-fiscal and passive-fiscal models, so a puzzle does not test one model vs. another. But the fact is still a puzzle. In the context of (3), there are two obvious possibilities. First, perhaps current deficits – negative \( s_t \) – are balanced by expectations of future surpluses – positive \( s_{t+j} \), to pay off the incurred debts. Perhaps, in fact, the expected future surpluses are even larger, resulting in the current unexpectedly low inflation. Perhaps – not at all plausibly in this instance, but at least it’s a possibility, and one we can look for in the data. Second, in 2008 real and nominal interest rates fell dramatically, and so, plausibly, did the expected real returns of government bonds. A lower expected return is a lower discount rate. It raises the value of unchanged, or even lower, expected surpluses, and thus is a deflationary force. Broadly, the innovation accounting following (3) finds this is the case on average.

The second term on the left hand side of (3) is a key and novel point of the analysis. For example, when there is a negative innovation to the present value of surpluses on the right hand side of (3), a decline in nominal long-term bond prices and consequent negative return \( \Delta E_{t+1}r^n_{t+1} \) can lower the real value of debt, in place of unexpected inflation \( \Delta E_{t+1}\pi_{t+1} \). In this
way, long-term debt can buffer fiscal shocks.

To evaluate this channel, I examine the size of responses \( \Delta E_{t+1} r^n_{t+1} \), and I break them down to expected future inflation vs. expected future real returns. With a geometric maturity structure, in which the face value of maturity \( j \) debt declines at rate \( \omega^j \), the Appendix develops the approximate identity

\[
\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} r^n_{t+1+j} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r^n_{t+1+j} - \pi_{t+1+j}) + \pi_{t+1+j} \right].
\] (4)

Lower bond prices correspond to higher bond expected nominal returns, which in turn are composed of real returns and inflation. I find that the bond return responses \( \Delta E_{t+1} r^n_{t+1} \) are large, and that they mostly correspond to changes in expected future inflation, not to changes in expected real returns.

To see the intuition of this channel, substitute (4) into (3), and consider the simple case with no growth \( g = 0 \), and constant expected returns \( E_t r^n_{t+1} = E_t \pi_{t+1} \). Then we have

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}.
\] (5)

With only one-period debt, \( \omega = 0 \), there is only one term on the left-hand side, \( \Delta E_{t+1} \pi_{t+1} \). Shocks to the present value of surpluses must be soaked up by a price-level jump, which absent default is the only way to devalue outstanding debt.

With long-term debt, \( \omega > 0 \), however, a shock to the present value of surpluses can result in a drawn out period of inflation, which slowly devalues outstanding long term bonds. This is a much more realistic vision of postwar US events. In the identity (3), the term \( r^n_{t+1} \) marks the future inflation to market.

In a fiscal theory of monetary policy, monetary policy controls the path of expected inflation. In the simplest frictionles model, \( i_t = E_t \pi_{t+1} \) so the interest rate target sets expected inflation directly. Thus, the monetary policy response to a fiscal shock controls whether the fiscal shock leads to an immediate price level jump or a larger but more drawn out inflation, perhaps even with no immediate inflation.

As we move to continuous time, all debt is long-term debt, and typical sticky-price models disallow price level jumps. In this case also, all the response to a fiscal shock comes from a drawn-out expected future inflation, rather than a price level jump. This is a fundamentally different and more realistic view of the world than the standard view that fiscal theory is all and only about price level jumps which devalue outstanding short-term debt.
With time-varying expected returns, interesting additional dynamics can emerge. Future expected inflation can be set off by a discount rate shock on the right side of the inflation decomposition (3). But the same time-varying returns, now weighted by the maturity structure of debt, affect the value of long-term bonds in (4). Monetary policy with sticky prices also affects real interest rates. However, in most of the analysis here I find expected inflation dominates the response of long-term bond returns (4) in response to the shocks I analyze. Discount rates are more important for the inflation decomposition (3) because most discount-rate variation happens at longer horizons, when the $\omega^j$ terms in (4) have diminished.

In the analysis of recession shocks such as 2008, in fact, the fall in discount rate is so large that on its own it would produce a large deflation. But the fall in expected bond returns provokes a rise in bond prices. So a large ex-post bond return $r_{t+1}^n$ term on the left hand side of decomposition (3) soaks up the extra deflationary force of the decline in expected bond returns on the right hand side.

Overall, then, I find two novel descriptions of the fiscal roots of inflation: First, inflationary and deflationary fiscal shocks come to a large extent from discount rate variation, not from shocks to expected surpluses. Second, fiscal and discount rate shocks translate via long-term debt to persistent movements in expected inflation. Strong current or forecastable surplus movements, or sudden debt-devaluing price level jumps, are not the dominant fiscal roots of inflation.

1.1. Literature

The technique in this paper is adapted from asset pricing. The general approach to linearizing the valuation identity follows Campbell and Shiller (1988). The Appendix relates impulse-response calculations to asset price variance decompositions. The summary of this literature in Cochrane (2011) and the treatment of identities in Cochrane (2007) are obvious precursors. The uniting theme in the former is that asset price and return variation is largely driven by variation in discount rates.

The analysis of government finances, how debt is paid off, grown out of, or inflated away, is a huge literature. Hall and Sargent (1997), Hall and Sargent (2011) are the most important precursors. Hall and Sargent focus on the market value of debt, as I do, not the face value reported by the Treasury, and consequent proper accounting for interest costs. I use data provided by Hall, Payne, and Sargent (2018).

This paper uses the innovation identity (3), to focus on inflation, paralleling VAR-based return decompositions from asset pricing such as Campbell and Ammer (1993). A companion
paper Cochrane (2019c) decomposes the value of government debt \( v_t \), starting from the value identity (2), paralleling price/dividend ratio variance decompositions.

The fiscal theory of monetary policy is the latest step in a long literature on the fiscal theory of the price level, starting with Leeper (1991), that integrates fiscal theory with sticky-price models and interest rate targets. Sims (2011), Cochrane (2017) and are immediate antecedents. Cochrane (2019a) works out such a model with the S-shaped surplus processes I find here, and calculates inflation decompositions and response functions in the model.

Much of the fiscal theory literature has pursued various theoretical controversies. A big point of this paper is to begin productively use fiscal theory to understand US data.

2. Data and VAR estimates

I use data on the market value of government debt held by the public and the nominal rate of return of the government debt portfolio from Hall, Payne, and Sargent (2018). I use standard BEA data for GDP and total consumption. I use the GDP deflator to measure inflation. I use CRSP data for the three-month Treasury rate. I use the 10-year constant maturity government bond yield from 1953 on and the yield on long-term United States bonds before that date to measure a long yield.

I infer the primary surplus from the flow identities. This calculation measures how much money the government actually borrows. NIPA surplus data, though broadly similar, does not obey the flow identity.

I measure the debt to GDP and surplus to GDP ratios by the ratios of debt and surplus to personal consumption expenditures, times the average consumption to GDP ratio. Debt to GDP ratios are often used to compare countries, but in our time-series application they introduce cyclical variation in GDP. We want only a detrending divisor, and an indicator of the economy's long-run level of tax revenue and spending. Potential GDP has a severe look-ahead bias. Consumption is a decent stochastic trend for GDP.

I infer the surplus from the linearized identity (1), at an annual frequency. By doing so, the data obey the identity exactly. Therefore VAR estimates of the decompositions add up exactly with no approximation error. The approximation errors are much smaller than sampling errors, so this choice just produces clearer tables.

To measure the accuracy of the linear approximation, I also infer the monthly real primary surplus from the exact nonlinear flow identity, Appendix equation (16). I then carry the surplus to the end of the year using the government bond return. This procedure produces an annual
series for which the nonlinear flow identity (16) continues to hold in annual data.

I approximate around \( r = g \) or \( \rho = 1 \). The variables are all stationary, impulse-responses and expected values converge, so downweighting higher order terms by something like \( 0.99^j \) makes little difference to the results. Since the value of the debt \( v_t \) is stationary, \( \lim_{T \to \infty} E_t v_{t+T} = 0 \) without \( \rho \) weighting. The parameter \( \rho \) is only the arbitrary point about which one takes a Taylor expansion of the one-period flow relation. It is not the long-run value \( r - g \) in the economy, and its choice does not determine whether present values converge, transversality conditions hold, the economy is dynamically efficient, government debt never needs to be repaid, and so forth.

One can linearize an economy that has \( r > g \) around \( r = g \). With \( \rho = 1 \), the same linearization applies to the surplus to value ratio, which we will shortly see is a bit more accurate. One can also view the unweighted identities as \( r \to g \) limits.

![Figure 1: Surplus](image)

Figure 1: Surplus. “Linear” is inferred from the linearized flow identity, and is the definition used in VAR analysis. “\( sv_t \)” is the exact ratio of the primary surplus to the previous year’s market value of the debt. “\( sy_t/e_v \)” is the exact ratio of surplus to consumption, scaled by the average consumption to GDP ratio and the average value of debt. Vertical shading denotes NBER recessions.

Figure 1 presents the surplus and compares three measures. The “Linear, \( s_t \)” line imputes the surplus from the linearized flow identity (1) directly at the one-year horizon, which is the measure I use in the following analysis. The “\( sv_t \)” and “\( sy_t/e_v \)” lines both infer the surplus from the exact nonlinear flow identity (16), as above. The “\( sv_t \)” line presents the ratio of the exact
surplus to the previous year’s value of the debt. The “$s_{yt}/e^v$” line presents the exact surplus to GDP ratio – actually, the ratio of surplus to consumption, times the average consumption to GDP ratio – scaled by the average value to GDP ratio $e^{E_t(v)}$. The Appendix shows that linearizing in terms of either concept leads to the same result, at a linearization point $r = g$.

I use a data sample 1947-2018 for the VAR analysis. The immense deficits of WWII would otherwise dominate the analysis, and one may well suspect that financing that war, and expectations and reality of paying it off follows a different pattern than fiscal-monetary policy in the subsequent decades of largely cyclical deficits. WWII also featured price controls, clouding inflation measurement. The vertical dashed line of Figure 1 indicates the post-1947 sample.

The first piece of news in Figure 1 is that there are primary surpluses. One’s impression of endless deficits comes from the deficit including interest payments on the debt. Even NIPA measures show regular positive primary surpluses. Steady primary surpluses from 1947 to 1975 helped to pay off WWII debt. The year 1975 started an era of large primary deficits, but also interrupted by the strong surpluses of the late 1990s. Postwar primary surpluses also have a clear cyclical pattern. The primary surplus correlates very well with the unemployment rate (not shown), a natural result of procyclical tax revenues, automatic (e.g. unemployment insurance) and discretionary countercyclical spending.

The three measures in Figure 1 are close. The graph is a measure of the accuracy of the linearized identity (1). The linearized identity is a slightly closer approximation to the surplus to value ratio $sv$. The difference is largest when the value of debt is far from its mean, both in WWII and in the 1970s.

2.1. Vector autoregression

Table 1 presents OLS estimates of the VAR coefficients. Each column is a separate regression. I orthogonalize shocks later, so the order of variables has no significance. The VAR includes the central variables for the inflation identity – nominal return on the government bond portfolio $r^m$, consumption growth rate $g$, inflation $\pi$, surplus $s$ and value $v$. I include the three-month interest rate $i$ and the 10 year bond yield $y$ as they are important forecasting variables for growth, inflation, and long-term bond returns.

It is important to include the value of debt $v_t$ in the VAR, even if we are calculating terms of the innovation identity (3) that does not reference that value. When we deduce from the present value identity (2) expressions $v_t = E_t(\cdot)$, we must include $v_t$ in the information set that takes the expectation. The surplus typically follows an s-shaped process, in which deficits today are followed by surpluses in the future. The process is not invertible, so will not be properly re-
covered by VARs that do not include the value of debt. (See Cochrane (2019b), Cochrane (2019a) for discussion and examples.)

I use a single lag. Adding the last variable, the long-term rate, already introduces slight wiggles in the impulse-response function indicative of overfitting. More lags are insignificant forecasters, and add additional wiggles.

\[
\begin{align*}
\Delta r^n_t & \quad -0.17^{**} & \quad -0.02 & \quad -0.10^{**} & \quad -0.32^{*} & \quad 0.28^* & \quad -0.08^* & \quad 0.04^* \\
g_t & \quad -0.27^{**} & \quad 0.20^* & \quad 0.16^* & \quad 1.37^{**} & \quad -2.00^{**} & \quad 0.28^{**} & \quad 0.06 \\
\pi_t & \quad -0.15 & \quad -0.14^* & \quad 0.53^{**} & \quad -0.25 & \quad -0.29 & \quad 0.09 & \quad 0.04 \\
s_t & \quad 0.12^{**} & \quad 0.03 & \quad -0.03^* & \quad 0.35^{**} & \quad -0.24* & \quad -0.04^* & \quad -0.04^{**} \\
v_t & \quad 0.01 & \quad -0.00 & \quad -0.02^{**} & \quad 0.04^* & \quad 0.98^{**} & \quad -0.01 & \quad -0.00 \\
i_t & \quad -0.32^* & \quad -0.40^* & \quad 0.29^* & \quad 0.50 & \quad -0.72 & \quad 0.73^{**} & \quad 0.36^{**} \\
y_t & \quad 1.93^{**} & \quad 0.54^{**} & \quad -0.17 & \quad -0.04 & \quad 1.60^* & \quad 0.11 & \quad 0.46^{**} \\
\end{align*}
\]

Table 1: OLS VAR estimate. Sample 1947-2018. One (two) stars means the estimate is one (two) Monte Carlo standard errors away from zero.

I compute standard errors from a Monte Carlo. The stars in Table 1 represent one or two standard errors above zero. Since we aren’t testing anything, stars are just a visual way to show standard errors without another table.

In the first column, the long-term bond yield \(y_t\) forecasts the government bond portfolio return \(r^n_{t+1}\). The negative coefficient on the three-month rate \(i_t\) means that the long-short spread also forecasts those returns. Since the \(y_t\) and \(i_t\) coefficients are not repeated in forecasting inflation and growth, the long rate and long-short spread forecast real, growth-adjusted, and excess returns on government bonds, as we expect from the long literature in which yield spreads forecast bond risk premia (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005)). The long rate \(y_t\) is thus an important state variable for measuring expected bond returns, the relevant discount rate for our present value computations.

Growth \(g_t\) is only very slightly persistent (0.20). The term spread \(y_t - i_t\) also predicts economic growth, a common finding, and reinforcing the importance of the interest rates as state variables. Inflation \(\pi_t\) is persistent (0.53). The interest rate also helps to predict inflation.

The surplus is somewhat persistent, with an own coefficient of 0.35. Growth \(g_t\) predicts higher surpluses, an important and realistic feedback mechanism. The surplus responds to the
value of the debt, (0.04). This coefficient should not be misinterpreted to measure a passive-fiscal regime. The active vs. passive fiscal question is how surpluses respond to changes in the value of debt induced by multiple-equilibrium inflation. We cannot measure off-equilibrium responses from data drawn from equilibrium. Even a completely exogenous surplus process, in which a government borrows, then raises surpluses as promised to pay off the resulting debt, will show this coefficient.

The value of the debt is very persistent, with an 0.98 own coefficient. It thus becomes the most important state variable for long-run calculations. A larger surplus $s_t$ results in less market value of debt, $v_{t+1}$, (-0.24), as one expects. The long-run yield $y_t$ forecasts a rise in the value of debt $v_{t+1}$, as we expect given its effect on the expected return $r^n_{t+1}$.

The short rate $i_t$ is also autocorrelated with an 0.73 own coefficient. The long yield $y_t$ does not forecast the short rate, again reflecting time-varying real returns. The long yield is also autocorrelated (0.46), again reflecting standard yield curve dynamics.

For calculations reported below, I use the standard notation

$$x_{t+1} = Ax_t + \varepsilon_{t+1}$$

(6)

to denote this VAR.

### 3. Responses and decomposition estimates

I start by examining the fiscal roots of a simple inflation shock, an unexpected movement in inflation $\Delta E_1 \pi_1$. I orthogonalize the inflation shock so that all other variables move contemporaneously to the inflation shock. I specify $\varepsilon^\pi_1 = 1$. I fill in shocks to the other variables by running regressions of their shocks on the inflation shock. For each variable $z$, I run

$$\varepsilon^z_{t+1} = b_{z, \pi} \varepsilon^\pi_{t+1} + \eta_{t+1}.$$ 

Then I start the VAR (28) at

$$\varepsilon_1 = -\left[ b_{r_1, \pi} \ b_{g, \pi} \ \varepsilon^\pi_1 = 1 \ b_{s, \pi} \ ... \right]'.$$

This procedure is equivalent to the usual orthogonalization of the shock covariance matrix, but it is more transparent and it generalizes more easily later. I denote the VAR innovations as the change in expectations at time 1, i.e. $\Delta E_1$, and thus the response of variable $x$, $j$ periods in the
future is $\Delta E_j x_j$.

Figure 2 plots responses to this inflation shock. The “inflation” column of Table 2 presents the terms of the decomposition (3) for impulse-response functions, i.e.

$$
\Delta E_1 \pi_1 - \Delta E_1 (r^n_1 - g_1) = - \sum_{j=0}^{\infty} \Delta E_1 s_{1+j} + \sum_{j=1}^{\infty} \Delta E_1 (r^n_{1+j} - \pi_{1+j} - g_{1+j}) .
$$

(7)

The top panel of Figure 2 also presents the main terms in this identity. As shown in the Appendix, these terms can also be interpreted as a decomposition of the variance of unexpected inflation – they answer the question, What fraction of the variance of unexpected inflation is due to each component? Table 3 presents quantiles of the sampling distributions of the terms of the inflation decomposition, discussed below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Inflation</th>
<th>Recession</th>
<th>Monetary</th>
<th>Fiscal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation $\pi_1$</td>
<td>1.00</td>
<td>-1.00</td>
<td>1.91</td>
<td>0.45</td>
</tr>
<tr>
<td>Bond return $(r^n_1 - g_1)$</td>
<td>-0.23</td>
<td>2.19</td>
<td>-0.99</td>
<td>-0.04</td>
</tr>
<tr>
<td>of which $r^n_1$</td>
<td>-0.56</td>
<td>1.19</td>
<td>-0.99</td>
<td>0.12</td>
</tr>
<tr>
<td>of which $g_1$</td>
<td>-0.33</td>
<td>-1.00</td>
<td>-0.00</td>
<td>0.16</td>
</tr>
<tr>
<td>Total current $\pi_1 - (r^n_1 - g_1)$</td>
<td>1.23</td>
<td>-3.19</td>
<td>2.90</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 2: Inflation identity terms. The top two panels present the terms of (7). The bottom panel presents the terms of the bond return identity (8).

<table>
<thead>
<tr>
<th>Component</th>
<th>Inflation</th>
<th>Recession</th>
<th>Monetary</th>
<th>Fiscal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation $\pi_1$</td>
<td>1.00 25%</td>
<td>-1.00 25%</td>
<td>1.91 25%</td>
<td>0.45 25%</td>
</tr>
<tr>
<td>Bond return $(r^n_1 - g_1)$</td>
<td>0.00 75%</td>
<td>2.40 75%</td>
<td>-0.95 75%</td>
<td>-0.43 75%</td>
</tr>
<tr>
<td>Future $\sum s$</td>
<td>-0.69 25%</td>
<td>-1.28 25%</td>
<td>-1.57 25%</td>
<td>-1.00 25%</td>
</tr>
<tr>
<td>Future $\sum (r - g)$</td>
<td>0.42 75%</td>
<td>-4.56 75%</td>
<td>-2.60 75%</td>
<td>-0.87 75%</td>
</tr>
</tbody>
</table>

Table 3: Inflation identity quantiles. 25 and 75 percent quantiles of the sampling distribution of the terms of the inflation identity, based on a Monte Carlo.

Looking at the top panel of Figure 2, the inflation shock coincides with a negative surplus shock $s$, which builds with a hump shape. One might think these persistent deficits account for inflation. But surpluses eventually rise to pay back almost all of the incurred debt. The sum of
Figure 2: Response to a 1% inflation shock.
all surplus responses is \(-0.06\), essentially zero.

The line marked \(r - g\) plots the response of the real growth-adjusted discount rate, \(\Delta E_1 (r_{1+j}^n - \pi_{1+j} - g_{1+j})\). These are plotted at the time of the ex-post return, \(1 + j\), so they are the expected return one period earlier, at time \(j\). The line starts at time 2, where the terms of the last, discount-rate, sum in the inflation decomposition (7) start. After two periods of no movement, this discount rate rises. The sum of all discount rate terms is 1.17%. When inflation \(\Delta E_1 \pi_1\) rises 1%, more than all of the corresponding decline in the value of government debt comes from a rise in discount rates.

The extra decline in the present value of debt, coming from the 0.06 surplus decline and the 0.17 extra discount rate rise, shows up in bond prices. The line \(r^n - g\) shows the change in the first term of (7), \(\Delta E_1 (r_1^n - g_1)\), which declines by 0.23%.

In sum,

- **The decline in present value of surplus corresponding to an inflation shock comes almost entirely from a rise in discount rate, and not from a change in expected surpluses.**

This is an important finding for matching the fiscal theory to data, or for understanding the fiscal side of passive-fiscal models. Thinking in both contexts has focused on the presence or absence of surpluses, not the discount rate.

- **A fourth of the decline in present value of surpluses associated with an inflation shock is soaked up by a decline in the growth-adjusted value of long-term bonds.**

The lower panel of Figure 2 plots the response of rates of return in more detail, to give some intuition for the discount rate behavior of the upper panel, and Table 2 includes some of the relevant numerical values.

The response of growth \(g\) is negative and persistent. The inflation shock is, on average in this sample, stagflationary. Below, I isolate a shock in which unexpected inflation coincides with more growth.

The return \(r_t^n\) falls for one period, but then rises. This is the picture of an unexpected rise in bond yields, which produces a one-period decline in bond prices, and a one-period negative ex-post return \(\Delta E_1 r_1^n\), but then produces a rise in bond expected returns \(\Delta E_1 r_{1+j}^n\). (The saw-tooth pattern comes from a slightly negative eigenvalue of the VAR, which is far below statistical significance.)

Both long and short bond yields rise throughout. The rise in discount rate, labeled \(r - g\) in the top panel, comes mostly from the rise in nominal return with the contributions of growth and inflation largely offsetting past year 4.
The -0.23% growth-adjusted ex-post return $\Delta E_1 (r^n_1 - g_1)$ in the top panel, consists of a large -0.56% negative bond return, mitigated by the negative of the -0.33% decline in growth. This movement in long-term bond return $\Delta E_1 r^n_1$, and its consequent ability to soak up fiscal shocks, is not a separate phenomenon. By the bond return identity (4)

$$\Delta E_1 r^n_1 = - \sum_{j=1}^{\infty} \omega^j \Delta E_1 \left[ (r^n_{1+j} - \pi_{1+j}) + \pi_{1+j} \right],$$

the bond return corresponds to the change in expectations of future inflation or real returns. I choose the parameter $\omega = 0.69$ that makes identity (4) hold. The last two rows labeled “$r^n_1$” in Table 2 then give the terms of the decomposition (4). The return $\Delta E_1 r^n_1 = -0.56\%$ corresponds to $-(0.03)\%$ expected future real rates, and $-(0.59)\%$ expected future inflation.

- The unexpected decline in bond returns that comes with an inflation shock comes almost entirely from expected future inflation.

The bond return could have come from variation in expected future real returns as well, requiring a more complex interpretation. The fact that inflation is persistent – the persistent response of inflation to the inflation shock in the top panel of Figure 2 – has so far been irrelevant to the analysis. Here we see that persistent inflation response feeds in to the negative impact response of bond prices, which in turn soaks up part of the fiscal shock.

- By maintaining a maturity structure with about three years duration, and by allowing interest rates and expected future inflation to rise when there are shocks to the present value of surpluses, the US spreads the inflationary impact of changes in the present value of surpluses forward, absorbing shocks to the present value of surpluses in long-term bond prices and a persistent inflation which slowly devalues long-term bonds.

This mechanism is not terribly important quantitatively in the point estimates so far. It is much more important in estimates that follow, which is why I stress the intuition now.

### 3.1. Discussion and disclaimers

The s-shaped surplus response is a crucial lesson. It means that early debts are repaid, at least in part, by following surpluses. The surplus does not follow an AR(1) like process.

Mechanically, the estimates produce this response via debt dynamics. Early deficits raise the value of debt through the flow identity. Higher subsequent debt raises surpluses through the
positive coefficient of surpluses on debt in the VAR. Thus, the finding is econometrically robust; it does not rely on a tenuous measurement of high-order surplus autocorrelations.

The VAR coefficient by which debt forecasts surpluses does not mean that the estimates measure a passive fiscal policy, however. Suppose, for example, that surpluses are completely exogenous. Suppose that the underlying surplus process is s-shaped. When a government borrows money (negative surplus) it commits to future positive surpluses, to repay bondholders, but the schedule of those surpluses is fixed, and in particular does not not respond to revaluations of later debt induced by unexpected inflation. Municipal bonds, that pledge a stream backed by a particular project, have this flavor. That’s active fiscal policy. Yet we observe deficits, which run up debts, and then surpluses which seem to “respond” to those debts.

The s-shaped moving average is a non-invertible representation. If we fit an autoregressive process, or any VAR that excludes the value of debt as a forecaster, we do not measure the underlying s-shape. If we specify a theoretical model with AR(1) surplus, we miss the crucial fact that governments do promise, and people do expect, subsequent surpluses to pay off debts. Cochrane (2019a) takes up this issue in detail.

I use the words “shock,” and “response,” which have become conventional in the VAR literature, and compactly describe the calculations. But the calculations do not imply or require a causal structure. In fact, my fiscal theory interpretation offers a reverse causal interpretation: It is news about future surpluses and discount rates that causes inflation to move. That news in turn reflects news about future productivity, fiscal and monetary policy and other truly exogenous or structural disturbances. A “shock” is only an “innovation,” a movement in a variable not forecast by the VAR. A “response” is a change in VAR expectations of a future variable coincident with such a movement.

Many VAR exercises do attempt to find an “exogenous” movement in a variable by careful construction of shocks, and “structural” VAR exercises aim to measure causal responses of such shocks. Not here.

Last, since we start with an identity (1) that holds ex-post, it holds ex-ante using any information set, so we do not implicitly assume that agents use only the information in the VAR in order to make the calculation. But “unexpected” here means relative to the VAR information set. People may see a lot more. A decomposition using larger information sets, survey forecasts, or people’s full information sets, may be quite different.
3.2. Recession shocks

We can use the same procedure to understand the fiscal underpinnings of other shocks. For any interesting $\varepsilon_1$, we can compute impulse-response functions, and thereby the terms of the decomposition (7). I show in the Appendix that we can consider the calculation as a decomposition of the covariance of unexpected inflation with the shock $\varepsilon_1$, rather the decomposition of the variance of unexpected inflation.

I start with a recession shock. The response to the inflation shock in Figure 2 is stagflationary, in that growth falls when inflation rises. Unexpected inflation is, in this sample, negatively correlated with unexpected consumption (and also GDP) growth. The stagflationary episodes in the 1970s outweigh the simple Phillips curve episodes.

However, it is interesting to examine the response to disinflations which come in recessions, following a conventional Phillips curve. Such events are common, as in the recession following the 2008 financial crisis. But they pose a puzzle for the fiscal theory. In a recession, deficits soar, yet inflation declines. How is this possible? As I outlined in the introduction, future surpluses or lower discount rates could give that deflationary force – needed whether fiscal policy is active or passive. Can we see these effects in the data, and which one is it?

To answer that question, we want to study a shock in which inflation and GDP go in the same direction. I simply create such a shock: I specify $\varepsilon_\pi^1 = -1$, $\varepsilon_g^1 = -1$. The model is linear, so the sign doesn’t matter, but the story is clearer for a recession.

Again, we want shocks to other variables to have whatever value they have, on average, conditional on the inflation and output shock. To fill out the other shocks, then, I run a multiple regression

$$\varepsilon_{I+1}^z = b_{z,\pi} \varepsilon_{I+1}^\pi + b_{z,g} \varepsilon_{I+1}^g + \eta_{t+1}$$

and I fill in the other shocks at time 1 from their predicted variables given $\varepsilon_1^\pi = -1$ and $\varepsilon_1^g = -1$.

I then start the VAR at

$$\varepsilon_1 = \left[ b_{r,\pi} + b_{r,g} \ varepsilon_1^g = 1 \ varepsilon_1^\pi = 1 \ b_{s,\pi} + b_{s,g} \ ... \right]^f.$$

Figure 3 presents responses to this recession shock, and Table 2 collects the inflation decomposition elements in the “Recession” column.

In the bottom panel of the figure, both inflation $\pi$ and growth $g$ responses start at -1%, by construction. Consumption growth $g$ returns rapidly, but does not much overshoot zero, so the level of consumption does not recover much at all. Consumption is roughly a random walk in
Figure 3: Response to a recession shock, $\varepsilon_1^{\pi} = \varepsilon_1^{\varphi} = -1$. 
response to this shock. The nominal interest rate $i$ falls in the recession, and recovers slowly, in parallel with inflation. Long-term bond yields $y$ also fall, but not as much as the short term rate, for about 4 years. The persistent fall in interest rate, inflation and the smaller fall in bond yield correspond to a large positive ex-post bond return $\Delta E_1 r^n_1$. In short, we see a standard picture of a recession.

In the top panel, the recession includes a deficit $s$, which continues for three years. These deficits, reinforced by the positive return shock $r^n - g$ imply a large rise in the value of debt, $v$. These are the deficits in recessions that puzzle a simplistic interpretation of the fiscal theory. Surpluses subsequently turn positive, paying down some of the debt. But the total surplus is still -1.15%. Left to their own devices, surpluses would produce a 1.15% inflation during the recession. A potential story that disinflation results from future surpluses more than matching today’s deficits is wrong.

Discount rates are the central story. After one period, expected real returns $r - g$ decline persistently (top panel) raising the value of debt by 4.34%. We can see the underlying forces in the bottom panel: At year 3, which are expected values at year 2, the expected nominal return $\Delta E_1 r^n_j$ falls more than inflation $\Delta E_1 \pi_j$, and persistently. The fall in expected nominal return follows unsurprisingly the falls in interest rates and bond yields.

Even after the -1.15% decline in surplus, the 4.34% discount rate effect is larger than the -1% fall in inflation. Left on their own, these forces would produce a 3.19% deflation. The current growth-adjusted bond return $\Delta E_1 (r^n_1 - g_1) = 2.19\%$ soaks up this extra deflationary pressure. That return derives from the -1% growth rate, defined in the shock, and the 1.19% positive bond return. The decomposition of bond returns in the bottom rows of Table 2 again reveals that the bond return is driven almost entirely by the persistently lower future inflation, visible in the figure. The 4.34%-1.15% deflationary fiscal shock is soaked up by the persistent disinflation.

In sum, rounding the numbers,

- **Disinflation in a recession, despite deficits, is driven by a higher discount rate. For each 1% disinflation shock, the expected return on bonds falls so much that the present value of debt rises by 4.3%. This discount rate shock overcomes a 1.1% percent inflationary shock coming from persistent deficits, and generates persistent disinflation.**

The opposite conclusions hold of inflationary shocks in a boom. Discount rate variation gives us a fiscal Phillips curve.

The relative magnitudes of the inflation and growth shocks that I used to define a “recession” are (obviously) arbitrary. Growth fell about twice as much as inflation in 2008, but inflation
fell a bit more than growth in 1982. Other recessions have been stagflationary. To produce a better number one must write a model and find an identification in the data to separate “supply” or “stagflationary” Phillips-curve shift shocks from “demand” or “movement along the Phillips curve” shocks, i.e. to explain why inflation doesn’t fall in all recessions, and to define precisely just what kind of events we seek to evaluate. Rather than belabor the point with such a calculation, or fill the paper with multiple graphs, I choose a simple and transparent value. The calculations report correctly “How did expectations change if we observe inflation and growth both decline by 1%?” The only quibble is whether that is an interesting question, or whether some other combination of numbers, prefaced by a long identification calculation, might be more interesting.

3.3. Monetary and fiscal shocks

Central banks move interest rates, but they cannot tax or spend. Therefore, I define here a monetary shock as one that moves interest rates $\Delta E_1 i_1$, but does not affect the sum of current and future surpluses, $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = 0$. I further orthogonalize the monetary shock so that it does not contemporaneously move the growth rate $g_t$, ascribing the contemporaneous positive correlation between growth and interest rate shocks as a Taylor-rule reaction of the Fed to growth and not the other way around.

Conversely, I define here a fiscal shock as a movement in current and expected primary surpluses $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j}$ that comes with no movement in the short-run interest rate $\Delta E_1 i_1 = 0$.

The response of the sum of future surpluses to a shock $\varepsilon_1$ is

$$\Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = a_s'(I - A)^{-1} \varepsilon_1.$$  

To calculate how other shocks respond instantaneously to a monetary shock, then, I run for each variable $z$ a multiple regression

$$\varepsilon_{t+1}^z = b_{z,i} \varepsilon_{t+1}^i + b_{z,ps} a_s'(I - A)^{-1} \varepsilon_{t+1} + b_{z,g} \varepsilon_{t+1}^g + \eta_{t+1}. \quad (9)$$

The monetary policy shock wants

$$\varepsilon_1^i = 1, \quad a_s'(I - A)^{-1} \varepsilon_1 = 0, \quad \varepsilon_1^g = 0.$$
Thus, I start the monetary policy impulse-response function with

\[ \varepsilon_1 = \left[ b_{\text{rev},i} \quad b_{g,i} = 0 \quad b_{\pi,i} \quad \ldots \quad b_{\pi,i} = 1 \quad \ldots \right]' . \]

For the fiscal shock I run the same regression without \( g_t \),

\[ \varepsilon_{z+1} = b_{z,i} \varepsilon_{t+1} + b_{z,pv} \varepsilon_{p+1} + (I - A)^{-1} \varepsilon_{t+1} + \eta_{t+1} , \]

I start the fiscal impulse-response function with

\[ \varepsilon_1 = \left[ b_{\text{rev},pv} \quad b_{g,pv} \quad b_{\pi,pv} \quad \ldots \right]' . \]

Figure 4 presents the responses to the monetary policy shock. Table 2 collects relevant contributions to the inflation identity (7).

The instantaneous movement of the nominal interest rate \( \Delta E_1 i_1 \) is 1% by construction, as is the zero instantaneous growth response \( \Delta E_1 g_1 = 0 \) and the zero response of the sum of surpluses \( \Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = 0 \). Consumption \( g \) declines, as one might expect from a positive monetary shock, and as a new-Keynesian Phillips curve predicts with declining inflation. Although the sum of surpluses does not change by construction, near-term surpluses increase, and long-term surpluses decrease, roughly paralleling the path of consumption. The surplus is procyclical.

Figure 4 shows that

- The response of inflation to this monetary policy shock is super-Fisherian, with inflation rising immediately.

A “Fisherian” response has come to mean that if the central bank raises the interest rate \( i_t \), then inflation rises, for example fulfilling the simple Fisher relation \( i_t = r + E_t \pi_{t+1} \). A “super-Fisherian” response is one in which raising the interest rate \( \Delta E_1 i_1 \) raises inflation \( \Delta E_1 \pi_1 \) immediately. That is the pattern shown in Figure 4. Here, inflation rises by even more than the nominal rate.

Why does inflation \( \Delta E_1 \) rise immediately? It’s not from deficits. Despite the string of deficits seen out to year 8 in Figure 4, surpluses eventually turn positive and pay off all these debts, exactly and by construction with \( \sum s_{1+j} = 0 \). The answer is discount rates, as it must be. After a rise in period 2, the growth-adjusted expected return falls as far as the graph can see, with a cumulative sum \( \sum (r - g) = 2.90 \), which on its own would lead to an immediate 2.90% inflation
Figure 4: Response to a monetary shock – a movement in the interest rate $y_1$ with no movement in the sum of future surpluses $\sum_1^{\infty} s_{1+j}$ or growth $g$. The dashed lines labeled “no g, s” show the inflation response with movement in growth and surplus.
In turn, the higher discount rate comes from a natural source: in response to an interest rate rise, interest rates, yields, and bond expected returns rise more than inflation. This higher discount rate lowers the value of government debt. The rise is concentrated at long horizons however. A mechanistic sticky price view might think interest rates would move more than inflation in the short run only. The actual pattern is the reverse. The negative of the persistent decline in growth $g$ adds measurably to the adjusted discount rate $r - g$, which becomes positive at year 3, one year earlier than the nominal $r^n$ rises above $\pi$.

Higher expected nominal bond returns mean a negative ex-post return $\Delta E_1r^n_1 = -0.99$, as bond prices fall. With $\Delta E_1g_1 = 0$ by construction, this bond return is the entire contemporaneous $r_1 - g_1$ term. The fall in long-term bond prices soaks up 0.99 percentage points of the 2.90% inflationary effect of the discount rate change, leading to the 1.91% inflation. Unlike the inflation and recession shocks, in this case the unexpected bond return $\Delta E_1r^n_1 = -0.99\%$ is not driven entirely by a change in expected inflation. The change in expected inflation, on its own, would produce an even larger $-1.95\%$ bond return, but the $-0.96\%$ decline in near-term (ω weighted) real rates offsets half of that rise.

In sum,

- A monetary shock, defined as an interest rate rise with no change in surpluses and growth, leads to an immediate and persistent increase in inflation. Unexpected inflation comes from a rise in discount rates, which lowers the value of government debt, overwhelming the negative response that comes from long-term debt and persistent inflation.

Figure 5 presents responses to a fiscal policy shock. I specify a negative shock to produce positive inflation, which is a clearer story. In the bottom panel, the contemporaneous interest rate response to the fiscal shock is $\Delta E_1i_1 = 0$, by construction. The interest rate then rises slightly. The fiscal shock gives rise to a positive and persistent inflation.

In the top panel, though the sum of all surplus terms is -1.00% by construction, near-term surpluses rise, and long-term surpluses fall even more. There are three years of negative discount rate movement, offsetting more than half of the surplus shock. This discount rate movement comes from the dynamics shown in the bottom panel. The long-term rate $y$ declines for one period, which gives rise to a large one-period expected return $r^n$. The remaining -0.50% shock to the present value of surplus results in 0.45% inflation, and a small -0.04% bond return term.

- A fiscal shock sets off a protracted inflation. Discount rate variation offsets about half of the fiscal shock.
Figure 5: Fiscal policy response. Response to a shock to expected surpluses $\Delta E_{t+1} \sum_{j=1}^{\infty} s v_{t+j} = 1$, with no interest rate shock $v_{t+1} = 0$. 
3.4. Orthogonalization and shock definitions

Most VAR estimates of the effects of a monetary policy shock find small inflation responses. These are often zero or positive in the short run, the “price puzzle.” When inflation does respond negatively, it typically drifts down only quite slowly, and even then after a long specification search (Ramey (2016)).

The main difference in these results is that I define a monetary policy shock differently. First, I allow a contemporaneous response of inflation to the interest rate shock. Many identification schemes assume that response is zero, in which case we can't measure the response by construction. Inflation that does not move at time 1 tends not to move much at time 2 either.

Second and most importantly, standard estimates do not measure fiscal variables or try to keep any measure of fiscal policy constant. In historical episodes, both monetary and fiscal authorities react to the same events. On their own, VARs will thus find monetary policy shocks that also move fiscal variables unless one defines and orthogonalizes the shock not to move fiscal variables, as I have.

Allowing such a contemporaneous fiscal movement is not a mistake. The standard new-Keynesian model produces its disinflationary effect of a temporary interest rate rise by assuming a fiscal shock, a rise in expected surpluses, contemporaneous with the interest rate rise. It takes a strong passive-fiscal view, then the fiscal authority always follows the central bank's equilibrium-selection desires, and the central bank can count on it to keep doing so.

But if one doubts this mechanism, if one wants to ask what would happen if monetary policy moved and fiscal policy did not follow, then an orthogonalization such as this one in which monetary policy does not coincide with a fiscal policy change is more interesting. The main innovation of this calculation, then, is try to measure the effects of conceptually separate monetary policy and fiscal shocks, each holding the other constant. No shock definition is right or wrong, they just answer different policy questions, and thus can be more or less interesting.

To evaluate this central difference, the dashed lines in the top panel of Figure 4 present the inflation response without restriction on surpluses. The line labeled “no s, no g” also removes the growth orthogonalization, allowing growth to move contemporaneously to the monetary policy shock. These inflation responses are much smaller. Surpluses in these cases (not shown) rise sharply, with a sum $\Delta E_1 \sum s_{t+j} = 3.20\%$ and $3.15\%$ respectively. We do see fiscal tightening contemporaneous with monetary tightening. The question is, do we want to include such a fiscal tightening as part of a “monetary” shock?

The positive response of inflation to an interest rate rise is a disappointment to fiscal theory. Sims (2011), Cochrane (2017), Cochrane (2018) and others put in a huge effort to overcome
the neo-Fisherian tendencies of the new-Keynesian model, especially when one rules out a fiscal contraction contemporaneous with a rise in interest rates.

The basic mechanism is (with 20/20 hindsight) straightforward. Consider the simplified model with constant expected real returns, (5), which I repeat here

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}.
\] (10)

If there is no change in surpluses, as we have assumed, a rise in expected inflation must give rise to lower current inflation. In a flexible price model with \( i_t = E_t \pi_{t+1} \), then, a persistent rise in interest rate will lead to a lower inflation. With somewhat sticky prices, that lower inflation is drawn out as well. Yes, sticky prices lead to a countervailing effect: The higher nominal interest rate is a higher real rate, which discounts surpluses more and leads to more immediate inflation. But in the above-cited models, this discount rate effect does not undo the prediction of lower inflation after an interest rate rise, which is also a long-standing belief.

Well, says this estimate, too bad. Inflation rises when interest rates rise with no change in expected surpluses.

This calculation is, however, unsophisticated regarding exogeneity and orthogonalization. The contemporaneous correlation between interest rate shocks and shocks to inflation, GDP, or other variables can result from the Fed reacting within the period to those variables, as described for example by a Taylor rule such as

\[
i_t = \theta_i \pi_t + \theta_x x_t + u_t^i \tag{11}
\]

instead of the contemporaneous reaction of the economic variables to the Fed’s shock, an innovation to \( u_t^i \). And perhaps even the apparent disturbance \( u_t^i \) is taken in response to news about future inflation or output, or other variables such as financial conditions that forecast inflation or output, news not captured by VAR variables.

I define the monetary policy shock so that growth does not respond contemporaneously, conservatively assigning all the correlation between the interest rate and growth to reverse causality from consumption to the interest rate. The correlation is positive, so otherwise we estimate that the interest rate rise raises growth. I can’t orthogonalize inflation the same way, or there is by definition no inflation shock left \( \Delta E_1 \pi_1 = 0 \), and one of the main questions of the analysis is ruled out.

Reality of course lies somewhere in between. Some of the correlation of inflation and in-
terest rate shocks within a year reflects the reaction of interest rates to inflation, so my Fisherian estimate is surely overstated. Some of the correlation of growth and interest rate shocks within a year surely reflects the response of growth to interest rates, not entirely the other way around, so that orthogonalization is overstated as well. A better estimate thus requires something more sophisticated than a recursive identification, i.e. assuming either that the Fed does not react within a year to specific economic variables, or that those variables do not react within a year to monetary policy shocks. This monetary shock is really just the best I can do with recursive identification in the annual VAR described above, in a paper that is mostly about inflation shocks. As in all the other responses, one is on much more solid ground thinking of the results as measuring the change in VAR expectations of future variables coincident with a change in interest rate not forecast by the VAR, along with no change in consumption growth and no change in VAR forecast surpluses, rather than measuring the Holy Grail, “what if the Fed capriciously raises interest rates for no reason?” (And, it does not directly affect fiscal policy.)

High frequency data, narrative approaches, or a detailed specification and estimation of the policy rule such as (11) may help to identify purer monetary policy shocks. However, Ramey (2016) shows that a half-century of effort has still not led to a clear standard answer or procedure, on to which one can layer a fiscal policy assumption. The deepest problem is that the Fed never explains any action as a random change. Rather, the Fed always describes every action (or inaction) as a response to something. The only hope is to find a Fed response to something orthogonal to inflation, output, or employment, or forecasts of those, but given those are the Fed’s mandate it’s hard to think what that object could be. And now we also want fiscal authorities not to respond to the same events, an orthogonalization so far untried in the VAR literature.

My shock definitions are crude relative to even simple models with policy rules. For example, in Cochrane (2019a) I write a monetary policy rule (11) and a fiscal policy rule

\[ s_t = \theta_{s\pi} \pi_t + \theta_{sx} x_t + \alpha v^*_t + u^*_t \]  

(12)

where the state variable \( v^* \) accumulates past deficits and surpluses. The \( \theta \) parameters in this rule capture the effects of output and inflation on fiscal surpluses, via standard effects such as proportional income taxation, and automatic and discretionary fiscal stabilizers. These are fiscal reactions that one might well wish an analysis of the effects of monetary policy to consider, rather than to hold surpluses constant.

Thus, ideally, one should define and estimate monetary and fiscal policy rules such as (11) and (12). One should then define a monetary policy shock as one that comes with no fiscal
policy disturbance \( u_t^* = 0 \), and one should define a fiscal shock as one that comes with no monetary policy disturbance \( u_t^* = 0 \), rather than define the monetary shock as having no change in surpluses themselves \( s_t \) and the fiscal shock as having no instantaneous change in the interest rate \( i_t \). Doing so would allow the endogenous responses of surpluses to output and inflation in the effects of a monetary policy shock, and it would allow the typical responses of interest rates to output and inflation in the effects of a fiscal shock. But doing so, though important, requires a lengthy theoretical, data, and econometric investigation (identifying policy rule parameters is hard) so I leave it as a suggestion for future research.

The shock definitions here are not wrong, they just answer different and potentially less interesting questions. The response to the monetary shock here answers the question, “how do we change our forecasts of the future if interest rates rise unexpectedly and fiscal policy and growth do not change,” rather than “... and fiscal policy follows its customary reaction to endogenous variables,” and likewise for the fiscal shock. That’s about the best one can do, I think, in an annual atheoretic VAR. Conversely, standard VARs should at least measure and report the fiscal responses to their shocks, and let us think whether shocks that include those fiscal movements are interesting.

4. Standard errors

I have delayed a discussion of standard errors because there is nothing important to test. Identities are identities. If \( x = y + z \) and \( x \) moves, \( y \) or \( z \) must move, and all we can do is to measure which one. In addition, unlike the case in asset pricing, no important economic hypothesis here rests on whether one of surpluses or discount rates do not move. (Asset pricing finds the hypothesis that expected returns are constant over time interesting to test.) Standard errors only give us a sense of how accurate the measurement is.

To evaluate sampling distributions I run a Monte Carlo. Most of the interesting statistics – variance decompositions, impulse response functions, \((I - A)^{-1}\), etc. – are nonlinear functions of the underlying data, and the near-unit root in value \( v_t \) also induces non-normal distributions. For these reasons, I largely characterize the sampling distribution by the interquartile range – the 25% and 75% points of the sampling distribution.

Table 3 collects the sampling quantiles for the variance decompositions of Table 2. Figure 6 presents the main components of the impulse-response function relevant to the inflation variance decomposition. The bands are 25% and 75% points of the sampling distribution, the dashed line is the median, and the solid line is the estimate.
As shown in Table 3, the -0.06 sum of future surpluses in the inflation decomposition has quartiles of -0.69 to 0.23. The 1.17 future return contribution has wider quartiles of 0.42 to 1.57. Even the instantaneous return contribution of -0.23 has quartiles of -0.45 to 0.00. That discount rates matter is a pretty solid conclusion, but negative surpluses may contribute more to unexpected inflation than the point estimate suggests.

There are several sources of this rather large sampling variation. First, the shocks are large. As shown in Table 1, the surplus innovation has a 4.08 percentage point standard deviation, and value 6.55 percentage points, compared to 1.12 percentage points for inflation. Our friend $\sigma / \sqrt{T}$ starts off badly.

Second, the shocks are imperfectly correlated. This matters, because in each case I find movements in other variables contemporaneous with the shock of interest by running a regression of the other shocks on the shock of interest. The sampling uncertainty of this orthogonalization adds to that of the VAR. We see a correspondingly wide band around the initial surplus response in Figure 6. There is hope in this observation, however. Higher frequency data can better identify shock correlations, at the cost that one must model the strong seasonal in primary surpluses. Moreover, other shock identifications may have better measured correlations.
Third, we measure sums of future surpluses and discount rates. The value of the debt $v_t$ is the main long-run state variable, and uncertainty about its evolution adds to the uncertainty about the sum of surpluses. The coefficient of value $v_t$ on its own lag is 0.98 in Table 1, so small variations in that value lead to large variation in $(I - A)^{-1}$ sums. The Appendix shows that the last two sources of variation contribute about equally.

Table 3 also presents 25% and 75% quantiles of the inflation decomposition for the recession, monetary, and fiscal shocks of Table 2. The -1.15 surplus response to a recession shock has quantiles -1.28 to 0.49, spanning zero, while the -4.34 discount rate response has quantiles -4.56 to -2.60. The conclusion that discount rate variation is a central part of the story for understanding disinflation during recessions is supported, despite its large sampling error. Similarly, the positive inflation effect and strong discount rate effects of monetary policy shocks are well away from zero, as is the positive inflation effect and discount rate effect of the fiscal shock. The quantiles reveal asymmetric sampling distributions. In many cases the point estimate is well to the edge of the 25%-75% quantiles. The 1.91 inflation effect of monetary policy is even outside the 0.85-1.47 interquartile range.

5. Concluding comments

This analysis evidently just scratches the surface. Quarterly or monthly data are attractive, offering potentially better measurement of correlations and shock orthogonalization but requiring us to model the strong seasonality in surpluses. Debt data go back centuries, allowing and requiring us to think what is the same and different across different periods of history. Inflation through wars and under the gold standard may well have different fiscal foundations than in the postwar environment. A narrative counterpart, especially for big episodes such as the 1970s and 1980s, awaits. Different countries under different monetary and exchange rate regimes and different fiscal constraints will behave differently. A parallel investigation of exchange rates beckons, following Jiang (2019a), Jiang (2019b). One could define shocks in many additional interesting ways. The treatment of debt can be refined in many ways. In particular, the maturity structure is not geometric, and varies over time.

I omitted analysis of the fiscal correlates of the remaining shocks in the VAR. A shock to any other variable, orthogonal to the inflation shock, can move all of the other terms of the inflation identity (3), (7). Such movements must offset: If a shock does not move inflation, but does move the sum of future surpluses, then it must also move the sum of future discount rates or the current bond return, in such a way that current inflation does not move. These additional effects
are large. The variation in $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j}$ when other shocks move is large; the corresponding movement in the discount rate term is also large, and the two movements are negatively correlated. The meaning of such orthogonalized movements in expected surpluses, matched by movements in discount rates, in response to these other shocks needs to be understood. I do not pursue this question for length, but also because it is much more interesting if one can give some structural or economic interpretation to the shocks to other variables, which requires a model.

Perhaps most of all, linking these theory-free characterizations to explicit fiscal theory of monetary policy models, or at least to explicit models of discount rates and long-term debt management, is an obviously important step.
References


Online Appendix to “The Fiscal Roots of Inflation”

A Derivation of the linearized identities

In this appendix I derive the linearized identities (1) (2) and (3),

\[ v_{t+1} = v_t + r_t^n - \pi_{t+1} - g_{t+1} - s_{t+1} \]  

\[ v_t = \sum_{j=1}^T s_{t+j} - \sum_{j=1}^T (r_{t+j}^n - \pi_{t+j} - g_{t+j}) + v_{t+1} \]

and

\[ \Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} (r_{t+1}^n - g_{t+1}) = - \sum_{j=0}^\infty \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^\infty \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j} - g_{t+1+j}) \]  

I also define the variables more carefully.

The symbols are as follows:

\[ V_t = M_t + \sum_{j=0}^\infty Q_{t+j} (t+1) B_{t+j} \]

is the nominal end-of-period market value of debt, where \( M_t \) is non-interest-bearing money, \( B_{t+j} \) is zero-coupon nominal debt outstanding at the end of period \( t \) and due at the beginning of period \( t+j \), and \( Q_{t+j} \) is the time \( t \) price of that bond, with \( Q_t^{(t)} = 1 \). Taking logs,

\[ v_t \equiv \log \left( \frac{V_t}{Y_t P_t} \right) \]

is log market value of the debt divided by GDP, where \( P_t \) is the price level and \( Y_t \) is real GDP or another stationarity-inducing divisor such as consumption, potential GDP, population, etc. I use consumption times the average GDP to consumption ratio in the empirical work, but I will call \( Y \) and ratios to \( Y \) “GDP” for brevity.

\[ R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^\infty Q_{t+j} B_{t+j}}{M_t + \sum_{j=1}^\infty Q_{t+j} B_{t+j}} \]

is the nominal return on the portfolio of government debt, i.e. how the change in prices overnight
from the end of $t$ to the beginning of $t + 1$ affects the value of debt held overnight, and

$$r_{t+1}^n \equiv \log(R_{t+1}^n)$$

is the log nominal return on that portfolio.

$$\pi_t \equiv \log \left( \frac{P_t}{P_{t-1}} \right), \ g_t \equiv \log \left( \frac{Y_t}{Y_{t-1}} \right)$$

are log inflation and GDP growth rate.

We can accommodate explicit default, and yes, the formulas also apply to countries in the Euro. An explicit default is a reduction in the nominal quantity of debt overnight. The $B_{t+j}^{(t+j)}$ in the numerator of (15) represents the post-default number of bonds outstanding, i.e. on the morning of time $t + 1$, while the $B_{t+j}^{(t+j)}$ in the denominator represents the pre-default number of bonds outstanding, i.e. in the evening of time $t$. A partial default then shows up as a low return. To handle default one would, of course, add notation distinguishing these two values.

Now, I establish the nonlinear flow and present value identities. In period $t$, we have

$$\sum_{j=0}^{\infty} Q_{t+j} B_{t-1}^{(t+j)} + M_{t-1} = P_t s_{t} + \sum_{j=0}^{\infty} Q_{t+j} B_{t}^{(t+1+j)} + M_t, \quad (16)$$

where $s_{t}$ denotes the real primary (not including interest payments) surplus or deficit. Money $M_t$ at the end of period $t$ is equal to money brought in from the previous period $M_{t-1}$ plus the effects of bond sales or purchases at price $Q_{t+j}$, less money soaked up by primary surpluses.

The left hand side of (16) is the beginning-of-period market value of debt, i.e. before debt sales or repurchases $B_{t+j} - B_{t-1}^{(t+j)}$ have taken place. It turns out to be more convenient here to express equations in terms of the end-of-period market value of debt. To that end, shift the time index forward one period and rearrange to write

$$\sum_{j=1}^{\infty} Q_{t+1+j} B_{t}^{(t+1+j)} + M_{t} = P_{t+1} s_{t+1} + \sum_{j=0}^{\infty} Q_{t+2+j} B_{t+1}^{(t+2+j)} + M_{t+1},$$

$$\sum_{j=1}^{\infty} Q_{t+1+j} B_{t+1}^{(t+1+j)} + M_{t+1} = P_{t+1} s_{t+1} + \sum_{j=1}^{\infty} Q_{t+2+j} B_{t+1}^{(t+2+j)} + M_{t+1},$$

$$\left( M_t + \sum_{j=1}^{\infty} Q_{t+j} B_{t}^{(t+j)} \right) R_{t+1}^n = P_{t+1} s_{t+1} + \left( M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1+j} B_{t+1}^{(t+1+j)} \right),$$
We can iterate this flow identity (17) forward to express the nonlinear government debt valuation identity as

\[
M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} = \frac{sp_{t+1}}{P_{t+1} Y_{t+1}} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.
\]  

(17)

The market value of government debt at the end of period \(t\), as a fraction of GDP, equals the present value of primary surplus to GDP ratios, discounted at the government debt rate of return less the GDP growth rate. (I assume here that the right hand side converges. Otherwise, keep the limiting debt term or iterate a finite number of periods.)

The nonlinear identities (17) and (18) are cumbersome. I linearize the flow equation (17) and then iterate forward to obtain a linearized version of (18). Write (17) as

\[
\frac{V_t}{P_t Y_t} R_t^n \frac{P_t}{P_{t+1} Y_{t+1}} = \frac{sp_{t+1}}{P_{t+1} Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}.
\]

Taking logs,

\[
v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left( \frac{sp_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right)
\]

(19)

I linearize in the level of the surplus, not its log as one conventionally does in asset pricing, since the surplus is often negative. To linearize in terms of the surplus/GDP ratio, Taylor expand the last term,

\[
v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log(e^v + sy) + \frac{e^v}{e^v + sy} (v_{t+1} - v) + \frac{1}{e^v + sy} (sy_{t+1} - sy)
\]

where

\[
sy_{t+1} \equiv \frac{sp_{t+1}}{Y_{t+1}}
\]

(20)

denotes the surplus to GDP ratio, and variables without subscripts denote a steady state of (19).

With \(r \equiv r^n - \pi\),

\[
r - g = \log \frac{e^v + sy}{e^v}.
\]

Then,

\[
v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[ \log(e^v + sy) - \frac{e^v}{e^v + sy} \left( v + \frac{sy}{e^v} \right) \right] + \frac{e^v}{e^v + sy} v_{t+1} + \frac{e^v}{e^v + sy} sy_{t+1}
\]
\[ v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[ v + r - g - \frac{e^v}{e^v + sy} \left( v + \frac{e^v + sy}{e^v} - 1 \right) \right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v} \]

\[ v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = [r - g + (1 - \rho)(v - 1)] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v} \tag{21} \]

where

\[ \rho \equiv e^{-(r-g)}. \tag{22} \]

Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is

\[ v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \rho \frac{sy_{t+1}}{e^v} + \rho v_{t+1}. \tag{23} \]

Iterating forward, the present value identity is

\[ v_t = \sum_{j=1}^{T} \rho^{j-1} \left[ \rho \frac{sy_{t+j}}{e^v} - (r^n_{t+j} - \pi_{t+j} - g_{t+j}) \right] + \rho^T v_T. \tag{24} \]

If we linearize around \( r - g = 0 \), then the constant in (23) is zero \( (sy = 0) \), and we obtain the linearized flow and present value identities (13) and (14), with the symbol \( s_t \) representing \( sy_t / e^v \).

There is nothing wrong with expanding about \( r = g \). The point of expansion need not be the sample mean.

To approximate in terms of the surplus to value ratio, write (19) as

\[ v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left( \frac{V_t}{P_t Y_t} \cdot \frac{s_{P_{t+1}}}{s_{P_{t+1}Y_{t+1}}} \cdot \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right) \]

\[ r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left( \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \cdot \frac{P_{t+1} Y_{t+1}}{P_{t+1} Y_{t}} \right) \]

\[ r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left( sv_{t+1} + e^{v_{t+1} - v_t} \right). \]

At a steady state

\[ r - g = \log (1 + sv). \tag{25} \]

\[ e^{r-g} = 1 + sv. \]

Taylor expanding around a steady state,

\[ r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log (1 + sv) + \frac{1}{1 + sv} \left( sv_{t+1} - sv + v_{t+1} - v_t \right) \]
\[ v_t + (1 + sv) \left[ r_{t+1}^n - \pi_{t+1} - g_{t+1} \right] = \left[ (1 + sv) \log (1 + sv) - sv \right] + sv_{t+1} + v_{t+1} \] (26)

The linearized flow identity (13) follows, with the symbol \( s_t \) representing the surplus to value ratio \( s_t = sv_t \), if we suppress the constant, using deviations from means in the analysis, or if we use \( r = g \) or \( sv = 0 \), as a point of expansion.

The linearizations in terms of the surplus to value ratio \( sv_t \) are more accurate. The units of the flow identities (13), (23) are rates of return. Dividing the surplus by the previous period's value gives a better approximation to the growth in value, when the value of debt is far from the steady state.

With stationary \( v_t \), the term \( v_{t+T} \) does not vanish in (14), where the term \( \rho^{T} v_{t+T} \) vanishes in (24). In this paper, the presence of the \( v_{t+T} \) term is not a difficulty. I study innovations \( \Delta E_{t+1} v_{t+T} \) and \( \Delta E_{t+1} v_{t+T} \rightarrow 0 \). For other purposes, one may wish to use the surplus to GDP linearization and \( r > g \) steady state, so that the limiting term vanishes.

A constant ratio of surplus to market value of debt for any price level path leads to a passive fiscal policy. An unexpected deflation raises the real value of debt. If surpluses always rise in response, they validate the lower price level. Thus, although on the equilibrium path one can describe dynamics via either linearization, if one wants to think about how fiscal-theory equilibria are formed, it is better to describe a surplus that does not react to price level changes, so only one value \( v_t \) emerges, as is the case in (24). For such purposes, the surplus to GDP definition is appropriate, as well as adopting a linearization point \( r > g \) and \( \rho < 1 \). It's also better to use the nonlinear versions of the identities for determinacy issues. The analysis of this paper is about what happens in equilibrium, and does not require an active-fiscal assumption, so the difference is irrelevant here.

I infer the surplus from the linearized flow identity (13) so which concept the surplus corresponds to makes no difference to the analysis. The difference is only the accuracy of approximation, how close the surplus recovered from the linearized flow identity corresponds to a surplus recovered from the nonlinear exact identity (19).

### B Variance Decomposition

I use the elements of the impulse response function and their sums to calculate the terms of the unexpected inflation identity (3). We can interpret this calculation as an decomposition of the variance of unexpected inflation. Multiply both sides of (3) by \( \Delta E_{t+1} \pi_{t+1} \) and take expectations,
\[
\text{var} \left( \Delta E_{t+1} \pi_{t+1} \right) - \text{cov} \left[ \Delta E_{t+1} \pi_{t+1}, \Delta E_{t+1} \left( r^n_{t+1} - g_{t+1} \right) \right] = - \sum_{j=0}^{\infty} \text{cov} \left[ \Delta E_{t+1} \pi_{t+1}, \Delta E_{t+1} s_{t+1+j} \right] + \sum_{j=1}^{\infty} \text{cov} \left[ \Delta E_{t+1} \pi_{t+1}, \Delta E_{t+1} \left( r^n_{t+1+j} - \pi_{t+1+j} - g_{t+1+j} \right) \right].
\]

Unexpected inflation may only vary to the extent that it covaries with current bond returns, or if it forecasts surpluses or real discount rate.

Dividing by \( \text{var} \left( \Delta E_{t+1} \pi_{t+1} \right) \), we can express each term as a fraction of the variance of unexpected inflation coming from that term. This decomposition adds up to 100%, within the accuracy of approximation, but it is not an orthogonal decomposition, nor are all the elements necessarily positive. Each term is also a regression coefficient of the other terms on unexpected inflation.

The two approaches give exactly the same result – the terms of (27) are exactly the terms of the impulse-response function, to an inflation shock orthogonalized last, i.e. a shock that moves all variables at time 1 including \( \Delta E_1 \pi_1 \).

To see this fact, write the VAR

\[ x_{t+1} = Ax_t + \varepsilon_{t+1} \]

so

\[ \Delta E_{t+1} \sum_{j=1}^{\infty} x_{t+j} = (I - A)^{-1} \varepsilon_{t+1}. \]

Let \( a \) denote vectors which pull out each variable, i.e.

\[ \pi_t = a_{\pi}^t x_t, \ s_t = a_s^t x_t, \]

etc. Then the present value identity (3) reads and may be calculated as

\[ a_{\pi}^t \varepsilon_{t+1} - (a_{r^n} - a_g)^t \varepsilon_{t+1} = -a_{s}^t (I - A)^{-1} \varepsilon_{t+1} + a_{rg}^t (I - A)^{-1} A \varepsilon_{t+1} \]

where

\[ a_{rg} \equiv a_{r^n} - a_{\pi} - a_g. \]

We can calculate the variance decomposition (27) by

\[ a_{\pi}^t \Sigma a_{\pi} - (a_{r^n} - a_g)^t \Sigma a_{\pi} = -a_{s}^t (I - A)^{-1} \Sigma a_{\pi} + a_{rg}^t (I - A)^{-1} A \Sigma a_{\pi} \]
where \( \Sigma = \text{cov}(\varepsilon_{t+1}, \varepsilon'_{t+1}) \), and then divide by \( a'\pi a\pi \) to express the result as a fraction,

\[
1 - (a'\pi - a\pi)' \frac{\sum a\pi}{a'\pi a\pi} = -a'\pi (I - A)^{-1} \frac{\sum a\pi}{a'\pi a\pi} + a'\pi (I - A)^{-1} A \frac{\sum a\pi}{a'\pi a\pi}.
\]  

(31)

To show that this variance decomposition is the same as the elements and sum of elements of the impulse-response function to an inflation shock, orthogonalized last, note that the regression coefficient of any other shock \( \varepsilon_z \) on the inflation shock is

\[
b_{\varepsilon^z, \varepsilon\pi} = \frac{\text{cov}(\varepsilon_{t+1}^z, \varepsilon_{t+1}^\pi)}{\text{var}(\varepsilon_{t+1}^\pi)} = \frac{a'\pi \sum a\pi}{a'\pi a\pi},
\]

so the VAR shock, consisting of a unit movement in inflation \( \varepsilon_{t+1}^\pi = 1 \) and movements \( \varepsilon_{t+1}^z = b_{\varepsilon^z, \varepsilon\pi} \) in each of the other variables is given by

\[
\varepsilon_{t+1} = \frac{\sum a\pi}{a'\pi a\pi}.
\]

We recognize in (31) the responses and sums of responses to this shock. Dividing (27) by the variance of unexpected inflation, or examining the terms of (31), we recognize that each term is also the coefficient in a single regression of each quantity on unexpected inflation.

In an analogous way, we can interpret the responses to other shocks as a decomposition of the covariance of unexpected inflation with that shock, based on

\[
\text{cov} (\Delta E_{t+1} \pi_{t+1} \varepsilon_{t+1}) - \text{cov} [\varepsilon_{t+1}, \Delta E_{t+1} (r_{t+1}^n - g_{t+1})] = - \sum_{j=0}^{\infty} \text{cov} [\varepsilon_{t+1}, \Delta E_{t+1} s_{t+1+j}] + \sum_{j=1}^{\infty} \text{cov} [\varepsilon_{t+1}, \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j} - g_{t+1+j})].
\]

This variance decomposition is similar in style to the decomposition of return variance in Campbell and Ammer (1993). To avoid covariance terms, however, it follows the philosophy of the price/dividend variance decomposition in Cochrane (1992), extended to a multivariate context. With \( x = y + z \), I explore \( \text{var}(x) = \text{cov}(x, y) + \text{cov}(x, z) \) rather than \( \text{var}(x) = \text{var}(y) + \text{var}(z) + 2\text{cov}(y, z) \).

C Formulas for geometric maturity structure

Here I derive the linearized identity

\[
r_{t+1}^n \approx \omega q_{t+1} - q_t,
\]
which leads to (4),

\[ \Delta E_{t+1}r_{t+1}^n = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r_{t+1+j}^n - \pi_{t+1+j}^n + \pi_{t+1+j}^n) \right]. \]

I also derive expectations-hypothesis bond-pricing equations. These are used in the sticky-price model Cochrane (2019a).

\[ E_{t+1}r_{t+1}^n = i_t \]
\[ \omega E_tq_{t+1} - q_t = i_t. \]

Suppose the face value of debt follows a geometric pattern, \( B_t^{(t+j)} = B_t \omega^{-j} \). Then the nominal market value of debt is

\[ \sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)} = B_t \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}. \]

Define the price of the government debt portfolio as

\[ Q_t = \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}. \]

The return on the government debt portfolio is then

\[ R_{t+1}^n = \frac{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)}}{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_{t+1}^{(t+j)}} = \frac{\sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}}{\sum_{j=1}^{\infty} \omega^{j-1} Q_{t+1}^{(t+j)}} = \frac{1 + \omega \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+1+j)}}{\sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}} = \frac{1 + \omega Q_{t+1}}{Q_t}. \]

I loglinearize as

\[ r_{t+1}^n = \log \left( \frac{1 + \omega Q_{t+1}}{Q_t} \right) = \log (1 + \omega e^{q_{t+1}}) - q_t \approx \log \left( \frac{1 + \omega Q}{Q} \right) + \frac{\omega Q}{1 + \omega Q} \tilde{q}_{t+1} - \tilde{q}_t \]

where as usual variables without subscripts are steady state values and tildes are deviations from steady state.

In a steady state,

\[ Q^{(t+j)} = \frac{1}{(1 + i)^j}, \]
\[ Q = \sum_{j=1}^{\infty} \omega^{j-1} \left( \frac{1}{1 + i} \right) = \left( \frac{1}{1 - \omega} \right) \left( \frac{1}{1 + i - \omega} \right) = \frac{1}{1 + i - \omega}. \]
The limits are $\omega = 0$ for one period bonds, which gives $Q = 1/(1 + i)$, and $\omega = 1$ for perpetuities, which gives $Q = 1/i$. The terms of the approximation (32) are then

\[
\begin{align*}
\frac{1 + \omega Q}{Q} &= 1 + i \\
\frac{\omega Q}{1 + \omega Q} &= \frac{\omega}{1 + i}
\end{align*}
\]

so we can write (32) as

\[
r^n_{t+1} \approx i + \frac{\omega}{1 + i} \tilde{q}_{t+1} - \tilde{q}_t.
\]

since $i < 0.05$ and $\omega \approx 0.7$, I further approximate to

\[
r^n_{t+1} \approx i + \omega \tilde{q}_{t+1} - \tilde{q}_t. \tag{34}
\]

To derive (4), iterate (34) forward to express the bond price in terms of future returns,

\[
\tilde{q}_t = -\sum_{j=1}^{\infty} \omega^j \tilde{r}^n_{t+j}
\]

Take innovations, move the first term to the left hand side, and divide by $\omega$,

\[
\Delta E_{t+1} \tilde{r}_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \tilde{r}^n_{t+1+j}
\]

then add and subtract inflation to get (4),

\[
\Delta E_{t+1} \tilde{r}^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (\tilde{r}^n_{t+1+j} - \tilde{\pi}_{t+1+j}) + \tilde{\pi}_{t+1+j} \right].
\]

The expectations hypothesis states that expected returns on bonds of all maturities are the same,

\[
E_t r^n_{t+1} = \hat{i}_t \\
i + \omega E_t \tilde{q}_{t+1} - \tilde{q}_t = \hat{i}_t \\
\omega E_t \tilde{q}_{t+1} - \tilde{q}_t = \hat{i}_t
\]

In the text, all variables are deviations from steady state, so I drop the tilde notation.
Table 4: Regression of other shocks on inflation shock, and correlation matrix of VAR shocks

<table>
<thead>
<tr>
<th></th>
<th>$r^n$</th>
<th>$g$</th>
<th>$\pi$</th>
<th>$s$</th>
<th>$v$</th>
<th>$i$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.56$</td>
<td>-0.33</td>
<td>1.00</td>
<td>-0.58</td>
<td>-0.65</td>
<td>0.24</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Std. err.</td>
<td>(0.24)</td>
<td>(0.17)</td>
<td>(0.00)</td>
<td>(0.53)</td>
<td>(0.74)</td>
<td>(0.14)</td>
<td>(0.09)</td>
</tr>
<tr>
<td></td>
<td>$r^n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>-0.25</td>
<td>-0.29</td>
<td>-0.27</td>
<td>0.63</td>
<td>-0.74</td>
<td>-0.93</td>
</tr>
<tr>
<td></td>
<td>$g$</td>
<td>-0.25</td>
<td>1.00</td>
<td>-0.24</td>
<td>0.39</td>
<td>-0.56</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>-0.29</td>
<td>-0.24</td>
<td>1.00</td>
<td>-0.14</td>
<td>-0.11</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>$s$</td>
<td>-0.27</td>
<td>0.39</td>
<td>-0.14</td>
<td>1.00</td>
<td>-0.88</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>0.63</td>
<td>-0.56</td>
<td>-0.11</td>
<td>-0.88</td>
<td>1.00</td>
<td>-0.63</td>
</tr>
<tr>
<td></td>
<td>$i$</td>
<td>-0.74</td>
<td>0.41</td>
<td>0.21</td>
<td>0.35</td>
<td>-0.63</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$y$</td>
<td>-0.93</td>
<td>0.20</td>
<td>0.31</td>
<td>0.26</td>
<td>-0.60</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 4: Regression of other shocks on inflation shock, and correlation matrix of VAR shocks

The yield $y_t$ on the government bond portfolio is the $i_t$ that solves (33) for given $Q_t$,

$$y_t = \frac{1}{Q_t} + \omega - 1$$

To find the yield as deviation from steady state, given the bond portfolio price as deviation from steady state, write

$$q_t = \log \frac{1}{1 + i - \omega} + \tilde{q}_t$$

$$y_t = e^{-\log \frac{1}{1 + i - \omega}} + \tilde{q}_t + \omega - 1$$

$$\tilde{y}_t = e^{-\log \frac{1}{1 + i - \omega}} + \tilde{q}_t - e^{-\log \frac{1}{1 + i - \omega}} = (e^{\tilde{q}_t} - 1)(1 + i - \omega).$$

**D Sources of sampling variation**

Table 4 includes the regression of other shocks on inflation shock that starts off the main inflation decomposition, and thus determines the instantaneous response in Figures 2 and 6. The table also includes the correlation matrix of the shocks.

To measure the relative contribution of the shock correlation and the long-run response function given the shock identification as sources of variation, Table 5 includes two other sampling calculations. The “no b” columns resample data using the original regression of shocks $\tilde{\varepsilon}_{t+1}$ on inflation shocks $\varepsilon_{t+1}$, the top row of Table 4, in each sample. The VAR coefficients still vary across samples, but the identification of the inflation shock does not. The “no A” columns
<table>
<thead>
<tr>
<th>Component</th>
<th>Fraction Estimate</th>
<th>25%</th>
<th>75%</th>
<th>25%</th>
<th>75%</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation $\pi_1$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond return $(r^n_1 - g_1)$</td>
<td>-0.23</td>
<td>-0.45</td>
<td>0.00</td>
<td>-0.23</td>
<td>-0.23</td>
<td>-0.45</td>
<td>0.00</td>
</tr>
<tr>
<td>Future $\Sigma s$</td>
<td>-0.06</td>
<td>-0.69</td>
<td>0.23</td>
<td>-0.60</td>
<td>0.14</td>
<td>-0.69</td>
<td>0.23</td>
</tr>
<tr>
<td>Future $\Sigma r - g$</td>
<td>1.17</td>
<td>0.42</td>
<td>1.57</td>
<td>0.63</td>
<td>1.37</td>
<td>0.42</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Table 5: Decomposition of unexpected inflation variance – distribution quantiles. No b holds the initial response constant across trials. No A holds the VAR regression coefficients constant across trials.

Likewise keep constant the VAR regression coefficients, but reestimate the shock regression in each sample. Turning off either source of sampling variation reduces that variation, but not as much as you might think. Sampling variation is still large in either case, and variances add, not standard deviations. Moreover the sampling variation associated with shock orthogonalization – the “no A” exercise – does not go away no matter how small the shocks. Both left and right hand sides of the shock on shock regressions get smaller at the same rate.