The Fiscal Theory of the Price Level

John H. Cochrane

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Preface

This book is a midpoint, I hope, of a long intellectual journey. It started in the fall of 1980 or so, drinking a beer and eating nachos on a gorgeous sunlit afternoon in Berkeley, with my good friends and graduate school study group partners, Jim Stock, Eric Fisher, Deborah Haas-Wilson, and Steve Jones. We had been studying monetary economics and what happens as speedier electronic transactions reduce the demand for money. When money demand and money supply converge on fast-moving electronic claims to a single dollar bill, framed at the Federal Reserve, will supply and demand for that last dollar really determine the price level and interest rates? If the Fed puts another dollar bill up on the wall, does the price level double? Jim and I, fallen physicists, were playfully thinking about a relativistic limit. Signals are limited by the speed of light, so maybe that puts a floor on money demand.

The conversation moved on, but the seed was planted. Clearly, long before we’re down to the last dollar bill, each of us holding it for a microsecond, at a nanodollar interest cost, the price level would become unhinged from money supply and demand. Is there a theory of inflation that continues to work as we move to electronic transactions and a money-less economy? Why is the price level apparently so stable as our economy moves in that direction? Or must economic and financial progress be hobbled to maintain money demand and thereby control inflation?

Berkeley was, it turns out, a great place to be asking such questions. Our teachers, and especially George Akerlof, Roger Craine, and Jim Pierce, mounted a sustained critique of monetarism, the view that the price level is determined by the quantity of money, MV=PY. They had their own purposes. George was, I think, anti-monetarist for traditional Keynesian reasons. Roger had, I think, lost the faith on more general intellectual grounds as he grappled with the rational expectations revolution that had recently upended big models.

But the critique stuck, and my search for an alternative, and in particular a theory
of inflation that could survive in a frictionless environment, continued. Berkeley also
gave us an excellent grounding in microeconomics and general equilibrium, for which
I thank in particular Rich Gilbert, Steve Goldman, and Gerard Debreu, together
with unmatched training in empirical economics and econometrics, for which I thank
especially Tom Rothenberg.

I was then supremely lucky to land a job at the University of Chicago. This was a
natural fit for my intellectual inclinations. I like the way standard economics works
– you start with supply and demand, and frictionless markets, get some basics, and
then add frictions and complications as needed. It also often turns out that if you
just work a little harder, a simple supply and demand story works to explain lots of
puzzles, and you don’t need the frictions and complications. I get great satisfaction
out of that. For my tastes, too many economists try to start the next revolution,
invent a new theory, or apply a sexy name to a puzzle, and too quickly proclaim that
no standard economic model can explain a given fact. Ninety-nine revolutions are
started for every one that succeeds.

These were also glory years for macroeconomics at Chicago. The Modigliani-Miller
theorem, efficient markets, Ricardian equivalence and rational expectations were just
in the past. Dynamic programming and time-series tools were cutting through long-
standing technical limitations. [Kydland and Prescott (1982) had just started real
business cycle theory, showing that you can make remarkable progress understand-
ing business cycles in a frictionless supply and demand framework, if you just try
hard enough and don’t proclaim it impossible before you start. For me, it was a
time of great intellectual growth, learning intertemporal macroeconomics and as-
set pricing, privileged to hang out with the likes of Bob Lucas, Lars Hansen, Gene
Fama and many others, and to try out my ideas with a few generations of amazing
students.

But monetarism still hung thick in the air at Chicago, so that larger question nagged
at me. I wrote some papers in monetary economics, skeptical of the standard stories
and the VAR literature that dominated empirical work. But even though I thought
about it a lot I didn’t find an answer to the big price level question.

A watershed moment came late in my time at the Chicago economics department. I
frequently mentioned my skepticism of standard monetary stories, and my interest
in frictionless models. The conversations usually didn’t get far. But one day Mike
Woodford responded that I really should read his papers on fiscal foundations of
monetary regimes (Woodford (1995), Woodford (2001)). There it was at last: a
model able to determine the price level in a completely cashless and frictionless
economy. I knew in that instant this was going to be a central idea I would work on for the foreseeable future. (I was vaguely aware of Eric Leeper’s original paper, Leeper (1991), but I didn’t understand it or appreciate what it meant. We explore knowledge through social connections.)

It is taking a lot longer than I thought it would! I signed up to write a Macroeconomics Annual paper (Cochrane (1998)), confident that I could churn out the fiscal history analogue of the Friedman and Schwartz (1963) monetary history in a few months. Few forecasts have been more wrong. That paper solved a few puzzles, but I’m still at the larger question two decades later.

I thought then, and still do, that the merit of the fiscal theory will depend on its ability to organize history, explain events, and coherently describe policy, not by theoretical disputation or some abstract test with 1% probability value, just as Milton Friedman’s MV=PY gained currency by is cogent description of history and policy. But my first years with the fiscal theory were nonetheless dragged into theoretical controversies. One has to get a theory out of the woods where people think it’s logically wrong or easily dismissed by armchair observations before one can get to the business of matching experience.

“Money as Stock” (Cochrane (2005)) (written the same year as ”Stocks as Money,” Cochrane (2003) an attempt at CV humor as well as to point towards a common theory that integrates fundamental value with transactions frictions) addressed many controversies. I guess I owe a debt of gratitude to critics who wrote scathing attacks on the fiscal theory, for otherwise I would not have had a chance to rebut the similarly wrong but more polite dismissals that came up at every seminar.

I then spent quite some time documenting the troubles of the apparently competing new-Keynesian paradigm, primary “Determinacy and Identification with Taylor Rules” (Cochrane (2011a)) and “the New-Keynesian Liquidity Trap” (Cochrane (2017c)). To change paradigms, people need the carrot of a new theory that plausibly accounts for the data, but people also need a stick, to see the flaws of the existing paradigm, and how the fiscal theory solves those problems. Both papers owe a deep debt to generations of students. I taught a Ph.D. class in monetary economics for many years, and I felt it was my duty to explain the standard new-Keynesian approach, which otherwise tended to be ignored at Chicago. Working through Mike Woodford’s book (Woodford (2003)), and working through papers such as Werning (2012), to the point of understanding their flaws, is hard work, and only the pressure and repeated inquiry of sharp graduate students prompted the effort. “Determinacy and identification” also owes a lot to my work as editor of several journals and es-
especially the JPE, as I was forced to understand new-Keynesian models while editing papers in the light of referee reports. For example, I grasped a central point late one night while working on Benhabib, Schmitt-Grohe, and Uribe (2002). Their simple elegant model finally made clear to me that these models assume the Fed deliberately destabilizes an otherwise stable economy. I immediately thought “That’s crazy.” And then “This is an important paper, I have to publish it.”

Matching the fiscal theory with experience turns out to be much more subtle than noticing correlations between M and PY as Friedman and Schwartz (1963) did. The present value of surpluses is much harder to independently measure. Obvious armchair predictions based on easy simplifying assumptions quickly go the wrong way in the data. For example, deficits in recessions correspond to less, not more inflation. I spent a lot of time working through these puzzles. For example, “Long term debt and optimal policy” (Cochrane (2001)) shows that expected future deficits are quite likely to move in the opposite direction of current deficits, so the theory does not naturally predict any sharp relationship between current deficits and current inflation. In retrospect, it’s a little embarrassing that this point took me a year and unnecessary playing with spectral densities to digest. Also, discount rates matter crucially. Inflation declines in recessions because the discount rate for government debt declines, not because there is great news about surpluses. Here, it turned out to be useful that I have spent most of my other research time on asset pricing. I recognized the central equation of the fiscal theory as a valuation equation, like price = present value of dividends, not an “intertemporal budget constraint.” And perhaps the most important thing I learned in finance is that prices move largely on discount rates rather than expected cashflow news. More generally, all the natural “tests of the fiscal theory” you want to try have counterparts in the long difficult history of “tests of the present value relation” in asset pricing. Dividend forecasts, discounted at a constant rate, look nothing like stock prices, so don’t expect surplus forecasts, similarly discounted, to look like inflation. This analogy let me cut through a lot of knots and avoid repeating two decades of false starts. My approach to the fiscal theory represents a lot of intellectual arbitrage from asset pricing. But again, it took me an embarrassingly long time to recognize such simple analogies sitting right in front of me.

Another example of a little interaction that led to a major step for me occurred at the Becker-Friedman Institute conference on fiscal theory in 2016. I had spent most of a year struggling to produce any simple sensible economic model in which higher interest rates lower inflation, without success. (I wrote up the list of failures in “Michelson Morley, Fisher and Occam” (Cochrane (2018)), which may seem self-
indulgent, but documenting that all the simple ideas you probably think work to this end fail is still useful, I think.) Presenting this work at a previous conference, Chris Sims mentioned that I really ought to read a paper of his, “Stepping on a rake,” (Sims (2011)) that had the result. Again, I was vaguely aware of the paper, but had found it hard and didn’t really realize he had the result I needed. After Chris nagged me about it, I did sit down to work through the paper. It took me six full weeks to read and understand Chris’ paper – to the point that I wrote down how to solve Chris’ model, in what became Cochrane (2017d). But there it is – he did have the result, and the result is a vital part of the unified picture of monetary policy I present below. Interestingly, Chris’ result is a natural consequence of the analysis in my own “Long Term Debt” paper Cochrane (2001). We really can miss things right in front of our own noses.

This event allowed me to complete a view that has only firmed up in my mind in the last year or so: The “fiscal theory of monetary policy” models expressed in the “rake” paper and this book. Here, the fiscal theory mostly neatly solves the determinacy and equilibrium selection problems of standard new-Keynesian models, and monetary policy remains crucially important. You don’t have to throw out everything you know and approach inflation data armed with debt and surpluses. You can approach the data armed with interest rate rules, and fiscal theory really only requires small and methodologically straightforward modification of standard new-Keynesian models. The results may change a lot however. But without the conference, and a nudging conversation to remind me of an earlier email to read a paper that really in the end just drew the proper conclusion from my own paper that I had forgotten about, it would not have happened.

My fiscal theory odyssey has also included essays, papers, talks, and blog posts trying to understand experience with the fiscal theory. This story-telling is an important prelude to empirical work, and an eventual summary. Friedman and Schwartz must have started with “I bet the Fed let the money supply collapse in the Great Depression.” Story-telling is hard too. As you will see, fundamental observational-equivalence theorems stand in the way of easy “tests of the fiscal theory,” just as Fama’s joint hypothesis theorem stands in the way of easy “tests of the present value relation,” and the equivalence of \( P = MV/Y \) with \( M = PY/V \) stand in the way of monetarism. Still, we have to start with a story. Is there at least a possible, and then a plausible story to interpret events via the fiscal theory, on which we can build formal tests? That’s what “unpleasant fiscal arithmetic” Cochrane (2011e), and “Michelson Morley, Fisher and Occam” Cochrane (2018) and “The fiscal roots of inflation” Cochrane (2019) attempt, building on “Frictionless View” (Cochrane
(1998), among others. You will see the challenges that bringing this theory to data present and these and many more stories in advance of tests.

These little anecdotes are the tip of an iceberg. My fiscal theory odyssey built on thousands of conversations with colleagues and students. More recently, running a blog has allowed me to try out ideas and have a discussion with a new electronic community. The whole Fisherian question – does raising interest rates maybe raise inflation? – developed there. My understanding has been shaped by being forced to confront different ideas and objections through teaching, editorial and referee work, seminar and conference participation and discussant work, writing promotion letters and so forth. I likewise benefitted from the efforts of many colleagues who took the time to write me comments, discuss my and other papers at conferences, write referee and editor reports, and contribute to seminars and many discussions. Much of this activity may seem like a waste of time, and many economists regard teaching and service as such. But occasionally a little spark comes. The sparks do not come without the work, and they cumulate over time. Economics is a conversation, and a social enterprise. Most of what I have written is a response to challenging thoughts of my colleagues, and an integration and expansion of their ideas. I have also been influenced by things I have read – often after a pointer from a colleague - whose idiosyncratic nature will reveal patterns of influence.

I owe debts of gratitude to institutions as well as to people. This work would not have happened without their combined influences and intellectual support. Without the Berkeley economics department I would not have become a monetary skeptic, or, probably, an economist at all. Without Chicago’s economics department and Booth school, I would not have learned the surrounding ideas. Without the Hoover Institution, I would not have finished the project, connected it to policy, or communicated it well.

Why tell you all these stories? Bob Lucas advises that how you got to a paper’s ideas is irrelevant. Save it for your memoirs, get on with theory and evidence. That’s good advice for an academic paper but maybe not necessarily so for an integrative book.

At least I must express gratitude for those sparks, and the personal effort behind them and the institutions that support them. By mentioning a few, I regret that I will seem ungrateful for hundreds of others. But, in my academic middle age, I also think it’s useful to document how one piece of work like this occurs. Teaching, editorial and referee service, conference attendance and discussing, seminar participation, reference letter writing, all are vital parts of the collective research enterprise, as is
the institutional support that lets all this happen.

My journey includes esthetic considerations as well. I have pursued fiscal theory in part because it’s simple and beautiful, characteristics which I hope to share in this book. That’s not a scientific argument. Theories should be evaluated on logic and their ability to match experience, elegance be darned. But it is also true that the most powerful past theories have also been simple and elegant. I have pursued fiscal theory in this way in part to emulate in monetary economics all the many past successes of economics in which simple and beautiful supply and demand produced powerful insights, and a foundation for more detail.

I have been attracted to monetary economics for many reasons. Monetary economics is (even) more mysterious at first glance than many other parts of economics, and thus more beautiful in its insights. If a war breaks out in the Middle East, and the price of oil goes up, the mechanism is no great mystery. Inflation, in which all prices and wages rise together, is more mysterious. If you ask the grocer why the price of bread is higher, he or she will blame the wholesaler. The wholesaler will blame the baker, who will blame the wheat seller, who will blame the farmer, who will blame the seed supplier and the worker’s demands for higher wages, and the workers will blame the grocer for the price of food. If the ultimate cause is a government printing up money to pay its bills, there is really no way to know this fact but to sit down in an office with statistics, armed with some decent economic theory. Investigative journalism will fail. The answer is not in people’s minds, but in their collective actions. No wonder that inflation has led to so many witch-hunts for “hoarders” and “speculators,” greedy “middlemen,” and other phantasms.

Monetary economics offers a surprisingly high ratio of talk to equations. We fancy ourselves a science in which equations speak for themselves. But they do not. They don’t in science either. You will see that throughout this book. The equations are quite simple. And there will be seemingly endless talk about what they mean and how to read them, which variable causes which. You will long for a short clear set of theorems, a few key equations that express the answers without interpretation. Alas, that will not happen.

0.1 This book

I am reluctant to write this book, as there is so much to be done. Perhaps I should title it “Fiscal theory of the price level: A beginning.” I think the basic theory is now
settled, and theoretical controversies over. We know how to include fiscal theory in standard macroeconomic models including realistic pricing, monetary and financial frictions. But just how to use it most productively, and then actually using it to explain episodes and data, and to guide policy, lies ahead.

First, we have only started to fit the theory to experience. This is as much a job of historical and institutional inquiry and story-telling as it is of model fitting and econometric testing. There isn’t a single “test of monetarism” in Friedman and Schwartz. It seems to have been pretty influential anyway! The fundamental equation of the fiscal theory holds essentially as an identity, and in all interesting models. There is no easy “test.” That is true in the end of most interesting economic models: without restrictions on preferences, technology, and expectations, general equilibrium makes few if any testable predictions on its own. But understanding just how the equation holds, how to construct plausible stories, and then evaluate those stories for various episodes is not easy. I offer a few beginnings here, but they are more efforts to light the way than claims to have concluded a trip.

Second, we have only started to apply the theory to think about how monetary institutions could be better constructed. How should the euro be set up? What kinds of policy rules should central banks follow? What kind of fiscal commitments are important for stable inflation? Can we set up a better monetary system with stable prices and without requiring central banks to divine the correct interest rate?

My bottom line is that an integration of fiscal theory with new-Keynesian models is a promising path forward. But just how do such models work exactly? Do they match data as well or better than standard models, as well as curing the pathologies of those models? The simple project of integrating fiscal theory with standard price stickiness models, models with financial frictions, or other connections of monetary to real affairs is only just starting. The international version, extending the theory to exchange rate determination, has barely begun.

Time will tell, and for years I put off writing this book because I wanted to finish the next step in the research program first. But life is short, and for each step taken I can see three others that need taking. It’s time to encourage others to take those steps, as well as to put down here what I understand so far so we can all build on it. You may find lots of this book chatty and speculative. Some sections are likely wrong, when flushed out with equations. I far prefer to read short, clear books, but sadly this is the one I know how to write in the current state of my understanding of the fiscal theory.

On the other hand, though the path is only half taken, every time I give a fiscal
0.1. THIS BOOK

theory talk, we have to go back to basics. That’s understandable. The basic ideas are spread out in two decades’ worth of papers, written by a few dozen authors, and simple ideas are often hidden in the less than perfect clarity of first academic papers on any subject. By putting what we know and have digested in one place, I hope we can move the conversation to the things we genuinely don’t know, and broaden the conversation beyond the few dozen of us who have worked intensely in this field.

Where’s the fire, you may ask? Great books in economics come with great historical events. Keynes wrote the General Theory to address the great depression. Friedman wrote in advance of the great inflation. Yet inflation is remarkably stable in the developed world, at least as I write. Well, economic theory is not always propelled just by big events. Inflation is actually too stable. Other than repeat the incantation that “expectations are anchored,” current economic theory doesn’t really understand the current quiet. And it’s increasingly obvious that current theory doesn’t hold together, or provide much guidance for how central banks should behave if inflation does break out. The worry that central banks have much less power over the economy than they think they do, and much less understanding of the mechanism behind what power they do have, is more and more common. So, sometimes theories fall apart in the face of experience. Sometimes theories just fall apart all on their own. In either case they need replacing. So the intellectual fire is there, and for once we have the luxury of contemplating it before a real fire is on our hands.

As it evolved this book took on an unusual organization. You will find that many issues crop up again and again. I found it rang better to introduce an issue in a simple context, develop it a bit further as the context developed, and then include a full development of the idea much later in the book. Ideas such as the negative autocorrelation of surpluses, the nature of active vs. passive policies, the contrast between fiscal and new Keynesian models are examples. That organization may seem repetitive to a careful reader, who may wish for each issue to be treated once. But I found that many of the important issues could be addressed very simply and quickly in the context of the very simple models early on in the book, and then treated in more depth and generality once I developed more general models. Finding the active vs. passive discussion delayed to about page 450 of an early draft convinced me that this organization was better. So be warned, you won’t find everything on an issue in one place. But if you read from front to back, you may find issues introduced when they are really easy to understand, and if you wish more generality, hang on, it’s coming later.
Part I

The Fiscal Theory
Chapter 1

Introduction

What determines the overall level of prices? What causes inflation, deflation, or currency devaluation? Why do we work so hard for pieces of paper? A $20 bill might cost one cent to produce, yet you can trade it for $20 worth of goods or services. And now, $20 is really just a few bits moved in a computer, for which we work just as hard. What determines the value of a dollar? What is a dollar, really?

The fiscal theory of the price level at heart answers these eternal questions in this way: Money is valued because the government accepts money for tax payments. If on April 15, you have to come up with these specific pieces of paper, or these specific bits in a computer, and no others, then you will work hard through the year to get them. You will sell things to others in return for these pieces of paper. If you have more of these pieces of paper than you need, others will give you valuable things in return. Paper money gains value in exchange because it is valuable on tax day. This seems pretty simple and obvious, but as you will see it leads to all sorts of surprising conclusions. (In fact, acceptance of money for tax payments is neither necessary nor sufficient for the fiscal theory. The government could accept goods or foreign currency and then sell those to soak up cash, But this is a good story to start with in order to understand the intuition, and we’ll generalize later.)

The fiscal theory gains interest by contrast with the more common current theories of inflation, and its simple insight solves the problems of those theories. Briefly, there are three main alternative theories of the price level. First, money may be valued because it is explicitly backed: the government promises 1/32 of an ounce of gold in return for each dollar. This theory no longer applies to our economy. We will also see that it is really just an interesting instance of the fiscal theory, as the
government must have the gold to back the dollars, or be able to get the gold by taxation. Second, money may be valued even though it is intrinsically worthless, if people need to hold some money to make transactions or for other needs – “money demand” – and if there is a restricted supply of that money. This is the most classic view of fiat money (money with no intrinsic value, redemption promise, or other backing), and it pervaded the analysis of inflation until about 10 years ago. But current facts challenge it: transactions require less and less non-interest-bearing cash, and our governments do not control money supply. Third, starting in the late 1970s, a novel theory emerged that inflation could be stable under an interest rate target if the interest rate varies more than one for one with inflation, following what became known as the Taylor principle. We will analyze the problems with this view in detail below. In particular, the fact that inflation has remained stable and quiet even when interest rates do not move in long-lasting zero bound episodes contravenes this theory.

The fiscal theory then is an alternative to these three great, and classic, theories of inflation. Other than the fiscal theory, then, there is no simple, coherent, economic theory of inflation that is vaguely compatible with current institutions. This may seem like an audacious claim, but I will back it up.

Macroeconomic models are built on these basic theories of the price level, plus extensive descriptions of people’s saving, consumption, and investment behavior, how labor markets work, and frictions in price or wage setting or financial markets. Such models are easily adapted to the fiscal theory instead of alternative theories of inflation, leaving the rest of the structure intact. Procedurally, changing this one ingredient is easy. But the results of economic models often change a lot if you change just one ingredient.

Let’s jump right in to see what the fiscal theory is and how it works, and then compare it to other theories.
Chapter 2

Simple models

This chapter introduces the fiscal theory with two very simple models. The first model lasts only one period. The second model is intertemporal, and includes a full description of the economic environment. Both models have perfectly flexible prices, constant interest rates, and no risk premiums.

2.1 A one-period model

We look at the one-period frictionless fiscal theory of the price level

\[ \frac{B_{T-1}}{P_T} = s_T. \]

In the morning of day \( T \), bondholders wake up owning \( B_{T-1} \) one-period zero-coupon government bonds coming due on day \( T \). Each bond promises payment of $1. The government pays bondholders by printing up new cash. People go about their business. They may use this cash to buy and sell things, but that is not essential to the theory.

At the end of the day, the government requires people to pay taxes \( P_T s_T \) where \( P_T \) denotes the price level. For example, the government may levy a proportional tax \( \tau \) on income, in which case \( P_T s_T = \tau P_T y_T \) where \( y_T \) is real income and \( P_T y_T \) is nominal income. Taxes soak up money.
Nobody wants to hold cash after the end of the day. In equilibrium, then, cash printed up in the morning must all be soaked up by taxes at the end of the day,

\[ B_{T-1} = P_T s_T \]

or

\[ \frac{B_{T-1}}{P_T} = s_T. \]  \hspace{1cm} (2.1)

Debt \( B_{T-1} \) is predetermined. The price level \( P_T \) must adjust to satisfy (2.1).

We just determined the price level. And this model has none of the customary frictions – there is no money demand, no sticky prices, and no other deviation from pure Arrow-Debreu economics. This is the fiscal theory of the price level.

### 2.2 Intuition of the one-period model

The mechanism for determining the price level can be interpreted as too much money chasing too few goods, as aggregate demand, or as a wealth effect of government bonds.

The fiscal theory does not feel at all strange to people living in it. The fiscal theory differs on the measure of how much money is too much, and the source of aggregate demand.

We quickly meet “passive” policy and confusions about budget constraints.

If the price level \( P_T \) is too low, more money was printed up in the morning than will be soaked up by taxes in the evening. People have, on average, more money in their pockets than they need to pay taxes, so they try to buy goods and services. There is “too much money chasing too few goods and services.” “Aggregate demand” for goods and services is greater than “aggregate supply.” Economists trained in either the Chicago or Cambridge traditions living in this economy would not, superficially, notice anything unusual.

The difference from the standard (Cambridge) aggregate demand view lies in the source and nature of aggregate demand. Here, aggregate demand results directly and only as the counterpart of the demand for government debt. We can think of the fiscal theory mechanism as a “wealth effect of government bonds,” again tying the fiscal theory to classical ideas. Too much government debt, relative to surpluses, acts like net wealth which induces people to try to spend, raising aggregate demand.
The difference from the standard (Chicago) monetary view lies in just what is money, what is the source of money demand, and therefore how much money is too much. Here, inflation results from more money in the economy than is soaked up by net tax payments, not by more money than needed to mediate transactions or to satisfy other sources of demand for money.

There are two classic theories of the value of paper – and now electronic – money. In the first, money is a unique liquid asset, which people demand despite a poor rate of return, for example to make transactions. When this demand is intersected with a limited supply, the price level is determined. The other classic theory of money posits that money is valued because it is backed. For example, in an idealized gold standard, the government promises to trade each dollar for 1/32 ounce of gold, and the government has enough gold or the ability to get gold to make good on that promise.

The fiscal theory of the price level is a backing theory. Dollars are backed by the government’s tax revenues. The mechanism of that backing can take many forms.

The fiscal theory generalizes the idea that money is valued because the government accepts its money in payment of taxes. As such, the fiscal theory has a long history. The requisite Adam Smith quote (thanks to Ross Starr):

“A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.” (Wealth of Nations, Vol. I, Book II, Chapter II).

My story about money printed up in the morning and soaked up in the afternoon helps to fix intuition, but it is not essential. People could redeem debt for money 5 minutes before using the money to pay taxes. Or they could just pay taxes with maturing government bonds.

How people make transactions is irrelevant. People could make transactions with maturing bonds, with foreign currency, or Bitcoins. People could make transactions with debit cards or credit cards linked to bank accounts, netted at the end of the day with no money changing hands, which is roughly how we do things today. The dollar can be a pure unit of account, with nobody ever holding actual dollars.

Paying taxes in dollars or maturing government debt that promises dollars is not essential. Dollars (paper) are freely convertible to reserves, accounts banks hold at the Fed. So taxes paid by check or credit card ultimately do deliver reserves to the Treasury’s account at the Fed. But people could pay taxes by delivering foreign
currency, gold coins, Bitcoins, or wheat and goats as in the old days. The government sells these commodities to soak up money or maturing government bonds. What matters is that the government uses real tax revenues in excess of spending to soak up any excess dollars and thereby maintain their value. Offering the right to pay taxes with money, or requiring such payment, is a useful way of communicating and pre-committing to fiscal backing, and we shall see that lots of institutions (such as the gold standard) are used in this effort.

The fiscal theory, like other backing theories, can determine the price level in a frictionless economy – an economy in which money has no extra value from use in transactions or other special features; one in which people do not carry around an inventory of a special low-interest asset, or one in which the government does not limit the supply of such assets. Since our economies are getting more and more frictionless and cashless, and our governments do not limit the supply of transactions-facilitating assets and technologies, this aspect makes the fiscal theory an empirically attractive starting point for monetary economics today. The alternative “cashless limit” in which the price level is still determined by a nearly zero demand for cash intersected with a tightly controlled but still nearly zero supply is obviously fragile.

One can and should add frictions. Cash and government debt may gain an extra value over their backing, or they can offer a lower rate of return than other assets, because they are especially useful in transactions and the financial system, if the government limits their supply. But the fiscal theory allows us to start to analyze the price level with a simple frictionless backing model and to add frictions on top of that. Conventional theories require frictions to even begin to talk about the price level.

2.3 Budget constraints and passive policies

I preview two theoretical controversies.

$B_{T-1}/P_T = s_T$ is an equilibrium condition, not a budget constraint. The government could leave cash $M_T$ outstanding overnight, it is people who don’t want it.

The government may choose to set surpluses $s_T$ so that $B_{T-1}/P_T = s_T$ for any $P_T$, and then the fiscal theory would not determine the price level. This is called a “passive fiscal” policy. This is a choice, however, not a budget constraint. It is also not a natural outcome of a proportional tax system.
This simple model helps us to quickly preview the resolution of some central theoretical quibbles.

First, isn’t equation (2.1) the government’s budget constraint? Shouldn’t we solve it for the surplus $s_T$ that the government must raise to pay off its debts, given the price level $P_T$? Budget constraints must hold for any price, and limit quantities given prices, not the other way around. You and I certainly can’t fix our real repayment, and demand that the price level adjust to bring the value of our nominal debts in line. Are we specifying, perhaps incorrectly or incoherently, that the government is some special agent that can threaten to violate its budget constraint at off-equilibrium prices?

No. Equation (2.1) is not a budget constraint. The government’s budget constraint is

$$B_{T-1} = P_T s_T + M_T$$

(2.2)

where $M_T$ is money left over at the end of the day. (Even this relation assumes the government does not default on its promise to exchange debt $B_{T-1}$ for money, and that people redeem all the debt $B_{T-1}$ for money.) If people decide to line their caskets with money at the end of the day $M_T > 0$, no budget constraint stops them from doing so, and no budget constraint forces the government to soak up the money with taxes. The government may leave money outstanding at the end of the day.

Consumer demand is why $M_T = 0$. Equation (2.1) results from the budget constraint (2.2) plus consumer demand. Equation (2.1) is thus an equilibrium condition, a market-clearing condition, a supply = demand condition, deriving from budget constraints and consumer optimization. Equilibrium conditions do not hold at off equilibrium prices, and prices adjust to make them hold. There is no reason that equation (2.1) should hold at a non-equilibrium price, any more than supply = demand for potatoes should hold at $10 per potato. As in this example, we must be careful not to substitute natural private-sector demands into an apparent “budget constraint” for the government.

A policy in which the government adjusts surpluses $s_T$ so as to make the equilibrium condition (2.1) hold for any price level $P_T$, following $s_T = B_{T-1}/P_T$, is known as a “passive” fiscal policy. If the government follows such a policy, $P_T$ cancels from left and right, and (2.1) no longer determines the price level. In essence the government’s supply curve lies directly on top of the private sector’s demand curve. A government that wishes to let the price level be set by other means, such as a foreign exchange peg, a gold standard, use of another government’s currency, or $MV = PY$ once the model is expanded to include money demand, follows a passive fiscal policy.
Standard theories of inflation thus do include equation (2.1), but they include it in this “passive” way, and specifying the fiscal policy that achieves that “passive” response is an important part of “monetary-fiscal policy coordination.” In these cases (2.1) does start to act somewhat like a budget constraint, in that it determines what surpluses must be given the price level to avoid default.

We will consider these alternative regimes, but for now the important point is that the government does not have to follow a passive fiscal policy. An “active” fiscal policy – one in which $s_T$ is set, potentially responding to the price level $P_T$, $s_T(P_T)$ or other variables, but in such a way that there is only one solution to (2.1), one $P_T$ such that $B_{T-1}/P_t = s_T(P_T)$ – is a logical and economic possibility, one that does not violate any of the rules of Walrasian equilibrium.

(The “active” and “passive” labels are due to Leeper (1991). The label is not perfect, as “active” fiscal policy here means leaving surpluses alone, and “passive” policy means adjusting them according to the price level, but the terminology is what it is. The same possibilities are sometimes called “money-dominant” vs. “fiscal-dominant,” which isn’t bad, or “Ricardian” vs. “non-Ricardian,” which is terrible. It is not true that fiscal-dominant regimes fail to display Ricardian equivalence, and government debt is a free lunch.)

Though a passive fiscal policy is a possible choice, there is nothing necessary or natural about a passive policy. In the simple case of a proportional tax on income, $P_Ts_T = \tau P_Ty_T$ the real surplus $s_T = \tau y_T$ is independent of the price level, and an active fiscal policy results. To engineer a passive policy, the government must change the tax rate after the fact as a function of the price level. For $s_T = B_{T-1}/P_T = \tau_T y_T$ we must have $\tau_T = B_{T-1}/(P_T y_T)$. This rule features a lower tax rate for a higher price level: A higher price level devalues nominal government debt, so the government lowers the tax rate. The U.S. tax code generates the opposite sign, raising tax rates when there is inflation. Inflation pushes people to higher tax brackets, generates capital gains, and devalues depreciation allowances and past losses carried forward. Governments facing inflation and desiring to fight it raise taxes and cut spending, not the other way around. So a passive policy is a deliberate choice, not a natural outcome of a proportional or progressive tax system.

Why can’t you and I “threaten to violate our budget constraints at off-equilibrium prices” and thus demand that the price level adjust? Because we don’t issue the currency that defines the price level. You and I are like a government that uses someone else’s currency. Ours are budget constraints, or more precisely no-default conditions.
2.4 A BASIC INTERTEMPORAL MODEL

2.4 A basic intertemporal model

We derive the simplest intertemporal version of the fiscal theory, the government debt valuation equation,

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}.
\]

The price level adjusts so that the real value of nominal debt equals the present value of future surpluses.

The one-period model is conceptually useful, but to begin to think about real economies we need a model that describes economies over time. It is also useful to fill out economic foundations to see a complete economic model. This model is simplified down to its essentials. We will generalize it in many ways later.

At the end of each time period \( t - 1 \) the government issues nominal one-period debt \( B_{t-1} \). Each nominal bond promises to pay one dollar at time \( t \). At the beginning of time \( t \), the government prints up new money to pay off the maturing debt. At the end of period \( t \), the government collects net taxes \( s_t \). Taxes must be paid in money. The government also sells new debt \( B_t \) at a price \( Q_t \). Both actions soak up money.

The government budget constraint is

\[
M_{t-1} + B_{t-1} = P_t s_t + M_t + Q_t B_t
\]

(2.3)

where \( M_{t-1} \) denotes non-interest paying money held overnight from the evening of \( t - 1 \) to the morning of time \( t \), \( P_t \) is the price level, \( Q_t = 1/(1 + i_t) \) is the one period nominal bond price and \( i_t \) is the nominal interest rate. Interest is paid overnight only, from the end of date \( t \) to the beginning of \( t + 1 \), and not during the day at time \( t \).

A representative household maximizes

\[
\max E \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

in a complete asset market. The economy has a constant endowment \( c \). Net taxes are a flat proportion of income

\[
P_t s_t = \tau_t P_t c_t.
\]
The household’s period budget constraint is the mirror of (2.3). Household money and bond holdings must be non-negative, $B_t \geq 0, M_t \geq 0$.

The consumer’s first order conditions and equilibrium $c_t = c$ then imply that the gross real interest rate is $R = 1/\beta$, and the nominal interest rate $i_t$ and bond price $Q_t$ are

$$Q_t = \frac{1}{1 + i_t} = \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right). \tag{2.4}$$

When $i_t > 0$ the household demands $M_t = 0$. When $i_t = 0$ money and bonds are perfect substitutes, so the symbol $B_t$ can stand for their sum. The interest rate cannot be less than zero in this model. Thus, we can eliminate money from (2.3), substitute for the bond price (2.4), and write

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R} B_t E_t \left( \frac{1}{P_{t+1}} \right). \tag{2.5}$$

In addition to the intertemporal first order conditions, household maximization and equilibrium $c_t = e$ imply the household transversality condition

$$\lim_{T \to \infty} \frac{1}{R^T} \left( \frac{B_{T-1}}{P_T} \right) = 0. \tag{2.6}$$

If the expected value $E_t$ of this condition is positive, then the consumer can raise consumption at time $t$, lower this terminal value, and raise utility. Non-negative debt $B_t \geq 0$ rules out a negative value.

As a result, we can iterate (2.5) to

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \tag{2.7}$$

The government sets debt and surpluses $\{B_t\}$ and $\{s_t\}$ via $\{\tau_t\}$. The government can fully pre-commit to its policy choices.

The surplus concept here – taxes collected minus spending – is the primary surplus in government accounting. The usual “deficit” or “surplus” includes interest payments on government debt, which are not included here.
2.5. DYNAMIC INTUITION

Debt $B_{t-1}$ is predetermined. Surpluses don’t respond to the price level by the assumption $s_t = \tau_t c_t$ and the assumption that the tax rate does not respond to the price level. (We’ll generalize that.) The real interest rate $R$ also does not respond to the price level. (We’ll generalize that too.) The right side of (2.7) does not depend on the price level. Therefore, the price level must adjust so that (2.7) holds – so that the real value of nominal debt equals the present value of real primary surpluses.

We have determined the price level, in a completely frictionless intertemporal model. Equation (2.7) is the simplest workhorse dynamic version of the fiscal theory of the price level.

2.5 Dynamic intuition

The fiscal theory is an instance of the basic asset pricing valuation equation. Nominal government debt is a claim to primary surpluses. The price level is like a stock price, and adjusts to bring the real value of nominal debt in line with the present value of primary surpluses, just as the stock price adjusts to bring the value of shares in line with the present value of dividends.

The right hand side of (2.7) is the present value of future primary surpluses, tax revenues less spending, not including interest payments on the outstanding debt. The left hand side is the real value of nominal debt. So, the fiscal theory says that the price level adjusts so that the real value of nominal debt is equal to the present value of primary surpluses.

We recognize in (2.7) the basic asset pricing equation, price per share $1/P_t$ times number of shares $B_{t-1}$ equals present value of dividends $\{s_{t+j}\}$. We quote the price level as the price of goods in terms of money, not the price of money in terms of goods, so the price level goes in the denominator not the numerator. Primary surpluses are the “dividends” that retire nominal government debt. In an accounting sense, nominal government debt is a residual claim to primary surpluses.

The fact that the price level can vary means that nominal government debt is an equity-like, floating-value, claim, not a debt-like, fixed-value claim. If the present value of surpluses falls, the price level can rise to bring the real value of debt in line, just as a stock price falls to bring market value of equity in line with the expected present value of dividends. Nominal government debt is “stock in the government.”
Continuing the analogy, suppose that we decided to use Microsoft stock as numeraire and medium of exchange. When you buy a cup of coffee, Starbucks quotes the price of a venti latte as 1/10 of a Microsoft share, and to pay you swipe a debit card that transfers 1/10 of a Microsoft share in return for your coffee. If that were the case, and we were asked to come up with a theory of the price level, our first stop would be that the value of Microsoft shares equals the present value of its dividends. Then we would add liquidity and other effects on top of that basic idea. That is exactly what we do with the fiscal theory.

This perspective also makes much sense of a lot of commentary. Exchange rates go up, and inflation goes down, when an economy does better, when productivity increases, when governments get their budgets under control. Well, money is stock in the government.

Back ing government debt by the present value of surpluses allows for a more stable price level than the one-period model suggests. If the government needs to finance a war or to counter a recession or financial crisis, it can soak up dollars by debt sales rather than a current surplus, and avoid inflation. For that to work, however, the government must persuade investors that more debt today will be matched by higher surpluses in the future.

Surpluses are not “exogenous” in the fiscal theory! Surpluses are a choice of the government, via its tax and spending policies and via the fiscal consequences of all its policies. Surpluses may react to events, for example becoming greater as tax revenues rise in a boom. Surpluses may also respond to the price level, by choice or by non-neutralities in the tax code and expenditure formulas. We only have to rule out or treat separately the special case of “passive” policy that surpluses react exactly one-for-one to the price level so that equation (2.7) holds for any price level \( P_t \).

I have started with the simplest possible economic environment, abstracting from monetary frictions, financial frictions, pricing frictions, growth, default, risk and risk aversion, quantity fluctuations, limited government pre-commitment, and so forth. We will add all these ingredients and more. But starting the analysis this way emphasizes that no additional complications are necessary to determine the price level.

As you can see, the fiscal theory is not an “always and everywhere” theory of inflation. It relies on specific institutions. The model shown here has its own currency and issues nominal government debt. We use maturing debt, or the currency it promises, as numeraire and unit of account. The government does not choose to follow a
2.5. DYNAMIC INTUITION

policy of raising tax rates systematically validate any price level, the “passive policy” special case, and the government chooses to inflate rather than default in the event of intractable deficits. (Default probabilities are easy to add.) In the end we need an equation with something nominal on one side, and something real on the other. We will generalize most of these assumptions, but not fully. This is not a theory of clamshell money, or of Bitcoins. It is a theory adapted to our current institutions: fiat money, rampant financial innovation, interest rate targets, governments that will generally inflate rather than explicitly default, and the consensus that short-term government debt is the safest asset in the economy, and the one around which it is sensible to build a monetary and financial system. The former have not been true at all times historically. The latter may be the weak point in our institutions going forward. If we experience a serious sovereign debt crisis, not only will the result be inflation, it will also be an unraveling of our payments, monetary, and financial institutions. Then, we shall have to write an entirely new book, of monetary arrangements that are insulated from sovereign debt. Let us hope that day does not come to pass anytime soon.

Back to the fiscal theory. Our central question is to find policies that allow the government to control the price level via (2.7). Clearly, if the government sets \( \{B_t\} \) and \( \{s_t\} \), then as long as \( (2.7) \) gives a positive result, it determines \( \{P_t\} \). But this is not a realistic policy. So the first step is to describe more realistic policies.
Chapter 3

Fiscal and monetary policy

This chapter introduces “monetary policy,” changes in debt $B_t$ with no change in surpluses, as opposed to “fiscal policy,” which changes surpluses. “Monetary” (no surplus change) and “fiscal” debt issues are analogues to share splits vs. equity offerings. This insight suggests a reason for the institutional separation between treasury and central bank. “Fiscal stimulus” can cause inflation.

Monetary policy can target the nominal interest rate. A fiscal theory of monetary policy emerges that looks much like standard new-Keynesian models. Therefore the “fiscal” theory of the price level does not require us to think about inflation in terms of debts and surpluses; we can approach the data very much as standard new-Keynesian modelers do, starting with interest rate targets. Technically, adapting standard new-Keynesian models to FTPL is straightforward. The answers are quite different however.

Distinguishing FTMP from new-Keynesian and monetarist alternatives introduces deep observational equivalence theorems. These are useful guideposts for thinking about how to approach data.

This chapter introduces these ideas in the context of the very simple model studied so far – one period debt, perfect price flexibility, an endowment economy with a constant real interest rate and no risk premium. Later chapters add price stickiness, discount rate variation, risk premiums and other realistic complications.
3.1 Expected and unexpected inflation; monetary and fiscal policy

I break the basic present value relation into expected and unexpected components, giving

\[
\frac{B_t}{P_t} (E_{t+1} - E_t) \left( \frac{P_t}{P_{t+1}} \right) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j},
\]

\[
\frac{B_t}{P_t} \left( \frac{1}{1+i_t} \right) = \frac{B_t}{P_t} R_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j},
\]

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t} \left( \frac{1}{1+i_t} \right) = E_i \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]

In this model, unexpected inflation results entirely from innovations to expected fiscal policy \( \{s_t\}\). Monetary policy – a change in \( B_t \) with no change in \( \{s_t\} \) – can determine the nominal interest rate and expected inflation. The government can also target nominal interest rates, and thereby expected inflation, by offering to sell any amount of bonds at the fixed interest rate. A rise in debt \( B_t \) accompanied by an equal increase in subsequent surpluses has no effect on the interest rate or price level. Such a debt issue raises revenue to fund a current deficit – lower \( s_t \).

Policy is described by two settings, nominal debt \( \{B_t\} \) and surpluses \( \{s_t\} \). We will spend some time thinking about their separate effects: What if the government changes nominal debt without changing surpluses, or vice versa? Almost all actual policy actions consist of simultaneous changes of both instruments, so this separation is not that useful to understanding historical episodes. But it will let us understand the mechanics of the theory more clearly to answer this conceptual question.

We will learn a lot by breaking the basic government debt valuation equation into expected and unexpected components. It will be clearer to move the time index forward and to start with

\[
\frac{B_t}{P_{t+1}} = E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}. \tag{3.1}
\]

Multiply and divide by \( P_t \), and take innovations \( (E_{t+1} - E_t) \) of both sides, giving

\[
\frac{B_t}{P_t} (E_{t+1} - E_t) \left( \frac{P_t}{P_{t+1}} \right) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}. \tag{3.2}
\]
3.2 Fiscal policy and unexpected inflation

As of time $t+1$, $B_t$ and $P_t$ are predetermined. Therefore, in this simple model,

- Unexpected inflation is determined entirely by changes in expectations about the present value of fiscal surpluses.

If people lower their expectations of future surpluses, the value of the debt must fall. People try to get rid of debt. With only one-period debt outstanding, and leaving aside default for now, the relative price or quantity of debt cannot fall, so all people can do is to try to buy goods and services. They drive the price level up until the value of the debt once again equals the expected value of surpluses. Unexpected inflation acts like a partial default. The same mechanism creates inflation if the discount rate $R$ applied to government debt rises, and we will see this mechanism is important in understanding events.

What’s missing is important in (3.2): $B_{t+1}$ and beyond. Unexpected inflation at time $t + 1$ depends *only* on revisions to expectations of surpluses from time $t + 1$ onwards, or their discount rates. Unexpected inflation is, in this model, completely unaffected by the decision $B_{t+1}$ at the end of time $t + 1$ to sell more or less debt, and similar later decisions.

3.3 Monetary policy and expected inflation

Next, multiply and divide (3.1) by $P_t$, and take the expected value $E_t$ of both sides, giving

$$\frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+1+j}.$$  

Multiplying by $1/R$, and recognizing the one-period bond price and interest rate in

$$Q_t = \frac{1}{1 + i_t} = \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right),$$  

we can write

$$\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \frac{1}{R_j} s_{t+j}.$$  

(3.4)
The first term is the revenue the government raises from selling bonds at the end of period $t$. The last term expresses the fact that this revenue equals the present value of surpluses from time $t + 1$ on. The outer terms thus express the idea that the real value of debt equals the present value of surpluses, evaluated at the end of period $t$. The inner equality tells us about expected inflation, the counterpart of the unexpected-inflation relation (3.2):

Now, examine equation (3.4), and consider what happens if the government sells more debt $B_t$ at the end of period $t$, without changing surpluses $\{s_t\}$. $P_t$ is already determined by (3.2) at time $t$. If surpluses do not change, the bond price, interest rate, and expected future inflation must move one for one with the debt sale $B_t$. Therefore,

- The government can control interest rates $i_t$, bond prices $Q_t$ and expected inflation $E_t(P_t/P_{t+1})$, by changing the amount of debt sold $B_t$ with no change in surpluses.

If the government does not change surpluses as it changes debt sales $B_t$, then it always raises the same revenue $Q_tB_t/P_t$ by bond sales. Equation (3.4) with constant surpluses describes a unit-elastic demand curve for nominal debt – each 1% rise in quantity gives a 1% decline in bond price, since the real resources that will pay off the debt are constant.

Bond sales without changing surpluses are like a share split. When a company does a 2-for-1 share split, each owner of one old share receives two new shares. People understand that this change does not imply any change in expected dividends, so the price per share drops by half and the total value of the company is unchanged. As of the morning of $t+1$, then, additional bonds $B_t$ with no more surplus are like a currency reform, and imply an instant and proportionate change in price level.

Rather than announce an amount of debt $B_t$ to be sold, the government can also announce the price or interest rate $i_t$ and then offer markets all the debt $B_t$ they want to buy at that price, while offering no change in surpluses. A horizontal rather than vertical supply curve of debt can intersect the unit-elastic demand for government debt. In that case, equation (3.4) describes how many bonds the government will sell at the fixed price or interest rate, and verifies that this quantity is not infinite, zero, negative, or otherwise pathological.

- The government can target nominal interest rates.

This is an initially surprising conclusion. You may be used to stories in which targeting the nominal rate requires a money demand curve, and reducing money supply
raises the interest rate. You might have thought that trying to peg the nominal rate in a frictionless economy would lead to infinite demands, or other problems. Equation (3.4) denies these worries. The debt quantities are not unreasonably large either. To raise interest rates one percentage point, the government has only to sell one percent more debt.

(Terminology: An interest rate *peg* usually means an interest rate that is constant over time and does not respond to other variables. A time-varying peg moves over time but does not respond systematically to other variables. An interest rate *target* means that the government sets the nominal interest rate, but may change that rate over time and also in response to other variables such as inflation and unemployment.)

Contrary intuition comes from different, and more common, assumptions. The proposition here is only that the government can fix its nominal rate. An attempt to fix the real rate in this model would lead to infinite demands. The proposition here says that surpluses are constant while the government sells more debt. If people always read into any debt sale an implicit promise of proportionally higher future surpluses, then again bond demand is either undefined, if the offered rate equals the real interest rate, or infinite one way or the other, if the offered rate is larger or lower than the current real interest rate.

Still, it is a classic doctrine that the government cannot peg an interest rate, and an attempt to do so will lead to unstable or indeterminate inflation. That view includes monetarism and both new and old Keynesian monetary theories. Here, we overturn that classic doctrine. Those theories assume passive fiscal policy; here we do not make that assumption.

I will use the label “monetary policy” to describe setting an interest rate target or changing the quantity of debt without direct control of surpluses. Central banks buy and sell government debt in return for cash, and our model represents the limit point of such operations when demand for overnight cash vanishes and prices are infinitely flexible. Central banks cannot, at least directly, change fiscal policy – they must always trade one asset for another. They may not write checks to voters; they may not drop money from helicopters. Those are fiscal policy operations. I will likewise use the label “fiscal policy” to describe changes in surpluses. We’ll spend some time later generalizing these ideas, including indirect surplus effects of central bank actions, and mapping these concepts to current institutions.

With this terminology, we now have a summary of this section so far:

- *Monetary policy can target the nominal interest rate, and determine expected...*
inflation, even in a completely frictionless model. Fiscal policy alone determines unexpected inflation.

You might have thought “fiscal theory” would lead us entirely to think about inflation in terms of debt and deficits. We now learn this is not the case. “Monetary policy,” either choosing nominal debt \( \{B_t\} \) or interest rates \( \{i_t\} \), with no effect at all on deficits and surpluses, fully controls expected inflation in this simple model, leaving fiscal policy only to control unexpected inflation. We will generalize this insight to a “fiscal theory of monetary policy” based on interest rate targets, starting in Chapter 4 in this frictionless context and then fully developed with sticky prices in Chapter 7.

### 3.4 Fiscal policy and debt

Monetary policy as I have defined it consists of setting interest rate targets, implemented by changing debt \( B_t \) without changing surpluses. Fiscal policy does change surpluses. But fiscal policy also changes debt while it changes surpluses. Fiscal policy does not consist of setting a sequence of surpluses and deficits \( \{s_t\} \), ignoring debts that finance deficits and are paid off by surpluses.

To gain a picture of standard fiscal policy operations, use the flow relation (2.3) and (3.4) to work back from the end of period \( t \) to its beginning, and to write

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{B_t}{P_t} = E_t \sum_{j=0}^\infty \frac{1}{R^j} s_{t+j}. 
\]  

(3.5a)

(As opposed to (3.4), the sum starts at \( j = 0 \) not \( j = 1 \).) Recall also three different ways of writing the middle term as bond price and expected inflation via (3.3).

Consider now what happens if the government unexpectedly sells more debt \( B_t \) at the end of period \( t \), and now \textit{does} change future surpluses \( \{s_{t+1}, s_{t+2}, \ldots\} \). Looking at (3.4), if the government raises future surpluses enough, then the rise in debt \( B_t \) can imply no change at all in the current interest rate \( i_t \), bond price \( Q_t \) and expected inflation. Now the government does raise greater revenue, \( Q_t B_t / P_t \) rises. But the far left side of (3.5a) \( B_{t-1}/P_t \) is unchanged, and so is the right side, including the \( j = 0 \) term. So raising revenue and future surpluses \( s_{t+j} \) means that the government can lower the current surplus \( s_t \).
This fiscal policy rise in debt $B_t$ with higher expected future surpluses is like an equity issue, as contrasted with a share split. In an equity issue, a firm also increases shares outstanding. But now it promises to increase future dividends. The revenue from the share issue are used to fund investments that will generate more dividends. So a share issue does raise revenue and does not change the stock price, where a share split raises no revenue and does change the stock price. The only difference between a share issue and a split is expected dividends.

This is regular fiscal policy. The government issues debt to fund a current deficit. When it issues debt, it promises, explicitly or implicitly, to raise future surpluses. By doing so, it raises revenue from the debt sales. The revenue raised is a direct measure of how much the government has, in fact, persuaded markets that it will raise future surpluses to pay off the debt. If investors are skeptical of promises, interest rates rise, bond prices fall, and the government raises less revenue. By raising revenue and future $s_{t+j}$ – which may start quite some time in the future – the government funds the current deficit $s_t$. Raising debt $B_t$ in this way implements a rearrangement of the sequence $\{s_t, s_{t+1}, s_{t+2}, \ldots\}$ that does not change their present value, and so does not imply any change in $P_t$.

In sum,

- **Normal fiscal policy consists of debt sales that finance current deficits. Such sales promise higher future surpluses, and do not change interest rates or the price level.**

Attractive as these conceptual experiments are, however, beware that most events and policy interventions mix the three possibilities. Data and events are unlikely to contain a pure “fiscal” or “monetary” policy shocks. Fiscal authorities are likely to respond to the same events as do monetary authorities. So we are likely to see $\{s_t\}$ shocks that cause unexpected inflation, interest rates shocks that affect expected inflation, and debt plus future surplus shocks that rearrange the surplus process, all at the same time.

### 3.5 The central bank and treasury

The institutional division that the Treasury conducts fiscal policy and the central bank conducts monetary policy works like the institutional division between share splits and secondary offerings. Treasury issues come with promises of future surpluses. Fed open market operations do not.
The central bank sets interest rates, and then the Treasury sells bonds given interest rates to finance deficits. In this two-step process the government overall sells debt at fixed interest rates, a flat supply curve.

To create a fiscal inflation, the government must persuade people that increased debt will not be paid back by higher future surpluses. That has proved difficult to accomplish.

The “monetary policy” debt sale and the “fiscal policy” debt sale of the last section look disturbingly similar. All we see is that the government sells more debt. One debt sale engenders expectations that future surpluses will not change. That sale changes interest rates and expected inflation, and raises no revenue. The other debt sale engenders expectations that future surpluses will rise to pay off the larger debt. It raises revenue with no change in interest rates or prices. Open market operations and Treasury issues even look the same: government debt goes one way, cash or reserves go the other way. How does the government achieve these miracles of expectations management?

Answering this question is important, first, to solidify our understanding of the simple frictionless model as a sensible abstraction of current institutions. Second, however, it is the first instance of something that we shall see many times: the importance of monetary institutions. In this case, and more dramatically following the observational equivalence discussion of the next chapter, it is important for governments (or, perhaps someday, private parties or cryptocurrency programmers) to form commitments and to communicate commitments about how they will behave in distant, infrequently, or even never-observed circumstances. Monetary institutions serve that role.

Stock splits and secondary offerings also look disturbingly similar. All we see is that the company issues more shares. A split engenders expectations that overall dividends will not change, so a 2:1 split cuts the stock price per share in half. An offering engenders expectations that total dividends will rise, so the price per share is unaffected and the company gets new funds for investment. (Yes, a long literature in finance studies small effects of offerings on price, as the decision to issue shares may reveal information about the company. Absent such information, though, the share price stays constant.)

Companies achieve this miracle of expectations management by issuing shares in carefully differentiated institutional settings, along with specific announcements and disclosures. Companies do not just increase shares and let the market puzzle out
their own expectations. The carefully differentiated institutional settings convey the clearly different expectations. The results then embody the intent of the company, either to change its price per share or to raise investment capital.

This parallel helps us to understand the institutional separation between central banks and treasuries. The Treasury conducts “fiscal policy” debt sales. Historically, many federal debt issues were passed by Congress for specific and transitory purposes, and backed by specific tax streams (see Hall and Sargent (2018)). That legal structure is an obvious aid to assuring repayment, i.e. to promising higher future surpluses. Many state and municipal bonds continue these practices. The gold standard also gave a promise to repay rather than inflate. It was not ironclad as governments could and did suspend convertibility, but it was helpful. U.S. Federal debt now has no explicit promises, but the Treasury, and Congress, have earned a reputation for largely paying back debts incurred by Treasury issues, going back to Alexander Hamilton’s famous assumption of revolutionary war debt. Large debts, produced by borrowing, produce political pressure to raise taxes or cut spending to pay off the debts, part of Hamilton’s point. Hall and Sargent (2014) note that Hamilton did not repay colonial currency, which largely inflated away. That seems like a default, but it also emphasizes different promises implicit in currency vs. debt. The implicit promise to repay debt has also not always been ironclad, but it has helped.

In the end, the idea that Treasury debt sales engender expectations that surpluses will eventually be raised to pay back additional debt issues, and thereby Treasury sales raise revenue rather than just create higher interest rates and expected inflation, is now standard – so much so that the possibility of an opposite share-split-like assumption may seem weird. Other governments are not so lucky, or have lost confidence and reputation. In times of fiscal stress, debt issues fail or push up interest rates. You can only signal so much, and reputations are finite.

“Monetary policy” is conducted by a different institution. The Federal Reserve’s legal authority roughly requires it not to change current or future surpluses. The Fed must always buy something in return for issuing cash or reserves. Other central banks have similar legal limitations. The list of securities central banks may buy is typically limited to high-quality fixed income securities, to avoid risk that eventually floats back to the Treasury.

Central banks are legally forbidden direct fiscal policy. They cannot alter tax rates or expenditures directly. At most central bankers can give speeches imploping fiscal authorities for stimulus or restraint. Though central banks are mandated to control inflation, central banks are legally forbidden from “helicopter drops,” perhaps the
most effective, means of inflating. Central banks cannot write checks to people or businesses, issuing money without a buying a corresponding asset. They can lend, not give. Central banks doubly cannot conduct a helicopter vacuuming, confiscating money from people and businesses without issuing a corresponding asset, though that is the most direct way to stop inflation! Only the Treasury may write checks to voters or confiscate their money, and for many good political as well as economic reasons. Central banks are mandated to control inflation, and not to meddle in fiscal affairs. The Treasury and Congress are expected to conduct fiscal policy and not to meddle with inflation. At most central bankers can give speeches promoting fiscal stimulus or advocating fiscal responsibility, and the rest of the government can plead or tweet for more or less inflation.

Yes, central bank actions have indirect fiscal implications. In the presence of non-interest-paying currency, inflation produces seigniorage revenue, and has fiscal effects through an imperfectly indexed tax code. Central bank purchases of risky assets expose the Treasury to losses, or gains when the bets pan out. The Federal Reserve can lend pretty freely: it can create money, send it to a bank, and call the bank’s promise to repay an asset. (Even so, the Fed can only lend to banks, an important legal limit on inflationary finance.) With sticky prices, interest rate rises change the Treasury’s real interest expense. This is an important channel in the presence of large debts. Many central banks are charged to keep government interest expense low, as was the US Fed through WWII and into the 1950s. When monetary policy affects output, tax revenues and automatic expenditures change. We can and will model many of these indirect effects and generalize the definition of “monetary policy” to account for them.

Still, a central bank open market operation is a clearly distinct action from a Treasury issue. The latter by definition and immediately funds a deficit, and the former does not. The restriction against fiscal policy is closer to holding than not.

Our legal and institutional structures have many additional provisions against inflationary finance, adding to the separation between Treasury and central banks, and helping to guide expectations. The Treasury cannot sell bonds directly to the Fed. The Fed must buy any Treasury bonds on the open market, ensuring some price transparency and reducing the temptation to inflationary finance. In 2008, there were serious proposals for the Treasury to create inflation by minting billion-dollar coins, as the Fed had no authority for helicopter drops and the Treasury no authority to finance debts by printing currency. The tradition of central bank independence adds to the precommitment against inflationary finance.
In sum, the institutional separation between Treasury and central bank serves many important functions. Since expectations of future surpluses are somewhat nebulous, and since Treasury issues do not come with specific tax streams, it is important to have one institutional structure for selling more debt without raising revenue, without changing expected surpluses, and in order to affect interest rates and inflation; and a distinct institutional structure for selling debt that does raise revenue, does change expected future surpluses, and does not affect interest rates and inflation. This structure mirrors the different institutional structures for secondary offerings vs. share splits.

However, these observations should not stop us from institutional innovation. The current structure has evolved by trial and error to something that has seemed to work. But it certainly was not designed with this understanding in mind. We can think about better institutional arrangements. To stabilize the price level, how can the government minimize variation in the present value of surpluses, and commit to those surpluses? When the government wishes to inflate or to stop deflation, how can it better commit not to repay debts? This is a pressing policy concern today, as many people wish to deliberately inflate, or worry about stopping uncontrollable deflation in the next recession. Our institutional structure did not evolve to stop deflation or to create mild inflation. Large advanced country institutional structures also did not evolve to mitigate a potential sovereign debt crisis, which large short-term debts and unfunded promises leave as an enduring possibility. The euro debt crisis is only perhaps the first example of others to come. Can we construct something better than implicit, reputation-based Treasury commitments, along with implicit state-contingent defaults via inflation? Can we construct something better than nominal interest rate targets following something like a Taylor rule? We’ll come back to think about these issues.

### 3.6 The flat supply curve

In our simple model, the government fixes interest rates and offers nominal debt in a flat supply curve. In reality the Treasury auctions a fixed quantity of debt. The Treasury sets the quantity of debt after seeing the interest rate, raising that quantity if the bond price is lower, and thereby generating a flat supply curve. The Treasury and central bank acting together, therefore, generate a flat supply curve.

The U.S. and most other Treasuries auction a fixed quantity of debt. The above
CHAPTER 3. FISCAL AND MONETARY POLICY

description, in which a government sets interest rates by offering any amount of debt at the fixed interest rate, while holding surpluses constant, does not look realistic. However, on closer look, the horizontal supply mechanism can be read as a model of our central banks and Treasuries operating together, taken to the frictionless limit.

The central bank sets the short-term interest rate. It currently does so by setting the interest rate it pays to banks on reserves, and the discount rate at which banks may borrow reserves. Reserves are just short-term – overnight, floating-rate – government debt. Central banks allow free conversion of cash to interest-paying reserves at the fixed rate. Thus, paying interest on reserves and allowing free conversion to cash really is pretty much already the world of a fixed interest rate and a horizontal supply for overnight debt. In reality, people still hold cash overnight, but that makes little difference to the model, as we will shortly see by adding such cash. It just lowers interest expense a bit. Shifting overnight cash up from 0 to $\bar{M}$, transparently would make no difference.

Historically, central banks controlled interest rates by open market operations rather than by interest on reserves. They rationed non-interest bearing reserves or controlled the quantity of money, affecting $i$ via $M$ in $MV(i) = PY$. But central banks reset the quantity limit daily, forecasting demand for reserves that would result in the interest rate hitting the target. So on a daily basis, reserve supply was flat at the interest rate target.

One could stop here, and declare that Treasury auctions involve longer maturity debt which we have not yet included. (We will, and auctions make sense.) But there is another answer: even if the Treasury were auctioning overnight debt, the Fed and Treasury together would produce a flat supply curve for that debt. This answer allows us to continue to use the simple model as a parable for horizons up to a year or so.

What matters for our story so far is that the central bank sets the interest rate, by any mechanism. The Treasury then decides how much debt to sell at the new bond prices in order to finance its deficits or partially to repay debts. Given bond prices $Q_t$, the price level $P_t$, and the surplus or deficit $s_t$ that the Treasury must finance, the flow condition (3.4),

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t},$$

describes how much nominal debt $B_t$ the Treasury must sell to roll over debt and to finance the surplus or deficit $s_t$. 

The Treasury decides how many bonds $B_t$ to sell after it observes the interest rate, price level, and bond price. If the central bank raises interest rates one percent, the Treasury sees one percent lower bond prices, and it raises the face value of debt it sells by one percent. This equation, solved for $B_t$ is now nothing more or less than the process that the Treasury accountants go through to figure out how much face value of debt $B_t$ to auction. In this two-step process, the government overall—central bank plus Treasury—thus really sells any quantity of debt at a fixed interest rate, though neither Treasury nor central bank may be aware of that fact.

Now real-world Treasuries do conduct an auctions, which do change interest rates by a few basis points. But if the resulting bond price is unexpectedly low, and revenue unexpectedly low, the Treasury must still fund the deficit $s_t$. So it goes back to the market and sells some more, adjusting the quantity. Moreover, if a change in Treasury debt rates spreads to interbank rates, the central bank steps in to re-set the interbank rate. In the end only the small spread between short-term Treasury and bank rates can change as the result of Treasury auctions, and that spread disappears in our model with no financial frictions. So in the frictionless model, the two-step process is equivalent to a flat supply curve of Treasury debt.

### 3.7 The time-series process of surpluses and debts

Return now to the picture of normal fiscal policy outlined in section 3.4. As governments fund occasional deficits, to finance wars, combat recessions, invest in infrastructure, or accommodate other temporary fiscal demands, without causing inflation, the vast bulk of debt issues will be of the latter, equity-issue kind, rather than the former, expected inflation-inducing, share-split kind.

A decline in current surpluses—a rise in current deficits—will thus typically be associated with a rise in future surpluses. We should expect that current surpluses $s_t$ will be negatively correlated with future surpluses $s_{t+j}$. The surplus will not be an AR(1) or similarly positively correlated process.

If the government wishes no inflation—if it wishes to pay back its debts, and not finance any surprise deficits from inflation-induced partial defaults—then there will be no surprise in the present value of surpluses

$$ (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j s_{t+j} = 0. $$
This means that the revision in expected future surpluses must have the opposite sign as the innovation in the current surplus:

\[(E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j s_{t+j} = -(E_{t+1} - E_t) s_t,\]

where the minus sign is the crucial point. An innovation in current surpluses usually carries in the same direction for a few years – recessions and wars last a while – so the change in expectation of long term surpluses must be correspondingly larger. If \(s_{t+j}\) up through \(k\) move in one direction, then,

\[(E_{t+1} - E_t) \sum_{j=k}^{\infty} \beta^j s_{t+j} = -(E_{t+1} - E_t) \sum_{j=0}^{k} \beta^j s_{t+j}.\]

In this case there will be no correlation between the current surplus/deficit \(s_t\) or accumulated debts and the price level or inflation rate. Large accumulated debts will seem to forecast (Granger-cause) surpluses, but this fact simply respects the keeping of promises made when the debt was sold – promises that, if they had not been believed, would have resulted in no revenue from the debt sale. The contrary propositions all seem like easy refutations of fiscal theory. But in fact, they are just what we expect of normal fiscal policy.

Likewise, even in a completely fiscal-theory economy, we are likely to see that higher debts lead to and forecast higher surpluses. This does not mean that the government is following a passive fiscal policy. To see this fact, write the flow condition

\[\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.\]

The second term is the real revenue raised from bond sales at the end of time \(t\). If the government wishes to run a deficit, a lower \(s_t\), without unexpectedly changing the price level \(P_t\), then it must fund that deficit by selling bonds, and raising revenue by doing so, and raising the value of debt.

If, to fund a lower deficit \(s_t\) today, the government does not increase expected subsequent surpluses \(s_{t+j}\), and does not induce an unexpected inflation in \(P_t\) that wipes out some of the outstanding debt \(B_{t-1}\), then the government does not raise revenue and it cannot run a larger deficit \(s_t\). If there was no surprise inflation in \(P_t\), if the bond issue \(B_t\) did in fact raise revenue to fund the deficit \(s_t\) and if the bond issue
3.8. **FISCAL STIMULUS**

did not raise interest rates \( i_t \), then we know bondholders, at least, believed future surpluses would rise. That is direct evidence of expected higher future surpluses accompanying current deficits.

Responsible governments in normal times borrow and then save in order to smooth taxes intertemporally, and only infrequently resort to inflation-induced default of outstanding debt. This means that most debt sales that we observe in such normal times will be of the “equity issue” type, not the “share split” type. It implies that

- **Surpluses are likely to include an element of negative autocorrelation – low current surpluses match higher later surpluses.**

and

- **Higher debts are likely to be followed by higher surpluses.**

Such governments are not likely to show us frequent “fiscal theory” experiments in the data. Fiscal theory is shown by the absence of inflation in the presence of varying surpluses.

### 3.8 Fiscal stimulus

A deliberate fiscal loosening creates inflation in the fiscal theory. However, to create inflation one must convince people that future surpluses will be lower. Current deficits per se matter little. The U.S. and Japanese fiscal stimulus programs contained if anything the opposite promises, not enough to overcome the long tradition of debt repayment.

In the great recession following 2008, many countries turned to fiscal stimulus. The fiscal stimulus was in part a deliberate attempt to create inflation. Japan tried these policies earlier. Even this simple fiscal theory has some interesting perspectives on this attempt.

Equation (3.2),

\[
\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left( \frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j},
\]

describes how looser fiscal policy can create inflation. However, the equation points
to the vital importance of future deficits in creating inflation. Larger current deficits really don’t matter per se.

The U.S. and Japanese fiscal stimulus programs, though massive, were largely failures, at least at the project of increasing inflation. This observation helps to explain why. The U.S. Administration promised debt reduction to follow once the recession is over, i.e. that the debt would be paid back. That is what a Treasury does that wants to finance current expenditure without creating current or expected future inflation. To create inflation, the key is to promise that future surpluses will not follow. (Even in a traditional Keynesian multiplier framework, which is how the U.S. Administration analyzed its stimulus, one wishes people to ignore future surpluses, to help break Ricardian equivalence.)

Even if the U.S. had tried to say the debt would not be paid back, good reputations and institutional constraints on inflationary finance are often hard to break. Once people are accustomed to the reputation that Treasury issues, used to finance current deficits, will be paid back in the future by higher surpluses, it turns out that it is hard to break them of that habit. The expectations involved in a small inflation are harder yet to create. A government might be able to persuade bondholders that a fiscal collapse is on its way, and create a hyperinflation. But how do you persuade bondholders that the government will devalue debt by 5%, and only by 5%?
Chapter 4

The fiscal theory of monetary policy, alternatives, and observational equivalence

In the last chapter, we established that the government can target the nominal interest rate, with something that looks like “monetary policy,” and that doing so controls expected inflation in the simple model under consideration. Here I take the next steps to build a fuller model, describing how control of the nominal interest rate propagates to the rest of the economy, in the new-Keynesian DSGE tradition. I call the result a “fiscal theory of monetary policy.”

The goal here is to create a model with the minimal departures from today’s new-Keynesian / DSGE structure. I want to open the door to fiscal theory to readers with a lot of investments in those models. So we start here with an interest rate target in a frictionless model. I compare it to the standard new-Keynesian approach in that model, and then we add long-term debt, pricing frictions, and the other elements of contemporary models.

This approach does not mean this is the only way to go however. Later chapters step away from interest rate targets, including a description of current “unconventional” policy including quantitative easing and forward guidance, a largely historical and history-of-thought look at monetary policy via control of monetary aggregates, and a few out-of-the-box ideas including a target for the spread between indexed and non-indexed debt. Here and later, I also stay within a very traditional new-Keynesian framework, with simple forward-looking IS and Phillips curves. Like everyone else,
I recognize the limitations of those ingredients, and I have my own views of how one should modify them to better describe the economy. But it’s best to add one ingredient at a time, so in this book I focus on adding fiscal theory to very standard, if unrealistic models.

In sum, I work here to show that you can analyze an interest-rate target based new-Keynesian model with a fiscal theory perspective, but you do not have to specify monetary policy that way, and you do not have to describe the economy that way.

### 4.1 The simplest FTMP model and its response function

Under an interest rate target, the model comes down only to

\[ i_t = r + E_t \pi_{t+1}, \]

\[ \pi_{t+1} - E_t \pi_{t+1} = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{b_{t+1}}{b_{t+1}} = -\varepsilon_{t+1}^s. \]

This is the simplest example of a fiscal theory of monetary policy. The interest rate target sets expected inflation, and fiscal news sets unexpected inflation.

Figure 4.1 presents the response of this model to an interest rate shock with no fiscal change, and a fiscal shock with no interest rate change. The interest rate shock is Fisherian – inflation rises one period later – as it should be in this completely frictionless model.

Let us start with the simplest example of a fiscal theory of monetary policy. The government targets the nominal interest rate, which controls expected inflation, and fiscal policy controls unexpected inflation. In the background, we still have one-period debt, completely flexible prices, and no monetary or financial frictions.

The connection to standard models is clearer by linearizing the equations of the last section, as standard models do. Monetary policy sets an interest rate target \( i_t \), and expected inflation follows from

\[ \frac{1}{1 + i_t} = E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1}} \right) \]

\[ i_t \approx r + E_t \pi_{t+1}. \] (4.1)
4.1. THE SIMPLEST FTMP MODEL AND ITS RESPONSE FUNCTION

Fiscal policy determines unexpected inflation via (3.2). Linearizing in the standard way, and denoting
\[ b_t \equiv B_{t-1}/P_t \]
the real value of the debt, we can write (3.2) at time \( t + 1 \) as
\[
\pi_{t+1} - E_t \pi_{t+1} = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{s_{t+1+j}}{b_{t+1}}
\]
(4.2)
\[
\pi_{t+1} - E_t \pi_{t+1} = - \varepsilon_{t+1}^s.
\]
(4.3)
Equation (4.3) defines the notation \( \varepsilon_{t+1}^s \) for the shock to the present value of surpluses.

Equation (3.4),
\[
\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{RE_t} \bigg( \frac{P_t}{P_{t+1}} \bigg) = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j},
\]
becomes a footnote. It determines debt \( B_t \), but has no further implications for inflation or anything else. We can think entirely in terms of the interest rate target \{i_t\} and let debt \{B_t\} and the mechanics of interest rate targeting drop from the analysis.

The combination (4.1) and (4.3) form the simplest example of a fiscal theory of monetary policy. We will expand on it a lot, adding interest-rate rules, long-term debt, discount-rate variation, price stickiness, quantity variation, and many other ingredients. Essentially all the other equations of a standard new-Keynesian or DSGE model can be imported directly.

Using
\[
\pi_{t+1} = E_t \pi_{t+1} + (E_{t+1} - E_t) \pi_{t+1},
\]
then, the full solution of the model – the path of inflation as a function of monetary and fiscal shocks – is
\[
\pi_{t+1} = i_t - r - \varepsilon_{t+1}^s.
\]
(4.5)
Using (4.3), Figure 4.1 plots the response of this model to a permanent interest rate shock at time 1 with no fiscal shock \( \varepsilon_1^s = 0 \), and the response to a fiscal shock \( \varepsilon_1^s \) at time 1 with no interest rate movement.

In response to the interest rate shock, inflation moves up one period later. The Fisher relation says \( i_t = r + E_t \pi_{t+1} \) and there is no unexpected time-1 inflation without a
fiscal shock. The response is the same if the interest rate shock is announced ahead of time, so I don’t draw a second line for that case. If $E_t \pi_{t+1}$ rises, then $E_t \pi_{t+1}$ rises. Many models offer different predictions for expected vs. unexpected policy, and expected policy changes can affect the economy on announcement. Expected and unexpected monetary policy have the same effects in this model, and there are no announcement effects except on long-term bond prices.

In response to the positive fiscal shock $\varepsilon^*_t$ with no change in interest rates, there is a one-time downward price-level jump, corresponding to a one-period disinflation. If the fiscal shock is announced ahead of time, the disinflation happens when the shock is announced, not when surpluses actually move. The shock is just a shock to news, and makes no mention of the timing of the actual surpluses. The response is the same for a shock to the surplus at time 1 and a $\beta^{-10}$ larger shock to the surplus at time 10.

These are boring and unrealistic responses. They are, on reflection, exactly what one expects of a completely frictionless model. That’s good news. The model is
really boring, it should have boring responses! It shows us that we can rather easily construct a fiscal theory of monetary policy, even in a completely frictionless model. It verifies that in a frictionless model, monetary policy is neutral, and makes specific just what neutral means. To get realistic and interesting dynamics, we have to add sticky prices, long term debt, cross-correlated and persistent policy responses, dynamic economic mechanisms in preferences, production, and capital accumulation, or other ingredients.

In particular, this graph gives a perfectly “Fisherian” monetary policy response. An interest rate rise leads to higher inflation, one period later. There is a long tradition of belief that higher interest rates lower inflation, at least temporarily, though the data are, in fact, ambiguous. We will look for specifications of the model that can produce that relationship. We can produce a temporary inflation decline here by combining the two shocks – an interest rate rise paired with an unexpected fiscal contraction. We will see that the new-Keynesian approach to this economic model works this way. And that pairing of shocks may happen in the data as monetary and fiscal authorities respond to the same underlying shocks, or to each other’s actions. But this model does not a negative response to a pure monetary policy shock. In fact, the opposite would be surprising. A model with no pricing, monetary or expectational frictions should be neutral.

This simple plot is best, then, for showing exactly how a totally neutral and frictionless world works – not realistic, but possible – and suggesting the kinds of frictions we need to add. It also shows us how absolutely simple the basic fiscal theory of monetary policy is, before we add such elaborations. Yes, there is something as simple as MV=PY and flexible prices, which also is neutral, on which to build realistic dynamics.

4.2 Interest rate rules

To the simple model

\[ i_t = E_t \pi_{t+1}, \]
\[ \pi_{t+1} - E_t \pi_{t+1} = -\varepsilon^s_{t+1}. \]

I add a Taylor-type rule

\[ i_t = \phi \pi_t + v_t \]
\[ v_t = \rho v_{t-1} + \varepsilon^i_t. \]
to find the equilibrium inflation process

\[ \pi_{t+1} = \phi \pi_t + v_t - \varepsilon_{t+1}^s. \]

Figure 4.2 plots responses to monetary and fiscal policy shocks in this model. The persistence of the monetary policy disturbance and the endogenous response of the interest rate rule introduce interesting dynamics, and show how monetary policy affects the dynamic response to the fiscal shock.

The standard analysis of monetary policy specifies Taylor-type interest rate rule,

\[ \begin{align*}
  i_t &= \phi \pi_t + v_t \\
  v_t &= \rho v_{t-1} + \varepsilon_t^i
\end{align*} \tag{4.6} \]

rather than directly specify the equilibrium interest rate process, as I did in the last section. The variable \( v_t \) is a serially correlated monetary policy disturbance: If the Fed deviates from a rule this period, it is likely to continue deviating in the future as well.

(Terminology: I use the word “disturbance” for deviations from structural equations that may be serially correlated or predictable from other variables, like \( v_t \), and I reserve the word “shock” and greek letters for variables that only move unexpectedly, like \( \varepsilon_t^i \) with \( E_{t-1} \varepsilon_t^i = 0 \). “Shocks” and “disturbances” need not be “structural.” For example, the fiscal policy “shock” \( \varepsilon_t^f \) reflects news about future surpluses, which in turn has structural roots in productivity, tax law politics, and so forth.)

The model is now

\[ i_t = E_t \pi_{t+1}, \]

\[ \pi_{t+1} - E_t \pi_{t+1} = -\varepsilon_{t+1}^s \]

and the policy rule (4.6). Eliminating the interest rate \( i_t \), the equilibria of this model are now inflation paths that satisfy

\[ \begin{align*}
  E_t \pi_{t+1} &= \phi \pi_t + v_t \\
  \pi_{t+1} - E_t \pi_{t+1} &= -\varepsilon_{t+1}^s
\end{align*} \tag{4.7} \]

and thus

\[ \pi_{t+1} = \phi \pi_t + v_t - \varepsilon_{t+1}^s. \tag{4.8} \]

The top lines of Figure 4.2 plot the response of inflation and interest rates to a unit monetary policy shock \( \varepsilon_1^i \) in this model, and the line “\( v_t, \text{FTMP} \)” plots the associated
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Figure 4.2: Responses to monetary and fiscal shocks. The top two lines graph the response of inflation $\pi_t$ and interest rate $i_t$ to a monetary policy shock $\varepsilon^i$ in the fiscal theory of monetary policy model. The monetary policy disturbance is labeled $v_t$, $FTMP$. The parameters are $\rho = 0.7$, $\phi = 0.8$. The bottom lines plot the response of inflation and interest rate to a unit fiscal shock $\varepsilon^s$ in the FTMP model. These lines are also the responses to a monetary policy shock $\varepsilon^i = 0.7$ in the new-Keynesian model, with parameters $\phi_{nk} = 1.5$, $\rho_{nk} = 0.8$. The corresponding monetary policy disturbance is labeled “$v_t$, NK.”

monetary policy disturbance $v_t$. I use a value $\phi < 1$ here, which keeps the responses stationary. The model can be solved with $\phi > 1$. $\phi < 1$ is not a necessary condition for a unique solution, as we will see $\phi > 1$ is needed in new-Keynesian models. But $\phi > 1$ leads here to explosive inflation, which is unrealistic at least for normal times.

The combination of two AR(1)s – the shock persistence $\rho$ and the interest rate rule $\phi$ – generates a pretty hump-shaped inflation response. Interest rates that move one period ahead of inflation – $i_t = E_t \pi_{t+1}$ are still part of the model, and the lack of a fiscal change in (4.2) means that $\pi_1$ cannot jump either way on the monetary policy news at time 1. We will continue to work towards a model in which higher interest
rates can produce lower inflation, but this isn’t it yet.

Comparing the top lines of Figure 4.1 and Figure 4.2, you can see the same model at work. If we had fed in the equilibrium \{i_t\} response of Figure 4.2 to the calculation (4.5) behind Figure 4.1, we would have gotten the same result as in Figure 4.2. The monetary policy rule is a mechanism to endogenously produce an interest rate path with interesting dynamics, and for us to ask questions of the economy in which we envision monetary policy reacting systematically to inflation.

The lower two lines of Figure 4.2 plot the response to a unit fiscal shock \(\varepsilon_1^s\). By definition, this disturbance is not persistent. The fiscal tightening produces an instant deflation, i.e. a downward price level jump, just as in Figure 4.1. Again, the endogenous \(i_t = \phi \pi_t\) monetary policy response now produces more interesting dynamics. However, we would get the same result if the plotted interest rate path were a time-varying peg, a dynamic response to the initial shock, rather than an endogenous response to inflation. Expected inflation follows the interest rate as before.

As (4.2) reminds us, fiscal policy alone sets unexpected inflation \(\pi_1 - E_0 \pi_1\). But what happens after that, \((E_1 - E_0) \pi_2\) and beyond, depends on monetary policy, via either interest rate target \(\phi \pi_t\) or a persistent disturbance \(v_t\). Monetary policy could return the price level to its previous value. Monetary policy could turn the event into a one-time price level shock, with no further inflation. Or monetary policy could let the inflation continue for a while, as it does here with \(\phi > 0\). When we add long-term debt and sticky prices, these future responses will have additional effects on the instantaneous inflation response \(\pi_1\). Monetary policy matters a lot in this fiscal model, to the dynamic path of expected inflation after the shock.

These responses are not ready to evaluate against data. The important lesson here is that we can produce impulse response functions of this sort, just as we do with standard models of interest rate targets. And we can compare them with the standard new-Keynesian analysis of the same model 4.3.

We are ready to add pricing frictions and other complications, the topic of Chapter 7. Here, however, I next consider the new-Keynesian and monetarist alternatives, and address controversies in trying to tell the models apart, all in the context of this very simple frictionless model.
4.3 The new-Keynesian approach

I present the standard new-Keynesian approach to price level determination. The new-Keynesian approach studies the same set of equations. It specifies $\phi > 1$, adds a rule against nominal explosions, and so determines unexpected inflation by solving the inflation equilibrium condition forward. The fiscal relation linking unexpected inflation to the revision in present value of surpluses is still present, but is satisfied by assuming “passive” fiscal policy. In this flexible-price setting, the new-Keynesian model producers a negative response of inflation and interest rates to a monetary policy shock, though observed equilibrium interest rates and inflation still go in the same direction. The new-Keynesian response to a monetary policy shock is the same as the FTMP response to a fiscal shock, our first taste of observational equivalence.

Now, let’s consider the standard new-Keynesian approach to the same model. The simplest form of the standard new-Keynesian model, as set forth for example in Woodford (2003), consists of exactly the same set of equations, (4.1), (4.2) and (4.6):

\begin{align}
    i_t &= E_t \pi_{t+1} \quad (4.9) \\
    i_t &= \phi \pi_t + v_t \quad (4.10) \\
    \pi_{t+1} - E_t \pi_{t+1} &= -\varepsilon_{t+1}^s. \quad (4.11)
\end{align}

(Here I set the real rate $r = 0$ for simplicity, or equivalently I study deviations from the steady state.)

Though the equations are the same, new-Keynesians solve them differently. New-Keynesian modelers specify a “passive” fiscal policy, that (4.11) determines surpluses \( \{\varepsilon_{t+1}^s\} \) for any unexpected inflation. To determine unexpected inflation in place of (4.11), new-Keynesian modelers specify $\phi > 1$, and they add a rule against nominal explosions. New-Keynesian authors then solve the equilibrium condition (4.7), $E_t \pi_{t+1} = \phi \pi_t + v_t$, forward to obtain

\begin{equation}
    \pi_t = -E_t \sum_{j=0}^{\infty} \frac{v_{t+j}}{\phi^{j+1}} = -\sum_{j=0}^{\infty} \frac{\rho^j}{\phi^{j+1}} v_t = -\frac{1}{\phi - \rho} v_t, \quad (4.12)
\end{equation}

where the last equality uses the AR(1) model of (4.6) for the monetary policy disturbance $v_t$. Equilibrium inflation follows the same AR(1) process as the shock $v_t$,

\begin{equation}
    \pi_{t+1} = \rho \pi_t - \frac{1}{\phi - \rho} \varepsilon_{t+1}^i. \quad (4.13)
\end{equation}
Figure 4.2 plots the response of this model to a monetary policy shock $\varepsilon_1 = 0.7$, using $\phi_{nk} = 1.5$ and $\rho_{nk} = 0.8$. (I use the $nk$ subscript to denote parameters used in the new-Keynesian version of the model, when confusion is possible.) The line marked “$v_t$, NK” plots the monetary policy disturbance, $v_t$ of this case. The inflation and interest rate responses are shown in the lower set of lines. Yes, the new-Keynesian response to this monetary policy shock is exactly the same as the FTMP response to a fiscal policy shock. The next section examines this “observational equivalence” result in detail. For now, this pair of lines gives us the standard new-Keynesian view of a monetary policy shock: In this simple model, a monetary policy shock $\varepsilon_1$ instantly lowers inflation $\pi_t$. Inflation then recovers back to its steady state with an AR(1) pattern, following the slow mean reversion of the disturbance $v_t$. Though we see a negative inflation response to the monetary policy disturbance $v_t$, observed interest rates and inflation still go in the same direction. It will take price stickiness and other ingredients to obtain a model in which interest rates and inflation go in opposite directions.

I emphasize that new-Keynesian models do include (4.11). It is one of the equilibrium conditions of the model, so the equations of the new-Keynesian and FTMP models are the same. New-Keynesian authors assume that it holds via “passive” fiscal policy, that (4.11) determines the shock $\varepsilon_{t+1}$ given unexpected inflation. Often, authors write that the government adjusts lump-sum taxes to pay off inflation-induced variation in real values of government debt. Since (4.11) doesn’t feed back into the rest of the model, it is often relegated to footnotes, so you have to look hard to find it. But it is there!

### 4.4 Observational equivalence and equilibrium selection

The standard new-Keynesian model is observationally equivalent to the fiscal theory model. The new-Keynesian model response to a monetary policy shock is the same as the FTMP response to a fiscal shock, with different parameters. Any observed time-series process for inflation and interest rates consistent with the economic model can equally well be described as the result of the new-Keynesian or FTMP foundations.

Writing the new-Keynesian monetary policy rule in equivalent form $i_t = i^*_t + \phi(\pi_t - \pi^*_t)$, the equilibrium condition becomes $E_t (\pi_{t+1} - \pi^*_{t+1}) = \phi (\pi_t - \pi^*_t)$, where * de-
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notes equilibrium values. Here we see that $\phi > 1$ and a rule against explosions selects $\pi_t = \pi^*_t$. We also see that in equilibrium where $\pi_t = \pi^*_t$, we cannot measure $\phi$. The new-Keynesian rule separates to a “monetary policy” rule $i^*_t$, describing how observed equilibrium interest rates move, and a completely separate “equilibrium-selection policy” rule $\phi (\pi_t - \pi^*_t)$ describing off-equilibrium threats.

We can symmetrically write the FTMP fiscal policy rule separating fiscal policy from equilibrium selection policy as

$$s_t = s_0 + \alpha \left( \frac{B_{t-1}}{P^*_t} \right) + \gamma \left( \frac{B_{t-1}}{P_t} - \frac{B_{t-1}}{P^*_t} \right).$$

Here $\alpha$ controls the fiscal policy rule, and $\gamma$ controls equilibrium selection. Surpluses may respond to debts in equilibrium, $\alpha > 0$, while in an active fiscal regime $\gamma = 0$ and surpluses do not respond to changes in the value of debt brought about by multiple-equilibrium inflation. Data from an equilibrium cannot measure $\gamma$.

Writing policy in this way, monetary policy and fiscal policy are identifiable in both models, but equilibrium-selection policies are not identifiable.

As we noted, in Figure 4.2, the inflation and interest rate responses of the new-Keynesian model to a monetary policy shock $\varepsilon^i = 0.7$ using $\phi_{nk} = 1.5, \rho_{nk} = 0.8$ are exactly the same as the responses of the FTMP model to a fiscal $\varepsilon^s = 1$ shock, using $\phi = 0.8, \rho = 0$. You can also see that fact analytically, comparing \[4.8\] to \[4.13\]:

- The new-Keynesian model responses to a monetary policy shock $\varepsilon^i = (\phi_{nk} - \rho_{nk})$ \[4.13\] are observationally equivalent to the fiscal theory of monetary policy model responses to a \[4.8\] fiscal shock $\varepsilon^s = 1$, under parameters $\phi = \rho_{nk}, \rho = 0$.

Observational equivalence goes beyond this response function. The joint equilibrium dynamics of inflation, interest rates, and surpluses are exactly the same. Inflation follows an AR(1). Interest rates equal expected inflation, and the innovation to fiscal surpluses is the negative of the inflation innovation. There is no way to tell the two models apart from data on inflation, interest rates, and fiscal surpluses. Data from an equilibrium cannot distinguish causal stories about equilibrium formation.

What about running a regression $i_t = \phi \pi_t + v_t$, you may ask? Can that not measure $\phi$? No, because the new-Keynesian model predicts that $v_t$ and $\pi_t$ are correlated. Via \[4.12\], they are perfectly (negatively) correlated. Using \[4.12\] and either \[4.9\]
or (4.10), the equilibrium relation between interest rates and inflation in the new-Keynesian model is

\[ i_t = \rho \pi_t \]  

(4.14)

with no error term. A regression of \( i_t \) on \( \pi_t \) produces \( \rho \) not \( \phi \). The new-Keynesian responses and \( \{ i_t, \pi_t \} \) dynamics are the same for any value of \( \phi \). The parameter \( \phi \) does not enter the observable dynamics.

The fiscal theory equilibrium \([4.8]\)

\[ \pi_{t+1} = \phi \pi_t + v_t - \varepsilon_{t+1}^s \]

\[ i_t = \phi \pi_t + v_t. \]

offers a bit more hope for measurement. If we know we are looking at the fiscal theory model, and if there are no monetary policy shocks, \( v_t = 0 \), then \( i_t \) and \( \pi_t \) are related in equilibrium by

\[ i_t = \phi \pi_t. \]  

(4.15)

Now the regression of \( i_t \) on \( \pi_t \) does measure the monetary policy rule \( \phi \). But we cannot tell that we are not looking at the new-Keynesian model and measuring the parameter \( \rho \).

The AR(1) is not central to this observational equivalence. Choose any process \( \{ \pi_t \} \). An equilibrium must have \( i_t = E_t \pi_{t+1} \). Choose any \( \phi \). Construct \( v_t = i_t - \phi \pi_t \). These reverse-engineered assumptions about \( \phi \) and \( v_t \) produce the chosen \( \{ \pi_t \} \) and its resultant \( \{ i_t \} \) as an equilibrium. Thus, observation of \( \pi_t \) and \( i_t \) cannot tell us \( \phi \) and \( v_t \), no matter what is the time series process of inflation.

### 4.4.1 Equilibrium selection policy

To understand observational equivalence more deeply, it is worth stepping back to view the models in a different way. In the end, new-Keynesian and FTMP models are both trying to solve the classic problem of "indeterminacy of rational expectations equilibria:" The economic model consists only of (4.9) and (4.10). If we assume passive fiscal policy, then (4.10) determines \( \varepsilon_{t+1}^s \) given \( \pi_{t+1} \). Inflation only needs to solve (4.9). But if the Fed fixes the interest rate \( i_t \), (4.9) leaves us multiple equilibria. Unexpected inflation is completely arbitrary.

The FTMP model responds by saying, well, you threw out one of the equations determining the price level, so no wonder you don’t have enough information to de-
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termine the price level. Do not make the passive fiscal assumption. Now unexpected inflation is determined again.

The new-Keynesian approach instead elaborates on the interest rate policy rule. To see the essence of the idea in this context, it will be clearer to write the new-Keynesian monetary policy rule in a form equivalent to (4.10) (due to King (2000)) as

\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*). \] (4.16)

Here, \( \{\pi_t^*\} \) is the inflation rate the central bank wishes to produce, and \( i_t^* = E_t\pi_{t+1}^* \) is the corresponding equilibrium interest rate satisfying (4.9). Again eliminate \( i_t \).

Equilibria are inflation processes \( \{\pi_t\} \) such that

\[ E_t(\pi_{t+1} - \pi_{t+1}^*) = \phi(\pi_t - \pi_t^*). \] (4.17)

Now, if the central bank follows \( \phi > 1 \), for all \( \pi_t \) except \( \pi_t = \pi_t^* \), the sequence of expectations \( E_t(\pi_{t+T} - \pi_{t+T}^*) \) grows geometrically with \( T \). New-Keynesians add a rule against such expected nominal explosions, or “non-local” equilibria, and conclude that \( \pi_t = \pi_t^* \) is the only (locally bounded) equilibrium left.

In sum, the central bank’s “active-money” policy rule \( \phi > 1 \) and the economists’ rule against hyperinflation or deflation together substitute for active fiscal policy to uniquely determine inflation.

The form of the policy rule (4.16) shows more clearly how this model works than the original form \( i_t = \phi\pi_t + v_t \). The central bank has a monetary policy rule, \( i_t^* \), a rule for what the actual, observed, equilibrium interest rate to be. The monetary policy rule determines expected inflation in equilibrium, \( E_t\pi_{t+1}^* = E_t\pi_{t+1} = i_t^* \), but it leaves unexpected inflation still indeterminate. The central bank additionally has an equilibrium-selection policy \( \phi(\pi_t - \pi_t^*) \) to control unexpected inflation. If inflation does not come out to the equilibrium value the Fed desires, then the Fed will respond with hyperinflation or deflation, as described by (4.17). The private sector, abhorring explosions, jumps to the equilibrium the Fed wishes to implement, with \( \pi_{t+1} = \pi_{t+1}^* \).

The parameter \( \phi \) does not enter monetary policy at all. The parameter \( \phi \) is only an equilibrium-selection threat. Monetary policy and equilibrium selection policy, \( \phi \) and \( \theta \), are are completely different objects. They have nothing conceptually to do with each other. Monetary policy is the question of how observed interest rates should move to guide economic dynamics. Equilibrium selection policy is the question of how to structure an off-equilibrium threat designed to select equilibria.
Clearest of all, in (4.16) we see that in equilibrium, where \( \pi_t = \pi^*_t \) always, the parameter \( \phi \) does not enter at all. The parameter \( \phi \) measures what the central bank will do out of equilibrium, what happens if \( \pi_t \neq \pi^*_t \). We cannot measure out-of-equilibrium behavior from data that are all taken from an economy in equilibrium, the sequences \( \{\pi^*_t, i^*_t\} \) etc. Moreover, the observed \( \pi_t = \pi^*_t \) is the same for any value of \( \phi > 1 \). If a threat to explode the economy is enough to get people to jump to the equilibrium you want, just how fast the threatened explosion never shows up in the data. If, to get the kids to eat spinach, you threaten no ice cream, and if the threat is effective, the data (spinach, ice cream) do not reveal whether the threat was no ice cream for a day, a week, or a month.

Now the observational-equivalence theorem can be clearer. Specify any stationary stochastic process for equilibrium inflation \( \{\pi^*_t\} \). Let us reverse-engineer assumptions that deliver it as the unique equilibrium of the new-Keynesian and then FTMP models. By example, we show those two models cannot be distinguished.

For the new-Keynesian model, assume that monetary and equilibrium-selection policies follow \( i_t = i^*_t + \phi(\pi_t - \pi^*_t) \), with \( i^*_t = E_t \pi^*_{t+1} \) and any \( \phi > 1 \). Then (4.17), \( E_t (\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t) \), plus a rule against nominal explosions determines \( \pi_t = \pi^*_t \) at all dates. Assume passive fiscal policy (4.11), that 
\[
\varepsilon^*_{t+1} = \pi_{t+1} - E_t \pi_{t+1}
\]
This assumption determines surpluses for any inflation, equilibrium or not. The two equilibrium conditions (4.9) (4.11) now hold, and given this policy specification and the rule against explosive equilibria no other inflation process satisfies the equilibrium conditions.

For the FTMP model, specify \( \varepsilon^*_{t+1} = \pi^*_{t+1} - E_t \pi^*_{t+1} \), and pick any surplus process with this innovation. This assumption on surpluses produces \( \pi_{t+1} - E_t \pi_{t+1} = \pi^*_{t+1} - E_t \pi^*_{t+1} \) as the unique equilibrium outcome for unexpected inflation. Let monetary policy simply set a time-varying peg \( i_t = i^*_t \). This assumption now produces expected inflation \( E_t \pi^*_{t+1} \) as the unique equilibrium outcome. Again the two equilibrium conditions (4.9) (4.11) hold, and given this policy specification no other inflation process satisfies the equilibrium conditions.

In both cases, I have written down simple and transparent policies that determine the equilibria, but these are sufficient, not necessary choices. We can also write more complex but more realistic policies, especially in the less-studied fiscal theory case. We can define \( v_t = i^*_t - \phi \pi^*_t \) and then write the new-Keynesian policy rule in more traditional form \( i_t = i^*_t + \phi(\pi_t - \pi^*_t) = \phi \pi_t + v_t \). In the FTMP case, surpluses may react partially to inflation, \( \varepsilon^*_{t+1} \) can be a function of \( (\pi_{t+1} - E_t \pi_{t+1}) \), so long as it is not a one-to-one function. In both new-Keynesian and FTMP cases, the monetary
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A policy rule can feed back from inflation, $i_t^* = \theta \pi_t^*$, react to output and other variables, or add disturbances. We shall look extensively at such policies.

4.4.2 On and off equilibrium in the fiscal theory

It is useful to state fiscal theory in a form parallel to (4.16), separating equilibrium and off-equilibrium responses.

A government can implement a passive fiscal regime by raising surpluses in response to all changes in the value of its debt. To see this proposition, suppose

$$s_t = s_0 + \alpha \left( \frac{B_{t-1}}{P_t} \right).$$  \hspace{1cm} (4.18)

Substitute this policy into the flow condition

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{1+r} \left( \frac{B_t}{P_{t+1}} \right).$$

(To keep it simple, I use perfect foresight.) Rearrange and debt grows as

$$\frac{B_t}{P_{t+1}} = (1-\alpha) (1+r) \frac{B_{t-1}}{P_t} - (1+r)s_0.$$

If $\alpha > 0$, then debt grows at less than the interest rate, the transversality condition holds, and the present value relation holds “passively,” for any value of the initial price level $P_t$. Intuitively, if you pay anything at all above each months’ charges on your monthly credit card statement, then your balance will grow slower than the interest rate.

(This is not the only way to achieve active vs. passive policy, and we will quickly generalize $s_0$ to a time-varying surplus that responds to economic variables. Related results: If $(1-\alpha)(1+r) < 1$, roughly if $\alpha > r$, then debt is bounded. This condition applies more credibly to you and me, who are not be able to run ever-growing debts. Intuitively, we have to pay at lest the interest on our debt every month. Governments can usually borrow up to a finite debt to GDP ratio, on the idea that their surpluses are bounded by GDP. Thus, governments are usually thought to be able to follow a policy with $\alpha > r - g$. If $r < g$, and this happy circumstance really will last forever, government debt never needs to be repaid with future surpluses, a free lunch. We’ll take up this case later.)
However, like the simple monetary policy rule $i_t = \phi \pi_t$, the surplus response (4.18) unites how surpluses respond to debts in equilibrium, and how surpluses might respond to off-equilibrium inflation. A government might well respond to large debts, accumulated at a constant price level by previous borrowing, but respond less or not at all to an increase in the real value of debt brought on by a sudden, unexpected, and unintended deflation, especially one that results from multiple-equilibrium jumps. The former is responsibly paying off the implicit or explicit promises made to bondholders when the borrowing occurred. The latter requires the government to tax hard-working people to pay an unexpected windfall to bondholders. A government that wishes to borrow in bad times needs to establish a reputation only for the former. A government that wishes to avoid multiple equilibrium inflation volatility needs to establish a reputation not to respond to the latter, and we will study mechanisms for doing so.

As one example, in response to the sudden and sharp deflation of the Great Depression, the U.S. along with many other countries defaulted on its gold standard promises in the wake of unexpected deflation, rather than “passively” raise taxes (Edwards (2018), Kroszner (2003)). Granted, I am breaking my own rules in reading this observed event as an observed reaction to an off-equilibrium event, but the episode emphasizes the plausibility that people may expect the government not to react to something that feels like an unexpected and undesired deflation, while still expecting the government to responsibly pay off debts at the desired or targeted price level.

As a contrary example, the US paid a unexpected real windfall in the late 1980s to people who bought bonds at the high expected-inflation-driven nominal rates of the late 1970s. The US desired lower inflation than those bondholders expected, and that windfall was the price of a default-free inflation stabilization.

We can formalize this observation in parallel with the * notation of the new-Keynesian policy rule (4.16). Let $\{P_t^*\}$ denote the desired equilibrium price level path. Write the surplus policy now as

$$s_t = s_0 + \alpha \left( \frac{B_{t-1}}{P_t^*} \right) + \gamma \left( \frac{B_{t-1}}{P_t} - \frac{B_{t-1}}{P_t^*} \right).$$

(4.19)

This form separates reactions in equilibrium from reactions to alternative equilibria. The active/passive question is the second term, whether the government will react to an off-equilibrium deviation of $P_t$ from $P_t^*$ by changing surpluses. Active fiscal policy is $\gamma = 0$, and passive policy is $\gamma > 0$. (I suppose $\gamma < 0$ is also active, though I can’t think of a practical application.)
The active/passive fiscal question is not whether the government raises surpluses to pay off debts at the equilibrium or expected and targeted price level $P_t^*$. It has nothing to do with about $\alpha$. We may have $\alpha > 0$ in an $\gamma = 0$ active-fiscal regime. We *should* have $\alpha > 0$ by any government that is faintly responsible, pays off incurred debts, and hopes to raise revenues to fund deficits by bond sales. The active/passive fiscal question is entirely whether the government responds to off-equilibrium prices, deviations of $P_t$ from $P_t^*$, as captured by $\gamma$. But in equilibrium, $P_t^* = P_t$. So, as before, we cannot measure the reaction $\gamma$ and the active/passive question from time-series data.

Here too we separate surpluses into “fiscal policy” and “equilibrium selection policy.” The fiscal policy part can and will have much more realistic responses to other variables. Surpluses respond to output and unemployment, and to secondary effects of monetary policy such as seigniorage. Equilibrium-selection policy, or (since nobody in government thinks this way) equilibrium-selection effects of fiscal policies, also can be more sophisticated. Many parts of the tax code and spending programs respond to changes in the price level, whether equilibrium or off-equilibrium. An off-equilibrium inflation will move us to higher tax brackets, and devalue nominal credits such as capital gains basis and depreciation allowances, every bit as much as an on-equilibrium inflation.

The point here: a fiscal-monetary regime will have both a fiscal policy and an equilibrium selection policy. They are different objects just as monetary policy is different from monetary equilibrium-selection policy. The heart of the fiscal equilibrium selection policy is to commit *not* to adjust surpluses in response to undesired inflation and deflation – to refuse the siren song of paying off nominal debt with less real taxation if inflation comes, and to refuse to pay real windfalls to bondholders in the event of deflation. It need not be a local commitment – we may have $\gamma > 0$ for small changes in price level, so long as eventually $\gamma = 0$. We will see may institutions that deliver this commitment. This fiscal equilibrium selection policy is separate from fiscal policy, conceptually and economically. It is not observed in equilibrium, and cannot be recovered by simple regressions, Granger causality tests, or any statistical processing of time series drawn from an equilibrium.

### 4.4.3 What we measure

Uniting the observations of the relation between interest rates and inflation in (4.14), (4.15), and the on- vs. off-equilibrium expressions of policy rules in (4.16), (4.19) It
appears that

- the “monetary policy rule” is not identified in the new-Keynesian model, but it is identified in the FTMP model, while the “fiscal policy rule” is not identified in the FTMP model, but it is identified in the new-Keynesian model.

I put the rules in quotes, however, because I think this conventional way of phrasing the observation is misleading. Yes, comparing (4.14) and (4.15), the parameter \( \phi \) is identified in the FTMP and not in the new-Keynesian model. But it plays a fundamentally different role in these models. If we write the monetary policy rule as,

\[
i_t = i_t^* + \phi(\pi_t - \pi_t^*) = \theta \pi_t^* + \nu_t + \phi(\pi_t - \pi_t^*),
\]

and if we agree to call the first part \( i_t^* = \theta \pi_t^* + \nu_t \) “monetary policy” – it is the relation between interest rates and inflation that we observe – and we agree to call the second part \( \phi(\pi_t - \pi_t^*) \) “equilibrium selection policy,” then a much clearer description of the situation would be that the “monetary policy rule” \( i_t^* = \theta \pi_t^* + \nu_t \) is (potentially) identified, while the “equilibrium selection policy” \( \phi(\pi_t - \pi_t^*) \) is not identified. Likewise, in (4.19), the parameter \( \alpha \) is identified but the parameter \( \gamma \) is not identified. If we could call the first part \( s_0 + \alpha (B_{t-1}/P_t^*) \) “fiscal policy” – it is the relation between surpluses and debt that we observe – and the second part \( \gamma (B_{t-1}/P_t - B_{t-1}/P_t^*) \) “equilibrium selection policy” then fiscal policy is identified in both models, while the FTMP “equilibrium selection policy” is not identified.

With this terminology, a much better summary of both models is this:

- Monetary policy – the relationship between observed interest rates, inflation and other variables – and fiscal policy – the relationship between observed surpluses, debts, and other variables – are identifiable in both models. Equilibrium-selection policies – how the interest rate or surpluses would respond to non-equilibrium inflation – are not identifiable in either model.

Again, this is demonstrated so far in a very simple model and policy environment, but we will see the same conclusions in more complex models later.

### 4.5 Monetarism and observational equivalence

Monetarism intersects money demand \( M_t V = P_t Y_t \) with a central bank that controls money supply \( M_t \) via open market operations. \( M_t V = P_t Y_t \) thus determines the price level, with a passive fiscal policy so that the present value relation holds. In
the presence of money demand, fiscal theory determines the price level from the present value relation, and then the central bank passively provides money needed to satisfy its demand. Central banks throughout history have frequently followed passive policies, an argument for fiscal theory. Once again, since all we see are the equilibrium conditions $M_t V = P_t Y_t$ and the present value relation in both regimes, the monetary and fiscal regimes are observationally equivalent.

Monetarism or the quantity theory starts with a demand for money $M_t V = P_t Y_t$, a special asset held despite an interest cost for transactions or similar reasons. The central bank controls the supply of money $M_t = M^*_t$. Intersecting demand and supply determines the price level. It is conceptually easier than the new-Keynesian model, so much so that many authors feel that $MV = PY$ still lies at the foundation of that model, not just a pure interest rate target. It is a vital piece of history, history of thought, and economics one should understand. I have deferred it as an alternative however, because it manifestly does not apply to current institutions. Who knows what “money” is anymore, and in any case our central banks do not control its supply. Despite the efforts of a group of remaining monetarists, it has largely disappeared from policy analysis.

Nonetheless, this is a book about theory and we should explore how fiscal theory is the same or different from monetarism. This section sketches the ideas informally. I return to a more detailed presentation with a more careful model in Chapter 17 below.

So far, we have specified that nobody wants to hold money overnight. Let us add a nonzero demand for money held from time $t$ to time $t+1$,

$$M_t V = P_t Y.$$  \hfill (4.20)

I specify here that money pays the same interest as bonds. Money $M_t$ is a special flavor of government bond that is liquid and useful for transactions, generating this demand, in a way that bonds are not. We will account in Chapter 17 for money that does not pay interest, or pays an interest less than bonds, which generates seigniorage revenue for the government. This complication is not important for the points here. The present value relation is then

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \hfill (4.21)$$

Our model now consists of equations (4.20) and (4.21). The monetarist, active-money, passive-fiscal, approach solves it this way: The central bank, by open market operations
trading $M_t$ for $B_t$ alters the composition of government debt between $M_t$ and $B_t$, without changing its total quantity $M_t + B_t$ and without changing surpluses. The quantity of money $M_t$ then determines the price level $P_t$. Fiscal policy is “passive” and once again adjusts surpluses as needed to make (4.21) hold for whatever value of $P_t$ the central bank chooses. If the central bank chooses a disinflation, the treasury raises taxes to pay off the windfall to bondholders.

Monetarists do not ignore the fiscal equation. Fiscal support for monetary policy, and the dangers of its absence is an important part of monetarist thought. Generalizing the model to include seigniorage revenues, as we will below, monetarists see that inadequate surpluses to pay off debts leads to too much money growth and then inflation. But in “normal times,” seigniorage is small and fiscal policy is assumed to take care of inflation-induced variation in the value of debt.

The fiscal theorist solves (4.20) and (4.21) in the opposite direction. Given preexisting stocks $M_{t-1}$ and $B_{t-1}$ fiscal policy determines the price level $P_t$. But then the central bank must act “passively,” supplying as much money as people demand so that (4.20) holds. Many central bank procedures implement such passive policy. The Fed was founded to “provide an elastic currency,” after all. Central banks have often “discounted bills,” allowing people to borrow newly-created money using illiquid private or government credit as collateral, at a fixed interest rate. An interest rate target, expressed as a discount rate, thus is a passive monetary policy.

If monetary and fiscal policy are both passive, then the price level is undetermined. For this reason monetarists have long bemoaned the fact that central banks so often follow apparently passive policies, discounting freely, i.e. following the “real-bills” doctrine, following interest rate targets, failing to control inside moneys, supplying money as needed to meet demand, providing “liquidity” to the banking system as needed, and so forth. That inflation is something, and often remarkably stable for long periods under such policies ought to cause more disquiet than it does in the face of a prediction that the price level is undetermined. All of these policies are consistent with a determinate price level in a fiscal theory.

Once again, with a money demand, all we see in equilibrium are the two equilibrium conditions, (4.20) and (4.21). Both conditions hold in both models, so the monetarist solution and the fiscal theory solution are observationally equivalent. You cannot tell them apart from time-series data.

Monetarism seems like a more attractive foundation than the new-Keynesian model, as $MV = PY$ and control of $M$ seems to determine the price level transparently, without the difficulties of multiple explosive equilibria. As we shall see in Chapter
however, even this is not the case. Money demand is interest-elastic – people hold less money when the interest cost of doing so is higher. Adding this fact to the model we have the same set of indeterminacy issues.
Chapter 5

Lessons of observational equivalence

Observational equivalence does not doom the fiscal theory enterprise! Nor, contrariwise, does it doom the new-Keynesian or monetarist enterprises. (Any such argument would cut both ways. Who came first doesn’t matter logically.) There are many observational equivalence theorems in economics and finance. For example, marginal utility and probability always enter together in asset pricing formulas, so “rational” and “behavioral” finance are observationally equivalent without further assumptions. This observation has not stopped both branches of finance from productive investigation and debate.

The next few sections set out what I see as the lessons of observational equivalence, and a vision of how we should proceed. The bottom line, really, is that anything unobservable can’t matter that much, so we should put these theoretical controversies in the rear view mirror and adopt a set of procedures in which the choice of regime is of minor importance. In the end, the government must produce a coordinated fiscal and monetary policy in the Ramsey style – government policy instruments must obey the constraints of private equilibrium conditions. Our description of how it does so should be faintly plausible. The stylized “regime” analysis of how it does so is, under the microscope, rather sterile. As I consider the alternatives, there really isn’t a set of equally plausible regimes to pick from. A lightly interpreted version of the fiscal regime is the only coherent model of the price level we have, that is vaguely consistent with current institutions.
5.1 Testing for regimes

Observational equivalence dooms formal general tests of active-fiscal vs. active-money regimes. Common ideas for tests don’t work: Inflation should Granger-cause surpluses, even in a fiscal regime. Debts should precede and Granger-cause higher surpluses, even in a fiscal regime. Larger surpluses should forecast declines in debt, even in a fiscal regime. One cannot test present value relations.

Easy armchair laugh-tests likewise fail. Deficits are higher in recessions, and lower in booms, yet inflation goes the other way, lower in recessions and higher in booms. What about Japan, and other countries with high debts and no inflation?

The fiscal theory does not predict a tight relationship between deficits or debt and inflation. For both cyclical and cross-country comparisons, variation in the discount rate may matter more than variation in expected surpluses to understand the price level.

One cannot reject a present value relation, without additional assumptions on surpluses or discount rates. And any rejection of a present value relation equally rejects the passively-achieved present value relation underlying monetarist or new-Keynesian models.

General tests of classes of models, and whether the common equilibrium condition is achieved “actively” or “passively” are fruitless. Measuring what combination of expected surpluses and discount rates account for inflation is a useful task.

Observational equivalence must quickly dampen one’s enthusiasm for “testing” fiscal theory vs. active-money regimes, by formal methods or by apparently easy armchair observations. Such a test must be entirely based on the auxiliary assumptions one introduces to gain identification.

Now, auxiliary assumptions are important. Evaluating the ability of models to account for data, models that include basic principles plus auxiliary assumptions, is what economists do. So observational equivalence leads us to pay close attention to those assumptions, and to focus on creating successful models rather than attempt general tests of classes of models. So, really, the problem is that the auxiliary assumptions required to surmount observational equivalence so far are not believable.

For example, one might assume that monetary policy reacts to inflation in equilibrium the same way it reacts to inflation off equilibrium. Write \( i_t = \phi \pi_t \), so
the rule (4.16) becomes $i_t = \phi \pi_t^* + \phi (\pi_t - \pi_t^*)$, with the same parameter governing monetary policy and equilibrium-selection policy. Then we can indeed measure equilibrium-selection policy by measuring monetary policy. This is how new-Keynesian models were first written down, when the distinction between monetary policy and equilibrium-selection policy was not clear. But it is an extra assumption, and the only source of testable content. And it’s not a particularly sensible assumption. Why should the in-equilibrium reaction of interest rates to inflation, designed to guide economic dynamics and smooth output, employment and inflation fluctuations, be the same as the off-equilibrium reaction of interest rates to inflation, designed to make multiple equilibria unpleasant and scare people away from them? They are entirely different policies with entirely different purposes. (Even then identification it’s not so easy. Section 16 reviews identification issues in more detail. But at least it’s not impossible.)

Since some notion of causal structure runs through the stories we have told, one is tempted to run Granger causality tests. In $MV = PY$, does money Granger-cause prices, or do prices Granger-cause money? In $i = \phi \pi$, does inflation Granger-cause interest rate changes? But these equilibrium relations, read with either causal structure, do not imply a time-series relation, that shocks to money precede shocks to prices or vice versa, or that shocks to inflation precede shocks to interest rates or vice-versa. (A Granger causality test checks whether innovations to one variable help to predict another variable. Money-nominal income causality is an issue with a long and illustrious history from Andersen and Jordan (1968), Tobin (1970), Granger (1969), to Sims (1980).)

That long-understood fact applies to fiscal theory as well. For example, one may be tempted to test whether shocks to surpluses Granger-cause (predict) inflation or whether shocks to inflation Granger-cause surpluses, and to read the results as a test of monetary vs. fiscal regimes. But this is not a valid test of regimes. The question is how policy reacts to a jump to a different equilibrium, not whether shocks to surpluses or inflation help to forecast the other in an equilibrium.

Moreover, we expect inflation to Granger-cause surpluses, even in a fiscal regime. The fiscal condition (4.11)

$$\pi_{t+1} - E_t \pi_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j s_{t+1+j} = \varepsilon_{t+1}^*$$

holds in equilibrium, of the data we observe. Suppose people get bad news about future surpluses. Inflation moves, and then we, studying the economy, see lower
surpluses or deficits ex-post that people in the economy learned of ex-ante. Inflation will help to forecast surpluses. The only possibility is that we have in hand all of people’s information about future surpluses. But then the race is a tie. This test is exactly the same as measuring whether innovations to stock prices help to forecast (Granger-cause) subsequent dividends – they do – and concluding that dividends “passively” adjust to random fluctuations in stock prices – they don’t. It is the same as measuring whether innovations to consumption help to forecast (Granger-cause) subsequent income – they do – and concluding that income “passively” adjusts to consumption choices. I wish.

One may be tempted to measure whether surpluses respond to the value of debt, and to call the result a test of an active vs. passive fiscal regime. Run regressions of surpluses on debt, or see if debts Granger-cause surpluses. Observational equivalence warns us that this too cannot work, especially once we learn to distinguish on- vs. off-equilibrium responses of surpluses to debt as in section 4.4.2. Moreover, the normal-times fiscal policy of section 3.7 has exactly this prediction even in a fiscal regime. The government borrows, promises higher surpluses in order to raise any revenue from bond sales, and then on average makes good on its promises. Regressions of surpluses on debt show positive coefficients, debts Granger-cause surpluses.

One may be tempted to test whether a shock to surpluses reduces subsequent debts, following Canzoneri, Cumby, and Diba (2001). If surpluses follow an AR(1), for example, then a larger surplus today would mean larger surpluses in the future. If the value of debt is forward-looking, as FTPL writes, then the value of the debt should rise, not fall, following a surplus shock. Unsurprisingly, in the data, higher surpluses pay down debts.

(Equations: If

\[ s_{t+1} = \rho s_t + \varepsilon_{t+1} \]

then

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \frac{1}{1 - \beta \rho} s_t \]

Debt is proportional to surplus, so debt rises when the surplus rises.)

Well, one answers, maybe surpluses do not follow an AR(1), the auxiliary assumption of this test. In particular, for the government to run a deficit without an inflation-induced default on outstanding debt every time it wishes to do so, the government must sell more debt, and to sell more debt and raise revenue by doing so, it must convince investors that subsequent surpluses will be larger. The most natural surplus
process is not an AR(1), as we realized in section 3.7. (We’ll investigate this time series process for surpluses in detail in section 9. The point is more general than an AR(1). It turns out surpluses cannot be forecast for this purpose from any regression.)

Moreover, observational equivalence means that if this argument were right, it would be equally devastating as a test of new-Keynesian and monetarist models, i.e. that their “passive” fiscal policy equilibrium condition is violated.

If testing one regime is hard, observational equivalence and non-identification tell us it’s even harder to measure time-varying or regime-switching models, in which an economy like the US switches between active-fiscal and active-monetary regimes. (Davig and Leeper (2006) estimate a regime-switching model; Clarida, Galí, and Gertler (2000) estimate a switch from passive to active monetary policy around 1980.) Again, everything relies on the identifying assumptions. Again, this is a lot clearer in retrospect than it was at the time these authors wrote.

5.1.1 Laugh tests

Aside formal testing, many commenters dismiss fiscal theory by apparently easy armchair rejections, or laugh tests. Recessions feature deficits and less inflation. Expansions feature surpluses and more inflation. Isn’t the sign wrong? Countries with large debts or deficits seem no more likely to end up with currency devaluation or inflation. What about Japan, with debt more than 200% of GDP, continuing fiscal deficits, and yet slight deflation? What about the US, with large debts and deficits, annual warnings from the CBO of yawning fiscal gaps to come, and no inflation? Contrariwise, many crashes, such as the late 1990s east Asian currency collapses, were not preceded by large deficits or government debts. Doesn’t this invalidate the fiscal theory?

No, as observational equivalence should indicate. There is an expectation of future surpluses and, especially, there is a discount factor that makes sense of all these observations. We have to at least state and examine those possibilities. And in fact underpinnings of all these observations are not implausible.

The fiscal theory does not predict a tight relationship, or even a positive correlation, between deficits or debts and inflation. First, the fiscal theory ties the price level to the present value of future surpluses, not to current surpluses, and there is good reason and evidence to suppose current deficits come with future surpluses. Second,
discount rates vary, and there is good reason and much evidence to suppose that
discount rates are lower in recessions and higher in booms, and lower in low-inflation,
high-debt countries such as Japan. Third, the valuation equation holds in all the
models we are contrasting, so such observations, like formal tests, even if cogent, are
not a refutation of fiscal theory in favor of passive-fiscal alternatives.

The valuation equation applies to the present value of all future surpluses, not to
current surpluses. Yes, you may say, but surpluses and deficits are persistent, so
shouldn’t we expect current deficits and surpluses to be correlated with those in
the future? Then current deficits are a signal of deficits and surpluses to come,
and inflations should be even more severe in recessions, and deflations in booms.
Moreover, if we look at surplus forecasts such as those provided by the CBO, there
is no evidence that surplus forecasts rise in recessions or after large deficits.

We have already seen this argument in its more formal form. It is completely natural
for a government that wants to finance a current deficit to issue debt, along with
explicit or implicit promises of future surpluses, and not always to finance deficits
by unexpectedly devaluing outstanding debt via inflation.

Yes, surplus and deficit forecasts often get worse in the recession. Those forecasts
indicate a huge fiscal crisis should be happening in the U. S. right now. But such
forecasts are often of the form “if the government does not raise taxes or cut spending,
here is what will happen.” CBO forecasts are by design conditional on current policy.
They are not unconditional means. Such forecasts are often issued precisely to prod
the government to action, to raising surpluses sooner rather than later and more
painfully. If it is to avoid a debt crisis, a fiscal adjustment must happen in the
U.S. The U.S. must pursue some combination of entitlement spending reform, a tax
increase via European-style middle class taxes, or greater tax revenue via greater
economic growth. It isn’t unreasonable to hope that this, rather than a catastrophic
debt crisis, will be the culmination of our era of deficits. And most of all, follow the
money: If bondholders believed the CBO forecasts as conditional means, the debt
crisis or inflation would have happened already, and the U.S. government would not
be able to raise revenue from debt sales.

Similarly, the fiscal theory makes no prediction that large debts must lead to inflation.
Large debts, resulting from deficits run to finance wars, financial crises, or as a result
of recessions, were incurred on the promise that they would be paid back, not inflated
away. So on average large debts, that raised revenue, should be followed by surpluses,
and should not forecast inflation. They must, on average be followed by surpluses,
or investors won’t lend in the first place.
Second and most importantly in accounting for data, \textit{discount rates matter}. The $R$ in
\begin{equation}
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j}
\end{equation}
varies over time. The previous argument only brought us to stable inflation in recessions and other times of high deficit or debts. With (to say the least) no obvious positive news about future surpluses, why does inflation actually decline with larger deficits in a recession, and vice versa in a boom? Well, real interest rates collapse in a recession, and rise in a boom. A lower real interest rate raises the value of a given set of surpluses, or even of lower surpluses. The right hand side of (5.1) rises, and so must the left – deflation or disinflation. Conversely, in a boom surpluses increase. Why does inflation increase, not decrease? Well, real interest rates rise, lowering the real value of government debt, an inflationary force.

Low real interest rates also offer a plausible resolution to the levels of debt we see in advanced economies together with low inflation, and to the comparison between Japan, Europe, and the US. Real interest rates on government debts are incredibly low and have been for a decade. Low discount rates make even small and far-future surpluses more valuable.

In fact, the puzzle, if there is one, is why we do not see even more deflation! Surpluses over the long run grow at the economy’s growth rate $g$. In steady state,
\begin{equation}
\frac{B}{PY} = \frac{1}{1/(r-g)}
\end{equation}
The growth rate $g$, though low, has been greater than the real interest rate $r$ on government debt for quite some time. Some commentators see this $r < g$ situation going on forever, implying there is no government budget constraint, or cost to borrowing and spending arbitrary amounts of money. If this were true, under the fiscal theory, we would see even more deflation! That we do not see more deflation means bond markets see an eventual return to the normal state $r > g$.

Why bond investors are willing to lend to the US, Japan, and Europe given our governments’ growth and fiscal prospects is a bit more of a mystery. A huge literature right now is exploring the reasons for low real rates and especially low rates for government debt. How long they will continue to do so an unsettling question. When $r$ is near $g$, $1/(r-g)$ is very sensitive to small changes in $r$. Thus, if we reconcile large debts with low inflation by a low real interest rate, that view raises the specter of substantial inflation should real interest rates rise or real growth rates
CHAPTER 5. LESSONS OF OBSERVATIONAL EQUIVALENCE

decline, even a bit. But the level of \( r \) – why investors demand such low returns – and what might change it are questions a bit outside the current investigation. Given the evident fact interest rates are very low, and bond holders for now willing to lend at such low rates, low inflation and large debt is not a puzzle.

These observations need quantification, and evaluation in the data, which I return to in section 10. The point for now is that obvious laugh tests of the fiscal theory, like formal tests, are not right, and that there is a plausible story for common observations that initially seem difficult.

Generalizing this line of thought, many apparent formal and armchair “tests of fiscal theory,” are just attempts to test the present value relation itself. “Surpluses, what surpluses? The US is going to run deficits forever!” is a common objection. If so, the present value of surpluses is negative and the present value relation is faulty.

Here, as well as in the quantification to follow, the lessons of a half-century quest to test present value relations in finance is useful. A theorem tells us that there won’t be easy tests:

- If the law of one price holds, there exists a discount factor \( \{\lambda_t\} \) which makes the present value relation

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\lambda_{t+j}}{\lambda_t} s_{t+j}
\]

hold. Absent arbitrage, the discount factor is non-negative \( \lambda_t \geq 0 \).

This theorem as I have stated it in its classic form is much more general than we need. Law of one price and absence of arbitrage apply when there are multiple assets. Here there is only one asset, so the theorem holds even more trivially. The ex-post return on the portfolio of government debt is such a trivial discount factor (section 6.3). “Absence of arbitrage” matters, in that there must be at least some possibility of positive surpluses someday to justify a positive price. And convergence of the sum matters. We will return to the \( r < g \) case in section 21, in which the sum might not converge, and hence the present value of surpluses is infinite and governments can borrow arbitrary amounts and not worry about paying it back. (Short answer: We should be so lucky.)

But for now, it is worth remembering the lessons of finance: without extra assumptions, in particular assumptions about how discount rates may vary, there is no test that refutes the present value relation – not volatility tests, not regression tests, not
the hundreds of anomalies and alphas. Let us not waste time rediscovering that fact in the context of government debt. Our task is the same as the task in finance: not easy tests, but quantitative assessment: do hard-to-measure expected surpluses and discount rates move in the right way to make sense of data on inflation?

5.2 Regimes and practice

Observational equivalence suggests that we modify procedures so that the choice of regime is, where possible, less important. By writing models in terms of observable equilibrium quantities, a researcher can keep most of the workaday practical use of the new-Keynesian model for understanding time series intact without taking a stand on regimes, and offer either interpretation.

However, one should at least calculate and examine implicit fiscal predictions. Even if one specifies an active-money regime, blatantly counterfactual “passive” fiscal policy can inform that specification. The regime question still impinges on policy calculations: If the central bank raises interest rates, should we include or not a contemporaneous fiscal shock? And it affects central doctrines: Can the central bank set an interest rate peg without causing volatile indeterminate inflation? Must a central bank raise observable interest rates more than one for one with observable inflation?

Observational equivalence and non-identification of equilibrium-selection policies seem like bad news for both fiscal theory and new-Keynesian, monetarist, or other passive-fiscal models. On the contrary, I think, these theorems can have positive and somewhat liberating implications, if rather than fight them with heroic identification efforts, and indulge in more extensive controversies, we accept them and adapt to their implications.

The theorems tell us that these foundational “regime” and “equilibrium selection” questions must be much less important than we make them seem. Regimes themselves must not be that meaningful. So we phrase and use our models in ways that, as much as possible, do not require a firm statement about regimes.

The equilibrium quantities, $i_t^*, \pi_t^*, D_t^*$ (or $s_t^*)$ are the only observables. So, let us write a model entirely in their terms, i.e. write just the equilibrium conditions and not equilibrium selection rules. In the absurdly simple model we are studying, all we
have is
\[ i^*_t = E_t \pi^*_{t+1}, \]  
\[ \pi^*_{t+1} - E_t \pi^*_{t+1} = \varepsilon^*_t, \]  
or, if one wishes,
\[ \pi^*_{t+1} - E_t \pi^*_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j s^*_t. \]

Rather than try to derive the equilibrium interest rate and surplus from underlying equilibrium-selection policies and a statement about regimes, e.g. \( i_t = \phi \pi_t + v_t \) or \( i_t = i^*_t + \phi(\pi_t - \pi^*_t) \) let us instead write directly the equilibrium and observed policy variables \( i^*_t \) and \( \varepsilon^*_t \) or \( s^*_t \), and then find the equilibrium endogenous variables, \( \pi^*_t \) here, consistent with those policy variables. In this case, given \( \{i^*_t\} \) and \( \{\varepsilon^*_t\} \) or \( \{s^*_t\} \), we quickly find a unique \( \{\pi^*_t\} \). Then, add a footnote stating that one could, of course, derive the same equilibrium by specifying either active monetary policy \( i_t = i^*_t + \phi(\pi_t - \pi^*_t) \) with any \( \phi > 1 \) and a passive fiscal policy \( \{\varepsilon^*_t\} \), financed by the usual lump-sum taxes, or by specifying active fiscal policy \( \{s^*_t\} \) and a passive monetary policy \( i_t = i^*_t \). (Werning (2012) innovated this clever strategy, including the footnote.) No generality is lost. The theorems reassure us that we can support any equilibrium by such assumptions, and that nobody can prove us wrong.

For example, we can simply describe Figure 4.2 as how inflation will evolve if observed, equilibrium interest rates follow the displayed interest rate path plus, in the top group, no change in fiscal policy, and, in the bottom group, a contemporaneous contractionary fiscal policy.

It appears that this procedure is a fiscal regime and a time-varying interest rate peg. But it is not, and the linguistic subtlety is important. We are specifying the equilibrium interest rate and surpluses, not fixed policy values. We are calculating endogenous variables conditional on those equilibrium interest rate and surpluses. Sure, the equilibrium may come from exogenous surpluses and a time-varying peg. But it just as easily may come from an active policy rule and passive fiscal policy. Observational equivalence says we cannot tell the difference, so why not just use the fiscal interpretation as an easy calculation device.

This approach does not require that we fix the equilibrium interest rate, surplus, or, later, other policy variables. We may describe policy in a way that responds to endogenous variables as well. We can write \( i^*_t = \theta \pi^*_t + v_t \) and plot the response to a serially correlated \( v_t \), with or without a contemporaneous fiscal shock. That is how
5.2. REGIMES AND PRACTICE

I produced the top line of Figure 4.2. We can, and will, write a fiscal policy such as $s_t = s_{0t} + \alpha(B_{t-1}^*/P_t^*)$ plus responses to responses to output and other variables. In doing so we describe how policy making institutions behave in terms of observed variables.

A die-hard new-Keynesian can thereby produce exactly the same results of current new-Keynesian models. If he or she loses faith in standard new-Keynesian equilibrium-selection rules, or if critics or referees complain about their implausibility, fiscal theory offers alternative active-fiscal equilibrium-selection assumptions for the same results. At a minimum, then, fiscal theory offers an alternative set of footnotes about equilibrium selection.

However, reconciliation with fiscal theory suggests a somewhat more serious study monetary-fiscal coordination, without needing to take a stand on regimes, and that study may undermine the plausibility of traditional new-Keynesian calculations. Since the “passive” fiscal support for unexpected inflation is still required, (5.3) and (5.4) are still there, one should at least calculate what the fiscal shock is corresponding to one’s choice of unexpected equilibrium inflation. One should calculate a response function for surpluses, think about how that path is achieved, and one should check that response in the data along with all the other responses of the model. A footnote about arbitrary lump sum taxes might pacify theorists, but it should raise eyebrows in applied work.

For example, suppose you were wondering why the sharp deflation that standard new-Keynesian models predict when the U. S. hit the zero bound in 2008 did not occur. Well, calculating that the model also predicts fiscal policy “passively” substantially tightened – raised taxes, cut spending – just as the stimulus and Obamacare were announced, and declining potential GDP made long-term budget forecasts even more dire, might be a signal that something is wrong in the equilibrium-selection rule. Without that tightening, the deflation cannot happen. One could fix the model by moving to a fiscal equilibrium-selection rule in place of a specification of central bank equilibrium-selection threats to follow the exit from the zero bound. No fiscal tightening, no deflation. But one could also use the implication of a large “passive” fiscal tightening as a wake-up call that one has the wrong monetary equilibrium-selection policy. Specify a monetary equilibrium-selection rule which does not imply a substantial fiscal tightening. By observational equivalence both trains of thought lead to the same place. (Cochrane (2017c) is an extensive analysis of this case, and I return to it in section 20. That paper advocates the fiscal selection. Werning (2012) changes the equilibrium-selection rule to the same effect, though by a completely different train of thought not involving monetary-fiscal coordination.)
Equilibrium-selection policies and regimes become more important when one wants to ask policy questions. For example, suppose that the data show contemporaneous monetary and fiscal shocks, as in the bottom group of Figure 4.2. If one simply wishes to fit the data, it does not matter whether we regard this outcome as an active-money equilibrium selection, with a passive fiscal contraction, or whether we regard this outcome as simultaneous changes in monetary and fiscal policy under FTMP, perhaps each policy responding to the same external events. But if one wants to ask, “what happens if the Fed raises interest rates?” a mild form of the regime question cannot be avoided. If the Fed raises interest rates, independent of an external event, will we see the simultaneous fiscal contraction?

Really the question is not mysterious “regimes,” it is merely what set of responses is it interesting to plot? Is it interesting to monetary policy makers to include a contemporaneous fiscal shock in one’s calculations of the effects of monetary policy? Yes, if they have total faith in their equilibrium-selection powers and passive fiscal responses. No otherwise. I can imagine Fed officials at least wanting to see the plot calculated both ways, with and without associated fiscal contraction.

Viewed another way, the policy question is different from the what-regime-are-we-in question. One might believe we are in an active-money passive-fiscal regime, yet one may want to ask the policy question, what happens if the Fed raises rates and fiscal policy does not react with a tightening? Essentially, what if the regime shifts for this policy experiment?

In sum, the operational distinction between new-Keynesian and FTMP approaches is small. If you have spent a lot of time on new-Keynesian models, the fiscal theory does not mean you have to throw all that investment out and start over, armed with the present value relationship, time series of debts and surpluses, ignore the Fed and start focusing on tax, spending, and debt-management policy! You can write down exactly the same model, and solve it with the same methods, as you do now. Interest rate policy remains central.

What you do may not change much, but the results you will regard as plausible will likely change. The model may look quite strange once you write down the actual “passive” fiscal policy and check its results. If you start to take fiscal and monetary coordination more seriously, you may well arrive at different historical analyses, i.e. what shocks account for the data. You may well arrive at different policy conclusions – what are interesting policy experiments, what the economy’s responses to policy experiments are.

You will also become open to different views of how alternative policy arrangements,
rules, and institutions work, and even basic doctrines that guide such thinking. For example, the FTMP in this model says an interest rate peg is possible, and leads to stable and determinate inflation. In the standard new-Keynesian approach to the same model, an interest rate peg leads to indeterminacy, and thus volatile unexpected inflation. In the standard new-Keynesian approach, it is important to have an equilibrium-selection component of monetary policy with $\phi > 1$ in order to render inflation determinate. In the FTMP $\phi > 1$ is possible, but would lead to unstable inflation. Regimes still do matter.

In sum, the fiscal theory of monetary policy maintains the substance of workaday practice with standard new-Keynesian models, while solving simply the many holes in its theoretical foundation. The observational equivalence theorems emphasize this fact, and encourage de-emphasizing scholastic debates about unobservable “regime” assumptions. Yet the FTMP opens a box of new things to do, questions to ask, and papers to write with that familiar toolkit!

5.3 Plausibility and other information

We can look at the plausibility of different regimes. I argue that the standard active interest-rate policy makes no sense. Central banks do not threaten to hyperinflate and deflate economies in order to select equilibria, and there is no reason to rule out non-local equilibria.

The pure “regimes” are a conceptually useful experiment, but a terrible model of how governments actually produce a coordinated fiscal and monetary policy. Emphasizing such coordination rather than focusing on a stylized game of chicken between a hypothetical treasury and central banks is a more productive path.

Institutions, rules, legal limitations are all ways that governments communicate off-equilibrium behavior in order to circumvent the observational equivalence and non-identification theorems. Their analysis along with the analysis of choices governments make in times of stress are revealing about the regime.

As in the other observational equivalence and non-identification theorems of economics, we can and should examine the plausibility of different regimes, off-equilibrium behaviors, and identifying assumptions, and attempt to use information beyond time-series analysis of equilibrium quantities and prices to inform us about the world. How would monetary and fiscal policy authorities respond to unexpected variation in the price level? How do people in the economy expect authorities to behave? What kinds
of behavior are encoded in the legal and institutional structures and restrictions of monetary and fiscal policy? Why were those structures chosen? When we see governments making hard choices, say between unpopular and distortionary taxation or spending cuts vs. inflation or devaluation, what to those choices tell us about the economic constraints governments perceive?

Continuing the analogy, behavioral and rational asset pricers (often grudgingly) admit observational equivalence, but then question (or lampoon) plausibility of the alternative interpretations. Sure, you can cook up a set of preferences to account for time-varying expected returns, but aren’t they pretty strained? Sure, you can posit that something deep in human psychology leads people to misunderstand the distribution of stock returns following an IPO, but isn’t that “explanation” awfully arbitrary and vacuous?

As an example, Canzoneri, Cumby, and Diba (2001), introducing their test described above, write

“it is quite difficult (and perhaps impossible) to develop formal tests that discriminate between R [active money] and NR [active fiscal] regimes, since (as Cochrane, 1998, points out) both regimes use exactly the same equations to explain a given data set.”

Of my discussion of the likely long-run negative autocorrelation of surpluses above, the more detailed examples in section 9 and the admittedly much more convoluted precursor to these examples in the contemporaneous Cochrane (2001), they write

“NR regimes offer a rather convoluted explanation that requires the correlation between today’s surplus innovation and future surpluses to eventually turn negative. We will argue that this correlation structure seems rather implausible in the context of an NR regime, where surpluses are governed by an exogenous political process.”

Well, here we are – there is a theorem stating one can’t test regimes. To overcome the theorem one needs auxiliary assumptions, here involving the time-series process of surpluses. And we can and must productively argue the plausibility of those assumptions. (In addition to the surplus process discussion, nothing in FTPL requires surpluses to be “exogenous,” and not respond to other economic variables. This observation also reflects almost 20 years of hindsight.)

Well, how plausible are the off-equilibrium stories? Looking at the new-Keynesian equilibrium condition in the form $E_t(\pi_{t+1} - \pi^*_t) = \phi(\pi_t - \pi^*_t)$, I object that no
central bank on this planet has ever said, “if inflation breaks out, we will adjust monetary policy to raise subsequent inflation explosively, and vice versa. Fear of this hyperinflationary or hyperdeflationary path will keep inflation from breaking out in the first place.” Central banks always say they will respond to inflation by reducing subsequent inflation. They view it as their job to produce stable dynamics with $\phi < 1$ in the latter equation.

An explosive threat is empty anyway (time-inconsistent, subgame-imperfect) as hyperinflation or deflation is so contrary to the central bank’s objectives. Central bankers explaining their actions during a hyperinflation have really never said “this is what we committed to do if inflation deviated from our target, so we will continue to destroy the economy.”

Central banks do worry about expectations, and “anchoring” them, perhaps with multiple-equilibrium ideas in mind that expected inflation will lead to actual inflation. But they do so with exactly the opposite sign: They describe anchoring by their commitment to stabilizing actions, to $\phi < 1$ dynamics, to always bringing inflation back should it wander off target. Moreover, what matters is private sector expectations of policy. Any reflection of multiple equilibrium selection policy via of a threatened hyperinflation or hyperdeflation is totally absent from popular economic commentary. It exists only in the equations of new-Keynesian models, and not always in the introductions and conclusions that describe those equations.

Yes, central bankers raise interest rates when they see inflation, but they view the world through old-Keynesian glasses in which higher interest rates produce lower inflation at all horizons. Their underlying commitment is to do what it takes to lower future inflation when today’s inflation is too high, and vice versa. Their quick adoption of “unconventional” policy, quantitative easing and forward guidance, when lower interest rates could no longer be applied proves the point, no matter how unsuccessful those policies were in the end to creating inflation.

Moreover, the new-Keynesian equilibrium selection story also requires us to adopt a rule against “non-local” inflation paths, in addition to the usual definitions of Walrasian equilibrium. $\pi_{t+1} - \pi^*_{t+1} = \phi(\pi_t - \pi^*_t)$ with $(\pi_t - \pi^*_t) \neq 0$ is simply ruled out by assumption. In the linearized model, this is a rule against hyperinflation or deflation. Tell it to a Venezuelan. Real transversality conditions rule out real explosions, but there is nothing in standard economics that rules out nominal explosions. The nonlinear version of new-Keynesian equilibrium selection is even more stringent and troublesome. Now the equilibrium condition is $\pi_{t+1} - \pi^*_{t+1} = \Phi(\pi_t - \pi^*_t)$ where $\Phi(\cdot)$ is a nonlinear function, that locally has $\Phi' > 1$ but not globally. There are
“non-local” equilibria that don’t explode, but merely drift boundedly away from a desired equilibrium $\pi_t = \pi^*_t$. In particular, there are equilibria that drift down to zero nominal rates and slight deflation. If $\pi^*_t = 2\%$, for example, and the real interest rate is $r = 1\%$, then we have to rule out paths that start at $\pi_t = 1.9\%$ and drift down to $\pi_t = -1\%$ and then stay there. Well, these equilibria are “non-local” to 2%, since $-1\%$ is outside an $\varepsilon$ ball of 2%, but that seems like a poor reason to rule out the possibility, especially given recent experience. By contrast, the fiscal theory equilibrium is immediately “globally unique.” It also offers a unified way of thinking about “normal times” monetary economics along with the inflations, currency crashes, unconventional policies, and other events in which the “fiscal” and debt management parts takes center stage.

In sum, my view, which I hope to persuade you of by the end of this book (section 16 in particular) if I have not done so already, that the fiscal theory is the only coherent theory of the price level we have that is consistent with current institutions. Equilibrium selection by interest rate policies simply makes no sense. Central banks don’t deliberately induce instability, nobody believes that they do, and there is nothing wrong with “non-local” equilibria anyway. Monetarism is a more theoretically coherent story. It could be true. It just isn’t. Our central banks do not limit money supplies, money demand is interest-elastic and therefore even controlling the quantity of money alone doesn’t control the price level (Chapter 17). Money demand has lost any meaning in a plethora of liquid assets and electronic transactions. An active-fiscal policy is the only coherent foundation we have for price level determination in an economy with our institutional features.

Fiscal theory critics offer similar critiques of the plausibility of “active” fiscal policy, and whether people could expect such a thing. We have discussed many, which I hope I have defused, and we will discuss and defuse many more.

More deeply, and back to the theme that observational equivalence means we have paid too much attention to these issues, pure “active fiscal - passive money” and “active money - passive fiscal” regimes, though useful for these conceptual exercises, don’t make sense as serious models of how monetary and fiscal policies are set. Attempting to test for them in data therefore makes even less sense. In the end, the government must produce a coordinated set of monetary and fiscal policies – a setting of interest rates or money supplies and surpluses that generates a unique equilibrium price level. When there is a possibility that two tools conflict, the government needs to figure out settings that do not conflict. This is old advice that goes well beyond fiscal theory. A government cannot set both a money supply target and an interest rate target. It cannot set both a tax rate and a tax revenue. The Ramsey description
of policy settings says that the government must take private sector equilibrium conditions as constraints, and not attempt to set policy levers that violate such constraints.

In this context, both “regime” stories reflect a stylized and unrealistic game-theoretic view of how a government, composed of two independent agencies, with conflicting objectives, and each with some power over the price level, forms a coordinated policy. In one, “monetary authorities” win a game of chicken, run everything, and fiscal authorities meekly follow. In the other, “fiscal authorities” win the game of chicken, run everything, and monetary authorities meekly follow. Most of the time in economics and public finance we do not bother modeling the inner workings of government or other agents. We don’t describe consumers with a little angel on one shoulder and a devil on the other playing an ultimatum game. When we do model inner workings, in public choice theory, we recognize that actual governments produce outcomes that reflect many conflicting voices in a complex interaction, not anything as simple as this two-agent chicken game. So, given that regimes are observationally equivalent, and that the regimes themselves are not remotely realistic descriptions of how real governments form policy, they are doubly not worth testing.

Moreover, the regime question narrows once we look more closely and understand the issue as equilibrium-selection policy only. The question is no longer “Who is in charge of the price level?,” but “Who is in charge of equilibrium selection?” And equilibrium selection is only the question of unexpected inflation in this simple model. The fiscal theory still requires a lot of monetary policy to determine expected inflation: We still need an interest rate target or some policy to determine the supply of debt \( \{B_t\} \) independent of surpluses, and that part is common to both “fiscal-dominant” and “money-dominant” regimes. The potential area for conflict between a treasury and a central bank trying to do equilibrium-selection is only over the question whether a central bank can control unexpected inflation by equilibrium-selection threats \( \phi(\pi_t - \pi^*_t) \) to force a fiscal reaction, or whether fiscal policy can forestall any such attempts. There is no monetary-fiscal conflict over expected inflation.

With \( MV = PY \) and seigniorage, the game of chicken interpretation is a bit more sensible – and this context is the original source of the regime idea (see Sargent and Wallace (1981)). When money supply does directly control seigniorage, one component of the surplus, one can ask whether a resolute central bank can refuse to monetize deficits of a profligate government, or conversely whether it can force inflation on an austere government. But once asked, this question also immediately jumps beyond simple “regime” game-of-chicken analysis to thinking about the government as a whole choosing between the distortions of an inflation tax and those
of the other taxes at its disposal, or the consequences of spending cuts, and in-
vites multiple-player political-economy questions far more interesting than a game of chicken between absolutes. (Sargent and Wallace (1981) were also concerned about the larger, non-monetary “starve the beast” game of chicken: Can refusing tax hikes, running large deficits, and putting the government in fear or a debt crisis force changes to government spending?)

But this game of chicken doesn’t apply to normal times of subdued inflation, or to new-Keynesian models, in which seigniorage revenues are an insignificant part of government surpluses. The relevant game of chicken is, if the central bank wishes to select an equilibrium with unexpected inflation or deflation, whether the fiscal authorities validate a deflation-induced rise in the value of nominal debt, or whether they will accept the gift of inflation-induced declines in the value of debt. The new-Keynesian game of chicken involves a different threat too. A central bank can, perhaps threaten to hold money supply constant, and one can see its action directly. Can a central bank credibly threaten to hyperinflatedeconomy? This issue has not received any analysis that I know of.

The same line of thought offers a path to thinking about regime kinds of ques-
tions, while acknowledging observational equivalence of time series taken from a given equilibrium, and going beyond plausibility debates which are unlikely to be resolved quickly. (The 50 year old behavioral vs. rational finance debate is exhibit A of that problem!) Times of fiscal or monetary stress show governments making choices, and those choices illuminates the nature of the economy they face. We shall look at many below. Monetary and fiscal policies are chock full of institutions, rules and commitments about government behavior that we can interpret as ways of committing to and communicating equilibrium-selection policies that cannot be directly observed. The gold standard, foreign exchange pegs, backing promises, currency boards, balanced budget rules, inflation targets, Taylor rules, the many legal restrictions against inflationary finance and central bank actions, the institutional separation of monetary and fiscal policy, and so on serve this purpose. Analysis of institutions that governments choose and how successful they are also illuminate the nature of the economy they face.

This kind of institutional and plausibility discussion may seem annoyingly chatty to those trained in modern formal methods. But observational equivalence means we have to talk about plausibility, and cannot rely on formal testing.
Chapter 6

A bit of generality

This chapter presents a few generalizations of the fiscal theory valuation formula: risk and risk aversion, long-term debt, continuous time, an expression in terms of debt to GDP and surplus to GDP ratios, and a version that includes non-interest bearing money. These are useful formulas for applications that need to recognize these generalizations, and they show that the simplifications of model so far are in fact just simplifications and not necessary assumptions. I also present two useful linearizations.

6.1 Long-term debt and the inflation response

With long term debt, the basic flow and present value relations become

\[ B_{t-1} = P_t s_t + \sum_{j=1}^{\infty} Q^{(t+j)}_t \left( B^{(t+j)}_t - B^{(t+j)}_{t-1} \right). \]

\[ \frac{\sum_{j=0}^{\infty} Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

A fiscal shock may be met by lower bond prices instead of a higher price level, i.e. by future rather than current inflation. A rise in nominal interest rates with no change in surpluses can result in a lower price level.
Long-term debt adds much to the fiscal theory. As we move to higher frequency observations, more debt is long-term, so its analytics become more important.

Denote by $B_{t-1}^{(t+j)}$ the quantity of nominal zero-coupon bonds, outstanding at the end of period $t-1$, that come due at time $t+j$. $B_{t-1}^{(t)}$ are the one-period bonds coming due at $t$ that we have studied so far. Denote by $Q_t^{(t+j)}$ the price at time $t$ of bonds coming due at time $t+j$. Continuing the constant real interest rate frictionless case, bond prices are

$$Q_t^{(t+j)} = E_t \left( \frac{1}{R^j} \frac{P_t}{P_{t+j}} \right). \tag{6.1}$$

The flow condition now includes sales or repurchases of longer-maturity bonds,

$$B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_t^{(t+j)} \right). \tag{6.2}$$

Money created to redeem maturing bonds must be soaked up by primary surpluses, or by debt sales, including sales of long term debt, which may be incremental sales. The present value condition now reads

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} \frac{P_t}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \tag{6.3}$$

The real market value of nominal debt equals the present value of primary surpluses.

We can derive (6.3) from (6.2) by iterating forward and applying the condition that the real value of debt not grow faster than the interest rate, as before. We can derive (6.2) from (6.3) by considering its value at two adjacent dates.

The present value condition (6.3) now allows a fiscal shock to be met by a decline in nominal bond prices $Q_t^{(t+j)}$ rather than a rise in the price level $P_t$. However, the bond pricing formula (6.1) tells us that this event means future inflation rather than current inflation. Chapter 8 studies when each of those outcomes occurs. The response of current and future inflation to debt sales becomes much more interesting as well, which is also studied in Chapter 8.

Long-term debt allows the model to produce a version of the classic belief that higher interest rates lower inflation, at least temporarily. In section 4.1, with one period debt, we concluded that in this frictionless model, a rise in interest rates
6.1. LONG-TERM DEBT AND THE INFLATION RESPONSE

Figure 6.1: Effect of an unexpected interest rate rise at time 0 with long-term debt outstanding.

with no fiscal shock had no immediate impact on current inflation, but uniformly raised subsequent inflation, as captured by Figure 4.1. Now, a surprise persistent rise in nominal interest rates lowers long term bond prices \( Q_{t+j} \) on the left-hand side of (6.3). Debt \( B_{t+1} \) is predetermined. With no change in surpluses, this change requires a decline in \( P_t \). Higher interest rates still require higher subsequent expected inflation. But we first see a drop in the price level.

The mechanism is “aggregate demand” as usual. If the price level does not change, then the real value of government debt to investors is greater than its real market value. People try to buy more government debt, and thus less goods and services. This lack of aggregate demand pushes the price level down. The deflationary force is the same as that which occurs if the real present value of primary surpluses \( \{ s_{t+j} \} \) increases. It is a “wealth effect” of government debt; the lower value of government debt makes people want to consume less, which in this endowment economy means a lower price level.

Figure 6.1 plots an example. I assume that the maturity structure of debt outstand-
CHAPTER 6. A BIT OF GENERALITY

ing at time 0 is geometric,

\[ B_{t-1}^{(t+j)} = \Omega^t B_{t-1}. \]

I use \( \Omega = 0.8 \), which roughly approximates the U.S. maturity structure. I start at a steady state \( B_{-1} = B, P_{-1} = P, i_{-1} = r \). Interest rates then rise suddenly and unexpectedly at time 0 from \( i = r = 2\% \) to \( i = 3\% \) and stay there forever. Since all interest rates rise the same amount, yields of all maturities rise from 2\% to 3\%.

The price level path for \( t > 0 \) is determined by the interest rate target, and displays greater inflation,

\[ P_t = \left( \frac{1 + i}{1 + r} \right)^t P_0, \quad t \geq 1 \]

The time-0 valuation equation (6.3) then determines the downward jump in the price level \( P_0 \). (Section 8.1.1 has the algebra.) The line labeled “log(\( P_t \))” shows in this case a roughly 3.5\% decline in the price level, followed by steady 1\% inflation.

The dashed line marked “Short debt; expected” in Figure 6.1 plots inflation in the \( \Omega = 0 \) case of only one-period debt. In this case, inflation starts one period after the interest rate rise, with no downward jump, as in Figure 4.1.

From (6.3), the size of this short-term disinflationary effect of an interest rate increase (and vice versa) depends precisely on how much the nominal market value of the debt changes. It is larger when bond price changes are larger, and when more long-term debt is outstanding. This state-dependence of the effects of monetary policy may be a useful restriction to test, to understand episodes, and to evaluate the effects of policies. Just watch the market value of government debt.

The expected path of interest rates matters more than the current rate in determining a deflationary force. A credible, persistent interest rate rise that lowers long term bond prices a lot has a stronger disinflationary effect than a tentative or transitory rate rise that induces smaller changes to long-term bond prices. In this way, this model gives an opposite picture from standard new-Keynesian models. The latter produce larger inflation declines for transitory AR(1) interest rate movements than for persistent interest-rate movements. In this model, convincing markets that interest rates really will be high for a long time is crucial to affecting inflation today. And, though I am trying to mimic the result, the mechanism here is entirely different from that in new-Keynesian, old-Keynesian, or monetarist models of interest-rate policy.

Before getting too serious in matching a response like this one to data, however, we need to consider sticky prices, discount rate variation, and potential coincident fiscal
6.2 Debt to GDP and the fate of a dollar

In terms of ratios to GDP, the basic valuation equation reads

\[
\frac{B_{t-1}}{P_{t}y_{t}} = E_{t} \sum_{j=0}^{\infty} \left( \frac{1}{R^{j}} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right).
\]

Debt to GDP is surplus to GDP discounted with a factor that subtracts the GDP growth rate from the interest rate. For the sum to converge, the interest rate (rate of return on government bonds) must exceed the GDP growth rate.

We can focus on inflation, rather than the value of all government debt, with

\[
\frac{1}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \frac{1}{R^{j}} \frac{s_{t+j}}{B_{t-1} y_{t+j}}.
\]

or

\[
\frac{1}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \frac{1}{R^{j}} \left( \frac{y_{t+j}}{y_{t}} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right) / \left( \frac{B_{t-1}}{y_{t}} \right).
\]

Debt, spending, and taxes scale with GDP over time and across countries, so ratios to GDP, consumption, or some other common trend are useful ways to keep data stationary. We can easily express the basic present value and flow equations in terms of ratios to GDP by multiplying and dividing by real GDP \( y_{t} \). Then we can write the government debt valuation equation to state that the debt-to-GDP ratio is equal to the present value of surplus to GDP ratios, with a “discount factor” composed of the real interest rate less the GDP growth rate,

\[
\frac{B_{t-1}}{P_{t}y_{t}} = E_{t} \sum_{j=0}^{\infty} \left( \frac{1}{R^{j}} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right).
\]

For this sum to converge, we require that the real interest rate is greater than the GDP growth rate. I consider the opposite possibility later.

This expression, like the basic valuation equation, expresses the value of all government debt. In the end, we are really interested in the price level, or the value of a
single dollar, a single share of government debt. We can focus on that issue with

\[
\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j B_{t-1}} s_{t+j}.
\]

(6.4)

Here, the value of a dollar today depends on future surpluses divided by today’s debt only.

This expression may seem counterintuitive – surpluses grow over time, and future surpluses will also be used to pay down debts incurred in the future. Why are we dividing by debt today? However, if debts are incurred in the future, and then paid off, any negative surpluses and increases in debt would be followed by positive surpluses and reductions in debt, so the time \( t \) present value of surpluses is unchanged. Today’s expected surpluses are, in fact, only those that pay off today’s debts! Similarly, the government might print up a lot of debt in the future \( B_{t+j} \), without raising surpluses, raising future inflation. But debt is rolled over, and the real value paid to current time \( t \) debtholders would not change.

Merging the two ideas, we can write an equation for inflation that recognizes stationary ratios to GDP as

\[
\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( \frac{y_{t+j}}{y_t} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right) / \left( \frac{B_{t-1}}{y_t} \right).
\]

6.3 Risk and discounting

With a general stochastic discount factor \( \Lambda_t \), e.g. \( \Lambda_t = \beta^t u'(c_t) \), we have

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}.
\]

(6.5)

We can also discount using the ex-post real return to holding government bonds,

\[
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}
\]

where

\[
R_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} = (1 + i_t) \frac{P_t}{P_{t+1}}
\]
6.3. RISK AND DISCOUNTING

To introduce risk formally, let the endowment of the model in the last chapters $c_t$ vary over time, and let

$$\frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

denote the stochastic discount factor. Then the price of the one-period nominal bond is

$$Q_t = E_t \left( \frac{\lambda_{t+1} P_t}{\lambda_t P_{t+1}} \right)$$

and the flow condition (2.5) becomes

$$\frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right) \frac{B_t}{P_t}.$$  

Iterating forward, and applying the transversality condition which now reads

$$\lim_{T \to \infty} E_t \left( \frac{\lambda_T B_{T-1}}{\lambda_t P_T} \right) = 0$$

we obtain the standard stochastically-discounted valuation formula:

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\lambda_{t+j}}{\lambda_t} s_{t+j}.$$  

As with the constant interest rate example, even though the government here only finances itself by one-period debt, the real value of that debt depends on a long string of future surpluses. That intertemporal linkage comes from the fact that the government rolls over debt rather than pay it off in finite time. If the government paid off the debt at date $T,$ so $B_T = 0,$ then the iteration would stop at that point and we would have instead

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{T} \frac{\lambda_{t+j}}{\lambda_t} s_{t+j}.$$  

Taking the limit as $T \to \infty$ of this case is an alternative way to understand the transversality condition. That condition can cause lots of technical headaches if we come at it only from the consumer’s side.

One can always discount by the ex-post return, $\frac{\lambda_{t+1}}{\lambda_t} = 1/R_{t+1}.$ This fact is useful empirically when one does not wish to specify a model connecting the discount factor
to other economic quantities. To verify this fact, write the one-period flow relation as

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + \frac{Q_t P_{t+1}}{P_t} B_t.
\]

now,

\[
R_{t+1} = \frac{1}{Q_t P_{t+1}} = (1 + i_t) \frac{P_t}{P_{t+1}}
\]

is the ex-post gross real return on one-period debt. Thus, we can write the flow condition

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R_{t+1}} B_t
\]

and iterate forward to

\[
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}.
\]

This equation holds ex-post; it does not require an expectation. What holds ex-post holds ex-ante, so we can also write

\[
\frac{B_{t-1}}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} \right].
\]

The expectation can refer to any set of probabilities, including sample frequencies. It is really just a transformation of accounting identities.

The same principles hold with long term debt. We just get bigger formulas. We discount using the ex-post return on the entire portfolio of debt,

\[
R_{t+1} = \frac{\sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} P_{t+1}^{(t+1+j)}}{\sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}} P_t.
\] (6.6)

This return reflects how the change in bond prices from \(Q_t\) to \(Q_{t+1}\) affects the market value of debt outstanding at the end of time \(t\). Then the flow identity is

\[
\sum_{j=0}^{\infty} Q_{t}^{(t+j)} P_{t-1}^{(t+j)} B_{t-1}^{(t+j)} = s_t + \frac{1}{R_{t+1}} \sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} P_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.
\] (6.7)

We iterate again to

\[
\sum_{j=0}^{\infty} Q_{t}^{(t+j)} P_{t-1}^{(t+j)} B_{t-1}^{(t+j)} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}
\]

but keep in mind the definition [6.6] for the real bond portfolio return that includes long-term debt.
6.4 The discount factor for government debt

We think about the discount rate, the expected return for nominal government debt.

\[ i_t \approx r_t^f + E_t \pi_{t+1} - \gamma \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) \]

Since the covariance of inflation and consumption growth is positive – bonds gain real value in recessions – the risk premium is likely to be negative, so nominal government bonds pay a lower expected real return than the real interest rate.

Surpluses are procyclical, so the present value relation would seem to predict a large risk premium, like the equity premium. But deficits today are generally followed by surpluses tomorrow, or lower discount rates, so the present value of surpluses is not obviously procyclical – which is why inflation can be procyclical.

One may question the need for general stochastic discounters. After all, this is government debt and we observe its yield. Expected inflation isn’t that hard to measure. However risk premiums matter for government debt.

The real risk free rate is

\[ \frac{1}{R_t^f} = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right), \]

so from the definition of covariance we can write the nominal bond price

\[ Q_t = \frac{1}{1+i_t} = \frac{1}{R_t^f} E_t \left( \frac{P_t}{P_{t+1}} \right) - \text{cov}_t \left( \frac{\Lambda_{t+1}}{\Lambda_t}, \frac{P_t}{P_{t+1}} \right) \]

\[ i_t \approx r_t^f + E_t \pi_{t+1} - \gamma \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) \] (6.10)

The bottom linearization uses the standard power utility with risk aversion \( \gamma \).

Thus, we can ignore risk and discount at the implied government bond real interest rate \( i_t - E_t \pi_{t+1} \) if unexpected inflation is not correlated with the stochastic discount factor, or consumption growth in the simple model. That usually is not the case. In the U.S., inflation is lower in recessions, when consumption growth is lower and the discount factor is higher. The covariance term is positive. Government bonds pay off well in real terms in recessions. Therefore, The real expected returns of nominal bonds should be (even) lower than those of real or indexed bonds. We should see

\[ i_t < r_t^f + E_t \pi_{t+1}. \]

Long-term bonds are even better recession hedges, because their nominal value also rises as interest rates fall in a recession.
In an economy subject to stagflation, such as the U. S. in the late 1970s, or in countries subject to frequent debt and currency crises and flights from rather than to local currency in bad times, higher inflation corresponds with worse economic outcomes, and the risk premium is reversed.

The bottom line from this consideration is that we can use observed nominal interest rates and expected inflation to guide us in measuring the discount rate for surpluses, but we should expect the correct discount rate to be different, even lower for the U.S. currently, but to vary across time and countries.

But why is the interest rate on government debt so low in the first place? Surpluses are strongly positively correlated with GDP and consumption growth. Surpluses look like stock market dividends, and fall in recessions. Shouldn’t the expected return on government debt also be high, just like the expected equity return? The resolution of this puzzle lies in the negative autocorrelation of surpluses, a recurring theme. Surpluses are not an AR(1). A deficit in a recession corresponds to higher subsequent surpluses. A negative shock to dividends lowers expected future dividends at all horizons.

To see the point, start with the case of a constant discount rate and constant price level, and think about what that constant discount rate should be. In this case, each deficit, each decline in $s_t$, corresponds to an exactly equal rise in future surpluses, $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$. (Notice the sum starts at $j = 1$.) So when the “dividend” component of the return $s_t$ falls in a recession, the “price” component of subsequent value rises $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$, leaving a riskless overall return. Contrariwise, the fact that government debt earns a remarkably low real return is more confirming evidence that bond investors do think current deficits and debt issues correspond to larger future surpluses.

Moreover, this calculation makes sense of the overall valuation formula, not the experience of individual investors. In this example, the government finances the deficit, lower $s_t$, by raising the end-of-period quantity of debt. $Q_t B_t / P_t = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ rises because the quantity of debt $Q_t$ rises. An investor still holds only one-period nominal debt, which is riskfree in real as well as nominal terms in this example. Individual investors do not experience a lower dividend $s_t$ and a higher price for a given investment. So, the investor only demands the risk free rate, not the equity premium.

The discount rate is not constant, and inflation does vary. But, adding the last two

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1I thank Bob Hall for raising this puzzle several times, as I was slow to figure out the answer.
stories together, these considerations only strengthen our understanding of a low expected return for government debt despite procyclical surpluses. In a recession, the discount rate drops, making the remaining stream of debt $E_t \sum_{j=1}^{\infty} s_{t+j}/R_{t,t+j}$ even more valuable. Government debt has a negative consumption beta, even to the individual investor!

The level and variation in the interest rate and risk premium are potentially important forces on the price level, and important for accounting for the variation in inflation over time and across countries. Measuring discount rates is challenging. We will return to these issues, and attempt to quantify some of these stories, in Chapter 10.

6.5 Money

When people want to hold non-interest-bearing money, the fiscal theory generalizes to

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right)$$

or

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right).$$

These equivalent expressions offer two different ways to account for seigniorage revenue.

When we add money demand to the fiscal theory, such as

$$M_t V = P_t Y$$

we also must specify a “passive” monetary policy, such as a rule allowing free conversion of non-interest bearing cash to interest-bearing reserves.

The cashless models are simplifications. We can easily add cash or interest rate spreads between assets of varying liquidity. We no longer have to do so in order to obtain a determinate price level, but we can if we wish to recognize the presence of such assets and investigate their impact.

Suppose that people want to hold some cash overnight. The flow equilibrium condition becomes

$$B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1+i_t} B_t + M_t.$$  \hfill (6.11)
CHAPTER 6. A BIT OF GENERALITY

$M_t$ stands here for non-interest-bearing government money, i.e. cash and any reserves that do not pay interest. Only direct liabilities of the government count in this $M_t$, not checking accounts or other inside money. $M_t$ is held overnight from period $t$ to period $t + 1$.

A good (better?) way to understand the frictionless model may be to start with the flow condition (6.11), with the understanding that the constraint on government choices is how much money $M_t$ people demand. Then, our frictionless flow condition is the limit (and limit point) that people don’t want to hold money overnight. It is an equilibrium condition, resulting from (6.11) and zero money demand. The answer to “so what if prices don’t move” is that “then you’ll end up holding more money than you want.” Since money has vanished in the frictionless flow equilibrium condition, it is harder to see that fact, but if prices did not move, the money would reappear.

I iterate forward in two ways, which give two useful intuitions:

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \left( \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j} M_{t+j}}{1 + i_{t+j} P_{t+j}} \right) \right)$$

(6.12)

and

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \left( \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right) \right)$$

(6.13)

where $\Delta x_t \equiv x_t - x_{t-1}$.

To derive (6.12), write the flow equation (6.11) as

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{B_t + M_t}{P_t} + \frac{i_t M_t}{1 + i_t P_t}$$

and iterate. To derive (6.13), write (6.11) as

$$\frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \right) \frac{B_t + M_t}{P_t} + \frac{i_t M_t}{1 + i_t P_t}$$

and iterate.

The presence of government-provided money, that people are willing to hold without receiving interest, introduces seigniorage revenue. In (6.12), we count seigniorage as
an interest saving on money, viewed as government debt that pays a lower interest rate. On the consumer side, people are willing to hold money because it provides liquidity services, an unmeasured dividend. In equilibrium, the value of liquidity services, the invisible “dividend” that money pays, is equal to the interest cost of holding money. In (6.13), we see seigniorage revenue as the direct ability to print up some money to pay bills.

It is interesting to track the case that money pays interest, as reserves now do pay interest. I hope we see further monetary innovation in the form of treasury-provided interest-bearing electronic money and wider access to interest-paying reserves. When money pays interest $i^m$, the flow condition becomes

$$B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + \frac{1}{1 + i^m_t} M_t.$$ 

Here I quote the interest on money $M$ on a discount basis, paralleling bonds. Proceeding the same way, the present value relation becomes

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left[ s_{t+j} + \frac{i_{t+j} - i^m_{t+j}}{(1 + i_{t+j})(1 + i^m_{t+j})} \frac{M_{t+j}}{P_{t+j}} \right]$$

or

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{1}{1 + i^m_t} \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right).$$

As usual, the formulas are prettier in continuous time, below.

---

2The intermediate steps:

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{(B_t + M_t)}{P_t} + \left( \frac{1}{1 + i^m_t} - \frac{1}{1 + i_t} \right) \frac{M_t}{P_t}$$

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + E_t \left( \frac{1}{R} \frac{(B_t + M_t)}{P_{t+1}} \right) + \left( \frac{1}{1 + i^m_t} - \frac{1}{1 + i_t} \right) \frac{M_t}{P_t}$$

3The intermediate steps:

$$B_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + \frac{1}{1 + i^m_t} M_t - M_{t-1}.$$ 

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R} E_t \frac{B_t}{P_{t+1}} + \frac{1}{1 + i^m_t} \frac{M_t - M_{t-1}}{P_t}.$$
6.6 Money, seigniorage, and fiscal theory

Seigniorage is small in most economies. In the presence of a rigid money demand $M_t V = P_t Y_t$, the fiscal theory must assume a passive money-supply policy, that the central bank accommodates the desired split of overall debt $B_t + M_t$ between $B_t$ and $M_t$. Monetary policy, as I have described it, the choice of $B_t + M_t$ or interest rate targets, remains. Money just distorts the interest rate on government debt, and thus the discount rate for surpluses. That fact does not alter the basic picture of fiscal price level determination. Seigniorage opens the door to interesting monetary-fiscal coordination issues in economies where seigniorage is large. The fact that real interest rates affect debt service, however, is a coordination issue that does not involve seigniorage. Seigniorage invites us to think more seriously about what fiscal reactions will occur in response to a monetary policy change.

Equations (6.12) and (6.13) seem to offer an interesting opportunity for fiscal-monetary interactions. By exchanging bonds for money in open market operations, the central bank affects fiscal surpluses. Before you get too excited however, recognize that for most advanced economies, in normal times, seigniorage is a small part of government finances, typically less than a tenth of the remaining surplus or deficit. The government-provided non-interest-bearing money stock, primarily physical cash, is a small part, typically less than a tenth, of the stock of outstanding government debt.

In the US, as I write, the currency stock is about $1.5 trillion, official federal debt is about $20 trillion, and the deficit about $1 trillion. The interest rate is about 2%, so seigniorage revenue counted as interest savings is about $30 billion, or 4.5% of the deficit. At a constant money/GDP ratio, even 5% growth of nominal GDP (2% inflation, 3% real) implies 5% growth of the money supply and thus $75 billion or 7.5% of the deficit. The amount by which these numbers change upon monetary policy actions is an order of magnitude smaller. If the Fed raised interest rates by one percentage point, and ignoring any decline in money holdings, that would only imply $15 billion of additional seigniorage revenue.

Even in times of high inflation in the U.S., direct seigniorage was a small part of the fiscal story. In the early 1980s, currency was only about $100 billion, GDP about $3-4 trillion, so currency/GDP about 3%. Higher nominal interest rates meant lower real money demand. Even at 10% interest rates, seigniorage was $10 billion or 0.3% of GDP. Currency was growing about 10% per year, giving the same answer. Federal debt was about $1 trillion, 33% of GDP, with deficits bottoming out $200 billion or
5% of GDP, and roughly 3% of GDP throughout the 1980s. Seigniorage represented less than a tenth of the deficit throughout the great inflation and its aftermath. Whatever caused that inflation, direct monetization of deficits wasn’t it.

Seigniorage does matter for many episodes and other countries, including most hyperinflations and currency collapses. Most large inflations result from issuing large amounts of non-interest-bearing money to cover fiscal deficits.

There is a potentially much larger fiscal effect of monetary policy. If prices are sticky so that nominal interest rate changes imply real interest rate changes, at least for a while, then raising the interest rate raises the government’s real cost of borrowing. A one percentage point rise in the real interest rate means the government must, as soon as the debt rolls over, pay 1 percentage point higher interest on its entire stock of outstanding debt, 1% × $20 trillion or $200 billion. This is a very large effect, which we will study elsewhere. Early post-WWII monetary policy in the U.S., for example, was explicitly devoted to holding down interest costs on the large debt. Given currently large debts, any desire of the Fed to substantially raise rates is likely to run into similar pressures. But this mechanism has nothing to do with money or money demand.

Suppose there is a money demand function

\[ M_t V = P_t Y_t. \] (6.14)

As we have seen, if the government or central bank fixes \( M_t \), this equation can, potentially, determine the price level. (“Potentially,” because interest elastic demand \( V(i) \) or inside moneys muddy that claim, issues I return to in Chapter 17.) Then fiscal policy must “passively” adjust surpluses to the monetary-determined price level. For now, our job is to generalize fiscal theory, so we will assume the opposite: The valuation equation (6.12) or (6.13) determines the price level, and the government must then “passively” provide the amount of money people demand by (6.14). The central bank must “passively” adjust the composition of government debt, the split of debt \( B_t + M_t \) overall between \( B_t \) and \( M_t \). For example, the central bank could allow banks to freely exchange interest-paying reserves \( B_t \) for cash \( M_t \), which is precisely what the Fed does. There is lots of “monetary policy” left even with this “passive” assumption. The \( B_{t-1} + M_{t-1} \) decision with fixed surpluses that I have called “monetary policy” remains, and an interest rate target that is controlled by this margin remains as well. So, to be clear, let us call the needed policy a “passive money supply” policy.

With the passive money supply assumption, the presence of non-interest-bearing cash
CHAPTER 6. A BIT OF GENERALITY

is a straightforward extension, and often a minor footnote, to the fiscal theory. Cash is just one of many flavors of government debt that bear small interest rate spreads. These yield differences are important for precise accounting, and for measurement of the discount rate for government debt. But those features not disturb the basic picture of price level determination. This is a claim that needs to be verified with equations, and I return to do that in Chapter 17 along with more detailed analysis of monetary-fiscal interactions.

The possibility of seigniorage, and more generally of yield spreads among various issues of government debt, invites us to think more carefully about fiscal-monetary interactions. So far I defined “monetary policy” as changes in the quantity or composition of government debt, or changes in interest rate targets, that leave surpluses constant. But suppose monetary policy or money supply policy creates seigniorage revenues. How will fiscal policy adapt? Fiscal authorities could ignore the seigniorage revenue in their tax and spending decisions, so primary surpluses $s_t$ are unchanged. Then seigniorage revenue repays debt, or its lack incurs more debt, modifying the right hand sides of (6.12) and (6.13). But fiscal authorities could also regard increased seigniorage as a dandy present, and use it to increase spending or reduce taxes by the amount of the seigniorage revenue. They could likewise view decreased seigniorage as an indication of their duty to raise taxes or cut spending in response, as they might in response to decreases in any other single small source of tax revenue. In this case, the total primary surplus including seigniorage – the entire terms on the right hand sides of (6.12) and (6.13) – are fixed, not the $s_t$ terms that ignore seigniorage.

Other fiscal reactions abound. Fiscal authorities may react to inflation directly. The tax code is not indexed, and both taxes and spending react to output variation that monetary policy may cause, even if fiscal authorities do not take any actions. Fiscal authorities also react predictably to inflation and output variation.

How then should we define “monetary policy?” There is no pure right answer. We just need to be aware of what question we are asking, and make sure it is interesting and appropriate to the context. Fiscal reactions matter, so pay attention to which ones you include and which you exclude.
6.7 Linearizations

I derive the linearized flow and present value relations,

\[ \tilde{v}_{t-1} + \tilde{r}_t^n - \tilde{\pi}_t - \tilde{\gamma}_t = \tilde{s}_t + \beta \tilde{v}_t, \]

\[ \tilde{v}_{t-1} = \sum_{j=0}^{\infty} \beta^j \left[ \tilde{s}_{t+j} - (\tilde{r}_{t+j}^n - \tilde{\pi}_{t+j} - \tilde{\gamma}_{t+j}) \right]. \]

Taking the innovation of the present value relation,

\[ (E_{t+1} - E_t) \pi_{t+1} - (E_{t+1} - E_t) (r_{t+1}^n - g_{t+1}) \]

\[ = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j \tilde{s}_{t+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \left( \tilde{r}_{t+1+j}^n - \tilde{\pi}_{t+1+j} - \tilde{\gamma}_{t+1+j} \right). \]

Here \( \tilde{v}_t \) is the log of the market value of government debt divided by GDP (or other divisor), \( \tilde{r}_t^n \) is the log nominal return on the portfolio of government debt, \( \tilde{\pi}_t \) is log inflation rate, \( \tilde{\gamma}_t \) is the growth of GDP (or other divisor), \( \tilde{s}_t \) is the surplus to GDP ratio divided by the steady state value of debt, variables with tildes are deviations from steady state, and \( \beta = e^{-(r-g)} \).

To linearize the flow and present value relations, I follow the same general procedure as Campbell and Shiller (1988): I linearize the one-period relation, essentially the definition of return, and then I iterate forward. Unlike Campbell and Shiller (1988), I linearize in the level of surplus, not its log, since surplus is often (!) negative.

6.7.1 Linearizing the flow condition

Start with the flow condition (6.2),

\[ M_{t-1} + \sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} = P_t s_t + M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)} \] (6.15)

In the one-period debt case, the left hand side of (6.15) is dated \( t - 1 \), as debt at the end of \( t - 1 \) is the same as debt at the beginning of time \( t \). But with long-term debt we don’t know its value until date \( t \) and \( Q_t \) are learned, so beginning of period
t debt is not the same as end of period $t - 1$ debt. For this reason it’s a little bit prettier to track the end-of-period value of debt

$$V_{t-1} \equiv M_{t-1} + \sum_{j=0}^{\infty} Q_{t-1}^{(t+j)} B_{t-1}^{(t+j)}.$$

I also scale by $y_t$ which can represent real GDP, real potential GDP, real consumption, or some other divisor that renders debt and surpluses stationary. Thus, multiply and divide to obtain

$$\frac{M_{t-1} + \sum_{j=0}^{\infty} Q_{t-1}^{(t+j)} B_{t-1}^{(t+j)}}{y_{t-1} P_{t-1}} \frac{P_{t-1} y_{t-1}}{M_{t-1} + \sum_{j=0}^{\infty} Q_{t-1}^{(t+j)} B_{t-1}^{(t+j)}} \frac{y_t}{P_t} = \frac{s_t}{y_t} + \frac{M_t + \sum_{j=0}^{\infty} Q_{t}^{(t+1+j)} B_{t-1}^{(t+1+j)}}{P_t y_t}.$$

Introducing notation, this equation is

$$\frac{V_{t-1}}{y_{t-1} P_{t-1}} R_t^n \frac{P_{t-1} y_{t-1}}{P_t y_t} = \frac{s_t}{y_t} + \frac{V_t}{y_t P_t} \tag{6.16}$$

where

$$R_t^n \equiv \frac{M_t + \sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{M_t + \sum_{j=0}^{\infty} Q_{t-1}^{(t+j)} B_{t-1}^{(t+j)}}$$

is the nominal return on the portfolio of government debt from time $t-1$ to $t$, and $y_t$ is real GDP or another stationarity-inducing divisor (consumption, potential GDP, population, etc.).

Write \[6.16\]

$$e^{v_{t-1}} e^{r_t^n - g_t - \pi_t} = \frac{s_t}{y_t} + e^{v_t} \tag{6.17}$$

with

$$v_t \equiv \log \left( \frac{V_t}{y_t P_t} \right); \quad r_t^n \equiv \log(R_t^n); \quad \pi_t \equiv \log \left( \frac{P_t}{P_{t-1}} \right); \quad g_t \equiv \log \left( \frac{y_t}{y_{t-1}} \right).$$

Define a steady state of variables without subscripts, with $r \equiv r^n - \pi$, and

$$e^{r - g} = 1 + \frac{s}{y e^v}.$$
Define
\[ \beta \equiv e^{-(r-g)} = \left(1 + \frac{s}{y e^v}\right)^{-1}. \]

Linearize (6.17) around this steady state,
\[ v_{t-1} + r^n_t - \pi_t - g_t = \log \left( \frac{s}{y} + e^v \right) + \frac{1}{\beta} \left[ \left( \frac{s_t}{y_t} - \frac{s}{y} \right) + e^v (v_t - v) \right] + \beta \left( \frac{s_t}{y_t V} - \frac{s}{y V} \right) + (v_t - v). \]

(6.18)

Define deviations from steady state,
\[ \tilde{v}_t \equiv v_t - v, \]
\[ \tilde{s}_t \equiv \frac{s_t}{\beta y_t V} - \frac{s}{\beta y V}, \]
etc. Then we have
\[ \tilde{v}_{t-1} + \tilde{r}^n_t - \tilde{\pi}_t - \tilde{g}_t = \tilde{s}_t + \beta \tilde{v}_t. \]

(6.19)

Equation (6.19) is the linearization of the flow condition with long-term debt (6.16). It says that the log value of debt to GDP ratio at the end of time \( t \) is its value at the end of time \( t - 1 \), increased by the real return less growth, and decreased by the surplus to GDP ratio divided by the steady state value of debt. With that interpretation in mind, we can write the linearization as
\[ \tilde{v}_t = R (\tilde{v}_{t-1} + \tilde{r}^n_t - \tilde{\pi}_t - \tilde{g}_t - \tilde{s}_t) \]

(6.20)

with \( R \equiv 1/\beta = e^{(r-g)}. \)

The linearization point \( r - g = 0 \) is particularly pretty, since then the constant terms in (6.18) are all zero \((r - g = 0, s = 0, \text{and the two } v \text{ on the right hand side cancel})\), so we can describe levels, not deviations from steady state
\[ v_t = v_{t-1} + r^n_t - \pi_t - g_t - \frac{s_t}{y_t V}. \]

A second form is often useful, as it parallels the beginning of period timing. Define
\[ b_t \equiv v_t + i_t. \]

(6.21)
then (6.19) becomes

\[ \tilde{b}_{t-1} + (\tilde{r}_{nt} - \tilde{\nu}_{t-1}) - \tilde{\pi}_t - \tilde{g}_t = \tilde{s}_t + \beta \left( \tilde{b}_t - \tilde{\nu}_t \right). \] (6.22)

This form more closely matches our nonlinear one-period debt formulas. With one-period debt, and without GDP growth, we have \( r^i_t = i_{t-1} \), and

\[ v_{t-1} = \log \left( \frac{Q_{t-1}B_{t-1}}{P_{t-1}} \right). \]

With \( i_t = -\log(Q_t) \), then in this case

\[ b_{t-1} = \log(B_{t-1}/P_{t-1}). \]

The one-period-debt linearized flow identity becomes

\[ \tilde{b}_{t-1} - \tilde{\pi}_t = \tilde{s}_t + \beta \left( \tilde{b}_t - \tilde{\nu}_t \right) \] (6.23)

an obvious counterpart to our friend

\[ \frac{B_{t-1}P_{t-1}}{P_{t-1}} \frac{P_t}{P_t} = s_t + \frac{1}{1+i_t} B_t = s_t + \frac{1}{1+i_t} B_t. \]

It can be convenient to write (6.23) in debt accumulation form,

\[ \tilde{b}_t = R \left( \tilde{b}_{t-1} - \tilde{\pi}_t - \tilde{s}_t \right) + \tilde{\nu}_t. \] (6.24)

The form (6.22) is practically useful with long-term debt when we pair it with the expectations hypothesis \( i_t = E_t r^i_{t+1} \). Then the quantity \( \tilde{r}^n_t - i_{t-1} \) is a pure expectational error, and quickly shows how the expectational error in the bond-pricing formula feeds in to the debt-accumulation equation. The quantity \( b_{t-1} \) is what the beginning of period market value of debt would be, if it earned the expected rate of return \( r^n_t = i_{t-1} \) overnight. That is a natural extension of the one-period debt concept of what the beginning of period value of the debt is.

Linearizing the flow identity is often enough. In studying new-Keynesian models, for example, we usually write down one-period equations and then solve them forward or backward as a system. (Chapter 7 will solve several models this way.) We don’t solve forward individual equations. Doing so loses a great deal of intuition – you
don’t see that the debt present value equation is picking the inflation equilibrium, for example. But it is necessary to solve larger systems.

It is also often much easier and prettier to write the model in continuous time, and linearize the continuous-time flow equation. This step simply requires taking a derivative, and all the headache about beginning vs. end of period timing disappears. However, continuous time requires one to think hard about what variables are continuous, diffusions, or jumps, and what variables are predetermined. Section 7.8 explores the model in continuous time, and derives a linearization analogous to (6.19).

### 6.7.2 A linearized present value equation and innovation identity

Iterating (6.19) forward,

\[ \tilde{v}_{t-1} = \sum_{j=0}^{T} \beta^j \left[ \tilde{s}_{t+j} - (\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j} - \tilde{\gamma}_{t+j}) \right] + \beta^{T+1} \tilde{v}_{t+T+1}. \]  

(6.25)

and when the last term vanishes,

\[ \tilde{v}_{t-1} = \sum_{j=0}^{\infty} \beta^j \left[ \tilde{s}_{t+j} - (\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j} - \tilde{\gamma}_{t+j}) \right]. \]  

(6.26)

The value of the debt to GDP ratio is the discounted sum of future primary surpluses, discounted by the real growth-adjusted return on government bonds.

These equations are linearized identities, without economic content. They hold ex-post – notice there are no expectation signs. We can take expectations of both sides to obtain ex-ante relations. Those expectations can be with any probabilities you want: true probabilities, sample probabilities, probabilities inferred from a VAR or other time-series model, behavioral probabilities, risk neutral probabilities. A great advantage of starting with an identity is that we do not have to assume in empirical work that agents have the same information set we do.

Taking time \( t \) innovations of (6.26), we can relate unexpected inflation to the current return, and innovations in expected future surpluses and discount rates. I also move
the time index forward, as has been our custom with innovations,
\[
(E_{t+1} - E_t) \tilde{\pi}_{t+1} - (E_{t+1} - E_t) \left( \tilde{r}_{t+1} - \tilde{g}_{t+1} \right)
\]
\[
= - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j \tilde{s}_{t+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \left( \tilde{r}_{t+1+j} - \tilde{\pi}_{t+1+j} - \tilde{g}_{t+1+j} \right).
\]

(6.27)

This form is particularly useful since the value of debt \( v_{t-1} \) drops out, and we relate inflation only to surpluses, rates of return, and growth.

In the case of one-period debt and no growth, the linearized present value relations becomes
\[
\tilde{b}_{t-1} - \tilde{\pi}_t = \sum_{j=0}^{\infty} \beta^j \left[ \tilde{s}_t - (\tilde{i}_{t+j} - \tilde{\pi}_{t+j+1}) \right].
\]

This is an obvious loglinearization of our old friend, the present value equation. It includes real interest rate variation, and thus discount rate variation.

In this case the innovation equation (6.27) becomes
\[
(E_{t+1} - E_t) \pi_{t+1} = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j \tilde{s}_{t+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j (\tilde{i}_{t+j} - \pi_{t+j+1}).
\]

Again, we obtain an easy discount-rate extension of the earlier idea that unexpected inflation depends only on surpluses. Now it depends on surpluses and real interest rates, which opens up many interesting mechanisms.

### 6.7.3 Comments on the linearized identities

Cochrane (2019) evaluates the accuracy of approximation, by comparing the surplus calculated from the nonlinear (6.16) to the surplus calculated from the linearized (6.19). I find it reasonably close outside of the extreme deficits of early WWII. How close is close is in the eye of the beholder. That paper shows we can get a closer approximation by defining \( \tilde{s}_t \) as the ratio of surplus to the previous period’s value,
\[
\tilde{s}_t = \frac{s_t}{\beta y_t V_{t-1}}.
\]

This definition leads to exactly the same linearized identities. The difference lies in the nonlinear versions, so the surplus calculated from the linearized identity is a
closer match to this nonlinearized concept. In empirical work, one can use original
surplus numbers, which do not obey identities; one can use the surplus inferred
from other quantities via nonlinear identities, which leaves an approximation error
in the linearized formulas, or one can simply use the surplus implied by the linearized
identities. I generally follow the latter procedure, so which linearization one chooses
is less important.

The entire effect of long term bonds, or of money or other liquidity premiums, is
captured in equations (6.26) and (6.27) by the nominal return $r^n_t$. This offers a
substantial simplification of what otherwise might have become a complex issue. But
that simple return captures a lot of interesting mechanisms, which we will explore in
detail below. In (6.27), higher nominal interest rates cause a negative ex-post return
$r_{t+1}^n$, which can then give disinflation $\pi_{t+1} - E_t \pi_{t+1} < 0$. We can thereby produce
this desired sign of monetary policy changes. It will be useful to understand when
long-term bond prices can soak up fiscal shocks, and allow fiscal shocks to be met
by a long-drawn out future inflation rather than just one inflation shock. It will
be useful to understand discount rate effects, for example my story that disinflation
comes in recessions because the discount rate on the right hand side of (6.27) declines.
Higher expected future real interest rates can inflation, even with constant surpluses,
because they depress the present value of surpluses. It will be also useful to generate
discount rate effects in sticky price models or models that add risk premiums.

It is convenient at times to linearize the left hand side of the present value identity
as well. From the definitions we can write

$$v_t \equiv \log\left(\frac{V_t}{P_t y_t}\right) = \log\left(\sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}\right) = \log\left(\sum_{j=1}^{\infty} e^{-j\varphi_t^{(j)}} B_t^{(t+j)}\right) - p_t - \log(y_t)$$

where $\varphi_t^{(j)}$ is the log nominal yield at time $t$ of $j$ year bonds. Now linearize to

$$\tilde{v}_t \equiv \sum_{j=1}^{\infty} \left(\frac{e^{-j\varphi_t^{(j)}} B_t^{(t+j)}}{\sum_{k=1}^{\infty} e^{-k\varphi_t^{(k)}} B_t^{(t+k)}}\right) \left(\frac{\tilde{B}_t^{(t+j)}}{B_t^{(t+j)}} - j\varphi_t^{(j)}\right) - \tilde{p}_t - \log(y_t).$$

I linearized in terms of debt $B$ rather than $\log B$ to more easily allow very small
values of $B$. Using a $\pi = 0$ and constant real rate $r$ steady state,

$$\tilde{v}_t \equiv \sum_{j=1}^{\infty} \left(\frac{e^{-jr B(t+j)}}{\sum_{k=1}^{\infty} e^{-kr B(t+k)}}\right) \left(\frac{\tilde{B}_t^{(t+j)}}{B(t+j)} - j\varphi_t^{(j)}\right) - \tilde{p}_t - \log(y_t).$$
6.8 Continuous time

Continuous time formulas are straightforward and often prettier analogues to the discrete time versions.

With a stochastic discount factor $\lambda_t$, the present value formulas are, with short term debt

$$V_t = \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\lambda_{\tau}}{\lambda_t} s_{\tau} d\tau;$$

with long-term debt

$$V_t = \int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} \frac{dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\lambda_{\tau}}{\lambda_t} s_{\tau} d\tau;$$

with money, either

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\lambda_{\tau}}{\lambda_t} \left( s_{\tau} + (i_{\tau} - i^m_t) \frac{M_t}{P_{\tau}} \right) d\tau;$$

or, in the case $i^m_t = 0$,

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\lambda_{\tau}}{\lambda_t} \left( s_{\tau} d\tau + \frac{dM_t}{P_{\tau}} \right).$$

Special cases include risk-neutral valuation at the interest rate

$$\frac{\lambda_{\tau}}{\lambda_t} = e^{-\int_{\tau=t}^{\tau} r_d dj}$$

or at a constant real interest rate

$$\frac{\lambda_{\tau}}{\lambda_t} = e^{-\tau r_r}.$$

Discounting with ex-post returns, we can write, for one-period debt

$$V_t = \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_{\tau}} s_{\tau} d\tau;$$

for long-term debt

$$V_t = \int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} \frac{dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_{\tau}} s_{\tau} d\tau.$$
and with money,

\[ \frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left[ (s_\tau + (i_t - i_t^m) \frac{M_\tau}{P_\tau}) \right] d\tau. \]

For \( i_t^m = 0 \), we can also write

\[ \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left( s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right) \]

In each case \( W_t \) is the cumulative real return on the value-weighted portfolio of government debt. We can also discount using the cumulative return on the portfolio of government debt including money,

\[ \frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t^m}{W_\tau} s_\tau d\tau \]

The flow conditions express the idea that money printed up in the morning must be soaked up in the afternoon by surpluses or by new debt sales. For one-period debt,

\[ \frac{B_t}{P_t} i_t dt = s_t dt + \frac{dB_t}{P_t}; \]

for long-term debt

\[ \frac{B_t^{(t)}}{P_t} dt = s_t dt + \int_{j=0}^{\infty} Q_t^{(t+j)} dB_t^{(t+j)} dj \]

and with money,

\[ \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt = s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t} \]

These flow conditions plus bond pricing allows us to track the evolution of the real value of debt. For short-term debt

\[ dV_t = d \left( \frac{B_t}{P_t} \right) = V_t dR_t - s_t dt. \]

for long-term debt

\[ dV_t = d \left( \int_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_t^{(t+j)}}{P_t} dj \right) = V_t dR_t^p - s_t dt, \]
where \( dR^p \) is the real return on the portfolio of all government debt, with \( dW_t/W_t = dR^p_t \). The equation is the same except for the nature of \( dR^p_t \).

It is useful to linearize these debt evolution equations, as

\[
dv_t = dR^p_t - \tilde{s}_t dt.
\]

where \( v_t = \log(V_t) \), \( dR^p_t \) is the relevant portfolio return, and \( \tilde{s}_t = s_t/V_t \) for an exact relation or \( \tilde{s}_t = s_t/V \) for an approximation.

As often is the case, continuous time formulas are much prettier, but they take a little more care to set up correctly. Continuous time formulas avoid many of the little timing conventions that are a distraction to discrete-time formulations. They also force one to think through which variables are differentiable, and which may jump discontinuously or move with a diffusion component. I use discrete time in this book largely to keep the derivations transparent, but it is really much more elegant and simple to use continuous time formulas once the logic is clear. The bottom lines are transparent analogues of the discrete time formulas. The only slight technical difficulties arise in translating between flow and present value formulations.

### 6.8.1 Short-term debt

In continuous time, it is easier to think of truly instantaneous debt as a floating rate perpetuity; the quantity is \( B_t \), it has a price of \( Q_t = 1 \) always, and it pays a flow of interest \( i_t dt \). Let \( s_t dt = (T_t - G_t) dt \) denote the flow of primary surpluses. The symbol \( d \) represents the forward-differential operator, loosely the limit as \( \Delta \to 0 \) of \( dP_t = P_{t+\Delta} - P_t \).

The nominal and real flow conditions are then

\[
B_t i_t dt = P_t s_t dt + dB_t \tag{6.28}
\]

\[
\frac{B_t}{P_t} i_t dt = s_t dt + \frac{dB_t}{P_t}.
\]

Interest paid on the debt must be financed by surpluses or by selling more debt. Since the first two quantities sport \( dt \), then \( B_t \) also must be differentiable, with neither jump nor diffusion components. The price level may have jumps or diffusions.
6.8. **CONTINUOUS TIME**

It’s useful to describe the evolution of the real value of government debt

\[
\begin{align*}
    d \left( \frac{B_t}{P_t} \right) &= dB_t + B_t d \left( \frac{1}{P_t} \right) \\
    d \left( \frac{B_t}{P_t} \right) &= B_t i_t dt - s_t dt + B_t d \left( \frac{1}{P_t} \right) \\
    d \left( \frac{B_t}{P_t} \right) &= \left( \frac{B_t}{P_t} \right) dR_t - s_t dt
\end{align*}
\]

\[dV_t = V_t dR_t - s_t dt\] \hspace{1cm} (6.29)

where

\[dR_t \equiv i_t dt + \frac{d(1/P_t)}{1/P_t}\]

is the real ex-post return on government debt, and

\[V_t \equiv \frac{B_t}{P_t}\]

is the real market value of debt. The real value of debt grows at the ex-post real return, less primary surpluses.

Linearizations are easier in continuous time. We can write from (6.29)

\[dv_t = dR_t \frac{s_t}{V_t} dt\]

where

\[v_t \equiv \log(V_t)\].

Thus, if we take

\[\tilde{s}_t = \frac{s_t}{V_t}\]

the surplus to value ratio as our measure of surplus, we have an exactly linear flow equation

\[dv_t = dR_t - \tilde{s}_t dt\] \hspace{1cm} (6.30)

Alternatively, we may linearize. Linearizing around any \(V = e^v\) and \(s = 0\), (implicitly, \(r = 0\) as well), we can write

\[\tilde{s}_t \equiv \frac{s_t}{V}\]
and then (6.30) applies to the surplus scaled by value, as in section 6.7.1. Adding GDP growth is straightforward, as well as writing
\[ dR_t = i_t - \pi_t \]

etc.

Let \( \lambda_t \) denote a generic continuous-time discount factor, e.g.
\[ \lambda_t = e^{-\rho t} u'(c_t). \]

The valuation equation in this case is then
\[ \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \lambda_{\tau} s_{\tau} d\tau. \]

The distinction between \( t \) and \( t+1 \) vanishes.

The risk neutral case and constant real interest rate case specialize quickly to
\[ \frac{\Lambda_{\tau}}{\Lambda_{t}} = e^{-\int_{\tau=0}^{\tau} r_j d\tau}, \quad \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} e^{-\int_{\tau=0}^{\tau} r_j d\tau} s_{\tau} d\tau \]
\[ \frac{\Lambda_{\tau}}{\Lambda_{t}} = e^{-r(\tau-t)}, \quad \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} e^{-r(\tau-t)} s_{\tau} d\tau. \]

We can also discount at the ex-post real return on nominal government debt, yielding
\[ \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_\tau} s_{\tau} d\tau \quad (6.31) \]
where \( W_t \) is the ex-post real cumulative return from investment in nominal government debt. It satisfies
\[ \frac{dW_t}{W_t} = dR_t = i_t dt + \frac{d(1/P_t)}{1/P_t}. \quad (6.32) \]

Integrating, we can define the cumulative return explicitly as
\[ \frac{W_t}{W_0} = e^{\int_{\tau=0}^{\tau} i_t d\tau} \frac{P_0}{P_t} \]

As in discrete time, equation (6.31) this equation holds ex-post, and therefore it also holds ex-ante with any set of probabilities.
To connect the flow and present value relations, note

\[
    r_t^f dt = -E_t \left( \frac{d\Lambda_t}{\Lambda_t} \right),
\]

\[
    i_t dt = -E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right].
\]

(6.33)

\[
    i_t dt = -\frac{d \left[ 1 / (P_t W_t) \right]}{1 / (P_t W_t)}.
\]

(The latter takes a few lines of algebra starting from (6.32).) Then, work either up or down,

\[
    \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau.
\]

\[
    \frac{\Lambda_t B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \Lambda_{\tau} s_{\tau} d\tau.
\]

\[
    d \left( \frac{\Lambda_t B_t}{P_t} \right) = -s_t \Lambda_t dt.
\]

\[
    \Lambda_t \frac{dB_t}{P_t} + B_t E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] = -\Lambda_t s_t dt.
\]

\[
    dB_t + B_t E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] = -P_t s_t dt.
\]

\[
    B_t i_t dt = P_t s_t dt + dB_t.
\]

Similarly, for the rate of return as discount factor, work either up or down,

\[
    \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_t} s_{\tau} d\tau.
\]

\[
    \frac{1}{W_t} \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{s_{\tau}}{W_t} d\tau.
\]

\[
    d \left( \frac{1}{W_t} \frac{B_t}{P_t} \right) = -s_t dt.
\]
\[
\frac{1}{W_t} dB_t + B_t d \left( \frac{1}{P_t W_t} \right) = - \frac{s_t}{W_t} dt
\]
\[
\frac{1}{W_t} dB_t - \frac{B_t}{P_t W_t} dt = - \frac{s_t}{W_t} dt
\]
\[-dB_t + B_t dt = P_t s_t dt.\]

6.8.2 Long term debt

The flow relation is
\[
B_t^{(t)} dt = P_t s_t dt + \int_{j=0}^{\infty} Q_t^{(t+j)} dB_t^{(t+j)} dj, \quad (6.34)
\]
or
\[
\frac{B_t^{(t)}}{P_t} dt = s_t dt + \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} dB_t^{(t+j)} dj}{P_t}.
\]

Where \(B_t^{(t+j)}\) is the quantity of debt due at time \(t+j\), i.e. between \(t+j\) and \(t+j+\Delta\), and \(Q_t^{(t+j)}\) is its nominal price. The relation says that debt \(B_t^{(t)}\) coming due between \(t\) and \(t+dt\) must be paid by primary surpluses or the issuance of additional long-term debt. (If not, a \(dM_t\) would emerge but people don’t want to hold money.) \(dB_t^{(t+j)}\) represents the amount of debt of maturity \(j\) sold between time \(t\) and \(t+dt\).

Here I simplify by writing debt that is paid continuously. One could add lumps of debt to be paid at specific instants. In particular, the instantaneous debt of section 6.8.1 is a continuously rolled over lump of debt at maturity zero which is why the limit (6.34) takes a bit of work to see.

The nominal market value of government debt is
\[
V_t = \int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj,
\]
so the present value relations are
\[
\int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_t}{\Lambda_t} s_\tau d\tau \quad (6.35)
\]
and
\[
\int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau \quad (6.36)
\]
6.8. CONTINUOUS TIME

Where $W_t$ denotes the cumulated real return on the value-weighted portfolio of all government bonds. The nominal return on a single bond is

$$dR^n_t \equiv \frac{dQ_t^{(t+j)}}{Q_t^{(t+j)}}$$

and the real return is

$$dR_t \equiv \frac{d \left( \frac{Q_t^{(t+j)}}{P_t} \right)}{\left( \frac{Q_t^{(t+j)}}{P_t} \right)}$$

so the cumulated real return obeys

$$dW_t \equiv R^n_t dt + \int_{j=0}^{\infty} \left[ \frac{d \left( \frac{Q_t^{(t+j)}}{P_t} \right)}{\left( \frac{Q_t^{(t+j)}}{P_t} \right)} \right] B_t^{(t+j)} dj$$

To express the evolution of the market value of debt, take the differential.

$$dV_t = d \left[ \int_{j=0}^{\infty} \frac{Q_t^{(t+j)}}{P_t} B_t^{(t+j)} dj \right] = \int_{j=0}^{\infty} \frac{Q_t^{(t+j)}}{P_t} dB_t^{(t+j)} dj + \int_{j=0}^{\infty} \left( \frac{Q_t^{(t+j)}}{P_t} \right) B_t^{(t+j)} dj - \frac{B_t^{(t)}}{P_t} dt$$

Using the flow relation (6.34),

$$dV_t = -s_t dt + \int_{j=0}^{\infty} \left( \frac{Q_t^{(t+j)}}{P_t} \right) B_t^{(t+j)} dj$$

and the definition of portfolio return (6.37),

$$dV_t = -s_t dt + V_t dR^p_t$$

or

$$\frac{dV_t}{V_t} = -\frac{s_t}{V_t} dt + dR^p_t.$$  (6.39)

The total real market value of government debt grows at its ex-post real rate of return, less repayment via primary surpluses.

Equation (6.39) leads to a convenient continuous-time version approach to the linearization (6.20)

$$dv_t = -\frac{s_t}{V_t} dt + dR^p_t$$
\[ dv_t = -\tilde{s}_t dt + dR_t^p, \]  
where \( v_t \equiv \log V_t \). If we define \( \tilde{s}_t = s_t/V_t \), then the equation is exact. This observation confirms the suggestion of section 6.7.1 that this definition leads to a better approximation. If we define \( \tilde{s}_t = s_t/V \) where \( V \) and \( s = 0 \) are points of linearization, we obtain an approximate identity. This identity corresponds to the linearization using \( r = g \) of section 6.7.1. To use this identity, we need to add a bond pricing model to find the ex-post return on government bonds. In section 7.8 I use the expectations hypothesis, an interest rate target, and geometric maturity debt.

From (6.39) we can connect the flow relation to the present value relation using the ex-post return as discounter. Write (6.39) as

\[ \frac{dV_t}{V_t} = -\frac{s_t}{V_t} dt + \frac{dW_t}{W_t}. \]

At non-jump points, this implies

\[ \frac{dV_t^2}{V_t^2} = \frac{dW_t dV_t}{W_t V_t} = \frac{dW_t^2}{W_t^2}. \]

Thus,

\[ d\left( \frac{V_t}{W_t} \right) = \frac{V_t}{W_t} \left( \frac{dV_t}{V_t} - \frac{dW_t}{W_t} \right) \]

\[ d\left( \frac{V_t}{W_t} \right) = \frac{V_t}{W_t} \left( \frac{dV_t}{V_t} - \frac{dW_t}{W_t} \right) \]

\[ d\left( \frac{V_t}{W_t} \right) = -\frac{V_t}{W_t} \frac{s_t}{V_t} dt = -\frac{s_t}{W_t} dt. \]

Integrating,

\[ \frac{V_t}{W_t} - \lim_{r\to\infty} \frac{V_T}{W_T} = -\int_{\tau=0}^{\infty} d\left( \frac{V_\tau}{W_\tau} \right) = \int_{\tau=0}^{\infty} \frac{s_\tau}{V_t} dt \]

\[ \frac{V_t}{W_t} = \int_{\tau=t}^{\infty} \frac{s_\tau}{W_\tau} d\tau. \]

From (6.41), \( V \) grows more slowly than \( W \), so the limit is zero.

Since jumps are discrete (not even order \( dt \)), at those points (6.41) implies that the jumps obey

\[ \frac{dW}{W} = \frac{dP}{P}. \]
At the jump points $d(V_t/W_t) = 0$ so they do not affect the integral \([6.43]\).

To go backwards, take the differential of the final integral. (The same steps allow us to express a stock’s price as the present value of its dividend stream, discounted by the ex-post return, in continuous time. Start with

$$\frac{dW_t}{W_t} = dR = \frac{D_t}{P} dt + \frac{dP_t}{P}. \tag{6.44}$$

Follow the same steps to conclude

$$\frac{P_t}{W_t} = \int_{\tau=t}^{\infty} \frac{D_\tau}{W_\tau} d\tau \tag{6.45}$$

and vice versa. )

To connect flow and present value relations using the discount factor, note that the definition of a discount factor $\Lambda_t$ implies the basic pricing relation

$$E_t[d (\Lambda_t W_t)] = 0$$

hence

$$E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dW_t}{W_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dW_t}{W_t} \right) = 0.$$

From (6.41), which in turn came from the flow relation, we have

$$\frac{dW_t}{W_t} = \frac{dV_t}{V_t} + \frac{s_t}{V_t} dt.$$  

So,

$$E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dV_t}{V_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dV_t}{V_t} \right) = -\frac{s_t}{V_t} dt$$

$$E_t[d (\Lambda_t V_t)] = -\Lambda_t s_t dt$$

$$V_t \Lambda_t = \int_{\tau=t}^{\infty} \Lambda_\tau s_\tau d\tau,$$

and vice versa.
6.8.3 Money

The nominal flow condition in continuous time, corresponding to the discrete time version (2.3), is

\[ dM_t = i_t B_t dt + i^m_t M_t dt - P_t s_t dt - dB_t. \]  

(6.46)

The government “prints” (or creates electronically) money to pay interest on nominal debt, to pay interest on money, and the government soaks up money with the flow of primary surpluses and new debt issues. In parallel with the other conditions, we can write this flow condition as

\[ \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i^m_t dt = s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}. \]

This form is also useful to digest the economics of the frictionless model (6.28). That equation is not a budget constraint of the government. Rather, it reflects zero money demand. We can regard (6.46) as the underlying accounting identity. It becomes an equilibrium condition with \( M_t \) resulting from money demand. When money demand goes to zero, we get the frictionless version (6.28). But the underlying off-equilibrium action behind (6.28) is to print up unwanted money \( dM_t \), and no budget constraint stops the government from doing so.

The sum \( dB_t + dM_t \) is of order \( dt \). To keep the analysis simple I also specify that each of \( dB_t \) and \( dM_t \) is of order \( dt \) rather than assume offsetting Ito terms or jumps.

To express seigniorage as money creation, specialize to \( i^m_t = 0 \), rearrange (6.46), and substitute (6.33):

\[ \frac{dB_t}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right] \frac{B_t}{P_t} = -s_t dt - \frac{dM_t}{P_t} \]

\[ \frac{\Lambda_t}{P_t} dB_t + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) B_t \right] = -\Lambda_t \left( s_t dt + \frac{dM_t}{P_t} \right) \]  

(6.47)

\[ E_t \left[ d \left( \frac{\Lambda_t B_t}{P_t} \right) \right] = -\Lambda_t \left( s_t dt + \frac{dM_t}{P_t} \right) \]

Now we can integrate, and impose the transversality condition to obtain

\[ \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right). \]
To express seigniorage in terms of interest cost, including the case that money pays interest \(0 < i_t^m < i_t\), we start again from (6.46), and write

\[
\frac{d (M_t + B_t)}{P_t} - i_t \frac{(B_t + M_t)}{P_t} dt = -s_t dt - (i_t - i_t^m) \frac{M_t}{P_t} dt
\]

\[
\frac{d (M_t + B_t)}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] (B_t + M_t) = -s_t dt - (i_t - i_t^m) \frac{M_t}{P_t} dt
\]

\[
\Lambda_t \frac{d (M_t + B_t)}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] (B_t + M_t) = -\Lambda_t \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

Integrating again,

\[
\frac{M_t + B_t}{P_t} = E_t \int_{\tau = t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau + (i_\tau - i_\tau^m) \frac{M_\tau}{P_\tau} \right) d\tau.
\]

To discount with the ex-post return, define \(W_t^n\) and \(W_t\) as the cumulative nominal and real values of investment in short-term debt, so \(dW_t/W_t\) is the ex-post real return. Then,

\[
\frac{dW_t^n}{W_t^n} = i_t dt
\]

\[
P_t W_t = W_t^n
\]

\[
d \left( \frac{1}{P_t W_t} \right) = - \frac{1}{W_t^n} \frac{dW_t^n}{W_t^n} = - \frac{1}{W_t^n} i_t dt = - \frac{1}{P_t W_t} i_t dt
\]

\[
i_t dt = -d \left( \frac{1}{P_t W_t} \right) \left/ \left( \frac{1}{P_t W_t} \right) \right.
\]

(6.48)

\((P_t \text{ and } W_t \text{ may jump, but } P_t W_t \text{ is differentiable.})\) Start again with the nominal flow condition (6.46), rearrange and divide by \(W_t\) to give.

\[
\frac{dB_t}{P_t W_t} - i_t \frac{B_t}{P_t W_t} dt = - \frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right).
\]

(6.49)

Substituting (6.48) for \(i_t\),

\[
\frac{dB_t}{P_t W_t} + d \left( \frac{1}{P_t W_t} \right) B_t = - \frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right)
\]
CHAPTER 6. A BIT OF GENERALITY

\[
d\left(\frac{1}{W_t} B_t \right) = -\frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right)
\]

Integrating,

\[
\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left( s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right).
\]

To discount at the ex post rate of return, expressing seigniorage as an interest saving, and allowing money to pay interest, start at (6.49), and write

\[
d\left(\frac{B_t + M_t}{P_t W_t} \right) - \frac{1}{P_t W_t} \left( B_t + M_t \right) dt = -\frac{1}{W_t} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

\[
d\left(\frac{B_t + M_t}{P_t W_t} \right) + d\left(\frac{1}{P_t W_t} \right) (B_t + M_t) = -\frac{1}{W_t} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

\[
d\left(\frac{B_t + M_t}{P_t W_t} \right) = -\frac{1}{W_t} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left( s_\tau + (i_t - i_t^m) \frac{M_\tau}{P_\tau} \right) d\tau.
\]

Perhaps a more revealing way to express this condition, looking ahead to a model with long term debt and debt with various liquidity distortions, is to write the discount factor as a rate of return that mixes the bond rate of return and the lower (zero) money rate of return. The demand for money allows the government to borrow at lower rates.

To pursue this idea, define \( W^{nm} \) and \( W^m \) as the cumulative nominal and real value of an investment in the overall government bond portfolio, now including money.

\[
\frac{dW^{nm}_t}{W^{nm}_t} = \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i^m_t dt
\]

\[
PW^m_t = W^{nm}_t
\]

\[
d\left(\frac{1}{P_t W^m_t} \right) = -\frac{1}{W^m_t} dW^m_t = -\frac{1}{P_t W^m_t} \left( \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i^m_t dt \right)
\]

\[
d\left(\frac{1}{P_t W^m_t} \right) = -\frac{1}{W^m_t} \frac{1}{B_t + M_t} \left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i^m_t dt \right)
\]

\[
(B_t + M_t) W^m_t d\left(\frac{1}{P_t W^m_t} \right) = -\left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i^m_t dt \right)
\]
Again start at (6.49), and substitute,

\[
\frac{d}{dt} \left( \frac{M_t + B_t}{P_t} \right) - i_t \frac{B_t}{P_t} dt - i_t^n \frac{M_t}{P_t} dt = -s_t dt
\]

\[
\frac{d}{dt} \left( \frac{M_t + B_t}{P_t W_t^m} \right) + (B_t + M_t) d \left( \frac{1}{P_t W_t^m} \right) = -\frac{1}{W_t^m} s_t dt
\]

\[
d \left( \frac{B_t + M_t}{P_t W_t^m} \right) = -\frac{1}{W_t^m} s_t dt
\]

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}^m}{W_t^m} s_\tau d\tau.
\]

Whether a distortion is a “cashflow” or a “discount rate” distortion is a bit in the eye of the beholder.
Chapter 7

Sticky prices

The models so far have been completely nominal, or representatives of the “classical dichotomy” that the price level moves unconnected with real quantities. Inflation is like measuring distances in feet rather than in meters. In reality, changes in the price level often appear to be connected to changes in real quantities, and monetary economics is centrally about studying ways that inflations and deflations can cause temporary booms and recessions.

There are lots of mechanisms that have been studied to describe nominal-real interactions. I work here with the standard and simple model that prices are a bit sticky. I’m no more happy about the assumption of sticky prices than anyone else who works in this area, or with the specification of common sticky price models. We certainly need a deeper understanding of just why monetary affairs seem often to have real effects – and sometimes none whatsoever as in currency reforms. But one should not innovate in two directions at once. Therefore, here I explore how the fiscal theory of the price level behaves if we combine it with utterly standard, though unrealistic, models of sticky prices. Equivalently, I explore how standard sticky price models behave if we give them fiscal underpinnings rather than the conventional “active” monetary policy assumption.

I also include long-term debt. That ingredient turns out to be key to get the model to produce a temporary decline in inflation when interest rates rise. This common belief may not be true, or to be true under certain circumstances, but it is certainly worth knowing whether the model can produce that result and under what conditions. Also, with sticky prices, real interest rates and the discount rate for government debt varies which adds another channel of monetary policy effect. If a change in
nominal rates changes the discount rate of future surpluses, but not the surpluses themselves, nonetheless that change can induce an immediate unexpected change in inflation.

I proceed by building models of increasing complexity, adding one ingredient at a time. Though it takes a bit more space, I find this approach easier to understanding the intuition, mechanisms, and practical application of a model than it would be to start with the most general case. It may also be useful for the reader to understand how to

#### 7.1 The simple new Keynesian model

We meet the standard new-Keynesian sticky-price model,

\[
    x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \\
    \pi_t = \beta E_t \pi_{t+1} + \kappa x_t.
\]

The point of this chapter is to add fiscal theory to this model of price stickiness.

I start by adding sticky prices to the simple fiscal theory of monetary policy model from section 4.

The standard new-Keynesian model is

\[
    x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \\
    \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{7.1}
\]

\[
    \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{7.2}
\]

These two equations generalize the simple model \( i_t = r + E_t \pi_{t+1} \) of section 4 to include sticky prices, which affect output. Equation (7.1) is the “IS” curve, which I like to call the Intertemporal Substitution curve. It shows how the allocation of consumption over time responds to the real interest rate. Equation (7.2) is the new-Keynesian Phillips curve. Inflation is high when output gap \( x \) is high. Expected future inflation shifts the Phillips curve.

To derive (7.1), start from

\[
    1 = E \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + i_t) \frac{P_t}{P_{t+1}} \right].
\]
Linearize and approximate to
\[ E_t (c_{t+1} - c_t) = \delta + \sigma (i_t - E_t \pi_{t+1}) \]
where \( \sigma = 1/\gamma \). Suppressing constants and with consumption equal to output \( c = x \) we get (7.1). Equation (7.2) comes from the first-order condition for monopolistically-competitive price setters, facing costs of changing prices. They set prices today knowing that prices will be stuck for a while in the future, so the expected future price level is the key ingredient for their decision.

While seemingly ad-hoc, the point of the new-Keynesian literature is that this structure has exquisite micro-foundations, which are summarized in [King (2000), Woodford (2003) and Gali (2015)]. There is an active debate on the right specification of the Phillips curve, including additional inflation dynamics (lagged inflation terms in particular, which the data seem to want) and the difference between output and marginal cost, which the theory says should go on the right hand side. The form (7.2) is just a simple and common textbook case, useful for our purpose of investigating how to mix price stickiness with fiscal theory and how doing so alters the most familiar model. Later contrasts between models hinge only on the stability and determinacy properties, which depend on whether eigenvalues are greater than or less than one, and thus are not affected by even quite large changes in model structure.

We can integrate the equations separately to express some of their intuition.

\[ x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1}) \tag{7.3} \]
\[ \pi_t = -\kappa E_t \sum_{j=0}^{\infty} \beta^j x_{t+j}. \tag{7.4} \]

Output is low when current and expected future real interest rates are high. Inflation is high when current and expected future output gaps are low. Each equation also typically has a disturbance term, which we will add later.

Equation (7.4) helps us to see that \( \kappa \to \infty \) is the frictionless limit. In that limit, output is the same for any value of inflation.

Our goal is to merge this simple model with the fiscal theory of the price level.
7.1.1 An analytical solution

The model can be written with inflation as a two-sided moving average of interest rates, plus a moving average of past fiscal shocks. We set the stage for impulse-response functions.

We can eliminate output $x_t$, leaving a relation between interest rates $i_t$ and leads and lags of inflation $\pi_t$.

$$
\pi_{t+1} = \left(1 + \frac{\lambda_1^{-1}}{1 - \lambda_1^{-1}} + \frac{\lambda_2}{1 - \lambda_2}\right)^{-1}\left[i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^{-j} E_{t+1-i_{t+j}}\right] + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}
$$

(7.5)

where

$$
\lambda_1, \lambda_2 = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2}.
$$

(7.6)

In words, inflation is a two-sided moving average of past and expected future interest rates. We have $\lambda_1 > 1$ and $\lambda_2 < 1$, so the moving averages as expressed converge. The term in front can be written more compactly as $\sigma \kappa / (\lambda_2 - \lambda_1)$, but this expression emphasizes that the sum of the coefficients is one — a permanent change in interest rate equals the permanent change in inflation. As usual, $\delta_{t+1} = (E_{t+1} - E_t) \pi_{t+1}$ is an expectational shock indexing multiple equilibria. (You get to (7.5) by first differencing (7.2), and substituting in (7.1), invert the lag polynomials and expand by partial fractions.)

As before, this is the solution for equilibrium inflation given the equilibrium interest rate. We do not have to regard (7.5) as the response of inflation to a time-varying peg. We can generate the interest rate from monetary policy rules or other considerations, and (7.5) will still describe the relationship between observed, equilibrium interest rates and inflation. As (7.3) and (7.4) integrated each equation separately to usefully express endogenous variables in terms of other endogenous variables, (7.5) integrates the pair (7.1)-(7.2). (Cochrane (2017b) gives the parallel solution for $x_t$, which we won’t need here.)

Recognize in (7.5) a generalization of the simple model’s

$$
\pi_{t+1} = i_t + \delta_{t+1}
$$

(7.7)

deriving from its “IS” curve, $i_t = E_t \pi_{t+1}$. It’s the same equation, with a moving average on the right hand side as a result of sticky prices. We can anticipate that
sticky prices will give us smoother dynamics more generally by putting a two-sided moving average in place of sharp movements. Smoother dynamics are more realistic. Now past expectational shocks also affect inflation today, again leaving more realistic delayed effects in place of the sudden jumps of the frictionless model.

Our next job is to tie down the expectational shocks \( \delta_t \) with the fiscal theory of the price level. Our task, conceptually, is to proceed exactly as in section 4. There, we united \( i_t = E_t \pi_{t+1} \) with \( (E_{t+1} - E_t) \pi_{t+1} = -\varepsilon_{t+1}^s \) to conclude

\[
\pi_{t+1} = -\varepsilon_{t+1}^s + i_t
\]

and we plotted responses to interest rate policies and fiscal shocks.

Conceptually, we do the same here. Unexpected inflation comes from the revision in present value of surpluses. In the linearized form (6.27),

\[
(E_{t+1} - E_t) \pi_{t+1} = -\varepsilon_{t+1}^s + \beta (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j (i_{t+j} - \pi_{t+j+1}) \tag{7.8}
\]

\[
\varepsilon_{t+1}^s \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.
\]

In a sticky price model, we will no longer have \( i_t = E_t \pi_{t+1} \) so the final term will not be zero. Discount rates now matter on the right hand side. A change in interest rates can provoke an unexpected inflation or deflation without any change in fiscal policy.

We might finesse the discount-rate issue by assuming “monetary policy” shocks leave the present value of surpluses unaffected, meaning that surpluses react to interest costs of the debt, and the second term is zero. Then we can just label \( \delta_{t+1} = -\varepsilon_{t+1}^s \) as the fiscal shock, and identify interest rate movements with no change in present value of surpluses as monetary policy. I keep the definition of monetary policy for now as changes in interest rates that leave surpluses alone. By this means, we see fiscal feedbacks of monetary policy. As always though, it’s important to specify just what question one is answering.

In fact, I solve the model by the more standard, more generalizable, but less transparent matrix methods.
7.1.2 Responses to interest rate and fiscal shocks

We add fiscal theory of the price level to the basic new-Keynesian model (7.1) (7.2) by adding the linearized flow equation for the real value of government debt \( b_t = R (b_{t-1} - \pi_t) - s_t + i_t \). I produce the response to monetary and fiscal policy shocks. These responses resemble those of the frictionless model, but with dynamics drawn out by price stickiness.

To merge new-Keynesian models with fiscal theory, I add the linearization (6.24) of the flow condition to (7.1) (7.2). The model is then

\[
\begin{align*}
x_t &= E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
b_{t+1} &= R (b_t - \pi_{t+1} - s_{t+1}) + i_{t+1}.
\end{align*}
\]

(7.9) (7.10)

Since all variables are deviations from steady state, I dispense with tildes. I solve the model with the standard matrix method, solving unstable eigenvalues forward and stable eigenvalues backward. I defer the algebra to section 7.3 after introducing one more generalization in the next section. \( R \geq 1 \) in equation (7.10) provides the additional forward-looking root needed to give a unique solution. One may use (7.10) at either \( t \) or \( t + 1 \), i.e.

\[
b_t = R (b_{t-1} - \pi_t - s_t) + i_t.
\]

The solutions are the same either way, and the only difference is a bit of esthetics in the algebra along the way.

Figure 7.1 presents responses to an unexpected permanent interest rate rise, with no changes to surpluses in this sticky-price model. Compare this figure to the responses in the frictionless model of Figure 4.1. There are two big differences and one disappointment. First, sticky prices are, well, stickier. The inflation response is drawn out slowly, and more realistically.

Second, there is an instantaneous and unexpected inflation response, \( \pi_1 > 0 \) on the same date as the interest rate shock, while previously, inflation did not move until period 2. How can inflation move instantly without a fiscal shock? This is a discount rate effect, as seen in equation (7.8). Expected interest rates rise, expected inflation does not rise by the full amount, so the real interest rate rises. A higher real interest rate raises the discount factor for unchanged future surpluses. The present value of surpluses falls, though surpluses themselves are unchanged. Equivalently, the higher debt service costs resulting from higher real interest rates and rolling over
7.1. THE SIMPLE NEW KEYNESIAN MODEL

Figure 7.1: Response to an unexpected permanent interest rate shock, with no fiscal shock, in the simple sticky price model. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$.

one-period debt add to the fiscal burden, and provoke the same response that a decline in surpluses would provoke.

This is an important effect, which we will see again and again below. It shows us that monetary policy can have indirect fiscal effects on inflation, even if central banks cannot change surpluses. And it emphasizes that discount rate changes rather than just surplus changes will be important to understanding inflation.

The plotted response depends on fiscal policy, and the nature of fiscal and monetary interactions as usual. I have defined a monetary policy shock as one that leaves surpluses unchanged. If the Treasury always raises taxes to cover interest costs “passively,” then the present value of surpluses would remain unchanged and this immediate inflation would not appear. So, the extra inflation from raising interest rates is likely to be seen from a government already having trouble to meet primary surpluses, one with a lot of short-term debt outstanding, and one in which fiscal policy does not typically adapt to interest costs. It offers a cautionary tale for fiscally stressed governments trying to battle fiscal inflation with higher interest rates alone.
CHAPTER 7. STICKY PRICES

The disappointment is that sticky prices do not lead to a negative response of inflation to interest rates. You might have thought higher nominal interest rates would mean higher real rates, which would depress aggregate demand, and via the Phillips curve lead to less inflation. That static ISLM thinking does not apply in this model.

In fact, stickier prices lead to more time-1 inflation in this model, as shown by the dashed line in Figure 7.1. As inflation becomes infinitely sticky, as $\kappa \to 0$, this model approaches an inflation jump at time 0.

Higher interest rates do lead to lower output. In this Phillips curve, output is low when inflation is low relative to future inflation. The expected increase in inflation corresponds to lower output. Equivalently, output is low when current and future real interest rates are high as in (7.3). So, this model agrees with the conventional wisdom that higher interest rates with sticky prices lower output.

Output does not return exactly to zero, as this model features a small permanent inflation-output tradeoff. From (7.2), permanent movements in $x$ and $\pi$ follow

$$x = \frac{1 - \beta}{\kappa} \pi$$

for $\beta$ near one, and $\kappa$ also near one, this effect is small. One way to eliminate it is to set $\beta = 1$. However, when we want to study lower values of $\beta$ or very sticky prices, low $\kappa$, this is an unpleasant feature. Another solution, which also helps to fit the data, is to include a lag of inflation. This can be rationalized as the effects of indexation.

$$\pi_t = (1 - \gamma) \pi_{t-1} + \gamma E_t \pi_{t+1} + \kappa x_t$$

Now there is no long-run output-inflation tradeoff.

Figure 7.2 presents the model’s response to a time-1 fiscal shock, with no change in nominal interest rates. Compare this response to the response to the same shock without price stickiness in Figure 4.1.

First, a fiscal tightening still lowers inflation. But price stickiness now leads to a drawn out response, where the fiscal shock led to a one-period response only without price stickiness.

Second, The 1% fiscal shock now only produces a -0.4% decline in inflation, not -1% as before. Again, price stickiness means higher real rates, and thus a higher discount rate and an inflationary force that battles the deflationary fiscal shock.

Third, low inflation relative to future inflation means low output. Conversely a negative fiscal shock – more deficits – would imply more inflation and more output. When
Figure 7.2: Response to a fiscal shock $\varepsilon_1^s$ with no interest rate movement in the sticky-price fiscal theory model. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$.

done deliberately, such expansion look a bit like “fiscal stimulus.” Again, however, the present value of future surpluses matters, not the current surplus or deficit. The usual promises of deficit today, but budget balance tomorrow, if believed, would have no effect in this model. When not deliberate, this graph or its opposite offers an interesting picture of a recession and disinflation or expansion and inflation that seems to come from nowhere, from “animal spirits,” i.e. a change in expectations.

Figure 7.3 presents the response to a fully expected rise in interest rates. In Figure 4.1 we found that expected and unexpected interest rates had exactly the same effect on inflation. That is no longer true. Inflation now moves ahead of the expected interest rate rise. The two-sided nature of the moving average in (7.5) shows up here. The two-sided moving average produces similar results for dates after the shock, but you can see now there is less inflation than in the unexpected interest rate rise of Figure 7.1. The two-sided moving average is the same, but the $\delta$ terms are not the same. In this case, there is no fiscal surprise, either in surpluses or discount rates. The expected interest rate rise also lowers output, but now output goes down in advance of the interest rate rise that causes it.
7.2 Sticky prices and policy rules

We add a policy rule $i_t = \phi \pi_t + u^i_t$ rather than a fixed interest rate target. As in the frictionless model, an endogenous policy response gives additional interesting dynamics, and shows how monetary policy can smooth the response to a fiscal shock.

New-Keynesian models are typically closed with a policy rule, for example

$$i_t = \phi \pi_t + u^i_t.$$  \hspace{1cm} (7.11)

As in section 4.2 we can calculate responses to a shock to $u^i_t$ in a formulation such as (7.11), or responses to other shocks when interest rates respond endogenously via that rule $\phi$.

So, let us consider the sticky-price model (7.1)-(7.2), the policy rule (7.11), and, for convenience, an AR(1) monetary policy shock. In sum, we generalize (7.9)-(7.10)
7.2.  **STICKY PRICES AND POLICY RULES**

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \]  
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]  
\[ b_{t+1} = R (b_t - \pi_{t+1} - s_{t+1}) + i_{t+1} \]  
\[ i_t = \phi \pi_t + u_t^i \]  
\[ u_{t+1}^i = \rho \varepsilon_{t+1}^i + \varepsilon_{t+1}^s \]  
\[ s_{t+1} = \rho_s s_t + \varepsilon_{t+1}^s \]

Figure 7.4: Response of the sticky-price model to a monetary policy shock. Parameters \( r = 0.01, \kappa = 0.25, \sigma = 1, \rho^i = 0.8, \phi = 0.8, \rho^s = 0 \).

I also include an AR(1) surplus, even though I argued for its unrealism above. A more realistic surplus process is coming. It will let us compute responses to monetary policy shocks that leave surpluses constant, and to fiscal shocks that do devalue debt. I used the symbol \( \varepsilon^s \) above to refer to innovations in the present value of future surpluses. Here it refers to innovations to the surplus itself. There are only so many Greek letters.

Figure 7.4 presents the response of this model to a monetary policy shock \( \varepsilon_t^i \) with such an endogenous response and \( \phi = 0.8 \). Compare this figure to the frictionless
case of Figure 4.2. The broad pattern is the same. This calculation approaches
the frictionless calculation one as we turn off price stickiness $\kappa \to \infty$. But there
are differences, due to stickier prices. The interest rate and inflation rate do not
move in lockstep any more. The real interest rate now varies, as plotted by the
line $r_t \equiv i_t - E_t \pi_{t+1}$. The inflation response is smoothed. As in the frictionless
case, the combination of the endogenous response and the persistent shock produce
interesting dynamics. However, remember we can get the same dynamics with the
$\phi = 0$ calculation of Figure 7.1 or any other value of $\phi$, by feeding in this interest
rate process.

Figure 7.5: Response to a fiscal shock in the simple model with sticky prices and a
policy rule. Parameters $r = 0.01$, $\rho_s = 0$, $\kappa = 0.25$, $\sigma = 1$, $\phi = 0.8$

Figure 7.5 shows the response of this model with a policy rule to a fiscal policy
shock, which we can compare to the frictionless case in the bottom of Figure 4.2.
The overall pattern is again similar to that case. If you look closely though, you
can see that inflation moves down more than interest rates decline. There is a slight
rise in the real interest rate in this case. And, with sticky prices, the deflationary
fiscal shock also lowers output $x_t$. Again, the new-Keynesian Phillips curve lowers
output when inflation is lower than expected future inflation or when real interest
rates are high. So, just as one would expect, sticky prices mean that a deflationary
fiscal shock also lowers output.
Absent the monetary policy reaction to the fiscal shock – the downward movement in \( i_t \) occasioned by \( i_t = \phi \pi_t \) in this case or its equivalent discretionary movement – the inflation response melts away more quickly, as seen in Figure 7.2. The output response is correspondingly larger. So, by moving interest rates down in response to a contractionary fiscal shock, the Fed induces a more persistent inflation response, and a correspondingly smaller output response. But, again, the Fed could do that with more \( u_t \) and less \( \phi \), or with an interest rate target that responds directly to the fiscal shock. A rule \( i_t = \phi \pi_t \) is a convenient way to capture such responses, and may be useful to convey expectations and commit to the policy, but it is not essential, or identified. From the perspective of the last section, this exercise combines a monetary policy response to a fiscal (no interest rate movement) shock, and shows how monetary policy can ameliorate fiscal shocks.

### 7.3 Matrix solution method

We write the discrete time models in standard form

\[
    z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}.
\]

Then, eigenvalue-decompose the matrix \( A \), solve unstable eigenvalues forward and stable eigenvalues backward. With as many forward-looking eigenvalues as there are expectational errors \( \delta \), we obtain a unique solution.

---

Here I present the solution method for the models of section 7.1.2 and 7.2. This standard method generalizes easily to models of arbitrary complexity.

To solve the system (7.12)-(7.16), I first express it in standard form

\[
    z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}
\]

(7.18)

I use the notation \( \delta_{t+1} \) to denote expectational errors in equations that only determine expectations. For example, I write

\[
    \pi_t = \beta E_t \pi_{t+1} + \kappa x_t
\]

as

\[
    \beta \pi_{t+1} = \pi_t - \kappa x_t + \beta \delta_{t+1}.
\]

All this equation says about \( \delta_{t+1} \) is that \( E_t \delta_{t+1} = 0 \). Solving the model means also finding \( \delta_{t+1} \) in terms of other variables.
Next, I substitute out the non-dynamic equations that just define one variable in terms of others, (7.15) here. I eliminate $i_t$, and write $s_t = u^s_t$. Putting the result in matrix form, we have

$$
\begin{bmatrix}
1 & \sigma & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & R - \phi & 1 & -1 & R \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{t+1} \\
\pi_{t+1} \\
b_{t+1} \\
u^i_{t+1} \\
u^s_{t+1}
\end{bmatrix}
=
\begin{bmatrix}
1 & \sigma \phi & 0 & \sigma & 0 \\
-\kappa & 1 & 0 & 0 & 0 \\
0 & 0 & R & 0 & 0 \\
0 & 0 & 0 & \rho^i & 0 \\
0 & 0 & 0 & 0 & \rho^s
\end{bmatrix}
\begin{bmatrix}
x_t \\
\pi_t \\
b_t \\
u^i_t \\
u^s_t
\end{bmatrix}
$$

In general we may invert the first matrix numerically. However, in this case we can either invert it analytically, or recursively substitute the equations of the model to obtain the form (7.18).

In general we may invert the first matrix numerically. However, in this case we can either invert it analytically, or recursively substitute the equations of the model to obtain the form (7.18).

$$
\begin{bmatrix}
x_{t+1} \\
\pi_{t+1} \\
b_{t+1} \\
u^i_{t+1} \\
u^s_{t+1}
\end{bmatrix}
=
\frac{1}{\beta}
\begin{bmatrix}
\beta + \kappa \sigma & \sigma (\beta \phi - 1) & 0 & \sigma \beta & 0 \\
-\kappa & 1 & 0 & 0 & 0 \\
\kappa (R - \phi) & -(R - \phi) & R \beta & \beta \rho^i & -R \beta \rho^s \\
0 & 0 & 0 & \beta \rho^i & 0 \\
0 & 0 & 0 & 0 & \beta \rho^s
\end{bmatrix}
\begin{bmatrix}
x_t \\
\pi_t \\
b_t \\
u^i_t \\
u^s_t
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\varepsilon^i_{t+1} \\
\varepsilon^s_{t+1}
\end{bmatrix}
$$

Next, eigenvalue decompose the transition matrix $A$, and transform the dynamics.

$$
z_{t+1} = QAQ^{-1}z_t + B\varepsilon_{t+1} + C\delta_{t+1}
$$

$$
Q^{-1}z_{t+1} = \Lambda Q^{-1}z_t + Q^{-1}B\varepsilon_{t+1} + Q^{-1}C\delta_{t+1}
$$

where $\Lambda$ is a diagonal matrix of the eigenvalues $\lambda_i$ of $A$, and $Q$ is the corresponding matrix of eigenvectors. Using hats to denote transformed variables, and $k$ to denote
elements of vectors, the system decouples into a set of scalar difference equations,

\[ \hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} \]  

(7.20)

Solving unstable roots forward with \( E_t \varepsilon_{t+j} = E_t \delta_{t+j} = 0 \), we now have

\[ \hat{z}_{k,t} = -\sum_{j=0}^{\infty} \frac{1}{\lambda_i^{j+1}} E_t \left( \hat{\varepsilon}_{k,t+j} + \hat{\delta}_{k,t+j} \right) = 0; \lambda_k > 1 \]

where the \( i \) index denotes the elements of vectors.

As a result, (7.20) implies

\[ \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} = 0; \lambda_k > 1 \]  

(7.21)

Equation (7.21) means that we can find the expectational errors in terms of the structural shocks;

\[ \hat{\delta}_{k,t+1} = -\hat{\varepsilon}_{k,t+1}; \lambda_k > 1 \]  

(7.22)

In order to have a unique solution, we need exactly as many unstable eigenvalues \( \lambda_i > 1 \) as there are expectational shocks \( \delta \). If we have too few such eigenvalues, there are still multiple equilibria of the kind we first encountered when studying how \( i_t = E_t \pi_{t+1} = \pi_{t+1} + \delta^\pi_{t+1} \) and an interest rate peg (and passive fiscal policy) leaves multiple equilibria. If we have too many such eigenvalues, then the system is potentially overdetermined, and there is no solution.

The eigenvalues of the transition matrix in (7.19) are

\[ \lambda_k = \frac{(1 + \beta + \kappa \sigma) \pm \sqrt{(1 + \beta + \kappa \sigma)^2 - 4 \beta (1 + \kappa \sigma \phi)}}{2 \beta}, R, \rho^j, \rho^s. \]

We have two expectational errors \( \delta^x_{t+1} \) and \( \delta^\pi_{t+1} \), so we need two eigenvalues greater than one. If \( \| \phi \| < 1 \), the first set of eigenvalues, coming from the \( x \) and \( \pi \) equations, has one eigenvalue greater and one less than one, as we found a two-sided analytic solution. New-Keynesian modelers adopt \( \phi > 1 \), and leave out the debt equation with \( R \). We use \( \| \phi \| < 1 \), and then \( R \geq 1 \) provides the second forward-looking eigenvalue.

Denote \( Q^{-1}_{\lambda < 1} \) a matrix composed of the rows of \( Q^{-1} \) corresponding to stable eigenvalues, and likewise \( Q^{-1}_{\lambda > 1} \) a matrix composed of the rows of \( Q^{-1} \) corresponding to unstable eigenvalues. Equation (7.22) then implies

\[ Q_{\lambda > 1}^{-1} D \delta_{t+1} = -Q_{\lambda > 1}^{-1} C \varepsilon_{t+1}, \]
and the assumption that there are as many explosive eigenvalues as \( \delta \) means we can invert,

\[
\delta_{t+1} = - \left[ Q_{\lambda > 1}^{-1} D \right]^{-1} Q_{\lambda > 1}^{-1} C \varepsilon_{t+1},
\]

and then write

\[
\hat{\delta}_{t+1} = -Q^{-1} D \left[ Q_{\lambda > 1}^{-1} D \right]^{-1} Q_{\lambda > 1}^{-1} C \varepsilon_{t+1}.
\] (7.23)

We can now write the system dynamics as

\[
\lambda_k < 1 : \hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1}
\]

\[
\lambda_k > 1 : \hat{z}_{k,t} = 0
\]

Then we find the original variables by

\[
z_t = Q \hat{z}_t.
\]

### 7.4 Long-term debt and sticky prices

I introduce long-term debt into the discrete-time sticky-price model. The model adds long term debt to the debt accumulation equation, and an expectations hypothesis model of bond prices:

\[
v_{t+1} = R (v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1})
\]

\[
E_t r_{t+1}^n = i_t
\]

\[
r_{t+1}^n = \omega q_{t+1} - q_t
\]

This modification gives a temporary inflation decline after a rise in interest rates.

Long term debt exists, and from section 6.1 we know long-term debt can produce a negative response of inflation to interest rates, without a contemporaneous fiscal shock. So, let us add long-term debt along with sticky prices to our fiscal theory of interest rate targets. As a reminder, in section 6.1 we found that with long-term debt, a rise in interest rates gave rise to a decline in the price level, Figure 6.1 in particular. In section 7.1 and Figures 7.1 and 7.4 in particular, we saw how sticky prices give rise to smooth dynamics of the response of output and inflation to interest rates, but higher interest rates led only to higher inflation. Putting the two
ingredients together, we can hope to produce smooth dynamics, and a temporary negative output and inflation response to interest rate rises.

The model consists of the usual IS and Phillips curve, (7.1)-(7.2), the linearized flow condition with long-term debt (6.19), and two bond-pricing equations to determine the rate of return $r_{t+1}^n$.

$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1})$$

(7.24)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

(7.25)

$$v_{t+1} = R \left(v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1}\right)$$

(7.26)

$$E_t r_{t+1}^n = i_t$$

(7.27)

$$r_{t+1}^n = \omega q_{t+1} - q_t$$

(7.28)

I leave the government debt equation in terms of $v_t$, the end of period market value of government debt. If we express it in terms of $b_t \equiv v_t + i_t$, we obtain

$$b_{t+1} = R \left[b_t + (r_{t+1}^n - i_t - \pi_{t+1} - s_{t+1}) + i_{t+1}\right].$$

Here we see it is exactly the same equation as (7.14) with one-period debt, except that the ex-post return $r_{t+1}^n$ may not equal the interest rate $i_t$. The entire effect of long-term debt on the model comes through this ex-post return. (See equation (6.21) and following discussion in Section 6.7.) We could use $b_t$ to maintain continuity, but it does not change the results so I use the simpler version in terms of $v_t$

We need to determine the ex-post nominal bond return $r_{t+1}^n$. To this end I assume the expectations hypothesis that the expected return on bonds of all maturity is the same in equation (7.27), and add a linearized bond pricing equation in (7.28).

I assume a geometric maturity structure,

$$B_{t-1}^{(t+j)} = \omega^j B_{t-1},$$

Define the end of period value of one share of the portfolio of government debt portfolio as $Q_t$

$$Q_t = \sum_{j=0}^{\infty} Q_t^{(t+1+j)} \omega^j.$$

Then the nominal return on the portfolio of government debt is

$$r_{t+1}^n = \log \left( \frac{\sum_{j=0}^{\infty} Q_t^{(t+1+j)} \omega^j}{\sum_{j=0}^{\infty} Q_t^{(t+1+j)} \omega^j} \right) = \log \left( \frac{1 + \omega Q_{t+1}}{Q_t} \right)$$

(7.29)
Linearizing, with \( q_t = \log(Q_t) \),
\[
\begin{align*}
    r_{t+1}^n &= \log (1 + \omega e^{\eta_{t+1}}) - q_t \\
    \tilde{r}_{t+1}^n &= \frac{\omega Q}{1 + \omega Q} \tilde{q}_{t+1} - \tilde{q}_t \\
    \tilde{r}_{t+1}^n &= \omega e^{\eta} \tilde{q}_{t+1} - \tilde{q}_t.
\end{align*}
\]
Here \( R = e^{\eta^n} = e^\eta \). Since \( \omega \approx 0.7 - 0.8 \), leaving \( e^{\eta^n} \) out does little harm. In (7.28), I again drop the tildes since all variables are linearized deviations from steady states.

Solving (7.28) forward by itself, and using (7.27), we obtain
\[
q_t = -\sum_{j=0}^{\infty} \omega^j r_{t+j+1}^n = -E_t \sum_{j=0}^{\infty} \omega^j \iota_{t+j}.
\]
(7.30)

The first equality just expresses the identity that bond prices are the sum of future returns. The second equality is the standard form of the expectations hypothesis that the bond yield is the average of expected future short rates.

This latter form is the key for our model. The extra forward-looking eigenvalue induced by the bond rate of return equation (7.28) determines bond prices, ex-post, by the time \( t+1 \) version of (7.30), and hence the ex-post return \( r_{t+1}^n \) in the long-term debt accumulation equation (7.26).

Again, however, I actually solve all the flow relations together by the same method outlined in Section 7.3. I substitute out \( r_{t+1}^n \) so the extra equations added to the model are
\[
\begin{align*}
    \iota_{t+1} &= R (\iota_t + \pi_{t+1} - s_{t+1}) + R \delta^r_{t+1} \\
    \omega q_{t+1} &= q_t + i_t + \delta^r_{t+1}.
\end{align*}
\]

Figure 7.6 presents the response to an unexpected permanent interest rate rise in this model. This is the generalization of the sticky price model in Figure 7.1 to long-term debt, using \( \omega = 0.8 \) which roughly approximates the US maturity structure. Where in Figure 7.1 inflation started rising immediately, now we have a disinflation first. Relative to the frictionless long-term debt case in Figure 6.1, we have a drawn out period of disinflation and then low inflation, rather than a one-time downward price-level jump. The temporary disinflation coincides with an output decline as well, capturing standard intuition.
Figure 7.6: Response to an unanticipated permanent interest rate rise, with sticky prices, no change in surpluses, and long term debt. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$, $\theta = 0.8$.

Though higher interest rates now give a temporary disinflation, higher inflation eventually reemerges. This model does not produce the standard belief that higher interest rates permanently reduce inflation. That result requires a contemporaneous fiscal shock, or an old-Keynesian, irrational-expectations model. Sims (2011) called the pattern of lower and then higher inflation “stepping on a rake,” advances it as a description of the 1970s, in which interest rate increases did temporarily reduce inflation, and cause recessions, but each time inflation came back more strongly.

The upper dashed line shows inflation with longer maturity structure, $\omega = 0.9$ rather than $\omega = 0.8$. A longer maturity structure produces a larger and more protracted disinflation from an interest rate increase.

The lower dashed line shows inflation in the frictionless case $\kappa = \infty$. Without sticky prices, as in Figure 6.1, the inflation decline lasts one period and then inflation rises immediately to 1%. (I cut off the line so it would not overlap with the others.)

More importantly, we see here that the decline in inflation is larger when prices are less sticky. Discount rate variation accounts for this effect. Look at the inflation
identity (6.27) in this case

\[(E_{t+1} - E_t) \pi_{t+1} = (E_{t+1} - E_t) r^n_{t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j (i_{t+j} - \pi_{t+j}).\]

In the frictionless model the right hand term is zero, so unexpected inflation must reflect the entire change in long-term bond prices of the \(r^n_{t+1}\) term. The negative ex-post return that long term bonds suffer when the interest rate rises means a negative ex-post inflation as well. With sticky prices, inflation does not match the interest rate rise of the final term, so real interest rates rise. Higher real interest rates discount surpluses at a higher rate, a countervailing inflationary effect.

One might think that sticky prices mean that higher nominal rates mean higher real rates, less aggregate demand, and via the Phillips curve less inflation. That stickier prices imply less inflation reminds us that even though the response functions capture common intuition, the mechanism is different. The disinflation is entirely a wealth effect of government bonds, as explained in the frictionless context. Price stickiness just smooths out the dynamics.

Long-term debt also has no effect on the response to a fully anticipated interest rate rise, so Figure 7.3 is also completely unchanged. Like a fiscal shock, only an unanticipated shock to bond prices can lower their value. More generally, the disinflationary effect of interest rate increases happens when they are announced, as in the frictionless case of Figure 8.1, not when the interest rates actually rise.

Long-term debt has no effect at all on the response to a fiscal shock with no change in interest rate – Figure 7.2 is completely unaltered. If current and expected future nominal rates do not respond to the fiscal shock, then long-term nominal bond prices do not respond to the fiscal shock, and the only reason in this model for a difference between long and short term debt disappears.

We can mix the two shocks. For example, monetary policy may try to offset the inflationary response to a fiscal shock. If the central bank raises interest rates in response to a fiscal shock, then it can substitute the long slow later inflation for the current inflation of the fiscal shock. A policy rule can achieve the same thing, as we will see shortly – if the central bank raises interest rates in response to inflation, then it will raise interest rates in response to a fiscal shock, and automatically perform this inflation-smoothing function. We will explore the quantity side of this idea, that long term debt helps to buffer fiscal shocks, in section 8.2.1.
7.5 Higher or lower inflation?

Do higher interest rates raise or lower inflation? Motivated by current events, I summarize the above investigation with a list of considerations: Is the interest rate rise permanent, or temporary; is it likely to be reversed if a fiscal shock or the long-term effect sends inflation temporarily in the opposite from the desired direction? Is there a lot of long-term domestic currency debt outstanding? Is the interest rate rise a surprise or widely anticipated? Are prices sticky? Is fiscal policy likely to react either to the same events or to the monetary policy intervention? How will fiscal policy react to larger interest costs? Each of these considerations is important to the sign of the effect of interest rates on inflation.

So, does raising interest rates raise or lower inflation, and conversely? The fiscal theory offers a loud “it depends.” There is no mechanistic answer, and sometimes you will observe a positive sign and sometimes a negative sign. That is useful, as we see conflicting evidence. If the theory is right – and if we are interpreting it right – the theory will help us to avoid exporting experience from one event to another where the preconditions for its result do not hold.

As I write in 2019, the issue is in the air. Japan and Europe, after long periods of near-zero interest rates and all sorts of fiscal stimulus, quantitative easing, and forward guidance, still have inflation below their targets. The U.S. by 2018, after a widely pre-announced set of interest rate increases, experienced a slow rise in inflation. In both cases, though policy circles do not question a rather mechanistic negative relation, many academics and commentators are starting to question that perhaps a steady, permanent, and widely pre-announced interest rate rise might raise inflation, at least eventually. In Argentina, going through another periodic fiscal crisis, the central bank has tried to defend the currency and to lower inflation by repeated sharp interest rate rises. Each one has seemed to quickly and perversely lower the exchange rate and result in more inflation. A range of opinion in Brazil and Turkey, each dealing with persistent inflation, has started to think perhaps lowering interest rates is the secret to lowering inflation. The question whether those economies have the preconditions for that to work is important. Yet the memory of 1980, in which a sharp interest rate rise is thought to have been crucial for lowering inflation, and the memory of the 1970s remains strong, in which too-low interest rates are thought to have raised inflation.

For an interest rate rise to lower inflation, in this simple model, the interest rate rise must first of all be *persistent*. It must lower long-term bond prices, and only a credibly
persistent interest rate rise will do that. It’s easy to write down a persistent process, but harder for the government to communicate that expectation. If people think this is a trial or experimental effort, or if they worry that the government will quickly back down if it doesn’t go right, then they will not perceive it as persistent. In this case, the preconditions for a negative effect differ from the standard new-Keynesian model, in which temporary interest rate rises have a larger negative inflation effect than permanent ones.

With sticky prices, the positive effect of interest rates on inflation is also larger when the rate rise is persistent, just as a result of the sluggish two-sided response that sticky prices induce. If you want a rise in interest rates to raise inflation, you really have to convince markets that the rate rise will be permanent so that the response of inflation to expected future interest rates kicks in. Here especially, you have to convince markets that if inflation temporarily goes in the opposite of the desired direction, due to the long-term bond effect, or due to an adverse fiscal shock, you won’t give up and abandon the experiment.

In sum, a credibly permanent rate rise raises the magnitude of the temporary inflation decline, but also raises the magnitude and brings about more quickly the subsequent rise.

For an interest rate rise to lower inflation, there must be long-term debt outstanding. The disinflationary effect can be measured by the change in total market value of long-term debt. Many countries in fiscal stress have moved to short-term financing, so there just isn’t that much long-term debt left, and they are then less likely to experience the temporary inflation decline.

Likewise, the interest rate rise only affects domestic currency debt. A government that has largely borrowed in foreign debt cannot change the value of that debt by interest rate rises. The denomination as well as the maturity structure of government debt is a crucial state variable.

The analysis of interest rate rises here presumes there is no contemporaneous fiscal shock, or a fiscal response to monetary policy. If fiscal authorities say, “whew, the central bank is going to solve inflation for us, we can relax,” or if the monetary tightening is itself a response to a fiscal shock, then we will see fiscal inflation, not disinflation. If the fiscal authorities cooperate with a joint monetary-fiscal contraction, then the inflation decline will be larger. The conventional new-Keynesian analysis pairs a fiscal tightening with the interest rate rise, and thereby produces lower inflation even without long-term debt. In this reading of those equations, the monetary authorities by raising interest rates convince the fiscal authorities to tighten, and
that tightening produces a reduction in inflation.

For an interest rate rise to lower inflation it must be a surprise. A pre-announced interest rate rise lowers the value of long-term debt when it is announced, not when it actually happens. Sudden shocks, like 1980, have greater disinflationary effects than widely pre-announced rate rises, like 2016-2018.

The discount rate, or interest cost effect can dampen the disinflationary effect, and thus lead rate rises to raise inflation more quickly. Interest rate rises lower inflation more, when prices are less sticky, the opposite of conventional intuition. (Reminder: When prices are sticky, the nominal interest rate rise translates to a rise in real interest rates. This rise in discount rate lowers the present value of surpluses, an inflationary force. This effect may be easier to understand and to communicate in flow terms. If the rise in nominal rates raises real interest rates, due to sticky prices, then it raises interest costs in the budget, which makes the budget situation worse.)

This channel is more important for highly indebted countries. At 100% debt to GDP ratio, each one percentage points rise in real interest rates adds 1% of GDP to interest costs. At 10% of GDP, the same rate rise only adds 0.1% of GDP to interest costs. So highly indebted countries, with much short-term debt and sticky prices are more likely to see higher interest rates translate into higher, not lower inflation.

Again, we are holding fiscal policy constant in this calculation. If fiscal authorities react to higher real interest costs by reducing primary deficits, that adds a deflationary effect. If fiscal authorities react to a reduction in real interest costs by postponing fiscal reforms, a reduction in rates that monetary authorities hope to create disinflation will fail to do so.

As a result, the political perception of fiscal policy matters. In the model, when prices are not sticky, higher interest rates do result in higher interest costs, but they are only nominally higher interest costs, matched by higher inflation. A government may not understand that the reportedly higher interest costs are only nominal. Accounting for interest costs is often far from economic concepts. If nominal higher interest costs provoke a fiscal response, then even a nominal rate rise will lead to disinflation by provoking a tightening of primary surpluses.

When thinking about fiscal policies, growth effects are larger than tax rate effects in the present value of future surpluses. “Austerity” plans may backfire if distorting taxes reduce long-run growth. The present-value Laffer curve peaks far to the left of the one-year-revenue curve. Conversely, a growth-oriented fiscal reform, lowering
marginal tax rates, can raise the present value of surpluses and thereby disinflate, even if it produces a few years of larger deficits.

### 7.6 A monetary policy rule and long-term debt

I plot responses to fiscal and monetary policy shocks with sticky prices, long term debt, and a policy rule $i_t = \phi \pi_t + u_t^i$.

As before, we can add a policy rule

$$i_t = \phi \pi_t + u_t^i$$

to the model (7.24)-(7.28) and plot the response to policy disturbances. Figure 7.7 plots the response to a monetary policy shock with this rule, generalizing Figure 7.4 to long-term debt. Comparing the two pictures, you use roughly the same pattern, except that now we have a disinflation following the interest rate rise. The dynamics
are smoothed by sticky prices and the persistence induced by the persistent policy shock and by the endogenous response of interest rates to inflation.

Figure 7.8: Response to a fiscal shock with long-term debt, sticky prices, and a policy rule. Parameters $r = 0.05$, $\sigma = 1$, $\kappa = 0.25$, $\phi = 0.8$, $\theta = 0.8$. The thin dashed line shows inflation for the $\theta = 0$ case.

Figure 7.8 presents the response to a fiscal shock with a policy rule $i_t = \phi \pi_t$. It includes the inflation response in the short-term debt case, from Figure 7.5, for comparison. The pattern is the same, but the magnitude of the disinflation is less, peaking at -0.3% rather than -0.7%. With a policy response, interest rates do change via $\phi > 0$, so this plot is equivalent to a combined fiscal - monetary policy shock with $\phi = 0$. With long-term debt, an interest rate decline raises inflation over what it otherwise would be. By lowering interest rates even more, the government could offset the disinflationary effect of the shock even more.

### 7.7 A better fiscal policy rule

The fiscal policy rules in the models so far are simplistic. In particular, they do not capture the feature that deficits today should imply at least some surpluses in
the future, and they do not reflect the distinction between on and off equilibrium responses that are useful conceptually, if not in fitting actual models. Here, I summarize a model from [Cochrane (2019)] that incorporates these features, along with sticky prices, and long-term debt with a geometric maturity structure.

The model consists of standard IS and Phillips equations (16.18), (16.19), fiscal and monetary policy rules, and equations tracking the evolution of the value of government debt,

\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \tag{7.31}
\]

\[
\pi_t = \beta E_t \pi_{t+1} - \kappa x_t. \tag{7.32}
\]

\[
i_t = \theta_{s\pi} \pi_t + \theta_{tx} x_t + u^i_t \tag{7.33}
\]

\[
s_t = \theta_{sx} \pi_t + \theta_{sx} x_t + \alpha v^*_t + u^s_t \tag{7.34}
\]

\[
[1 + R\alpha (1 - \gamma) / \gamma] v^*_{t+1} = R (v^*_t + i_t - E_t \pi_{t+1} - s_{t+1}) \tag{7.35}
\]

\[
v_{t+1} = R (v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}) \tag{7.36}
\]

\[
E_t r^n_{t+1} = i_t \tag{7.37}
\]

\[
r^n_{t+1} = \omega q_{t+1} - q_t \tag{7.38}
\]

\[
u^i_{t+1} = \rho^i u^i_t + \varepsilon^i_{t+1} \tag{7.39}
\]

\[
u^s_{t+1} = \rho^s u^s_t + \varepsilon^s_{t+1} \tag{7.40}
\]

Everything is the same as the last model, except for the surplus process (7.33) - (7.34). I allow surpluses to respond to the output gap, as they do, with procyclical tax receipts and countercyclical stabilizers and stimuli. I allow surpluses to respond to inflation as well, which one can view either as a policy decision or imperfect indexation.

We can regard (7.34) and (7.35) as a way to write compactly and intuitively a dynamic surplus process in which a government running deficits promises future surpluses. It writes an otherwise complex non-invertible MA process in a VAR(1) form by introducing a latent state variable $v^*_t$. To see this point, consider the simple case with $\theta_{sx} = \theta_{s\pi} = 0$ and $r_t = i_t - E_t \pi_{t+1} = 0$. Substituting the surplus response (7.34) into the $v^*$ process (7.35) we can write the $v^*$ process as

\[
\left[1 + \left( \frac{1}{\gamma} - 1 \right) R\alpha \right] v^*_{t+1} = R (v^*_t - \alpha v^*_{t+1} - u^s_{t+1})
\]
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\[ v_{t+1}^* = -\frac{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}}{1 - \left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}} u_{t+1}^*. \]

Substituting this result back into (7.34), the system (7.33) - (7.34) is equivalent to the surplus process

\[ s_{t+1} = \alpha v_{t+1}^* + u_{t+1}^* \]

\[ s_{t+1} = \left[ 1 - \alpha \frac{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}}{1 - \left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}} L \right] u_{t+1}^* = a(L)u_{t+1}^* \]

where the last equality defines notation. Writing it out,

\[ s_{t+1} = u_{t+1}^* - \frac{\alpha}{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)} u_{t+1}^* - \frac{\alpha}{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^2} u_{t}^* - \frac{\alpha}{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^3} u_{t-1}^* - ... \]

A deficit shock, a negative \( u_t^* \), is followed by a string of small positive surpluses, which pay back some of the debt.

Suppose that the shock \( u_{t+1}^* = \varepsilon_{t+1}^* \) is unexpected. Then

\[ (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} = a \left(\frac{1}{R}\right) = \left[ 1 - \alpha \frac{\left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}}{1 - \left(\frac{1}{R} + \frac{\alpha}{\gamma}\right)^{-1}} \right] = (1 - \gamma) \]

With \( \gamma = 1 \), the government pays back all of the surprise deficit \( s_{t+1} \), and there are no shocks to the expected sum of future surpluses. The parameter \( \gamma < 1 \) allows us to model a surplus process in which the government promises to pay back part of the deficit rather than all of the deficit, letting unexpected inflation soak up the rest by devaluing outstanding bonds. Intuitively, with \( \gamma < 1 \), (7.35) describes what the value of the debt would be if someone else came along with a surplus that pays \((\gamma^{-1} - 1)\alpha \) of the debt.

We can also regard the \( v^* \) term in (7.34) and (7.35) as a way to express the distinction between fiscal responses to on and off equilibrium price levels from Section 4.4.2. This pair of equations says that the surplus responds to what the value of the debt would be without inflation, i.e. to debts accumulated from past deficits or from changes in the ex-ante real interest rate \( r_t = i_t - E_t \pi_{t+1} \). However, fiscal policy ignores changes...
in the value of debt that arise from unexpected and especially multiple-equilibrium inflation and deflation. This specification gives us a fiscal policy that is “active,” and rules out multiple equilibria, but nonetheless pays off debts accumulated from past deficits, as the government promised to do, implicitly or explicitly, when it sold debt.

Like the rest of the model, this surplus process can and should still be generalized towards realism in many ways. In particular, the $v^*$ process can respond to one particular value of unexpected inflation, rather than the strict zero-inflation target here. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks.

If one wished a model that could be solved in standard form, and fully incorporates the philosophy of Section 4.4.2, one could also write monetary policy in a parallel form, distinguishing on and off equilibrium responses. Since my purpose is just to construct a fiscal theory model, I leave out that complication.

### 7.7.1 Responses

I solve this model with the same matrix technique as in section 7.3. The detailed algebra including huge matrices is in the Appendix to Cochrane (2019).

The top panel of Figure 7.9 presents the response of this model to an AR(1) fiscal policy disturbance $u^t_s$, in the case of no policy responses to endogenous variables $\theta = 0$. The interest rate $i$ and long-term bond return $r^n$ do not respond in this case. Inflation rises and decays with an AR(1) pattern, not the one-period price-level jump that the frictionless model produces in this case. Output rises mirroring the path of inflation, following the forward-looking Phillips curve that output is high when inflation is greater than future inflation.

We can see here several aspects of the surplus process (7.34) - (7.35) at work. The surplus $s_t$ and the AR(1) surplus disturbance $u^t_s$ are not the same. The surplus initially declines, but these deficits raise the $v^*$ latent variable, which accumulates past surpluses. A long string of small positive surplus responses on the right side of the graph then partially repays the incurred debt.

This graph warns us of the empirical challenges ahead, and against many apparently easy rejections of fiscal theory. It would be hard to distinguish the surplus $s$ from the disturbance $u^s$ in the data, as they differ only in the long run. The surplus seems
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Figure 7.10 plots responses to the same fiscal and monetary shocks, but now adds endogenous policy responses, modifying (7.33)-(7.34) to

\begin{align}
  i_t &= 0.5 \pi_t + 1.0 x_t + u_t^i \\
  s_t &= 0 \pi_t + 0.5 x_t + 0.2 v_t^s + u_t^s.
\end{align}

(7.41)  
(7.42)

I choose the coefficients to make the graphs clear, not in an attempt at realism.

In the bottom monetary policy response of Figure 7.10 we see that the interest rate \( i \) is no longer the same as the disturbance \( u^i \). The positive response of the interest rate target to inflation and output, and the decline in inflation and output after the shock, pull down the actual interest rate from its disturbance. I held down the coefficient \( \theta_{\pi} = 0.5 \), rather than a more traditional larger value, to keep the interest rate response from being negative, the opposite of the shock. Interest rates that go in the opposite direction from monetary policy shocks are a common feature in new–Keynesian models of this sort. (Cochrane (2018) p. 175 shows some examples.) But they are confusing, and my point here is to illustrate mechanisms. The interest rate then rises gradually, along with inflation, before settling down long past the right end shown in the figure. Long-term bonds again suffer a negative return on impact, as the yield rises, and then follow the interest rate as before.

The surplus behaves quite differently than before. In Figure 7.9 the surplus rises, reacting only to the larger value of debt coming from a positive expected return. Now, the output decline following the monetary policy shock leads to a large deficit. The surplus eventually rises to partially, but not totally, pay off that debt.

Just how one defines and orthogonalizes monetary and fiscal policy is a subtle matter. Here I define a monetary policy shock that does not affect the fiscal shock \( u^t \). But monetary policy nonetheless has fiscal consequences: The fiscal rule responds to output, (potentially) to inflation, and to real-interest-rate-induced rises in the value of debt. This is not “passive” fiscal policy in the traditional definition, since it does not respond to multiple-equilibrium unexpected-inflation induced variation in the value of the debt. But nonetheless these are sensible fiscal consequences of monetary policy which an analysis of “what if we raise interest rates and the Treasury behaves as normal” should – and, the central point here, can – include. There is no right or wrong here, there is only more or less interesting. Different definitions and orthogonalizations just ask different policy questions.
The top panel of Figure 7.10 presents the response to a fiscal shock, holding constant the monetary policy disturbance $u_t$ but now allowing a monetary policy rule. The fiscal shock causes inflation and output to rise. Monetary policy raises interest rates persistently in response to the inflation and output rise, and greater output gives larger fiscal surpluses. Higher nominal interest rates also occasions a fall in bond prices, which soaks up some of the fiscal shock.

This is not a realistic example, in part because of the form of the IS and Phillips curves. Fiscal shocks in the 1970s appear stagflationary, lowering output. The point is the question, not the answer: endogenous fiscal and monetary policy responses modify the economy’s response to all shocks in important ways, in these models as in standard models.
Figure 7.9: Responses of the sticky-price long-term debt model, with no policy responses, to AR(1) surplus shock (top panel) and AR(1) monetary policy shock (bottom panel). Parameters are $\theta = 0$, $\omega = 0.7$, $\gamma = 0.8$, $\sigma = 0.5$, $\kappa = 0.5$, $\rho^i = 0.9$, $\rho^s = 0.7$, $\alpha = 0.2$. 
Figure 7.10: Response to a fiscal (top) and monetary (bottom) policy shock in the sticky-price long-term debt model, with endogenous policy responses. Parameters add $\theta_{ix} = 1$, $\theta_{i\pi} = 0.5$, $\theta_{sx} = 0.5$, $\theta_{s\pi} = 0$. 
7.8 Continuous time

I introduce the model with sticky prices in continuous time.

It is useful to express the model in continuous time. Continuous time avoids or forces us to clarify timing conventions, leading to simpler formulas. Continuous time techniques are becoming more common. Continuous time also lets us – forces us – to think more carefully about which variables can and can’t jump. The price level jumps of the frictionless model are unattractive. Do we need them? The answer turns out to be no, a major point of this section. But just what that means needs to be shown. (The model in this section and the following is drawn from Cochrane (2017d), which expands on the model in Sims (2011). The appendix to the former has a more detailed derivation.)

I start with the continuous-time equivalent of the standard IS and Phillips curve model, with only instantaneous (short-term) debt.

\[
\begin{align*}
    dx_t &= \sigma(i_t - \pi_t) dt + d\delta_{xt} \quad (7.43) \\
    d\pi_t &= (\rho\pi_t - \kappa x_t) dt + d\delta_{\pi t} \quad (7.44) \\
    dp_t &= \pi_t dt \quad (7.45) \\
    db_t &= [b(i_t - \pi_t) + rb_t - s_t] dt \quad (7.46) \\
    di_t &= d\varepsilon_{mt} \quad (7.47) \\
    ds_t &= d\varepsilon_{st} \quad (7.48)
\end{align*}
\]

Here \(dx_t = x_{t+\Delta} - x_t\) is the forward-differential operator used in continuous time with either diffusion or jump shocks. Equations (7.43) and (7.44) are the continuous-time equivalents of the IS and Phillips curves (7.1) and (7.2). The \(d\delta_t\) shocks are expectational shocks. Equation (7.43), for example, is the consumer’s first order condition and usually reads \(E_t dx_t = \sigma(i_t - \pi_t) dt\).

The quantity \(d\delta_t\) is the difference between actual and unexpected growth, which this equation does not pin down. The \(d\varepsilon_t\) shocks are structural shocks. Both \(d\delta\) and \(d\varepsilon\) may be jumps or diffusions. In this section I only study responses to “MIT shocks,” one-time unexpected shocks \(d\delta_0\) and \(d\varepsilon_0\) at time 0, and perfect foresight thereafter.

Letters without subscripts \(b\) and \(r\) are steady-state values. Each of these equations is linearized.
Equation (7.45) stresses that though inflation may jump, the price level must be continuous. If a fraction $\lambda dt$ of producers changes prices each instant $dt$, the price level cannot jump. It seemed that one-period debt models relied a lot on the idea that a jump in the price level devalues nominal debt. That mechanism is ruled out, and our first task will be to see what takes its place. (Preview: discount rates.) Equation (7.46) tracks the evolution of the real market value of government debt. Debt grows with the real interest rate, and declines with primary surpluses.

I start here with very simple and unrealistic policy settings. At time 0, the government announces a new path for interest rates and the primary surplus. Section 7.10.2 discusses each equation in more detail.

### 7.9 An analytic solution

I solve the simplest model analytically. We see that the price level is continuous but the inflation rate jumps. The present value relation selects equilibria. The present value relation holds from discount rate variation, not a price level jump. However, the frictionless limit of discount rate variation to a price level jump is smooth.

In this section, I solve this simplest version of the model, with short-term debt and no policy rules, analytically. I pursue the continuous-time equivalent of the analysis of Section 7.1.1. As in that section, this analytical solution is useful to understanding how the model works. Most of all, this solution shows how the continuous time model can works with no price level jumps, and how it smoothly approaches a frictionless solution that does have a time-0 price level jump.

I specialize to short-term debt, and no policy rules. Again, I solve the model with perfect foresight for $t > 0$ after one unexpected initial jump $(d\delta_0, d\xi_0)$ at time 0. In the perfect foresight region, we solve

$$
\frac{dx_t}{dt} = \sigma(i_t - \pi_t) \tag{7.49}
$$

$$
\frac{d\pi_t}{dt} = \rho \pi_t - \kappa x_t \tag{7.50}
$$

$$
\frac{db_t}{dt} = [b(i_t - \pi_t) + rb_t - s_t]. \tag{7.51}
$$

The debt equation provides one extra explosive root in place of the usual policy rule with $\phi > 1$. 

The solutions to the pair (7.49)-(7.50) are

\[ \pi_t = C_0 e^{-\lambda_2 t} + \frac{\kappa \sigma}{\lambda_1 + \lambda_2} \left[ \int_{\tau=0}^{\tau=t} e^{-\lambda_2 \tau} i_{t-\tau} d\tau + \int_{\tau=0}^{\infty} e^{-\lambda_1 \tau} i_{t+\tau} d\tau \right] \]  

(7.52)

where

\[ \lambda_1 \equiv \frac{\rho + \sqrt{\rho^2 + 4 \kappa \sigma}}{2}; \quad \lambda_2 \equiv \frac{-\rho + \sqrt{\rho^2 + 4 \kappa \sigma}}{2} \]

and \( C_0 \) is an arbitrary constant. (Algebra below.) As in Section 7.1.1, equilibrium inflation is a two-sided average of equilibrium interest rates, plus an exponentially decaying transient. There is a family of stable solutions, indexed by \( C \), or equivalently by the initial value \( \pi_0 \),

\[ \pi_0 = C + \frac{\kappa \sigma}{\lambda_1 + \lambda_2} \int_{\tau=0}^{\infty} e^{-\lambda_1 \tau} i_{t+\tau} d\tau. \]

We use the fiscal theory to pick the initial condition \( \pi_0 \) or \( C \). The similar solution to (7.51) at time 0 is

\[ b_0 = \int_{\tau=0}^{\infty} e^{-\rho \tau} \left[ s_{t+\tau} - (i_t - \pi_t) \right] d\tau. \]  

(7.53)

This is our usual linearized present value formula. The real value of nominal debt is the present value of surpluses, discounted at the real interest rate. We substitute (7.52) into (7.53) to solve for the initial \( \pi_0 \).

Before solving an example, reflect on what we are doing, which is really the whole point of this exercise. In the frictionless case, (7.52) becomes \( \pi_t = i_t \), (7.53) becomes \( b_0 = \int_{\tau=0}^{\infty} e^{-\rho \tau} s_t d\tau \), so we must have a price level jump at time 0 to accommodate the latter. The denominator of \( b_0 = \log(B_0/P_0) \) jumps with \( B_0 \) predetermined.

With price stickiness, at least in this model of price stickiness, we no longer have price level jumps. An inflation jump at time \( t = 0 \) is sufficient. How can an inflation jump restore present value balance? Each of the possible inflation paths in (7.52) implies a different path of real rates in (7.53). The discount rate path on the right hand side of (7.53), not the initial price level on the left hand side of that equation, adjusts to pick the initial inflation rate.

So, from a present value point of view, a period of inflation implies low real interest rates. The low real rates raise the discount factor for government debt, just offsetting the reduced surpluses, so the present value at time 0 \( b_0 \) need not change.
From the bondholder’s point of view, an unexpected price level jump can devalue nominal debt. In the sticky price model, a protracted period of low real interest rates devalues nominal debt instead. This is an important point in judging the realism of fiscal theory:

- **With sticky prices, a fiscal shock leads to a protracted inflation, and a protracted period of low real interest rates. The low real interest rates slowly devalue even instantaneous debt.**

In discrete time with price stickiness, both discount rate and price level jump effects were present, and we weighed their importance. You see here an advantage of continuous time. It forces us to recognize that in fact the discount rate effect, the smooth rise in inflation over multiple periods, is the heart of the adjustment. Fiscal theory really does not essentially rely on price level jumps to devalue outstanding debt. (Since the real rate is high for a discrete amount of time, one could even have a continuous inflation path.)

To work out a simple example, let $i_t$ and $s_t$ each be constants. I.e., we will have a permanent “monetary policy” shock from 0 to $i$ at time 0, and a permanent “fiscal policy” shock from 0 to $s$ at time 0. (You could work out AR(1) just as easily.) Then (7.52) and (7.53) become

$$
\pi_t = (\pi_0 - i) e^{-\lambda_2 t} + i
$$

$$
b_0 = \frac{s}{r} + (\pi_0 - i) \frac{1}{r + \lambda_2}.
$$

With no price level jumps, $b_0$ is predetermined, so we solve the second equation for $\pi_0$ and then we have the unique path for inflation. The solution is

$$
\pi_t = (r + \lambda_2) \left( b_0 - \frac{s}{r} \right) e^{-\lambda_2 t} + i. \quad (7.54)
$$

If $b_0 = s/r$, then the response to the interest rate change is perfectly Fisherian, $\pi_t = i$ immediately, despite price stickiness. We have seen this result in discrete time as well. It’s much prettier now, as there is no implied price level jump. With $b_0 = s/r$, discounting surpluses at $r$ already works so we do not need any additional discounting due to real rate variation to make the fiscal present value hold.

The response to a fiscal shock, a decrease in $s$ below $s = b_0/r$, with no interest rate change, results in a transitory rise in inflation, but no price level jump. The *discount rate* for government debt makes the present value relation hold at time 0, and adapt to the fiscal shortfall.
One may worry about a model that has no price level jump for finite price stickiness, but requires a price level jump for the frictionless case. In fact, the frictionless limit is well behaved. As \( \kappa \to \infty \), \( \lambda_2 \to \infty \). The inflation path (7.54) has a larger and larger jump in inflation, but one that lasts a shorter and shorter time. The price level path smoothly approaches the jump of the truly frictionless model. The cumulative inflation is

\[
\int_{t=0}^{\infty} \pi_t dt = (r + \lambda_2) \left( b_0 - \frac{s}{r} \right) \int_{t=0}^{\infty} e^{-\lambda_2 t} dt = \left( \frac{r}{\lambda_2} + 1 \right) \left( b_0 - \frac{s}{r} \right)
\]

so

\[
\lim_{\kappa \to \infty} \pi_t dt = b_0 - \frac{s}{r},
\]

exactly the size of the price-level jump of the frictionless model.

In reality one does not solve the model this way, solving forward one or groups of equations at a time. One uses matrix methods on the system (7.49)-(7.51), solving the unstable roots of the whole system forward and the stable roots backwards, as detailed in Section 7.12. One ends up at the same solution of course, but not an analytic expression. This analytic excursion helps only to understand how the fiscal theory adapts in continuous time, and can operate with inflation jumps and no price level jumps.

### 7.9.1 Algebra

Differentiating (7.50) and using (7.49) to eliminate \( x_t \),

\[
\frac{d^2 \pi_t}{dt^2} - \rho \frac{d \pi_t}{dt} - \kappa \sigma \pi_t = -\kappa \sigma i_t.
\]

To solve this differential equation, express it as

\[
(D - \lambda_1)(D + \lambda_2)\pi_t = -\kappa \sigma i_t; \quad D \equiv d/dt.
\]

with

\[
\lambda_1 = \frac{\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}; \quad \lambda_2 = \frac{-\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}.
\]

(Note, \( \lambda_1 \lambda_2 = \kappa \sigma \), and \( \lambda_1 - \lambda_2 = \rho \) come in handy.) Now solve it as

\[
\pi_t = -\frac{1}{(D - \lambda_1)(D + \lambda_2)} \kappa \sigma i_t = -\frac{1}{\lambda_1 + \lambda_2} \left[ \frac{1}{(D - \lambda_1)} - \frac{1}{(D + \lambda_2)} \right] \kappa \sigma i_t. \quad (7.55)
\]
Note if 
\[(D - a)y_t = z_t,\]
i.e. 
\[\frac{dy_t}{dt} = ay_t + z_t,\]
then we solve forward, and the stationary solution is 
\[y_t = -\int_{\tau=0}^{\infty} e^{-\alpha \tau} z_{t+\tau} d\tau\]
If, on the other hand 
\[(D + b)y_t = z_t,\]
then we solve backward, and the stationary solution is 
\[y_t = Ce^{-bt} + \int_{\tau=0}^{t} e^{-br} z_{t-\tau} d\tau\]
The solution to (7.55), and thus to the pair (7.49)-(7.50), is the sum of the last two integral expressions.

## 7.10 Long-term debt and an interest rate policy

add long-term debt and a monetary policy rule to the model.

Next, I add two more important elements, long-term bonds and a monetary policy. Long-term bonds are the key to obtaining a negative response of inflation to an interest rate increase. A monetary policy reaction is an important realism, and I include it to emphasize that one can include monetary policy rules, and solve completely standard new-Keynesian model including policy rule under fiscal theory. The model is a continuous time expression of the model in (7.24)-(7.28):

\[
\begin{align*}
    dx_t &= \sigma (i_t - \pi_t) dt + d\delta_{xt} \\
    d\pi_t &= (\rho \pi_t - \kappa x_t) dt + d\delta_{\pi t} \\
    dp_t &= \pi_t dt \\
    dy_t &= r (y_t - i_t) dt + d\delta_{yt} \\
    db_t &= [b (i_t - \pi_t) + rb_t - s_t] dt - \frac{b}{r} d\delta_{yt} \\
    di_t &= -\rho_t (i_t - \phi_n \pi_t - \phi_x x_t) dt + d\varepsilon_{mt} \\
    ds_t &= d\varepsilon_{st}
\end{align*}
\]
The model now has long-term debt, consisting of nominal perpetuities. The perpetuity yield is $y_t$, and the price of perpetuities is $1/y_t$. The real value of nominal perpetuities is $b_t$. Equation (7.56) is the term structure of interest rates. Solved forward, it says that the perpetuity yield is the forward-looking average of expected interest rates.

Equation (7.57) tracks the evolution of the real market value of government debt. Debt grows with the real interest rate, and declines with primary surpluses. A shock to yields $d_y$ is a negative shock to bond prices, and so appears as a negative shock to the real value of debt. The only difference between this perpetuity case and the instantaneous debt case in this equation is the appearance of this final term $b/r d_y$.

It should be straightforward to generalize the model to include both instantaneous debt and perpetual debt by keeping track of two $b$’s, or geometric or other maturity structures. This generalization also opens the door to maturity-management policies such as quantitative easing.

Equation (7.58) is a monetary policy rule. The parameter $\rho_t$ describes a partial-adjustment process, in which interest rates move slowly towards the policy rule. Equivalently, it effectively adds a lagged interest rate in the policy rule. It is in discrete time

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) (\phi_\pi \pi_t + \phi_x x_t) + \varepsilon_t.$$  

(7.60)

This formulation is important in continuous time. For example, recall the discrete time frictionless model with $i_t = \phi_\pi \pi_t$ and $i_t = E_t \pi_{t+1}$, so dynamics are $E_t \pi_{t+1} = \phi_\pi \pi_t$. In continuous time with differentiable prices, as here, $i_t = E_t \pi_{t+1}$ becomes just $i_t = \pi_t$, so this approach gives $\pi_t = \phi \pi_t$. OK, you can conclude $\pi_t = 0$ as the dynamics are infinitely fast, but that doesn’t make much sense. Instead, write $d i_t = -\rho_i (i_t - \phi_\pi \pi_t) dt$ together with $i_t = \pi_t$, we have $d \pi_t = -\rho_i (1 - \phi_\pi) \pi_t dt$ and thus $\pi_t = \pi_0 e^{-\rho_i (1 - \phi_\pi) t}$, a more sensible dynamics. This is a nice example of how continuous time helps to clarify ideas and distinguish economics from timing conventions.

One could, and to be reasonable should, add a persistent forcing shock,

$$d i_t = -\rho_i (i_t - \phi_\pi \pi_t - \phi_x x_t) dt + v_{mt},$$

$$d v_{mt} = -\rho_v v_{mt} + \delta \varepsilon_{mt}.$$  

and add a $v_t$ rather than $\varepsilon_t$ to (7.60). The two kinds of persistence are not the same.
Equation (7.59) allows a very simple fiscal policy shock. It grossly violates the point I've stressed many times, that near-term deficits should be balanced by long-term surpluses, and one should distinguish between on-and off equilibrium responses to debt. We'll generalize that later. It allows us to track what happens when there is bad fiscal news that the government will meet by inflation, not by borrowing, but it would not match time series of debts and deficits which mostly consist of the latter mechanism.

7.10.1 Response functions

I plot responses to monetary and fiscal shocks in continuous time, with long-term debt. The basic patterns are the same as in discrete time but prettier. The price level does not jump, but the model has a smooth frictionless limit that approaches the jump. Rather than a price level jump, the model gives a period of inflation or disinflation, greater than interest rate changes, that move the discount rate and hence present value of surpluses. As a result, either fiscal or monetary shocks give a drawn out period of inflation or disinflation to revalue government debt, in place of jumps. This pattern is much more realistic.

Here I compute responses to the full model (7.43)-(7.48), including long-term debt and policy responses. Mirroring the discrete-time treatment, I solve the model by writing it in standard form,

\[ dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t. \]

Solving the unstable eigenvalues forward we find \( d\delta_t \) in terms of \( d\varepsilon_t \), and then we have a standard autoregressive representation driven by the structural shocks \( d\varepsilon_t \).

Figure 7.11 and Figure 7.12 show the response to expected and unexpected permanent interest rate increase. They are not much different than the corresponding Figures 7.6 and Figure 7.3 for discrete time, only smoother since we have a value at every point, and since shocks are true jumps.

Figure 7.13 plots the price level response to the unexpected interest rate increase for a variety of price-stickiness parameters. The \( \kappa = 0.20 \) line plots the price level for the same parameters as the previous two graphs. The period of disinflation shown in Figure 7.12 results in the protracted price level decline, which recovers when the disinflation turns to inflation. Sensibly, as prices become stickier, as \( \kappa \) declines, the period of disinflation lasts longer.
As prices become less sticky, and $\kappa$ increases the price level response approaches downward jump followed by inflation shown in Figure 7.6. This is an important point. Here, and in other calculations, the fiscal theory of monetary policy has a smooth frictionless limit. As we will see, several standard models do not have this property - the models blow up as you remove price stickiness, though the frictionless limit point is well behaved.

More directly, the smooth frictionless limit means that the simple frictionless models do provide a useful approximation, a baseline from which one can start to think about monetary policy. The frictionless model generates a downward price level jump, followed by inflation. The model with price stickiness gives a period a deflation followed by slowly emerging inflation – price stickiness just drags out (to reasonable values) the dynamics suggested by the stark frictionless model.

As in the simple model, there is no price level jump, and instead a period of inflation changes discount rates to satisfy (and be selected by) the fiscal theory. With perfect foresight so we don’t have to worry about where to put the expectations, the
In continuous time with this sticky-price model that precludes price-level jumps. As in the simple model, now (7.61) picks an inflation path, where the higher discount rate over a period of time lowers the right hand side of the present value just enough to match the effect of the lower bond price $Q_t$. Figure 7.6 shows that as prices get less sticky, the period of low inflation gets shorter and more dire, approaching a price level jump, even though the price level at time 0 never moves.

Figure 7.14 presents the response to a fiscal policy shock. With the interest rate held constant, this response is the same with or without long-term debt. It is nearly identical to the discrete time case, Figure 7.2, though again slightly prettier. Unless one can observe the event that changes fiscal expectations, a recession and disinflation seemingly come out of nowhere, as they often do.
The inflation and output responses to this fiscal shock look a lot like the inflation and output responses to the monetary shock of Figure 7.11. The same basic mechanism is at work. The monetary policy shock reduced the nominal value of government debt, the numerator on the left hand side of $Q_t B_t / P_t = EPV(s)$. The fiscal shock reduced the right hand side. The pressure on $P_t$ is the same.

Again, here inflation jumps down but the price level does not jump at time 0, unlike discrete time which combines the two effects. A graph of the price level is visually nearly identical to that of Figure 7.13. If there is a downward jump in expected surpluses on the right hand side of (7.61), and either with instantaneous debt or a fixed interest rate monetary policy so that $Q_t$ does not change, the discrete-time model generates a price level jump. The price level cannot jump in the sticky price model. Instead, we get a period of inflation higher than the interest rate, lowering the discount rate of government debt so that the present value is unchanged despite the change in surpluses. As we reduce price stickiness, the period of high inflation gets shorter and more dire, smoothly approaching the price level jump.
Figure 7.14: Response to a fiscal policy shock, with no change in interest rate. Continuous time model with or without long-term debt, parameters $r = 0.05, \kappa = 0.20, \sigma = 0.5$.

Though the response functions smoothly approach the frictionless value, the price level never actually jumps, so it may be better to use the sticky price intuition always. For example, take again the case of short term debt or a constant interest rate policy so $Q_t = 1$, and a fiscal shock. The jump intuition says the price level must jump to affect $x$ of a partial default. You can’t smoothly default on short term debt, as they instantly charge a higher interest rate to compensate for any announced default. With sticky prices, we can say instead that bondholders lose wealth through a protracted period of lower than expected real interest rates, but they never suffer an unexpected price level shock. The period of negative rates is unexpected - they suffer a rollover shock of poor investment opportunities.

Price level jumps, or substantial inflations in a period shorter than the roll-over frequency of government debt, are rare. Many historical inflations happen slowly, and so the fiscal theory based on unexpected inflation equals unexpected fiscal shock seems to have a lot of trouble matching those episodes.

Now we can describe a fiscal shock as setting off a protracted period of inflation,
with low nominal rates, that slowly devalues the value of government bond holdings. That intuition clarifies that with sticky prices the period of inflation can be quite drawn out, and last much longer than the maturity structure of the debt. That starts to sound like the 1970s. Fiscal shocks in the fiscal theory do not just describe unrealistic price level jump debt revaluations.

We started way back in Figure 7.2 in discrete time with one-period debt. A fiscal policy shock provoked a pure jump in the price level to suddenly devalue debt. When we added sticky prices in Figure 7.2 we thought of it as that downward jump, smoothed out in time. But the subsequent inflation, with nominal rates held constant, had a crucial effect – it led to further reductions (in the case of inflation) in the value of debt. Now, in Figure 7.2 the latter mechanism has completely taken over. There is no sudden reduction, default through inflation. There is only a drawn-out period of low (in the inflationary case) or high (here) real returns that slowly and predictably eat away or add to the value of debt. With sticky prices, a sudden fiscal shock can eat away at the value of government bonds for a long time.

An event such as graphed in Figure 7.2 or its inflationary opposite, might well flummox observers. Out of nowhere seems to come a long period of high or low real rates, an inflation or disinflation seemingly having nothing to do with monetary policy and nominal interest rates.

Clearly, adding together fiscal shocks, monetary policy responses $i_t = \phi_\pi \pi_t + \phi_x x_t$ or equivalent disturbances $i_t = v_i^t$, and the expected/unexpected distinction, we can produce quite interesting inflation and output dynamics.

### 7.10.2 Model details

I derive the continuous-time model equations, with focus on the evolution of the yield of long-term bonds and the market value of those bonds.

Equations (7.44) and (7.45) are the continuous time version of the new-Keynesian Phillips curve. If we integrate forward to

$$\pi_t = -\kappa E_t \int_{s=0}^{\infty} e^{-\rho s} x_{t+s} ds$$

the analogy to the discrete time version (7.4) is clearer. Inflation is high if there are low future output gaps. As $\kappa \to \infty$, output variation becomes smaller for given inflation rate variation, so this is the frictionless limit.
CHAPTER 7. STICKY PRICES

Equation (7.45) clarifies something left hanging in the discrete-time formulation: inflation can jump, but the price level cannot jump (or have a diffusion component). In the underlying model, a fraction $\lambda \Delta$ of firms can change prices in time interval $\Delta$. As $\Delta$ shrinks, the price level must be continuous. The inflation rate can jump.

Equation (7.43) is the consumer’s first order conditions in continuous time, linearized, avoiding risk premiums, and using the absence of price level jumps. Again it is easiest to see the analogy to (7.24) by integrating forward, and writing

$$x_t = -\sigma E_t \int_{s=0}^{\infty} (i_{t+s} - \pi_{t+s}) ds.$$

Equation (7.50) is the term structure relation between long and short rates. It expresses the condition that the expected return on long-term bonds should be the same as the short term interest rate. Government debt is all perpetuities. The perpetuity has nominal yield $y_t$, nominal price $Q_t = 1/y_t$ and pays a constant coupon $1dt$. Equation (7.57) is the continuous time flow condition. The quantity

$$b_t \equiv \frac{Q_t B_t}{P_t}$$

is the real market value of government debt. The common $d\delta_{yt}$ term tells us that shocks to asset prices also shock the market value of government debt.

Our first step on the way to (7.56) - (7.57) is to derive their nonlinear versions,

$$dQ_t = Q_t (i_t - y_t) dt + Q_t d\delta_{Qt} \tag{7.62}$$

$$db_t = [b_t (i_t - \pi_t) - s_t] dt + b_t d\delta_{Qt}. \tag{7.63}$$

Equation (7.62) stems from the condition that the expected nominal perpetuity return should equal the riskfree nominal rate. The perpetuity pays $1dt$ coupon, so

$$i_t dt = \frac{1dt + E_t dQ_t}{Q_t}$$

and introducing an expectational error,

$$\frac{E_t d (Q_t)}{Q_t} = (i_t - y_t) dt$$

and

$$dQ_t = (i_t - y_t) dt + \delta_{Qt} \tag{7.64}$$
To derive (7.63), start by differentiating $b_t$:

$$
\frac{db_t}{dt} = \frac{d}{dt} \left( \frac{Q_t B_t}{P_t} \right) = \frac{Q_t}{P_t} dB_t + b_t \frac{dQ_t}{Q_t} - b_t dp_t,
$$

(7.65)

where $p_t = \log P_t$. In the last term I use the fact that there are no price-level jumps or diffusions. Now, to evaluate $dB_t$ in this equation, use the flow condition that the government must sell new perpetuities at price $Q_t$ to cover the difference between coupon payments $\$1 \times B_t$ and primary surpluses $s_t$,

$$
\frac{Q_t}{P_t} dB_t = \frac{B_t}{P_t} dt - s_t dt.
$$

(7.66)

Substituting (7.66) into (7.65), with $\pi_t dt = dp_t$, we obtain

$$
\frac{db_t}{dt} = \left[ (y_t - \pi_t) b_t - s_t \right] dt + b_t \frac{dQ_t}{Q_t}.
$$

Substituting from (7.64), we obtain (7.57).

Our next step is to linearize (7.62)-(7.63) to obtain (7.56)-(7.57), We linearize around a steady state with $\pi = 0$ and hence $i = r = y$.

$$
dQ_t = Q_t (i_t - y_t) dt + Q_t d\delta Q_t
$$

$$
d(1/y_t) = \frac{1}{y_t} (i_t - y_t) dt + \frac{1}{y_t} d\delta Q_t
$$

$$
-\frac{1}{y^2} d\tilde{y}_t \approx \frac{1}{y} (\tilde{i}_t - \tilde{y}_t) dt + \frac{1}{y} d\delta Q_t
$$

$$
d\tilde{y}_t \approx r (\tilde{y}_t - \tilde{i}_t) dt - r d\delta Q_t.
$$

Define

$$
d\delta y_t \equiv -r d\delta Q_t.
$$

Dropping the tildes and the approximation sign, we have (7.56)

$$
dy_t = r (y_t - i_t) dt + d\delta y_t.
$$

Next, from (7.63),

$$
\frac{db_t}{dt} = [b_t (i_t - \pi_t) - s_t] dt + b_t d\delta Q_t,
$$

we linearize,

$$
\frac{d\tilde{b}_t}{dt} \approx \left[ r \tilde{b}_t + b(\tilde{i}_t - \tilde{\pi}_t) - \tilde{s}_t \right] dt - \frac{b}{r} d\delta y_t
$$

and dropping tildes and approximation sign we have (7.57).
7.11  Sims’ model

I add habit persistence in consumption, a policy rule that reacts to inflation and output, surpluses that react to output growth. The result is more realistic impulse-response functions.

Clearly, this effort needs to expand to a full, serious, calibrated/estimated model that attempts to match impulse-responses from the data — including yet-to-be estimated response functions that try to isolate fiscal shocks or to orthogonalize monetary and fiscal policy shocks. Sims (2011) is an important step in that direction.

Sims’ model is, in my notation, and after linearization

\[
\begin{align*}
    d\pi_t &= (\rho \pi_t - \kappa c_t) dt + d\delta_{\pi t} \tag{7.67} \\
    dy_t &= r(y_t - i_t) dt + d\delta_{yt} \tag{7.68} \\
    ds_t &= \omega \dot{x}_t dt + d\varepsilon_{st} \tag{7.69} \\
    db_t &= [b(\tilde{i}_t - \pi_t) + rb_t - s_t] dt - \frac{b}{r} d\delta_{yt} \tag{7.70} \\
    d\lambda_t &= -(i_t - \pi_t) dt + d\delta_{\lambda t} \tag{7.71} \\
    dx_t &= \dot{x}_t dt \tag{7.72} \\
    d\dot{x}_t &= [\psi \lambda_t + \sigma \psi x_t + r\dot{x}_t] dt + d\delta_{\dot{x}t}. \tag{7.73}
\end{align*}
\]

Equation (7.67) is a policy rule, now featuring responses to inflation, output, and output growth. Sims specifies that the policy rule reacts to output gap growth.

\[di_t = -\rho_t (i_t - \phi_t \pi_t - \phi_x x_t) dt + d\varepsilon_{mt}\]

I use a more conventional response to the output gap itself. Equation (7.68) is the Phillips curve. Equation (7.69) describes the perpetuity yield. Fiscal policy (7.70) now responds to output growth. As we saw, surpluses are higher in expansions and lower (deficits) in recessions. This is a conceptually important extension. No, fiscal surpluses do not have to be “exogenous,” and this feature gives us feedback from monetary to fiscal policy. Equation (7.71) is the fiscal flow condition with long term debt, as before.

I use \(\phi < 1\) as this is a fiscal theory model, and otherwise solutions are explosive. The on-equilibrium monetary-policy (not equilibrium-selection policy) parameter \(\phi\) can be measured in fiscal theory, so regression evidence is relevant. But the evidence for \(\phi > 1\) in the data, such as Clarida, Galí, and Gertler (2000) is tenuous, needing specific lags, instruments, and a sample period. Also, with more complex specifications, one can create a passive-money model in which regressions of interest rates
on inflation have a coefficient greater than one (Cochrane (2011c)). But, as with
the other parameters of the model, I leave estimation along with detailed realistic
specifications for another day.

The last three equations are the main novelty. They generalize preferences to
include a sort of habit. Preferences include a cost of quickly adjusting consumption.
Equation (7.72) describes the evolution of the marginal utility of wealth. But now
it is connected to output via (7.73) and (7.74). The appendix to Cochrane (2017d)
contains a derivation. A term of this sort is a common ingredient to generate hump-
shaped dynamics in this sort of model. (I use $\psi$ in place of Sims’ $1/\psi$ to make the
equation prettier.)

Figure 7.15: Response to an unexpected monetary policy shock in the modified Sims
model with habit persistence in consumption.

Figure 7.15 and Figure 7.16 present responses to an unexpected monetary policy
shock and to a fiscal shock respectively in this model. You can see similar qualitative
lessons of previous graphs, but with pretty dynamics especially in output. The
monetary policy shock leads to a nice hump-shaped output response. The fiscal
shock leads to a recession with disinflation, along with an endogenous interest rate
movement. The Fed lowers interest rates to fight the recession, and in this model
that does bring inflation up over what it would otherwise be, reducing the output
This sort of response function starts to look very much like what comes out of standard new-Keynesian model building exercises. The point is not a dramatically new qualitative lesson but rather to show that one can quickly and productively solve new-Keynesian models with fiscal theory foundations, and obtain results that are interesting, plausible, and potentially novel.

### 7.12 Continuous time model solutions

The continuous-time linear models are in the form

\[ dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t \]

where \( d\varepsilon_t \) are structural shocks and \( d\delta_t \) are expectational errors. We find the expectational errors in terms of the structural shocks, and then find an autoregressive and then a moving average representation for the equilibrium \( x_t \).
7.12. CONTINUOUS TIME MODEL SOLUTIONS

Eigenvalue decomposing the transition matrix $A$,

$$A = QΛQ^{-1}$$

Defining $\tilde{z}_t \equiv Q^{-1}z_t$,

$$d\tilde{z}_t = Λ\tilde{z}_t dt + Q^{-1}Bdξ_t + Q^{-1}Cdδ_t \quad (7.75)$$

I offer two notations for the answer. First, defining by a $+$ and $-$ subscript rows corresponding to explosive eigenvalues and stable eigenvalues (real part greater than zero or less than or equal to zero), we have

$$\tilde{z}_{+t} = 0,$$

an autoregressive representation

$$d\tilde{z}_{-t} = Λ_{-t}\tilde{z}_{-t} dt + Q_{-1}^{-1} \left[ I - C \left[ Q_{+1}^{-1}C \right]^{-1} Q_{+1}^{-1} \right] Bdξ_t,$$

and a moving average representation

$$\tilde{z}_{-t} = e^{Λ_{-t}\tilde{z}_0} + \int_{s=0}^{t} e^{Λ_{-s}Q_{-1}^{-1} \left[ I - C \left[ Q_{+1}^{-1}C \right]^{-1} Q_{+1}^{-1} \right]} Bdξ_{t-s}.$$ 

Reassembling $\tilde{z}_t$ and with $z_t = Q\tilde{z}_t$ we have the solution.

Second, defining matrices $P$ and $M$ that select rows of $Q^{-1}$ corresponding to explosive and non-explosive eigenvalues, we can express the whole operation as an autoregressive representation

$$d\tilde{z}_t = Λ^*\tilde{z}_t dt + M'MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bdξ_t,$$

and moving average representation,

$$\tilde{z}_t = e^{Λ^*t}\tilde{z}_0 + \int_{s=0}^{t} e^{Λ^*s}M'MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bdξ_{t-s}$$

where

$$Λ^* \equiv M'MΛM'M.$$

The linear models we study can all be written in the form

$$dz_t = Az_t dt + Bdξ_t + Cdδ_t.$$
where \( d\varepsilon_t \) are structural shocks and \( d\delta_t \) are expectational errors. Eigenvalue decomposing the transition matrix \( A \),

\[
A = QAQ^{-1}
\]

we can premultiply by \( Q^{-1} \) and defining \( \tilde{z}_t \equiv Q^{-1}z_t \) we have

\[
d\tilde{z}_t = \Lambda\tilde{z}_t dt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t
\]

The goal of this section is an autoregressive and then a moving average representation for \( \tilde{z}_t \) and consequently \( z_t = Q\tilde{z}_t \).

**Selection notation**

We partition the system into the rows with explosive (real part greater than zero) eigenvalues and the rows with stable (real part less than or equal to zero) eigenvalues. Let \( Q^+_{+} \), \( \tilde{z}^+_{+} \) denote the rows of these matrices corresponding to explosive eigenvalues, and \( \Lambda_{+} \) the diagonal matrix with positive eigenvalues. Then, the explosive eigenvalues obey

\[
d\tilde{z}^+_{+} = \Lambda^+_{+}\tilde{z}^+_{+} dt + Q^+_{+}^{-1}Bd\varepsilon_t + Q^+_{+}^{-1}Cd\delta_t.
\]

To have \( E_t\tilde{z}^+_{+j} \) not explode, we must have

\[
\tilde{z}^+_{+} = 0
\]

and hence

\[
Q^+_{+}^{-1}Cd\delta_t = -Q^+_{+}^{-1}Bd\varepsilon_t
\]

\[
d\delta_t = -[Q^+_{+}^{-1}C]^{-1}Q^+_{+}^{-1}Bd\varepsilon_t.
\]

The explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there as are many explosive eigenvalues as there are expectational errors, i. e. \( [Q^+_{+}^{-1}C] \) is invertible.

The rows with stable eigenvalues then give us

\[
d\tilde{z}_{-} = \Lambda^-_{-}\tilde{z}_{-} dt + Q^-_{-}Bd\varepsilon_t + Q^-_{-}Cd\delta_t
\]

\[
d\tilde{z}_{-} = \Lambda^-_{-}\tilde{z}_{-} dt + Q^-_{-}Bd\varepsilon_t - Q^-_{-}C[Q^+_{+}^{-1}C]^{-1}Q^+_{+}^{-1}Bd\varepsilon_t
\]

\[
d\tilde{z}_{-} = \Lambda^-_{-}\tilde{z}_{-} dt + Q^-_{-}I - C[Q^+_{+}^{-1}C]^{-1}Q^+_{+}^{-1}Bd\varepsilon_t.
\]
This gives us an autoregressive representation for the \( \tilde{\varepsilon}_t \) with stable eigenvalues. Integrating, we have a moving average representation

\[
\tilde{\varepsilon}_{-t} = e^{\Lambda_{-t}} \tilde{\varepsilon}_{-0} + \int_{s=0}^{t} e^{\Lambda_{-s}} Q_{-1}^{-1} \left[ I - C \left[ Q_{+1}^{-1} C \right]^{-1} Q_{+1}^{-1} \right] B d\tilde{\varepsilon}_{t-s}.
\]

Note, \( e^{\Lambda t} \) is not an element by element power. In this case of a diagonal \( \Lambda \) matrix, its meaning is

\[
\begin{bmatrix}
  e^{\lambda_1 t} & 0 & 0 \\
  0 & e^{\lambda_2 t} & 0 & \cdots \\
  0 & 0 & e^{\lambda_3 t} & \cdots \\
  \vdots & & & \ddots
\end{bmatrix}.
\]

A computer program will typically interpret a statement such as \( \exp(\text{Lambda}^* t) \) as an element-by-element multiplication, putting ones in the diagonal that don’t belong. We reassemble \( \tilde{\varepsilon}_t \) from \( \tilde{\varepsilon}_{-t} \) and \( \tilde{\varepsilon}_{+t} = 0 \). Then, the original values are

\[
z_t = Q \tilde{z}_t.
\]

**A matrix expression**

The matrix carpentry of this solution may seem inelegant. At the cost of a bit of notation we can do the same thing with matrices and obtain somewhat more elegant formulas. To do this, let \( N_v \) denote the number of variables – \( A \) is \( N_v \times N_v \) –, let \( N_{\varepsilon} \) be the number structural shocks so \( B \) is \( N_v \times N_{\varepsilon} \), and let \( N_{\delta} \) be the number of expectational errors, so \( C \) is \( N_v \times N_{\delta} \). There are \( N_{\delta} \) explosive eigenvalues with positive real parts. Then let \( P \) be a \( N_{\delta} \times N_v \) matrix that selects rows of \( Q^{-1} \) corresponding to eigenvalues with positive real parts, and \( R \) an \( (N_v - N_{\delta}) \times N_v \) matrix that selects rows corresponding to eigenvalues with non-positive real parts. For example, if

\[
\Lambda = \begin{bmatrix}
  0.1 & 0 & 0 \\
  0 & -0.1 & 0 \\
  0 & 0 & 0.2
\end{bmatrix}
\]

then

\[
P = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & 0 & 1 \\
  0 & 1 & 0
\end{bmatrix}
\]

and

\[
M = \begin{bmatrix}
  0 & 1 & 0
\end{bmatrix}.
\]
selects the first and third row, and \( M \) selects the second row. In terms of the notation of the last section, \( Q^{-1}_+ = PQ^{-1} \), \( \tilde{z}_{t+1} = P\tilde{z}_t \), etc. The matrices \( P' \) and \( M' \) then put things back in the original rows, so \( P'P + M'M = I_{N_v} \). We start again from (7.75),

\[
d\tilde{z}_t = \Lambda\tilde{z}_tdt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t
\]

\[
Pd\tilde{z}_t = P\Lambda\tilde{z}_tdt + PQ^{-1}Bd\varepsilon_t + PQ^{-1}Cd\delta_t
\]

to have \( E_t\tilde{z}_{t+1} \) not explode, we must have

\[
P\tilde{z}_t = 0
\]

and hence

\[
PQ^{-1}Cd\delta_t = -PQ^{-1}Bd\varepsilon_t
\]

\[
d\delta_t = -[PQ^{-1}C]^{-1}PQ^{-1}Bd\varepsilon_t.
\]

Again, the explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there are as many explosive eigenvalues as there are expectational errors, i.e. \( PQ^{-1}C \) is invertible.

The rows with stable eigenvalues then give us from (7.75),

\[
Md\tilde{z}_t = MA\tilde{z}_tdt + MQ^{-1}Bd\varepsilon_t + MQ^{-1}Cd\delta_t
\]

\[
Md\tilde{z}_t = MA\tilde{z}_tdt + MQ^{-1}Bd\varepsilon_t - MQ^{-1}C[PQ^{-1}C]^{-1}PQ^{-1}Bd\varepsilon_t
\]

\[
dM\tilde{z}_t = MA(P'P + M'M)\tilde{z}_tdt + MQ^{-1}\left[I_{N_v} - C [PQ^{-1}C]^{-1}PQ^{-1}\right]Bd\varepsilon_t.
\]

With \( P\tilde{z}_t = 0 \),

\[
d(M\tilde{z}_t) = MA M'(M\tilde{z}_t)dt + MQ^{-1}\left[I_{N_v} - C [PQ^{-1}C]^{-1}PQ^{-1}\right]Bd\varepsilon_t
\]

We can reassemble the whole \( \tilde{z} \) vector with

\[
d\tilde{z} = (P'P + M'M)d\tilde{z}
\]

\[
d\tilde{z} = M'Md\tilde{z}
\]

\[
d\tilde{z}_t = \Lambda^*\tilde{z}_tdt + M'MQ^{-1}\left[I_{N_v} - C [PQ^{-1}C]^{-1}PQ^{-1}\right]Bd\varepsilon_t
\]

where

\[
\Lambda^* \equiv M'MAM'M
\]
is the $N_v \times N_v$ matrix of eigenvalues, with zeros in place of the explosive eigenvalues.

This is the autoregressive representation of $\tilde{z}$. The moving average representation, useful for impulse response functions, is

$$
\tilde{z}_t = e^{\lambda t} \tilde{z}_0 + \int_{s=0}^{t} e^{\lambda t} M' M Q^{-1} \left[ I_{N_v} - C \left[ PQ^{-1} \right]^{-1} PQ^{-1} \right] B d \xi_{t-s}
$$

Then, the original values are

$$
z_t = Q \tilde{z}_t.
$$

### 7.13 The future

These example brings us about to the frontier of quantitative fiscal-theory model building.

One hungers, of course, for a model that one can bring to data, estimate parameters, and formally match impulse-responses to structural and policy shocks. One wishes, in the end, at least a Smets and Wouters (2007), or Christiano, Eichenbaum, and Evans (2005), adapted to fiscal theory as I adapted the textbook new-Keynesian model above, and eventually one wants a more ambitious model incorporating the latest in financial frictions, zero bounds, and so forth. This chapter shows that one can construct such models, and quite easily from a technical standpoint. Any new-Keynesian model can be quickly merged to fiscal theory: Add (really, restore) the government debt equations, specify passive equilibrium policy by the central bank, and go. It explores many of the novel mechanisms that solving new-Keynesian models with fiscal theory produce. But I stop short of producing “the” model, one that fits the data.

Though computing solutions to fiscal theory of monetary policy models is easy, the challenges to finding the right model are large. My monetary policy rule is simplistic, needing at least lags and a zero bound, plus matching policy rule regressions in data. Estimating the fiscal policy rule is a challenge of similar order, not yet started, and made even more challenging by the fact that any sensible rule, such as this one, has subtle but crucial long-run responses, or a latent state variable. Then one must confront all the usual carpentry of DSGE models – the empirical troubles of the IS and Phillips curves, habits or other dynamic preferences, heterogeneity, the evident
variation in risk premia over the cycle, labor market and investment frictions, and so on.

Though the models are fundamentally unchanged, so the fit is likely to be the same, the impulse response functions one finds interesting will likely change. Moreover, the existing model are the result of a long specification search. If one takes a second round of specification search after changing the foundations to fiscal foundations, new results still are likely.
Chapter 8

Long-term debt dynamics

Long-term debt adds many wrinkles to the fiscal theory, and is important to understanding policy choices, episodes, and patterns in the data.

As we have already seen, long-term debt is also important to producing a fiscal theory of monetary policy that captures the common belief that higher interest rates reduce inflation, at least temporarily. Here I return to the frictionless model, and I explore long-term debt in greater detail. I start by analyzing forward guidance, promises of future interest rates. I then analyze how changes in the quantities of long-term debt can affect the path of inflation. The result is a unified theory of interest rate targets, forward guidance, quantitative easing, and fiscal stimulus, that can produce standard beliefs about the signs of these policies’ effects. I examine these effects in the totally frictionless constant real interest rate model. Pricing frictions should give output effects and more realistic dynamics, and will introduce interesting real interest rate and discount rate variation. Monetary frictions or liquidity effects of government bonds should add to those. But these are still waiting for investigation. As usual it is interesting to see how many effects don’t require frictions, though frictions can make dynamics more realistic.

The mechanism behind the effects is utterly different from standard models, however, as are some of the ancillary predictions. For example, this fiscal theory model predicts that persistent interest rate rises have stronger disinflationary effects than transitory ones, contrary to new-Keynesian model predictions. It predicts that anticipated interest rate rises have no disinflationary effects, and it ties the disinflationary effect to the maturity structure of outstanding debt. As always, the nature, presence or absence of fiscal policy (\(\{s_t\}\)) actions and reactions is also central to model
predictions, unlike in standard models.

The basic tools are simple. As a reminder, with long-term debt, the basic flow relation becomes (6.2),

\[ B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right), \]

and the basic present value relation becomes (6.3)

\[ \sum_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \]

We can also substitute in the constant real interest rate bond pricing formula (6.1),

\[ Q_t^{(t+j)} = E_t \left( \frac{1}{R^j} \frac{P_t}{P_{t+j}} \right), \]

to express the flow and present value relations between debt and price levels directly,

\[ \frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \left( \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{R^j} \right) E_t \left( \frac{1}{P_{t+j}} \right). \] (8.1)

\[ \sum_{j=0}^{\infty} \frac{B_t^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \] (8.2)

This is a useful step to understanding the relationship between debt quantities and the price level directly.

As with one-period debt, we seek to understand the evolution of the price level \{P_t\}. We study policies in which the government adjusts quantities of debt \( B_t^{(t+j)} \) and policies in which the government sets prices of debt via interest rate or bond price targets. What happens if there is news about surpluses \( (E_t - E_{t-1}) s_{t+j} \) with no change in debt? What happens if the government sells debt with no change in surpluses? And if we specify monetary policy via interest rate targets, now including long-term rates, how does the economy respond?
8.1 Forward guidance and bond price targets

Announcements of future interest rate changes can lower bond prices $Q_{t}^{(t+j)}$ and thus change the price level $P_{t}$ today. In this sense the model captures forward guidance.

An announcement whose horizon exceeds the maturity of all outstanding bonds has no effect on the price level at time $t$. In this sense, fully expected interest rate increases have no temporary disinflationary effect.

The central bank can also peg the yields of all bonds across the yield curve to obtain the desired inflation effect.

We have seen how a rise in an interest rate target can create higher expected inflation. With long-term debt we have seen how an unexpected interest rate rise produces a one-period disinflation. Here, we investigate forward guidance: If the central bank can credibly commit to higher or lower interest rates in the future, that announcement alone will change long-term bond prices, and it will have an immediate impact on inflation, even if it has no effect on the current short-term interest rate.

Figure 8.1 picks up where Figure 6.1 left off. Figure 8.1 plotted the effects of an interest rate rise. Figure 8.1 plots a “forward guidance” policy. At time 0, the government announces that interest rates will rise starting at time 3. This anticipated rise in interest rates induces long-term bond yields at time 0 to rise as indicated by “yields at $t=0$” (yields are plotted as a function of maturity, interest rates as a function of time).

The price level jumps down at time 0. However, the price level drop in Figure 8.1 is smaller than that in Figure 6.1 because fewer bonds change price, and those that do change price by a smaller amount.

- A given interest rate change in the form of forward guidance has less disinflationary effect than the same change immediately. The maturity structure of outstanding debt controls how quickly the effect of forward guidance falls with announcement horizon.

An announcement today of a future interest rate change only affects the value of debt whose maturity exceeds the time interval before rates change. Forward guidance eventually loses its power altogether once the guidance period exceeds the longest outstanding bond maturities.
Figure 8.1: Price level response to a forward guidance interest rate change. At time 0, the government announces that interest rates will rise at time 3. Long term debt with a geometric maturity structure is outstanding.

To see these points, suppose that at time 0, the government announces that interest rates will rise starting at time $T$ onward, and bonds of maturity up to $k$ are outstanding (30 years in the US). Now, inflation starts in period $T + 1$, and only bond prices of maturity $T + 1$ or greater are affected. In the present value relation

$$\frac{\sum_{j=0}^{T} Q_0^{(j)} B_{-1}^{(j)} + \sum_{j=T+1}^{k} Q_0^{(j)} B_{-1}^{(j)}}{P_0} = E_0 \sum_{j=0}^{\infty} \frac{s_j}{R^j},$$

(8.3)

only the second term in the numerator on the left hand side is affected by the forward guidance. Furthermore, for given interest rate rise, bond price declines in that second term are smaller: For a permanent rise from $r$ to $i$ starting at time $T$, the prices of bonds that mature at $j \leq T$ are unaffected, and the the prices of bonds that mature at $T + j$ are

$$Q_0^{(T+j)} = \frac{1}{(1 + r)^T (1 + i)^j} > \frac{1}{(1 + i)^{T+j}}.$$

If $T > k$, and forward guidance exceeds the longest outstanding maturity, the price level $P_0$ does not decline.
In Figure 8.1, the price level stays at the new lower level until the interest rate actually rises. On the date 3 that the interest rate rises there is no further price-level jump. Inflation then rises following the higher nominal rate.

- The negative response of the price level to higher interest rates happens when the interest rate rise is announced, not when the interest rate rise happens. Fully-expected interest-rate rises have no disinflationary effect.

The line labeled “expected” in Figure 6.1 emphasizes the latter point, plotting the inflation response to an interest rate rise announced before the oldest outstanding bond was sold. The model thus has some of the feel of rational-expectations models in which only unexpected monetary policy actions have effects, though all effects here are purely nominal.

Though the answer reflects some of what forward guidance advocates hope for, the inflationary or deflationary force of an interest rate change in this model has really nothing to do with the contemporaneous interest rate. There is no variation in real interest rates, no Phillips curve, no intertemporal substitution reacting to current or future interest rates, and so forth. The time-zero disinflation is entirely a “wealth effect” of aggregate demand stemming from the value of government debt.

The inflation effects here all result from the time-path of interest rates and long-term bond prices. The central bank could also implement the long-term bond prices directly, by offering to freely buy and sell long-term bonds at fixed prices, with no change in surpluses, in exactly the same way that offering to buy and sell short-term bonds at a fixed rate. (We analyze such operations in section 8.2.3.)

Thus we can read Figure 8.1 as the answer to a different question. Rather than promise (and, troublingly, try to commit to) the plotted path of future short-term rates, suppose the central bank at time 0 announces a full set of bond prices or the plotted yields as a function of maturity, and offers to buy and sell bonds of any maturity at those prices. By doing so, the central bank immediately creates the plotted yield curve, and obtains the plotted disinflation.

A bond price targeting policy has the advantage that it does not require promises about future behavior, and in particular promises that are not time-consistent and hence a bit hard to believe. The disadvantage is that the central bank has to trust the prediction that if it pegs long-term bond prices, it will not suffer immense demands.
8.1.1 Geometric maturity formulas

A geometric maturity structure $B^{(t+j)}_t = \Omega^j B_{t-1}$ in discrete time and $B^{(t+j)}_t = \omega e^{-\omega j} B_t$ in continuous time is analytically convenient. I present formulas for the examples in Figure 6.1 and Figure 8.1.

Geometric maturity structures are not passive. To maintain the geometric structure, the government must roll over debt, and gradually sell more debt of each coupon as its date approaches.

A geometric maturity structure $B^{(t+j)}_{t-1} = \Omega^j B_{t-1}$ is analytically convenient. A perpetuity is $\Omega = 1$, and one-period debt is $\Omega = 0$. I use these formulas in Figure 6.1 and Figure 8.1.

Suppose the interest rate $i_{t+j} = i$ is expected to last forever, and suppose surpluses are constant $s$. The bond price is then $Q^{(t+j)}_t = 1/(1+i)^j$. The valuation equation becomes

$$\sum_{j=0}^{\infty} \frac{Q^{(j)}_0 \Omega^j B_{-1}}{P_0} = \sum_{j=0}^{\infty} \frac{\Omega^j B_{-1}}{(1+i)^j P_0} = \frac{1+i}{1+i-\Omega} \frac{B_{-1}}{P_0} = \frac{1+r}{r}s. \quad (8.4)$$

Start at a steady state $B_{-1} = B$, $P_{-1} = P$, $i_{-1} = r$. In this steady state we have

$$\frac{1+r}{1+r-\Omega} \frac{B}{P} = \frac{1+r}{r}s. \quad (8.5)$$

Therefore, we can express (8.4) as

$$\frac{P_0}{P} = \frac{(1+i)(1+r-\Omega)}{(1+r)(1+i-\Omega)}. \quad (8.6)$$

These formulas are prettier in continuous time. The valuation equation is

$$\int_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_t}{P_t} dj = E_t \int_{j=0}^{\infty} e^{-rj}s_{t+j} dj.$$

With maturity structure $B^{(t+j)}_t = \omega e^{-\omega j} B_t$, and a constant interest rate $i_t = i$,

$$\omega \int_{j=0}^{\infty} e^{-ij} e^{-\omega j} dj \frac{B_t}{P_t} = \omega \frac{B_t}{i+\omega} \frac{P_t}{P_t} = \frac{s}{r}. \quad (8.7)$$
Here $\omega = 0$ is the perpetuity and $\omega = \infty$ is instantaneous debt. They are related by $\Omega = e^{-\omega}$. $B_t$ is predetermined. $P_t$ can jump.

Starting from the $i_t = r$, $t < 0$ steady state, if $i_0$ jumps to a new permanently higher value $i$, we now have

$$\frac{P_0}{P} = \frac{r + \omega}{i + \omega} \tag{8.8}$$

in place of (8.6).

In the case of one-period debt, $\Omega = 0$ or $\omega = \infty$, $P_0 = P$ and there is no downward jump. In the case of a perpetuity, $\Omega = 1$ or $\omega = 0$, (8.6) becomes

$$P_0 = \frac{1 + i}{1 + r} P. \tag{8.9}$$

and (8.8) becomes

$$P_0 = \frac{r}{i} P. \tag{8.10}$$

The price level $P_0$ jumps down as the interest rate rises, and proportionally to the interest rate rise.

This is potentially a large effect; a rise in interest rates from $r = 3\%$ to $i = 4\%$ occasions a $25\%$ price level drop. However, our governments maintain much shorter maturity structures, monetary policy changes in interest rates are not permanent, and they are often pre-announced, each factor reducing the size of the effect. With $\Omega = 0.8$, the permanent interest rate rise graphed in Figure 6.1 leads to a $3.5\%$ price level drop. The forward guidance of Figure 8.1 leads to a $1.6\%$ price level drop. A mean-reverting interest rate rise will have a smaller effect still. Price stickiness also makes the effect smaller, because higher real interest rates also devalue the right hand side of the valuation equation, a countervailing inflationary effect.

When the government announces at time 0 that interest rates will rise from $r$ to $i$ starting at time $T$, equation (8.3) reads

$$\sum_{j=0}^{T} \frac{\Omega^j}{(1 + r)^j} + \sum_{j=T+1}^{\infty} \frac{\Omega^T}{(1 + r)^T} \frac{\Omega^{(j-T)}}{(1 + i)^{(j-T)}} B_{-1} \frac{P_0}{P} = \frac{s}{1 - \beta}$$

and with a bit of algebra

$$\frac{P_0}{P} - 1 = \left( \frac{\Omega}{1 + r} \right)^T \left[ \frac{(1 + i)}{(1 + r)} \frac{(1 + r - \Omega)}{(1 + r)} - 1 \right].$$
generalizing (8.6). In continuous time, we have
\[
\int_0^T e^{-rj} e^{-\omega j} dj + \int_T^\infty e^{-rT-i(j-T)} e^{-\omega j} dj \frac{B_0}{P_0} = \frac{s}{r},
\]
leading to
\[
\frac{P_0}{P_0} - 1 = e^{-(r+\omega)T} \left( \frac{r + \omega}{i + \omega} - 1 \right),
\]
generalizing (8.8).

The price level $P_0$ still jumps – forward guidance works. Longer $T$ or shorter maturity structures — lower $\Omega$ or larger $\omega$ – give a smaller price-level jump for a given interest rate rise. As $T \to \infty$, the downward price level jump goes to zero.

A geometric maturity structure is not passive, except in a knife edge case that surpluses are also nonstochastic and geometric. The government generically has to roll over long term debt as it did short term debt, and has to sell new debt to finance deficits or repurchase debt in times of surplus. It also has to readjust the maturity structure if it wishes to keep the geometric shape.

To see the needed bond sales, write
\[
B^{(t+j)}_t = B^{(t+j)}_{t-1} + \left( B^{(t+j)}_t - B^{(t+j)}_{t-1} \right).
\]

Thus, to maintain a steady state,
\[
\left( B^{(t+j)}_t - B^{(t+j)}_{t-1} \right) = \Omega^{j-1} B_t - \Omega^j B_{t-1}.
\]

In order to pay off maturing debt $B_{t-1}$, in addition to the current surplus $s_t$, the government must issue new debt. It issues debt across the maturity spectrum, in the same geometric pattern as debt outstanding. Equivalently, the government issues more and more of each bond as it approaches maturity, again with a geometric pattern. This is roughly what our governments do.

### 8.2 Bond quantities

We now consider bond quantities $B^{(t+j)}_t$. What price paths follow from given bond quantities? What bond quantities support a given price path?
Now we analyze bond quantities. What are the effects of long-term bond sales given surpluses, and what is the effect of surplus shocks with fixed long-term bond supplies? We also look for bond sales and purchases behind the movements in interest rate and long-term bond yield targets, and their effect on price levels.

The answers to these questions turn out to be algebraically challenging in the presence of long-term debt. The objective is to solve the sequence (for each \(t\)) of flow or present value conditions

\[
\frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) \quad (8.11)
\]

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j} \quad (8.12)
\]

for \(P_t\), given \(\{s_t\}\) and \(\{B_t^{(t+j)}\}\). Alternatively, given a path of \(\{P_t\}\) and \(\{s_t\}\) we search for corresponding debt policies \(\{B_t^{(t+j)}\}\). As with one-period debt, we can interpret either calculation as finding debt policies that generate the desired \(\{P_t\}\), or as debt policies that emerge endogenously if the government targets bond prices by freely offering to buy and sell debt at given interest rates.

In the one-period bond case, the present value relation (8.12) by itself provided such a solution – there was only one price level, \(P_t\), on the left hand side, so we could find the price level given debt and surplus policy settings. Now we have to solve the system of such equations simultaneously at each date to find such as solution.

These operations are not mathematically hard – these are linear equations. But the general cases don’t lead to much intuition, so I start with a set of examples that isolate some important channels. I turn on three important pieces of long-term debt policy one by one. First, I consider a government that inherits a maturity structure \(\{B_t^{(j)}\}\) and simply pays off outstanding long-term debt as it matures.

Next, I consider the effects of purchases or sales at time 0, \(\{B_0^{(j)}\}\) or \(\{B_0^{(j)} - B_{-1}^{(j)}\}\) in the presence of outstanding long-term debt, but still with no future purchases and sales. Last, I consider the effects of expected future purchases and sales \(\{B_t^{(t+j)}\}\) or \(\{B_t^{(t+j)} - B_{t-1}^{(t+j)}\}\). Then I present some general-case formulas.
8.2.1 Maturing debt and a buffer

The government inherits a maturity structure \( \{ B_{-1}^{(j)} \} \) and pays off outstanding long-term debt as it matures. The price level each period is then determined by that period’s surplus and maturing debt only. Bond prices in the present value of nominal debt, reflecting future prices, adjust completely to news in the present value of future surpluses, and the current price level no longer adjusts. In this way, long-term debt can be a buffer against shocks to expected future surpluses.

I start with a very simple case: turn off all sales or repurchases – the right hand side of the flow condition (8.11). The government just pays off outstanding long term bonds \( \{ B_{-1}^{(t)} \} \) by surpluses \( \{ s_t \} \) at each date. Figure 8.2 illustrates the example.

\[
B_{-1}^{(2)} \\
B_{-1}^{(1)} \\
B_{-1}^{(0)} \\
\downarrow \quad M \\
\downarrow \quad M \\
\downarrow \quad M \\
s_0 \\
s_1 \\
s_2
\]

Figure 8.2: Example with outstanding debt, and no subsequent sales or purchases.

Without subsequent sales or repurchases, today’s two-period bonds become tomorrow’s one-period bonds, and so forth, so the debt coming due on date \( t \) is the same as the date \( t \) debt outstanding at the beginning of period 0,

\[
B_{t-1}^{(t)} = B_{-1}^{(t)}.
\]

The price level at each date \( t \geq 0 \) is then set by debt coming due at that date only, and that period’s surplus,

\[
\frac{B_{-1}^{(t)}}{P_t} = \frac{B_{t-1}^{(t)}}{P_t} = s_t. \tag{8.13}
\]

Each date becomes a version of the one-period model.
There is still a full spectrum of bonds outstanding, \( \{ B^{(t+j)}_{t-1} \} \) at each date. Their presence just doesn’t affect the price level at time \( t \). There is a stream future of future surpluses and deficits \( \{ s_{t+j} \} \) at each date too, but they don’t affect the price level at time \( t \) either. The linkage between the price level and future surpluses seems to have disappeared in this example! What’s happening? The present value condition is still valid,

\[
\sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R_j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+j}.
\]

From (8.13), bad news about a future surplus \( s_{t+j} \) raises the future expected price level, lowering \( E_t (1/P_{t+j}) \) and hence lowering the bond price \( Q^{(t+j)}_t \). So the real value of nominal debt at time \( t \) still equals the present value of future surpluses at time \( t \). But in this case the market value of debt does all the adjusting to lower future surpluses, needing no help from the price level in the dominator. Formally, we now have the innovation version

\[
(E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R_j} E_t \left( \frac{1}{P_{t+j}} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+j}.
\]

In this case, all of the impact of future surpluses shows up in today’s bond prices, and none of it shows up in the price level, the exact opposite of the case with one-period debt that is constantly rolled over. A surprise fall in the present value of surpluses still results in an unexpected “default” of bondholder value. But that “default” shows up entirely in bond prices today and future inflation, rather than showing up entirely in today’s inflation.

In this way, long-term debt can be a useful buffer against shocks to future surpluses, allowing their affects to be absorbed by bond prices rather than by the price level. However, this effect is not general. It depends on the commitment not to buy or sell long-term debt in the future. As we will see, expected future bond sales can undo this effect.

### 8.2.2 Intertemporal linkages, runs and defaults

With the opposite long-term debt case in front of us, in which future surpluses have no effect on today’s inflation, we return to the mechanics of inflation under one-period debt. Future surpluses affect today’s inflation through a roll over process.
People become concerned about repayment in year 30. They then fear bond sales in year 29, and thus inflation in year 29. This process works its way back so that people try to sell government debt today on fear the government will not be able to roll it over tomorrow. The mechanism is the same as a financial crisis or run, and its fiscal roots are hard to see.

It is initially puzzling that short term debt leads to a present value formula, and long term debt, at least in this example, leads to a one-period formula. We are used to thinking of long-term assets leading to a long-term present value relation, and short-term assets valued by short-term present value relations.

![Figure 8.3: Short term debt, rolled over.](image)

The key by which short-term assets lead to a long-term present value relationship is that the short-term bonds are rolled over. Figure [S.3](image) reminds us of the flow of money, and offers a contrast to the long-term debt examples in similar figures.

If the government did not roll over debt, then we would have \( B_{t-1}^{(t)}/P_t = s_t \) even for short term debt. The present value relation comes from the flow relation

\[
\frac{B_t^{(t)}}{P_t} = s_t + Q_t^{(t+1)} \frac{B_t^{(t+1)}}{P_t} = s_t + E_t \left( \frac{1}{R} \frac{B_t^{(t+1)}}{P_{t+1}} \right),
\]

(8.14)

In words, suppose people become worried that there will be no surpluses at time \( s_T \) far in the future. They then worry that the government will print up money to pay off debt, that \( B_{T-1}^{(T)}/P_T = s_T \) result in a high \( P_T \). Given that fear, they reason that there won’t be investors around willing to pay a lot for that debt at time \( T-1 \),
so revenue from bond sales at time $T - 1$ will be disappointing. That means that with

$$
\frac{B_{T-2}(T-1)}{P_{T-1}} = s_{T-1} + E_{T-1} \left( \frac{B_{T-1}(T)}{P_{T}} \right) = s_{T-1} + E_{T-1} \left( \frac{s_T}{R} \right),
$$

disappointing revenue from bond sales (the second term) will lead to a greater price level at time $T - 1$. Working backwards, investors are reluctant to hold government bonds at time 0 because they know that the government will have trouble rolling them over at time 1, so people at time 0 try to get rid of the bonds and drive up the time 0 price level.

In sum, the link between the current price level and future surpluses results as people consider the likely results of rolling over short-term debt.

This is a fragile mechanism. If people start to suspect that other people will start to suspect that surpluses will not be forthcoming, they start to suspect that debt will be hard to roll over, and they try to spend it now. The apparently soothing present value formula hides a great fragility. All financial crises come from problems in rolling over short-term debt. Inflation here becomes another such run on short term debt. Inflation in the short-term debt present-value formula can start from a rumor, an expectation that 29 years from now the government may have trouble rolling over its debt. Such inflations can seem to come from nowhere, to be “bubbles,” or “sudden stops,” like all seemingly unforecastable financial crises.

Bad opinions about future deficits shows up as poor revenue from bond sales, which is the proximate source of the inflation at time $t$. People see trouble brewing in the future. They fear that if they buy bonds today, those bonds will be defaulted on or inflated away in the future. People are then less willing to lend new money $Q_t B_t / P_t$. Having to print up money to pay off the current bonds, and unable to sell enough new bonds to soak up that money, inflation breaks out today.

The triggering fear may not be future inflation. If people fear a future explicit default or sudden wealth taxation that includes government debt to address a future debt crisis, they will try to get out of government debt today. Inflation results when the government prints money rather than default today, but the fiscal theory extends to fears of future default in place of future monetization. It is not true that the fiscal theory requires a permanent commitment to inflate rather than default.

The mechanism is really a rollover crisis. As usual, it is easy to miss its fiscal roots, and commenters, not seeing obvious “fundamental” news will be tempted to attribute the inflation to sunspots, “self-confirming expectations,” multiple equilibria, “con-
tagions,” irrational markets, or other chimera. We might ask why the central bank
doesn’t just run open market operations, soaking up the excess money with more
debt. Or we might ask why the government doesn’t sell more debt to stop the crisis.
But the real revenue raised from bond sales equals the real value of surpluses that
will pay it off. To soak up money with debt, the government must persuade people
that surpluses will be there to retire the debt.

This run-like nature of inflation is useful when thinking about events. Why does
inflation seem to come so suddenly and unexpectedly? Well, for the same reason
that financial crises come suddenly and unexpectedly. If people expect a fiscal in-
flation tomorrow, it happens today. Why, conversely, can economies go on for years
with economists scratching their heads over large debts and deficits, but no inflation
breaking out? Well, like Greek debt or short-term debt backed by mortgage-backed
securities in 2006, it all looks fine until suddenly it doesn’t. The US, Europe, and
Japan have the means to pay off our debts if we choose to do it. The question in
front of bond markets is whether we will choose to do it. 25 years from now.

8.2.3 Bond sales and interest rates

Now we consider sales or repurchases of long-term debt at time 0, $B_0^{(j)} - B_1^{(j)}$, but
still no future sales or purchases.

- **If there is no long-term debt outstanding, $B_{-1}^{(j)} = 0$ for $j > 0$, then the real
  revenue raised by selling debt $B_0^{(j)}$ with no change in surplus $s_j$ is independent
  of the amount of debt sold. Such a sale lowers bond prices $Q_0^{(j)}$, and it causes
  future inflation $E_t (1/P_j)$, but it has no effect on the current price level $P_0$.

- The government can target long-term bond prices $Q_0^{(j)}$, by offering to freely buy
  or sell long term debt at fixed prices.

In this model, we have

$$
\frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \left( \frac{B_0^{(j)} - B_{-1}^{(j)}}{B_0^{(j)}} \right) E_0 (s_j) \frac{R^j}{R^j}.
$$

- In the presence of outstanding long-term debt, $B_{-1}^{(j)} > 0$, additional debt sales
  with no change in surplus do raise revenue, and therefore such sales can lower
  the price level $P_0$ immediately.
Monetary policy can target long-term rates as well as short-term rates. Bond purchases, such as those of quantitative easing programs, can lower long-term interest rates, and they can “stimulate” additional inflation right away. The government can also use unexpected debt sales to smooth surplus shocks.

- **An active or state-contingent debt policy, unexpectedly buying or selling long-term debt** $B_0^{(j)} - B_{-1}^{(j)}$, **can offset surplus shocks and stabilize inflation** – **though at the cost of future expected inflation.**

Now, suppose the government buys or sells some extra long term debt $B_0^{(j)} - B_{-1}^{(j)}$ at time 0, potentially on top of outstanding debt $B_{-1}^{(j)}$. For now, I still suppose that the government never buys or sells debt at subsequent dates. Figure 8.4 illustrates the example.

![Figure 8.4: Long term debt example. The government may buy or sell debt at time 0, but not subsequently.](image)

We have for $j > 0$

$$\frac{B_0^{(j)}}{P_j} = s_j$$

(8.15)
and hence

\[ Q_0^{(j)} = \frac{1}{R^j} E_0 \left( \frac{P_0}{P_j} \right) \]  

(8.16)

\[ \frac{B_0^{(j)} Q_0^{(j)}}{P_0} = \frac{E_0 (s_j)}{R^j} \]  

(8.17)

Equation (8.17) tells us that if surpluses are fixed, the total real value of date-\( j \) debt is independent of the amount sold. This result naturally extends our original one-period model in which the total real value of date 1 debt was independent of the amount sold.

The \( t = 0 \) flow condition is now

\[ B_{-1}^{(0)} = P_0 s_0 + \sum_{j=1}^{\infty} Q_0^{(j)} \left( B_0^{(j)} - B_{-1}^{(j)} \right). \]

Substituting bond prices from (8.16) and (8.15),

\[ \frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \left( \frac{B_0^{(j)} - B_{-1}^{(j)}}{P_0^{(j)}} \right) \frac{E_0 (s_j)}{R^j}. \]  

(8.18)

\( B_0^{(j)} \) is the total amount of \( j \)-period debt at the end of time 0, including both debt \( B_{-1}^{(j)} \) outstanding at the beginning of 0, \( B_0^{(j)} \), and debt \( B_0^{(j)} - B_{-1}^{(j)} \) sold or, if negative, repurchased at \( t = 0 \). The right hand term in (8.18) is then real revenue raised at time 0 by selling additional debt. We want to find the effects of the bond sales \( B_0^{(j)} - B_{-1}^{(j)} \).

No outstanding debt

Start with the case that no long-term debt is outstanding, so \( B_{-1}^{(j)} = 0 \) for \( j > 0 \). One-period debt is outstanding, but any long-term debt is sold at period 0. Equation (8.18) reduces to

\[ \frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \frac{E_0 (s_j)}{R^j} \]  

(8.19)

(assuming \( B_0^{(j)} > 0 \) for all \( j > 0 \)). With no long-term debt outstanding, \( P_0 \) is still determined by fiscal shocks alone, independently of any sales \( B_0^{(j)} \). We then have a natural generalization of the one-period results:
If there is no long-term debt outstanding, \( B^{(j)}_{-1} = 0 \) for \( j > 0 \), then the real revenue raised by selling debt \( B^{(j)}_0 \) with no change in surplus \( s_j \) is independent of the amount of debt sold. Such a sale lowers bond prices \( Q^{(j)}_0 \), and it causes future inflation \( E_t(1/P_j) \), but it has no effect on the current price level \( P_0 \).

We also have in \((8.19)\) again the familiar present value statement of the fiscal theory with one period debt, even though the government now rolls the one period debt over to debt of arbitrarily long maturities. We learn that statement \((8.19)\) is the special case of one-period debt outstanding. It is unaltered if debt sales are long-term.

Via \((8.17)\), selling more bonds \( B^{(j)}_0 \) raises the future price level \( P_j \), and lowers the bond price \( Q^{(j)}_0 \). The demand for total nominal debt of maturity \( j \) slopes down in this example, and has a unit elasticity.

- Purchases of long-term bonds can drive up the future price level and drive down long-term interest rates, and vice versa.

This operation resembles quantitative easing. There is a sense in which the nominal debt market appears completely “segmented” in this example. Each bond maturity is a claim to a specific surplus, and no other. The government can change, say, the 10 year bond price, with no effect on the 9 year price or the 11 year price. These results depend on the assumption that the government does not change surpluses \( s_j \) along with a debt sale. The usual theory of bond markets makes the opposite assumption, which is why it usually sees flat demand curves. The usual theory also concerns real, not purely nominal, interest rate variation.

Sales of maturity-\( j \) debt \( B^{(j)}_0 \) reduce maturity-\( j \) bond prices \( Q^{(j)}_0 \). Conversely, then, the government can fix long-term bond prices by offering to sell any amount of debt at fixed prices, and the resulting demands will be finite:

- The government can target long-term bond prices \( Q^{(j)}_0 \), by offering to freely buy or sell long term debt at fixed prices. Equation \((8.17)\) then tells us how much debt the government will sell.

In quantitative easing, central banks changed bond supplies \( B^{(j)}_0 \) with the hope of changing interest rates. It is a bit puzzling that they did not just announce the interest rate they wanted, and offer to freely buy and sell long-term bonds at that rate. They may have been worried that huge demands would have ensued. This observation again reassures us that fixed bond prices can result in finite, and quite limited bond sales. A one percentage point bond price change implies a one percent different stock. However, this proposition depends crucially on fixed surpluses. If
people think higher bond sales come with higher surpluses, then the demand curve really is flat. This flat demand for real debt lies behind the quantity worry. As before, communicating fixed surpluses is not easy.

**Outstanding debt**

Now suppose there is some long-term debt is outstanding at time 0 as well, \( B_{-1}^{(j)} > 0 \). We have an additional effect: Long-term bond sales, with no change in surpluses, can raise revenue, and can affect the price level \( P_0 \). Equation [8.18] offers this novel result:

- **In the presence of outstanding long-term debt,** \( B_{-1}^{(j)} > 0 \) \( j > 0 \), **additional debt sales** \( B_0^{(j)} - B_{-1}^{(j)} \) **with no change in surplus raise revenue, and therefore such sales lower the price level** \( P_0 \) **immediately, as well as raising future price levels.**

With no outstanding debt, each additional dollar of debt sold \( B_0^{(j)} \) lowers the bond price just enough that the product \( Q_0^{(j)} B_0^{(j)} \) is unchanged, and revenue is independent of the total amount of debt sold. But the percentage increase in new debt is larger than the percentage increase in total debt. Therefore, a one percent increase in new debt sales lowers the bond price by less than one percent, and raises revenue. The source of revenue is that new long-term debt sales dilute existing long-term debt as a claim to future surpluses. That revenue helps to pay off existing debt at time 0, lowering the time 0 price level.

This debt operation adds a second important element of quantitative easing, or tightening. A debt purchase at time 0 lowers long term interest rates. In the presence of outstanding debt it can also stimulate inflation at time 0.

The immediate effect on the price level in the presence of outstanding debt is the same as we saw in Figure 6.1 and Figure 8.1. Debt sales or purchases can implement the pattern of bond prices, and hence price levels shown in those figures, as well as an interest rate target and forward guidance.

With outstanding debt, one might worry that debt sales \( B_0^{(j)} - B_{-1}^{(j)} \) could affect the price level \( P_0 \) as well as \( E_0 (1/P_j) \), in just offsetting amounts so the government loses the ability to target bond prices \( Q_0^{(j)} \). However, a debt sale raises \( P_j \) and lowers \( P_0 \), so bond sales still control long-term bond prices, and vice versa. The effect is the opposite – outstanding debt and an effect on \( P_0 \) makes the curve relating bond prices to sales becomes steeper, not flatter.
8.2. BOND QUANTITIES

In the presence of outstanding long-term debt, additional debt sales can also help to fund the surplus at time 0, without needing future surpluses. With one period debt, and without changing future surpluses, debt sales $B_0^{(1)}$ raise no revenue and cannot help to fund $s_0$. The innovation version of (8.18) is

$$\frac{B_{-1}^{(0)}}{P_{-1}} (E_0 - E_{-1}) \left( \frac{P_{-1}}{P_0} \right) = (E_0 - E_{-1}) s_0$$

$$+ \sum_{j=1}^{\infty} \frac{1}{R_j} (E_0 - E_{-1}) \left\{ \frac{B^{(t+j)}_t - B^{(t+j)}_{t-1}}{B^{(t+j)}_t} E_t (s_{t+j}) \right\}$$

- An active or state-contingent debt policy, unexpectedly buying or selling long-term debt $B_0^{(j)} - B_{-1}^{(j)}$, can offset surplus shocks and help to stabilize inflation - though at the cost of future expected inflation.

8.2.4 Future sales

Expected future bond sales with no change in surpluses affect price levels and interest rates. By doing so, they affect the proceeds of bond sales today, and therefore can affect the price level today in the presence of long-term debt.

Current $B_0^{(T)}$ and expected future $B_t^{(T)} - B_{t-1}^{(T)}$ sales enter symmetrically to determine the price level $P_T$. Thus, the effects of any bond sale or purchase $B_0^{(T)}$ on the price level $P_T$ can be undone by expected future bond sales or purchases. To evaluate the effects of long-term bond purchases or sales on future price levels and therefore interest rates, we must specify expectations of future purchases and sales.

For expected future debt sales to affect the price level $P_0$, there must be long-term debt outstanding, just as there must be for time-0 bond sales to affect $P_0$.

For expected future debt sales to affect the price level $P_0$, there must be sales at time 0 as well. Expected future sales have an interaction effect on $P_0$, modifying the dilution effects of time-0 sales.

Expected future sales $B^{(2)}_t - B^{(2)}_0$ have offsetting positive and negative effects on the price level $P_0$. These effects depend on how much time 1 vs. time 2 debt is being sold at time 0, relative to the amount outstanding. If the government sells proportionally
more time 1 debt than time 2 debt,

\[
\frac{B_0^{(1)} - B_0^{(1)}}{B_0^{(1)}} > \frac{B_0^{(2)} - B_0^{(2)}}{B_0^{(2)}}
\]

then expected future debt sales \( B_1^{(2)} - B_0^{(2)} > 0 \) lower the price level \( P_0 \), and vice versa.

In sum, evaluation of QE-like bond purchases or sales must carefully consider expected future bond purchases or sales as well.

Next, how do expected future bond sales affect current prices, future prices, and hence long-term interest rates? The algebra quickly gets more tedious than enlightening, so I pursue a three-period example. Figure 8.5 illustrates.

![Figure 8.5: Long term debt example, illustrating the effects of future purchases and sales.](image)

Figure 8.5: Long term debt example, illustrating the effects of future purchases and sales.

To solve this example, start at the final period 2. Debt \( B_1^{(2)} \) is outstanding, so the price level is determined by

\[
\frac{B_1^{(2)}}{P_2} = s_2.
\] (8.20)

The flow condition for period 1 now gives us \( P_1 \), the same as [8.18],

\[
\frac{B_0^{(1)}}{P_1} = s_1 + \frac{B_1^{(2)} - B_0^{(2)}}{B_1^{(2)}} E_1(s_2) \frac{R}{R}.
\] (8.21)
These period-1 bond sales or purchases, $B_1^{(2)} - B_0^{(2)}$, become the expected future purchases that will modify our view of period 0. That effect is our central focus here.

The period 0 flow condition is

$$\frac{B_0^{(0)}}{P_0} = s_0 + \frac{Q_0^{(1)}}{P_0} \left( B_0^{(1)} - B_{-1}^{(1)} \right) + \frac{Q_0^{(2)}}{P_0} \left( B_0^{(2)} - B_{-1}^{(2)} \right).$$

$$\frac{B_0^{(0)}}{P_0} = s_0 + \frac{1}{R} E_0 \left( \frac{1}{P_1} \right) \left( B_0^{(1)} - B_{-1}^{(1)} \right) + \frac{1}{R^2} E_0 \left( \frac{1}{P_2} \right) \left( B_0^{(2)} - B_{-1}^{(2)} \right).$$

Substituting in the prices from (8.20) and (8.21),

$$\frac{B_0^{(0)}}{P_0} = s_0 + \frac{\left( B_0^{(1)} - B_{-1}^{(1)} \right)}{B_0^{(1)}} E_0 \left[ \frac{s_1}{R} + \frac{\left( B_1^{(2)} - B_0^{(2)} \right)}{B_1^{(2)}} s_2 \right] + E_0 \left[ \frac{\left( B_0^{(2)} - B_{-1}^{(2)} \right)}{B_1^{(2)}} \frac{s_2}{R^2} \right].$$

Equation (8.22) groups terms by the effect of selling time-1 debt $B_0^{(1)} - B_{-1}^{(1)}$, and then time-2 debt $B_0^{(2)} - B_{-1}^{(2)}$. We can also collect terms in surpluses,

$$\frac{B_0^{(0)}}{P_0} = s_0 + \frac{\left( B_0^{(1)} - B_{-1}^{(1)} \right)}{B_0^{(1)}} E_0 \left( s_1 \right) +$$

$$E_0 \left[ \frac{\left( B_0^{(1)} - B_{-1}^{(1)} \right)}{B_0^{(1)}} \frac{\left( B_1^{(2)} - B_0^{(2)} \right)}{B_1^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right)} \frac{\left( B_0^{(2)} - B_{-1}^{(2)} \right)}{B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right)} \right] \frac{s_2}{R^2}$$

The last expression is not the most compact, but it is in the end the most elegant. I expand $B_1^{(2)} = B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right)$ to express the final amount of debt in terms of its sales at date 0 and date 1. We could expand further to $B_1^{(2)} = B_{-1}^{(2)} + \left( B_0^{(2)} - B_{-1}^{(2)} \right) + \left( B_1^{(2)} - B_0^{(2)} \right)$ to include initial outstanding debt and time 0 purchases as well. I highlight the central question, expected future debt sales $B_1^{(2)} - B_0^{(2)}$, in boldface.

To make sense of these expressions, I consider a few special cases of this special case.
No outstanding long-term debt

Suppose there is no long-term debt outstanding, \( B_{-1}^{(1)} = 0 \); and \( B_{-1}^{(2)} = 0 \). First, find the effect on \( P_0 \). Equation (8.22) or (8.23a) reduce once again to

\[
\frac{B_{-1}^{(0)}}{P_0} = s_0 + E_0 \left[ \frac{s_1}{R} + \frac{s_2}{R^2} \right].
\]

• For expected future debt sales to affect the price level \( P_0 \), there must be long-term debt outstanding, just as there must be for time-0 bond sales to affect \( P_0 \).

Expected future sales just like current sales must devalue already outstanding debt in order to provide revenue that can be applied to the time 0 debt.

Next, find the effect on \( P_2 \). From (8.20), both initial \( B_0^{(2)} \) and future \( (B_1^{(2)} - B_0^{(2)}) \) sales affect \( P_2 \), and therefore also the interest rate \( Q_0^{(2)} \). Only total two-period debt \( B_1^{(2)} = B_0^{(2)} + (B_1^{(2)} - B_0^{(2)}) \) affects \( P_2 \), however. Expected future sales at time 1 \( (B_1^{(2)} - B_0^{(2)}) \) enter symmetrically with sales \( B_0^{(2)} \) at time 0.

• Expected future bond sales and purchases \( B_1^{(2)} - B_0^{(2)} \) enter symmetrically with time zero sales \( B_0^{(2)} - B_{-1}^{(2)} \) in determining the price \( P_2 \), and, in the absence of outstanding debt, the long term bond price \( Q_0^{(2)} \).

Current and expected future sales are equivalent for \( P_2 \), whether or not there is outstanding long-term debt. When there is outstanding long-term debt they have potentially different effects on \( P_0 \), and thus they are not exactly equivalent for the bond price \( Q_0^{(2)} \).

This fact has an important implication:

• The effects of any bond sale or purchase \( B_0^{(2)} \) on the price \( P_2 \) and the long-term bond price \( Q_0^{(2)} \) can be undone by expected future bond sales or purchases.

This result qualifies optimism you may have felt in the last section about the power of quantitative-easing style bond purchases or sales to affect interest rates, future inflation, and, later, when we add back outstanding debt, the price level \( P_0 \): To be effective, such purchases must come with an expectation that they will not be undone in the future.

This fact easily generalizes. In place of period 2, consider a generic period \( T \). The
8.2. BOND QUANTITIES

flow condition is

\[
\frac{B_{T-1}^{(T)}}{P_T} = s_T + \sum_{j=1}^{\infty} \frac{B_{T}^{(T+j)} - B_{T-1}^{(T+j)}}{R^j} \cdot E_T \left( \frac{1}{P_{T+j}} \right).
\]

Now, imagine selling more debt \( B_0^{(T)} \) vs. selling more debt \( B_T^{(T)} - B_{T-1}^{(T)} \) for some \( t > 0 \). Holding constant time \( T \) sales, \( B_T^{(T+j)} - B_{T-1}^{(T+j)} \), only the total amount of debt \( B_{T-1}^{(T)} \) arriving at time \( T \) matters for \( P_T \), not the timing of current \( B_0^{(T)} \) vs. expected future \( B_T^{(T+j)} - B_{T-1}^{(T+j)} \) sales. Thus, the last two bullet points generalize past \( T = 2 \), and generalize past the special case that there are no sales at time \( T \).

Last, find \( P_1 \). Expected future debt sales \( B_1^{(2)} - B_0^{(2)} \) can affect \( P_1 \). From equation (8.21),

\[
B_0^{(1)} E_0 \left( \frac{1}{P_1} \right) = E_0 \left[ s_1 + \frac{B_1^{(2)} - B_0^{(2)}}{B_1^{(2)}} s_2 \right].
\] (8.24)

Time 0 bond sales \( B_0^{(1)} \) on the left-hand side matter for \( P_1 \) via the usual share-split effect. But expected future sales \( B_1^{(2)} - B_0^{(2)} \) also matter. If the government sells some time-2 debt at time 0 \( B_0^{(2)} > 0 \) and then sells some additional debt at time 1, \( B_1^{(2)} - B_0^{(2)} > 0 \) the dilution effect provides revenue at time 1 and lowers \( P_1 \).

- **If there is some-long term debt outstanding at time 1 \([ B_0^{(2)} > 0 \] , then expected sales \([ B_1^{(2)} - B_0^{(2)} \] of additional long-term debt can lower the expected price level \( P_1 \), and therefore raise the bond price \( Q_0^{(1)} \).**

We know selling debt when some debt is outstanding can lower the price level. That was \( P_0 \) in the previous section. Here, can ask how such bond sales look ex-ante, and we find that even an expected dilution can raise revenue and lower the price level on the day it happens.

The fact that the dilution is expected shows up in lower bond prices, and less revenue raised at time 0. The time 0 real price of time 2 bonds is

\[
\frac{Q_0^{(2)}}{P_0} = \frac{1}{R^2} E_0 \left( \frac{1}{P_2} \right) = E_0 \left( \frac{1}{B_1^{(2)} R^2} \right) = E_0 \left[ \frac{1}{B_0^{(2)} + (B_1^{(2)} - B_0^{(2)}) R^2} \right].
\]

This price depends on the total amount of time 2 debt \( B_1^{(2)} \), including expected future...
sales. Revenue from time 0 sales is then

\[
B_0^{(2)} Q_0^{(2)} = E_0 \left[ \frac{B_0^{(2)}}{B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right) s_2 / R^2} \right] \tag{8.25}
\]

The larger the expected future sales, the less the government raises by selling \( B_0^{(2)} \).

Why then, however, do expected future sales not affect \( P_0 \)? The answer: Because as expected future sales \( B_1^{(2)} - B_0^{(2)} \) rise, \( P_1 \) also declines, the bond price \( Q_1^{(1)} \) rises, and the government earns *more* from the same sales \( B_0^{(1)} \) of time 1 debt. Revenue from time 1 debt is, from (8.24)

\[
B_0^{(1)} Q_0^{(1)} / P_0 = B_0^{(1)} E_0 \left( \frac{1}{R P_1} \right) = E_0 \left[ \frac{s_1}{R} + \frac{\left( B_1^{(2)} - B_0^{(2)} \right) s_2 / R^2}{B_1^{(2)}} \right]. \tag{8.26}
\]

As expected future sales of time 2 debt \( B_1^{(2)} - B_0^{(2)} \) rise, the price of time 1 debt \( Q_0^{(1)} \) rises and so does revenue from time 1 bond sales \( B_0^{(1)} \). Adding together (8.25) and (8.26) and a bit of algebra, you will find that total revenue from time 0 bond sales \( B_0^{(1)} \) and \( B_0^{(2)} \) is independent of expected time 1 sales \( B_1^{(2)} - B_0^{(2)} \).

In sum, an expected future sale of time 2 debt at time 1, \( B_1^{(2)} - B_0^{(2)} \), has the same effect as selling more time 2 debt at time 0, \( B_0^{(2)} \) (raises \( P_2 \)), and selling less time 1 debt at time 0, \( B_0^{(1)} \) (lowers \( P_1 \)). It is a complex way of accomplishing a time 2 share split, which raises \( P_3 \), and a time 1 reverse split, which lowers \( P_1 \), but this has no effect on total time 0 revenue, and hence \( P_0 \).

The same result has a different implication. In the previous bullet point, it seemed that when the government sold \( B_1^{(2)} \) didn’t matter – current sales \( B_0^{(2)} \) and expected future sales \( B_1^{(2)} - B_0^{(2)} \) enter symmetrically in determining \( P_2 \); they had no effect on \( P_0 \), and consequently the timing of time 2 debt sales had no effect on the bond price \( Q_0^{(2)} \). Here we see that selling time 2 debt in period 0 vs. period 1 does affect the period 1 price \( P_1 \) and the time 0 price of period 1 debt \( Q_0^{(1)} \).

- The timing of bond sales and purchases affects intermediate price levels and bond prices, even though it has no effect on the terminal price level and its time-0 price.
8.2.5 Outstanding long-term debt

Now to the main event. Suppose long-term debt is outstanding at time 0, \( B_{-1}^{(j)} > 0 \). The big question is how expected future sales \( B_1^{(2)} - B_0^{(2)} \) affect \( P_0 \). I repeat (8.23a):

\[
\frac{B_0^{(0)}}{P_0} = s_0 + \left( \frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} E_0 s_1 \right) + \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right)} + \frac{B_0^{(2)}}{B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right)} \frac{s_2}{\varepsilon^2}
\]

If the government sells no debt at time 0, \( B_0^{(1)} - B_{-1}^{(1)} = 0; B_0^{(2)} - B_{-1}^{(2)} = 0 \), then we revert to the coupon case, \( B_0^{(0)}/P_0 = s_0 \), losing intertemporal linkages, and expected future sales have no effect. As we have seen, expected future sales have no effect if there is no time-1 debt outstanding either. Thus,

- Expected future sales only have an interaction effect on \( P_0 \), modifying the dilution effects of time-0 sales in the presence of outstanding debt.

All the \( B_1^{(2)} - B_0^{(2)} \) multiply a \( B_0^{(j)} - B_{-1}^{(j)} \) term.

The last term of (8.27a) is the straightforward dilution effect for period 2 debt. That’s easier to see if we write it as

\[
\frac{B_0^{(0)}}{P_0} = ... + E_0 \left[ \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)} + \left( B_1^{(2)} - B_0^{(2)} \right) + \left( B_0^{(2)} - B_{-1}^{(2)} \right)} \frac{s_2}{\varepsilon^2} \right]
\]

Selling additional debt at time 0 when there is debt outstanding \( B_0^{(2)} - B_{-1}^{(2)} \) can raise revenue and affect the price \( P_0 \). The twist is that the denominator includes expected future debt sales as well as outstanding debt. Dilution occurs relative to all expected claims, even future ones.

The second-to last term of (8.27a) is more interesting. Here, expected future debt sales modify the dilution effect for period 1 debt. The first part \( (B_0^{(1)} - B_{-1}^{(1)})/B_0^{(1)} \) just expresses the dilution of outstanding time 1 debt, how much is sold at 0 vs. the total amount that competes for period 1 resources. The second part reflects the mechanism we saw above, by which expected future sales generate revenue for period 1 and affect total resources at period 1.
CHAPTER 8. LONG-TERM DEBT DYNAMICS

- Expected future long-term debt sales $B_1^{(2)} - B_0^{(2)}$ lower the price level $P_1$, as we have seen, and make date 1 debt more valuable. With outstanding debt and date-0 sales, diluting date 1 debt then generates more revenue and has a larger effect on $P_0$.

However, the last two terms of (8.27a) partially offset. The last term declines in expected future debt sales, $B_1^{(2)} - B_0^{(2)}$, while the first term rises in that quantity. The weights of the two terms are the fractions of each maturity’s debt outstanding at the end of time 0 that was sold at time 0. When those two fractions are equal, when

$$\frac{(B_0^{(1)} - B_{-1}^{(1)})}{B_0^{(1)}} = \frac{(B_0^{(2)} - B_{-1}^{(2)})}{B_0^{(2)}}$$

the last two terms offset, and expected future debt sales have no effect.

- The effect of expected future debt sales $(B_1^{(2)} - B_0^{(2)})$ on $P_0$ depends on how much time 1 and time 2 debt is being sold at time 0, relative to the amount outstanding. If the government sells proportionally more time 1 debt than time 2 debt,

$$\frac{(B_0^{(1)} - B_{-1}^{(1)})}{B_0^{(1)}} > \frac{(B_0^{(2)} - B_{-1}^{(2)})}{B_0^{(2)}}$$

then expected future debt sales $B_1^{(2)} - B_0^{(2)} > 0$ lower the price level $P_0$, and vice versa.

8.2.6 A general formula

I display a general, but complex, formula for finding the price level $P_t$ given paths of debt $\{B_t^{(t+j)}\}$ and surpluses $\{s_t\}$.

The reader is doubtless anxious for a pretty and general formula. Substituting bond prices (6.1) into (6.2) and (6.3) we have a present value relation

$$\frac{B_t^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \frac{1}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right)$$

(8.28)
and a flow relation
\[
\sum_{j=0}^{\infty} E_t \left( \frac{1}{P_{t+j}} \right) B_{t-1}^{(t+j)} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\] (8.29)

We want to solve these equations for \( P_t \) on one side and all the \( B \) and \( s \) on the other side. The equations are equivalent, so either one will do, but one must solve the one we choose at each value of \( t \) simultaneously.

In the case of long-term debt and no buying and selling, the flow relation (8.28) provides such a solution as its right-hand term is absent. In the case of one-period debt, (8.29) provides a solution as no future prices \( P_{t+j} \; j>0 \) are present. In general, with an arbitrary maturity structure and current and expected future buying and selling of debt, finding a solution is not so pretty.

The problem is not mathematically difficult. These are linear equations. In the perfect certainty case, to avoid keeping track of the \( E_t \), we can write (8.29) as

\[
\begin{bmatrix}
B_0^{(1)} & B_0^{(2)} & B_0^{(3)} & B_0^{(4)} & \ldots \\
B_1^{(1)} & B_1^{(2)} & B_1^{(3)} & B_1^{(4)} & \ldots \\
B_2^{(1)} & B_2^{(2)} & B_2^{(3)} & B_2^{(4)} & \ldots \\
B_3^{(1)} & B_3^{(2)} & B_3^{(3)} & B_3^{(4)} & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
1/P_0 \\
1/P_1 \\
1/P_2 \\
1/P_3 \\
\vdots
\end{bmatrix}
= \begin{bmatrix}
1 & 1/R & 1/R^2 & 1/R^3 & \ldots \\
1 & 1/R & 1/R^2 & \ldots \\
1 & 1/R & \ldots \\
1 & \ldots \\
\vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
s_0 \\
s_1 \\
s_2 \\
s_3 \\
s_4
\end{bmatrix}.
\]

We could write this equation as
\[
Bp = Rs
\] (8.30)
and hence write its solution as
\[
p = B^{-1}Rs.
\]

The problem is just that the inverse \( B \) matrix doesn’t yield very pretty answers.

My best attempt, from Cochrane (2001) has the form of a weighted present value:

\[
\frac{B_{t-1}^{(t)}}{P_t} = E_t \sum_{j=0}^{\infty} W_t^{(j)} s_{t+j} R^{-j}.
\] (8.31)

The weights are defined recursively. Start by defining the fraction of time \( t+j \) debt sold at time \( t \),

\[
A_t^{(t+j)} = \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{B_t^{(t+j)} - B_{t+j-1}^{(t+j)}}.
\]
Then, the weights are

\[
\begin{align*}
W_t^{(0)} &= 1 \\
W_t^{(1)} &= A_t^{(t+1)} \\
W_t^{(2)} &= A_t^{(t+1)} W_t^{(1)} + A_t^{(t+2)} \\
W_t^{(3)} &= A_t^{(t+2)} W_t^{(2)} + A_t^{(t+3)} W_t^{(1)} + A_t^{(t+3)} \\
W_t^{(j)} &= \sum_{k=0}^{j-1} A_t^{(t+j)} W_t^{(k)}.
\end{align*}
\]

These formulas likely hide additional interesting insights and special cases.

One can see just from the fact that \( B \) is a matrix and \( P \) is a vector that

- There are many debt policies that correspond to any given price level path.

We have already seen how either expected sales of one period debt or initial sales of long-term debt can determine any sequence of expected price levels, and many paths involving dynamic buying and selling of debt exist as well. This insight leads me to focus on interest rate targets, and once we have reassurance that there is at least one debt policy that supports the target, to spend less attention on the forward question, what are the effects of given debt operations.

### 8.3 Constraints on policy

The present value condition

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}
\]

acts as a “budget constraint” on the price level sequences that debt policy – changes in \( \{B_t^{(t+j)}\} \) with fixed surpluses – or interest rate policy – changes in \( \{Q_t^{(t+j)}\} \) – can accomplish. There is a debt policy and interest rate policy that achieves any price level path consistent with this formula, and debt policy cannot achieve price level paths inconsistent with this formula.

The unexpected version

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(j)}}{R^j} (E_0 - E_{-1}) \left( \frac{1}{P_j} \right) = (E_0 - E_{-1}) \sum_{j=0}^{\infty} \frac{s_j}{R^j}
\]
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tells us how a state-contingent debt sale can stabilize the price level $P_0$ in the face of surplus shocks, but by transferring inflation to the future.

Moving the index forward, the expected version

$$\sum_{j=1}^{\infty} B_0^{(j)} Q_0^{(j)} \frac{P^j}{P_0} = \sum_{j=1}^{\infty} B_0^{(j)} \left( \frac{1}{P_0} \right) = E_0 \sum_{j=1}^{\infty} \frac{s_j}{R^j}.$$  

shows that the real end of period value of nominal government debt is still a constant, independent of the quantity $B_0^{(j)}$ sold. With long term debt, however, $P_0$ can change as well as expected future prices.

What price level paths can debt policy – changes in debt without changes in surplus – accomplish? The present value condition provides this general result directly:

$$\sum_{j=0}^{\infty} \frac{Q_0^{(j)} B_{-1}^{(j)}}{P_0} = \sum_{j=0}^{\infty} B_{-1}^{(j)} \left( \frac{1}{P_0} \right) = E_0 \sum_{j=0}^{\infty} \frac{s_j}{R^j}.$$  

(8.32)

- **Fixing surpluses**, there is a debt policy – a set of debt sales or purchases with no change in surpluses – that achieves any path of price levels consistent with (8.32). There is no debt policy which can achieve a price level path inconsistent with (8.32).

The maturity structure of outstanding debt acts as a “budget constraint” for the sequence of expected future price levels achievable by debt policy or interest rate policy. This is the only constraint on debt policy – there is a debt policy that can achieve any sequence of expected price levels consistent with (8.32). In fact, there are many. There is no other equation limiting what debt policy can achieve.

If only one-period debt is outstanding at time 0, then $B_0^{(0)}/P_0$ is the only term on the left-hand side. The government can achieve any sequence of price levels $E_0(1/P_0^{t+j})$ it wants in the future. But changes in future price levels have no effect on the time-0 price level $P_0$. Only surplus shocks can change the price level $P_0$.

If long-term debt $B_{-1}^{(j)}$ is outstanding, then (8.32) describes a binding tradeoff between future and current price levels. I have typically used it to find the implied jump in $P_0$ that results from the government’s choices of $\{E_t(1/P_{t+j})\}$, since the latter are unconstrained.

The interest rate policy and forward guidance examples of Figures 6.1 and 8.1 involved raising $\{P_j\}$ and thereby lowering $P_0$, and vice versa. We see in (8.32) attractive generalizations of those results. For example, if you want to create a quantitative
easing policy that raises the price level for some interval of time between 0 and \( T \), (8.32) shows what is and is not possible.

A QE policy that raises short term price levels with no decline in future price levels is not possible. We could construct an example in which the price level moves slowly down too, rather than take the sharp jump of the figures. But (8.32) warns that the price level must then rise in the end. Equation (8.32) generalizes Sims (2011) and Cochrane (2017d) “stepping on a rake” characterization, that a lower price level today must result in higher price levels in the future, to say that lower price levels at some dates must be accompanied by higher price levels at some other dates, all weighted by the maturity structure of outstanding debt.

If we fix \( P_0 \), (8.32) implies a constraint between price levels at different future dates. For example, we could use (8.32) to describe future inflation that must follow from a surplus shock, if \( P_0 \) is held constant by debt policy.

The attractive part of this statement is what’s missing. It is an existence proposition. It tells you there is a debt policy that achieves a given set of expected price levels, and there is no debt policy that deliver others, but it does not tell you which debt policy generates the sequence of price levels. In general, there are many: one can achieve a price level sequence consistent with (8.32) by time 0 sales of long-term debt, by expected future sales of long and short term debt, or by combinations of those policies. Similarly, it tells you that there is an interest rate policy that achieves the given set of price levels – a set of interest rate or bond price targets \( Q_{t+j}^{t+j} = E_t(P_t/P_{t+j})/R^j \), enforced by passive bond sales at those targets – without specifying just which bonds the government must offer to sell – that achieves the price level path.

To prove an existence theorem, we can just give an example. We already have two examples of a debt policy that generates any price level sequence \( E_0(1/P_j); \ j > 0 \): First, sell long run debt at the end of period 0 in the quantity \( B_0^{(j)} \) given by

\[
B_0^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0(s_j); \ j > 0
\]

and then don’t buy or sell any more. Second, sell all the outstanding long-term debt \( B_{-1}^{(j)} \) at time 0, and roll over short-term debt in the right quantity to set \( P_1, P_2, \) etc. as desired via

\[
\frac{B_{-1}^{(j)}}{P_j} = E_j \sum_{k=0}^{\infty} \frac{s_{j+k}}{R^k}.
\]
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More realistic alternatives exist between these two extremes. But to prove an existence theorem, one of them is enough.

Given this sequence of price levels \( P_1, P_2, \text{etc.} \), which may extend past the maturities outstanding \( \{ B^{(j)}_{-1} \} \) – the present value relation (8.32) tells us what the price level \( P_0 \) must be. Any debt policy that generates a given \( \{ E_0(1/P_j); j > 0 \} \) must generate this \( P_0 \). By construction, these policies satisfy the period \( j \) flow and present value constraints for every \( j \), so there are no other constraints.

8.3.1 Unexpected prices; smoothing surplus shocks

As before, it’s worth splitting the analysis into expected and unexpected components. The innovation of equation (8.32) reads:

\[
\sum_{j=0}^{\infty} \frac{B^{(j)}_{-1}}{R^j} (E_0 - E_{-1}) \left( \frac{1}{P_j} \right) = (E_0 - E_{-1}) \sum_{j=0}^{\infty} \frac{s_j}{R^j}.
\] (8.33)

If there are no changes in surpluses, this equation tell us what innovations to expectations of debt sales \( B^{(t+j)}_t \) and beyond can achieve. In particular, they tell us how unexpected current \( B^{(j)}_0 \) and expected future \( B^{(t+j)}_t \) debt sales can create an unexpected price level change \( (E_0 - E_{-1})(1/P_0) \).

We saw that active debt policy can offset surplus shocks. How far can such policy go? Equation (8.33) tells us that with outstanding long-run debt, active debt sales can completely offset the effect of surplus shocks on today’s price level \( P_0 \) – but by accepting shocks to expected future price levels. The maturity structure of debt gives the tradeoff. To emphasize this interpretation, we can write

\[
B^{(j)}_{-1} (E_0 - E_{-1}) \left( \frac{1}{P_0} \right) + \sum_{j=1}^{\infty} \frac{B^{(j)}_{-1}}{R^j} (E_0 - E_{-1}) \left( \frac{1}{P_j} \right) = (E_0 - E_{-1}) \sum_{j=0}^{\infty} \frac{s_j}{R^j}
\]

thereby focusing on time 0. Debt policy can only postpone and smooth through time the inflationary impact of surplus shocks. The surplus shock has to be absorbed by inflation somewhere. In this sense, debt policy can affect the timing of fiscal inflation, but cannot eliminate it entirely.

Section 8.2.1 showed how long-term debt can be a passive buffer, absorbing surplus shocks into the price of bonds rather than the price level, and thereby postponing the
inflationary effect of surplus shocks. Here we see a complementary “active buffer” mechanism as well. By actively selling long term debt in response to shocks, the government can achieve a similar result.

8.3.2 Expected prices

The expected version of equation (8.32) reads

$$\sum_{j=0}^{\infty} \frac{B^{(j)}_{-1}}{R^j} E_{-1} \left( \frac{1}{P_j} \right) = E_{-1} \sum_{j=0}^{\infty} \frac{s_j}{R^j}. \quad (8.34)$$

This equation tells us that

- In the presence of long-term debt, changes in expected debt policy – sales and purchases at time 0 and later, expected at time -1 with no change in surpluses – can influence the expected price level at time 0, and later.

In the last section, we established that with long term debt, an unexpected sale at time 0 and beyond could influence $P_0$, not just $P_t$. Now we verify in general, as we saw in examples, that an expected debt sale at time 0 can also affect price levels at $P_0$, not just $P_1$ and beyond. As in the examples, the dilution effect does not rely on surprising bondholders.

For example, suppose that by some debt policy of the sort we have studied – say announcing a dilutive debt sale that will take place at time 4 – the government at time -1 changes $E_{-1}(1/P_5)$, and no other price except $E_{-1}(1/P_0)$, and also does not change time -1 debt sales. Then,

$$B_{-1}^{(0)} \Delta E_{-1} \left( \frac{1}{P_0} \right) = -\frac{B_{-1}^{(5)}}{R^5} \Delta E_{-1} \left( \frac{1}{P_5} \right).$$

In the absence of long term debt, the expected price level at time 0 would not change.

Other uses of (8.34) center on the effects of immediate rather than future debt sales. Since we usually think about those occurring at time 0, it will be easier to push the time indices forward and divide by $R$, obtaining

$$\sum_{j=1}^{\infty} \frac{B_0^{(j)} Q_0^{(j)}}{P_0} = \sum_{j=1}^{\infty} \frac{B_0^{(j)}}{R^j} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=1}^{\infty} \frac{s_j}{R^j}. \quad (8.35)$$
In this expression, we examine the end-of-period value of government debt, rather than the usual beginning-of-period value based on $B^{(j)}_t$ and $Q^{(j)}_0$ as in (8.32).

In the one-period debt case, we had

$$\frac{B^{(1)}_0}{P_0} Q^{(1)}_0 = \frac{B^{(1)}_0}{R} E_t \left( \frac{1}{P_1} \right) = E_0 \sum_{j=1}^{\infty} \frac{s_j}{R^j}.$$

We concluded that the government could arbitrarily set $E_0(1/P_1)$ by debt policy $B^{(1)}_0$, and thus set the interest rate $Q^{(1)}_0$, since $P_0$ was already determined. The first equality states that the real end of period value of nominal government debt is a constant, independent of the quantity $B^{(1)}_0$ sold.

In equation (8.35) the end-of-period real market value of government debt is again a constant independent of both quantity and maturity structure. Now, however, $P_0$ can change as well as expected future prices – which will also depend on expected future debt sales – and interest rates.

### 8.4 Quantitative easing and friends

Short-term stimulus or cooling from monetary policy can be implemented by forward guidance about interest rate targets, by a path of promised future one-period debt sales, by a set of time-zero long-term debt sales (together with promises not to undo those sales in the future) in the form of quantitative easing, or by a set of direct long-term yield targets, or by direct offers to buy and sell long-term bonds at fixed prices.

I construct two simple examples: In one, the government uses only current and future short-term debt. In the other, it uses only long-term debt and no future debt sales or purchases.

I construct a more realistic example with an outstanding geometric maturity structure. The central bank modifies this maturity structure with one-period debt sales and purchases, and quantitative-easing long-term bond sales and purchases that respect the geometric maturity structure. The resulting intervention, combining long-term bond purchases, short-term issues, and promises not to repurchase the long-term debt and on the path of interest rates, looks like quantitative easing.
In quantitative-easing policies, central banks buy long-term debt, issuing short-term debt (interest-paying reserves) in return. They hope to lower long-term interest rates, and to stimulate current aggregate demand and inflation by so doing. Central banks offer stories for this policy firmly rooted in frictions – segmented bond markets, preferred habitats, and ISLM style aggregate demand. Still, let us ask to what extent and under what conditions the simple frictionless model here can offer something like the hoped-for effects of a quantitative easing policy, or to what extent we obtain neutrality results on which to build models with frictions.

Open-market operations are similar to quantitative easing. In both cases, the government buys bonds and issues reserves. The conventional story told for open market operations is different: They increase reserves, and thereby increase the money stock. Now monetary frictions, $MV = PY$, and changes in the supply of money, rather than bond market frictions and changes in the supply of bonds, are thought to affect aggregate demand.

Here, with neither monetary nor bond market frictions, the effects of these policies will be closely related. The major difference is that open market operations are usually thought of as a way to change short-term interest rates immediately, as in Figure 6.1, while quantitative easing is usually thought of as a way to change long-term interest rates as in Figure 8.1. Open-market operations typically buy short or medium-term debt, where quantitative easing operations focus on longer-term debt. Again, though the model so far is missing the usual ingredients for stories of the impact of open market operations, let us see how far it goes, and to what extent it provides instead neutrality results that frictions must overcome.

(Interest rate changes are not, in fact, associated with immediate open market operations, though that is the textbook story. Interest rates typically rise when central banks announce a higher target, and reserves only adjust slowly after that. For now, I focus on textbook stories.)

Suppose the government wishes to follow the policies graphed in Figure 6.1 or Figure 8.1 with given paths for interest rates in the presence of long-term debt. What are the debt policies that lie behind this operation? How could the government implement these price level paths by buying and selling debt? What debt sales or purchases emerge if the government implements the interest rate targets by offering debt for sale at fixed rates? In particular, is there a debt policy that features an immediate (time 0) lengthening of the maturity structure, an exchange of short-term debt for long-term debt, as in a QE or open market operation?

So, given the path of expected price levels graphed in Figure 6.1 or Figure 8.1 our
job is to reverse-engineer debt policies that produce those price levels. This reverse engineering is a generally useful approach, worth pointing out. Rather than specify a set of debt policies \( \{ B_t^{(t+j)} \} \) and ask what price levels come out, specify instead a sequence of price levels \( \{ P_t \} \), and ask what debt policies \( \{ B_t^{(t+j)} \} \) support them. In the \( Bp = Rs \) rubric, find the set of possible \( B \) given \( p, R, s \) rather than attempt \( p = B^{-1}Rs \) for a given \( B \), and then fish around for \( B \) that give the \( p \) we want.

The present value condition gives us a general recipe:

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)} Q_t^{(t+j)}}{P_t} = \sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}.
\] (8.36)

It’s useful also to describe end-of period values as in (8.35),

\[
\sum_{j=1}^{\infty} B_{t}^{(t+j)} Q_t^{(t+j)} = \sum_{j=1}^{\infty} B_{t}^{(t+j)} \frac{1}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=1}^{\infty} \frac{s_{t+j}}{R^j}.
\] (8.37)

In the latter, \( B_t, P_t \) and \( E_t \) all have the same time subscript. For a given stream of expected price levels or bond prices, these formulas gives us a recipe to produce a maturity structure \( \{ B_t^{(t+j)} \} \) at each \( t \).

To keep it simple, specify a sequence of perfect-certainty price levels, \( \{ P_t, t = 1, 2, 3... \} \), as in the figures after the time-0 shock, and specialize to a constant surplus \( s \). Given the initial maturity structure of the debt \( \{ B_{-1}^{(j)} \} \) the price level \( P_0 \) follows by (8.36) at \( t = 0 \). With prices determined, we know all interest rates \( \{ Q_t^{(j)}, t = 0, 1, 2... \} \) as well. We want to find debt policies \( \{ B_t^{(t+j)}, t = 0, 1, 2.. \} \) that support the price levels, meaning that (8.36) holds at each date.

There are many such debt policies. Start with our two familiar extreme examples. First, a pure short-term debt policy sets at each date \( t \)

\[
\frac{B_{t-1}^{(t)}}{P_t} = \frac{1 + r}{r} s.
\] (8.38)

We are now solving for or reverse-engineering \( B_{t-1}^{(t)} \) given \( P_t \), and this equation tells us the answer. This policy has the flavor of forward guidance. The government
announces what debt actions it will take at dates in the future. These are also the
debt sales that would emerge under forward guidance about interest rate targets,
implemented with a flat supply curve of one-period debt at each future date.

Second, a pure coupon policy sets long term debt at the end of time period 0,

\[
\frac{B_0^{(j)}}{P_j} = s; \quad j = 1, 2, \ldots
\]  

and then does not buy or sell debt in the future. Again, this is now a recipe for \( \{ B_0^{(j)} \} \)
reverse-engineered to produce a given \( \{ P_j \} \). This policy has a flavor of quantitative easing: To lower \( P_j \) and therefore lower the time \( j \) interest rate, the government buys
long term debt \( B_0^{(j)} \) today. This is also the debt quantity that would result under
long-term interest rate targets at time 0, implemented by sales of long-term debt at
fixed prices. Thus, it gives us reassurance that if the central bank were to explicitly
target long-term rates, offering to buy and sell arbitrary quantities, it would not be
swamped with demand – so long as people understand that additional debt in and
out of the central bank does not imply anything about surpluses.

In sum, we have seen that the time 0 stimulus or cooling represented in the price
level paths of Figure 6.1 or Figure 8.1 can be implemented by forward guidance
about interest rate targets, by a path of promised future one-period debt sales as in
(8.38), by a set of time-zero long-term debt sales together with promises not to undo
those sales in the future as described by (8.39), or by a set of direct long-term yield
targets, enforced by flat supply curves of long-term debt.

\subsection{A geometric maturity structure example}

Policies (8.38) and (8.39) are both unrealistic. In each case, the government restructures
the entire stock of debt. In the first case, it rolls all debt into one-period bonds,
and maintains that structure. In the second case it issues very long-term debt and
forswears any further action.

Intermediate and more realistic cases are straightforward to construct however. As
before, the choice of maturity structure, plus expectations of future purchases and
sales give us too many options, not too few. Here I construct an example that is still
simple but a bit more realistic.

Suppose the treasury keeps a geometric maturity structure \( B_{t-1}^{(t+j)} = \Omega^j B_{t-1} \). Suppose
the central bank adjusts this structure by selling or buying long term debt \( \tilde{B}_t^{(t+j)} \),
and by issuing or borrowing reserves $M_{t+1}^{(t+1)}$. Reserves here are just additional one-period debt, with face value $M_{t+1}^{(t+1)}$ payable at time $t+1$. I use the notation $\tilde{B}_t$ and $M_t$ to distinguish the central bank’s modifications of the debt from the treasury’s original issues. Start at a steady state with $\tilde{B}_t = 0$ and $M_t = 0$. From (8.36), the steady state obeys

$$\sum_{j=0}^{\infty} \frac{B\Omega^j}{P} = B \frac{R}{P} \frac{R}{R - \Omega} = \frac{R}{R - 1},$$

and hence

$$\frac{B}{P} = \frac{R - \Omega}{R - 1}. \tag{8.40}$$

Suppose that the treasury keeps this debt quantity unchanged so $B_t = B_{-1} = B$, and all adjustments to the new price level path come from the central bank’s $M_t$ and $\tilde{B}_t$ modifications. As in the coupon example, let the central bank engage in long term bond sales once at time 0, and then let them roll off,

$$\tilde{B}_t^{(j)} = \tilde{B}_{t-1}^{(j)} = \tilde{B}_0^{(j)}, j = 1, 2, 3...$$

In sum, the central bank’s decision is a time-zero quantitative-easing, maturity-structure rearrangement $\tilde{B}_0^{(j)}$, and a sequence of monetary policies $M_t$.

Putting these ingredients together, (8.36) and (8.37) read

$$\frac{M_{t-1}^{(t)}}{B} \frac{P}{P_t} + \sum_{j=t}^{\infty} \frac{1}{R^{j-t}} \left( \Omega^{j-t} + \frac{\tilde{B}_0^{(j)}}{B} \right) \frac{P}{P_j} = \frac{R}{R - \Omega}. \tag{8.41}$$

(I drop $E_t$ in front of $P_j$ as we are looking at a perfect foresight path after a one-time shock. That specification simplifies formulas a bit.) Now, we can reverse-engineer policies to support a given price level path. We can specify a debt policy $\{\tilde{B}_0^{(j)}\}$ – a set of quantitative easing purchases – and find the corresponding monetary policy $\{\tilde{M}_{t-1}^{(t)}\}$. We could also specify a monetary policy $\{M_t^{(t)}\}$ and find the corresponding debt policy $\{B_0^{(j)}\}$. I work these cases out analytically for the exercises of Figure 6.1 and 8.1.

In Figure 6.1, inflation jumps from 0 to $\Pi$ at time 0. Equation (8.36) at $t = 0$ (where $M_{t-1}^{(t)} = 0; B = 0$) implies that $P_0$ jumps by

$$\frac{P_0}{P} = 1 - \Omega \frac{\Pi - 1}{R\Pi - \Omega}.$$
(This is the same as (8.6).) For other dates,
\[
\frac{M_{t-1}^{(t)}}{B} + \sum_{j=t}^{\infty} \frac{1}{R^{j-t} \Pi^{j-t}} \tilde{B}_0^{(j)} = \frac{R \Pi}{R \Pi - \Omega} (\Pi^{j} - 1) .
\] (8.42)

Figure 8.6: Debt policies to support a delayed interest rate decline, or forward guidance, with steady geometric long-term debt outstanding. “All M” gives the path of \(M_{t-1}^{(t)}\) with no debt sales \(\tilde{B}_0^{(j)}\). The “B” line plots debt sales – long term debt sold at time 0 \(\tilde{B}_0^{(j)}\) as a function of maturity \(j\). (The negative value means a debt purchase.) “M” gives the path of \(M_{t-1}^{(t)}\) with debt sales as given by “B.” The “All M” or the combination of “M” and “B” policies are alternatives that produce the same price level path “\(\log(P_t)\).” \(M\) and \(B\) are expressed as percentages of the steady state nominal market value of debt, \(B \sum_{j=0}^{\infty} \frac{R^j}{R^j - \delta} = \frac{R B}{R - \delta}\).

Figure 8.6 plots two debt policies corresponding to a forward guidance policy, with lower future interest rates. This scenario is the negative of Figure 8.1 “easing” not “tightening.” The “\(\log(P_t)\)” line plots the price level we are trying to produce. The lower inflation starting in period 3 produces lower long-term interest rates, and therefore an immediate upward price level jump or stimulus from time 0 to time 3.
The “All M” line produces this price level path by short term debt \( \{ M_{t-1}^{(t)} \} \) alone, i.e. (8.41) with \( \tilde{B}_0^{(j)} = 0 \).

The “M” and “B” lines are a quantitative easing-like alternative. Here the central bank at time 0 buys zero-coupon bonds that mature at times 4, 5, 6, and 7, and lets them mature. The “B” line graphs the face value of these bonds as a function of their maturity at time zero, \( B_0^{(j)} \) as a function of \( j \). The B line is negative, since the policy is a bond purchase. The “M” line displays the monetary policy \( M_t^{(t)} \) at each date \( t \) required along with these debt purchases to produce the given price level path, by (8.41). The central bank purchases long term debt \( \{ \tilde{B}_0^{(j)} \} \) and it issues one-period debt \( \{ M_{t-1}^{(t)} \} \), as in a quantitative easing operation. As the long-term debt rolls off, the central bank returns to standard monetary policy implemented with short-term debt \( M_t \) alone to target interest rates.

To understand these examples it helps to watch the market values of debt, not just the face values plotted in 8.6. The left-hand terms of (8.36) and (8.37) give us a simple rule for reverse-engineering debt policies in this constant-surplus and constant-discount-rate environment: Construct a debt policy so that the market value of debt follows the desired price level \( P_t \) at each date. However, that rule applies to the market value of all debt, including the treasury’s geometric maturity debt as well as the central bank’s modifications. Specializing (8.36), these market values are the left hand sides of

\[
\begin{align*}
    t \leq T & : \frac{M_{t-1}^{(t)}}{B} + \frac{R}{R - \Omega} \left[ 1 + \frac{(1 - \Pi)}{(R\Pi - \Omega)} \left( \frac{\Omega}{R} \right)^{T-t+1} \right] + \frac{P_0}{P} \sum_{j=t}^{\infty} \frac{1}{R^{j-t}} \frac{\tilde{B}_0^{(j)}}{B/P_j} = \frac{P_0}{P} \frac{R}{R - \Omega}. \\
    t > T & : \frac{M_{t-1}^{(t)}}{B} + \frac{R\Pi}{R\Pi - \Omega} + \sum_{j=t}^{\infty} \frac{1}{R^{j-t}} \frac{\tilde{B}_0^{(j)}}{B} = \frac{R}{R - \Omega} \frac{P_t}{P}.
\end{align*}
\]

Figures 8.7 and 8.8 present the market values of each component of debt at each date. I plot \( M_{t-1}^{(t)} \) at date \( t \). (I add the \( t \) notation as a reminder that this is debt coming due at time \( t \).)

Figure 8.7 gives the market value of debts corresponding to the “All M” line of Figure 8.6. This line may have been puzzling. With a pure short-term debt policy, (8.38), with no long-term debt outstanding, the equivalent of \( M_{t-1}^{(t)} \) \( (B_{t-1}^{(t)} \) in (8.38)) follows the \( \log(P_t) \) line. Short-term debt rises at time 1, plateaus, and then starts falling
CHAPTER 8. LONG-TERM DEBT DYNAMICS

Figure 8.7: Decomposition of the market value of debt, for the “All M” policy that implements the price level path using only one-period debt. “θ debt” gives the market value of the treasury’s geometric-maturity long-term debt. “M” shows the contribution of the Fed’s one-period debt. “All debt” is the sum, the market value of all debt, whose pattern mirrors the price-level path.

once the disinflation kicks in. In Figure 8.6 by contrast, the monetary contraction starts right away, even though disinflation waits several periods. What’s going on? The one-period debt is now in addition to the existing geometric maturity structure long-term debt.

Figure 8.7 explains. The line marked “Ω debt” gives the market value of the treasury’s geometric maturity structure debt. When long-term interest rates decline at time 0, this long-term debt jumps up in value. As the day of disinflation and lower short-term rates get closer, its value is more affected by interest rates, so that value rises further. It approaches a new higher plateau once all long-term rates have fallen to their new values.

If we wish, then, to produce the desired price level path, then monetary policy $M_t^{(t)}$ must not only induce the disinflation when it comes, it must offset this higher value of long-term debt that the future disinflation creates, so that the total market value
8.4. QUANTITATIVE EASING AND FRIENDS

Figure 8.8: Decomposition of the market value of debt, for the “Both” policy that includes sales of additional long-term bonds. “θ debt” gives the market value of the treasury’s geometric-maturity long-term debt. “B” gives the contribution of the additional long-term bonds. “M” shows the contribution of the Fed’s one-period debt. “All debt” is the sum, the market value of all debt, whose pattern mirrors the price-level path.

of the debt follows the price level path. The gap between the “Ω debt” line and the “All debt” line is the contribution of monetary policy $\{M_{t-1}\}$. You can see it declines immediately, and then declines even faster once the period of disinflation comes in.

Figure 8.8 presents the analogous decomposition of the nominal market value of debt for the quantitative-easing, “B” an “M” case of Figure 8.6. Again, the decline in long-term bond prices raises the value of the treasury debt in the “Ω debt” line, and monetary policy, both $M$ and $B$, must offset this rise so that the total market value of debt mirrors the desired price level path. The “Ω + B” line adds the market value of the central bank bond purchases $B_{0}^{(j)}$. Since these are purchases, they reduce the market value of the debt. But at first they reduce it too much, requiring increased short term debt $M_{t-1}^{(l)}$. As the $B$ purchases roll off, they reduce the market value of the debt too little, and monetary policy eventually takes over in driving the desired
pattern of total debt.

There is nothing special about the debt policy \( \tilde{B}_0^{(j)} \) assumed in these figures. There are lots of combinations of \( \{ \tilde{B}_0^{(j)} \} \) and \( \{ M_{t-1}^{(t)} \} \) that produce the same price level path. One can find a set of debt purchases \( \tilde{B}_0^{(j)} \) that produce the price level path without monetary policy \( M_{t-1}^{(t)} \). Just solve (8.41) for \( \{ \tilde{B}_0^{(j)} \} \) with \( \{ M_{t-1}^{(t)} = 0 \} \). I do not present these policies because they are uninteresting. Like the pure debt case (8.39), they involve very long-term debt.

Now stop and savor the result: We have an example in which the government stimulates activity at time 0 – higher price level – with no change in short-term interest rates between time 0 and time 3, by buying long-term bonds \( B_t \), sending long-term interest rates lower, and issuing a lot of reserves \( M_t \). This looks a lot like quantitative easing.

The rise in money or reserves \( M_t \) is not equal to the change in value of debt \( \tilde{B}_t \). You might hope for a model of quantitative easing or open market operations in which the central bank buys bonds and issues reserves in exactly the same quantity, getting away from the simple model we started with in which the central bank increases the amount of debt and just drives up interest rates. But the point of open market operations or quantitative easing is to change prices. So a successful model of open market operations and quantitative easing must involve some element of price pressure, not just exchanges at given prices. Some element of increasing the overall amount of debt and watching its price go down must remain. (In an accounting sense, one can write the operation as an exchange in debt for money at fixed prices, and then a change in value due to changing prices.)

The mechanism is quite different from that which central banks talk about. In particular, these examples tie the decrease in long-term interest rates to expectations of lower future short-term rates. In many central banks’ stories for QE, bond buying alters long term interest rates by changing the risk premium in long term bonds. Either mechanism has the same effect on the time - 0 market value of long-term debt, and so on the stimulative effect of QE. Under the risk-neutral measure, a decline in risk premium is the same thing as a decline in expected future short term rates, so we can regard this exercise as describing risk-neutral expectations. Also central banks did give forward guidance of lower interest rates and try to lower long rates by direct expectations hypothesis mechanisms.
8.4.2 Comments on quantitative easing

A neutrality theorem, its limits, and why QE may be useful anyway. Why actual QE may have had smaller effects than we seem to see here.

A summary: We have a unified theory of interest rate targets, forward guidance, QE, and open market operations, that can operate even in a completely frictionless model. However familiar the answers, the mechanisms are completely different from standard models built on frictions.

In these examples, there are lots of ways to produce the same price level path. The government can follow policies that only use short-term debt, (8.38), by policies that only vary short term debt with long-term debt outstanding, the “All M” policy, by policies that use only long-term debt such as (8.39), and by policies that lie in between or have long-term and short-term debt moving in opposite directions, as in the “M” and “B” quantitative easing example. Going back to the left hand expression of (8.36) or (8.37), which I repeat here,

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)} Q_{t}^{(t+j)}}{P_t} = \sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right)}{R^j} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}
\]

\[
\sum_{j=1}^{\infty} \frac{B_{t}^{(t+j)} Q_{t}^{(t+j)}}{P_t} = \sum_{j=1}^{\infty} \frac{B_{t}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right)}{R^j} = E_t \sum_{j=1}^{\infty} \frac{s_{t+j}}{R^j}
\]

We see right away a neutrality theorem for the maturity structure of debt, or for QE and open market operations:

- The only restriction on debt in this reverse-engineering exercise with constant surplus and discount rate is that the total nominal market value of debt at each date move proportionally to the desired price level $P_t$. For given total market value, the maturity structure is irrelevant.

Moreover, the maturity structure at one date $t$ is irrelevant to subsequent price levels, and therefore to interest rates. Buying more long term bonds today does not introduce any state variable for tomorrow’s decisions.

The maturity structure at time $-1$ matters, but only to determine the price level jump $P_0$. After that, the price level sequence $\{P_t\}$ depends only on subsequent debt.

However, there are a few ways in which maturity structure does matter. First, a maturity structure rearrangement alters the timing of debt policy, when the Fed
takes actions, and thus may help it to offer some pre-commitment. Contrast the coupon example (8.39), in which the government sells long-term bonds at time 0, with the short-term debt example (8.38), in which the government adjusts the price at each time \( t \) with debt at \( t - 1 \). Yes, both examples produce the same price level path, showing that nothing per-se about long vs. short-term debt is vital to a given price level path. But the short-term debt policy (8.38) requires future action: future interest rate targets, or future direct higher debt sales. The coupon policy using long-term debt (8.39) is a “fire and forget” policy. It requires no future action.

The quantitative easing example in Figure 8.6 and 8.8 has a similar though more muted flavor. To produce the decline in long-term interest rates that pushes up the current price level using monetary policy \( M_t \) alone, the government must promise a contraction that starts slowly and then really gets going four periods later. The debt purchase policy bakes in some of that contraction, requiring in fact a short run monetary expansion.

Lack of commitment is a central problem with forward guidance, or any aspect of monetary policy that depends on expectations. The central bank may say, in the depths of a recession, and facing a zero bound, that it will keep interest rates low after the recession is over, lower than it will in fact prefer to do ex-post once the recession is over. But central banks have relatively little ability to pre-commit to actions that they would rather not take ex-post. By implementing the same policy with long-term debt, the government takes a concrete action, that left alone, will produce or help to produce the desired price level path.

It’s not quite so easy, of course. The QE policies require not just that the government buy long-term debt, but that it commit not to undo that policy later, either by selling off the long-term debt or by more expansionary short-term debt policies. As we have seen, there is no action the government can take at time 0 regarding the price level at time \( t \) that it cannot undo later. But it is plausibly easier to commit not to undo an action taken today, than it is to commit to take an action tomorrow that may seem ex-post undesirable.

Moreover, these examples suggests it is important for QE operations to live along with a forward guidance statement about interest rates, and for the central bank to state that it will let QE bonds mature – or even reinvest them – rather than re-sell them the moment the central bank thinks the time is right. Both promises were prominent features of the QE operations, and make sense here.

The coupon example (8.39) suggests that QE works by a segmented markets mechanism. There is a separate nominal demand curve for each maturity, so operations
that change maturity structure have important and direct effects on interest rates and the price level. (It’s a nominal segmentation, not a real one.) The fact that the government can undo any current bond sale or purchase by subsequent ones, so only the total current market value of debt matters to each period’s price level, pushes us in the opposite direction, towards a neutrality result that the maturity structure doesn’t matter. The latter observations lead more to the “signaling” view: QE works because it is a pre-commitment device, a signal that interest rates really will be lower than otherwise, rather than having direct effects on bond markets.

But the neutrality result is also more delicate than it seems. The fact that we are finding debt policies consistent with the price level at time $t$ while holding constant all other prices is crucial to the result. It remains true that selling, say, an additional $B_{t+j}$, and taking no other action, raises the price level $P_{t+j}$ and lowers the bond price $Q_{t+j}$, and thus a maturity rearrangement with more $B_{t+j}$ and less $B_{t+k}$ would affect both price levels. Maturity structure matters. By holding future price levels constant, the neutrality of maturity structure at time $t$ implicitly assumes that, if the government sells additional $B_{t+j}$, it will also take some future action, buying back that debt, to have no effect on $P_{t+j}$. So, among debt policies that produce the same sequence $\{P_t\}$, yes, the maturity structure at each time $t$ is irrelevant. But strike the first clause and you strike the conclusion. Really, this irrelevance theorem says again that to understand the effects of any debt policy today we must understand expectations of future policies.

So the maturity structure irrelevance result says that a change in maturity structure that does not affect current or future price levels ... does not affect current or future price levels. Its message is that there are such changes in maturity structure, that the set of debt policies consistent with a given sequence of price levels includes a range of maturity structures.

But the point of open market operations is to change interest rates or price levels. So, again, a successful model of open market operations and quantitative easing must involve some element of price pressure – not just exchanges at given prices. Changes in the overall quantity of debt changes prices, and changes in maturity structure without exactly countervailing future changes do so as well.

Finally, we are only considering the impulse-response function question, how expectations of the future adapt to a single shock. A longer maturity structure changes the response of the price level to future shocks, which are set to zero in a response function calculation.
With this theory in mind, we might wonder why actual quantitative easing in the US, Europe, and Japan seemed so ineffective. It is hard to see any lasting effect of QE on either bond prices or inflation. Central banks argue, naturally, that without their courageous action things would have been worse, but that is a weak argument to explain apparently ineffective policies.

We started with a strong QE: \[ B_0^{(j)} = P_j s_j \] in (8.39) means a one percent decrease in bond supply gives a one percent decrease in price level and a one percentage point decrease in bond price. The subsequent analysis gives plenty of reasons for a weaker QE. Though the Federal Reserve in its quantitative easing operations announced its plan to let long-term debt roll off the balance sheet naturally, and would keep interest rates low for a long time, people may have believed that QE would be reversed or that central banks would use monetary policy (M) to raise interest rates at the customary rate ex-post. Surely if conditions improve, the hawks at the Fed will press for selling off the bond portfolio before it matures. They did so argue, in fact.

Moreover, as usual, debt policy of this sort requires people to expect that changes in debt quantities do not correspond to changes in surpluses. Left out of the analysis here, QE operated on top of variation in surpluses – huge during the great recession – and the debt sales and maturity rearrangements of the treasury, which also came with the usual talk about eventual deficit reduction, i.e. higher future surpluses. I argued above that the institutional separation between central bank and treasury is useful to send different signals. Central bank actions are like share splits, that affect interest rates and not surpluses. Treasury actions are like secondary offerings, that correspond to future surpluses and do not change interest rates. The traditional separation that central banks change the amount of overnight debt relative to very short-term treasury debt, and treasuries set the quantity of long-term debt, can also help to keep those expectations separate. A central bank operating in long-term debt markets leaves open the question whether the large QE changes in long-term debt do or don’t correspond to changes in surpluses. (Among others in the literature, Greenwood et al. (2015) show that Fed purchases have a larger effect on bond prices than the same bonds issued by the Treasury.) In addition, with sticky prices, changes in nominal interest rates move real interest rates, so even if surplus expectations were unaffected by QE, the present values of those surpluses are affected. We will add this effect later.

A final puzzle: If central banks wanted lower long term interest rates, why did they not do so directly? Why did the Fed buy a fixed number of bonds, rather than announce a target for long-term interest rates, say 1.5%, and buy and sell freely at that price? It may have worried that it would have been swamped with near-infinite
demands, and lose control of the balance sheet. This analysis says that no, the bond
demand would have been finite, and in fact small. However, that analysis presumes
people really understand that Fed sales of long term bonds do not come with changes
in fiscal surpluses. If people think of the Fed as acting as an agent for the treasury,
and that every bond sold occasions a rise in future surplus to pay off that bond at
unchanged prices, then the demand curve is indeed horizontal and the Fed would be
swamped by a fixed price offer. Real vs. nominal debt are different things, and work
quite differently. The Fed cannot, in this model, peg a real interest rate.

In sum, the fiscal theory offers a framework that can begin to describe quantitative
easing and open market operations, in the same breath as it can describe interest
rate targets and forward guidance about those targets, even before we add price
stickiness, monetary frictions or liquidity premiums for special assets, or financial
frictions. It offers insights – why promises not to quickly re-sell debt are important,
why combining quantitative easing with forward guidance is important, and that
long-term nominal interest rate targets could work.

The mechanism for quantitative easing here has nothing to do with the usual motiva-
tion. The usual motivation is that via segmented markets for real debt, central bank
bond-buying lowers long-term interest rates even though future surpluses rise one
for one with debt sales. Markets are just unsegmented enough, however, that those
lower long-term treasury rates leak to corporate and household borrowing rates and
stimulate investment, and thereby output. The mechanism here is entirely a wealth
effect of government debt. And the different mechanism makes important predictions
– stimulative effect requires outstanding long-term debt, for example.

8.5 A last word on debt sales

Even the simple three-period examples I explored in this chapter reveal complex
relationships between debt sales, outstanding debt, and expected future debt sales.
All of the effects we studied earlier are substantially modified by expected future
debt sales, including how long-term debt may be a buffer to surplus shocks, and how
current debt sales affect current and future price levels and bond prices. Moreover,
expected future debt sales are important. Our governments maintain relatively short
maturity structures, and therefore roll over debt on a regular basis. They also ac-
tively borrow more in bad times. Yet debt sales, and expected future debt sales in
particular, have subtle and complex effects on the term structure of interest rates
and on current and expected price levels, at least until someone else finds clearer lessons in these formulas than I have found.

This observation leads me to focus more attention on policies that fix interest rates rather than specific debt policies. In addition even this complexity emerged in a set of radically simplified and unrealistic example. Few real-world debt policies or events move debt without changing surpluses, and few policies move current debt, or debt of one maturity, without changing expected future sales and purchases.
Chapter 9

Debt, deficits, discount rates and inflation

In section 3.7 we considered the question, how can it be that inflation is lower in recessions, when deficits are high and often persistently high, and lower in booms, when deficits are low? Likewise, we asked how can it be that inflation is often low in high-debt countries like Japan and the contemporaneous US and Europe, and more generally debts and deficits are poor predictors of inflation. We came up with three potential answers: First, the vast majority of debt is likely issued to fund deficits, and so comes with a promise of higher future surpluses. The surplus process is likely to have negative autocorrelation such that current deficits have no information about the present value of surpluses. Second, the discount rate varies. Real interest rates are lower in recessions, which raises the present value of surpluses and thus is a deflationary force; and real interest rates are higher in booms. Discount rate variation has the potential to generate a Phillips curve even in a frictionless fiscal theory model. Third, the valuation equation holds in equilibrium in all models, a repeated warning on observational equivalence, so a test of the valuation equation would not be interesting if it were possible. We can ask the question which of surpluses or discount rates accounts for specific episodes however, and learn whether the discount rate variation story fits the data.

Those observations were short and verbal. Here, I fill in the details with both data and explicit calculations. This is to some extent a “story” as in the last chapter, but the importance of the issue and need for serious analysis elevate it to a chapter on its own.
This analysis is a prelude to a larger question. How do we account for the historical pattern of inflation, over time, in the postwar period, including its rise in the 1970s and decline in the 1980s, and its regular business cycle pattern? How do we account for cross-country patterns of inflation? The theoretical analysis has been rich in possibilities: We told three stories of debt and inflation, unexpected fiscal news causing unexpected inflation, debt sales with no change in surplus implementing interest rate targets that lead to expected inflation, and debt sales with changes in surplus that lead to more debt and no inflation. We added important dynamic elements: long term debt can serve as a buffer, smoothing the inflationary impact of fiscal shocks over time, it allows the government to actively smooth surplus shocks over time with debt sales, and it gives rise to a temporary negative sign in the response of inflation to unexpected shocks to current or expected future interest rates. We added sticky prices, which also change the dynamic view of surpluses and inflation, most deeply by replacing price level jumps that devalue debt with long periods of real interest rate change that instead let the discount rate equate debt and surpluses. How do we put these possibilities together, to figure out which story maps to which episode?

9.1 A look at the maturity structure

Figure 9.1 presents the maturity structure of US Treasury debt in 2014, on a zero-coupon basis. (Data from Hall, Payne, and Sargent (2018).) The US sells long-term bonds, which combine a large principal and many coupons. I break these up here to their individual components. This is the quantity $B_{t+j}$ of the theory, expressed as a fraction of the total, i.e. $B_{t+j}^{(t+j)} / \sum_{j=1}^{\infty} B_{t+j}^{(t+j)}$. These are face values, not market values $Q_{t+j} B_{t+j}^{(t+j)}$.

The maturity structure is relatively short, with 22% of the debt due in a year or less, and half the debt due, i.e. rolled over, every three years. The bump on the right are principal payments 30 year debt issued in the several prior years of large deficits. The graph also suggests that a geometric maturity structure $B_{t+j}^{(t+j)} = \Omega^j B_t$ is not a terrible first approximation.

Figure 9.2 presents the cumulative maturity structure, the fraction of debt with maturity less than or equal $k$ for each $k$, i.e. $\sum_{j=1}^{k} B_{t+j}^{(t+j)} / \sum_{j=1}^{\infty} B_{t+j}^{(t+j)}$. This graph is a little smoother and thus easier to compare across dates. It shows that the maturity structure has varied quite a bit over time. At the end of WWII, the maturity structure was relatively long, as the US financed the massive WWII debt with a
9.1. A LOOK AT THE MATURITY STRUCTURE

Figure 9.1: Face value of US treasury debt by maturity, on a zero coupon basis, $B_t^{(t+j)}$ in 2014.

A lot of relatively long term bonds. By 1955, the maturity structure had shortened, as the WWII debt got younger, to something like its current state. By 1975, as the WWII debt was largely paid off, the maturity structure was very short. 50% of the debt was one year or less maturity, and over 70% of three year or less maturity. The dynamics of inflation in the 1970s may well have been affected by this short maturity structure. The maturity structure lengthened again however, with the beginning of structural deficits. By 1985, it was longer, again about where it is now.

Just how bad an assumption is the convenient one-period debt model? Is it really important to carry around long-term debt? These graphs suggest that if one considers a “period” to be a few years or more, then one-period debt is not a terrible approximation. If a period is a day, then we really have long-term debt.

In absolute terms, the maturity structure of US debt is quite short. The duration of the assets – present value of surpluses – is very long. So the US has a classic maturity mismatch, rolling over short term debt in the face of a very long-term asset. For example, if the US issued perpetuities, the first graph would be completely flat, and the second graph would increase linearly.
On a scale of several years, then, one might well worry that US inflation dynamics can display the run-like instability associated with short-term debt.

Put another way, the US does not have much of the “buffer” associated with long-term debt. Expected inflation can’t wipe out debt that comes due before the inflation comes. So, for example, even a complete hyperinflation that wiped out all debt in year 3, would leave about 45% of the debt, which pays off before year 3, unscathed. For inflation to devalue one year debt, it must come within one year. Only a very sharp unexpected inflation would do much to lower the value of US debt.

9.2 US surpluses and debt

I plot surpluses and debt for the US. Most variation in US surpluses is related to variation output. Higher output at a given tax rate generates more surpluses. There is little visible correlation between debt, deficits and surpluses. What correlation there
is at business cycle frequencies goes the “wrong” way – higher deficits in recessions correspond to less inflation, and vice versa in booms.

One’s first reaction to the fiscal theory may be, “Surplus, what surplus? We seem to have only perpetual deficits. The right hand side of the valuation equation is negative!” Figure 9.3 plots the US federal surplus in the postwar period. Indeed, except for a few brief years in the late 1990s, the Federal government has run steadily increasing deficits since 1960, even as a percent of GDP.

Figure 9.3: Surplus, unemployment, and recession bands. “Surplus” is the US federal surplus/deficit as a percentage of GDP. “Primary surplus” is that surplus plus interest costs, also as a percentage of GDP. The graph plots the negative of the unemployment rate. Vertical bands are NBER recessions.

However, the valuation equation wants primary surpluses, i.e. not counting interest costs. The “primary surplus” line in Figure 9.3 shows that the US has historically run small primary surpluses on a regular basis.
The difference between the usual surplus/deficit and the primary (net of interest) figure is important to understanding the history of fiscal policy. For example, much of the “Reagan deficits” of the early 1980s represented interest payments on existing debt, as interest rates rose sharply, not large tax and spending decisions.

The primary surpluses in Figure 9.3 follow a clear cyclical pattern, shown by their correlation with NBER recession bands and the unemployment rate. Surpluses fall – deficits rise – in recessions, and then surpluses rise again in good economic times. Surpluses, like unemployment, are related to the level of economic activity, where recessions are defined by negative growth rates. Thus, surpluses correlate well with the negative of unemployment. (The GDP gap, (GDP - potential GDP)/ potential GDP, not shown, looks just about the same as the negative of unemployment in the plot.)

This surplus movement has two primary sources. When income (GDP) falls, tax revenue = tax rate \times income falls. Automatic stabilizers such as unemployment insurance increase spending, and to some extent the government embarks on discretionary countercyclical spending. The business-cycle variation in surpluses has very little to do with variation in tax rates or tax policy, despite media and too many economists’ preoccupation with that issue, e.g. “President x raised taxes.”

In sum, most of the variation we see in primary surpluses is regularly and reliably related to the business cycle, and caused by movements in output. That means most of it is transitory, and does not necessarily tell us all that much about the present value of all future surpluses that appears in the fiscal theory.

(The NIPA debt, surplus, and interest expense measures I use in this graph are poor matches to the quantities in our formulas. For example, interest expense includes only coupons on government bonds, and debt is the face value, not market value of debt. However, proper measurement is not vital for points here, and the readers should know how standard data sources behave before we embark on a project of better measurement. In following sections I measure surpluses and deficits directly from debt sales – how much did the government actually borrow – which creates a series that satisfies identities. Graphically, it is not much different from the NIPA surplus plotted here.)

Figure 9.4 presents the primary surplus along with debt, both as percentages of GDP, and CPI inflation.

The US debt-to-GDP ratio started at 90% at the end of World War II. It declined slowly to 1975, due to a combination of surpluses, inflation, and growth. The down-
ward trend ended with the large (at the time) deficits of the 1970s, and reversed itself with the high interest rates of the 1980s. Debt really rocketed up again starting in the 2008 great recession, rising from 35% of GDP to 75% of GDP and with no end in sight.

The figures make abundantly clear that the dominant story surrounding debt and deficits is that debt is sold in times of temporary need, primarily recessions; it promises higher future surpluses. Thereby, it raises revenue which funds deficits; and then in good times the higher surpluses pay down the debt, at least relative to GDP. The stories involving inflation – debt sold without future surpluses to raise or lower expected inflation, unexpected inflation changing the real value of debt – will have to be seen, if they can be seen at all, through this dominant pattern.

Looking now at inflation in Figure 9.4 overall, fiscal correlations do not jump out of
the graph. On top of the regular cyclical pattern, one can see that primary surpluses declined overall in the low-growth 1970s. Decade-long low GDP growth led to lower tax revenues, and this decline coincided with the large inflation of the 1970s. The economic boom that started in 1982 resulted in large primary surpluses, and the sudden end of inflation. We will pursue the fiscal side of the rise and fall of US inflation later.

But that's it for obvious correlations of debt or deficits with inflation. Primary surpluses have turned into immense primary deficits since 2000, driven by another two-decade growth slowdown, the great recession, and the inexorable expansion of entitlement programs. Long-term fiscal forecasts, such as the Congressional Budget Office's long-term outlook, describe ever rising deficits if policy does not change. Yet inflation has so far continued its slow decline.

There is a clear positive correlation between surpluses and inflation. In each recession, the budget turns to deficit, and inflation falls. In each recovery, the budget turns toward surplus and inflation rises. This is exactly the “wrong” sign for a simpleminded interpretation of the fiscal theory.

9.3 Interpretation

One’s first instinct on seeing our basic fiscal equation

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j} \]

might be to fit a simple model to surpluses, such as an AR(1), use that model to forecast surpluses, and see if variation in the price level matches variation in the surplus. If we fit, say,

\[ s_t - s = \phi (s_{t-1} - s) + \varepsilon_t, \]

then we might calculate a prediction

\[ \frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} s + \phi^j (s_t - s) = \frac{R}{R-1} s + \frac{R}{R-\phi} (s_t - s) \quad (9.1) \]

and see if it holds, either directly or via inflation shocks

\[ \frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left( \frac{P_{t-1}}{P_t} \right) = \frac{R}{R-\phi} \left[ (s_t - s) - \phi (s_{t-1} - s) \right]. \]
Likewise, one might find deficit forecasts, such as from the Congressional Budget Office, plug those on the right hand side, calculate a present value and see if it lines up with the value of debt.

The graphs make it clear just how fruitless this approach would be. Equation (9.1) predicts that the real value of the debt should be positively correlated with surpluses – perfectly correlated in fact. Figure 9.4 shows that this prediction is false. High surpluses produce declining debt, not high values of debt. Low surpluses produce rising debt. Deficits happen in recessions along with less inflation, not more inflation.

Similar points hold across countries. Current debt and deficits are by and large poor forecasters of inflation or devaluation. “What about Japan?” with 200% debt to GDP ratio and deflation, is a common question.

The graphs also suggest a more subtle puzzle. In the fiscal theory, the price level adjusts so that the value of the debt equals the present value of future surpluses. In a passive fiscal regime, surpluses adjust instead to the value of the debt. Figure 9.4 suggests that pattern: The high surpluses of the 1950s to 1960s paid off much of the WWII debt, and look like reactions to that debt. The surpluses of the late 1990s paid off much of the rise in debt from 1980 to then, and look like reactions to that debt. Much hope for avoiding an inflation or debt crisis in the near future rests on the idea that Congress will react to spiraling debts by increasing surpluses. One might more formally test for Granger-causality – do surprises in the value of the debt lead to subsequent surpluses?

One can point out that both approaches are inappropriate. On the first, the observational equivalence theorem tells us that the basic fiscal equation holds, in equilibrium, in every model. If one could reject the equation, it would reject all models, not the fiscal theory.

We know how the first exercise comes out in the long history of empirical asset pricing. If you fit an AR(1) to dividends, or use analysts forecasts, and discount them back at a constant rate, you get asset-price predictions that have essentially no correlation with actual asset prices. This fact has not stopped price equals present value of dividends from being a productive framework for thinking about asset prices for half a century.

This modeling approach assumes that agents have the same information as us who study the economy. One should always be on the lookout for this assumption, and insist on tests of a theory that are valid when agents in the economy have better information than we are.
Economics and finance are littered with tests that fail this criterion. For example, early tests of the permanent income hypothesis modeled income as an AR(1), and then calculated consumption as the present value of that income. In a simplified version, if you model income as

\[ y_t = \rho y_{t-1} + \varepsilon_t, \]

and the quadratic utility PIH predicts

\[ c_t = c_{t-1} + r\beta \sum_{j=0}^{\infty} \beta^j (E_t - E_{t-1}) y_{t+j}, \]

you predict a tight relation between consumption and income,

\[ c_t - c_{t-1} = r\beta \sum_{j=0}^{\infty} \beta^j (E_t - E_{t-1}) y_{t+j} = \frac{r\beta}{1 - \beta \rho} (y_t - \rho y_{t-1}) \]

which is easy to reject. But consumers have more information than the econometrician’s AR(1) model, which destroys this restriction. Modern tests of consumption and asset pricing models following Hall (1978) and Hansen (1982) are (usually) robust to agents that have more information than the econometrician.

Causality tests are classic cases that are sensitive to this information problem. For example, though we think of changes in expected dividends and discount rates as the fundamental causes of changes in asset prices, the asset prices happen first and reveal some of that agent information. Granger causality tests tell you that an unpredictable asset price increase helps to predict higher subsequent dividends or lower returns, and thus asset prices Grange-cause dividends and returns. Likewise, a surprisingly good Friday weather forecast and Granger-causes good weather over the weekend, but the weather forecaster has no causal effect on the weather.

And likewise, if people learn from reading the newspaper rather than fitting an AR(1) that surpluses will be poor, if they rush to sell government bonds and drive up the price level, this decline in the value of government debt will forecast poor surpluses. But causality goes from surpluses to price level, not the other way around.

More deeply, the fiscal vs. monetary regime question is (at best) whether treasury or central bank wins a stylized game of chicken to produce a consistent fiscal and monetary policy. That game need not leave any signs in the time-series patterns of equilibrium debt, surpluses, and inflation.
But this is whining. The fiscal theory, like any theory, must have a first-order plausible story to tell about graphs such as these. We must have a clear quantitative view of the world and its history that accounts for the data, not just general theorems and what-ifs about superior information. The purpose of this section is to take the first step in that direction.

The punchline:

- We do not expect a strong correlation between debt, deficits, and inflation.
- The AR(1) or any similar model that forecasts long-run surpluses from its past is deeply flawed, and cannot estimate the true surplus process, even with infinite data.
- We expect debt to lead, and help to forecast, i.e. to Granger-cause surpluses, even if surpluses are completely exogenous.

Investigating this question, finding what a reasonable surplus process does look like, is important background for interpreting events in the light of the fiscal theory, for bringing the fiscal theory to data, and for constructing realistic fiscal-theory models.

### 9.4 A class of moving average models

Governments who must finance temporary deficits, but do not want unexpected inflation, will choose a negatively correlated surplus process – more deficit today is financed by more debt, which promises more surpluses in the future. If the surplus follows a moving average process $s_t = a(L)\varepsilon_t$, then to avoid inflation the government must set taxes and spending so that $\sum_{j=0}^{\infty} a_j \beta^j = a(\beta) = 0$, which means the $a_j$ must change sign. Here, shocks to current surpluses have no information about the present value of surpluses.

Governments do not like inflation. So let us consider what a surplus process looks like, for a government that does not wish inflation, but occasionally borrows money to fight a war or recession, and then pays it off. Adapting to temporary exigency by tax-smoothing, borrowing rather than temporarily raising taxes, is a time-honored principle of good public finance.

Consider the extreme example of a fiscal-theory government that desires no inflation at all. What would its surplus process look like? When it borrows money to finance
a deficit $s_t$, the government must raise expected future surpluses $\{s_{t+j}\}$. If it did not do so, the government would only create future inflation, and it would raise no revenue. Thus a decline in today’s $s_t$ must be met by a *rise* in subsequent $\{s_{t+j}\}$. The surplus process cannot follow an AR(1).

To formalize this intuition and create some simple models, write a general surplus process as

$$s_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} = a(L)\varepsilon_t.$$  \hspace{1cm} (9.2)

The $\{a_j\}$ form the impulse-response function of surpluses to structural shocks $\varepsilon_t$. The $\{a_j\}$ paint how a shock to surpluses today changes expectations of future surpluses i.e.

$$(E_t - E_{t-1}) s_{t+j} = a_j \varepsilon_t.$$  

If a government wishes no inflation at all, in our simple frictionless fiscal theory model, it uses an interest rate target to set expected inflation to zero, $i_t = r$. Then it arranges its surpluses to generate no variation in unexpected inflation:

$$(E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j} = 0; \beta \equiv 1/R.$$  \hspace{1cm} (9.3)

A surplus process generates no inflation shocks then if and only if

$$\sum_{j=0}^{\infty} a_j \beta^j = a(\beta) = 0.$$  \hspace{1cm} (9.4)

(This lovely formula is due to [Hansen, Roberds, and Sargent (1992)](https://doi.org/10.1017/CBO9780511814100) p. 127; [Sargent (1987)](https://doi.org/10.1017/CBO9780511898806) p. 381-385.)

Condition (9.4) says that the *impulse response function must change sign*. With the normalization $a_0 = 1$, there must be a string of negative following responses $a_j < 0$. The AR(1), in which $a_j$ are all of one sign, is a singularly bad model for surplus processes. The AR(1) cannot have $a(\beta) = 0$. It will fail by construction.

This statement embodies common sense. A government that runs a string of deficits, and does not wish to finance these deficits by unexpected inflation, i.e. defaulting on existing debt, must then run a string of surpluses, to induce people to lend more to finance the deficits. Surplus shocks that do not cause unexpected inflation must involve only a rearrangement of surpluses over time, weighted by $\beta_j$. 
More generally, if a government targets no unexpected inflation in this way, shocks to current surpluses have no information at all about shocks to the present value of future surpluses. The shock to current surplus is $a_0 \varepsilon_t$. The shock to the present value of surpluses is, from [9.3], $a(\beta) \varepsilon_t = 0$.

Well, why beat up on the AR(1)? Just fit a more flexible time-series process, for which $a(\beta) = 0$ is possible. The problem is deeper: *One cannot recover any surplus process that has $a(\beta) = 0$ from an autoregression.* The central problem is that $a(L)$ must by $a(\beta) = 0$ have a root inside the unit circle. Autoregressions can only recover invertible moving averages that have roots outside the unit circle. The project of fitting a time series process to surpluses and discounting its forecast is irretrievably doomed. This point is easier to see in specific examples first. I then return to the general case.

### 9.5 An MA(1) example

I explore an MA(1) example. The surplus follows $s_t = \varepsilon_t + \theta \varepsilon_{t-1}$. A government that wishes to avoid inflation must set $\theta = -R$. The resulting surplus process is not invertible – a regression of surpluses on past surpluses recovers the wrong coefficient and the wrong shock. If one calculates the present value of surpluses from such an estimate, it falsely predicts volatile inflation. Forecasting surplus using debt, one can recover the structural process. Debt Granger-causes – helps to forecast – surpluses, though by construction surpluses cause variation in the value of debt.

The MA(1) gives a simple though unrealistic example. Suppose the surplus follows

$$s_t = a(L) \varepsilon_t = \varepsilon_t + \theta \varepsilon_{t-1}.$$ 

Directly, the value of debt is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \varepsilon_t + \theta \varepsilon_{t-1} + \beta \theta \varepsilon_t = (1 + \beta \theta) \varepsilon_t + \theta \varepsilon_{t-1}. \quad (9.5)$$

If a government wishes no unexpected inflation, the surplus process must follow [9.3],

$$a(\beta) = 1 + \beta \theta = 0.$$
Therefore, the process must follow $\theta = -R$,

$$\begin{align*}
    s_t &= \varepsilon_t - R\varepsilon_{t-1} \\
    \frac{B_{t-1}}{P_t} &= \varepsilon_t - R\varepsilon_{t-1} + \beta R\varepsilon_t = -R\varepsilon_{t-1}. 
\end{align*}$$ (9.6) (9.7)

In words, this government issues debt $B_{t-1}/P_t = \varepsilon_{t-1}$ at time $t - 1$ to fund the surplus shock $\varepsilon_{t-1}$, and then pays it back one period later, with interest $R$. The process (9.6) features negative correlation – a deficit today is followed, on average, by surplus tomorrow.

But the moving average (9.6) is not invertible, so it cannot be estimated by an autoregression. If we try to invert the true process (9.6),

$$\frac{s_t}{1 - RL} = \varepsilon_t$$

$$s_t + Rs_{t-1} + R^2s_{t-2} + ... = \varepsilon_t$$ (9.8)

we see exploding coefficients on the left-hand side. An autoregression of a stationary process always produces convergent coefficients.

What would happen if you ran autoregressions of surpluses from data generated by (9.6)? The answer is, you would recover the wrong coefficient, the wrong error, and you would predict inflation volatility where there is none. Run the autoregression

$$b(L)s_t = \delta_t$$

i. e.,

$$s_t = \sum_{j=1}^{\infty} (-b_j) s_{t-j} + \delta_t.$$ 

Here I use the letter $\delta$ for shocks, as they are regression errors which are not necessarily the same as structural shocks $\varepsilon_t$, and turn out to be different in this case. From this regression you recover a stationary and invertible $b(L)$. When you invert it, you find an MA(1),

$$s_t = \delta_t - \beta\delta_{t-1}$$ (9.9)
Comparing (9.6) and (9.9), you recover a moving average coefficient \( \theta = -\beta = -1/R \) not the correct \( \theta = -R \), and you recover \( \delta_t \neq \epsilon_t = (E_t - E_{t-1}) s_t \), the wrong shock.

Most of all, using (9.5), (9.9) implies

\[
(E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j} = (1 - \beta^2) \delta_t.
\]

You infer that the model predicts unexpected inflation volatility where there is none. The model seems to falsely predict inflation (lower \( 1/P_t \)) in times of unexpected deficits – exactly the apparent prediction that started this whole investigation. This false “failure” of the fiscal theory is exactly what this model predicts you will find in the data! (The mistake, really, is using the same symbol \( E_t \) to mean expectation conditional on agent’s information, which includes the \( \epsilon_t \), and expectation conditional on our information, just the set of current and past \( s_t \).)

1To demonstrate the answer (9.9), we match autocovariances. From the true model,

\[
\begin{align*}
\text{var}(s_t) &= (1 + R^2) \sigma^2 \\
\text{cov}(s_t, s_{t-1}) &= -R \sigma^2 \\
\text{cov}(s_t, s_{t-j}) &= 0; \quad j > 1.
\end{align*}
\]

Since autocovariances past the first are zero, the regression model will also be an MA(1). Write it

\[
s_t = (1 - \theta L) \delta_t.
\]

Matching autocovariances, \( \theta \) and \( \sigma_\delta^2 \) solve

\[
(1 + \theta^2) \sigma_\delta^2 = (1 + R^2) \sigma^2 \\
\theta \sigma_\delta^2 = R \sigma^2
\]

Thus,

\[
\frac{\theta}{1 + \theta^2} = \frac{R}{1 + R^2},
\]

or \( \theta = R \) or \( R^{-1} \). The regression model picks the stable root \( R^{-1} \) not the correct but explosive root \( R \). Then

\[
R^{-1} \sigma_\delta^2 = R \sigma^2 \\
\sigma_\delta^2 = R^2 \sigma^2.
\]

The regression error is also larger than the true error. The true errors \( \epsilon_t \) are a linear function of future regression errors \( \delta_t \). The information set for conditional expectation \( E_t \) is larger than the autoregression information set spanned by current and past \( s_t \). \( \delta_t = s_t - E(s_t|s_{t-1}, s_{t-2}...) \neq \epsilon_t = s_t - E_{t-1}s_t. \)
Now, consider the joint process of surplus and debt. From (9.6) and (9.7), when the government follows a surplus process that eliminates unexpected inflation, the fundamental (using structural shocks $\varepsilon$) joint moving average is

$$s_t = \varepsilon_t - R\varepsilon_{t-1} \tag{9.10}$$

$$\frac{B_t}{P} = -R\varepsilon_t. \tag{9.11}$$

(Here, since $P_t = P$ constant, I locate $B_t/P_{t+1}$ in the time $t$ information set, which clarifies the example. You can also express the example with $B_{t-1}/P_t$ as the time $t$ variable.) More formally,

$$\begin{bmatrix} s_t \\ B_t/P \end{bmatrix} = \begin{bmatrix} 1 \\ -R \end{bmatrix} \varepsilon_t + \begin{bmatrix} -R \\ 0 \end{bmatrix} \varepsilon_{t-1}. \tag{9.12}$$

Inverting this moving average, we find an autoregressive representation

$$s_t = \varepsilon_t + B_{t-1}/P$$

$$\frac{B_t}{P} = -R\varepsilon_t.$$

Or, to be super explicit,

$$\begin{bmatrix} s_t \\ B_t/P \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_{t-1} \\ B_{t-1}/P \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ -R\varepsilon_t \end{bmatrix}. \tag{9.13}$$

The terms here do converge unlike (9.8). The right hand variables are uncorrelated with the error term. So OLS regressions uncover exactly (9.13). You can work backwards: If you run this vector autoregression, including debt on the right hand side,

$$\begin{bmatrix} s_t \\ B_t/P \end{bmatrix} = A \begin{bmatrix} s_{t-1} \\ B_{t-1}/P \end{bmatrix} + \begin{bmatrix} \delta^s_t \\ \delta^b_t \end{bmatrix},$$

this is a consistent estimate of the structural VAR (9.13). Inverting that VAR you can estimate the structural impulse response function (9.10) or (9.12). Equation (9.11) provides the key – the value of debt, which we can observe, reveals to us agents’ information about the structural shock, just as the value of equity reveals to us a slice of agents’ information about future dividends.

Debt shocks help to forecast surpluses, so debt Granger-causes surpluses, even though by construction surpluses are exogenous and cause debt.
9.6 A permanent / temporary example

I explore a tractable and useful example more realistic than the MA(1). The surplus has a permanent and transitory component, \( s_t = z_t + c_t \); \( (z_t - z) = \phi (z_{t-1} - z) + \nu_t \); \( c_t = \rho c_{t-1} + \varepsilon_t \), with \( \phi > \rho \). The model generates a pretty response in which temporary deficits are financed by long-lasting increases in later surpluses, shown in Figure 9.5. Again, the surplus process is not invertible – a regression of surpluses on past surpluses recovers a positively correlated process that falsely predicts inflation volatility. Again, forecasting surplus using debt, one can recover the structural process, and debt Granger-causes – helps to forecast – surpluses, though by construction surpluses cause variation in the value of debt.

The MA(1) is unrealistic. General theorems about \( a(\beta) \) are not salient or useful for working out theoretical models. This section gives a somewhat realistic but still simple surplus process that can capture these ideas.

Suppose the surplus (or surplus/consumption ratio) has a permanent component as well as a transitory AR(1) component.

\[
\begin{align*}
  s_t &= z_t + x_t \\
  (z_t - z) &= \phi (z_{t-1} - z) + \nu_t \\
  x_t &= \rho x_{t-1} + \varepsilon_t.
\end{align*}
\]

Think of the cyclical component \( x_t \) as resulting from largely exogenous temporary events like recessions, wars, or economic booms like the late 1990s. These events result from temporary spending needs or fluctuations in GDP with a fixed tax code. Think of \( z_t \) as set by tax rates or the structure of entitlement programs. These changes are much more permanent.

Thus, in a war or recession, the government has large deficits – negative \( x_t \). To fund the deficits, it issues debt, promises persistently higher taxes to pay off the debt after the war or recession is over – positive \( z_t \). I allow \( \phi < 1 \) to avoid a pure random walk in the surplus/consumption ratio, but \( \phi = 1 \) simplifies formulas even more and does little harm. Think of \( \phi \) as a very large number, however, and \( \rho \) as a smaller number.
With this time-series model,
\[
E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \sum_{j=0}^{\infty} \beta^j \left[ z + \phi^j (z_t - z) + \rho^j x_t \right] = \\
\frac{R}{R-1} z + \frac{R}{R-\phi} (z_t - z) + \frac{R}{R-\rho} x_t.
\]

(9.17)

and
\[
(E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j} = \frac{R}{R-\phi} \nu_t + \frac{R}{R-\rho} \varepsilon_t.
\]

Now, if the government wants to avoid unexpected inflation, the government must choose surpluses so that
\[
\nu_t = -\frac{R - \phi}{R - \rho} \varepsilon_t.
\]

(9.18)

(Again, the interest rate target can set expected inflation to zero, so fiscal policy need only set unexpected inflation to zero.) In words, the government raises persistent taxes or cuts persistent spending in order to fund negative shocks to the transitory part of the deficit.

Restriction (9.18) means that we only have one shock. We can then write a univariate \(s_t\) process as a function of this one shock.
\[
s_t = z - \frac{R - \phi}{R - \rho} \frac{\varepsilon_t}{1 - \phi L} + \frac{\varepsilon_t}{1 - \rho L}
\]

\[
s_t = z + \frac{(\phi - \rho)}{(R - \phi)(1 - \rho L)(1 - \phi L)} \varepsilon_t.
\]

(9.19)

With the \((1 - RL)\) in the numerator, we have \(a(\beta) = 0\).

Figure 9.5 presents the response function (9.19). I plot the response to a unit negative \(\varepsilon_t = -1\) shock, a deficit. As you can see, the time of deficits is persistent. It passes, however, and turns to surpluses to pay back the accumulated debts. The positives exactly counterbalance the negatives in that the weighted sum of these responses \(\sum_{j=0}^{\infty} a_j \beta^j = a(\beta) = 0\).

This is the sort of surplus process we should expect from a government that is trying to stabilize the price level, and largely paying off debts as they come and go.

Unexpected fiscal inflation, or expected monetary policy inflation that raises nominal \(B_t\) but not real \(B_t/P_{t+1}\) debt, will operate on top of such a surplus process. For
Figure 9.5: Surplus impulse-response function for the permanent-transitory model. The AR response is what one would infer from a regression of surpluses on past surpluses. $\phi = 0.975$, $\rho = 0.7$, $R = 1.05$.

example, people may distrust that the persistent component of surpluses will rise quite as much as needed to fully pay off the debt. Then some of the deficit shock is met by an unexpected inflation. The government may choose such a response, meeting the bad news with an effective state-contingent default. Or, the required permanent component may run into long-run Laffer limits – permanent taxes reduce the growth rate of the economy enough that the present value of revenues does not increase. But the sort of response plotted in figure (9.6), not an AR(1), is the baseline we should keep in mind and modify as needed.

Canzoneri, Cumby, and Diba (2001) write of examples like Figure 9.5, “NR [fiscal-theory] regimes offer a rather convoluted explanation that requires the correlation between today’s surplus innovation and future surpluses to eventually turn negative.” The point here is that this sort of response is not at all convoluted. It is exactly what one expects from the classic tax-smoothing theory of public finance (Barro (1979)). Borrow now, and promise higher surpluses later to pay off the debt. If you do not or cannot make that promise, you do not raise any revenue from borrowing.

One cannot recover the surplus response from running autoregressions of surpluses
on their past values, as this is also a non-invertible representation. No matter how complex a time-series model you allow, you will get it wrong. If you run autoregressions or fit an ARMA model to data generated by the model (9.19), you will recover an estimated model

\[ s_t = z + \frac{(\phi - \rho)(1 - \beta L)}{(R - \rho)(1 - \rho L)(1 - \phi L)} \delta_t \]  

(9.20)

rather than (9.19), where the \( \delta_t \) are residuals from the regression of \( s_t \) on lagged \( s_{t-j} \). You recover \( \beta = 1/R \) not \( R \) in the moving average term, and the regression error \( \delta_t \) is not the true shock \( \varepsilon_t \). In the not-unreasonable case \( \beta = \phi \), you recover exactly an AR(1) response function with coefficient \( \rho \), and miss all of the negative responses.

Figure 9.5 also presents this implied estimated response function (9.20), the response to a single unit \( \delta_t = -1 \) shock. (The variance of the \( \delta \) shocks is also larger, so one will also mis-estimate the size of a one-standard-error shock. I graph the response to a unit shock to focus on the shape.) The response functions are broadly similar, but this one, fitted by a regression of surpluses on lagged surpluses, misses most of the rise in surpluses that pays off the debt. Hence, it predicts counterfactual surprise inflation associated with deficits.

Moreover, both response functions in Figure 9.3 and given in (9.19) (9.20) are a type that bedevil conventional time-series modeling for the purpose of measuring present values and long-run responses. The long string of positive responses, equivalently a long string of small negative autocorrelations, are crucial to paying off debt; to bringing the cumulative response \( \sum_{j=0}^{\infty} \beta^j a_j \) back to zero. Conventional time-series techniques focus on fitting the first few autocorrelations well, which are the dominant contributors to one-step ahead mean-square error which is the conventional goal of model-fitting. Equivalently, autoregressive and moving average roots which nearly cancel have very little impact on short-run forecasts which time-series techniques aim to improve, but are crucial for long-run forecasts. The responses in Figure 9.5 would be very well fit by an AR(1), and you would likely never know how disastrously wrong the AR(1) is for its long run implications. These dangers of long-run implications of short-run forecasts are known in the time-series literature (Campbell and Mankiw (1987), Cochrane (1988)), though they are easy to forget, and the long-run risks

\[ 2 \text{To show that the autoregression results in (9.20) in this more general case, it’s easier to match spectral densities rather than autocorrelations. The spectral density of a moving average process is } S(\omega) = a(e^{-i\omega})a(e^{i\omega})\sigma^2. \text{ The invertible process has all roots of } a(z) \text{ outside the unit circle. So, we can find the invertible from the true process by just inverting all of its roots.} \]
literature uses highly parametric models to infer crucial long-run properties from short-run dynamics.

Figure 9.6: Simulation of the permanent-transitory surplus model. Parameters $z = 1.1$, $\rho = 0.8$, $\phi = 0.95$, $\sigma_z = 2$.

Figure 9.6 presents a simulation of this permanent-transitory model. I picked parameters by eye to roughly match the dynamics of Figure 9.4. And you see that it does match that graph. There is no simple relation that debt or the price level is proportional to surpluses. Instead, when surpluses are positive, debt falls. When surpluses are negative, debt rises. The government seems to run surpluses to pay off debts, following a passive fiscal policy, though the example is constructed under the explicitly opposite assumption. The only thing missing is that inflation is completely constant here. Generating countercyclical inflation is the next step.

As in the MA(1) example, you can estimate the surplus process, if you use debt data. To that end, find how debt behaves in the example. Let tildes denote de-meaned variables, i.e.

$$\tilde{B}_t \equiv \frac{B_t}{\bar{P}} - \frac{R}{R - 1} \tilde{z}$$
$$\tilde{s}_t \equiv s_t - \tilde{z}.$$
Use (9.17) and (9.18) to obtain

\[
\bar{B}_{t-1} = -\frac{R}{R - \rho} \frac{\varepsilon_t}{1 - \phi L} + \frac{R}{R - \rho} \frac{\varepsilon_t}{1 - \rho L}
\]

\[
\bar{B}_{t-1} = -\frac{R}{(R - \rho)(1 - \rho L)(1 - \phi L)} \varepsilon_t
\]

(9.21a)

The structural autoregressive representation of debt is then

\[
(1 - \rho L)(1 - \phi L) \left( \frac{B_{t-1}}{P} - \frac{R}{R - 1} \varepsilon_t \right) = -\frac{R(\phi - \rho)}{(R - \rho)} \varepsilon_{t-1}.
\]

Substituting out \(\varepsilon_t\) in the surplus process from (9.19) from (9.21a), it is

\[
\bar{s}_t = -\frac{(1 - RL) \bar{B}_t}{R}.
\]

In sum, the structural vector autoregressive representation of debt and surpluses is

\[
\frac{\bar{B}_t}{P} = (\rho + \phi) \frac{\bar{B}_{t-1}}{P} - \rho \phi \frac{\bar{B}_{t-2}}{P} + \delta_t
\]

(9.23)

\[
\bar{s}_t = -\frac{1}{R} \frac{\bar{B}_t}{P} + \frac{\bar{B}_{t-1}}{P}
\]

(9.24)

where

\[
\delta_t = -\frac{R(\phi - \rho)}{(R - \rho)} \varepsilon_t.
\]

The structural vector moving average representation, uniting (9.21a) and (9.19), is

\[
\frac{\bar{B}_t}{P} = \frac{1}{(1 - \rho L)(1 - \phi L)} \delta_t
\]

(9.25)

\[
\bar{s}_t = -\frac{1}{R (1 - \rho L)(1 - \phi L)} (1 - RL) \delta_t.
\]

(9.26)

Now, the vector autoregression is finite, and the inverse of the structural moving average. The VAR errors \(\delta_t\), recovered from the regression of debt on past debt, are, up to a constant, the same as the structural regression errors \(\varepsilon_t\), and since \(\rho, \phi, \) and \(R\) are observable, the \(\varepsilon_t\) can be recovered from the \(\delta_t\). Past debt shocks forecast surpluses, so debt Granger-causes surpluses, even though by construction exogenous surpluses cause variation in debt.
9.7 A general moving average

I make the same points as in the last two sections in the context of a general moving average \( s_t = a(L)\varepsilon_t \). No surprise inflation means \( a(\beta) = 0 \). \( \beta \) is inside the unit circle, so the moving average is not invertible. A surplus autoregression falsely predicts inflation volatility. Again, forecasting surplus using debt, one can recover the structural process, and debt Granger-causes – helps to forecast – surpluses, though by construction surpluses cause variation in the value of debt.

I warn that the apparently promising avenue to include debt in the VAR does not generalize to time-varying discount rates, which are the empirically important case.

Now, let’s see the same points in the general analysis of section 9.4. The surplus follows

\[ s_t = a(L)\varepsilon_t \]

with \( a(\beta) = 0 \). One cannot estimate \( a(L) \) and test \( a(\beta) = 0 \) from any autoregression. The condition for a moving average \( a(L) \) and test \( a(\beta) = 0 \) from any autoregression is that all the zeros of \( a(z) \) lie outside the unit circle. The condition \( a(\beta) = 0 \) means that a zero lies inside the unit circle, so this structural representation is not invertible. Second, debt Granger-causes surpluses even though by construction surpluses are exogenous.

When we factor \( a(L) \),

\[
a(L) = \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - RL)}{(1 - \delta_1 L)(1 - \delta_2 L)...}, \tag{9.27}
\]

for \( a(\beta) = 0 \), one of those factors must be \((1 - RL)\) as written. But since \( R > 1 \), you can’t write this surplus process in autoregressive form

\[
\frac{(1 - \delta_1 L)(1 - \delta_2 L)...}{(1 - \lambda_1 L)(1 - \lambda_2 L)} \frac{1}{(1 - RL)} s_t = \varepsilon_t
\]

since the \((1 - RL)\) root blows up going backwards. If you do run an autoregression you recover a representation that is invertible, by the Wold decomposition theorem. If all the other \( \delta \) and \( \lambda \) in the structural representation are appropriately less than one, an autoregression yields

\[
\frac{(1 - \delta_1 L)(1 - \delta_2 L)...}{(1 - \lambda_1 L)(1 - \lambda_2 L)} \frac{1}{(1 - \beta L)} s_t = \delta_t.
\]
You have both the wrong root, $\beta$ not $R$, and the regression error is not the structural shock $\delta_t \neq \varepsilon_t$. After you invert, the resulting estimate of $a(L)$ does not have $a(\beta) = 0$ even in infinite data. Thus, it implies innovations to inflation that do not exist. Though the surplus follows an exogenous univariate process, the whole procedure of estimating a surplus process and discounting it is wrong.

To see that debt Granger-causes surpluses, we need to find the vector autoregressive representation for surpluses and debt together, and then verify that regression shocks to debt help to forecast surpluses.

Given a surplus process with $a(\beta) = 0$, debt follows the structural moving average representation

$$\frac{B_{t-1}}{P_t} = \frac{a(L)\varepsilon_t}{1 - \beta L^{-1}} = \frac{s_t}{1 - \beta L^{-1}} = \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

(9.28)

Yes, debt equals the ex-post as well as expected present value of surpluses. We can get to (9.28) by

$$\frac{B_{t-1}}{P_t} = s_t + \beta B_t E_t \left( \frac{1}{P_{t+1}} \right)$$

and iterate forward. The fact that people know the government will adjust surpluses $\{s_{t+j+1}\}$ to offset shocks to $s_{t+1}$ to give a constant price level is the key in this example.

More elegantly, for general $a(L)$ the value of the debt follows the Hansen and Sargent.
9.7. A GENERAL MOVING AVERAGE

(1981) prediction formula for geometric sums

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \frac{a(L) - a(\beta)\beta L^{-1}}{1 - \beta L^{-1}} \varepsilon_t \tag{9.29}
\]

Thus, if \(a(\beta) = 0\) we have \(9.28\).

Now, express \(9.28\) using the factor representation of \(a(L)\), \(9.27\)

\[
\frac{B_{t-1}}{P} = \frac{a(L)\varepsilon_t}{1 - \beta L^{-1}} = -\frac{a(L)RL\varepsilon_t}{(1 - RL)}
\]

\[
= -\frac{R}{(1 - RL)} \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - RL)}{(1 - \delta_1 L)(1 - \delta_2 L)...} \varepsilon_{t-1}
\]

or, moving the time index forward,

\[
\frac{B_t}{P} = -\frac{R}{(1 - \delta_1 L)(1 - \delta_2 L)...} (1 - \lambda_1 L)(1 - \lambda_2 L)... \varepsilon_t. \tag{9.30}
\]

The debt, though it is a strange-looking present value of future surpluses, is in fact a proper function of current and past shocks \(\varepsilon_t\), because the surplus process wipes out any shocks to that present value. The non-invertible root cancels – the debt is an invertible moving average of the structural shock. Thus, one can recover the structural shock from an autoregression of debt on past debt.

\[\text{For a simple derivation, write out the terms of}\]

\[
\sum_{j=0}^{\infty} \beta^j s_{t+j} = \sum_{j=0}^{\infty} \beta^j \sum_{k=0}^{\infty} a_k \varepsilon_{t+j-k}
\]

and collect terms in each \(\varepsilon_t\):

\[
\begin{array}{cccccccc}
... & \varepsilon_{t+2} & \varepsilon_{t+1} & \varepsilon_t & \varepsilon_{t-1} & \varepsilon_{t-2} & \varepsilon_{t-3} & ... \\
\beta^0 a_0 & \beta a_1 & a_2 & a_3 & ... \\
\beta^2 a_0 & \beta^2 a_1 & \beta^2 a_2 & \beta^2 a_3 & \beta^2 a_4 & ... \\
\end{array}
\]

The innovation result \(9.3\) is the \(\varepsilon_t\) column. The result \(9.29\) comes by subtracting the future terms from the total, leaving only the terms in \(\varepsilon_t, \varepsilon_{t-1}\) and so on.
Equation (9.30) is therefore also the moving average representation of that autoregression of debt on past debt, up to a normalization of the size of the shock $\varepsilon_t$. It, together with (9.2) and (9.27),

$$s_t = a(L)\varepsilon_t = \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - RL)}{(1 - \delta_1 L)(1 - \delta_2 L)...} \varepsilon_t$$  \hspace{1cm} (9.31)

are now the moving average representation of the debt and surplus VAR. And VAR shocks to debt $\varepsilon_t$ help to forecast surpluses, so debt Granger-causes surpluses.

One might get excited by these VAR and Granger-causality examples. Yes, estimating a surplus process that excludes debt and discounting it will not work. But it seems one can run an autoregression that includes debt, recover the structural shocks $\varepsilon_t$, run a regression of surpluses on current and past debt shocks as in (9.31), and test whether $a(\beta) = 0$. \cite{Hansen, Roberds, Sargent} (1992) propose this test, and generalize to the case that some of the other zeros of $a(L)$ are inside the unit circle.

Alas, this test does not extend to a time-varying discount rate, and time-varying discount rates are central to making sense of the data, so this idea does not work either with a time-varying discount rate. The next section 9.8 brings current methods for evaluating present value relations with time-varying discount rates to the problem.

9.8 What to do?

Pure tests of the present value relation are not worth pursuing. Do not repeat the decades of controversy it took macroeconomics and finance to get to this result. The interesting question is how discount rate variation can account for variation in the value of debt and for inflation.

In order to fit the pattern of debt, deficits, and inflation in US data, we need a model past the permanent-transitory example of section 9.6. That model produces reasonable debt and deficit paths, but with a constant price level it does not recover the main point of this whole enterprise, a theory of inflation. The natural first approach would be to generalize the model by loosening the perfect correlation between permanent and transitory surplus shocks (9.18). But to produce less inflation in a recession, we would have to assume that the permanent component of surplus increased more than needed to stabilize the price level, in the middle of a recession.
9.8. **WHAT TO DO?**

This seems hardly likely, so there is no point in pursuing the suggestion. A more likely path to understanding data is to introduce time-varying discount rates – real interest rates fall in recessions, which raises the present value of surpluses. Really matching the time series naturally will require a model of sticky prices as well.

If fitting a time series model to surpluses and discounting it is not a valid test of the present value relation, than how does one compute such a test? Once again, the fiscal theory need not (yet) innovate as this set of problems pervades macroeconomics and finance. We just need to avoid repeating the missteps of the past, and apply current technique, at least before innovating technique. This is a hallowed question, going back to volatility tests in asset pricing, tests of the permanent income hypothesis in macroeconomics, and tests of government intertemporal budget constraints presuming real rather than nominal debt.

The question is, what are the testable implications of a present value relation, allowing for discount rate variation? The short answer: The only almost-testable implication of the present value relation per se (allowing discount factors to vary, but not introducing an explicit discount factor model) is the transversality condition, which states (loosely) that real debt is not expected to grow faster than the real interest rate. If the real interest rate is greater than the growth rate of surpluses, then the debt to GDP ratio must not grow arbitrarily. Historically, debt to GDP ratios don’t exceed a factor of about 2.5 before default, inflation, or a return to surpluses, so the proposition passes the eyeball test. Formal tests are essentially unit root tests, with the usual finite-sample uncertainties of those tests.

If the real interest rate is forever less than the growth rate of surpluses, \( r < g \) then the transversality condition also fails. Here the debt to GDP ratio does not grow, though debt itself grows faster than the interest rate. The theoretical foundations of this view are subtle, so postpone it to in section 21 below. If it’s true, and scalable, then government debt is a fountain of prosperity as it never needs to be repaid. It’s a view that has been tried many times in the past, with uniformly disastrous results. Moreover, it does not lead to an interesting analysis of inflation.

Once we admit the transversality condition, the present value relation, using the ex-post rate of return as a discount rate becomes an identity. More generally, the theorem that there always exists a discount factor to make a present value relation hold appears. Hope of testing the present value relation, and only the present value relation, vanishes. The hope of testing present value models in macroeconomics (permanent income) and asset pricing (volatility tests) without discount rate models largely evaporated in the mid 1990s. And since the government debt valuation
equation is part of the equilibrium of every model, again, testing it isn’t interesting in
the first place. Testing the out-of-equilibrium or equilibrium-formation mechanism
of the equation is, let’s say, a more advanced subject.

The more interesting question, which we can answer, is whether variation in the
value of debt, and variation in inflation, corresponds to news about future surpluses,
or to news about future discount rates. We can measure those discount rates directly
as the ex-post returns on government debt, or we can see if models of the discount
rate based on ex-ante data provide useful signals. That is the subject of the next
section.
Chapter 10

Discount rates

In asset pricing, we have learned that discount rate variation, and risk premium variation in particular, is central to understanding the time-series and cross-sectional variation of asset prices. The same lesson is likely to hold for the value of government debt. We are likely to be fortunate, however, in that the discount rate of government debt is likely to be measurable by observed nominal and real interest rates to a greater extent than is the discount rate for riskier assets like stocks. As previewed several times, for example, the decline in inflation during a recession is likely to correspond to a reduced discount rate, which we will learn a lot from by observing a low real interest rate.

10.1 The risk premium for government debt?

A puzzle: The fiscal theory seems to predict a high discount factor, as surpluses are a risky process, while the real interest rate on government debt is low. This seems like one more armchair refutation of the fiscal theory.

Write the government debt valuation formula

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j}.
\]

(10.1)

\[\text{I'm grateful to Bob Hall, who brought up this puzzle.}\]
Figure 9.3 shows that surpluses fall in recessions, when consumption falls, and just as dividends fall in recessions. If government debt is a claim to a procyclical payoff, it should have a lower value and higher expected return than risk free assets. Yet we observe the opposite.

The kind of surplus process graphed in Figure 9.5 holds the key to the puzzle. When the current surplus falls in a recession, expected future surpluses rise. The current "dividend" falls, but expected future dividends rise. If surpluses were a dividend, the price would rise in recessions, so the overall return would rise, not fall, in recessions.

In fact, long-term government bond prices rise in recessions, and the price level falls in recession, so all government bonds get unusually good real returns as we fall into recession. This negative beta generates the low average return on government bonds – which makes sense with a negatively autocorrelated surplus such as graphed in the structural line of Figure 9.5. Again, the univariate autoregression line of that graph hides the negative serial correlation, which also hides the crucial response to this paradox.

To look at an equation, start by breaking (10.1) up

\[
\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j}.
\]

Since \( s_t \) is known at time \( t \), it will be more fruitful to look at the end of period value,

\[
\frac{Q_t B_t}{P_t} = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} s_{t+1} \right] + E_t \left[ \sum_{j=2}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \right] = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} s_{t+1} \right] + E_t \left[ \beta \frac{u'(c_{t+1}) Q_{t+1} B_{t+1}}{Q_{t+1} B_{t+1}} \right].
\]

The first term, and more generally the first few such terms, generates the apparent paradox. The surplus \( s_{t+1} \) is positively correlated with consumption \( c_{t+1} \), and thus negatively correlated with marginal utility growth. That negative correlation lowers the value on the left hand side, and thus raises the required return. But \( \{ s_t \} \) is negatively autocorrelated. So, when consumption \( c_{t+1} \) declines, the value \( Q_{t+1} B_{t+1} / P_{t+1} \) rises, and it is this rise which generates the low required return.

To see this point explicitly, we need to examine the real return to an investor who holds a single bond, rather than the evolution of the entire value of government
A VALUE DECOMPOSITION

10.2. A VALUE DECOMPOSITION

The value of government debt increases when the government issues more debt to fund a deficit, even if the rate of return to individual investors is completely unaffected. The individual investor does not receive the cashflow \( s_t \). He or she receives the promised $1. Surpluses only enter as they affect the price level.

Since I have simplified to one period debt, the real return to a bond investor is just

\[
R_{t+1} = \frac{1}{P_t} \frac{P_{t+1}}{Q_t P_{t+1}}.
\]

Thus, if the government controls the surplus in such a way that the price level is constant, then government debt is riskfree. The negatively autocorrelated surplus process is how the government achieves a predictable price level.

In the case that the price level is constant, a deficit \( s_t \) represents a flow of new revenue, received from new investors. It is then paid off by the larger \( s_{t+j} \) paid to those new investors. Fluctuation in the value of government debt \( B_t/Q_t P_t \) or \( B_{t-1}/P_t \) reflects entirely variation in the total quantity of debt, not fluctuation in the rate of return to an individual investor.

If the government fixed the surplus process, or followed an autoregressive surplus process, allowing frequent and large surprises to the price level to devalue debt when deficits grow, then yes, inflation would be large in recessions when marginal utility is high, the consumption beta of government debt would be positive, and it would bear a positive risk premium. That we do not see a high risk premium in normal times, advanced economy debt, is one more sign that the surplus process is in fact negatively autocorrelated as illustrated in Figure 9.5. It is not perfectly so – there is surprise inflation, the permanent component of surpluses evidently does not exactly cancel the transitory component and discount rate fluctuations also matter. But it is closer to that extreme than to the other. Countries or times that frequently allow unexpected inflation (or exchange rate devaluation) in bad times can expect higher risk premiums on their debt.

10.2. A value decomposition

Here, I use the linearized present value identity from section 6.7 to decompose variation in the value of government debt into terms reflecting the effect of surpluses and of discount rates.
Starting with the present value identity for long term debt,

\[ \sum_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} \]

section 6.7 scaled by GDP and linearized, obtaining (6.25),

\[ \tilde{v}_{t-1} = \sum_{j=0}^{T} \beta^{j+1} \tilde{s}_{t+j} - \sum_{j=0}^{T} \beta^{j} (\tilde{r}_{t+j}^{n} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j}) + \beta^{T+1} \tilde{v}_{t+T+1}, \quad (10.2) \]

or, taking the limit,

\[ \tilde{v}_{t-1} = \sum_{j=0}^{\infty} \beta^{j+1} \tilde{s}_{t+j} - \sum_{j=0}^{\infty} \beta^{j} (\tilde{r}_{t+j}^{n} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j}) . \quad (10.3) \]

Most simply, we can in the style of Shiller (1981) just calculate the terms of (10.2). This calculation tells us how the value of debt resolved ex-post – by greater surpluses or by lower rates of return, and if the latter by more inflation or lower interest rates. It gives us an “ex-post rational” interpretation of history. If people knew exactly what was going to happen, this is why debt had the value it did. That’s unrealistic, of course, but agents always know more than we do, so it is an upper bound on what they could have known.

We can also use (6.26) to decompose the value of the debt, following Campbell and Shiller (1988) or Cochrane (1992). From (6.26), and advancing the time index by one,

\[ \text{var} (\tilde{v}_t) = \sum_{j=1}^{\infty} \beta^j \text{cov}(\tilde{v}_t, \tilde{s}_{t+j}) - \sum_{j=1}^{\infty} \beta^{j-1} \text{cov} (\tilde{v}_t, \tilde{r}_{t+j}^{n} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j}) \]

Dividing by \(\text{var} (\tilde{v}_t)\), or directly by regressing left and right hand sides of (10.2) on \(\tilde{v}_t\),

\[ 1 = b \left[ \sum_{j=1}^{\infty} \beta^j \tilde{s}_{t+j}, \tilde{v}_t \right] - b \left[ \sum_{j=1}^{\infty} \beta^j \left( \tilde{r}_{t+j}^{n} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j} \right), \tilde{v}_t \right] \]

where \(b[y, x]\) is the regression coefficient of \(y\) on \(x\). In this way, forecasting regression coefficients provide a variance decomposition of the value of debt. Debt can only vary if it forecasts either higher surpluses or lower discount rates, and the regression coefficients tell you how much.
Since (6.26) is an identity, it also holds in expected value using any information set.

\[ \tilde{v}_t = \sum_{j=1}^{\infty} \beta^j E_t (\tilde{s}_{t+j}) - \sum_{j=1}^{\infty} \beta^{j-1} [E_t (\tilde{r}^n_{t+j}) - E_t (\tilde{r}_{t+j}) - E_t (\tilde{\pi}_{t+j})]. \]

This formula lets us examine various methods for measuring expected surpluses and discount rates to see if they account for the value of debt. Though using interest rates to measure expected returns on stocks does not work in this context, expected returns on government debt may well be measurable from interest rates.

A quick look at Figure 9.4 warns us that this procedure will run into difficulty however. The story of the value of debt in the last century in the US consists of two or three data points. WWII borrowing led to a huge increase in debt. It declined, and we can profitably characterize the contributions of surpluses, interest rates, inflation and growth to that history. Debt increased again starting in the 1980s, and how that will be resolved has yet to be determined. Taking variances and long-term forecasts of such a time series is not likely to be profitable.

One wishes for a decomposition of the cyclical components of debt, ignoring these large secular trends. To do that, we can apply any filter to both sides of e (10.2)

\[ a(L)\tilde{v}_t = \sum_{j=1}^{T} \beta^j a(L)\tilde{s}_{t+j} - \sum_{j=1}^{T} \beta^{j-1} a(L) (\tilde{r}^n_{t+j} - \tilde{r}_{t+j} - \tilde{g}_{t+j}). \]

For example, we can understand growth in debt \( v_t - v_{t-1} = (1 - L)\tilde{v}_t \) in terms of growth of the surplus, and changes in rates of return.

### 10.3 A decomposition for inflation

The calculations of the last section address the value of the debt, but our central concern really is the determinants of inflation. They are two quite different objects. For example, as we have seen many times, governments may borrow to fund current deficits, promising future surpluses; thereby raise debt, and then pay off the debt with those surpluses. We see an increase and then decrease in the value of debt, all due to variation in future surpluses, but inflation is constant through the episode. Likewise, a decline in real interest rates increases the value of debt, which then declines by paying back at lower than expected interest rates, despite no variation in surplus or inflation at all. Measuring these scenarios is important for public finance, but not for
our cause of understanding inflation. With the frictionless model in mind, we want to look at unexpected inflation and the corresponding revision in the value of debt, ignoring the above mechanisms; we want to look at expected inflation and “share split” increases in the value of the debt only.

To this end, we write (10.3)

$$\tilde{v}_{t-1} + (\tilde{r}_t^n - \tilde{\pi}_t - \tilde{g}_t) = \sum_{j=0}^{\infty} \beta^{j+1} \tilde{z}_{t+j} - \sum_{j=1}^{\infty} \beta^j (\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j}). \tag{10.4}$$

Taking innovations, with any information set that includes \(\tilde{v}_{t-1}\), that variable drops and

$$\left(E_t - E_{t-1}\right) \left[\tilde{\pi}_t - (\tilde{r}^n_t - \tilde{g}_t)\right] = -\left(E_t - E_{t-1}\right) \left[\sum_{j=0}^{\infty} \beta^{j+1} \tilde{z}_{t+j} + \sum_{j=1}^{\infty} \beta^j (\tilde{r}_{t+j} - \tilde{g}_{t+j})\right]. \tag{10.5}$$

The first term is unexpected inflation, which is the object we’ve been looking at this whole book. The second term is the unexpected return on the portfolio of government bonds. With one period debt and no growth, this would be zero, as \(r^t_t = i_{t-1}\) is known ahead of time. With long-term debt, this term measures how much news about the present value of surplus shows up in bond prices, and thus in the frictionless model in future rather than current inflation. The first term on the right hand side is our friend, the revision in a constantly discounted present value of surpluses. The second term on the right hand side is the discount rate effect: A rise in the expected return of government bonds lowers the value of future surpluses.

The different treatment of the time-t term and the future terms looks strained at first glance. One is tempted to treat all the inflationary terms together, and write the model as a constraint on a long-term average of inflation \(\sum_{j=0}^{\infty} \beta^j \tilde{\pi}_{t+j}\) instead. But separating out this term makes sense. In the frictionless model, \(E_t \left(\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j}\right) = 0\) and \(E_t \left(\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j}\right) = 0\) always. Expected inflation and nominal bond returns move one-for-one. That is not true of the first term. For example, in the one-period debt model \((E_t - E_{t-1}) (\tilde{r}_t^n) = 0\), while there can be lots of unexpected inflation. The revision in the value of expected future inflation and returns play a distinct role to the revision in the ex-post value of current inflation and returns.
10.4 A decomposition for inflation in a sticky-price model

The last section explored the mechanisms of inflation in the frictionless model – how unexpected inflation came only from revisions to fiscal policy or discount rates; how expected inflation came only from increases in debt that did not move surpluses; how surprises in bond prices could lead to an unexpected inflation. The dynamics of these mechanisms are, however, tied to the quite unreasonable frictionless price assumption. In a sticky price model, for example, we found that a surplus shock with constant real interest rates leads to a protracted period of inflation. At the moment of learning of the shock, the price level does not jump. Instead the extra inflation relative to nominal interest rates lowers the discount rate. So we see a fiscal shock and a discount rate movement at the same time, offsetting each other to produce no price level shock. Its the same mechanism, but fundamentally different dynamics.

To address these questions we step outside of single equation identities. We write a full model, with fiscal shocks, monetary policy shocks, and so forth; with an explicit sticky price mechanism; and then we investigate what shocks account for the historical time series.

[INCOMPLETE]

10.5 A fiscal Phillips curve

How do we account for the fact that inflation is low in recessions, when deficits are large, and inflation is high in booms, when deficits are low? As I have previewed, the natural way to account for this fact is via discount rates. Interest rates are low in a recession, and risk premiums are high driving demand for government debt. This move drives the price level down. Interest rates are high in booms, and risk premiums are low, as people demand less government debt and more risky assets. This move drives the price level up.

Yes, most economics needs models with extensive frictions to generate a Phillips curve. But we can at least see if it is possible to generate the curve with this simple model – and perhaps to account for the Phillips curve’s frequent failures.
This section evaluates the fiscal Phillips curve story quantitatively. Do real interest rates and risk premiums move enough? Enough to offset the contrary surplus news?

[INCOMPLETE]

10.6 Japan, US and Eurozone at the long and debt-laden lower bound

Going from time-series to cross-section, how is it that Japan has little inflation despite 250% debt to GDP ratios? How does the US and Europe have low inflation despite 100% debt to GDP ratios? And future deficits as far as the eye can see as well? Why is this fiscal situation different from the 1970s, which also featured rising, though lower, debt, rising surpluses, and an emerging productivity slowdown?

Again, I have offered discount rates as a potential explanation. Japan and Europe borrow at negative nominal rates! Real rates in the US are exceptionally low as well. Just why interest rates are so low is beyond this book. But does this story hold up quantitatively? Is the cross-country variation in interest rates sufficient to justify low inflation in these high debt countries? What happens if interest rates change?

[INCOMPLETE]

10.7 Dynamic inefficiency, \( r_{i,g} \), and a safe-asset premium for government debt

[INCOMPLETE]
Part II

Monetary doctrines and institutions
Our understanding of monetary affairs often can be captured by a set of doctrines—statements about the effects of various monetary arrangements or policy interventions. Examples include “interest rate pegs are unstable,” “the government must control inside money production to control inflation.” “I use the word “doctrines” to capture the fact that many of these propositions are not tied to particular models. They are sets of beliefs handed down in a largely verbal tradition, much like military or foreign policy “doctrines”. They are also more robust than specific models that embody them.

This discussion also helps to understand how fiscal theory matters. As we saw earlier, the mechanisms of conventional models are present in the fiscal theory. Inflation comes from “too much money chasing too few goods,” excess “aggregate demand,” or a “wealth effect of government bonds.” A follower of these schools would not notice the fiscal theory in operation by casual observation—which is a good thing, since those casual observations carry much experience. And the observational equivalence theorems make it sound like the whole exercise might be vacuous, or a scholastic exercise in arguments about off-equilibrium threats.

However, the operation of monetary policy, the outcomes of different policy arrangements, the “doctrines” of monetary policy, are quite different under the fiscal theory. So the fiscal theory is not boring, obvious, or empty. Moreover, experience is putting many classic doctrines to the test. The distinction between “money” and “bonds” is vanishing; interest rates stuck at zero did not move to offset inflation for nearly a decade in the US and EU, and nearly a quarter century in Japan. Yet inflation remains quiet. This experience can provide nearly experimental, or cross-regime, evidence on fiscal vs. classic theories of inflation that time-series tests within a regime cannot, by the observational equivalence theorems, easily distinguish.

This chapter contrasts core doctrines under the Fiscal theory with their nature under classic monetary theory, in which the price level is determined by $MV = PY$ and control of the money supply, and under interest-rate targeting theory, in which the price level is determined by an active interest rate policy and the cashless limit (but not limit point) of $MV = PY$. I develop those alternative theories in detail in later chapters. However, since the point now is to understand what the fiscal theory says rather than to understand those alternative theories in detail, since these doctrines are likely familiar to most readers and stand apart from specific models, we can proceed now to discuss classic doctrines and fill in details of the monetary and interest rate views later.
Chapter 11

Monetary policies

The monetarist tradition states that $MV = PY$ sets the price level $P$. Fiscal policy (if mentioned) “passively” adjusts surpluses to pay off unexpected inflation or deflation-induced changes in the value of government debt. This theory does not just require a money demand – an inventory demand for special liquid assets. It also requires a restricted supply of such assets.

Monetarist tradition says that the split of government liabilities $M$ vs. $B$ is important to the price level, not the overall level of such liabilities, because only the $M$ part cause inflation. Monetarist tradition also emphasizes that private money such as bank accounts can provide $M$, and thus inside money and credit need to be controlled. Therefore, monetary doctrine has long argued against monetary policies that do not restrict supply of government money or its substitutes, as such policies seemingly unmoor the price level.

The fiscal theory does not rely on $MV = PY$ for price level determinacy. With $P$ determined, in fact, money supply must be “passive” (unless velocity is also able to move to accommodate price level changes.) So, fiscal theory rehabilitates passive money supply. That’s a good thing, as passive money policies easily and increasingly characterize our world.

To fix ideas, it’s worth remembering (6.12)

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right).$$

(11.1)

The split between $B$ and $M$ only affects $P_t$ through usually small seigniorage effects,
when money pays less interest than other government debt. And those go the other way. If money demand increases, the government makes more seigniorage revenue and, fixing surpluses, the price level declines.

An “uncoordinated regime” or a “game of chicken” emerges if the government both controls money supply, and does not adapt fiscal policy to inflation, i.e. if both money and fiscal policy are “active.” That often happens when the Treasury is in trouble but the central bank tries to fight a fiscally induced inflation with money supply restrictions, financial repression, or high interest rates. One must give way. We’ll think more about when one or the other wins later. The point for now is that a passive money policy is possible, and need not give rise to uncontrolled inflation or deflation.

“Passive money” comes in many guises, and most of the point of this discussion is to illustrate the variety of such policies that have been followed in the past or are advocated now, and how the fiscal theory rehabilitates them.

11.1 An elastic currency to meet the needs of trade

- Classic doctrine: Elastic money supply does not determine the price level, so leads to unstable inflation or deflation.

- FTPL doctrine: Elastic money supply is consistent with a determinate price level.

Suppose monetary policy consists of open market operations (buy $B$, issue $M$), but the central bank decides on open market operations “passively.” It measures $PY$, estimates $V$, and issues the appropriate $M$. From a monetarist perspective, you can see the flaw. If the price level starts to rise, the central bank will issue more money, the price level keeps rising, and so forth. Any $P$ is consistent with this policy.

Yet even the title of the 1913 Federal Reserve act states that the first purpose of the Federal Reserve to “furnish an elastic currency,” and says nothing about the control of inflation or money supply. “Passive” money supply is exactly what Congress had in mind. The price level was, at the time, considered to be determined in the long run by the gold standard. But it was viewed that banks, issuing notes, private markets and the Treasury’s currency issues did not sufficiently move money supply to match demand. There were strong seasonal fluctuations in interest rates, such as
around harvest time, and a perceived scarcity of money. Financial crises also smelled of a lack of money. So, quite naturally, the Fed’s directive was to supply money as needed.

In a more modern and general sense, it is desirable for money supply to accommodate changes in real income $Y$, so that higher output need not cause deflation. And it is desirable for money supply to accommodate shifts in money demand – shifts in $V$ – rather than force those to cause inflation or deflation as well. The trouble is as always distinguishing just where a rise in money demand comes from, reacting to the “right” ones, but not to rises in money demand that just mean higher inflation.

Fiscal theory frees us from this conundrum. The price level is fixed by fiscal policy, and the supply of debt either directly or via an interest rate target. These passive money policies do not threaten the classic reaction of money to inflation.

11.2 Real bills

- Classic doctrine: A real bills policy leads to an uncontrolled price level.
- FTPL doctrine: A real bills policy is consistent with a determinate price level. It provides necessary passive money without the central bank having to measure velocity and its shocks.

The real bills doctrine states that central banks should lend money against high-quality private credit, as well as government credit. Bring in a “real bill,” either as collateral or to sell to the central bank, and the central bank will give you a new dollar in return, expanding the money supply. The Federal Reserve Act’s second clause says “to afford means of rediscounting commercial paper,” indeed suggesting something like real bills wisdom.

A real bills doctrine endogenizes the money supply as well, and in classic monetarist thought it therefore destabilizes the price level. As $P$ rises, people need more $M$. They bring in more real bills, either private or government, to get it, and $M$ chases $PY$.

Under the fiscal theory, a real bills doctrine does not destabilize the price level. The price level is determined by (11.1). Allowing private parties to determine the split of government liabilities between $M$ and $B$ – accepting treasury debt as a “real bill” makes no difference to the price level, up to second-order seigniorage effects, which are not the focus of the traditional argument anyway.
In fact, the real bills doctrine is a good monetary policy. The demand for money,

\[ M_t V(t, t) = P_t Y_t \]

contains many variables, represented as \( t \), that shift money demand around. Some are obvious – money demand is higher at Christmas and April 15. Most are not, and are essentially unmeasurable by the central bank. So, by allowing people to get money any time they need it, in exchange for debt, the central bank accomplishes the passive money that must accompany the fiscal theory; it “provides an elastic currency,” to “meet the needs of trade,” without itself having to measure velocity or decide on open market operations.

(Much motivation for real bills doctrine also concerns the supply of credit, and avoiding financial panics. We can think of a panic as a sharp decline in \( V \), which would otherwise cause a sharp decline in credit as well as money. A real bills doctrine is a sort of automatic lender of last resort.)

If the central bank accepts private “real bills,” that seems to expand total government liabilities on the left side of (11.1). But it also expands the supply of real government assets, thus providing exactly the required additional stream of surpluses on the right side of (11.1) so that the price level is unchanged.

The classic logic of the real bills doctrine has some of this flavor. Extra money is not inflationary because it is backed by real assets, the real bills of the doctrine. Backed money can be supplied elastically and retains its value.

However, there remains the nagging question of just what interest rate the government should offer on its real bills loans, or at what discount should it accept the real bills that it takes. So real bills are really an interest rate target, combined with a passive balance sheet.

In sum we have

- **Classic doctrine**: A real bills policy leads to an uncontrolled price level.
- **FTPL doctrine**: A real bills policy is consistent with a determinate price level. It is a good way to provide the necessary passive money without the central bank trying to measure velocity and its shocks.
11.3 Open market operations

- Classic doctrine: Open market operations cause interest rates to decline and then inflation. The composition, not quantity, of government debt matters for inflation. The size of the central bank’s balance sheet drives the price level.

- FTPL baseline: Open market operations have no first-order effect on the price level or interest rates. The composition of government debt ($B$ vs. $M$) is irrelevant. The size of the central bank’s balance sheet is irrelevant.

Seigniorage and liquidity demands for different kinds of government debt modify the latter baseline.

The primary instrument of classical monetary policy is an “open market operation.” In such an operation, the central bank buys bonds, issuing new money in return.

Central banks today issue reserves – electronic accounts that banks have at the Fed – but reserves can be freely exchanged for cash, so conceptually the central bank issues newly printed cash.

This is not the only way that central banks have to change the supply of base money (cash + reserves). They also can lend to banks, issuing “newly printed money,” and considering the value of the loan as the corresponding asset. Much of current Federal Reserve policy consists of repurchase agreements. The central bank buys a bond, but agrees to reverse the transaction later. This operation is equivalent to lending newly-printed money for a fixed period using the bond as collateral. Historically, central banks did a lot of “rediscounting,” either buying private securities or lending with those securities as collateral.

Central banks do not just print money and hand it out. Many monetary models specify “injections” or “transfers” of cash, but that’s not how our world works. Milton Friedman coined the humorous idea of dropping cash from helicopters if one wanted to create inflation. But central banks don’t do this. Handing out money without receiving something in return is fiscal policy, and central banks do not do it, and are legally not allowed to do it.

This omission is not a mistake. In a monetarist view, $MV = PY$ sets the price level. Whether the money supply increases because the Fed buys bonds, buys stocks, lends it out to banks, or simply drops it from helicopters makes no difference at all.

In particular, in the monetarist view, any interest rate effect of monetary policy comes from the quantity of money and interest elastic money demand $MV(i) = PY$. 
It does not come from bond supply. You might think that buying bonds reduces the supply of bonds, and that drives up interests rate. That’s the conventional idea behind quantitative easing. But in the classic monetary analysis, this is not true. In part, the changes in bond supply for normal open market operations are tiny relative to the stock of bonds.

In sum, the central instrument of classic monetary policy is an open market operation, an exchange of \( B \) for \( M \) with no change in the total quantity of debt. It is a change in the composition, in the maturity and liquidity structure of government debt that does not change the overall quantity of government debt.

In the classic monetarist view, \( MV(i) = PY \) an open market operation changes \( M \) so strongly affects interest rates and the price level. Fiscal policy, when mentioned, is passive – the government adjusts surpluses ex-post to pay off whatever windfalls that unexpected inflation or deflation incurs on bond-holders.

From

\[
\frac{M_{t-1} + B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}
\]

the natural first interpretation of the fiscal theory is that open market operations are completely irrelevant.

Equivalently, the size of the central bank balance sheet is irrelevant. In most governments all money is a liability of the central bank, backed by government debt which are the assets of the central bank. The monetary base is equal to the size of the central bank balance sheet.

Think of money as green M&Ms, and debt as red M&Ms. Taking some red M&Ms and giving you back some green M&Ms has no effect on your diet. To monetarists, only the green M&Ms make you fat, so exchanging your green M&Ms for red M&Ms really does affect your diet.

- Classic doctrine: Open market operations cause interest rates to decline and then inflation. The composition, not quantity, of government debt matters for inflation. The size of the central bank’s balance sheet drives the price level.
- FTPL: Open market operations have no effect on the price level or interest rates. The composition of government debt (\( B \) vs. \( M \)) is irrelevant. The size of the central bank’s balance sheet is irrelevant.

Now, let me quickly qualify the latter statements.
We should distinguish the case that money pays full market interest, so we are satiated in liquidity. That is approximately the case as I write. In this case $M$ and overnight debt $B$ are perfect substitutes, and the above statement goes through.

However, when money pays less interest than overnight debt, or when an open market operation involves a maturity rearrangement, we do not have exactly zero effect.

When money and bonds are not perfect substitutes, we need to consider just what an open market operation means. If we add money demand $M_t V = P_t Y$ to the fiscal theory, that condition must hold as well as the valuation equation (11.2). If we exchange $B_t$ for $M_t$ and proclaim no effect on $P_t$ from the fiscal theory, we have ignored that $M_t V$ has to equal $P_t Y$. An active fiscal regime must have a passive money regime, and an active money regime (fixed $M$ with $MV = PY$) must have a passive fiscal regime, so within this set of equations, an open market operation doesn’t make immediate sense.

The previous analysis of the money-less regime extends to describe the result. Think of the simple frictionless model with one-period debt. We considered as an increase in debt $B_t$ with no change in surpluses, and found that it would raise the nominal interest rate $i_t$ and expected future price level and inflation. The central bank sells $B_t$ in return for cash. But since people do not want to hold more cash overnight — $V = \infty$ — they come back with extra cash at the end of the day and bid up bond prices.

That flavor of analysis still applies. Modify the model so that all securities trading and tax payments happen in the morning. Cash used for transactions at date $t$ must then be held overnight from time $t$ to time $t+1$, $M_t V = P_t Y$. (We will study a cash in advance model with this timing below.) For simplicity, assume a constant surplus.

We have

$$\frac{B_t + M_t}{P_{t+1}} = \frac{1}{R} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} = \frac{R}{R-1}$$

$$\frac{B_t + M_t}{P_t} E_{t-1} \left( 1 + \frac{P_t}{R P_{t+1}} \right) = \frac{B_t + M_t}{P_t} \frac{1}{1 + i_t} = \frac{1}{R} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} = \frac{1}{R-1}$$

$$M_t V = P_t Y; \quad M_{t+1} V = P_{t+1} Y.$$
Now, however, the government must raise $M_{t+1}$ in the next period’s morning. So, there is an open market operation – a rise in $M_{t+1}$ at the expense of $B_{t+1}$. We can view the events then as an announcement at time $t$ that the open market operation will take place at $t+1$, this announcement changes the expected price level at $t+1$ and thus interest rates and bond prices at $t$, and then changes the quantity $B_t$ of bonds the government will sell to raise the same real revenue. Or we can view the events as a bond sale or interest rate target at $t$, followed by accommodative monetary policy. As usual, active vs. passive are observationally equivalent and hence must not be that well defined anyway.

The important point – even in this stark frictionless example, if there is a money demand, we will see events in which open market operations are connected to inflation. A coordinated fiscal-monetary regime must respect both money demands and fiscal backing.

This discussion so far also ignores seigniorage effects that occur when money pays less than full interest. One might also hope that a fiscal theory would rescue traditional views, but by emphasizing the fiscal consequences of monetary policy. However, seigniorage effects are typically small, as we have seen. (Hyperinflations are a different story, which we will return to.) Seigniorage effects are also typically the “wrong” sign. Increasing the money supply with constant $V$ raises seigniorage revenue. Like any rise in the surplus, then, it reduces the price level. For example, start with the expression (6.13) that counts seigniorage from money creation,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right). \quad (11.3)$$

Consider a steady state with no money creation,

$$\frac{B}{P} = 1 + \frac{r}{r} \tilde{s}.$$

Consider a one-time open market operation at time $t$, trading less $B_t$ for increased $M_t$. With $\Delta M_t = M_t - M_{t-1}$, (11.3) becomes

$$\frac{B_{t-1}}{P_t} = \frac{B}{P_t} = 1 + \frac{r}{r} s + \frac{\Delta M_t}{P_t}.$$

Dividing the last two equations,

$$\frac{P}{P_t} = 1 + \frac{\Delta M_t / P_t}{B/P} = 1 + \frac{\Delta M_t P}{B P_t}.$$
11.3. OPEN MARKET OPERATIONS

\[ \frac{\Delta P_t}{P} = -\frac{M \Delta M_t}{B \cdot M}. \]

Higher money growth leads to lower inflation. But the effect is small. With \( M \approx 1.5 \) trillion (and usually much less) and \( B \approx 20 \) trillion, \( M/B \approx 0.075 \) so 10% money growth means -0.75% inflation. With interest-elastic demand on the left of the corresponding Laffer curve, seigniorage is even less.

All of these examples suffer from an extreme and unrealistic money demand specification. Even a die-hard monetarist would not take seriously the prediction from \( M_t V = P_t Y_t \) that if the money supply increases at 12:00 PM Monday morning that nominal GDP will rise proportionally on Monday afternoon. There are “long and variable lags.” Velocity is only “stable” in a “long run.” Short-run elasticities are different than long-run elasticities. It takes a while for people to adjust their cash management habits. More money just leads to less velocity, at least for a considerable time, and then pressure on nominal GDP rises. If you want a serious analysis of what happens under an open market operation in the fiscal theory, you really have to pursue a model with sticky prices plus a reasonable money demand function of this sort. The simple fact that money demand is interest elastic \( V(i_t) \), and the interest rate is related to expected inflation, \( i_t \approx E_t \pi_{t+1} = E_t p_{t+1} - p_t \) generates an interesting monetary-fiscal interaction, the famous Cagan dynamics, a puzzle solved by fiscal theory. Chapter 17 explores these issues in more detail.

Today almost all “money” pays interest. Traditional cash and 1960s bank accounts are special, in that they do not pay interest. Then, when the interest spread between money and bonds must adjust to greater or lesser quantities of liquid assets, the interest rate on bonds and expected inflation must adjust to generate the needed spread. But even bank accounts now can pay interest, and almost all liquid assets used as cash in the legal economy pay interest. Now we should think of \( MV(i - i^m) = PY \) where velocity depends on the interest cost of holding money, \( i_t - i^m \).

Now, variation in the composition of government debt of varying liquidity values, including reserves as well as on the run vs. off the run, or popular vs. unpopular issues, will just result in a change in the money vs. non-money spread – the interest rate \( i \) need not change so that \( i - i^m \) may change. Since we are observing in overnight rates \( i^m \) not \( i \), the observation that \( i^m \) changes when quantities change may simply reflect this mechanism, but \( i \), connected to inflation, is unaffected.

These thoughts suggest a fun, serious evaluation of the effect of open market operations in the fiscal theory, with a coordinated fiscal-monetary regime, a money supply rule, and money that pays less than market interest, a variety of liquid assets. I don’t
pursue examples along this line mainly because I do not think they are interesting in
the current policy environment. (They may be quite relevant to economic history.)
Our central banks follow interest rate targets, and as I will argue below the whole
concept of money demand has turned to mush.
Chapter 12

Interest rate targets

In fact, central banks almost always follow interest rate targets, or exchange rate targets, they do not control monetary aggregates. Interest rate targets are criticized by traditional doctrine, as letting inflation get out of control. The fiscal theory gives us a complete theory of inflation under interest rate targets, that allows us to analyze times when central banks follow targets.

12.1 Interest rate pegs

- Classical doctrine: An interest rate peg is either unstable, leading to spiraling inflation or deflation, or indeterminate, leading to multiple equilibria and excessively volatile inflation.

- Fiscal theory: An interest rate peg can be stable, determinate, and quiet (the opposite of volatile).

An interest rate peg can be thought of as another form of passive money supply, that much standard monetary theory has long held leads to a loss of price level control.

First, as crystallized by [Friedman (1968)](1968), an interest rate peg leads to unstable inflation. In a section titled “What Monetary Policy Cannot do,” the first item on Friedman’s list is “It cannot peg interest rates for more than very limited periods.”
Intuitively, Friedman also starts from the Fisher relationship \( i_t = r_t + E_t \pi_{t+1} \). One of the two great neutrality propositions of his paper is that the real interest rate is, in the long run, independent of the rate of inflation. (The other proposition is that the unemployment rate is also in the long run independent of inflation.)

But to Friedman, the Fisher equation describes an unstable steady state. The Fed cannot fix \( i_t \) and expect expected and thus actual inflation to follow. Instead, if (say) the interest rate peg \( i_t \) is just a little bit too low, the Fed will need to expand the money supply to keep the rate down. This will increase will lead to more inflation, more expected inflation, and when the real interest rate settles down, a target that is even more too low. The Fed will then need to create even more money. Eventually it must give up and raise the interest rate peg, bringing back the Fisher equation at a higher level of interest rate and inflation.

Standard ISLM models with adaptive expectations give the same result, though through a different mechanism. In that view, the real interest rate directly affects aggregate demand. So a too low nominal rate implies a too low real rate. This low rate spurs aggregate demand, which via the Phillips curve produces more inflation. When expectations catch up, via the Fisher equation the real rate is lower still, and off we go.

A deflation spiral when interest rates are effectively pegged by the zero bound, widely feared in 2008, is the same mechanism with an opposite sign.

Adaptive expectations are a common element of these views. (We will study models of this phenomenon in section 16.) When rational expectations came along a different problem with interest rate pegs became standard doctrine. Under rational expectations, as in the models we have studied, the Fisher equation is stable – \( E_t \pi_{t+1} \) does settle down to \( i_t - r_t \) when \( i_t \) is pegged. But, as crystallized by Sargent and Wallace (1975), an interest rate peg leads to indeterminate inflation. The Fisher equation \( i_t = r + E_t \pi_{t+1} \) nails down expected inflation, but unexpected inflation \( \pi_{t+1} - E_t \pi_{t+1} \) can be anything. Such indeterminacy leads to excess inflation volatility.

As we have seen, the fiscal theory of monetary policy contradicts these doctrines. An interest rate peg can leave the price level stable and determinate, and inflation can be quiet. Even a peg at zero could work. A slight deflation would emerge, producing a positive real rate of interest.

I emphasize “can” here, because a stable, determinate, and quiet peg requires fiscal policy as well as the interest rate peg. In the frictionless model, bad fiscal news leads to unexpected inflation. In the model with price stickiness, bad fiscal news leads
to long periods of inflation. Countries with unsustainable deficits cannot just lower interest rates and expect inflation to follow! Countries with volatile fiscal policies, or who suffer volatile discount rates, will see volatile unexpected inflation under a peg.

Also, though a peg may be possible, it is not likely to be optimal. Under a peg, variation in the real rate of interest \( r_t \), due to variation in the marginal product of capital for example, must express itself in variation in expected inflation. When prices are sticky, such variation in expected and therefore actual inflation will produce unnecessary output and employment volatility. A central bank that could assess variation in the natural rate \( r_t \) and raise and lower the interest rate in response to such variation could produce quieter inflation and by consequence output.

12.2 Taylor rules

The Taylor rule – interest rates should vary more than one for one with inflation – makes inflation stable under interest rate targets in adaptive expectations models, and it is thought to make inflation determinate in rational expectations models. Thus, the modern statement of conventional doctrine is that pegs and passive policy – interest rates that react less than one-for-one to inflation – lead to instability or indeterminacy.

The fiscal theory gives stable and determinate inflation under passive interest rate targets.

A third doctrine of interest rate targets emerged in the early 1980s, after Friedman and Sargent and Wallace wrote. The Taylor rule that interest rates should vary more than one for one with inflation cures instability in the first case (adaptive expectations) and indeterminacy in the second (rational expectations).

(McCallum (1981) is the first paper I know of in the modern literature with the result that raising interest rates more than one for one with inflation either stabilizes the price level or renders it determinate. Carozzi and Taylor (1985) is the first Taylor paper with the result, (I asked Taylor) in the form that the interest rate should increase with the price level to stabilize the price level. As Woodford (2003) emphasizes, that idea traces to Wicksell (1965), originally Wicksell (1898), so it’s been around a while.

Taylor suggests the general idea was in the air previously, in the form that real
interest rates should rise with inflation. The Taylor principle is also a likely property of a constant money growth rule, so not inconsistent with Friedman’s view, though he would have preferred money rather than interest rate as the operating target. Taylor’s later work made clear how important the result is, as the key to allowing an interest rate target to work. It is a genuinely new theory of the price level. That the same principle brings determinacy to the already stable new-Keynesian apparatus – a dramatically different result – is central to that model as summarized in Woodford (2003).

So, standard doctrine now goes beyond pegs, and states that interest rate targets cause instability (adaptive expectations) or indeterminacy (rational expectations) when interest rates cannot or do not follow the Taylor rule. In this view, the 1970s is a classic example of interest rate targets that do not respond enough to inflation. Chapter 16 looks at all the problems of the new-Keynesian view, and argues that it really doesn’t work. Also, while somewhat adaptive expectations are a reasonable modeling choice for fitting dynamics in normal times or unusual events, requiring adaptive expectations for the basic stability and determinacy properties of inflation leaves monetary economics on perilous foundations. If people ever catch on, the castle crumbles.

For the discussion here, though, we should qualify the last section to say that classic doctrine now says that interest rate targets are unstable or indeterminate when the targets do not vary sufficiently with inflation. That condition includes a peg, but also policies in which interest rates react to inflation but not enough. The fiscal theory contradicts this doctrine, allowing such policies to leave stable and determinate, hence quiet, inflation. The contrast between fiscal theory and active (interest varies more than one for one with inflation) interest rate rules is about other issues – whether one wants to found all monetary theory on adaptive expectations, and whether the equilibrium-selection stories of new-Keynesian models make any sense, which I defer to Chapter 16.

The three views on interest rates do not argue with each other because they came sequentially. Friedman wrote before rational expectations and the Taylor rule had been dreamt of. Sargent and Wallace wrote before the Taylor rule. It is only in the last decade or so that the quite different properties of a Taylor rule in adaptive (stability) vs. rational (determinacy) expectations models has become clear. So do not look for a reconciliation of these views. Friedman did not qualify his views on interest rate targets with a Taylor rule proviso, or distinguish between a peg and a target, because nobody including him had thought of Taylor rules.
12.3 History and theory of interest rate targets

How can we disagree on such basic doctrines? Friedman was influenced by the lessons of the 1950s, but those episodes also have fiscal roots. Friedman did not see Japan’s 25 years at the zero bound. The postwar pegs were part of an evaporating gold standard, which is an instance of fiscal theory.

Central banks follow interest rate targets, often arguably passive. We need a theory for such regimes. At a minimum, the fiscal theory provides one.

Notwithstanding all this, good policy in normal times may well look much like a Taylor rule.

How could we disagree on such basic doctrines? How dare we contradict Friedman, Sargent and Wallace? Isn’t the historical evidence clear? No, actually.

Friedman was strongly influenced by the failure of interest rate pegs in the early post WWII period. But in retrospect, armed with the fiscal theory, one can’t help but note that the postwar pegs fell apart coincident with fiscal problems, and therefore perhaps due to such problems. The US, still paying down large WWII debts, ran into a recession and Korean War deficits. European countries had severe fiscal problems in the wake of WWII. All were following capital repression policies to try to force people to hold their money and debt. Friedman did not live to see the 25 years Japan spent at zero interest rates, with slight deflation, or the long period of zero rates in the US, UK, and Europe with quiet inflation. Always an empiricist, he too might have changed his mind with this remarkable experience in the rear view mirror. Moreover, one might have wondered at the remarkable persistence of essentially fixed interest rates, from 1930 to 1950, and the decades of interest rate targeting under the gold standard. (Under the gold standard, central banks did not just offer gold for money. They also set interest rates, typically the rate at which they “discounted” bills, i.e. lent money using private short term debt as collateral. They manipulated the discount rate to manage gold flows.)

Even as a matter of theory, Friedman left out the fact that the postwar interest rate targets were adopted in the shadow of the gold standard under Bretton Woods. A real gold standard is an instance of fiscal theory, as it commits governments to raise taxes to get needed gold. With fiscal theory, an interest rate peg is stable, as we have seen. But the postwar period was one of lip service to a gold standard with increasing restrictions making it more a pretence than a standard, and it fell apart completely in 1972 under fiscal and inflationary pressure. So one may view the
failures of interest rate pegs in the 1950s as early failures of the fiscal commitments underlying the gold standard

Except for a brief experiment in the early 1980s, monetary policy has operated by fixing interest rates, not by targeting monetary aggregates. And even the 1980s are debated – was the Fed really targeting nonborrowed reserves, or was that just cover for high interest rates?

Central banks follow interest rate targets for good reasons. Monetarists felt that money demand is much more stable than money supply, and that most volatility of output and inflation was due to money supply errors. Central banks think that money demand is more unstable than money supply – the \( V \) in \( MV(i) = PY \) varies, in ways that the central bank cannot measure directly. If the central bank holds \( M \) constant than variation in \( V \) leads to needless variation in \( PY \). Though Friedman argued forcefully and successfully that monetary policy shocks contributed a lot to the great depression and early postwar volatility, more recent literature finds that monetary policy shocks – unexpected changes in interest rates, deviations from a rule – do not contribute much to output and inflation volatility. Attention has shifted to the systematic part of policy, the rule, whether reactions to inflation and unemployment are right. (My small contribution is Cochrane (1994). Ramey (2016) is an excellent recent survey.) In the standard interpretation, the minute the Fed started actually controlling monetary aggregates in the early 1980s, velocity started moving. Push on the \( M \) in \( MV = PY \) and \( V \), not \( PY \), moves.

Whether or not these conclusions are correct, central banks do follow interest rate targets, and those targets are often remarkably passive-looking or pegs. Given that fact, it’s a testament to monetarists’ intellectual force that for so much monetary analysis pretends that central banks target the money supply, and studies money supply shocks. To describe the world we live in, we need a theory of interest rate targets.

The Taylor rule in either adaptive expectations or new-Keynesian models at least provides such a theory. But, especially in the latter case, it does not describe what happens under passive policy (inflation varies less than one for one with inflation) such as the 1970s or a peg. In my view it suffers the same failure to describe the recent zero rate experience, but there is a debate on that point, again covered later in Chapter 16. Just saying that well, inflation is indeterminate so there are multiple sunspot equilibria and the Fed should have run more active policy is not a satisfactory theory. At a minimum, the fiscal theory gives us a theory to analyze such episodes and think about policy regimes that switch between active money and active fiscal.
In my view, the active-money specification doesn’t make sense, and the observational equivalence theorems make detecting a regime switch dicey at best, issues deferred to Chapter 16. The point here, even if one disagrees with the latter point, we need a theory that can describe abundant passive-money episodes, and the fiscal theory provides it.

Do not over-interpret the last two sections as an attack on Taylor rules. Notwithstanding everything said so far, an interest rate target that follows something like a Taylor rule may be a very good policy. Again, though a peg is possible, it is likely not to be desirable because then variation in the natural rate of interest have to be met by inflation. It is plausibly better for the central bank to vary the nominal interest rate as the natural rate varies. But divining the natural rate is hard. So a good rule will respond to aggregates, rising in good times when the natural rate is high and declining in bad times when it is low – i.e. when inflation and output are low. True, a more than one-for-one response to inflation is not needed for stability or determinacy, and carried to extremes would actually cause problems in the fiscal theory. But stability and determinacy are really about extreme cases, about off-equilibrium behavior that we do not see in the course of day-to-day policy.

Moreover, Taylor’s main point is about a rule, any rule, rather than discretionary policy. Rule-based behavior helps people to form expectations and reduces unneeded surprises. It also precommits an overactive Fed against too much micro management.
Chapter 13

Monetary institutions

If the price level is determined by the intersection of money supply and demand, the government must engage in a certain amount of financial repression: It must ensure a substantial demand for base money, it must control the creation of inside-money substitutes, and it must restrict financial innovation that would otherwise reduce or destabilize the demand for money. None of these restrictions are necessary with fiscal price determination.

Money demand is thought to come from many sources, including transaction demand, precautionary demand, and so forth. Anathema to a monetarist, fiat-money, MV=PY view is one in which people can use assets they hold entirely for other reasons, without suffering any loss of rate of return, to accomplish transactions and other goals of money demand. If, for example, you held $100,000 of stocks and bonds in your retirement portfolio, you need $10,000 of assets as a buffer to make transactions, but you can costlessly wire around claims to the retirement portfolio to make those transactions, or to satisfy whatever the other demands motivate money holding – then price level determination falls apart. We are rapidly approaching that world.

13.1 Controlling inside money

- Classic doctrine: Since inside money satisfies transactions demands as well as the monetary base, the government must control the quantity of inside money or the price level becomes indeterminate.
Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements and limitations on the creation of liquid inside assets are not needed for price-level determination.

Government-provided, or base money, the sum of currency and reserves (accounts that banks have at the Federal Reserve) are not the only assets that people can use for transactions and other money-related activities. Checking accounts are the earliest and easiest example of “inside money.” A check is a promise to pay in the future, essentially an i.o.u. If I write an i.o.u, say “I’ll pay you back $5 next Friday,” you might be able to trade that i.o.u for a beer this afternoon, and your friend collects from me. Sufficiently safe i.o.u’s can start to trade as money. In the 19th century banks issued notes, which functioned much like today’s currency. Banks today “create money” by creating checking accounts. When a bank makes a loan, it flips a switch and creates a larger amount in a checking account, essentially writing a tradeable i.o.u. Money market funds offer money-like assets, backed by portfolios of securities.

Recognizing this fact, we can write money demand as

\[(Mb + Mi) V = PY,\]

distinguishing between the monetary base \(Mb\) and inside money \(Mi\), and allowing them to have different velocities. Here I treat the inside moneys as perfect substitutes for base money. More sophisticated treatments recognize that highly liquid assets are imperfect substitutes. More “liquid” assets pay less interest rate and turn over more quickly. To keep the discussion here simple, we’ll use the standard idea that there are only two kinds of assets, “liquid” money and money-like assets, all perfect substitutes, and “illiquid” assets that are useful savings vehicles but can’t be used for transactions.

Again, the monetarist, or fiat-money, view, determines the price level from the intersection of such a money demand with a limited supply. To that end, it is not enough to limit the supply of the monetary base \(Mb\). The government must also limit the supply of inside-money substitutes \(Mi\). To that end, for example, we have reserve requirements. To create a dollar of checking account, the bank must have a certain amount of base money. If, for example, the reserve requirement is 10%, then to create a dollar of checking accounts, the bank must have 10c of reserves. The total money supply is limited to be 11 times the amount of reserves.

Other kinds of inside money are controlled by regulation. For example, bank notes are simply illegal. Many countries pass legal tender laws forbidding the use of foreign
13.1. CONTROLLING INSIDE MONEY

money to make transactions.

In sum,

- Classic doctrine: Since inside money satisfies transactions demands as well as the monetary base, the government must control the quantity of inside money or the price level becomes indeterminate.

In the fiscal theory, clearly, the price level is already determined. Hence, not only may the government allow the monetary base to be passive, but there is no need, on price determinacy grounds, to limit inside money at all.

- Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements and limitations on the creation of liquid inside assets are not needed for price-level determination

As we shall study in detail later, this is fortunate. Inside moneys have exploded. Reserve requirements are tiny, and don’t realistically control inside money creation. Before 2008, reserves were on the order of 10 billion dollars. After 2008, reserves exploded to 3 trillion dollars, but the money supply did not increase nearly as much. Instead, reserve requirements became slack, so that the money supply can vary arbitrarily without changing the quantity of reserves.

The point here is narrow. There are many reasons to limit inside moneys. A bank that issues a lot of notes against illiquid assets, or a bank that issues more notes than it has assets at all, is prone to a run. The point here is only price level determination, not financial stability. Governments also engage in financial repression to enhance demand for their non-interest-bearing or low-interest liabilities, to help fund deficits.

This contrast illuminates a key distinction between the fiscal theory, or any theory based on backing, and a fiat money theory based on transactions demand. One might look at \( MV = PY \) and \( B = P \times EPV(s) \) and conclude they are basically the same. In place of money we have all government debt, and in place of a transactions demand related to the level of output, we have the present value of surpluses. But here we see a big difference: Only direct government liabilities appear on the left-hand side of the fiscal theory, while private liabilities also appear in \( M \).

By analogy, consider the question, does opening futures and options markets affect the value of a stock? By uniting a put and call option, you can create a synthetic share of the stock, and both options and futures allow you to bet on the future of a company without buying shares. Do these “inside stock shares” compete with
“real stock shares” to drive down the value of stocks? Well, in the baseline frictionless theory of finance, no. The company splits its earnings among its real owners only, and doesn’t owe anything to the owners of inside shares. Therefore, we begin the theory of valuation with price times company issued shares = present value of dividends, ignoring inside shares.

Likewise, primary surpluses are split only among the holders of actual government debt, not among those who have bought private claims denominated in shares of government debt, such as checking accounts. (Ignoring deposit insurance.) So, to first order, the value of government debt is not affected by arbitrary inside claims.

Equivalently, for every private buyer of inside money, an i.o.u. promising payment in government debt, there is a private issuer of this claim. The “wealth effect” of government debt only applies to the net amount. Money helps to grease the wheels of the economy even if it has no net value, and for every owner there is an issuer.

Now, stocks occasionally gain a “liquidity value” when they are in short supply relative to trading needs on top of their “fundamental value,” and in these situations “inside stock” supply in the form of options and futures markets, and short-selling, can affect stock prices (For example, Lamont and Thaler (2003) Cochrane (2003)). So too, the fiscal theory is not absolute. Government debt can have a liquidity value, which will show up in a lower rate of return than comparable private assets, and private substitutes for that liquidity value will lower the value of government debt and affect the price level. Even under the gold standard, there were occasional shortages of currency, leading to temporary deflations. But the point here is, as usual, whether such frictions are essential to basic price level determination, or whether they give second-order effects on top of a functional frictionless theory.

The news is that there exists a frictionless theory in which inside claims do not matter, and that stylized model can serve as a benchmark on which to add frictions. The frictionless fiscal theory and the theory of pure fiat money are two extremes. Real situations combine liquidity values and backing. Though we start with a frictionless fiscal theory model, we do not need to stop there. I phrased the fiscal theory doctrine carefully to allow for liquidity effects on top of the basic backing mechanism.
13.2 Controlling financial innovation

- Classic doctrine: For the price level to be determined, regulation must limit the introduction of new transactions technologies.

- Fiscal theory doctrine: The price level is determined with arbitrary financial innovation, and even if no transactions are accomplished using the exchange of government liabilities.

For \((Mb + Mi) V = PY\) to determine the price level we need to keep \(V\) from exploding, and we need to limit the production of inside money \(Mi\). We need constraints on financial innovation, which provides alternatives to transactions or other demands for money. I focus here on transactions demands.

Yet our economy is evolving with rampant financial innovation, much of which reduces the need for non-interest-bearing money to make transactions.

Checking accounts are already an example of an early money-saving, transactions-facilitating innovation. If I write you a $100 check, and we use the same bank, they just flip a (proverbial) switch, raise your account by $100 and lower mine by the same. No money ever changes hands. If we have different banks, our banks are most likely to also net our $10 payment against someone else’s $10 payment going the other way, as well as inter-bank security purchases and sales. They transfer the remainder by asking the Fed to similarly increase one bank’s reserve account by $10 and decrease the others’. That still requires some reserves, some money holding. But banks were able to accomplish all the transactions in the (then) $10 trillion economy, including the massive volume of financial transactions, with only $10 to $20 billion of non-interest paying reserves, an impressive velocity indeed. Basically, when non-interest-bearing reserves must be transferred, institutions get them seconds before the transfer, and dispose of them seconds after the transfer, to hold as little as possible.

As I write reserves are enormous, but reserves pay interest even greater than short-term treasury bills. So we truly have no money at all, and banks settle their claims with assets that they hold entirely for portfolio reasons. We have entered the interest-paying money regime of section 13.3. And inflation has remained remarkably quiet.

Credit cards, debit cards, money market funds, overnight repurchase agreements, Bitcoin, all are innovations that allow us to accomplish the same transactions, as well as to enjoy the other features of “money,” without holding government money,
and without suffering the lost interest that an inventory of money represents. Many of these institutions are essentially interest-paying money held outside of the banking system, and so doubly toxic from the point of view of classic theory – interest-paying as well as not subject to whatever quantity control that remaining reserve requirements imposed.

As a first abstraction, our economy looks a lot more like an electronic accounting system, transferring and largely netting inside claims to a vast quantity of government debt, held mainly for portfolio reasons, than it looks like an economy with “transactions” media (cash, checking accounts), held in limited quantity and suffering an interest cost, and provided in limited supply, rigidly distinguished from highly illiquid “savings” media (bonds, savings accounts).

But, to a diehard monetarist, all this must be stopped. If $V$ or $M_i$ go through the roof, then $(Mb + Mi)V = PY$ can no longer determine $P$. Chicago-based monetarists were pretty free-market, but not in this circumstance. The fiscal theory liberates us to be free-market even in the provision of transactions and financial services.

Sure, in theory, as $V$ increases, $M$ can decrease, from $10$ billion to $1$ billion, and finally to an economy of quickly circulating electronic claims to the last $1$ bill, framed in the office of the Federal Reserve Chair’s office, the puzzle that started for me this whole quest. But I think it’s clear that as velocity explodes, the power of money to control the price level must also disappear. If you no longer hold the dog, nor the dog’s tail, but just hold the last hair on the end of the dog’s tail, pretty soon it is no longer the dog that wags. If the Fed nails the price of chewing gum on the news stands at O’Hare Airport’s terminal C, though general equilibrium arbitrage arguments say all other prices in the economy must adjust, it’s pretty clear that arbitrage just falls apart. A theory that works at the limit point, not just in the limit, is better adapted to an economy taking that limit.

The fiat money, $MV=PY$ story is nice story, and it may well have described the economy of the 1960s or 1930s. (Even there, one must worry that money supply was not controlled.) But not today. If you drop an economist down from Mars and ask him or her to choose a simple model to describe our financial system, and the choice is Baumol-Tobin vs. Apple pay, linked to a cashless electronic netting system based on short-term government debt, I bet on Apple pay. The same economist likely would have chosen Baumol-Tobin and Friedman up to the 1960s.

More deeply, advances in communication, transactions, computation, and financial technology are destroying the need for us to hold money – any asset with fixed
nominal value, floating rate, and less than market rate of return, and whose supply is controllable by the government. In the 1930s, if you wished to buy a cup of coffee with a share of stock, that was impossible: at the coffee shop you don’t know the current price of stock (communications), you can’t quickly calculate how many shares to transfer (calculation), and selling stock takes delivery of physical certificates after a few days. Moreover individual stocks suffer from large bid-ask spreads due to adverse selection – why are you offering RCA, not GM, for your coffee? Only a fixed-value claim could be liquid. Instant communications, the possibility of millisecond transactions, and the creation of asymmetric information free index funds all mean that we could, if we wished to do so, have a financial system in which you pay for coffee by apple-pay linked to a stock index. Or, even more undercutting traditional banking, a long-only mutual fund containing mortgage-backed securities. Today’s computation undercuts the accounting difficulties of holding fixed-value securities for transactions.

Yes, a great deal of cash remains, about $1.5 trillion as I write. But more than 70% of cash is in the form of $100 bills, and most is held abroad. Cash supports the illegal economy, tax evasion, undocumented workers, illegal drugs, and is a store of value around the world where governments tax rapaciously and control capital movement. One could, I suppose, found a theory of the price level on the illegal demand for non-interest bearing cash. But it would be distasteful, especially as we want to understand the price level in the legal economy. (The bureau of labor statistics does not sample illegal transactions.) It would be even more distasteful, as the already delicate position of arguing for financial repression to control the price level would have to extend to arguing for continued illegal activity to the same end. Last, and perhaps most importantly, governments do not limit the supply of cash. They offer free exchange of cash for reserves, so even if we embarked down the questionable path of a price level theory based on illegal cash demand, we would be instantly faced with a flat supply curve.

Transactions or broader liquidity demands for particular assets including cash still exist. They will enter our theory to drive interest rate spreads between those assets and other assets with identical cash flows. But they will not be central to price level determination, and the fiscal theory of price level determination does not require them to persist, as the fiat money theory does.

In sum,

- Classic doctrine: For the price level to be determined, regulation must stop the introduction of new transactions technologies, which threaten to explode V.
• Fiscal theory doctrine: The price level is determined with arbitrary transactions technologies, and even if no transactions are accomplished using the exchange of government liabilities.

13.3 Interest-paying money and the Friedman rule

• Classical doctrine: Money must not pay interest, or at least it must pay less interest than risk-free short-term bonds, and its quantity must be rationed to maintain this interest differential. If the interest rate is zero, or if money pays the same interest as bonds, the price level becomes undetermined. We cannot have the Friedman-optimal quantity of money. Money and liquid assets must be artificially scarce to obtain price level control.

• Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, and if the central bank offers to freely exchange money for bonds at that equal rate. That interest rate can be zero, or money may pay the same interest as bonds. We can live the Friedman-optimal quantity of money, satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

The possibility of zero interest rates, or the equivalent, that money pays the same interest as bonds, is a grave threat to $MV = PY$ price level determination. When there is no interest cost to holding money, money and bonds become perfect substitutes. Now $V$ is no longer a constant, it is $PY$ divided by whatever $M$ happens to be. When money and bonds are perfect substitutes, then a switch of $M$ for $B$ really has no effect at all. It is like exchanging red M&M’s or green M&Ms, and defining velocity as calories eaten divided by green M&Ms only. As a function of interest rates, when money pays the same interest as other assets, money demand ceases to be a function, but is instead a correspondence, crawling up the vertical axis.

In the classic view, the government must maintain a separation between “transactions” or more generally “liquidity provision” assets, held in an inventory despite an interest cost, and “savings” assets like bonds, held to save (say) for retirement, but not liquid enough to be used in day to day transactions. If one can use “savings” assets to make transactions, then the economy is essentially a barter economy and price level determination disappears.

The fiscal theory offers the opposite conclusion. If $M_t$ pays the same interest as
13.3. INTEREST-PAYING MONEY AND THE FRIEDMAN RULE

$B_t$, then $M$ and $B$ are perfect substitutes, and we're simply back to $B_t/P_t = E_t \sum_{j=0}^{\infty} s_{t+j}/R^j$ with no money, no seigniorage, and no other change. The price level is easily determined.

The famous “Friedman rule” describes the optimum quantity of money: full satiation. Since making more money costs society nothing, we should have as much of it as we want. Before you get too excited, or dubious, keep in mind, this means more money and less bonds – the composition of government debt, and private assets, should give us all the money we want. The proposition is not that the government should shovel money out of helicopters until people don’t pick it up any more.

Money is like oil in the car, and there is no point to deliberately starving the car of oil.

With cash that cannot pay interest, or with money-like assets such as traditional checking accounts that do not pay interest, the nominal interest rate should be zero, with slight deflation giving a positive real rate of interest. Zero also means no hurry to collect on bills or other contracts that do not include interest clauses, and no need to write interest clauses into such contracts. And people should be free to have as much money (in exchange for government bonds) as they want. All of the cash management we do to use less money, and thereby save on interest costs, is a social waste.

As money becomes interest-bearing checking accounts, money-market funds, and transactions become all electronic using such funds, we can generalize the Friedman rule to say that the optimal quantity of money is achieved when such money-like assets pay the same return as illiquid assets, minus, at most the small costs of offering transactions services. And people should be free to have as much liquid interest-paying assets, in exchange for government debt, as they want We should be fully satiated in money-like liquid assets.

But Friedman did not argue for an interest rate peg at zero, nor passive money supply, nor for interest-paying money. He never took the optimal quantity of money seriously as a policy proposal. Why not? Because, if the price level comes from money supply and money demand, it would become unmoored by such arrangements. So society must endure the costs of an artificial scarcity of the medium of exchange, and an artificial scarcity of “liquid” assets useful in so many other ways, in order to keep inflation under control. If the car has the gas pedal stuck to the floor, you have to control its speed by rationing the quantity of oil.

The fiscal theory denies this doctrine.
Summing it all up,

- Classical doctrine: Money must not pay interest, or at least it must pay less interest than riskfree short-term bonds, and its quantity must be rationed to maintain this interest differential. If the interest rate is zero, or if money pays the same interest as bonds, the price level becomes undetermined. We cannot have the Friedman-optimal quantity of money. Money and liquid assets must be artificially scarce to obtain price level control.

- Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, and if the central bank offers to freely exchange money for bonds at that equal rate. That interest rate can be zero, or money may pay the same interest as bonds. We can live the Friedman-optimal quantity of money, satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

Again, this is a fortunate prediction because our world looks less and less like one that meets the classical requirements.

It is largely impractical for cash to pay interest, positive or negative, though many clever proposals have been advocated to do so. Until the early 1980s, checking accounts were forbidden by regulation to pay interest. Most of our money these days is electronic, and transactions made by electronic means. Now it is possible to pay interest on such money, calculated to the millisecond. So where the discussion previously was only about zero interest rates, now the question expands to money that pays the same interest as bonds.

The Federal reserve currently pays interest on reserves. The Fed offers money market funds the option to invest in reserves through its reverse repo program. So the possibility of interest-paying money is not just an academic what-if, but a pressing reality. Checking accounts and money market funds all pay interest; and large corporations use repurchase agreements or money market accounts to get interest-paying cash. We are very close to an economy of (both inside and outside) fully interest-paying, electronic money, for all (at least legal) transactions.

There is a small theoretical distinction arising from models of money. It is possible, as I have outlined verbally, that at some finite level of money, money and bonds become perfect substitutes. People are satiated with liquidity; the money demand curve runs in to the vertical axis at a finite if large amount of money, and then crawls up the vertical axis.

It is also possible that people are never fully satiated with liquidity, that the money
13.4  

The separation of debt from money

- Classical doctrine: Bonds must be kept deliberately illiquid, and separate from money, or the price level will not be determined. They may not be issued in small denomination, discount, bearer or transferable in low-cost electronic form.

- Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-value, floating-rate, electronically transferable debt.

In $MV = PY$, we need to have a definite separation between “liquid,” or transactions-facilitating assets $M$ and “illiquid” savings vehicles $B$. Control of the former gives control over the price level. This is the reason for banning interest-paying money, in section 13.3 Here, I discuss the complementary doctrine: For this reason, it is considered important to deliberately limit the liquidity of public and private debt issues. Bank notes are illegal, though they are just zero-maturity, zero-interest, small denomination bearer bonds issued by banks. Corporations may not issue small-denomination bearer bonds that might circulate. And the US Treasury does not issue bonds in denomination less than $1,000 – only recently reduced from $10,000 – and not in bearer form, and it does not (yet, I hope) issue fixed-value floating rate debt. All treasury securities fluctuate in value. That deliberate illiquidity keeps “bonds” separate from “money.”

- Classical doctrine: Bonds must be kept deliberately illiquid, and separate from money, or the price level will not be determined. They may not be issued in
small denomination, discount, bearer or transferable in low-cost electronic form that might be used for transactions demand.

This doctrine is really just an expression of the general proposition that the government must control the supply of inside moneys. Here I emphasize that control through legal restrictions on the form of financial contracts, rather than restrictions on the amounts issued given the form, such as reserve requirements on checking accounts.

The fiscal theory denies this proposition. The liquidity, denomination, and bearer form of inside or outside debt makes no difference in the frictionless model. To the extent that such features lower the interest rate markets require of Treasury debt overall, so much the better for government finances and liquidity provision to the economy.

- Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-value, floating-rate, electronically transferable debt.

In a more detailed proposal, Cochrane (2015), I argue that the Treasury should offer to all of us the same security the Fed offers to banks: fixed-value, floating-rate, electronically transferable, debt, in arbitrary denominations. This is essentially the same security as reserves at the Fed, but available to everyone not just banks. It is treasury electronic money, indistinguishable from Treasury floating rate debt. I also argue that the treasury should supply as much of this as people demand, leaving the split between this debt and longer term debt to the public. The rate would be set either as the rate the Federal reserve offers on excess reserves, or be set by a daily auction. This move would passively and automatically supply any liquidity demands. (The treasury can manage its duration and interest rate risk exposure with swaps.) This would be a simple way to live the Friedman rule. Such a proposal is anathema in a monetarist view, as the price level would be completely unmoored – both the relative quantity of $B$ and $M$ would become endogenous, and the character of $B$ and $M$ (reserves) would become identical.

Again, private issue of money-like assets remain a problem for financial stability, and I’m not endorsing it. Banks that can issue notes, backed by illiquid assets, tend to issue to many of them. The government’s monopoly of currency issue is a good thing for financial stability, and I think (Cochrane (2014)) that the government should extend that monopoly to all fixed-value, floating-rate, electronically-transferable debt. But financial stability and the chance of private sector runs and defaults are not the issue in this book.
13.5 A frictionless benchmark

- Classical (fiat money) doctrine: We must have monetary frictions to determine the price level.

- Fiscal theory doctrine: We can have a well-defined price level in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar’s taxes is valuable even if there are no dollars.

The fiscal theory does not stop with frictionless models. It is a benchmark on which we build models with frictions as necessary. But unlike standard monetary economics, frictions are not necessary to describe an economy with a determinate price level. And the very simple frictionless model can provide a first approximation to reality.

In classical monetary theory, based on completely fiat money, some monetary friction is necessary to determine the price level. In a completely frictionless economy, with no money demand, money would have no value.

As we have seen several times, the fiscal theory can determine the price level even in a completely frictionless economy. We have already determined the price level without liquidity demands, transactions demands, speculative demands, precautionary demands, incomplete markets, dynamic inefficiency, price stickiness, wage stickiness, irrational expectations, and so forth. Such ingredients make macroeconomics fun, and realistic. We can and will add them later. But the fiscal theory is that we do not need those ingredients to determine the price level.

We can even get rid of the “money” in the stories I told above. It did not enter into the frictionless model equations. Return to the “day” of Chapter 2 in which the government printed money in the morning to redeem bonds, and then soaked up that money with tax payments and bond sales in afternoon. Suppose that people instead use maturing government bonds to make transactions during the day, to pay taxes, and to buy new government bonds, and money vanishes entirely from the story. Electronic transfers of money market accounts, backed by short-term treasuries, are close to this vision today. Nothing changes. Bonds give the right to a dollar, but there is no point in exercising that right if you can do everything you want with a maturing treasury bill.

Nothing changes if people make transactions in Bitcoin, by transferring shares of
stock, or by an accounting system. The “dollar” can be a pure unit of account, and government debt can promise to pay a “dollar,” even dollars and government debt are not used at all as medium of exchange, and if nobody ever holds any dollars at all. The right to be relieved of one dollar’s worth of tax liability establishes its value as numeraire.

This frictionless view is not just a frictionless limit. For example, the preface, and more formally Woodford (2003), describe a limit in which velocity increases, money supply decreases, and the price level remains determined by the demand for the last dollar relative to its supply. But that story fails at the limit point when there is no cash at all, and one must wonder if it actually works when the price level of a $20 trillion dollar economy is supposedly pinned down by the demand for one last dollar intersected with its supply. The fiscal theory applies also to the limit point when there is no money at all, which also gives us more confidence that it still applies with small amounts of remaining money.

This frictionless valuation property is a property of a backing theory of money. If dollars promised to pay gold coins, and were 100% backed by gold coins, then we could establish the value of a dollar equal to one gold coin, also even if nobody used dollars in transactions. In a backing theory, money may gain an additional value if it is specially liquid and limited in supply, or it may pay a lower rate of return. In a backing theory, a fundamental value remains when the liquidity value or limited supply disappear. Entirely fiat money loses all value in that circumstance.

To summarize, continuing my list of doctrines,

- Classical (fiat-money) doctrine: We must have some monetary frictions to determine the price level.

- Fiscal theory doctrine: We can have a well-defined price level in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar’s taxes is valuable even if there are no dollars.

This observation really sums up previous ones – interest-paying money, abundant inside money not constrained by reserve requirements, debt that can function as money, and financial innovations that allow us to make transactions and satisfy other demands for money without holding money are all different aspects of the march to a frictionless financial system.
13.6 **Fiat vs. fiscal: History, esthetics, philosophy and frictions**

Monetary economics always recognized fiscal backing, and fiscal theory can include a money demand. But it is understandable that monetary economics focused on money when those frictions were larger. As frictions recede, the fiscal backing is left, and a simplification that ignores them falls apart.

The facts have changed. The failure of monetary targeting, the successful control of inflation under an interest rate target, and the experience of the long zero bound were not there when monetarism was developed.

Monetarism was once philosophically useful, but is no longer. The fiscal theory, by allowing free financial innovation, may even replace some of its usefulness.

The frictionless version of the fiscal theory is only a foundation, on which to build realistic descriptions of events and policies. All of the simplifying assumptions can be generalized. In this way it acts like many of the classic neutrality results of modern macroeconomics.

I have described stark views for clarity, but neither economic nor intellectual history is so stark. The equations $M_t V_t = P_t Y_t$ and $B_t^{-1}/P_t = E_t \sum_{j=0}^{\infty} s_{t+j}/R^j$ both hold in a monetary regime, in which control of $M$ delivers price level stability and the fiscal authorities adjust surpluses to pay off inflation-induced revaluations of debt.

The same equations hold in a fiscal regime in which the latter equation determines the price level, and money is passively supplied to satisfy the former. Both regimes involve aggregate demand, and both sources of aggregate demand must be present. Observational equivalence means you can’t immediately tell if the too much money you see chasing too few goods is relative to a transactions demand or to fiscal expectations.

Monetary theorists recognized that this fiscal backing of monetary policy was vital. The purely passive fiscal policy assumption – arbitrary lump-sum taxes to make $B_t^{-1}/P_t = E_t \sum_{j=0}^{\infty} s_{t+j}/R^j$ hold ex-post – is a technical assumption useful for writing papers, but any serious analysis studied and questioned fiscal foundations, and saw inflation dangers when those foundations were lacking.

But with reasonably small debts, inflation rates under 10% or so, and governments not facing important fiscal pressures, with plenty of taxing power remaining on the left side of the Laffer curve - one might view the fiscal consequences of monetary
policy as second-order for postwar US finance, and fiscal coordination an issue to be left for smaller countries with exchange rate and deficit problems, or for wartime, when monetary theorists all understood the vital importance of fiscal policy for inflation control.

Furthermore, in an economy in which you really could not go out to dinner on a weekend if you did not physically go to a bank on Friday afternoon and cash a check, in front of a human teller, monetary frictions loomed larger.

The emphasis on monetary rather than fiscal foundations is an understandable matter of degree, and a theory of pure fiat money without spelling out fiscal foundations just the sort of abstraction, simplification, and focus on the most important feature that produces clear economics.

But when considering larger questions, such as what happens if financial innovation eliminates transactions demand, the slight abstraction of ignoring fiscal foundations, though fine for day to day analysis is fatal.

Moreover, even in evaluating current events, the monetary frictions disappear. It becomes clear that policy doesn’t restrict money supply at all. Rampant financial innovation occurs, and makes the money vs. bonds story look about as relevant as backing by gold coins. As the frictions disappear, the habit of approximating our economy as pure fiat money without backing becomes fatal. In the end, only the fiscal backing remains, and long before the end the fiscal backing becomes first-order important.

The fiscal theory, and the opportunity to base a theory of the price level on a perfectly frictionless supply and demand model, on which we build frictions as necessary, is also esthetically pleasing. Everywhere else in economics, we start with simple supply and demand, and then load on frictions as needed. Monetary economics has not been able to do so. Now it can. Esthetically pleasing models tend to be right.

In this way, the fiscal theory also closes an open philosophical loop. It is initially puzzling that monetarism and free markets both came from Chicago. The same philosophy that generally pushes hard towards a simple, supply and demand explanation of economic phenomena, and generally tries to arrive at solutions to social problems based on private exchange and property rights, starts its macroeconomics with one big inescapable friction separating money from bonds, and then is forced to advocate both a powerful Federal Reserve, and many restrictions on free exchange and financial innovation, to the point of deliberately hobbling the economy from liquidity.
Well, that philosophy does make historical sense: the Chicago view was on the whole less interventionist than the contemporary Keynesian view. And there was no alternative.

But now there is. Philosophically, or ideologically, the fiscal theory can be more Chicago than Chicago!

I worry in writing this book that Friedman is rolling over in his grave at its repudiation of monetarism. But perhaps not. The failure of monetary targeting, the evident conquest and subsequent stability of inflation under interest rate targets, and then the stability of inflation at a long-lasting zero bound, despite immense reserves and financial innovation might give him pause. Friedman, and his colleagues, were in the end devoted to *empirical* inquiry. The historical experience – experience they could not possibly have foreseen – really does force a change of view, and they of all people were likely to change views when experience decisively demanded it. And the ability to write monetary theory in a way that allows a free-market financial system and all of us to live the Friedman rule might have won him over.

Theories prosper when they describe data. But they also prosper when they are useful to a larger debate. Monetarism was tremendously *useful* to the free-market philosophy of Chicago in the 1960s. In the face of the then-dominant static Keynesian paradigm, tied to a general aristocratic dirigisme that favored government microeconomic as well as macroeconomic planning, Friedman could not hope to succeed by advancing the idea that recessions are the normal work of a frictionless market. Kydland and Prescott (1982) were a long way away. Nobody had the technical skills to build their model, and the verbal assertions of the 1920s were generally dismissed with derision. The intellectual climate demanded that the government do *something* about recessions, and demanded a simple uni-causal theory. Monetarism provided one.

But as the set of facts we must confront has changed dramatically since the 1960s – from the monetary and financial missteps of the Great Depression and the failure of postwar interest rate pegs (in the face of fiscal pressures, below) to our quite different history and circumstances – the intellectual environment has changed too. We don’t *need* monetarism any more. Yes, new Keynesian economics is also ideologically tied to a desire for greater macroeconomic direction, now expressed in the push for “macroprudential policy” and similar merging of financial regulation with macroeconomic direction. But controlling monetary aggregates and leaving the rest alone is no longer a *useful* reply.

Neither is the fiscal theory. As we have seen and will see more, it really fixes and
rescues rather than attacks much of the new Keynesian description of monetary policy, so the practical and ideological debate over macroeconomic dirigisme will have to take place on its own. If it is useful for other agendas, I will let other more objective observers figure out what they are.

So, I hope that even Friedman, a practical and empirical economist if there was one, might change his mind if he were around today. The fiscal theory fits much of his philosophical purposes in today’s environment, even if it turns many monetarist propositions on their heads.

Esthetic and philosophical considerations don’t make a theory right. But they shouldn’t be ignored either. A theory that is philosophically consistent with so much else that is right is more likely to be right. Keynesianism was popular in part because it fit well with a view that technocratic dirigisme is necessary to the economy. But Keynesianism in the 1930s can also be praised for saving capitalism. Against the common view then that only Soviet central planning, fascist central inspiration, or at best Rooseveltian NRA micromanagement could save an economy, Keynesianism said no – if you fix “aggregate demand” with one magic elixir, such as fiscal expansion, all will be well. Even if one regards that as an economic fairy tale, embodying in one place most common fallacies, one must admit it was an immensely useful theory in its day, and if a historian of economic thought says it saved capitalism, one may smile.

_The fiscal theory does not stop at frictionless models, however._ I emphasize this point because of the common contrary misperception. The frictionless theory is a useful benchmark, on to which we add frictions.

Some cash remains. Government debt, and especially short term debt, seems to have an extra liquidity value because it is specially useful for transactions and as a riskless short term asset in portfolios. Prices are sticky, some assets are special, bond markets might be slightly segmented, and so forth.

I spend quite some time exploring the frictionless version, yes, because it is unfamiliar, and its predictions are unusual relative to intuition formed by thinking about the equally unrealistic pure fiat alternative. But that does not mean we will stop there.

Adding liquidity values in the fiscal theory is relatively straightforward. If government debt or some kinds of government debt have a liquidity value in addition to the frictionless backing value, that shows up as a reduction in the discount rate, a lower $R$ in $B_{t-1}/P_t = E_t \sum_{j=0}^\infty 1/R^j s_{t+j}$. Such discount rate variation is very important
in thinking about the price level. But it modifies rather than replaces the basic structure, just as adding a liquidity value on top of a metallic value of coins does. Adding sticky prices and other interesting frictions is equally straightforward. But the frictions become frosting on the cake. In the opposite extreme, the frosting is the cake.

The models I have used so far are much simplified in many other dimensions, but all those elaborations are straightforward to add. I have used constant discount rates; clearly risk and risk aversion matter so we need a better understanding of discounting. We need to add real debt indexed debt, risk premiums, the non-neutrality of tax and spending systems, government assets, real liability promises such as social security and health, partial endogeneity of surplus decisions, distorting taxes, sticky prices, and so on. If you don’t like rational expectations, add whatever theory of expectations you like. To reiterate the point, “fiscal theory” does not preclude one from adding those elaborations. Quite the opposite. The extremely stylized model here is the foundation on which we build all these elaborations, as needed.

In this sense, the fiscal theory is also related to the great neutrality propositions of economics. These include the Modigliani-Miller theorem, that firm value is independent of debt and equity; the Ricardian equivalence theorem, that deficit financing has no effect on the economy; the Modigliani-Miller theorem for open market operations (Wallace (1981)) that the composition of government debt is irrelevant; rational expectations and efficient markets, in which demand curves for securities are flat and asset prices incorporate all available information about value; and the neutrality of money propositions that real interest rates, unemployment rates, real output and other real quantities are eventually independent of inflation.

All of these theorems are false as descriptions of the world. They make “friction-less” assumptions, and our world has frictions. But they’re not as false as they seem. In each case, typical intuition suggested the frictions were all-important. Of course firm value depends exquisitely on debt vs. equity financing. Of course deficits “stimulate.” Of course open market operations matter. Of course stock prices are nuts, demand curves slope down, and it’s easy to make money on markets. Or so common intuition went. In each case, no. The unexpected theoretical proposition is first of all surprising as a possibility, as the proposition that the price level in terms of dollars can be well defined in an economy with no dollars at all. Moreover, in all these cases, the neutrality proposition turns out to be closer to true than false, and in each case the unexpected theoretical proposition is now our baseline starting point. Sure, debt vs. equity financing matters, but less than you thought, and just which Modigliani-Miller assumption fails provides the entire intellectual framework
for corporate finance.

So, the final description of the world will include fiscal backing, but also include monetary and financial frictions, and a role for policy in mitigating those frictions, and the potential to exploit those frictions. The emphasis will be different in different times and places. Still, the foundations matter, as the above list of doctrines reveal. The presence of the fiscal backing means that when we think about large or structural changes we get quite different answers than if we ignore it.

The fiscal theory I describe here is limited in a few ways. It is not an always-and-everywhere kind of monetary policy. It centrally requires that we use government money, or its equivalent, maturing government debt, as numeraire. It does not apply to an economy that uses gold coins, clamshells, Bitcoin, or another government’s money as numeraire. The Greek price level in the euro does not depend on Greek debt and surpluses, but on the fiscal backing of the euro. If we suffer a sovereign default and must rebuild a financial system around another definition of numeraire, we’ll need a different theory. (Bitcoin is built around the pure fiat money model; intrinsically worthless, but valued from the intersection of limited supply and a demand that withstands lower rate of return than comparable assets. I suspect that won’t last, and a successful non-government money will be based on some backing.)

The valuation equation must have something real in it ($s$, in the simple version) and something nominal ($B$, in the simple version) and policy must not be of the “passive” form described below. Basically, $P$ must not drop from the valuation equation. That allows us a lot of freedom – some real, indexed or foreign currency debt; some dependence of surpluses on the price level – but it does rule out a few special cases. Debt must at least sometimes and partially inflate rather than default.

We’ll explore these variations in what follows. The point here – the fiscal theory is, like all monetary theories, somewhat limited to some sets of institutions. These are our current institutions, but they have not always been, and may not always be our institutions.
Chapter 14

Stories

A few simple stories and conceptual experiments quickly come up when we think of fiscal theory. For any new theory one naturally jumps to conceptual experiments or stylized episodes to rule it out. The observational equivalence theorem bites hard on that enterprise, but it’s important to see how it bites, how the fiscal theory in fact is consistent with standard monetary stories.

14.1 Helicopters

The fiscal theory also predicts that prices rise under a helicopter drop. A helicopter drop is a device for communicating a fiscal commitment, that surpluses will not be raised to pay off the new debt.

Milton Friedman famously proposed the idea of dropping money from helicopters. If the government were to drop money from helicopters, people will run out and spend the money, driving up prices. Doesn’t that prove that in the end money is behind inflation?

No. Remember, the fiscal theory equation

\[
\frac{M_{t-1} + B_{t-1}}{P_{t}} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j}s_{t+j}.
\]

Dropping \( M \) from helicopters with no change in surpluses \( s \) and no change in debt \( B \) raises the price level \( P \) in the fiscal theory too! The sign of the response to this
conceptual experiment does nothing to distinguish monetary from fiscal theories of inflation.

The helicopter drop is a good conceptual experiment. First of all, recognize this is not what central banks do. Central banks always take back bonds when they give out money. The right question is, suppose the government comes in the middle of the night, takes $1,000 worth of your Treasury bond mutual fund and gives you $1,000 of cash – or $1,000 in your checking account – instead. How much would that make you spend? Suppose they took your $20 bills and gave you two $5 and a $10 bill for each one, an open change operation. The smaller bills are more liquid. How much would that make you spend? Phrased that way the answer is not so obvious!

The helicopter drop carefully combines a “wealth effect,” increasing the overall amount of government liabilities, and increasing private wealth at the current price level, with the composition effect – more money relative to bonds. This is not dishonest – to a monetarist, only $MV = PY$ matters, and whether the extra money comes with less bonds or not is irrelevant. But your intuition may be guided by the wealth effect! In the fiscal theory the effect of a helicopter drop is entirely a wealth effect.

A helicopter drop is not monetary policy, as conventionally defined. The Fed may not, by law, distribute money without buying something of equal value. A helicopter drop is fiscal policy, or at least a joint fiscal-monetary policy operation. In the US, a helicopter drop is accomplished by the Treasury borrowing money, issuing say 90 day Treasury bills in return; the Treasury then writes checks to voters. Then the Fed buys the Treasury bills, so the private sector has more reserves (now paying interest) and less 90 day bills. It is not so obvious that the last step is important!

Put another way, imagine that the Treasury drops the same quantity of newly printed three-month Treasury bills from the sky. Would that have much different effect on spending, stimulus, and eventual inflation than dropping the corresponding cash? The frictionless fiscal theory would say no. The monetary interpretation says this distinction is vitally important.

Imagine instead that the Treasury drops cash from the sky, with a note. “Good news: We have dropped $1 billion dollars from the sky. Bad news: Next week taxes will go up $1 billion dollars. See you in a week! With love from your public servants at the US Treasury.” Now how much will people spend? In the fiscal theory, this is a parallel rise in $M_{t-1}$ and $s_t$, which has no impact on the price level.

Now, we see, I think, the true meaning of the helicopters and the cash and why the
parable is so potent. The helicopters and cash are a brilliant way of communicating a fiscal expectation – we’re dropping this government debt on you, and we will not raise surpluses to pay it off. We’re so spendthrift we’re not even mailing checks and going through the pretense of balanced budgets – we’re shoveling the money out of the doors of helicopters! This is a share split, not a secondary offering. You will not have to pay more taxes, so go spend it. Had the government dropped bonds, or spent newly printed money in conventional ways, people might have inferred that this operation is like all bond issues, and comes with an implicit commitment to raise future taxes.

We can also consider the magnitude of the helicopter drop effect. Suppose you currently have $100 in your pocket in cash, and $10,000 in savings, in government bonds. Plus, the present value of your future earnings is $1,000,000. (Roughly, $50,000 per year for 20 years). The government drops another $100 in your front yard. How much do you try to spend? How much must the price level rise when everyone faces the same situation?

Under $MV = PY$, you spend the whole $100, and the price level must double. You personally may try to save the $100, by buying more bonds. But so is everyone else, and the money remains a hot potato in someone’s pocket until the price level doubles.

In the fiscal theory, the price level rises by $100/$10,000 = one percent. The extra money dilutes all government bonds as claim to unchanged primary surplus.

### 14.2 Hyperinflations and currency crashes

Hyperinflations all involve intractable fiscal problems. A central bank that refused to print money would not likely stop a fiscal hyperinflation.

Hyperinflations involve printing huge amounts of money. Doesn’t that prove that money printing is at the heart of inflation?

Every single hyperinflation has occurred because governments print money to finance intractable deficits. Hyperinflations end when the underlying fiscal problem is solved. The ends of large inflations typically involve printing more money, and interest rates that decline immediately. (Sargent [1983].) Hyperinflations are an entirely fiscal problem.
As a conceptual experiment, imagine that a central bank of a hyperinflation-ridden country refused to print any more money, and the government funded its deficits by printing up one-month bonds instead, paying suppliers with such bonds, and rolling over old bonds with new bonds directly. Would that stop the inflation? Probably not! If inflation did not occur, people would see a real default coming, and try to unload government debt by buying goods and services.

Similarly, imagine that a central bank of a country with fiscal deficits and facing pressure on its exchange rate fights the move by selling bonds, or refusing to buy new bonds and keeping control of the money supply. Would that stop the exchange rate collapse?

To be fair, monetarist analysis has long recognized that there are fiscal limits, and that successful control of the money supply requires a solvent fiscal policy. But therefore, the fact that hyperinflating countries do typically print up a lot of money does not tell us that money printing alone causes inflation.
Chapter 15

Assets, institutions and choices

Societies can choose a wide range of assets and institutions to run their fiscal and monetary affairs. In this chapter, I examine some possibilities, how the fiscal theory generalizes to include these possibilities, and some thoughts on which choices might be better than others in different circumstances.

The latter is the hardest question. The fiscal theory puts inflation squarely in the middle of public finance. Optimal fiscal and monetary policy faces many trade-offs. A government facing a fiscal shock chooses inflation, explicit partial default, partial defaults on different classes of debt held by different investors (money vs. debt, for example), raising distorting taxes, or cutting spending. Each of those has welfare and political costs. Governments also face temptations to inflate or default ex-post. Each decision is also dynamic, as actions taken this time influence expectations of what will happen next. Precommitment, time-consistency, reputation, moral hazard, and asymmetric information are important considerations in a monetary and fiscal regime. Fiscal-monetary policy is a regime mediated by institutions, not a string of decisions.

A theme recurs throughout this section. In

\[
\frac{B_{t-1}}{P_t} = E_t \sum \frac{1}{R^j} s_{t+j},
\]

the government could stabilize the price level if it could find a way to commit itself to a set of surpluses. The trouble with the fiscal theory so far is that the expectation on the right hand side is nebulous and potentially volatile. The government would like to precommit and communicate that it will manage surpluses to defend a stable
price level – no more, and no less. That stock prices are much more volatile than inflation suggests that governments have been able to make such commitments, at least implicitly.

Actually, it might like a more sophisticated commitment, that it will manage surpluses to defend a stable price level, but with rare escape clauses in war, deep recession, and so forth when it might like to implement a state-contingent default via inflation.

In this chapter, we will see a variety of structures, from indexed debt, foreign debt, exchange rate pegs, gold standard, and so on, to some suggestions for the future. Some of these use legal contracts as precommitments – precommitting to legal costs of default, say – others are policy regimes that try to mimic some of those commitments – an exchange rate peg, say. All of them can be seen as ways to make and communicate fiscal and monetary commitments that stabilize inflation, with escape clauses for times of fiscal stress, and to overcome the usual contracting problems in the way of that quest.

15.1 Indexed debt, foreign debt

I extend fiscal theory to include real debt – indexed debt, debt issued in foreign currency. Such debt acts as debt, where nominal debt acts as equity. If the government is to avoid explicit default, it must raise surpluses sufficient to pay off real debt, and the price level is not determined by its valuation equation – passive fiscal policy.

Governments often issue indexed debt or debt issued in a large-country foreign currency. Such debt acts as debt, where nominal debt acts as equity.

Indexed debt pays $P_t$ rather than $1$. Denote the quantity of one-period indexed debt issued at time $t - 1$ and coming due at time $t$ by $b_t$. The government must then pay $b_{t-1}P_t$ dollars at time $t$; it collects $P_ts_t$ dollars from surpluses. Each bond sold at the end of $t$ promises $P_{t+1}$ dollars. With a constant real rate and risk neutral pricing, the flow condition becomes

$$b_{t-1}P_t = P_ts_t + E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1}} (P_{t+1}) \right) b_t$$

$$b_{t-1} = s_t + \frac{1}{R} b_t$$
so iterating forward we obtain

\[ b_{t-1} = E_t \sum_{j=0}^{\infty} \frac{1}{R_t^j} s_{t+j}. \]  

(15.1)

The price level has disappeared, so long as real surpluses \( s_t \) are independent of the price level. Something else must determine the price level. The fiscal theory is not an always and everywhere theory. For the fiscal theory to determine a price level, we need an equation with something nominal and something real in it. (We will, however, explore mechanisms by which \( s_t \) is a function of \( P_t \), which can restore fiscal price level determination even with completely real or indexed debt.)

If the government is to avoid default, equation (15.1) now describes a restriction on surpluses – the government must raise enough surpluses to pay off its debt. Fiscal policy must be “passive.” In (15.1), debt is incurred by running deficits in the first place, so this condition only means that surpluses must rise to pay off debts as promised.

If we add real interest rate variation, with \( R_t \) the real interest rate from time \( t \) to time \( t+1 \), we have

\[ b_{t-1} = E_t \sum_{j=0}^{\infty} \left( \prod_{k=0}^{j-1} \frac{1}{R_{t+k}} \right) s_{t+j}. \]

(15.2)

Now, to avoid default, the government must respond to real interest rate increases, which may unexpectedly raise its cost of funding the debt.

Suppose the government borrows entirely in foreign currency, and uses foreign currency completely. This case can be handled with the usual equation, denominating everything in foreign currency:

\[ \frac{B_{t-1}}{P^*_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R_t^j} s_{t+j}. \]  

(15.3)

Now, \( P^*_t \) represents the price of goods in terms of the foreign money, and \( s_t \) is the surplus measured in the same units.

As with (15.1), (15.3) is now a constraint on surpluses which the government must run in order to avoid default. Fiscal policy must be passive. Additional events not of the governments own doing will affect budgets: the government must adapt surpluses to changes in the foreign price level, or equivalently to changes in the real exchange
rate. If the foreign country price level goes down unexpectedly, our country must raise real surpluses or default. Moreover, it now needs to raise foreign exchange.

The equation no longer directly determines the price level $P_t$. The price level is determined by the foreign country, potentially with its fiscal theory. As price levels vary within a country, so the price level in such a country will vary from that of the country whose currency it is using. But fiscal and monetary policy have no part in that variation.

The same logic applies to a country in a currency union, such as the members of the euro. Greece uses Euros, and agrees to pay its debts in Euros, and (ideally) the rest of the EU is not responsible for Greece’s debts. Therefore, (15.3) requires that Greece either run surpluses to pay its debts, or default. The European price level does not adjust in response to Greece’s debts, as long as the rest of Europe commits that it will not print up money to pay o. (Applying the fiscal theory to understand the determination of the value of the Euro itself takes a little more effort, which I take up below.)

In both cases, these conclusions assume that the foreign country takes no responsibility for paying off our country’s debts, and in particular will never print up money to do so. The situation is the same as the private debt of a company, denominated in dollars. As long as the government takes no responsibility for paying off the company’s debt, private debt issue, repayment, and potential default have nothing to do with the price level. The $B_t$ in the fiscal theory is only direct liabilities of the government, and the surpluses $s_t$ only its revenues.

This assumption is frequently violated, both domestically and internationally. Implicit or explicit foreign debt guarantees can create international linkages of inflation and currency values, as well as moral hazard, as implicit or explicit bailout guarantees can cause domestic inflation. The design and imperfect operation of the Eurozone is all about this question; and our understanding of episodes including the East Asian currency crashes of the late 1990s centrally involve such guarantees.

\section*{15.2 Assets and liabilities}

What about other assets and liabilities, like social security, pensions, health care and so on? What about the national parks or other assets?
While generalizing and applying the theory, I turn to two issues on the fiscal side of the equation.

What about all the other assets and liabilities of the government? Social security, pensions, medicare, medicaid, Social security and pensions are all promises to pay people that act in some ways like government debt. Adding them up, depending on how one takes present values, one can get numbers of $70 to $300 trillion, dwarfing the official $20 trillion debt.

The federal government also has a lot of state contingent promises. It offers deposit insurance, and it is likely to bail out banks, so in fact perhaps we should count M1 as indirectly a government liability. It is likely to bail out private (ERISA) and state and local pension funds, at least in part. It offers formal credit guarantees, including those on home mortgages that pass through Fannie and Freddie. (The Federal Reserve Bank of Richmond’s “bailout barometer” is a nice compilation of some of these contingent promises.) “Automatic stabilizers” such as unemployment insurance and social security disability create additional spending in recessions. The government has assets as well. What are the national parks worth? More important financially, it owns vast swaths of the western states. Where do we put all this?

If one wishes a complete accounting, I think it wiser to add these considerations in to the flow of surpluses. Formal financial assets, such as some countries have in a sovereign wealth fund, are easier to measure and to sell, so we can add them productively to the right hand side in a separate category.

Social security, health, and pensions are promises to pay, as coupon and principal payments are promises to pay. However, the government can at any time reduce those promises without formal default. Governments around the world reform pension and health payment systems all the time, in response to fiscal pressures. More importantly, they are not marketable debt, and they are long-term debt. As we have seen, current inflation responds to future deficits when debt is short-term and rolled over, in a run-like mechanism. There is no way to run on promised pension and health care payments. They cause inflation if short-term investors in formal government debt spy a default or inflation on the horizon due to unresolved entitlements, but not directly. Many of the promised payments and credit guarantees are option-like and not debt-like as well – they kick in only in bad states of the world. Figuring out a market value in good states of the world and treating them like debt is not that productive. They will make matters dramatically hard in bad states of the world, more than a debt calculation would reveal. A good analysis of their effect on inflation should retain their put-option characteristic.
Forecasts of future health and retirement payments are clearly not forecasts in the traditional sense, but “here is what will happen if you don’t do something about this soon” warnings. The US government, with its current tax system, simply cannot make the promised payments. Even defaulting on or inflating away the entire current debt would do no good, since future tax revenues are nowhere near capable of funding future payments. What unsustainable eventually does not happen, so the forecasts simply tell us that somewhere down the road the US must fundamentally reform its spending plans, its tax system, its tradeoff of growth for protective regulation, and likely all three. So adding up the exploding deficits under current law, treating them as debt, and puzzling over the price level, is not a productive exercise.

For all these reasons, I think it’s more productive to treat these considerations as elements of future state-contingent surplus forecasts, with uncertainty, rather than to add up forecasts or expected values, discount them, and treat them as debt.

The state-contingency of future surpluses is important, and especially their sensitivity to inflation. The extent to which surpluses depend on the price level matters a lot in the fiscal theory. To draw out the point, write

\[
\frac{B_t}{P_t} + b_t = a_t + \frac{A_t}{P_t} + \mathbb{E}_t \sum_{j=0}^{\infty} \frac{A_{t+j} (P_{t+j}, z_{t+j})}{A_t} s_{t+j} (\{ P_t, z_t \}).
\]  

(15.4)

Here \( z_t \) is vector of other economic variables, and \( \{ P_t, z_t \} \) indicates that the sequence of price levels and other variables can influence the time \( t + j \) surplus, not only \( P_{t+j}, z_{t+j} \) on date \( t + j \). \( a_t \) and \( A_t \) represent real and nominal assets of the government. The central idea of the fiscal theory is that the sequence of price levels adjusts so that (15.4) holds on every date.

Now, clearly, how surpluses depend on the price level matters a lot. For example, if government worker salaries are nominal – as they are – and if they are not indexed for inflation, then a little bit of inflation also reduces government deficits. If medical care prices are administered by the government – as they are – and they are sticky to respond to inflation, then a little bit of inflation reduces government deficits. Non-neutralities in the tax code, including progressive tax brackets that are not indexed, taxation of nominal capital gains, and the fact that depreciation schedules are not indexed, all mean that inflation helps government finance, at least once – until people demand better indexation – through its positive effect on surpluses. If we count these as debt on the left hand side, they would count as debt that is at least partly nominal, and thus reduced by inflation. Or we can count these as a positively sloping \( s(P) \) function on the right hand side.
15.3. **DEBT AND EQUITY**

On the other hand, social security payments are aggressively indexed for inflation, so social security is at least a real debt, whose value does not change with inflation, or even a debt whose value increases with inflation, or an \( s(P) \) function that is flat or negatively sloping on the right hand side.

These considerations are all important for figuring out how sensitive inflation is to fiscal and other shocks, and how tempting it will be for the government to inflate rather than reform or default when in trouble. The state contingent nature of inflationary and other shocks to surpluses matter too, as I have indicated with the \( z_t \) variables. Poor economic growth, rather than tax rates, are, as we have seen, the greatest source of surplus variation. Poor economic growth is also associated with higher marginal utility.

Clearly, a proper accounting of all these state-contingent and price-level contingent features of government finances is important. A much better job of this analysis would advance fiscal policy. Understanding the contingencies involved would, I think, help an analysis of just how exposed are we to the danger of inflation, or default.

However, it clearly is unlikely to be *productive* to try to mash all this into present values, and try to predict the current price level, in particular to understand the last percent or two of inflation and its timing.

### 15.3 Debt and equity

Real debt – indexed or foreign – act like corporate debt. The government must raise the required surpluses or default. Nominal debt acts like corporate equity. Its value can adjust to respond to surplus news.

Indexed debt and foreign debt are debt. Like corporate debt, the government must either adjust surpluses to pay back the debt, or default. If the price level declines, if interest rates rise, or if the foreign price level falls, the government must adjust surpluses or default, just as a corporate issuer must pay more to bondholders and less to equity holders or default in these circumstances. The price of default-free debt does not change, so we cannot determine the price level from the valuation equation of a default-free government that issues only real liabilities.

Government-issued nominal debt functions like corporate equity. Its price can adjust, just as corporate equity prices can adjust when there is a decline in expected dividends. As a corporation does not have to adjust its dividends upward to match
an increase in its stock price, neither does a government that has issued nominal
debt have to adjust surpluses to follow changes in the price level.

This distinction lies at the heart of much confusion over the fiscal theory of the
price level. Equation (15.1) is often called the “intertemporal government budget
constraint.” Even that word is inappropriate, as default is possible and a true “budget
constraint” does not have an escape clause. (This is a subtle issue about on and
off equilibrium prices, which I return to below.) But it does function much like a
constraint, in that if the government wishes to avoid default it must rearrange its
surpluses so that equation (15.1) holds. But with nominal debt, government debt
no longer functions as debt, it functions as equity. The valuation equation is not a
constraint. As stock prices fall to equate the value of stock to the present value of
dividends, so the price level can fall to equate the value of nominal debt to the present
value of surpluses. Yet continued use of the word “intertemporal budget constraint”
when nominal debt replaces real debt has resulted in a lot of confusion.

Real debt is a precommitment device. The legal structure of real debt, and the costs
default imposes, commits the government to arrange surpluses so as to defend the
price level, and to suffer the costs of explicit default if it should not do so. The
trouble with the set of legal commitments underlying real debt – purely indexed
debt, or full dollarization – is that they commit the government to repay the debt
for any price level, not just its target price level. We want a set of commitments
that defend a target price level or inflation rate, but not unexpected inflations or
deflations, and we will see commitments that achieve this result shortly.

All of these institutions lie on a spectrum between pure “debt” and pure “equity,”
involving different degrees of precommitment to change surpluses ex-post. None is
as inviolable as the “budget constraint.” And no wise government lets surpluses be
a purely exogenous process, letting the price level go where it may.

15.4 Default

The fiscal theory can incorporate default. An unexpected partial default substitutes
for inflation in adapting to a fiscal shock. A preannounced partial default is an
interesting way for governments to create moderate fiscal inflation. Default is costly
ex-post, which helps to enforce the precommitment to pay debts and not inflate.

The fiscal theory is often characterized as applying to nominal government debt,
and a commitment that the government will always inflate rather than default; it
15.4. DEFAULT

will in extremis print up money as needed to pay off maturing nominal debt. This characterization is not quite right.

The fiscal theory can incorporate defaults. Suppose, for example, that the government at date $t$ unexpectedly writes down its debt: It says, for each dollar of promised debt, we pay only $D_t < 1$ dollars. Now, we have

$$B_{t-1} (E_t - E_{t-1}) \left( \frac{D_t}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \quad (15.5)$$

The price level is still determined. In fact, this unexpected partial default allows the government to adapt to a negative surplus shock with less or no inflation. A greater haircut, lower $D_t$ implies a smaller rise in $P_t$ in response to a negative surplus shock. Of course, defaults and inflations are both costly, so the choice of default or inflation is a subtle one.

A pure expected partial default has no direct effect, but can influence future inflation by its influence on monetary policy. If people expect a partial default, we have

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j} \quad (15.6)$$

$$Q_t = \frac{1}{1 + i_t} = E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1}} D_{t+1} \right). \quad (15.7)$$

If at time $t$ the government announces a partial default $D_{t+1}$, with no change in surpluses, this change has no effect on the current price level $P_t$, by (15.6). The effect on the future price level $P_{t+1}$ depends on monetary policy – how much debt $B_t$ the government sells, or the interest rate target $i_t$. By (15.6), if surpluses are unchanged, then the quantity $Q_t B_t$ is unchanged. If the government allows the interest rate to rise, fully reflecting the default risk probability, so in (15.7) $D_{t+1}$ and $Q_t$ change one-for-one, then neither $P_t$ nor $P_{t+1}$ is affected by the announced partial default. This is the case that expected default has no effect. If the government sticks to the interest rate target, leaving $i_t$ and $Q_t$ unchanged, then the expected future price level $P_{t+1}$ declines.

But an announced partial default with no surplus news is a strange and unrealistic intervention, just as a debt issue with no surplus change is unusual for a treasury. Let’s go back to the beginning. Suppose that a government wants to inflate a little, as many governments at the zero bound and undershooting inflation targets wanted to do. A pure announcement that future $s_{t+j}$ will be lower, is likely not to be believed.
It’s not clear what the government will do to create inflation. Skeptical markets say, if you are still undershooting the inflation target, what will you do then? Give more speeches? Long-term US surplus forecasts are already disastrous enough, and having no visible effect on inflation. And conversely, routine politician promises about higher future surpluses not backed up by actions or precommitments have little effect.

Unbacked fiscal expansion, essentially helicopter money, adds a deficit today to this scenario: a lower surplus today $s_t$ raises debt $B_t$, while promising less or no change in future surplus $s_{t+j}$. But the extra debt $B_t$ only raises revenue to fund lower $s_t$ if people believe there will be more future surplus $s_{t+j}$. As before, having gained a reputation for paying back debt, it’s hard for the treasury to persuade markets that this time is different.

It’s harder still to persuade markets that this time is just a little bit different. It may be possible to convince markets that we’re adopting Venezuela’s fiscal policies, and induce a hyperinflation. But how do you convince markets that exactly 2% of a fiscal expansion will be unbacked?

Consider then an announcement, that next year there will be a 2% debt haircut. The above announcement of this haircut without any change in surpluses is remarkable and unusual. Why would the government take on the costs of partial default if there is no news about surpluses? So suppose the haircut announcement comes with an announcement, or simply causes an expectation, that following surpluses will be 2% lower. That is, after all, the most natural interpretation of the haircut – we’re defaulting because we don’t intend to pay it back! That is how a government issuing real debt would have to behave, and much of the art here is for a government with nominal debt, and therefore determining price level by the fiscal theory, to act like a government with real debt, and thereby committing to fiscal operations that lead to its desired price level.

So, suppose at time $t$, the government announces a 2% haircut for $t+1$, $D_{t+1} = 0.98$, and at the same time that surpluses from $t + 1$ onwards $E_t \sum_{j=1}^{\infty} s_{t+j}/R^j$ will be 2% lower. Since surpluses including $s_t$ on net approximately 2% lower, the price level $P_t$ rises 2%, via (15.5).

As usual, monetary policy then determines the expected future price level. If the government allows the interest rate to rise, to follow the increased default premium, then by (15.7), the expected price level at $t+1$ is also 2% higher. In this way, the announced partial default allows monetary policy to move away from a zero bound. If the government keeps the interest rate constant, however, then the price level at $t+1$ is unchanged, and the fiscal inflation only lasts one period.
Default also has costs. That is pretty much its point. Rather than just promise a set of surpluses, by offering indexed or foreign currency debt, the government precommits to large costs should it default. Greece is a great example: By joining the Euro, so its bonds were supposed to default if it could not pay them back, Greece precommitted against default, as default would involve large costs. That precommitment allowed Greece to borrow a lot of Euros at low interest rates, and to avoid the regular bouts of inflation and devaluation that it had suffered previously. Alas, when Greece finally did default, it discovered just how large those costs were.

15.5 Currency Pegs and Gold Standard

Exchange rate pegs and the gold standard are really fiscal commitments. Reserves don’t matter to first order, as no government has reserves to back all of its nominal debt. If people demand foreign currency or gold, the government must eventually raise taxes, cut spending, or promise future taxes to obtain or borrow reserves. The peg says “We promise to manage surpluses to pay off the debt at this price level, no more and also no less.” The peg makes a nominal debt (equity) act like real debt (debt). Unlike full dollarization, a peg gives the country the right to devalue without the costs of explicit default. But the country pays the price for that lower precommitment. Both gold and foreign exchange rate pegs suffer though, that the relative price of goods and gold, or foreign currency, may vary.

In an exchange rate peg or under the gold standard, the country issues its own currency, and can borrow in its own currency. But the government promises to freely exchange its currency for foreign currency or for gold, at a set value. (By “currency,” I mean the unit of account and bank reserves at the central bank. Actual currency may fade away.)

The exchange rate peg or gold standard sound like monetary policy, and suggest that money gains its value from the promised conversion rate. But they are in fact fiscal commitments, and the value of the currency comes ultimately from that fiscal commitment.

Analysis of the gold standard and exchange rate pegs often focuses on the question of reserves, whether the government has enough gold or foreign currency to stand behind its conversion promise. Enough has never been enough, and both gold promises and foreign exchange rate pegs have seen “speculative attacks” and devaluations. (And once, that I know of, Switzerland 2015, an attack leading to rise in currency value.)
A currency board takes the reserves logic to its limit: it insists that all currency – all central bank liabilities – must be backed 100% by foreign currency assets. 100% gold reserves for all currency issues are a similar idea.

But reserves are, to first order, irrelevant. It is the ability to get reserves when needed that counts. No country, even those on currency boards, has ever backed all its debts with foreign bonds or gold. If a country could do so, it wouldn’t have needed to borrow in the first place. When those debts come due, if the government cannot raise surpluses to pay them off, it must print unbacked money or default. Moreover, no government has ever had reserves against all its future borrowing needs. When the government runs in to fiscal trouble, abandoning the gold standard or currency board and seizing its reserves will always be tempting. Argentina’s currency board fell apart this way in a time of fiscal stress. Moreover, if people see that grab coming, they will run immediately.

Conversely, if the government has few reserves, but ample ability to tax or borrow reserves as needed, credibly promising future taxes or spending cuts, then it can maintain convertibility easily. Just tax or borrow the reserves when needed.

Sims (1999) provides a nice historical example:

“From 1890 to 1894 in the US, gold reserves shrank rapidly. US paper currency supposedly backed by gold was being presented at the Treasury and gold was being requested in return. Grover Cleveland, then the president, repeatedly issued bonds for the purpose of buying gold to replenish reserves. This strategy eventually succeeded. From one point of view, it was simply an open market operation: sale of bonds to absorb high-powered money. But at the time, the US had no central bank. Cleveland issued the bonds under dubious legal authority, without consulting Congress, and there resulted a major legal and political dispute – luckily after the fact. The argument of Cleveland’s opponents, which was surely correct in principle, was that while the issuance of the bonds was not directly a purchase of goods and services, it nonetheless imposed fiscal obligations, and Congress was constitutionally charged with deciding such issues.”

Reserves may matter to second order, if financial frictions or other constraints make it difficult for the government to raise money quickly. But they only matter for that short window. Likewise, solvent banks do not need lots of reserves because they can always borrow reserves or issue equity if needed. Insolvent banks run out of reserves quickly.
The government debt valuation still holds,
\[
\frac{B_{t-1}}{P_t} = G_t + E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]

Here, let \( P_t \) be the price of gold or foreign currency in terms of domestic currency, and let \( G_t \) denote the value of gold or foreign currency reserves. Other real government assets that can be sold to soak up debt go on the right hand side of the valuation equation. For now, assume that the relative price of gold and goods is always one, and purchasing power parity, i.e. that the relative price of domestic and foreign goods is always the same. We’ll generalize those assumptions in a minute.

Here we see explicitly how reserves per se are irrelevant. They are one source of fiscal resources to back the issue of currency and nominal debt, but they enter in parallel with the present value of surpluses.

The foreign exchange peg or gold standard are thus primarily a fiscal commitment. If \( P_t \) is going to be constant, then the government must adjust surpluses \( s_t \) on the right side as needed. Free conversion helps to enforce and make visible this commitment. The peg says, “We will manage our taxes and spending so that we can always pay back our debts in foreign currency or gold at this fixed exchange rate, no more and no less.” When that promise is credible, it removes the uncertainty of a present value of surpluses and stabilizes the price level.

This sort of fiscal commitment is valuable. Good fiscal-monetary policy regimes have such commitments. Much of the point of this section is to study such commitments and find better ones. The present value of surpluses is potentially as volatile as stock prices. If the government left the price level to the vagaries of investor’s expectations about long run surpluses and their time-varying discount rates and risk premiums, inflation could be as volatile as stock prices. But surpluses are not as exogenous as the profits of profit maximizing companies. If governments could offer and communicate a commitment, that surpluses will be adjusted to defend a given price level, and debt will be paid off at that price level; no lower but no higher either, inflation could be much more stable. Such an arrangement produces what looks like a passive fiscal policy at a price level given elsewhere, but is in fact an active fiscal policy arranged to determine a steady price level.

Conversely, abandoning the gold standard or revaluing an exchange rate peg offers a fiscal commitment that can create inflation or deflation as required. If the government says, rather than $20 per ounce, the dollar will now be worth $32 per ounce, that means that surpluses will only be raised in order to pay off existing debt at $32 per
ounce, not $20. Otherwise, as we have seen, generic announcements that surpluses will be lower may fail to move inflation. A devaluation is a way of credibly announcing the sort of partial default I described above, and its exact amount.

More subtly, governments on the gold standard, with the UK being the prime example, could suspend convertibility during a war or other crisis, but then return to convertibility at par afterwards. The return to convertibility, though fiscally expensive, gave bondholders the confidence to hold debt and paper money during the war.

Thus the gold standard or pegs offer a fiscal commitment with escape clauses. The government can enjoy in normal times the advantages of a fiscal precommitment, giving a steady price level and anchored long-term expectations, while leaving open the option of state-contingent default achieved through devaluation and inflation. This is a useful system if small state-contingent defaults are not desirable, because there are default costs and because there are moral hazard and time-consistency problems. Of course, the government also pays the price of an interest rate premium when it does not exercise its options to default.

The peg or gold standard also allows the country to keep any seigniorage resulting from liquidity value of its currency or debt, or resulting from legal restrictions forcing its use.

As usual, we start with a simple frictionless model, and add important frictions to describe reality. The most important friction regarding the gold standard is that the relative price of goods and gold varies. Pegging the currency in terms of gold, there have still been unpleasant inflation and deflations. Exchange rate pegs have this difficulty as well, in that the real exchange rate – the relative price of domestic and foreign goods – may vary, as well as the fact that there may be inflation or deflation in the foreign good. Pegging the value of the currency to the foreign currency, there still have been unpleasant inflations and deflations.

To incorporate these features let $P_t^*$ denote the foreign price level, in units of foreign currency per foreign good. Let $e_t^r$ denote the real exchange rate, i.e. the how many domestic goods it takes to buy one foreign good. In the simplest case of purchasing power parity, $e_t^r = 1$. Let $e_t$ denote the nominal exchange rate, how many units of
domestic currency it takes to buy a unit of foreign currency. Then
\[ e_t^e = e_t \frac{P_t}{P_t^*}; \]
\[ \text{euro goods} = \frac{\text{euros}}{\text{dollars}} \frac{\text{dollars}}{\text{US goods}}. \]

Likewise, let \( e_t \) denote the gold price, in units of ounces of gold per dollar. Let \( P_t \) be dollars per good, and \( P_t^* \) be ounces of gold per good. In this case, and we have
\[ e_t^r = e_t \frac{P_t}{P_t^*}; \]
\[ 1 = \frac{\text{goods}}{\text{goods}} = \frac{\text{ounces}}{\text{dollars}} \frac{\text{dollars}}{\text{ounces}}. \]

The valuation equation now reads
\[ \frac{B_{t-1}}{P_t} = \frac{B_{t-1}}{P_t} \frac{e_t}{e_t^r} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}, \tag{15.8} \]
with \( e_t^r = 1 \) in the case of gold.

For a floating exchange rate or gold price, we can read this equation as a fiscal theory of exchange rates or (less interesting) gold price. When the government pegs the nominal exchange rate \( e_t \) or gold price, there is nothing endogenous on the left hand side, and we see again that this equation describes how surpluses must adjust to pay off debt in order to defend the peg.

Equation (15.8) crystallizes the main trouble with the gold standard or foreign exchange rate peg. By fixing the value of the currency in terms of gold or foreign exchange, the government does not fix the price level. Variation in the real exchange rate \( e_t^r \) or the relative price of domestic and foreign country goods, the “terms of trade,” forces a fiscal change on the right hand side of (15.8) and thus forces a pointless inflation or deflation. If the price of gold relative to goods rises, the government must raise the present value of taxes and produce – or accommodate depending on your view – a deflation. If the relative price of domestic goods relative to foreign declines – if demand for a country’s commodity exports declines, for example – the government must tighten and produce or accommodate a deflation. The government must commit to pay back debt at the value of gold or foreign currency, not the value of goods. Or abandon the peg.
That’s pretty much what happened in the 1930s – the price level fell; currencies were tied to gold. Countries either revalued or abandoned the gold standard, which meant abandoning the fiscal commitment to repay dollar debt in gold. This step occasioned lawsuits, that went to the Supreme court. The court said, in essence, yes, the US is defaulting on gold clauses; yes, this means the US does not have to raise taxes to pay you back, and yes, the US has the constitutional right to do that. (Kroszner (2003), Edwards (2018).)

This story combines the downside of the gold standard, that it can induce unintended deflation, with the advantage of a standard or peg: When a country devalues, it makes clear the fiscal loosening that attempts at unbacked fiscal expansion during the recent zero-bound era were not able to communicate. Tying yourself to a mast has the advantage that it is very clear when you tie yourself to a shorter mast.

A successful gold standard or peg makes nominal debt (equity) look and act like real debt (debt) – the government adjusts surpluses ex-post to keep the price level target (gold or foreign currency) steady. But it remains nominal debt, and its value is determined by its fiscal backing. However, it really is a fiscal theory and a fiscal commitment of the value of currency in terms of gold or foreign currency. It is not a fiscal theory of the price level, the value of currency in terms of goods, or fiscal commitment to a stable price level directly, as it imports the relative value of gold or foreign currency and goods from elsewhere. Mostly it counts on other economic forces, and foreign inflation discipline in the case of a peg, as the additional step to control the price level. But those are often imperfect mechanisms.

The gold standard or peg take important steps away from gold coins or dollarization. The government gets to keep seigniorage it otherwise loses from any liquidity value of its liabilities including currency. When there is a money demand $MV = PY$ or similar liquidity demand for debt, the government can meet that demand, as note and debt issue exceeds foreign and gold reserves. The option to devalue, as above, can be an important escape hatch allowing a more stable price level in terms of currency than in terms of gold or foreign exchange itself, without the costs both to the government and to its financial system of a formal partial default. That option comes at a price of course, of less precommitment – more incentive to devalue when raising distorting taxes or cutting spending might be better; less clarity about just what the present value of surpluses will be, and thus higher ex-ante interest costs.

I implicitly assumed here that the real exchange rate $e^r_t$ is not affected by fiscal policy (the size of debt or future surpluses). That seems like a reasonable assumption, but
if it is false, it also offers an interesting next step in the project to add frictions to a fiscal theory of exchange rates. Trade frictions, financial frictions, multiple goods, price stickiness are all valuable next steps as well, which I will suggest but not take.

Similarly, actual analysis of the gold standard should take into account its many frictions – the costs of gold shipment; the way gold coins often traded above their metallic content value (Sargent and Velde (2003)), the limits on convertibility, and so forth. Gold standard governments also ran interest rate policies, and raised interest rates to attract gold flows. That combination merits analysis in the same way we added interest rate targets to the fiscal theory.

An obvious question regarding gold: What determines the value of gold in the first place? We often tell a story that the value of gold is determined by its industrial uses independent of monetary policy. But this is clearly false. In fact, the gold standard was built on economies that used gold coins. Gold coins are best analyzed, in my view, as a case of $MV = PY$, rather than a case in which money has value because it carries its own backing with it, but the value of gold vs. goods is determined by non-monetary forces. Gold is in sharply limited supply, with few substitutes especially for large-denomination coins. A transactions demand for gold coins – or gold bars, for large transactions – then gives gold its value. This pure monetary version ignoring all industrial uses seems more compelling especially of the pre-industrial age.

That fact helps to understand several features of the gold era. First, the price level was stable over centuries, but not over years. Well, short run variation in money demand cannot be met by a rigid supply. Flexible note issue, not constrained by a binding reserve requirement, is an improvement. Second, and more importantly, there is an obvious feedback not present in my above analysis. Paper money substitutes at least to some extent for gold coins.

$$MV + GV_g = PY,$$

if you will (though imperfect substitutability is likely a better model). This means that monetary policy – exchanging money for debt, or money for gold with less than 100% reserves – affects the price of goods in terms of gold, which I took as exogenous.

I write all of this only to disclaim the limitations of the analysis. Yes, the gold standard and exchange rate peg is part of the fiscal theory, and they work by offering a fiscal commitment. But I do not claim that PPP and constant gold price of goods are the end of the analysis!
The gold standard had many faults, and I do not advocate its return, despite its enduring popularity as a way to run a transparent rules-oriented monetary policy that (mostly) forswears inflation. A gold standard or peg works when there is an economic force that brings the price level we do want to control into line with the commodity that can be pegged. In the gold standard era, gold coins and bars continued to circulate. If the price of gold relative to other goods rose, i.e. if there was deflation, then people had more money than they needed, and in their effort to spend it the price level would return. But if the price of gold relative to other goods rises now, this mechanism to bring their relative prices back in line is absent. Gold is just one tiny commodity, and tying down its price is going to stabilize inflation about as well as if the New York Fed operated an ice-cream store on Maiden lane and decreed that a scoop shall always be a dollar. Well, yes, a network of general equilibrium relationships tie that to the CPI... but not very tightly, and certainly not as tightly as when gold was still an important part of money, and an inventory of money was still crucial for making transactions.

Foreign exchange rate pegs for small countries suffer some of the same disadvantage. The economic force that pulls real exchange rates back, purchasing power parity, is weak. At a minimum, that’s why countries peg to their trading partners. On the other hand, relative prices in a currency union are much less volatile than relative prices across countries with independent monetary policies, so the case for larger currency areas is worth considering. I return to this issue later.

More generally, I followed conventional analysis by implicitly predicting that if we move back to a gold standard, the CPI would inherit the current volatility of gold prices. The relative price of gold and other goods would remain constant, with other prices being as volatile as gold is now. This prediction is not at all obvious. If the Treasury returned to pegging the price of gold, it is instead quite possible that it, well, pegs the price of gold, and forever quiets the relative price of gold and other goods. The stickiness of goods prices alone suggests this outcome, though it is likely deeper. However, its converse is also true. By pegging the price of gold, with gold no longer tied to the rest of the economy, does the Treasury do anything to stabilize the CPI? Likely not.

An obvious last question: How can we have the advantages of a gold standard or currency peg, without the unwanted inflation when the relative price of gold or foreign currency moves? How can a government peg the consumer price index? One’s first thought goes to a larger commodity standard. But most of the commodities in the CPI are not tradeable, so the government cannot just open a huge Wal-Mart and trade the components of the CPI for money. Narrower commodity money proposals
suffer the same weakness as gold or foreign exchange, that their relative prices vary a good deal, and might vary more if monetary policy were pushing on them. I find a partial answer in inflation targeting regimes, below, and investigate ways to commit by offering to buy and sell inflation linked securities in place of the actual goods.

15.6 The corporate finance of government debt

I import concepts from corporate finance of equity vs. debt to think about when governments should issue real (indexed or foreign currency) debt, when they should have their own currencies and nominal debt, and when they might choose structures in between, like an exchange rate peg or gold standard which can be revalued without formal default. Governments must issue more debt-like instruments when they cannot precommit not to inflate or devalue, and when their institutions and government finances are more opaque. To issue equity, governments must offer something like control rights. In modern economies, the fact that inflation damages private contracts so much means that voters are mad about inflation, which helps to explain that stable democracies have the most successful currencies.

Should a government choose real – indexed, foreign currency – or nominal debt? Or contracts and institutions that are somewhat in between, such as the gold standard or price level target, which is like debt with a somewhat less costly default option. Corporations also fund themselves with a combination of debt, equity, and intermediate securities (convertible debt), so a good place to start is simply by importing that analysis.

Governments typically issue a combination of real and nominal debt. The latter may include just the currency itself. With such a combination, the valuation equation becomes

\[ b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \pi_{t+j}. \]

The price level is determined again.

A corporation that borrows more increases the volatility of its stock returns. Nominal debt is a buffer, like corporate equity. That analogy suggests that other things held constant (an important qualification!) the larger the fraction of the debt is real, the more inflation volatility will result from a given surplus variation, and that nominal
debt is preferable in order to reduce inflation volatility. In addition, more nominal
debt, like equity, makes formal default less likely and reduces overall costs of default
as a result.

Putting the question in terms of optimal fiscal policy, the government faces shocks
to its finances and trade-offs between the three ways of addressing those shocks:
formal default \((b)\) default via inflation \((B/P)\) and raising taxes or cutting spending
\((s)\). Formal default is costly. Unexpected inflation and deflation is also destructive
with sticky prices, nominal rigidities or unpleasant effects of surprise redistributions
between lenders to borrowers. Distorting taxes are costly, and governments may
regard “austerity” spending cuts as costly too. Lucas and Stokey (1983) argue for
state-contingent partial defaults, to minimize tax distortions. Schmitt-Grohé and
Uribe (2007) add price stickiness and argue for more tax variation and less inflation
variation. But clearly the optimum is an interior combination depending on these
three costs.

However, given that some fiscal stress is met by unexpected inflation, the more
fiscal bang for the inflation buck, the better. That consideration suggest that the
government issue more nominal debt – maybe even issuing extra nominal debt and
buying real assets \(b < 0\), as in countries that have substantial sovereign wealth
funds.

On the latter basis, Sims (2001) argued against Mexico adopting the dollar or issuing
lots of dollar denominated debt. Full dollarization means fiscal problems must be
met with distorting taxes, spending cuts, or costly explicit default. A Peso allows for
subtle devaluation via inflation. Moreover more Peso debt allows Mexico to adapt
to adverse fiscal shocks with less inflation – and lower still costs of explicit default
or devaluation.

The same argument lies behind a fiscal-theoretic interpretation of the widespread
view that countries like Greece should not be on the Euro – currency devaluations
implement state-contingent defaults, perhaps less painfully than explicit default or
“austerity” policies to raise surpluses. (The conventional arguments for local curren-
cies involve central banks’ ability to offset negative shocks with inflationary stimulus,
an entirely different story.)

When add the possibility of runs or on short-term debt to costly formal default,
the case for equity-like government finance grows stronger still, just as more equity
rather than short term debt is an easy cure for costly financial crises.

So why choose a debt-like security, in the form of indexed or foreign debt, or a
gold standard? In corporate finance, debt helps to solve moral hazard, asymmetric information, and time-consistency or precommitment problems. An entrepreneur may not put in the required effort; he or she may be tempted steal some of the cashflow, or he or she may not be able to credibly report what the cashflow is. Debt leaves the risk and incentive in the entrepreneur’s hands, helping to resolve the moral hazard problem. The fixed value of debt payments requires no information.

So, in fact, the theory of corporate finance predicts widespread use of debt. Equity only works when the issuers can certify performance, through accounting and other monitoring, and by offering shareholders control rights.

The same ideas apply to countries. Own-currency debt will work better when government accounts are more trustworthy and transparent. But what are the control rights of government equity? Most naturally in the modern world, voters. If nominal government debt gets inflated away, a whole class of powerful voters is really mad. Inflation is even more powerful than explicit default in this way. If the government defaults, only bondholders lose, and a democracy with a universal franchise may not care. Or the bondholders may be foreigners. If the government inflates, every private contract is affected. The government’s effective default triggers a widespread private default, and everyone on the losing end of that default suffers. Why do we use government debt as our numeraire, thus exposing private contracts to the risks of government finances? Well, the fact that we do, and we vote, means that there is a very large group of voters who don’t like inflation. That’s a good example of control rights.

So, what governments should or are forced to use indexed or foreign currency debt, and what governments should use their own currency? The standard ideas of corporate finance suggest that countries with precommitment problems, and with poor institutions including poor fiscal institutions and government accounts, who tend to issue and then default or inflate, must issue real debt, and explicit default must be painful to them. Countries who can precommit better, and stable democracies with a widespread class of lenders and others who prefer less inflation, are able to issue government equity, i.e. have their own currencies and borrow in it.

Confirming this view, foreign currencies, currency pegs, indexed and foreign debt are common in the developing and undemocratic world. Successful non-inflating currencies and large amounts of domestic currency debt seem to be the province of stable democracies.

Precommitment is, I think, a major push toward real debt. Surpluses are different from corporate profits in that the latter are (hopefully) the result of profit-maximizing
and hence at least somewhat less under the corporation's control. A country has more choice over surpluses. As I have emphasized above, a choice to finance a greater proportion of debt by indexed or foreign debt can be a useful device for committing to more stable surpluses. A corporate analogy: Corporations with more unionized workforces seem to choose more debt financing, likely to enhance their bargaining power relative to unions, see Bronars and Deere (1991).

And there is a spectrum of assets between real debt and pure equity. All governments with stable price levels somehow commit to stabilize surpluses. The gold standard and peg – with their escape clauses – are good examples of intermediate institutions. They precommit in some sense against “unnecessary” inflation. In normal times, the peg both commits the government to make fiscal adjustments, and communicates that commitment. (The latter may be more important.) The escape clause is used for huge, visible shocks – wars, historically, or Great Depressions – but not so often that people lose faith in the normal times commitments.

This section does not conclude with a firm one is better than the other. Clearly, the choice of securities, like the choice of policy regimes, depends on the circumstances of the country. This is good, as we see very different regimes. But now we have some guidelines about trying to understand why some countries choose some regimes and others choose other regimes, and when those choices might change over time. That sort of analysis also helps to confirm a fiscal-theoretic view, and overcome the observational equivalence theorems that hold inside regimes. If you can successfully predict when governments change regimes by observing changes in these kinds of circumstances, that helps to validate the underlying theory of inflation.

We started with a simple equation that suggested if other things are constant, more nominal debt means less inflation volatility. But the fact that different governments choose different debt structures is one of the many reasons that we should not expect such a simple correlation to test the theory. Governments who need to precommit will choose debt, and this will produce more stable surpluses for them. Governments with less volatile surpluses have less need for state-contingent default via inflation. And so forth.

This discussion makes light of an enormous literature on sovereign debt, and also long historical experience. Again, this is a place to start, and we should build by integrating fiscal theory with that theoretical end empirical literature. The sovereign debt literature studies the extent to which reputation and other punishments can induce repayment. Much foreign country debt includes rights to seize assets, and adjudication by third country legal systems for these reasons, or other costs to the
borrowing country of formal default. In the history of government finance, it took centuries for governments to be able to *borrow*, somewhat credibly promising repayment, something only achieved with some dependability in the early 19th century. The development of fiat inflatable currencies took another one to two hundred years, depending on when you count success with that ongoing project. Real government debt is full of institutions that help to precommit to repayment. The bank of England and Parliamentary approval for borrowing and expenditures were 1700s institutions for that purpose. Alexander Hamilton is justly famous for the insight that a democracy needs widespread ownership of government debt, by people with the political power to force repayment. Today, sovereign debt includes many institutions beyond reputation to try to force repayment, including third-country adjudication and the right of creditors to seize international assets – with only partial success, given the repeated foreign debt crises of the last several decades. Our monetary system is full of institutions that prevent inflationary finance, including the prohibition on the Federal Reserve buying debt directly from the Treasury, and on the Treasury printing money to spend it. The humorous suggestions for the Treasury to inflate by issuing trillion dollar coins to deliberately inflate only show how strong the institutional constraints on unbacked fiscal expansion.

## 15.7 Long vs. Short

We explore the choice between long term and short term debt.

As governments must choose between real – indexed, or foreign currency debt – and nominal debt, they must choose a maturity structure: Issue largely long-term debt, or issue largely short-term debt?

Decisions have varied a great deal through time. Queen Victoria’s empire was largely financed by perpetuities, the longest debt imaginable, and tied to the gold standard. The current U.S. government has, as above, a quite short maturity structure, rolling over about half the debt every two years. Governments in fiscal trouble find themselves pushed to shorter and shorter maturities.

The usual consideration in this choice is the search for low risk premiums in the term structure. The treasury tries to find maturities where yields are abnormally low, and issue in those yields. I will abstract entirely from this consideration.

Instead, let us take inflation stability as a goal of the government, and characterize
in what situations long or short term debt is advantageous.

15.7.1 Surplus shocks and debt as a buffer

Chapter 8 analyzed long term debt as a buffer against surplus shocks. Here, I assemble some key results to think about the choice of maturity structure.

From section 8.2.1 we saw how long term debt can offer a buffer against surplus shocks. Again, with one-period debt,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}$$

(15.9)

shocks to future surpluses affect the price level today, while in the example with long term debt and no future debt sales,

$$\frac{B_{t-1}}{P_t} = s_t$$

(15.10)

only immediate surpluses have any effect on the price level. Shocks to expected future surpluses are absorbed by bond prices, the numerator on the left hand side of the valuation formula

$$\sum_{j=0}^{\infty} Q^{(t+j)}_t B^{(t+j)}_{t-1} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}$$

rather than the denominator.

Does this mean long-term debt is better for inflation stability? Not necessarily. It is better in these examples when the volatility of current surpluses is lower than the volatility of the present value of future surpluses (15.9). It would take a lot of negative serial correlation of surpluses to make the right hand side of (15.9) less volatile than the right hand side of (15.10). But, as we have seen several times, such negative serial correlation is a hallmark of well-run fiscal policy. A government that wishes to borrow, to run a negative surplus $s_t$, must promise higher surpluses in the future $s_{t+j}$ to do so. Section 9 describes surplus processes with no variation in the present value of surpluses and lots of variation in actual surpluses, and section 15.9 describes fiscal rules designed to produce that result.

Thus, if a government is running a good fiscal policy, in which negative shocks to today’s surplus are matched by positive shocks to future surpluses, then it is quite
possible that short term debt via (15.9) produces less inflation volatility than long
term debt via (15.9).

Moreover, the long-term debt example is far too stylized. For the government can sell
debt in the future, and governments do so. If the government has a large perpetuity
outstanding, and is paying off coupons, following $B_{t-1}/P_t = s_t$, if a bad surplus $s_t$
comes along, the government need not suffer a one-period price level increase; it
can sell new debt, promising to raise future surpluses. And this is exactly what
governments do.

The government can also sell additional debt and not raise future surpluses. In
a long analysis from section 8.2 to section 8.5, we saw how governments can alter
the time-path of inflation by buying and selling debt when there is long-term debt
outstanding, with no change in surpluses. Extra sales dilute existing long-term bonds
as a claim to surpluses, and move inflation from period to period. The presence of
outstanding long term debt then opens the way to such later inflation-smoothing
options. QE does not work, in this frictionless analysis, unless there is long-term
debt outstanding. Issuing long term debt comes with the option to dilute it or
strengthen its value in the future.

One source of such sales is an interest rate target. When the government follows
an interest rate target, in this frictionless analysis, that target pins down expected
inflation. Implicitly, debt sales occur to break the $B_{t-1}/P_t = s_t$ examples. With an
interest rate peg, only one-period unexpected inflation matters to the volatility of
inflation. To think about the volatility of inflation with long term debt under an
interest rate target, then, start with the long term debt valuation formula

$$\sum_{j=0}^{\infty} Q^{(t+j)}_t B^{(t+j)}_{t-1}/P_t = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}$$

With our usual risk neutral valuation,

$$Q^{(t+j)}_t = \frac{1}{R^j} E_t \left( \frac{P_t}{P_{t+j}} \right),$$

we have

$$\sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R^j} (E_t - E_{t-1}) \left( \frac{1}{P_{t+j}} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}$$

Now, if the government follows a price level target (which is desirable in many ways),
then the future terms on the left hand side disappear, and

\[
\frac{B^{(t)}_{t-1}}{R^j} (E_t - E_{t-1}) \left( \frac{1}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}
\]

The price level target turns off the possibility that a surplus shock is met by future inflation. Instead, it must all be met by current inflation. The longer the maturity structure of the debt, the smaller one-period debt coming due \( B^{(t)}_{t-1} \), and the more sensitive the price level is to surplus innovations. Long-term debt and a price level target are therefore best suited to a fiscal policy that makes sure not to run shocks to the present value of surpluses.

If the government follows an interest rate target, then, approximately, it allows a one-period price level jump;

\[
(E_t - E_{t-1}) \left( \frac{1}{P_{t+j}} \right) = (E_t - E_{t-1}) \left( \frac{1}{P_t} \right).
\]

It thus devalues bonds of all maturity and

\[
\left( \sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R^j} \right) (E_t - E_{t-1}) \left( \frac{1}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]

So, with an interest rate target, the sensitivity of inflation to surplus shocks is independent of the maturity structure of the debt.

Of course, active interest rate targets and QE offer interesting extensions. If a rise in inflation this period leads to a rise in the interest rate target, then in this frictionless model, long term bonds take even more of the fiscal pressure and once again long term debt leads to less inflation volatility than short term debt.

### 15.7.2 Interest rate shocks

Long term debt can insure the budget against shocks to current and expected future real interest rates, and thus insure inflation against such shocks.

Long term debt leaves the budget, and hence the price level, less exposed to real interest rate variability. (Though we have mostly analyzed surplus shocks, in fact real interest rate shocks are vital to accounting for the patterns of debt and inflation.)
If the government borrows short term, then a rise in the interest rate raises interest costs in the budget and necessitate tax increases or spending decreases, or result in inflation. In the extreme of overnight or completely floating rate debt, the government must pay any increased interest times the entire stock of debt immediately.

If the government borrows long-term, then the increase in interest cost only affects the government very slowly, as new debt is issued to finance new surpluses, or as long-term debt is slowly rolled over. In the extreme that the government runs no new deficits, it may quietly pay off the coupons of a perpetuity and be completely insulated from any budgetary effects of interest rate increases.

The mechanism is familiar to any homeowner choosing between a fixed and floating rate mortgage. If interest rates rise, the floating rate borrower has to pay more immediately. The fixed rate borrower pays the same amount no matter what happens to interest rates, at least until he or she refines or borrows more. The homeowner must adjust spending; for a government the result can be inflation if surpluses do not respond.

For equations to illustrate these ideas, we can return to the simplest cases. For short-term debt, perhaps the continuous time version is clearest,

\[
\frac{B_t}{P_t} = E_t \int_0^{\infty} e^{-\int_{\tau=0}^{\tau} r_t + \delta \gamma_s t \tau} d\tau.
\]

Here we see that not only current interest rates, but changes in expected future interest rates will affect the price level. Persistent interest rate shocks that affect long term bonds have more effect than transitory shocks.

For long-term debt, the cleanest example is the case of a perpetuity, paid off with no new debt sold. In that case each day’s coupons are paid by that day’s surplus,

\[
\frac{B_{t-1}^{(t)}}{P_t} = s_t
\]

and the price level is unaffected by real interest rates. Real interest rate changes change the value of debt, but in this case also change the value of surpluses equally.

We can unite the two polar cases with the continuous time present value relation

\[
\int_0^{\infty} \frac{Q_{t+t+\gamma}}{P_t} B_{t+t+\gamma} d\gamma = E_t \int_0^{\infty} e^{-\int_{\tau=0}^{\tau} r_t + \delta \gamma_s t \tau} s_t \tau d\tau.
\]
Under the expectations hypothesis,

\[
\int_{j=0}^{\infty} E_t e^{-\int_{\tau=0}^{\tau=t} (r_{t+\tau} + \pi_{t+\tau})d\tau} B_t^{(t+\tau)} d\tau = E_t \int_{\tau=0}^{\infty} e^{-\int_{\tau=0}^{\tau=t} r_{t+\tau}d\tau} s_{t+\tau} d\tau.
\]

\[
\int_{j=0}^{\infty} E_t e^{-\int_{\tau=0}^{\tau=t} r_{t+\tau}d\tau} \frac{P_t}{P_{t+\tau}} B_t^{(t+\tau)} d\tau = E_t \int_{\tau=0}^{\infty} e^{-\int_{\tau=0}^{\tau=t} r_{t+\tau}d\tau} s_{t+\tau} d\tau.
\]

\[
\int_{j=0}^{\infty} e^{-\int_{\tau=0}^{\tau=t} r_{t+\tau}d\tau} \frac{B_t^{(t+\tau)}}{P_{t+\tau}} d\tau = E_t \int_{\tau=0}^{\infty} e^{-\int_{\tau=0}^{\tau=t} r_{t+\tau}d\tau} s_{t+\tau} d\tau.
\]

Now, the mismatch between the maturity of debt and the (usually much longer) maturity of the surplus process determines how sensitive the price level is to interest rate variation, and when that effect surfaces. A persistent interest rate rise lowers the right hand side. If debt is very short term, the interest rate rise does not much affect the numerator on the left hand side, and prices must rise; furthermore they must rise in the near term, for small \( \tau \). As debt becomes longer term, the interest rate rise starts to affect the left hand side as it does the right hand side, and smaller and more delayed inflation results. When the maturity of the debt matches exactly the maturity of the surplus, as in the perpetuity example, then the interest rate term cancels.

### 15.7.3 Runs and crises

Section 8.2.2 emphasized how the intertemporal linkages of the present value relation come from rolling over debt. Bad news of surpluses 25 years from now causes inflation today, because people know that 24 years from now the government will find it hard to sell bonds, so there will be inflation 24 years from now. But then 23 years from now the government will find it hard to sell bonds, and so on. That backward recursion is fragile. Short-term debt leads to runs and crises here as everywhere else. Greece got in to trouble, not because it could not finance one year’s deficits, but because it could not find new borrowers to roll over debt. A roll-over crisis, or run on real debt causes a default or financial panic. A roll-over crisis or run on nominal debt causes a sudden inflation or devaluation, which seems to come from nowhere or at least to be far outsized compared to the usual straw-that-broke-the-camel’s back piece of news that precipitates it.

All financial crises are runs on short term debt. This consideration argues for a much longer maturity structure.
15.8 Inflation targets

Inflation targets have been remarkably successful. I interpret the inflation target as a fiscal commitment. The target commits the legislature and treasury to pay off debt at the targeted inflation rate, no more and no less, and to adjust fiscal policy as needed, as much as it commits and empowers the central bank. This interpretation explains why the adoption of inflation targets led to nearly instant disinflation, and central banks have almost never been tested to sharply raise interest rates and exercise the toughness that conventional analysis of inflation targets says is their key. An inflation target is an instance of fiscal theory because the legislature commits to pay off debt at the target inflation rate, not any actual inflation rate.

Inflation targets have been remarkably successful. Figures 15.1, 15.2, 15.3 show inflation around the introduction of inflation targets in New Zealand, Canada, and Sweden. On the announcement of the targets, inflation fell to the targets pretty much instantly, and stayed there, with no large recession, period of high interest rates or other monetary stringency. Just how was this miracle achieved?

Inflation targets consist of more than just announcements by the central bank. Central banks make announcements and promises all the time, and markets regard such statements with skepticism well seasoned with experience. Inflation targets include an agreement between central bank, treasury, and government. The conventional story of their effect, however, revolves around central banks. The inflation target agreements free and empower the central bank to focus only on inflation, give it independence, and hold its feet to the fire to produce low inflation.

But these stories are wanting. Did previous central banks just lack the guts to do what’s right, in the face of political pressure to inflate? Moreover, just what does the central bank do to produce low inflation? One would have thought, and pretty much everyone did think, that the point of the inflation targeting agreement was to insulate the bank from political pressure during a long period of monetary stringency. To fight inflation, the central bank would have to produce high real interest rates, and a severe recession such as accompanied the US disinflation during the early 1980s. And the central bank would have to repeat such unwelcome medicine regularly. For example, that is exactly the diagnosis repeated by McDermott and Williams (2018).

Berg and Jonung (1999) discuss Sweden’s price level target of the 1930s. It called for systematic interest rate increases if the price level increased and vice versa. Like the modern experience, the central bank apparently never had to do it, and actually pegged the exchange rate against the pound during the period.
Figure 15.1: Inflation surrounding the introduction of a target in New Zealand. Source: McDermott and Williams (2018)

Figure 15.2: Inflation surrounding Canada’s introduction of an inflation target. Source: Nakamura (2018), Murray (2018)

the source of my New Zealand graph, of the 1970s and 1980s.

But nothing of the sort occurred. Inflation simply fell like a stone on the announce-
15.8. **INFLATION TARGETS**

Figure 15.3: Inflation target in Sweden. The vertical line marks January 1993, when the inflation target was announced.

...ment of the target, and the central banks were never tested in their resolve to raise interest rates, cause recessions, or otherwise squeeze out inflation; not in the initial nearly instant decline, nor later. Well, “expectations became anchored,” by the target, people say, but just why? The long history of inflation certainly did not lack for pleasant speeches from politicians and central bankers promising future toughness on inflation. Why were these speeches so effective now? Do we really think that one more agreement between politicians cemented everyone’s view that from now forward central banks would always be tough on inflation, so tough that inflation never breaks out in the first place? Anyone with children knows that unpleasant threats never tested are not believed.

A hint is provided in the first graph with the “GST introduced” and “GST increased” notations. Each one of these inflation targets emerged as a part of a package of reforms including fiscal reforms, spending reforms, and market liberalizations. Yes, the agreement grants the central bank autonomy, and suggests that it focus on inflation alone. **But it also binds the legislature and treasury.** Even [McDermott and Williams (2018)](https://doi.org/10.1017/9781108504129), though focusing on central bank actions, write “A key driver of high inflation in New Zealand over this period [before the introduction of the inflation target] was government spending, accommodated by generally lose monetary policy.”

I therefore read the inflation target as a bilateral commitment. It is equally a commitment by the *legislature and treasury* to 2% (or whatever the target is) inflation. They commit to run *fiscal* affairs to pay off debt at 2% inflation, no more, and no less, just as the gold standard or exchange rate peg commit the legislature and treasury...
to pay off debt at a prestated real value, no more and no less.

In the simplified frictionless model, the central bank commits to keep expected inflation at 2%, and the government commits to take fiscal actions so there is no unexpected inflation around 2%. As we have seen, price stickiness draws out both dynamics, and a sticky price model of the advent of an inflation target would be an interesting exercise.

I read the success of inflation targets as an instance of the pattern of ends of inflations noted by Sargent (1983). As Sargent showed, when the fiscal problem is solved, credibly, inflation drops on its own almost immediately. There is no period of monetary stringency, no high real interest rates moderating aggregate demand, no recession. At the end of hyperinflations, money supply actually increases: People know inflation will end so they demand more money. \( MV(i) = PY \), and \( V(i) \) declines quickly when the nominal interest rate falls.

The inflation target commits the legislature and treasury to back debt at the price level target, but not to pay back changes in the real value of debt due to changes in the price level away from the target. This is vitally important, and part of the genius of the inflation target. If, looking at

\[
\frac{B_{t-1}}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+j}, \tag{15.11}
\]

the fiscal authorities commit to adjusting \( s_{t} \) to pay off the debt for any price level, we have the case of passive fiscal policy, and the valuation equation no longer determines the price level. Should an unexpected deflation break out, the legislature must raise taxes or cut spending to finance a real windfall to bondholders. But the commitment in an inflation target is only to support the target \( P_{t}^{*} \), not the actual price level. Surpluses adjust so that

\[
\frac{B_{t-1}}{P_{t}^{*}} = E_{t} \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+j}. \tag{15.12}
\]

Equation (15.12) describes fiscal policy; it determines \( \{s_{t}\} \) given \( P_{t}^{*} \). Then equation (15.11) describes how the price level \( P_{t} \) is determined, and says the price level must equal the target \( P_{t} = P_{t}^{*} \).

Fiscal policy looks passive, in equilibrium. And fiscal policy acts passively in equilibrium, responding to increases or decreases in debt, paying off that debt at a stable price level. But fiscal policy is not passive, and the fiscal theory still determines the price level, because fiscal policy does not respond passively to non-equilibrium price

\[
\text{(15.11) and (15.12)}
\]
15.9 Fiscal rules

I model fiscal rules to capture the intuition of the inflation target. Surpluses can depend on the price level in a sort of fiscal Taylor rule

\[
\frac{B_{t-1}}{P_t} = s_t(P_t)
\]

The fiscal rules and commitments underlying my interpretation of inflation targeting suggest more general and important tools for successful monetary-fiscal policies.
There are two related ideas: First, the inflation target amounts to a commitment that the government will pay off accumulated nominal debts at the price level target, but not at other price levels. That commitment allows the government to accumulate debt, to borrow when needed, and to reassure investors that their debts will be paid off, without leaving a passive fiscal policy that does not determine the price level. But it also determines the price level, which a fully passive policy would not do, by refusing to pay off nominal debts that come from deflation or inflation away from target. Second, the government could raise surpluses in response to inflation, and decrease in response to disinflation or deflation, in a sort of fiscal Taylor rule. This provision turns out to be a more general case of the first one, and allows for price level determination even in the absence of nominal debt.

The basic ideas are easiest to see in the simple one-period model. Write the one-period model

\[ \frac{B_{t-1}}{P_t} = s_t(P_t). \]  

(15.13)

I write the right hand side \( s_t(P_t) \), as the dependence of the surplus on the price level is the central question. I started with a simple example of a constant tax rate and no spending, \( P_t s_t = \tau P_t y_t \), to establish that the real surplus does not naturally have to depend on the price level. But surpluses can and do depend on the price level, and the central question of this section is how designing surplus variation with the price level can lead to more stable inflation. Immediately, we see that the price level depends not just on the downward slope of the left hand side, but the functional relationship on the right hand side as well.

The tax code features many non-neutralities. For example, tax brackets, capital gains, and depreciation allowances are not indexed. Spending features many non-neutralities too: government salaries, defined-benefit pensions, and medical payments are at least somewhat nominally sticky. All of these forces generally result in somewhat higher surpluses with inflation \( s_t'(P_t) > 0 \). More importantly, the central idea in this section is that the government can intentionally vary surpluses vary with inflation or the price level to improve price level control, as central banks following a Taylor rule intentionally vary the interest rate with inflation or the price level to improve their control.
15.9. FISCAL RULES

15.9.1 Paying back debt at the price level target

I build on passive fiscal policy. If surpluses are set so \( s_t(P_t) = B_{t-1}/P_t \), then debts are always paid, for any price level, and the fiscal theory ceases to determine the price level. Instead, we specify a fiscal rule \( s_t(P_t) = B_{t-1}/P^*_t \) where \( P^*_t \) is the price level target. The government commits to pay back any debts actually incurred, but not to validate changes in the price level away from target. This policy determines the price level, and sets it equal to the target \( P_t = P^*_t \), while also committing the government to pay back any debts accumulated by previous real deficits.

I build on a passive fiscal policy. The classic passive fiscal policy is generated by a surplus that depends negatively on the price level, \( s_t(P_t) = s_0t/P_t \), and the nominal surplus \( P_t s_t \) is independent of the price level. If \( P_t s_0t = B_{t-1} \), i.e. if

\[
s_t(P_t) = \frac{B_{t-1}}{P_t},
\]

then the valuation equation holds for any price level. Money printed to pay off nominal debt is always soaked up; a deflation-induced rise in the real value of debt provokes higher taxes and lower spending. (If \( s_0t < B_{t-1} \), the price level rises to infinity; more money is printed to pay off debt than can ever be soaked up, or the government defaults on the debt. If \( s_0t > B_{t-1} \), the private sector must default on tax obligations.)

Such passive policy results from a commitment that the government will always pay back its nominal debts for any price level. That characterization makes it a reasonable potential description of policy. It’s how a gold standard or foreign exchange rate peg work, and as we have seen a problem with those arrangements is precisely that governments are forced to respond to price level variations, not just to pay off debts as promised. Despite the “passive” label, however, there is nothing passive about it, in the normal English sense of the word of not taking action. The government must legislate tax rate changes, say \( \tau = \kappa/P_t \), or spending changes, in reaction to changes in the price level, so that the natural approximate neutrality of surpluses with respect to the price level does not emerge. Passive policy is already an example of a price-level dependent fiscal policy rule.

Rather than promise bondholders that the government will always pay back its nominal debts, it is attractive for the government to promise always to pay back its real debts. Bondholders do not obviously want the inflation risk of nominal debts. Thus,
consider the surplus rule

\[ s_t(P_t) = \frac{B_{t-1}}{P_t^*} \]  

(15.15)

where \( P_t^* \) is the price level target. This policy is a commitment that the surplus will \textit{not} depend on the price level. (The point of the notation is that there is no \( P_t \) on the right hand side.)

This policy looks like the passive policy (15.14), but it restores price level determinacy in the presence of nominal debt. Moreover, this policy enforces the inflation target. The valuation equation now delivers

\[ \frac{B_{t-1}}{P_t} = s_t(P_t) = \frac{B_{t-1}}{P_t^*} \]

and thus \( P_t = P_t^* \).

Yet this policy retains attractive features of the passive policy. At time \( t - 1 \), bondholders understand that a greater bond issue \( B_{t-1} \) will result in greater surpluses to pay off the debt, not a higher price level. Thus, the government gets the bond-issuing benefits of passive policy. It can run “regular” cyclical or wartime deficits and surpluses. But it does not commit, say, to raise taxes to pay off deflation-induced windfalls to bondholders. It also gets the price-level determination benefits of active fiscal policy. The government can get price-level stability, if it wishes. It can also deliberately inflate, by changing the price level target. It communicates what the present value of surpluses will be, eliminating fiscal uncertainty.

I write equations in terms of a price level target for simplicity, but the same policy could enforce an inflation target rather than a price level target. Just let \( P_t^* = P_{t-1} \Pi_t^* \) where \( \Pi_t^* \) is the inflation target.

This policy also illustrates observational equivalence. As in many cases we have seen, the government appears, in equilibrium, and in its actions, to be following a passive policy, raising surpluses to pay off debts. But it is not doing so.

### 15.9.2 A surplus rule

Now, we consider fiscal policies that directly respond to inflation, raising surpluses in response to inflation, in a sort of fiscal Taylor rule,

\[ s_t(P_t) = s_0 t(P_t) - \alpha \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right) \]
This fiscal rule also determines the price level. If \( s_{0t}(P_t) \) is either passive or semipassive, promising to pay back outstanding nominal debt, then this fiscal rule also sets the price level equal to the target \( P_t = P_t^* \). This fiscal policy can also determine the price level at when there is no nominal debt, and all debt is real.

Now, suppose that the government also deliberately raises real surpluses with the price level or with inflation. I also add indexed debt \( b_{t-1} \), so the one period model equilibrium condition becomes

\[
b_{t-1} + \frac{B_{t-1}}{P_t} = s_t(P_t),
\]

with \( s_t'(P_t) \neq 0 \). (Each indexed bond promises to pay \( P_t \) dollars.) For example, consider a policy of the form

\[
s_t(P_t) = s_{0t}(P_t) - \alpha \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right). \tag{15.16}
\]

Think of \( s_{0t}(P_t) \) as a “regular” fiscal policy, and the second term as an additional fiscal Taylor rule provision designed to help stabilize the price level.

Suppose that \( s_{0t} \) is passive, promising to pay off debt at any price level,

\[
s_{0t} = b_{t-1} + \frac{B_{t-1}}{P_t}.
\]

Now adding the fiscal Taylor rule part of (15.16) implies that inflation must still equal the target.

The choice

\[
\alpha = B_{t-1}
\]

yields

\[
b_{t-1} + \frac{B_{t-1}}{P_t} = b_{t-1} + \frac{B_{t-1}}{P_t^*},
\]

so the previous policy (15.15) is a special case. Any value of \( \alpha \) enforces that the price level equals the target \( P_t = P_t^* \), and therefore enforces the idea that the government promises in equilibrium to pay nominal debt off at the target price level. The choice (15.15) \( \alpha = B_{t-1} \) means that the government makes this promise out of equilibrium: For any \( P_t \neq P_t^* \), the government pays off nominal debt as if \( P_t = P_t^* \) had occurred. Different \( \alpha \) values mean that the government makes stronger or weaker promises about repayment out of equilibrium. The policy \( \alpha = 0 \) with passive \( s_{0t} \) means that
the government promises to pay off nominal debt at $P_t$, no matter what that $P_t$ is.

We can also merge the two examples. Suppose that $s_{0t}$ commits to pay off nominal debt only at the price level target,

$$s_{0t} = b_{t-1} + \frac{B_{t-1}}{P_t^*}.$$ 

Now (15.16) implies

$$b_{t-1} + \frac{B_{t-1}}{P_t} = b_{t-1} + \frac{B_{t-1}}{P_t^*} - \alpha \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right).$$

With $\alpha \neq 0$, both components of the fiscal policy contribute to price level determination. Each component is sufficient alone. The left side is a negative function of the price level $P_t$. A flat function of the price level $P_t$ would do, but a choice $\alpha > 0$ makes the right side a positive function of $P_t$, strengthening the force behind price level determination. A sign $\alpha < 0$ will also work, up to the passive policy point $\alpha = -B_{t-1}$, where the price level again drops from the equation.

The inclusion of such a fiscal Taylor rule $\alpha > 0$ works even when all debt is indexed, $B_{t-1} = 0$, whereas the previous proposal (15.15) could not determine the price level in that case. To see how this works, note that with only real debt, and if real surpluses are independent of the price level – for example $P_t s_t = \tau P_t y_t$ – the price level $P_t$ cancels,

$$b_{t-1} = s_t,$$ \hspace{1cm} (15.17)

and we cannot determine the price level. This, the passive case for real debt, is a constant real surplus, not a constant nominal surplus which generates passive policy for nominal debt. (Everything is real in (15.17) because it is indexed. There are still nominal claims floating around, and there is still a price level to be determined somehow. In this case, money printed up in the morning increases with the price level, and so does money soaked up in the afternoon.)

Now, suppose that real surpluses respond to the price level, $s_t = s_t(P_t)$ with $s'_t > 0$, or equivalently that the nominal surplus $P_t s_t$ is not exactly proportional to the price level. Now we have again

$$b_{t-1} = s_t(P_t).$$

As the price level increases, the government raises taxes and cuts spending, until once again all the dollars are soaked up at only one price level.
The price level can be determined by fiscal policy, even with no nominal debt. The principle that to determine the price level you need something nominal and something real is satisfied by the dependence of the real surplus on the nominal price level.

The force that brings the price level in line – the supply curve; the off-equilibrium threat – is to leave unbacked money outstanding, or to demand such money that the private sector cannot provide. For example, review the case

\[ s_{0t} = b_{t-1} + \frac{B_{t-1}}{P^*_t}. \]  \hspace{1cm} (15.18)

In the morning, the government prints up money \( P_t b_{t-1} + B_{t-1} \), to pay off outstanding debt, $1 for each nominal bond and \( P_t \) dollars for each real bond. In the afternoon, the government soaks up money with the surplus \( s_{0t} \) as given by (15.18) and debt sales Thus the government’s budget constraint (emphasis to distinguish it from the equilibrium condition) says that, at a price level \( P_t \), money outstanding at the end of the day is

\[
M_t = (P_t b_{t-1} + B_{t-1}) - \left( P_t b_{t-1} + \frac{P_t}{P^*_t} B_{t-1} \right) \\
= \left( \frac{P^*_t - P_t}{P^*_t} \right) B_{t-1}.
\]

If \( P_t < P^*_t \), then there is extra money sitting around at the end of the day. People don’t want to hold money, so they try to spend it, raising aggregate demand and the price level. If \( P_t > P^*_t \), people don’t have enough money to pay taxes; they lower aggregate demand to get it and lower the price level. The equilibrium condition \( M_t = 0 \), together with this budget constraint, implies \( P_t = P^*_t \)

15.9.3 Intertemporal models

I express the fiscal rules of the last section in intertemporal models. This extension introduces a new and important idea – passive vs. active debt issue policy, whether variation in the price level induces a roll-over of debt that a semi-passive fiscal policy will then pay off.

The central idea is that fiscal policy rules can control how the right hand side of the valuation equation responds to the price level

\[
b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j} = PV_t(P_t). \]  \hspace{1cm} (15.19)
The current surplus can respond, \( s_t(P_t) \), as in the one-period examples, but surpluses can respond with a lag as well, \( s_t(P_t, P_{t-1}, \ldots) \). Thus future surpluses can respond to the price level today \( s_{t+1}(P_{t+1}, P_t, \ldots) \) and the present value thereby responds to \( P_t \).

We want to see some simple examples that build towards realism. We also want to consider how the government distinguishes between “regular” surpluses and deficits, and commits to pay off such debts, but also commits to unbacked fiscal expansion or contraction to stabilize inflation. The trouble is analogous to the usual practical trouble in the Taylor rule, \( i_t = \phi_\pi (\pi_t - \pi^*_t) + \phi_y y_t + v_t \). How do you separate in practice interest rate changes due to the “rule” component that gives price level stability, \( \phi_\pi (\pi_t - \pi^*_t) \) from changes in the inflation target \( \pi^*_t \) and responses to real events \( \phi_y y_t + v_t \)? How do you know when a central bank is deviating from a rule or just addressing some temporary concern? (The trouble is even deeper in a model in which equilibrium leads to \( \pi_t = \pi^*_t \)!

However, inflation targets may be successful in part by *not* making this distinction formal or rule-based. They say, essentially, “trust us, when we see more inflation we’ll start raising taxes to soak up the money, and you’ll be able to tell the difference between those taxes and taxes used for other purposes.” But rules clarify thinking and are often valuable when such faith is in question, and at least we can describe the policy by a rule.

### A two-period model

We study a two-period model. As in the one-period model, fiscal policy follows a semi-passive rule \( s_T = B_{T-1}/P^*_T \). Debt policy at time \( T - 1 \) also follows a semi-passive policy, rolling over debt at the price level target \( P^*_T \), but not rolling over revaluations in debt caused by variation in the price level away from target. Debt \( B_{t-1} \) is set so

\[
\frac{1}{R} \left( \frac{B_{T-1}}{P^*_T} \right) = \frac{B_{T-2}}{P^*_T} - s_{0T-1},
\]

with \( P^*_T \) rather than \( P_{T-1} \) on the right hand side. Thus, variation in the price level away from target at time \( T - 1 \) also leaves unbacked money outstanding, or demands more than there is. Now we have \( P_{T-1} = P^*_T \) at time \( T - 1 \) as well.

To see how such a dynamic example can work, it is useful to work backwards from a terminal period. As above, suppose the terminal surplus is set to pay off outstanding
15.9. **FISCAL RULES**

debt at the price level target,

\[ s_T = \frac{B_{T-1}}{P_T^*}. \]  \hspace{1cm} (15.20)

Then, the last period equilibrium condition is

\[ \frac{B_{T-1}}{P_T} = s_T = \frac{B_{T-1}}{P_T^*}, \]  \hspace{1cm} (15.21)

and we have \( P_T = P_T^* \) as the unique equilibrium price level.

Now, work one period backwards. Suppose the surplus at time \( T-1 \) is simply fixed, \( s_{T-1} = s_{0T-1} \). In the morning of time \( T-1 \), the government prints up money in the quantity \( B_{T-2} \) to pay off maturing nominal debt. It then soaks up money with surpluses \( P_{T-1} s_{0T-1} \), and by bond sales \( B_{T-1} \). The flow budget constraint then says money left over at the end of the day is

\[ M_{T-1} = B_{T-2} - P_{T-1} s_{0T-1} - Q_{T-1} B_{T-1}. \]  \hspace{1cm} (15.22)

With \( P_T = P_T^* \), we have

\[ Q_{T-1} = \frac{1}{R} \frac{P_{T-1}}{P_T^*}. \]

Let the government sell bonds in the amount \( B_{T-1} \), determined by

\[ \frac{1}{R} \frac{B_{T-1}}{P_T^*} = B_{T-2} - P_{T-1} s_{0T-1}. \]  \hspace{1cm} (15.23)

In words, these are the bonds it sells in equilibrium, if it is to redeem outstanding debt at the price level \( P_T^* \), and fund the surplus or deficit \( s_{0T-1} \). But, crucially, this debt sale policy does not roll over any increase or decrease in value of debt \( B_{T-1}/P_T \) that occurs from a non-equilibrium price level. The flow budget constraint now implies that money outstanding at the end of the day is

\[ M_{T-1} = B_{T-2} - P_{T-1} s_{0T-1} - P_{T-1} \left( \frac{B_{T-2}}{P_T^*} - s_{0T-1} \right) \]

\[ M_{T-1} = \left( \frac{P_{T-1}^* - P_{T-1}}{P_{T-1}^*} \right) B_{T-2} \]

Since people do not want to hold money \( M_{T-1} \), again the equilibrium price is now \( P_{T-1} - P_{T-1}^* \) as well. If the price level is lower, \( P_{T-1} < P_{T-1}^* \), then people end the day with unwanted money in their pockets, and vice versa.
In sum, we have $P_T = P_T^*$ and $P_{T-1} = P_T^*$. The policy that achieves this result is, at time $T$ a commitment to run whatever surpluses are required to redeem outstanding debt at the price level target, but not for other price levels. For other price levels, the government will monetize debt and leave unbacked money outstanding, or demand such money that people don’t have. At time $T-1$, the policy is a commitment to issue debt sufficient to redeem outstanding debt and fund the current surplus or deficit at the target price level, but no more or less. For other price levels, the government will again monetize debt and leave that money outstanding, this time refusing to soak it up by debt sales as well as to soak it up by surpluses, or conversely to demand money that people don’t have, and therefore had better go out and get.

This model clearly gets in trouble with the interest rate is zero, so money and bonds are perfect substitutes. Now people are willing to hold $M_{T-1}$. Indeed, inflation targeting regimes seemed to have trouble getting inflation up at the zero bound. We will look for different policies below that can work at the zero bound as well.

**An instructive mistake**

We explore the importance of the semi-passive debt rule in more detail. The semi-passive surplus rule $s_T = B_{T-1}/P_T^*$ responds to increases in debt $B_{T-1}$. Thus, if the government sets debt to always follow the equilibrium condition at time $T-1$,

$$\frac{1}{R} \frac{B_{T-1}}{P_T^*} = \frac{B_{T-2}}{P_{T-1}} - s_{0T-1},$$

with $P_{T-1}$ not $P_T^*$ on the right hand side, variation in $P_{T-1}$ will result in more debt $B_{T-1}$, and more surplus $s_T$. We have a passive fiscal policy. An active fiscal policy with $s_T = B_{T-1}/P_T^*$ also requires the semi-passive debt policy that leaves money outstanding at $T-1$ for alternative price levels.

The nature of the bond policy (15.23) merits additional attention, for it hides a seductive trap. The bond policy at time $T-1$ is crucial to avoiding a passive fiscal policy; the surplus policy is not enough.

In this two-period model, the flow *equilibrium condition* – for $T-1$ is

$$\frac{B_{T-2}}{P_{T-1}} = s_{0T-1} + \frac{1}{R} E_{T-1} B_{T-1}/P_T^*,$$

and substituting from the last period (15.21),

$$\frac{B_{T-2}}{P_{T-1}} = s_{0T-1} + \frac{1}{R} B_{T-1}/P_T^* \tag{15.24}$$
With fixed surpluses \( s_T = s_{0T} \), as we analyzed previously, when the government raises \( B_{T-1} \), that raises \( P_T \). But now \( P_T = P_T^* \) is determined. When the government raises \( B_{T-1} \), by (15.20) it also raises surpluses \( s_T \), and therefore the government raises more revenue from the bond sale. Therefore, the equilibrium condition (15.24) determines how much nominal debt \( B_{T-1} \) must be sold in equilibrium,

\[
\frac{1}{R} \frac{B_{T-1}}{P_T^*} = \frac{B_{T-2}}{P_{T-1}} - s_{0T-1}.
\] (15.25)

We lose the option to choose equilibrium \( P_T \) by different \( B_{T-1} \).

However, if the government follows the policy (15.25) for any price level, it becomes fully passive policy at time \( T - 1 \): The price level \( P_{T-1} \) is not determined. A lower price level \( P_{T-1} \) induces by (15.25) an increase in the real value of debt \( B_{T-2}/P_{T-1} \). That increase in real value of debt is rolled over; it triggers an increase in nominal debt sold at the end of \( T - 1 \), \( B_{T-1} \). The commitment to pay off \( B_{T-1} \) at the price level \( P_T^* \) means the lower \( P_{T-1} \) is validated by subsequent surpluses at time \( T \).

Though the government commits only to pay off nominal debt \( B_{T-1} \) at the price level target \( P_T^* \), it does not distinguish between nominal debt incurred from a low surplus \( s_{0T-1} \), that should be paid off, and nominal debt incurred by a too-low price level \( P_{T-1} \), and then rolled over. Equivalently, nominal debt \( B_{T-2}/P_{T-1} \) enters symmetrically with \( s_{0T-1} \) in (15.24). Any commitment to fund the regular deficit \( s_{0T-1} \) and pay off the subsequent debt will apply equally well to shortfalls (or windfalls) arising from different price levels \( P_{T-1} \) and revaluation of the debt \( B_{T-2} \).

The source of passive policy here is to use the equilibrium condition (15.24) or (15.25) to also describe how the government acts for non-equilibrium price levels. The actual bond policy (15.23) is the same as (15.25) in equilibrium \( P_{T-1} = P_T^* \), and \( M_{T-1} = 0 \), but differs out of equilibrium. To describe a policy that determines the price level, we must describe a policy that does not react to the price level in a passive way.

We are used to the idea that surpluses cannot be passive; that the equilibrium condition equating surpluses to the value of debt cannot also describe out of equilibrium behavior; that it cannot hold for non-equilibrium price levels. In this dynamic example, with a semi-passive fiscal policy that reacts to the amount of nominal debt, debt issues must behave the same way. Rolling over changes in debt values induced by off-equilibrium prices is also a passive policy that must be ruled out.
An intertemporal model

I construct a fully intertemporal model building on the above ideas. If the surplus follows the semi-passive policy

\[ s_t = s_0 + \alpha \frac{B_{t-1}}{P_t^*} \]

with \( \alpha > 0 \) and debt \( B_t \) is set by the semi-passive policy

\[ \frac{B_{t-1}}{P_t^*} + \frac{M_{t-1}}{P_t^*} = s_0 + \alpha \frac{B_{t-1}}{P_t^*} + \frac{Q_t}{P_t} B_t \]

then the price level is determined and equal to \( P_t = P_t^* \). The government also commits to pay back any debt incurred by variation in surpluses \( s_0 \). The semi-passive policies each modify fully passive policies by placing the equilibrium and target price level \( P_t^* \) in place of arbitrary price levels \( P_t \). Thus they commit the government to pay off debts, but only at the target price level.

With this understanding of debt policy, a complete dynamic model will make sense. As in the one period models, we mimic a passive policy, and then specify that it only holds at the desired price level target.

Let the surplus follow

\[ s_t = s_0 + \alpha \frac{B_{t-1}}{P_t} \quad (15.26) \]

with \( \alpha > 0 \).

Policy (15.26) is inspired by the passive policy

\[ s_t = s_0 + \alpha \frac{B_{t-1}}{P_t} \quad (15.27) \]

With this passive policy, the transversality condition

\[ \lim_{T \to \infty} \frac{1}{R} \left( \frac{B_{T-1}}{P_T} \right) \]

holds for any \( P_t \); the present value condition

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j} \]
holds for any $P_t$. Section 19 contains the algebra to show these points. The $\alpha B_{t-1}/P_t$ term means that the overall surplus $s_t$ inherits the negative autocorrelation property described in section 9, so debts are paid off.

As in the two-period model, the policy (15.26) modifies the passive policy by substituting the price level target $P^*_t$ for the actual price level $P_t$. Surpluses rise in response to increases in the value of the debt, accumulated from past surpluses, but only evaluated at the price level target. The surplus does not respond to changes in the value of debt induced by deviations from the price level target.

The flow budget constraint is

$$\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = s_{0t} + \alpha \frac{B_{t-1}}{P^*_t} + \frac{Q_t}{P_t} B_t + \frac{M_t}{P_t}. \quad (15.28)$$

Let the debt policy then specify that debt sales $B_t$ soak up money, so as to leave $M_t = 0$ if $P_t = P^*_t$ but not otherwise. Select $B_t$ so that

$$\frac{B_{t-1}}{P^*_t} + \frac{M_{t-1}}{P^*_t} = s_{0t} + \alpha \frac{B_{t-1}}{P^*_t} + \frac{Q_t}{P_t} B_t. \quad (15.29)$$

Substituting into (15.28),

$$\left(\frac{1}{P^*_t} - \frac{1}{P_t}\right) (B_{t-1} + M_{t-1}) = \frac{M_t}{P_t}.$$

The equilibrium condition $M_t = 0$ then implies

$$P_t = P^*_t.$$

As in the two-period model, the combination of a semi-passive fiscal policy (15.26) and debt policy (15.29) together mean that the price level is determined at the price level target, yet the government issues debt and commits to repay it in response to fluctuations in the basic surplus $s_{0t}$.

One may wonder why we need $\alpha > 0$. There are three important equilibrium conditions: zero money $M_t = 0$, the intertemporal bond pricing condition $Q_t = 1/R \times E_t (P_t/P_{t+1})$, and the transversality condition $\lim_{T \to \infty} B_t/P_t = 0$. Together with the other equilibrium conditions, $\alpha > 0$ is a sufficient condition for the transversality condition to hold. For example, if $\alpha = 0$ and $s_{0t} = 0$ as well, then debt grows at the interest rate and is never paid back, and the transversality condition is violated.
As in the two-period example, however, the semi-passive surplus reaction (15.26) is not sufficient on its own. We also have to specify debt policy so that debt rollovers do not induce a passive policy.

15.9.4 Fiscal price reactions

I extend the model to include explicit reaction of the surplus to the price level,

\[ s_t = s_{0t}(P_t) - \alpha_P \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right). \]

Again, with a semi-passive or fully passive \( s_{0t}(P_t) \), that promises to pay back debt at the target, or at any price level, we obtain \( P_t = P_t^* \). This rule extends to the case that only real debt \( b_t \) exists, and no nominal debt \( B_t = 0 \).

As in the single-period model, the fiscal commitment can include an explicit reaction to the price level or inflation. Such a reaction strengthens the fiscal commitment, and allows us to extend the model to include indexed, foreign currency, or other real debt.

Let the surplus follow

\[ s_t = s_{0t} + \alpha_N \frac{B_{t-1}}{P_t^*} + \alpha_R b_{t-1} - \alpha_P \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right), \]

with \( \alpha_N > 0, \alpha_R > 0 \) and \( \alpha_P \geq 0 \). The flow budget constraint, money left over equals money outstanding or printed up, less money soaked up by surpluses and bond sales, is

\[ b_{t-1} + \frac{B_{t-1}}{P_t^*} + \frac{M_{t-1}}{P_t} = \left[ s_{0t} + \alpha_N \frac{B_{t-1}}{P_t^*} + \alpha_R b_{t-1} - \alpha_P \left( \frac{1}{P_t} - \frac{1}{P_t^*} \right) \right] + \frac{1}{R} b_t + \frac{Q_t}{P_t} B_t + \frac{M_t}{P_t}. \]

Let debt sales \( B_t \) and \( b_t \) soak up money so as to leave \( M_t = 0 \) if \( P_t = P_t^* \) but not otherwise. Select \( B_t \) and \( b_t \) so that

\[ b_{t-1} + \frac{B_{t-1}}{P_t^*} + \frac{M_{t-1}}{P_t^*} = \left[ s_{0t} + \alpha_N \frac{B_{t-1}}{P_t^*} + \alpha_R b_{t-1} \right] + \frac{1}{R} b_t + \frac{Q_t}{P_t} B_t. \]

The split between real and nominal debt is irrelevant. Here, we do not let debt roll over surpluses induced by the price level response rule. Substituting into the budget constraint,

\[ (B_{t-1} + M_{t-1} - \alpha_P) \left( \frac{1}{P_t^*} - \frac{1}{P_t} \right) = \frac{M_t}{P_t}. \]
The equilibrium condition $M_t = 0$ then implies

$$P_t = P^*_t.$$

As in the two-period model, the explicit price level surplus rule strengthens the fiscal commitment behind the price level target. It also allows us to generalize to the case of entirely indexed debt. If $B_{t-1} = 0$, and in the equilibrium that $M_{t-1} = 0$, (15.30) still leads to a determinate price level $P_t = P^*_t$.

A reaction to the price level will also rescue fiscal price determination and the price level target if the remainder of fiscal policy is fully passive. Let the surplus follow

$$s_t = s_{0t} + \alpha_N \frac{B_{t-1}}{P_t} - \alpha_P \left( \frac{1}{P_t} - \frac{1}{P^*_t} \right)$$

with $\alpha_N > 0$ and $\alpha_P \geq 0$. (Note $P_t$ rather than $P^*_t$ following $\alpha_N$.) The flow budget constraint is

$$\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = \left[ s_{0t} + \alpha_N \frac{B_{t-1}}{P_t} - \alpha_P \left( \frac{1}{P_t} - \frac{1}{P^*_t} \right) \right] + \frac{Q_t}{P_t} B_t + \frac{M_t}{P_t}.$$

Let debt sales $B_t$ satisfy

$$\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = \left[ s_{0t} + \alpha_N \frac{B_{t-1}}{P_t} \right] + \frac{Q_t}{P_t} B_t.$$

Again, the $P^*_t$ are absent – this debt roll-over policy would lead to passive fiscal policy even if the surplus policy still had $\alpha_N B_{t-1}/P^*_t$. Substituting into the budget constraint,

$$\alpha_P \left( \frac{1}{P_t} - \frac{1}{P^*_t} \right) = \frac{M_t}{P_t}.$$

The equilibrium condition $M_t = 0$ then implies

$$P_t = P^*_t.$$
15.10 Separating fiscal and monetary policy

Suppose the treasury issues only real debt, and always pays it off. The central bank holds a pot of this real debt, and issues nominal debt. Now the “surpluses” of the fiscal theory are the earnings on the portfolio of real debt held by the central bank. This arrangement is a similar fiscal commitment; it separates and clarifies what resources back money actively, and makes clear that the cyclical component is repaid passively.

These fiscal policy rules are in some sense too successful, at least as a description of current institutions. They determine the price level at all dates, and allow no role for monetary policy. They do not allow the basic “monetary policy” operation of changing $B_t$ with no effect on $s_{t+1}$, and thus changing expected inflation and nominal interest rates.

We would like to find a less powerful fiscal rule. We would like changes in $B_{t-1}$ to be possible, in equilibrium, and still to control expected inflation $E_{t-1} (1/P_t)$ and hence the nominal interest rate. We would like the fiscal policy rule simply to control

$$ (E_t - E_{t-1}) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j} = 0, $$

For example to promise that any deficits incurred at time $t$, $s_t$, will be followed by surpluses to pay off the accumulated debt.

The previous examples could be extended to incorporate a monetary policy and interest rate target by adding a money demand function $M(t, i_t) = P_t y_t$. Then the threat to leave money outstanding would correspond to traditional open market operations and not leave $M_t = 0$ in equilibrium. But it would be nicer to find a completely frictionless description to build on, given that reserves pay full interest in most modern economies.

As those examples suggest, one way to build to such a description is to separate the debt that is used for smoothing cyclical fluctuations in revenue, that is backed and will be paid back, from the debt that is used by the central bank to implement the inflation and interest rate target, and is unbacked. A clean option is to let the treasury issue real debt, and the central bank issue nominal debt. A less clean, but more realistic option lets the treasury issue long-term debt while the central bank issues only one-period debt.
Let the treasury issue real debt $b_t$, and let it follow a rule that reacts to debt,

$$s_t = s_0 + \alpha b_{t-1}$$

with this rule, the real value of debt equals the present value of surpluses,

$$b_t = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.$$ 

Let the central bank own a constant amount $b_{CB}^t = b_{CB}$ of the Treasury’s debt. The central bank then issues nominal debt, backed by the interest payments on this real debt. (Since here the real debt is also one-period debt, each period the central bank redeems debt $b_{t-1} = b^{CB}$, getting $P_t b^{CB}$ dollars, and buys new debt worth $Q_t^r b^{CB}$ where $Q_t^r$ is the nominal price of real bonds – real bonds pay $P_{t+1}$ dollars each at time $t + 1 - Q_t^r = P_t b^{CB} / R$, so the central bank it gains from the treasury nominal dollars $P_t b^{CB} - P_t b^{CB} / R$ and real value $(R - 1) / R \times b^{CB}$. These dollars come from taxpayers, and are soaked up by the treasury via surpluses, then passed on to the central bank. Thus, the surplus devoted to paying back nominal debt is $(R - 1) / R b^{CB}$ each period.) The valuation of the central bank’s liabilities is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{(R - 1)}{R} b^{CB} = b^{CB}$$

This arrangement separates the price level from fiscal policy, and once again allows the central bank to run monetary policy, to follow an interest rate target, and to set expected inflation.

You can see the outlines of something like the euro emerging. Fiscal authorities commit to paying their debts; they endow the central bank with a pot of government securities. The central bank issues short-term nominal debt, and manages the price level with it. In this model however, the government securities are explicitly indexed, giving rise to a clearly real flow on the right hand side.

We can quickly generalize the example. The securities $b^{CB}$ could be long-term debt that varies in value, or that suffers credit losses. If so, the fiscal authorities have to agree to top up the central bank as needed, so that there is no innovation in the present value of central bank backing for the currency. Likewise, and more importantly for day to day operations, the central bank now makes seigniorage revenue from any currency it issues, or other liquidity advantage of its debt. It can rebate
that profit to the government, as the Fed rebates the treasury for its interest earnings, just enough to keep the right hand side of (15.31) constant.

This little example makes a general point: the surpluses that appear in the fiscal theory do not have to be general government surpluses. The fiscal theory is ultimately a theory of backing, and the art of setting up a monetary system is to provide a stable source of backing, period.

15.11 Targeting the spread

Rather than target the level of the nominal interest rate, the central bank can target the spread between indexed and non-indexed debt. This policy determines expected inflation, while letting the level of interest rates rise and fall according to market forces. The policy is equivalent to offering inflation swaps at a fixed rate, which may be an easier implementation.

Rather than target the level of the nominal interest rate, suppose the government targeted the spread between indexed and non-indexed debt. The nominal rate equals the indexed rate plus expected inflation, \( i_t = r_t + E_t\pi_{t+1} \). So, by targeting \( i_t - r_t \), the government could target expected inflation directly.

This target could be implemented like an exchange rate peg: Bring us any nominal government bond, and we will give you an indexed bond that pays \( E_t\pi_{t+1} \) less interest, or vice versa. For example, bring in a one-year, zero-coupon nominal bond, which promises to pay $1 at maturity. You get in return \( 1/\Pi^* \) indexed zero coupon bonds, each of which pays the gross inflation rate \( \pi_{t+1} \) at maturity.

Why? In much of the discussion, I have simplified by leaving out real interest rate variation, and treating the real interest rate as known. To target expected inflation, all the central bank has to do is to add the real rate \( r \) to its inflation target \( E_t\pi_{t+1} \), and set the nominal interest rate at that value. But in reality, the real rate varies over time. The real rate is naturally lower in recessions – more people want to save than want to invest; consumption growth is low; the marginal product of capital is low. The real rate is naturally higher in booms, for all the opposite reasons. And there is currently a big discussion over lower-frequency variation in the real rate, whether “\( r^* \)” is lower.

Moreover, there is no easy way to measure this real rate. With sticky prices, the real rate varies as the central bank varies the nominal rate, so the bank controls the thing
it wants to measure. For this reason, monetary economists speak of an ill-defined concept of the “natural” real rate of interest, which is even more unmeasurable. So, the central bank inevitably ends up in the business of setting the real interest rate. Economic planners have had a tough time setting the just price for centuries, and interest rates are no exception. Moreover, prices, once free, all move in response to myriad information that planners do not see.

In this context, then, the central bank could target the spread between indexed and non-indexed debt and leave the level of interest rates entirely to market forces. The second half is as exciting as the first. Finally, it leaves the central bank or government in charge of the nominal price level only. Of course, the proposal will not be exciting to those (especially inside central banks) who would like central banks to set real rates, exchange rates, and turn to a broader macroeconomic and financial dirigisme. It is more exciting to those who would like to find a way for the central bank to accomplish its mission of price level control without controlling vast other parts of the economy, or without trying to figure out what any real price ought to be. But even without stepping into this contentious arena, the possibility that the central bank can control inflation without having to divine the natural or real rate of interest is exciting.

Yes, the spread between indexed and nonindexed debt is the risk neutral expected inflation, or equivalently, contains an inflation risk premium. I’ll ignore this refinement. Perhaps it is good enough to control the risk-neutral rather than true-measure expected inflation. If the monetary-fiscal regime plus sticky prices produce little inflation volatility, the difference will not matter much. And maybe risk-neutral expectations are the right goal for policy. Indexed debt in the US also currently is rather illiquid, producing liquidity spreads, and suffers a complex tax treatment. Simplifying the security would make it far more liquid and transparent. (Cochrane (2015) contains a detailed proposal for simplified debt.)

Targeting the spread is really only a tiny step from the analysis so far. If the government can target the nominal interest rate \( i_t \), and then expected inflation will adjust in equilibrium to \( E_t \pi_{t+1} = i_t - r_t \) with \( r_t \) the real interest rate determined elsewhere in a frictionless model, then targeting the spread is really not fundamentally different from the interest rate target. As we will see in a minute, implementing the target by offering to trade indexed and nominal debt at a fixed price is quite like implementing the interest rate target by offering to sell nominal debt at a fixed price.

Moreover, the practical difference for monetary policy, in equilibrium, and in response to the usual shocks, may not be great. If the central bank follows a Taylor rule,
\( i_t = \pi^* + \phi_\pi \pi_t + \phi_y y_t \), and if in equilibrium the real interest rate tracks \( \phi_\pi \pi_t + \phi_y y_t \), then the Taylor rule is equivalent to the spread target. And second condition is not unreasonable – real interest rates should be higher in economic booms \( y_t \), and with a Phillips curve, in equilibrium, inflation may also be a signal of a high real rate. The difference may largely be the out of equilibrium question – getting to essentially the same equilibrium by targeting the spread is much clearer. Stating the same result that way may help in the all important job of clarifying expectations. And, more importantly, targeting the spread may produce a rule that performs better when the economy is hit by a different set of shocks. For example, “stagflationary” shocks in the 1970s arguably changed the habitual relationship between inflation and real rates, and following inflation as a guide is then misleading. Rules developed from history and experience have a certain wisdom and robustness, but often fail when conditions change from those of the history and experience.

This idea seems lunatic from the perspective of standard monetary analysis, especially in the context of ISLM modeling common in central banks. Sure, they might say, \( i_t - r_t = E_t \pi_{t+1} \), but causality goes from left to right. If the government targets the spread, and offers people to freely exchange real for nominal bonds at a fixed rate, first, the volume of bonds will explode, and then inflation will spiral away. Yes, \( i_t - r_t = E_t \pi_{t+1} \) is a steady state, but it is an unstable steady state.

But the same analysis says that the nominal interest rate peg is unstable. And we have discovered from our investigation of fiscal theory, from even non-fiscal theory rational expectations models, and from 10 years of experience in the West and 25 in Japan, that \( i_t = r + E_t \pi_{t+1} \) is a stable steady state. Peg \( i_t \) and sooner or later \( E_t \pi_{t+1} \) will settle down. It follows immediately that pegging \( i_t - r_t \) is also a stable steady state, and \( E_t \pi_{t+1} \) will settle down. The stability of an interest rate peg holds if and only if the interest spread peg is stable. Both ideas might be wrong, but if so major parts of the rest of this book are wrong. (The error would not really be the fiscal theory part, the error would be the forward-looking nature of expectations in the consumer’s first order conditions.)

The same idea can implement a price-level target. Loosely, just vary the inflation target to suck out any past deviations from the price level.
15.11.1 Forming the equilibrium

I explore the supply and demand mechanics of the spread target. If the government offers indexed debt in return for nominal debt at a higher rate than the market price, people will take the offer. The larger real debt must be paid out of real surpluses, lowering the surpluses available to pay nominal debt. But the lesser nominal debt means less surpluses are needed. The latter effect dominates, driving down the expected price level. The offer to exchange indexed for nominal debt at a fixed rate is stable, and drives expected inflation to the target.

Writing down $i_t - r_t = E_t \pi_{t+1}$ and concluding that the central bank or government can peg the left side, offering to trade bonds at a fixed price, and the right side will adjust may seem straightforward. Or it may seem crazy. Assurance against the latter merits a look into how the equilibrium forms.

To analyze the proposal, start with the present value relation with both real and nominal debt,

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+j}.$$  

The real interest rate is constant here, which will hide the usefulness of the idea, but clarifies the mechanics. We can add time-varying interest rates later. As usual, express the equation in terms of expected values,

$$b_t + B_t E_t \left( \frac{1}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+1+j}$$  \hspace{1cm} (15.32)

If the government exchanges each real bond for $E_t(1/P_{t+1})$ nominal bonds, then the left hand side does not change. With a line of algebra you can show this is the same thing as exchanging the bonds at market prices. Exchanging the bonds at market prices, the real vs. nominal structure of the debt is irrelevant to the expected price level. (Less nominal debt makes unexpected inflation more sensitive to unexpected surpluses, but that’s a separate issue.)

But if the government offers a different tradeoff, then the left hand side does depend on the real-nominal split. Suppose the government offers to sell $b_{0t}$ and $B_{0t}$ real and nominal debt unconditionally, and then offers $P^*$ nominal bonds in return for each real bond, and vice versa, so

$$-(B_t - B_{0t}) = (b_t - b_{0t}) P^*.$$
\[ b_t = b_{0t} - \frac{B_t - B_{0t}}{P^*}. \]

Now, plug in to (15.32),

\[ b_{0t} - \frac{B_t - B_{0t}}{P^*} + B_tE_t \left( \frac{1}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} \]

\[ b_{0t} + B_tE_t \left( \frac{1}{P_{t+1}} \right) + (B_t - B_{0t}) \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} \]

It’s easiest to see the effect of exchanging debt by taking derivatives,

\[ dB_t \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] + B_t \left[ dE_t \left( \frac{1}{P_{t+1}} \right) \right] = 0 \]

\[ dE_t \left( \frac{1}{P_{t+1}} \right) = - \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] dB_t / B_t. \]

As before, if \(1/P_t = E_t(1/P_{t+1})\) then the expected price level is independent of the real/nominal split. If \(1/P^* < E_t(1/P_{t+1})\) – if the government offers more nominal bonds per real bond than the market offers – then as \(B_t\) rises, \(E_t(1/P_{t+1})\) falls, i.e. the future price level rises. The previous description of monetary policy was in effect \(P^* = \infty\); the government simply increased nominal debt with no decline in real debt, and that change resulted in next-period inflation. This case is a generalization. But if if \(1/P^* > E_t(1/P_{t+1})\), if the government offers fewer nominal bonds per real bond than the market offers, then increasing \(B_t\) raises \(E_t(1/P_{t+1})\), i.e. lowers the future price level.

Individuals do not see the effect of their actions on aggregates. Individually, it’s worth exchanging a real bond for a nominal bond in the first case, if \(1/P^* < E_t(1/P_{t+1})\) and if the government gives more nominal bonds per real bond than offered by the market. Since an individual can sell the bonds in the market and repeat the process, this is an arbitrage opportunity. But as people exchange real bonds for nominal bonds, they drive down \(E_t(1/P_{t+1})\), until \(1/P^* = E_t(1/P_{t+1})\) and the arbitrage opportunity disappears. Likewise if \(1/P^* > E_t(1/P_{t+1})\), then people will exchange nominal bonds for real bonds, driving up \(E_t(1/P_{t+1})\) until \(1/P^* = E_t(1/P_{t+1})\) again.

In sum, we have shown that offering to freely exchange real debt for nominal debt at the rate \(P^*\), while not changing surpluses, drives the expected price level to \(E_t(1/P_{t+1}) = 1/P^*\).
15.11. TARGETING THE SPREAD

To express an inflation target just multiply by $P_t$ and let $\Pi^* = P^*/P_t$. Define a real bond to deliver a payment of $P_{t+1}/P_t$ dollars at time $t + 1$. This is, in fact, the institutional arrangement of TIPS, unlike the promise to deliver $P_{t+1}$ that I have used. Then, the government offers to give one real bond in exchange for $\Pi^*$ nominal bonds and vice versa. In a similar way, we find that this policy delivers $E_t \Pi_{t+1} = \Pi^*$.

15.11.2 Implementation

Trading an indexed for a nominal bond is the same thing as a CPI swap contract. It may be better for the central bank to offer swap contracts than to directly trade in debt. The idea can be extended out the yield curve – offer to trade nominal for indexed bonds at any maturity, setting long-run inflation expectations.

Many other implementations of the same basic idea are possible, and the pegging of one-year zero-coupon bonds I described is unlikely to be one that central banks use in practice.

First, the idea can be extended throughout the yield curve: offer to trade indexed for non-indexed debt in relative quantities derived from the inflation target at any maturity. This is especially simple for a constant price level target or inflation target: Offer always to trade indexed for non-indexed debt in the same quantity. Bring us one 5 year 3% coupon bond, and we’ll give you a 5 year 3% coupon indexed bond, period.

Second, the idea would most likely be implemented via swaps rather than by trading underlying bonds. In an inflation swap, parties agree to pay or receive the difference between realized inflation and breakeven rate set at the beginning of the contract period, $P_{t+1}/P_t - \Pi^*$. No money changes hands today; the reference rate $\Pi^*$ adjusts to clear the market. Clearly, the inflation swap is the same thing as buying one indexed bond, defined as a bond that pays $P_{t+1}/P_t$ in one period, and selling $\Pi^*$ nominal bonds. (Or, buying $1/P_t$ indexed bonds that pay $P_{t+1}$ next period.) Inflation swaps are in fact defined as the difference between bond payments – receive or pay the difference between the fixed vs. indexed payment.

Thus, the government could implement the expected CPI target by simply offering to buy and sell inflation swaps at a fixed reference inflation. This would avoid convulsions in debt markets.

In equilibrium, when $E_t(1/P_{t+1}) = 1/P^*$, people are indifferent to real vs. nominal
debt. Thus, we can describe the model by a value of the spread, \( i_t - r_t \), and either one of \( B_t \) or \( b_t \), or their ratio, and surpluses. The expected valuation equation then describes demand for the other of \( B_t \) or \( b_t \) or each of them subject to the ratio, and the unexpected valuation equation describes unexpected inflation. Though the government offers people to freely exchange \( B \) for \( b \), the equilibrium ratio between them remains under the government’s control.

### 15.11.3 Not a CPI standard

Targeting the spread, or CPI swaps or futures, starts to look like an implementation of a commodity standard. It is not. It only targets expected inflation, and actual inflation depends on fiscal innovations. Fiscal surpluses, tied to the general price level, have an interesting advantage over commodity standards, which are necessarily based on a subset of commodities, whose relative price may vary.

The spread target or CPI swap target has some of the flavor of a gold standard or commodity standard: Bring us something nominal, and we give you back something real, at a set conversion rate. Unlike the gold standard, it targets the entire CPI, eliminating problems induced by volatility in the relative price of gold and other goods. But it only targets the expected CPI and debt of a substantial maturity, say one year or more. There still can be unexpected inflation, corresponding to unexpected changes in surpluses. Unlike a promise to convert money itself to something of real value, the spread target does not automatically include a fiscal commitment to tame unexpected inflation.

This is an important distinction. To avoid volatility of the value of gold relative to other goods, many authors have suggested commodity standards: in return for one dollar you get a basket of traded commodities – wheat, pork bellies, oil, metals, and so on. But commodity values are also volatile, and only a bit more connected to the general price level than is the price of gold. Like gold, targeting commodity values might stabilize those values, but not have much effect on the overall price level.

It seems that one should be able to come up with a clever device to replace commodities with a cash-settled CPI-linked contract, and thus to mimic a commodity standard that applies to the whole CPI. This isn’t it, and I haven’t been able to come up with one. Suppose you exchange money for a cash-settled CPI spot contract – in return for $1, you get back $1 times \( P^*/P_t \). Then, yes, if \( P_t < P^* \), people will bring in money, and exchange it for more money. And repeat, very quickly! The intra-day money supply will explode, which, with constant surpluses and hence real...
value of debt sales will push up the price level. But this idea does not generalize to any amount of price stickiness. The CPI cannot change in a day, and all you do is send the money supply to infinity.

The basic structure of the fiscal theory neatly solves the commodity standard conundrum. Taxes are based on the entire bundle of goods and services, not one or a few specific goods. Thus the essential promise of the fiscal theory, bring us a dollar and we relieve you of a dollar’s worth of tax liability, functions as a commodity standard weighted by the whole bundle of goods, without requiring delivery of that bundle.

**15.11.4 The spread target with sticky prices**

Here I treat the spread target in a simple model with sticky prices and in continuous time. As we saw above, though there is a smooth frictionless limit, the picture painted with these two ingredients can be much more realistic. For example, as in section 7.10.1 a fiscal shock that induces a price-level jump in discrete time induces a period of smooth inflation with no price level jump in continuous time with sticky prices. This model also incorporates variation in real rates, which motivates the whole exercise.

[INCOMPLETE]

**15.12 The present value Laffer curve**

There is always a fiscal limit, at which governments can no longer run surpluses needed to contain inflation. Usual discussion of the Laffer curve, the tax rate that maximizes the flow of revenue, is static, and centers on the tradeoff of work vs. leisure. The fiscal theory responds to the present value of surpluses. Small effects of tax rates on growth have large effects on the present value of surpluses, even if tax rates have no effect on the immediate flow surplus. Considering the effects of distorting taxes on growth can result in a considerably lower fiscal limit than standard flow analysis suggests.

As we think about surpluses and fiscal rules it is natural to jump to taxing and spending decisions. In fact, for the present value of surpluses that matters in the fiscal theory, economic growth is far more important. Figure 9.3 reminds us that
output is the primary determinant of the surplus – tax revenue grows in expansions and falls in contractions. Economic policies that change growth by a small amount can cumulate to large changes in tax revenue. Conversely, economic policies that damage long-run growth can lead to large changes in the present value of surpluses even with little short-run impact.

Poorly crafted “austerity” policies in particular run this danger: raising marginal tax rates may bring a short run revenue increase, but by decreasing growth over the longer run can lower the present value of future surpluses. The present value Laffer curve may bite before the usual flow Laffer curve, and for different kinds of taxes.

The usual Laffer curve analysis considers only labor supply. Higher marginal tax rates discourage work effort. Less labor supply means less income and less tax revenue. Counter to this effect, however, we usually find little short-run work effort effect of higher tax rates, or wages. A higher tax rate has both income and substitution effect on labor supply – people who are poorer work more, people offered a lower marginal after tax wage work less. These largely offset. Highly progressive tax increases can have more substitution disincentive than wealth effect, but they don’t raise much revenue. Moreover, most people have settled in to careers and jobs; labor market regulation and custom make it hard for most employees to raise or lower work hours without taking additional jobs. The extensive margin of people joining or leaving the labor force is small in the short run.

But in the long run, there is room for much larger adjustment. A more progressive tax system may not cause a doctor, lawyer, or entrepreneur to change hours of work that much. But it will influence people’s career choices, to take unpleasant college majors, invest in graduate education, start businesses, rather than skip school or take more fun majors, or settle in to easier jobs. That margin takes a generation to take effect. Disincentives to start businesses, innovate new products, invest in businesses, and so forth also take a long time to kick in.

Raising taxes on capital is a classic temptation – as it hits a fixed investment today, it generates revenue. But it removes the incentive to create tomorrow’s capital, and thus may reduce the present value of tax revenue. Progressive taxation is essentially a tax on human capital, with the same tradeoff.

For a simple calculation, consider proportional taxes at rate $\tau$. The conventional Laffer curve calculation asks for the effect on tax revenue of a change in the tax
rate:

\[ \frac{\partial \log(\tau Y)}{\partial \log \tau} = 1 + \frac{\partial \log Y}{\partial \log \tau}. \]

The second term is negative, as a higher tax rate lowers output and therefore lowers tax revenue from what it would otherwise be. For example, at a 50% tax rate, a ten percentage point increase in tax rate is a 20% increase. (Remember, what matters in these calculations is the overall steady state permanent tax rate, including Federal, state, local; sales taxes, business taxes and so on.) If output were constant, that would imply that tax revenue increases from 50% to 60% of GDP, also a 20% increase, and the elasticity would be one. However, if output declines by 20%, the second term is -1 and there is no revenue. Most economists think the output decline is not so severe. But this is largely static thinking.

Suppose output grows at the rate \( g \). Write the present value of tax revenue

\[ PV_t = \int_s^\infty e^{-rs} \tau Y_{t+s} ds = \tau Y_t \int_s^\infty e^{-(r-g)s} ds = \frac{\tau Y_t}{r - g}. \]

Now the elasticity of the present value of surpluses is

\[ \frac{\partial \log (PV_t)}{\partial \log \tau} = 1 + \frac{\partial \log Y}{\partial \log \tau} + \frac{1}{r - g} \frac{\partial g}{\partial \log \tau}. \]

In addition to the static effect, we now have a dynamic effect. Since \( r - g \) is a small number, small growth effects can have a big impact on the fiscal limit. For example, if \( r - g = 0.01 \), \( dg/d \log \tau = -0.01 \) puts us at the fiscal limit immediately, even with no level effect. Thus, if a rise in \( \tau \) from 50% to 60%, a 20% rise, only implies \( 0.01 \times 20 = 0.2 \) percentage point reduction in long-term growth, then we are at the fiscal limit already.

The point here is not one side or another - the point is that the present value of surpluses describe fiscal limits. The present value Laffer curve may be quite different than the static curve most commonly discussed.

Protective economic regulation is probably a larger disincentive to growth than tax policy. Keeping the taxi monopoly in and Uber out does not help government finances. In thinking about the fiscal theory, then, we must broaden our vision from just tax and spending policies. Pro-growth economic and financial reforms are likely to raise the present value of surpluses – even if they reduce current surpluses – and thereby quickly lower inflation. This seems like part of the story for New Zealand, for example.
15.13 Interest rates, growth and runs

Lower growth may come with lower interest rates, partially offsetting the present-value Laffer curve effect. However, higher real interest rates without higher growth pose an independent danger to inflation. The debt crisis mechanism that causes default and currency crashes can also cause inflation.

Now, higher growth $g$ may bring higher real interest rates $r$, and conversely the ill effects of lower growth may be tempered by lower interest rates.

Real interest rates rise when growth rises. When interest rates are higher, people have an incentive to consume less today and more tomorrow, hence consumption growth is higher. Formally, the consumer’s first order condition says that the real interest rate equals the subjective discount rate plus the inverse of the intertemporal substitution elasticity times the per-capita growth rate,

$$r = \delta + \gamma (g - n).$$

Higher growth usually comes with a higher marginal product of capital, which also translates to higher interest rates. The simplest fiscal theory in steady state says

$$\frac{B}{P} = s \frac{r - g}{r - g}.$$

So, more growth will induce an interest rate rise, and this effect tempers the long-run or present-value Laffer effect. If $\gamma = 1$, then $r$ and $g$ rise one for one, and higher growth has no effect on the value of debt. The growth effect in the present value Laffer curve turns off. Expected surpluses, which benefit from more growth, rise just as the discount rate also rises. In fact, a simultaneous decline of $r$ and $g$ is my central story for why we have not seen inflation during the Great recession.

This conclusion requires $\gamma = 1$, which is the case for log utility. If $\gamma < 1$ – if people respond to a one percentage rise in interest rates so much that consumption growth rises by more than 1 percentage point – then $r$ rises less than $g$. More growth means only slightly higher interest rates, and thus less inflation; the present value Laffer curve reasserts itself though in moderated form. If $\gamma > 1$, however, if people respond little to the incentives to defer consumption offered by higher interest rates. Now $r$ rises more than $g$, so more growth can actually imply more inflation, and less growth less inflation. The present value Laffer curve goes the wrong way – distorting taxes or counterproductive policies that reduce economic growth, and thereby lower surpluses,
will lower interest rates so much that they raise the present value of surpluses, which implies less inflation.

These observations are tempered however, by the many other sources of interest rate variation. Growth that comes from larger population has no interest rate effect, but does raise surpluses, immediately and in present value. A rise in discount factor $\delta$ simply discounts the future more heavily, producing inflation, or a lower value of government debt, without any rise in surpluses.

Like other prices, higher interest rates can be a sign of good things, or a cause of bad things. Every price can move if a supply curve shifts or if a demand curve shifts, and the two movements have different interpretations.

The interest rate can rise without any change in growth, if the discount rate rises or if markets assign a higher risk premium to government debt. If higher interest rates that reflect higher growth are “good” rises in interest rates, these are “bad” ones. They lower the present value of surpluses with no offsetting beneficial effect.

The standard economics of debt crises points even to multiple equilibria. These analyses apply to countries that borrow in foreign currency. At low interest rates, they can pay off their debt. At high interest rates they cannot and must default. A rise in default premium leads to higher interest rates, which raise the default premium, and so on. The fiscal theory alerts us that the same thing can happen to a country that issues its own currency, only that inflation and devaluation are the result rather than default.

As I write, the US still benefits from amazingly low interest rates, about 2.5% on long-term government bonds. This is roughly zero in real terms, so the real interest rate is below the growth rate. However, should interest rates rise, the US, having borrowed largely short-term, will soon feel a major fiscal pressure that could amount to a crisis. With $20 trillion of debt outstanding, each one percentage point rise in the real interest rate means $200 billion of extra interest costs. Since we roll over about half of the debt within two years, that interest cost shows up on the budget quickly.
15.14 Europe

The Euro is a case of FTPL too. Euros are backed by the ECBs assets, primarily government debt. The ECB also rebates profits to member governments, and can call for recapitalization. The Euro is a useful fiscal commitment—it makes clear the assets and backing for money and separates those assets from general government surpluses and deficits and their cyclical variation. It even insulates the monetary unit from national defaults.

Initially the Euro seems like a different structure, with a central bank separated from fiscal policies, with price stability generated by its independent ability to set an interest rate target. But it is in fact, as I briefly alluded to above, a clever instance of the fiscal theory.

The European central bank issues Euros, and interest-paying Euro reserves. It holds assets, primarily European sovereign bonds. It is tempting to apply the ideas of section 15.10 directly: The value of the euro is equal to the value of the European Central Bank’s assets. As such, segregating a set of assets that back money is a good way to separate the value of money from the vagaries of government surpluses and deficits, like many of the other fiscal commitments we have studied. It’s like a currency board in which member governments cannot grab the assets backing money when they are having difficulty paying their debts.

It’s not quite so easy. The ECB’s assets are nominal government bonds, not indexed bonds. Maybe they should be indexed, but they aren’t. So, it would seem that inflation devalues both right and left hand sides of the valuation equation equally, up to second-order and dynamic maturity structure questions. To determine a price level we need something nominal and something real; as purely real bonds and purely real surpluses (which do not respond to the price level) do not work, likewise purely nominal assets and purely nominal liabilities don’t work either.

However, the structure of the ECB contains many provisions to stabilize the real value of its assets, almost as if it held default-free indexed debt as assets. The ECB, like the Fed, ultimately rebates profits to European governments. It’s website explains that profits are used first of all to fund its operations, and then simply held inside the bank as a provision against future losses.

“But after that, any remaining ECB profits go to the national central banks of the euro area countries, as the shareholders of the ECB. The

\[2\text{https://www.ecb.europa.eu/explainers/tell-me-more/html/ecb-profits.en.html} \]
central banks may save some of this money or use some in their work, but profits usually go to the country’s government, thus contributing to its budget. This benefits euro area taxpayers.”

In the other direction, should the ECB ever lose a lot money, it has the right to call up member governments and demand recapitalization. This provision would also address a substantial sovereign default imperiling its assets. Yes, the Eurobonds, collective responsibility of all euro members, have been invented – they are Euro reserves, interest-paying ECB liabilities.

In this way, we can in fact think of the ECB as an institution with a stable set of real assets that back its nominal liabilities. And as such, the real assets are designed to be separated from and more stable than the entirety of national budgets. Even if some countries default on some of their debts, the countries of the Eurozone are collectively committed to making up the losses the ECB would feel on its bond portfolio.

Part of this arrangement is supposed to be that the ECB does not monetize debts. In a currency union without fiscal union, insolvent countries default, just as insolvent corporations default, or obtain direct fiscal support from generous neighbors, or from the IMF. And, again, if the ECB suffers enough that inflation may result, that country and the others recapitalize the ECB to make good its losses, ahead of other investors. This provision was always a bit in doubt. Companies are not required to have debt and deficit limits to operate in the dollar zone, because we all understand they default if in trouble. (Well, ideally. Big banks are starting to look like the exception to that rule, both in being repetitively bailed out by the Fed and by being required to have debt and deficit limits.) That the Eurozone put such limits in place was already a sign that a hard-nosed attitude toward sovereign default might not prevail, and they would rather not face the temptation. The Greek affair and “whatever it takes” clearly show the rule against monetizing debts in practice may be more elastic. If so, the ECB will become a more classic fiscal theory of the price level operation, not one with a segregated and more solid asset base; though one with many actors racing to the bottom of deficits.
15.15 Backing

The fiscal theory is at heart a backing theory of money. The present value of future surpluses is long duration, and can be made very stable if the government is below its fiscal limit. This system has advantages over previous backing, such as bank issued money backed by real estate loans and stabilized by an equity tranche.

The fiscal theory is, at heart, a backing theory of money. It does not deny a liquidity demand and consequent liquidity premium for money or for government securities, but we build those as distortions on a basic model of backing.

Many different kinds of backing have been tried to give value to paper money and similar liabilities. Gold coins are, in a sense, money that carries with it its own backing. 19th century bank notes, and 20th century checking accounts are privately-provided money, backed by the loans and real estate collateral that constitute bank assets, less the value of bank equity that stands as a buffer before those money-like liabilities are exhausted. In these cases, the money is a promise to deliver government currency or gold coin, so the price level is set by monetary arrangements. But the backing still serves to maintain the value of the private money in terms of that government currency, so we can consider the question of how bank notes and checking accounts keep their value relative to government currency as we study the fiscal theory.

Many other backing schemes have been tried over time. For interesting examples, Sargent and Velde (1995) describe a number of monetary innovations in the French revolution, including Assignats: The revolutionary government had seized church property. It needed revenue, but it would take time to sell off the church property. It issued assignats, a form of paper money, backed by the proceeds of asset sales. Unlike many of the governments previous monetary experiments, this one did not immediately lose value, at least until the government printed more assignats then the backing would allow. John Law’s previous effort to back paper money by the gold that would soon be discovered in Louisiana failed when that backing proved illusory.

Backing money by loans and mortgages is a reasonably good arrangement. There are a lot of loans and mortgages – real estate is the largest element of the capital stock, so a large quantity of money and other liquid assets can be issued backed by real estate. Furthermore, two layers of equity make the value of resources promised to back money very stable. Banks assets are loans, collateralized by real estate, and the bank itself has an equity claim. Loans and real estate are also a very long lived
The fiscal theory of the price level describes a government money, backed by the present value of future fiscal surpluses. That backing has considerable advantages over backing by real estate loans. First, it is even longer lived – the present value of government surpluses is one of the few assets with longer duration than mortgages.

Second, mortgages are notoriously illiquid. If the time comes that people test the backing, banks have to sell mortgages or foreclosed property. Solvent banks can borrow against their assets – but it’s hard to tell illiquid from insolvent, and in any case this expedient does not increase the overall stock of assets in a systemic run. This asset illiquidity is a central ingredient in all our financial crises.

The government, by contrast, has a unique ability to raise the revenue stream that backs its money, so long as there is some space on the left of the Laffer curve or some political and economic space to cut expenditures. The present value that backs bank liabilities is made stable by carving out two equity tranches – the homeowner’s and the banks’. The present value that backs FTPL government money is made stable, in the first instance, by the government’s ability to raise and lower surpluses as needed. The various fiscal rules and commitments studied above implement that ability. That attribute allows the government to promise a steadier path of surpluses than any backing by private assets could do. In particular the events in which real estate loans default, and bank equity is wiped out, so bank money loses value are more common than the events in which the government cannot raise surpluses and its money must inflate. Or so we hope.

Third, government debt is only a promise to pay more government debt. It is uniquely free of explicit default. It can be its own currency and unit of account. Bank deposits must promise payment in some other currency, they can’t define their own currency.

Fourth, government debt is, now, in exceedingly abundant supply. One might have worried in the past that there simply was not enough government debt to supply liquidity needs, that banks were necessary on top of a government currency to “transform” illiquid real estate assets into liquid liabilities. No longer. US federal debt alone is twice the value of all bank assets.

All of these are good reasons that we have evolved from money backed by loans, defined by gold, to short term government debt as numeraire, backed by fiscal surpluses.
But primary surpluses are not a perfect backing either. Governments occasionally default or inflate; if they are on the top of the Laffer curve or face political constraints, they may not be able to keep surpluses equal to the value of debt.

The general principle of the fiscal theory remains – a numeraire can be valued by its backing – but perhaps we can find sources of backing are better than the arrangement we seem to have evolved toward, that short-term nominal government debt is numeraire. The euro is already an innovation relative to national currencies, and at least in its original design provided a second buffer between the assets pledged to back money and general government surpluses. Backed cryptocurrencies offer an interesting possibility. I explore both issues next.

15.16 After government money

There is nothing really special about the government. A private currency could also define a standard of value, backed by a portfolio of liquid assets as government money is backed by fiscal surpluses. Currently cryptocurrency proposals are not backed, and achieving a potential numeraire is harder than achieving a stable value cryptocurrency. It requires both backing, and a nominal anchor.

We have converged on a monetary system in which short-term nominal government debt is the numeraire, unit of account and by and large medium of exchange. Most transactions that are not simply netted (more and more) involve the transfer of interest-paying reserves, which are government debt. Government debt is the “safe asset” and best collateral in financial transactions. I have structured most of the fiscal theory discussion around this institutional reality.

It was not always so, and it may not always remain so. Monetary systems based on government debt are imperfect, and have failed before. They may fail again. I doubt that our economy will transition to another system before another monetary crisis, as it is deep in human nature not to embark on grand adventures when the current system is working reasonably well. But in the event that happens, or in the rare event that innovation precedes a crisis, it’s worth thinking about alternative arrangements, and not just better ways for governments to manage a system built on their nominal debts as the rest of this book imagines.

A failure of our fiscal-monetary arrangements is not unimaginable. We have had inflation before. Governments either choose or are forced to abandon promises to
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maintain the present value of surpluses. Other advanced countries have had severe inflations. Many countries, even advanced western countries have had debt crises and exchange rate crises. It can happen again, and it can happen here. Advanced countries all have in the neighborhood of 100% debt to GDP ratios, persistent deficits, health care and pension promises that they cannot keep, and emerging slow growth. We all live on the $r-g$ cusp. Debt and debt service are not a problem with $r$ as low or lower than the discouraging $g$, but a rise in $r$ would leave us in dire fiscal straits, as outlined above. A rise in $r$ with a decline in $g$ would be worse.

Moreover, the aftermath of the 2007 crisis seems to be that any financial crisis will be met by immense government credit guarantees and stimulus. Yet, what happens when sovereign debt is the cause of the crisis, and the firehouse has burned down? Imagine that a new global recession leads to defaults by, say, Italy, China, and US states. Now, the US government needs to borrow another few trillion dollars for bank and business bailouts, more trillions for state and local governments, pension funds, retirement funds, Fannie and Freddy, more trillions for stimulus, plus, as usual, rolling over something like half the stock of debt per year, all in a recession, and while other countries may be selling their treasury reserves? But this time, all starting from 100% or more debt to GDP ratio, with large deficits, unreformed entitlements and even less of a clear idea how it will be paid back. At some point bond markets say no, even to the US.

If the result were only inflation, we might be lucky. Even so, a sharp inflation, which would sharply devalue government debt, would likely cause a profound restructuring of monetary and financial arrangements. An actual default, even a small haircut, on US treasury debt would cause chaos in a financial system that treats such debt as safe collateral, and provoke radical change. And if the government fails to bail out as expected, the ATM machines could go dark.

It’s unlikely, but it could happen. Our monetary system has evolved from its predecessors, but evolution is not perfection, and many past monetary systems ended with rather spectacular failures.

Less darkly, perhaps a spirit of free-market reform will take over, or competition in financial arrangements will lead to the emergence of an alternative standard, as the cryptocurrency advocates suggest.

So, what are the alternatives to a monetary system based on short-term nominal debt as numeraire, backed by general government surpluses, managed by a central bank following an interest rate target?
The most obvious modifications further separate monetary backing from general government finance. Already, the US legal system has many barriers in the way of inflationary finance. For example, the Federal Reserve may not buy debt directly from the Treasury. Helicopter drops turn out to be essentially illegal. When a recent debt limit default loomed, many commentators looking around for a way for the Fed to finance an essentially technical default found just how hard it is. The most humorous idea was for the Treasury to issue coins worth a billion dollars, since the Treasury retains the power to issue coins. The Euro represents a stab at separating government finance from monetary backing, as explained above. Governments are supposed to default on their debts if they can’t pay them, and mechanisms are in place to try to ensure the value of the government debts backing the euro even in the event of default or inflation.

Additional institutions separating monetary backing from general government default, government equity in the form of GDP linked bonds, additional fiscal precommitments to ensure monetary backing, are all obvious avenues for improvement if inflation and sovereign default threaten the monetary system. Money can be backed by a pool of assets, and the pool of assets backing money can be more segregated and more stable than generic government surpluses. Central banks owning corporate bonds or indexed bonds or even stocks are, from this point of view, useful ideas, though government purchases of private assets raise lots of other problems, as much political as economic.

In the end, I think the most likely response to a future sovereign debt crisis or inflation will continue to be a government-provided currency, but with a more potent separation of fiscal from monetary affairs. The official Meter sits in Paris, defining the unit of length, the official Euro sits in Frankfurt, defining the unit of value, well backed, and insulated from government finances. Sovereigns default if they get in trouble, or offer more equity-like securities that fluctuate in real value without the legal distress of actual default.

Of course, this vision both eschews the corporate-finance advantage of an equity-like nominal debt described above, and the conventional arguments for local monetary policy to offset local shocks by inflation and devaluation. Après le déluge, perhaps devaluation and stimulus will not seem such useful tools, and price stability may reappear as a primary goal of monetary institutions, as it was for centuries. If sticky prices are a problem, perhaps governments will be encouraged to undergo microeconomic reforms to remove the legal restrictions that make prices sticky, rather than to encourage central banks to manipulate stickiness to our supposed benefit. Or, countries can establish pegs to the standard of value and devalue when they think
appropriate. Or, the same sort of standard of value can apply on a country by country basis, still managed by central banks but more remote from fiscal affairs.

But what other alternatives can we think of? Can a private standard of value function? (I admit this is, to most readers, a bit of libertarian fantasizing. But it is a line of thought brought to the fore by the cryptocurrency movement, as well as by the worthy efforts of libertarian writers – it is at least worth pondering if a completely private standard of value can work in a modern economy, or whether it is some essentially government function.)

The basic lesson, I think, of the fiscal theory and the last few hundred years’ experience is that only a backed money can have a long-term stable value, and especially so in our era of rampant financial innovation.

Bitcoin and other similar cryptocurrencies are the latest innovation, and they and their promoters seem to be learning this lesson slowly. Bitcoin’s is entirely a fiat money. Bitcoin has no intrinsic value. There is some demand, similar the transactions demand for money, though in this case fueled as much by the anonymity of Bitcoin transactions rather than its convenience. And, crucially, supply is limited.

Bitcoin already, visibly, suffers the first defect of gold, that its relative value to goods and services fluctuates wildly. That might change a bit if sticky prices were quoted in Bitcoins, but not entirely, as the gold standard era taught us. More deeply, though its supply is limited, there is no limit on the supply of its competitors or of derivative claims. You cannot freely create more Bitcoins, but you can create Ethereum, Ripple, Bitcoin Cash, EOS, Stellar, Litecoin, Basecoin and so forth. And you can create Bitcoin derivatives, promises to pay Bitcoin that every bit as liquid (or more) than Bitcoin itself. So, with a flat supply curve at marginal cost of zero for perfect substitutes, the long history of unbacked money suggests the long term value of a given unbacked cryptocurrency must be zero.

Cryptocurrency innovators are beginning to understand this, and to offer cryptocurrencies that are backed or partially backed. They are reinventing the 19th century bank, which issued fixed-value notes backed by loans and other investments, with an equity cushion to stabilize the value of resources backing the notes. As that long history teaches us, the safest and most stable value backing is government bonds, a narrow bank. Other sources of backing eventually run out and runs can develop.

Reinventing the bank or the Federal money market fund, or the Federal reserve itself – electronic money backed by short-term government bonds – remains an interesting
innovation, if the cryptocurrency can offer better transactions facilitation than their current versions can do. Cryptocurrency startups by and large have not arrived at that hard realization, as the profits from printing unbacked money are so much larger.

In any case, the value question which is the focus here is separate from the transactions technology question which occupies most of the attention. Whether transactions are handled in a distributed ledger, each Bitcoin self-validating by carrying along its entire history, or by a central ledger, each dollar validated by its presence on the Federal Reserve’s computer, is not really relevant to its value, at least away from the apocalypse that the Fed’s computer is hacked.

But a backed stable-value cryptocurrency, stops being a potential separate unit of account. Like 19th century banks, they can expand the money supply, or create useful new transactions media (bank notes on top of coins, cryptocurrency on top of reserves and dollars). But a cryptocurrency that promises to deliver one US dollar per share, whether backed by private assets or backed by government securities, cannot replace US dollars as numeraire.

How could we set up a private standard of value, whether transactions are handled by blockchain or by a conventional centralized ledger? We are back to the search for a backed money; some pot of real assets backing a set of liquid liabilities in a way that one such liability can become numeraire. Such a pot needs to be large, and have a stable real value, and there must be some real/nominal distinction, some promise to trade something nominal for something real, some nominal anchor. The latter is the crucial challenge.

Suppose that Apple continues to expand, and the next generation phone lets you transact in their stock. You buy coffee, and not only swipe your phone to sell a unit of Apple stock. Apple stock could become a medium of exchange and a cryptocurrency. It might not have a stable value, as the relative value of Apple shares and goods and services would vary, but the value would not asymptote to zero as an unbacked non-monopoly cryptocurrency will do. Could it become numeraire? The price of the cup of coffee could be quoted as one centishare of Apple stock (currently worth about $250) rather than $2.50. Can we generalize? Apple could charge 4 shares for its phones rather than $1,000, and it could pay its employees and providers in shares. It seems that Apple’s profit stream could soak up shares in the same way that fiscal surpluses soak up money.

On second thought, though, Apple stock cannot become numeraire, because it lacks a nominal anchor. If Apple shares are claims to more Apple shares, then any relative
price of Apple shares to other goods and services will clear the market.

As a simple but clear example, suppose on the last day \( E_{t-1} \) Apple shares are outstanding. The price level \( P_t \) describes how many shares at time \( t \) it takes to purchase one unit of a homogenous good. Apple sells goods \( \tau_t \) worth \( T_t = P_t \tau_t \) shares, and buys input goods \( g_t \) worth \( G_t = P_t g_t \) shares to its suppliers, for a net profit of \( S_t = T_t - G_t \) shares. Apple can take \( g_t \) goods and turn them into \( \tau_t \) goods for a profit measured in goods terms \( s_t = \tau_t - g_t \). It then returns the profits to shareholders by paying them the \( S_t \) shares. The shares \( E_{t-1} \) are still outstanding. A higher or lower price level does not result in more or fewer shares being soaked up.

Apple could announce that it will sell the new Iphone XVII at 1 share per phone, and thus introduce a nominal anchor. But like gold, then the CPI would vary as the relative price of Iphones and other goods varies. We desire a monetary system that stabilizes the entire CPI. Fiscal surpluses, based on general purchasing power, are a good anchor for this reason.

On the other hand, there is nothing really special about the government in all our previous descriptions of the fiscal theory. So we could construct a private standard of value by mimicking the fiscal theory setup.

Suppose a large company issues three classes of liabilities. The first is a non-interest bearing claim that will become the standard of value. Let’s call them Apple Dollars for now. The second is interest-bearing debt; let’s call them Apple Bonds. Each bond promises future payment of one Apple Dollar. The third is equity. It functions as a risk absorber, allowing the debt claim to be backed by a flow with a stable present value, buffering so losses do not result in inflation and profits do not result in deflation.

The day proceeds as before. In the morning, Apple issues Apple Dollars to pay off Apple debt. Apple requires payment in Apple Dollars, and pays its employees in Apple Dollars. (This being an electronic future, they can convert Apple Dollars to Google Dollars or Bitcoins or a stock mutual fund in milliseconds.) By the end of the day, Apple has made a profit, on average, and soaked up some Apple Dollars from the market. It sells new Apple Bonds which soak up additional Apple Dollars. The price level adjusts until Apple Dollars at the end of the day equal any money demand for such Dollars, including zero in the frictionless case. (Again, the budget constraint says that Dollars at the end of the day are those at the beginning plus and minus flows, the equilibrium condition adds the constraint on how many of those dollars are outstanding.)
Let $B_{t-1}$ denote supply of Apple Bonds outstanding at the beginning of the day, which Apple pays with new Apple Dollars. Apple sells $T_t$ Apple Dollars’ worth of goods at value $T_t = P_t \tau_t$, paying $G_t = P_t g_t$ to its suppliers, for a profit of $S_t = T_t - G_t = P_t s_t$ Apple Dollars. Here too I specify a single homogenous good, and Apple’s profit is its ability to turn $g_t$ input goods into $\tau_t$ output goods, for a profit measured in goods terms $s_t = \tau_t - g_t$.

But now, the equilibrium condition that nobody wants to hold Apple Dollars overnight determines the price level. The equations are the same as those of the fiscal theory applied to government finance.

That could end the story, except that stated this way we can foresee that innovations to the value of Apple’s profits will lead to price level uncertainty, equal to the innovations in Apple’s stock value today. So include in Apple’s payments $G_t = P_t g_t + P_d d_t$ a dividend payment, in Apple Dollars, to its stockholders. Now the value of Apple Dollars will be

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} (\tau_{t+j} - g_{t+j} - d_{t+j})$$

Now Apple has the same ability as the government to manage its debt, selling more or less without changing surpluses, as well as the ability via $d_{t+j}$ payments to stabilize the value of its debt backing. Managing debt $B_t$ and dividends $d_t$ also can make Apple Dollars stable in terms of all goods and services, not just the bundle that Apple buys and sells.

These Apple Bonds are a different legal animal than current debt. They promise only payment of Apple Dollars. That contract needs to be written for the system to work.

None of this is optimal for Apple – I have not derived a stable price level $P_t$ as a natural consequence of profit maximization. And it likely is not.

This structure is not immune to inflation. Like governments who may run out of taxing ability or desire to keep the present value of surpluses up to back their debt, any private structure backed by real assets runs the risk that the value of real assets falls at some point below the value of the debt and currency. At that point, it must inflate or default. Whether contractual restraints, a large equity cushion, and potentially derivative guarantees to bring funds in from other institutions prove more durable than government anti-inflationary will is a good question.

It is unlikely that any private currency would take on itself the task of macroeconomic management that central banks pursue.
Many such currencies could emerge and compete.

The currency as I have described it is immune to runs, unlike a backed money that promises payment in some other currency. It can always inflate. There could be a run forcing inflation, if the assets cannot be sold quickly enough to soak up all the money and debt.

Basecoin is an interesting cryptocurrency that has some of these features. Seeing the problem of unstable currency demand and therefore unstable value, Basecoin also has basecoin debt, which promises only future Basecoins. Thus its operators can manage its value by selling debt, reducing the currency outstanding. However, there remains no backing. Basecoin debt only pays off if there is someday a demand for more Basecoins to be printed. If nobody ever wants more basecoins, the debtholders lose. Moreover, there is no obvious reason that the debt should be less liquid than the coin. If basecoin debt, paying interest, can be transferred just as easily as basecoin itself, people will hold the debt, and the money demand will vanish.

However, what I describe is quite close to a backed basecoin. Alas, the need for backing means that Basecoin’s operators cannot get the first-order benefits of seigniorage.

A private backed currency can provide stable value – almost, but it will be more difficult to provide a large currency. Of course, large is not necessary to provide a standard of value. Likewise, a small government – Switzerland, say – could set up its business to offer a very stable currency, backed by a managed pot of assets, and committing its fiscal resources to stabilizing that pot. However, there are not enough Swiss Francs to go around, as there were not enough gold coins to go around in the 19th century. Thus, the demand for liquid zero nominal risk assets – electronic money – will be met by derivative claims; bank deposits backed by debts that promise to pay Swiss Francs. And, as in the banking crises of the 19th century through the financial crisis of 2007, when everyone wants to run from those derivative claims to the real thing, there isn’t enough to go around and a crisis happens. While we’re dreaming, a fully backed system that avoids financial panics is worth dreaming about.

The wonder of large government debts allows pretty much the entire demand for zero risk liquid assets to be direct claims to government debt. We could avoid financial crises completely if we wished to do so. A return to a small issue of the underlying currency and a large stock of derivative inside moneys seemingly opens us up to more crises.

On the other hand, modern technology offers a possibility to avoid this lot. There
is no longer any fundamental economic reason why our transactions and financial system requires such a large stock of nominally risk free assets. You could pay for that double cappuccino with a cellphone, which sells an S&P500 index fund, and transfers the resulting Apple Dollars, basecoin, or Swiss Francs, to the seller’s mortgage-backed security fund, all in 20 milliseconds. We needed fixed value to provide liquidity in the 1930s, and in the 1960s. But we do not need it today. If all derivative claims were floating-value claims, we could base a monetary system on a very small pot of real assets issuing a very small amount of liquid claims, without suffering financial crises. Communications, computational, and financial technology – the exchange traded fund – open up this possibility. Huge obstacles remain: regulation and accounting demand fixed-value assets, which I believe accounts for their continued demand. (For more on this vision, see Cochrane (2014).)

On the other hand, a private currency can be potentially large, having assets including stocks, corporate bonds, and mortgage-backed securities.
Part III

Money, interest rates, and regimes
Having described the fiscal theory of the price level, we turn to the alternatives. The two big non-fiscal theories of inflation are fiat money with a controlled supply, and interest rate targets. Each of these theories specifies an “active” monetary policy together with a “passive” fiscal regime, while the fiscal theory specifies some important “passivity” of these monetary arrangements, either letting \( M \) vary freely or \( i = \phi \pi \) with \( \phi < 1 \) along with its “active” fiscal regime. (I’ll argue later that the active-passive distinction is actually not a useful concept, but it’s useful to see it first and then realize its emptiness.)

We have met both theories already, as well as the “regime” question. We have also met the observational equivalence theorems. I return to all these issues in a more systematic way here.

Our first question is, can these alternative theories determine the price level or at least the inflation rate? With one exception, I come to a negative conclusion. That one exception is \( MV = PY \), with fixed money supply \( M \) and a fixed velocity \( V \), even at zero interest rates. Otherwise, the alternative theories leave multiple equilibria on the table. Clearly, they have left out some important ingredient.

Adding back the fiscal theory, as above, supplies that missing ingredient. We obtain interesting joint monetary-fiscal policies, such as that described above, in which an interest rate target sets expected inflation and an active fiscal policy sets unexpected inflation.

An enormous literature tries valiantly to specify alternative extra ingredients, in the form of extra specifications of the policy regime, to get rid of multiple equilibria without invoking fiscal theory. In a review, I will argue that none of these work. The fiscal theory is all we have, at least for now.

In addition to the question, are they possible – do these alternative theories determine the price level or inflation rate – we can ask whether they are plausible. Even choosing equilibria in the way specified by alternative theories, do the theories describe modern institutions in a vaguely plausible way? Can they apply to our economy? Surveying this question, I come again to a negative conclusion. Our central banks do not make anything like the equilibrium-selection threats specified by new-Keynesian models, and they do not restrict the quantity of money as specified by \( MV=PY \). Our financial arrangements have thoroughly blurred the distinction between money and debt.

The conclusion of this section is that the currently available alternatives don’t work. The fiscal theory is all we have, at least for now. There are indeed many challenges in applying it to the world, but until another theory comes along, the task at hand is to
figure out how the fiscal theory works, not to test it against a viable alternative.
Chapter 16

Interest rate targets

Classic monetary doctrine says that an interest rate target will not successfully control inflation. Under adaptive expectations, inflation will be unstable, spiraling away to hyperinflation or deflation, as described by Friedman (1968), and formalized below. Under rational expectations, inflation will be indeterminate, bouncing around unpredictably and uncontrollably following “sunspots,” as described by Sargent and Wallace (1975).

We have already seen the latter argument. From the Fisher equation,

\[ i_t = r + E_t \pi_{t+1}, \]

if the central bank pegs the interest rate \( i_t \), that action can determine expected inflation \( E_t \pi_{t+1} \), but unexpected inflation \( \delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1} \) can be any unpredictable random variable, \( E_t \delta_{t+1} = 0 \).

We have seen how the fiscal theory solves the indeterminacy problem. Intuitively, an unexpected disinflation raises the value of nominal debt. If surpluses do not rise to pay this windfall to bondholders, the unexpected disinflation cannot happen. But if surpluses rise to match changes in the value of government debt resulting from any change in the price level, even sunspots – a fully passive fiscal policy – we are left with indeterminate inflation.

Starting in the 1980s, a new theory emerged to overcome indeterminacy. If the interest rate target moves actively, following a rule such as \( i_t = \phi \pi_t \) with \( \phi > 1 \), then the instability or indeterminacy that follows from a fixed interest rate peg would be overcome.
Key innovators include John Taylor (Carlozzi and Taylor (1985) is, I think, the first, Taylor (1993) and Taylor (1999) perhaps more famous), who showed how the rule bearing his name induces stability under adaptive expectations, and McCallum (1981) for rational expectations. Mike Woodford, summarizes his own and many others’ contributions in the seminal Woodford (2003). Woodford credits Knut Wicksell, Wicksell (1898), republished as Wicksell (1965), with the basic idea that systematically raising or lowering the interest rate in response to inflation or the price level can determine the latter.

A genuinely new theory of the price level, only the third in history (backing, fiat money, interest rate target), is a remarkable achievement. It stands on a par with that of Irving Fisher and Milton Friedman, who described how a well-managed unbacked fiat money might not quickly inflate, overcoming the contrary standard doctrine of the 19th century. However, I will argue that the effort does not work in the end, in the rational-expectations context. Inflation remains just as indeterminate with “active” policy as it does under an interest rate peg. (This section draws from Cochrane (2011a).)

16.1 The simplest model

I revisit the simple new-Keynesian model,

\[ i_t = r + E_t \pi_{t+1} \]
\[ i_t = r + \phi \pi_t + \nu_t \]
\[ \nu_t = \rho \nu_{t-1} + \varepsilon_t. \]

The equilibrium condition is

\[ \pi_{t+1} = \phi \pi_t + \nu_t + \delta_{t+1}. \]

where \( \delta_{t+1} \) with \( E_t \delta_{t+1} = 0 \) are multiple equilibrium or sunspot shocks. With \( \| \phi \| < 1 \) that’s it – equilibria are stable, but indeterminate. With \( \| \phi \| > 1 \) all equilibria but one explode going forward. Ruling out such explosions, we have the unique locally bounded equilibrium

\[ \pi_t = - \sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t (\nu_{t+j}) = - \frac{\nu_t}{\phi - \rho}. \]
But there really is no reason to rule out nominal explosions. This summary motivates additional ingredients to rule out multiple equilibria.

Start, following Woodford (2003), with the simplest model, a linearized Fisher equation (consumer first order conditions) and a Taylor rule describing Fed policy,

\[ \dot{i}_t = r + E_t \pi_{t+1}, \]  
(16.1)

\[ \dot{i}_t = r + \phi \pi_t + v_t. \]  
(16.2)

We studied this model briefly in section 4.4. We return to it here in more detail, and to understand its own logic rather than emphasize fiscal theory ideas.

The monetary policy disturbance \( v_t \) represents variables inevitably left out of any regression model of central bank behavior, primarily responses to other variables. The disturbance is not a forecast error, so it is serially correlated,

\[ v_t = \rho v_{t-1} + \varepsilon_t. \]  
(16.3)

For this reason I use the word “disturbance” rather than “shock.”

Substituting out the nominal interest rate, we have a single equilibrium condition

\[ E_t \pi_{t+1} = \phi \pi_t + v_t. \]  
(16.4)

Equation (16.4) has many solutions. We can write the equilibria of this model as

\[ \pi_{t+1} = \phi \pi_t + v_t + \delta_{t+1}; \quad E_t (\delta_{t+1}) = 0, \]  
(16.5)

where \( \delta_{t+1} \) is any conditionally mean-zero random variable. Multiple equilibria are indexed by arbitrary initial inflation \( \pi_0 \), and by the arbitrary random variables or “sunspots” \( \delta_{t+1} \).

If \( \|\phi\| < 1 \), this economy is stable. Expected inflation \( E_t \pi_{t+j} \) converges going forward for any initial value. But it remains indeterminate. “Sunspot” shocks \( \delta_t \) can erupt at any time, and fade away.

If \( \|\phi\| > 1 \), all of these equilibria except one eventually explode, i.e. \( E_t (\pi_{t+j}) \) grows without bound. If we disallow explosive solutions, then a unique locally-bounded solution remains. “Locally bounded” is the important adjective – it means solutions that are expected always to stay within some finite region. Its use recognizes that
there are other solutions. We can find this unique locally-bounded equilibrium by solving the difference equation \(16.4\) forward,

\[
\pi_t = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t (v_{t+j}) = -\frac{v_t}{\phi - \rho}.
\] (16.6)

Equivalently, by this criterion we select the variables \(\pi_0, \{\delta_{t+1}\}\) which index multiple equilibria, as

\[
\pi_0 = -\frac{v_0}{\phi - \rho}; \delta_{t+1} = -\frac{\varepsilon_{t+1}}{\phi - \rho}.
\] (16.7)

Thus we have it: if the central bank’s interest rate target reacts sufficiently to inflation – if \(\|\phi\| > 1\) – then it seems that a pure interest rate target can determine at least the inflation rate.

If one wishes to determine the price level, let the interest rate policy rule be

\[i_t = r + \phi_p (p_t - p^*) + v_t.\]

Now, substituting in to the Fisher equation \(16.1\),

\[E_t (p_{t+1} - p^*) - (p_t - p^*) = \phi_p (p_t - p^*) + v_t\]

\[E_t (p_{t+1} - p^*) = (1 + \phi_p) (p_t - p^*) + v_t\]

Again, we have multiple equilibria. But if \(\phi_p > 0\) and if we rule out explosive equilibria, then we have a unique locally-bounded equilibrium price level,

\[p_t = p^* - \sum_{j=0}^{\infty} \frac{1}{(1 + \phi)^{j+1}} E_t (v_{t+j}).\]

Woodford calls this a “Wicksellian” regime, in honor of [Wicksell][1898], though since Wicksell lived decades before rational expectations, it is a genuinely new idea.

But what’s wrong with equilibria that are not locally bounded? Transversality conditions can rule out real explosions, but not nominal explosions. Hyperinflations are historic realities. The restriction to non-explosive equilibria didn’t come from any economics of the model. I conclude there’s nothing wrong with the explosive equilibria, so this model does not eliminate multiple equilibria and hence does not determine inflation or the price level.
Clearly, we need some additional ingredient. As we have seen, restoring the government debt valuation equation, assumed away by the passive fiscal policy assumption, is one such ingredient. An active fiscal policy specification picks one value of \( \delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1} = -\varepsilon_{t+1}^s \) and solves the problem. We will tour many other attempts to add something else to the model, additions to the specification of monetary policy or alternatives to the \( \phi \pi \) threat, and we will discover that none of them work.

### 16.2 A full nonlinear model

We derive and consider a full nonlinear model, not the linearized version of the last section. Figure [16.1] plots the set of equilibria. The previous linearized analysis bears out near the active equilibrium \( \Pi^* \). Multiplicity was not an artifact of the previous model’s simplifications. The zero bound forces us to consider another equilibrium \( \Pi_L \). This equilibrium must violate the Taylor principle, and hence has multiple locally bounded equilibria even with an active rule around \( \Pi^* \). In turn, the multiple equilibria to the left of \( \Pi^* \), though not locally bounded, are globally bounded, reducing further any reason to rule them out. We can always slip back to the zero bound. Consideration of the full nonlinear model has only made multiplicity worse.

The new-Keynesian model recognizes the government debt valuation equation, and specifies a passive fiscal policy, which will accommodate any price level. Adding FTPL determines a single equilibrium.

One may worry that this simple example is linearized and not fully spelled out. Let’s write down a full model, and make sure there is not some left-out ingredient. (Here I simplify standard sources, Benhabib, Schmitt-Grohé, and Uribe [2002] and Woodford [2003]. Again, this discussion is based on Cochrane [2011a].)

The setup is similar to that throughout this book. The government issues one-period nominal debt, \( B_{t-1} \), and levies lump-sum real primary surpluses \( s_t \). Consumers maximize a utility function

\[
\max E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}).
\]

Consumers receive a constant nonstorable endowment \( Y_t = Y \). Markets clear when \( C_t = Y \). Consumers trade in complete financial markets described by real contingent
claims prices $\Lambda_t$. The nominal interest rate is related to contingent claim prices by

$$\frac{1}{1 + i_t} = E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \right].$$

Consumers face a present-value budget constraint,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} (S_{t+j} + C_{t+j} - Y_{t+j}). \quad (16.8)$$

This constraint derives from the flow constraint plus a transversality condition, and I skipped a step by imposing optimal money holdings $M_t = 0$.

The consumer’s first order conditions state that marginal rates of substitution equal contingent claims price ratios, and equilibrium $C_t = Y$ implies a constant real discount factor,

$$\beta \frac{u_c(C_{t+1})}{u_c(C_t)} = \frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{u_c(Y)}{u_c(Y)} = \beta. \quad (16.9)$$

Therefore, the real interest rate is constant,

$$\frac{1}{1 + r} = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) = \beta.$$

The interest rate then follows the nonlinear Fisher relation,

$$\frac{1}{1 + i_t} = \beta E_t \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{1 + r} E_t \left( \frac{1}{\Pi_{t+1}} \right). \quad (16.10)$$

From the consumer’s present value budget constraint (16.8), and using contingent claim prices from (16.9), equilibrium $C_t = Y$ also requires

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t (s_{t+j}). \quad (16.11)$$

The value of government debt is the present value of future net tax payments. This is not a “government budget constraint,” it is an equilibrium condition, an implication of supply = demand or $C_t = Y_t$ in goods markets, as you can see directly by looking at (16.8).

The Fisher equation (16.10) and the government debt valuation equation (16.11) are the only two conditions that need to be satisfied for the price sequence $\{P_t\}$.
to represent an equilibrium. If they hold, then the allocation $C_t = Y$ and the
resulting contingent claims prices (16.9) imply that markets clear and the consumer
has maximized subject to his or her budget constraint. The equilibrium is not yet
unique, in that many different price or inflation paths will work. Unsurprisingly,
we need some specification of monetary and fiscal policy to determine the price
level.

The new-Keynesian analysis maintains a passive fiscal regime. Government surpluses
$s_{t+j}$ are assumed to adjust so that the government debt valuation equation (16.11)
holds given any price level. (See Woodford (2003). p. 124.) It also specifies a policy
rule following the Taylor principle for interest rates.

We have answered the first question needed from this explicit model: yes, solutions of
the simple model consisting of a Fisher equation and a Taylor rule (16.1)-(16.2), as
I studied above, do in fact represent the full set of linearized equilibrium conditions
of this explicit model. My simple example didn’t leave anything out.

Are the non-locally-bounded equilibria really globally valid? Is there some reason
to rule them out in this full model? Here I follow the standard sources, in part to
emphasize agreement on these points: Woodford (2003) Ch. 2.4, starting p. 123,
and Ch. 4.4 starting on p. 311, with a review; and Benhabib, Schmitt-Grohe, and
Uribe (2002).

To keep the equations as simple as possible, restrict attention to perfect foresight
equilibria. In the linear model, $E_t \pi_{t+1} = r + \phi \pi_t$ led to two kinds of indeterminacy,
$\delta_{t+1} = (E_{t+1} - E_t) \pi_{t+1}$ and $\pi_0$, which is really $\delta_0 = \pi_0 - E_{-1} \pi_0$. There was no
extrinsic uncertainty. But each date could still see shocks, based on random variables
like sunspots having nothing to do with the economy. By looking at perfect foresight
equilibria, we consider only the determination of $\pi_0$ and we ignore sunspot equilibria
at other dates $(E_{t+1} - E_t) \pi_{t+1}$. But $\pi_0$ and subsequent perfect foresight teaches
us how $\delta_{t+1}$ and $(E_{t+1} \pi_{t+j})$ behave at each date after that. Adding uncertainty
(sunspots) can only increase the number of equilibria.

Consider an interest rate rule
\[ 1 + i_t = (1 + r) \Phi(\Pi_t); \quad \Pi_t \equiv P_t / P_{t-1}. \tag{16.12} \]
\(\Phi(\cdot)\) is a function allowing nonlinear policy rules. With perfect foresight, the con-
sumer’s first order condition (16.10) reduces to
\[ \Pi_{t+1} = \beta (1 + i_t). \tag{16.13} \]
We are looking for solutions to the pair $\text{(16.12)}$ and $\text{(16.13)}$. As before, we substitute out the interest rate and study the equation

$$\Pi_{t+1} = \Phi(\Pi_t). \quad (16.14)$$

We have a nonlinear, global (i.e., not local), perfect-foresight version of the analysis of the last section.

Nonlinearity and global solutions make one big difference immediately: we must respect the zero bound. (If interest rates are much below zero, people will all try to hold cash instead. It’s now called the ELB or “effective lower bound” in policy circles, as central banks have discovered they can charge 1% or so negative interest without sparking wide scale flight to cash. Much more than that seems unlikely. To keep the discussion simple, I refer just to a zero bound here.) A policy rule with slope globally greater than one cannot apply globally to an economy in which consumers can hold money, because nominal interest rates cannot be negative. From $\text{(16.13)}$, that means we cannot have $\Pi_{t+1} < \beta$. Thus, if we want to specify a policy rule with $\Phi_\pi > 1$ at some point, we must consider the situation as illustrated in Figure $\text{16.1}$. The equilibrium at $\Pi^*$ satisfies the Taylor principle $\Phi_\pi > 1$, and is a unique locally bounded equilibrium. Any value of $\Pi_0$ other than $\Pi^*$ leads away from the neighborhood of $\Pi^*$ as shown. With a lower bound on nominal interest rates, and thus a lower bound on inflation, however, the function $\Phi(\Pi)$ must also have another stationary point, labeled $\Pi_L$. This stationary point must violate the Taylor principle, and have $\Phi_\pi < 1$. Therefore, many paths lead to $\Pi_L$ and there are “multiple local equilibria” near this point.

(Yes, $\Pi^*$ is considered the “good” equilibrium and $\Pi_L$ is considered the “bad” equilibrium in this literature. The point is to find determinacy by ruling out multiple equilibria. $\Pi^*$ is a unique locally-bounded equilibrium. “Stability” near $\Pi_L$ comes with “indeterminacy.” That disadvantage is thought to outweigh the Friedman-rule advantages of low inflation.)

All of the paths graphed in Figure $\text{16.1}$ are perfect-foresight equilibria. Since these paths satisfy the policy rule and the consumer’s first-order conditions by construction, all that remains is to check that they satisfy the government debt valuation formula $\text{(16.11)}$, i.e. that there is a set of ex-post lump-sum taxes that can validate them and hence ensure the consumer’s transversality condition is satisfied. There are lots of ways the government can implement such a policy. We only need to exhibit one: If the government simply sets net taxes in response to the price level as

$$s_t = \frac{r}{1 + r} \frac{B_{t-1}}{P_t} \quad (16.15)$$
then the real value of government debt will be constant, and the valuation formula will always hold.

To see why this is true, start with the flow condition that proceeds of new debt sales + taxes = old debt redemption,

\[ \frac{B_t}{1 + i_t} + P_t s_t = B_{t-1}. \]

With \( 1 + i_t = (1 + r)P_{t+1}/P_t \), this expression can be rearranged to track the real value of the debt,

\[ \frac{B_t}{P_{t+1}} = (1 + r) \left( \frac{B_{t-1}}{P_t} - S_t \right). \] (16.16)

Substituting the rule (4.6) we obtain

\[ \frac{B_t}{P_{t+1}} = \frac{B_{t-1}}{P_t}. \]
We’re done. With constant real debt and the flow condition (16.16) the transversality condition holds, and (16.16) implies (16.11). All the inflationary equilibria of the last section are valid.

The nonlinear model actually makes things worse. Deflationary equilibria that approach $L$ are also valid equilibria, as is $L$ itself. These equilibria are now “bounded,” though not “locally bounded around $*$.” Arguments against “locally unbounded” equilibria that are nonetheless “globally bounded” are a step harder to make. And there is a new class of equilibria around $L$: both locally and globally bounded. Indeterminacy is back full force.

The price level and inflation rate are uniquely determined in this model if we strengthen, rather than throw out, the government debt valuation equation – if the government follows an active fiscal regime, as before. As the simplest example, suppose fiscal policy sets the path of real net taxes $\{s_t\}$ independently of the price level. The initial face value of one-period government debt $B_{-1}$ is predetermined at date 0. Then, (16.11) determines the price level $P_0$,

$$ \frac{B_{-1}}{P_0} = \sum_{j=0}^{\infty} \beta^j s_{t+j}. $$

and therefore the initial inflation rate $\Pi_0 = P_0/P_{-1}$. The government implements the interest rate rule, by offering to sell debt at fixed interest rates, as in section 3.1. The price level is not hostage to the whims of fate. By choosing the path of surpluses at time 0, $\{s_t\}$, the government selects which of these equilibria will hold. The key “active” specification is that the government will not change its fiscal policy in response to the emergence of alternative equilibria. Thus, as before, the government will appear passive, and to act to pay off its debts, on the equilibrium path. If the explosive price level paths on the right seem counterintuitive, that is because no government in its right mind follows this monetary policy $\Phi$, deliberately inducing a hyperinflation.

But with passive fiscal policy, indeterminacy remains, even in this fully spelled out nonlinear model. The attempt to fully determine the price level or inflation rate by interest rate targets alone does not work, even if the targets are “active” with $\Phi_\pi > 1$. 
16.3 Price stickiness and adaptive expectations

We add price stickiness. I write a simple general model that includes adaptive vs. rational expectations.

\[ x_t = -\sigma (i_t - \pi_t^e) \]
\[ \pi_t = \pi_t^e + \kappa x_t. \]

The model’s equilibrium condition is

\[ \pi_t = -\sigma \kappa i_t + (1 + \sigma \kappa) \pi_t^e. \]

Add a policy rule

\[ i_t = \phi \pi_t + v_t, \]

and the equilibrium condition becomes

\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_t^e - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} v_t. \]

We’re ready to add rational \( \pi_t^e = E_t \pi_{t+1} \) or adaptive \( \pi_t^e = \pi_{t-1} \) expectations.

Most discussion of interest rate targets takes place in models with price stickiness. The intuitive ideas that motivated Taylor and others to advocate active interest rate targets, targets that react to inflation, rely on price stickiness (as well as adaptive expectations): The Fed raises nominal interest rates, with price stickiness that action raises real interest rates, higher real interest rates lower aggregate demand, and via a Phillips curve inflation declines. Most historic and policy discussion of interest rate targets operates with price stickiness and adaptive expectations, for example the stylized history that Fed Chairman Paul Volcker brought US inflation under control with a period of high interest rates. As we will see, sticky prices and adaptive expectations assumptions are not frosting on the cake, epicycles that give a bit better empirical fit, but rather lie deep in the basic mechanisms of inflation control. A lot of confusion has resulted from mixing adaptive expectations intuition with rational expectations models.

Write the standard new-Keynesian model, from section 7.1, as

\[ x_t = E_t x_{t+1} - \sigma (i_t - \pi_t^e) \]
\[ \pi_t = \pi_t^e + \kappa x_t \] (16.18)
\[ \pi_t = \pi_t^e + \kappa x_t \] (16.19)
Equation (16.18) is the “IS” curve, representing here intertemporal substitution between current and future consumption. Equation (16.19) is an expectations-adjusted Phillips curve, in which output is higher when inflation is higher. The symbol $\pi^e$ stands for expected inflation, which I will let be less than fully rational.

To keep the algebra very simple, I delete the $E_t x_{t+1}$ term in (16.18), so our model becomes

$$
x_t = -\sigma (i_t - \pi^e_t) \tag{16.20}
$$

$$
\pi_t = \pi^e_t + \kappa x_t. \tag{16.21}
$$

Now we have the static Keynesian IS curve, in which output is lower when the real interest rate is higher. I return later to verify that none of the conclusions depend on this specification, so all it does is to simplify the equations. Now we can eliminate $x_t$ and describe equilibria by

$$
\pi_t = -\sigma \kappa i_t + (1 + \sigma \kappa) \pi^e_t. \tag{16.22}
$$

Add a monetary policy rule of the Taylor type,

$$
i_t = \phi \pi_t + \nu_t.
$$

Eliminating $i_t$, our equilibrium condition becomes

$$
\pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi^e_t - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} \nu_t. \tag{16.23}
$$

16.3.1 Adaptive vs. rational expectations; new vs. old Keynesian models

With adaptive expectations, the equilibrium condition is

$$
\pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_{t-1} - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} \nu_t.
$$

An interest rate peg or passive policy $\phi < 1$ produces unstable, determinate inflation. A central bank following the Taylor rule $\phi > 1$ stabilizes an otherwise unstable economy.
With rational expectations, the equilibrium condition is

\[ E_t \pi_{t+1} = \frac{1 + \sigma \kappa \phi}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} v_t. \]

Now, an interest rate peg or passive policy \( \phi < 1 \) produces \textit{stable, indeterminate} (multiple equilibrium) inflation. A central bank following the Taylor principle \( \phi > 1 \) \textit{destabilizes} the economy, to render it locally determinate.

The models are diametrically opposite. Much confusion results from mixing them up, and mixing up stability vs. determinacy.

Now, assume expectations are adaptive,

\[ \pi_t^e = \pi_{t-1}. \]

The equilibrium condition (16.23) becomes

\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_{t-1} - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} v_t. \quad (16.24) \]

Under adaptive expectations, an interest rate peg \( \phi = 0 \) or passive policy \( \phi < 1 \) produces \textit{unstable, determinate} inflation. The coefficient on lagged inflation is above one. Inflation or deflation generically spiral away. There is only one solution and no multiple equilibrium problem. In this case the Taylor rule \( \phi > 1 \) \textit{stabilizes} an otherwise unstable economy. Raising \( \phi \) to a value greater than one, the coefficient on lagged inflation in (16.24) becomes less than one. Any shocks, such as induced by \( v_t \), eventually die out. There is still only one equilibrium.

This model captures in its simplest form the way Taylor introduced the rule, and how Taylor rules are thought to operate in policy circles. If inflation gets too big, then the central bank raises the nominal interest rate more than one for one with inflation. Via (16.20) that action lowers aggregate demand and output \( x_t \), which via the Phillips curve (16.21) lowers inflation. Indeterminacy just isn’t an issue.

Under rational expectations,

\[ \pi_t^e = E_t \pi_{t+1}. \]

The equilibrium condition (16.23) becomes

\[ E_t \pi_{t+1} = \frac{1 + \sigma \kappa \phi}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} v_t. \quad (16.25) \]
(The frictionless case \(16.4\), \(E_t \pi_{t+1} = \phi \pi_t + v_t\), is the \(\kappa \to \infty\) limit.) Now, an interest rate peg \(\phi = 0\) or passive policy \(\phi < 1\) produces stable, indeterminate inflation. The coefficient on lagged inflation is below one. Any inflation or deflation is expected to melt away on its own. But here the indeterminacy problem appears. Unexpected inflation \(\delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1}\) can be anything.

In this case a central bank following the Taylor principle \(\phi > 1\) takes an economy that is already stable, and deliberately makes it unstable, in order to try to make it determinate. For all but one value of \(\delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1}\), the central bank deliberately leads the economy to hyperinflation. If we add a rule against hyperinflations as equilibria, then there is only one equilibrium. This example generalizes the examples of the previous two sections to include price stickiness.

I have generally avoided calling \(\phi > 1\) the “Taylor rule” in a rational expectations context, and called it a “policy rule” following the “Taylor principle” instead. The Fed in \cite{Taylor1999}, for example, stabilizes inflation by raising interest rates more than one for one. It does not deliberately introduce instability to ward off indeterminacy.

In sum, in the old-Keynesian model, the economy under a peg is unstable and indeterminate, and \(\phi > 1\) renders it stable, and still determinate. In the new-Keynesian model, the economy under a peg is stable and indeterminate, and \(\phi > 1\) makes it unstable and therefore determinate. Their dynamic properties are exactly opposed. The Fed’s role in old-Keynesian models is to bring inflation back under control. The Fed’s role in new-Keynesian models is to make unpleasant threats about what would happen in alternative equilibria in order to get the economy to settle on one of many equilibria. They are night and day different models. Well, moving a subscript from \(\pi_{t-1}\) to \(E_t \pi_{t+1}\) changes the sign of the corresponding eigenvalue, which controls stability and determinacy properties.

Much of the confusing debate about new-Keynesian models comes, in my view, from confusing them with old-Keynesian models. In part, I think researchers genuinely thought that the new-Keynesian framework would end up just providing optimization and market-clearing foundations for old-Keynesian ISLM and monetarist intuition. Introductions that interpret new-Keynesian results with old-Keynesian intuition were a frequent result. It took quite some time to realize that the actual models are quite distinct, and in fact diametrically opposite on many issues.

For example, the old-Keynesian model says that \(\phi < 1\) in the 1970s led to unstable inflation, and a switch to \(\phi > 1\) in 1980 stabilized inflation, giving the economy eigenvalues less than one. Persistently high interest rates slowly beat down inflation.
The new-Keynesian model says that $\phi < 1$ in the 1970s was stable, but *indeterminate*, the poor inflation performance of that decade came from shifts between sunspot equilibria. The switch to $\phi > 1$ in the 1980s led the economy to *determinate* inflation, removing multiple equilibria. The new-Keynesian model only produces a decline in inflation from a transitory AR(1) monetary policy disturbance, and therefore does not produce the standard view of the 1980s. That this is what the equations actually say took a long time to figure out, given the widespread old-Keynesian intuition, and you can see this confusion in much of the debate.

### 16.3.2 Fed destabilization?

No central bank on the planet says it deliberately destabilizes the economy, or worries centrally about selecting from multiple equilibria. The $\phi > 1$ threat is disastrous ex-post for the central bank, so people are not likely to believe the bank will do it.

With this contrast in mind, we can see a second central problem with the active interest rate specification.

Let us grant for the moment a rule against non-locally-bounded equilibria, both the inflationary equilibria to the right of $\Pi^*$ and those that slip off to the zero bound to the left of $\Pi^*$ in Figure 16.1. To produce those equilibria, the central bank commits that if inflation gets going, the bank will increase interest rates, and by doing so it will *increase* subsequent inflation, without bound. Likewise, should inflation be less than the central bank wishes, it will drive the economy down to the liquidity trap.

No central bank on this planet describes its inflation-control efforts this way. They uniformly explain the opposite. Should inflation get going, the bank will increase interest rates in order to *reduce* subsequent inflation. It will induce stability into an unstable economy, not the other way around. I have not seen selecting among multiple equilibria on any central bank’s descriptions of what it does.

People discount all sorts of central-bank pronouncements of course. But one way or another people must believe this is how they will behave. That the central bank will react to inflation by pushing the economy to hyperinflation seems an even more tenuous statement about people’s beliefs, today and in any sample period we might study, than it is about actual central bank behavior.

Among many reasons to doubt it is that the threat is, ex-post, disastrous for the
central bank’s objectives. It is not “subgame-perfect.”

Note in this simple model $\phi < -1$ is as good as $\phi > 1$ to produce local determinacy. The central bank threatens oscillating hyperinflation and deflation. Well, if the economy abhors growing inflation, and will not choose such equilibria, oscillating inflation and deflation are even ghastlier. (King (2000), p. 78.) This example though should drive home that the central bank is not “stabilizing” inflation, raising interest rates to tamp down future inflation, but “destabilizing” in order to induce determinacy. Subsequent models also have strange parameter regions of this sort, and a variety of unpleasant threats that the central bank might make to trim equilibria.

### 16.4 Fixes

That the new-Keynesian model suffers multiple equilibria, and that $\phi > 1$ is not a completely satisfactory answer, both because economics does not rule out inflationary and liquidity-trap equilibria, and because this behavior is not a plausible description of central bank policy, is now a well-known problem. It has attracted an enormous number of attempts to fix it, while retaining the passive fiscal policy assumption that wipes out the government debt valuation equation.

There are several broad categories of such attempted fixes. (This section draws on the broader discussion in Cochrane (2011a).) Each fix adds something else to the policy regime to get rid of multiple equilibria.

#### Reasonable expectations and minimum state variables

Several authors argue that it is unreasonable for people to expect hyperinflation or deflation, so multiple equilibria should not break out. But what is unreasonable in our world is not so unreasonable in the model. If everyone believed central banks really were committed to react to inflation with ever-increasing inflation, then it would be much more reasonable for people to expect such inflation. The unreasonableness of these expectations signals that it is unreasonable for people to believe that central banks deliberately destabilize the economy, i.e. follow $i = \phi \pi$ in a Fisherian world in which that action leads to spiraling inflation.

One approach is to restrict expectations to what seems reasonable. For example, Woodford (2003) (p.128) argues that expectations should “coordinate” on the locally
unique equilibrium ($\Pi^*$) in Figure 16.1:

“The equilibrium ..$\Pi^*$.. is nonetheless *locally* unique, which may be enough to allow expectations to coordinate upon that equilibrium rather than on one of the others.”

Moreover,

“The equilibria that involve initial inflation rates near (but not equal to) $\Pi^*$ can only occur as a result of expectations of *future* inflation rates (at least in some states) that are even *further* from the target inflation rate. Thus the economy can only move to one of these alternative paths if expectations about the future change significantly, something that one may suppose would not easily occur.”

Similarly, King (2000) (p. 58-59) writes:

“By specifying $[\phi > 1]$ then, the monetary authority would be saying, ‘if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy.’ If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.

This seems a rather weak foundation for the basic economic question, what determines the price level? Is economics on its own really incapable of answering that question?

Woodford makes a good point. It does seem unlikely that people wake up one morning and believe, with no other news, that a hyperinflation is coming so they should raise prices just a little today. It’s a good deal more plausible that they wake up and decide that another slide to the zero bound is coming so they should lower prices just a little today, but even that case requires a bigger shift in expectations about the future than the instantaneous move.

But if we are to appeal to common intuition about reasonable beliefs, what seems unreasonable in this model is that people believe the central bank would react to inflation by blindly driving the world to hyperinflation without end.

In this model, expected increases in interest rates raise inflation. Our central banks, and our world, are populated by people who think increases in interest rates lower inflation, as described by the adaptive expectations old-Keynesian model. If the that world is true, and people believe the central bank will respond to inflation by raising
interest rates more than one for one, then indeed people would be unlikely to wake up and think hyperinflation is coming. But that’s not the world of this model. If fiscal theory underlies price level determination, the theme of this book, then people are also unlikely to wake up believing there will be a hyperinflation or deflation with no outside news. But that’s not the world of this model.

In this model, the central bank is absolutely committed to raising interest rates more than one for one with inflation, forever, no matter what. In this model, the central bank is centrally committed to selecting equilibria, and must loudly proclaim that intention, since we never see its threat in equilibrium. In this model, expected increases in nominal rates raise expected inflation, precisely the opposite of the stabilizing language in the Federal Reserve’s account of its actions. If we lived in the world of this model, it does not seem at all unreasonable that people could wake up expecting hyperinflation or a slide to the zero bound. If we think such expectations are “unreasonable,” that intuition means we don’t believe this model describes the world in which we live. Expectations that are far-fetched in our intuitive understanding of our world are not necessarily so far-fetched for agents in this model, once we recognize that this model may not represent our world.

Economics takes multiple equilibria seriously in many contexts, for example bank runs as in Diamond and Dybvig (1983). It seems just as easy to say people would think it unreasonable that everyone else runs, or that they would “coordinate” expectations on the non-run equilibrium.

A related idea: In a series of papers, summarized in McCallum (2003), McCallum argues for a “minimal state value” (MSV) criterion to pick from multiple equilibria. Endogenous variables in an economic model should only depend on the fundamental state variables of that model. This criterion is also a good technique for finding solutions to complex models, especially when state variables are Markovian – look for $x_t = f(v_t)$ where $v_t$ is a list of the state variables. This criterion can be invoked to rule out the explosive and sunspot solutions of this model. In the simple linearized model of section 16.1, the only exogenous variable is the monetary policy disturbance $v_t$, and it is Markovian, so it contains all information about future exogenous states of the economy. Hence, the minimum state variable criterion says to pick $\pi_t = f(v_t)$. In this case, the only such solution is the locally bounded choice (16.6) $\pi_t = -v_t/\phi$. The ideas are related. Really, minimum state variables argues that reasonable expectations of future inflation should be related to real state variables, thus ruling out sunspot equilibria. However, McCallum (p. 1154) states that his proposal does
not apply to selecting among nominal indeterminacies, and only apply to models with multiple real paths. Therefore, it appears, he would not apply them to the frictionless models on which I have focused.

To be clear, the point is not to defend the multiple explosive equilibria generated by the simple model with $\phi > 1$. My point is that if the simple model were true then we should take seriously these multiple explosive equilibria. Since, we agree, the multiple explosive equilibria don’t make a lot of sense, then it follows that the simple model with $\phi > 1$ is wrong. The $\phi > 1$ policy, in this Fisherian model where higher expected interest rates lead to higher expected inflation, is an unreasonable characterization of what central banks do, what they say they do, what they should do, and what people believe they do.

Both of these approaches add something else to economics, to the definition of equilibrium, applicable to all models.

### 16.4.1 Stabilizations and threats

I survey attempts to cut off multiple equilibria by adapting proposals to stop hyperinflations or deflations, by switching to a money growth target, commodity standards, or similar means. But if an inflation breaks out, and the government stops it, that path remains an equilibrium. In fact, it is now more plausible since inflation does not increase to infinity. These proposals in fact stop equilibria by specifying a period of inconsistent policy, in which equilibrium can’t form, because the policy settings force a violation of private first order or equilibrium conditions. They are “blow up the world” threats. But it is not plausible that governments would do such a thing, or even that they can do such a thing.

Why not just blow up the world directly, rather than as part of an otherwise sensible stabilization? That these proposals modify sensible proposals to stop stabilize inflations reveals a source of confusion about new-Keynesian models.

The next set of suggestions add something else, beyond $i = \phi \pi$, to the policy regime to try to prune multiple equilibria, while maintaining passive fiscal policy and the conventional set of equilibrium selection rules.

These approaches adapt common ideas for stopping hyperinflations, deflations or liquidity traps. If an inflation or disinflation breaks out, governments switch to another policy regime, including a money growth target, a commodity standard or foreign exchange peg, or an active fiscal regime in order to stop the inflation
or deflation. (Examples include, [Woodford (2003), section 4.3, Atkeson, Chari, and Kehoe (2010), Minford and Srinivasan (2011), and Christiano and Takahashi (2018).] Hyperinflations do typically end with a joint monetary-fiscal reform, and zero bound episodes seem to involve a lot of fiscal expansion.

It’s a natural idea: Speculative inflations and deflations are the problem. If we add an off-the-shelf policy prescriptions to stop inflations and deflations, we should solve the problem, no?

No. If a multiple-equilibrium inflation or deflation breaks out, and if the government successfully stops the inflation or deflation by these means, and is expected to do so, then the inflation or deflation and its end remain an equilibrium. If anything, such proposals make matters worse. To the extent that the prospect of never-ending hyperinflation or perpetual liquidity trap made expectations of such events “unreasonable,” or “coordinated” expectations against them, expectations that the government would likely stop the inflation or deflation makes the paths more reasonable for people to expect in the first place. Bringing inflation back, means that the multiple equilibria are once again bounded.

To stop a multiple-equilibrium inflation or disinflation from breaking out in the first place, one must change the policy configuration so that at some place along the path an equilibrium cannot form. Policy must be such that private-sector first order conditions, budget constraints, or market clearing conditions must be violated.

As we look carefully at these proposals we see that is exactly what they do. There is at least one period $T$ of overlap between inflation and its stabilization, in which the central bank commits both to an interest rate rule $i_T = \phi \pi_T$ with still high $\pi_T$, requiring a high nominal interest rate, and to a low money growth target, commodity standard, or active fiscal policy, to lower $\pi_{T+1}$, that requires a low nominal interest rate $i_T$ or low money growth. Since the interest rate and money growth cannot be simultaneously high and low, since an interest rate target and an inconsistent money growth target or commodity standard cannot coexist, “equilibrium cannot form” in such periods. In a rational-expectations dynamic economy, the equilibrium path leading to this event cannot form either.

It is these periods of inconsistent policy that rules out the equilibrium, not the underlying idea of stopping an inflation or deflation on which the proposals build. Stopping inflation does not need inconsistent policy. If the government separates by one period the inflation and its stabilization, then the inflation is stopped, and equilibrium can form each period on the way. That’s how inflations are stopped, with no period in which equilibrium “doesn’t form” along the way.
Conversely, to rule out an equilibrium, there is no need to appeal to the policies that stop inflations and deflations. Just set an inconsistent policy somewhere along the way. An inconsistent policy that increases inflation, $M_{T+1}/M_T$ huge with $\dot{i}_T$ only $i_T = \phi \pi_T$, would rule out the equilibrium path just as well. Or set $M_T = 0$, to really stop equilibrium from forming by removing all money from the economy.

Atkeson, Chari, and Kehoe (2010) recognize this fact, and offer a range of “sophisticated” policies to trim multiple equilibria without the smokescreen of inflation-stabilization policy changes. Their point is more general. The active policy $\phi > 1$ itself is designed to select equilibria, not to stabilize the economy in old-Keynesian fashion. One can make additional threats to select multiple equilibria in $\phi < 1$ stable multiple equilibrium economies too.

Once this point is understood, the objections are natural.

What does it mean for a government to set policy so that “equilibrium cannot form?” Presumably it means that all economic activity stops? Even reversion to barter is an equilibrium of sorts. For this reason, I think of this sort of policy as a “blow up the world” or “crash the economy” threat.

But what government on earth would ex-post embark on a policy so draconian that “no equilibrium can form,” whatever that means? Once the threat has failed, carrying it out is disastrous for the central bank’s, and the government’s, objectives. Technically, this is not a subgame perfect threat, and even more so than deliberately leading the economy to hyperinflation. Therefore, ex-ante, there is no reason for people to believe such a threat. And our central banks emphatically do not make such threats. They promise to stop and stabilize inflations, but they promise to do so in the smoothest possible way, not to set policy so “no equilibrium can form” on the date of the stabilization.

Is it even possible for the central bank to follow a policy that forces agents to violate first order conditions, or markets not to clear; for equilibrium not to form? What would actually happen if the central bank were to announce simultaneously an interest rate target requiring high money growth and a money growth target demanding low money growth, or an interest rate target together with a commodity standard requiring free exchange of money for the commodity? One instrument cannot achieve two targets, especially when they are at wide odds.

We usually think of governments acting in markets, just like everyone else. Governments may have monopoly powers, but even monopolies must respect demand curves and budget constraints. In the Ramsey tradition, most public finance studies
government policy settings, taking private first order conditions and market clearing as constraints. If the central bank wants to raise interest rates, it must respect the money demand curve. It is simply impossible for the central bank, with one instrument, to simultaneously target interest rates and money growth at values inconsistent with private first-order conditions, budget constraints, and market clearing. If it’s *impossible* for the central bank to do it, as well as disastrous for its objectives, that people would expect such a thing, and hence rule the inflationary path out of their expectations, seems even more dubious.

Moreover, these are at best proposals for how some future central bank might act, not proposals for how we model our central banks, our governments, expectations people have now, or expectations people had during a sample period we are studying. So proposals involving setting policy so equilibrium cannot form are not useful for studying history, data or current policy choices.

For example, [Woodford (2003)](#) section 4.3 p. 138 studies proposals to cut off inflationary equilibria to the right of $\Pi^*$:

...self-fulfilling inflations may be excluded through the addition of policy provisions that apply only in the case of hyperinflation. For example, Obstfeld and Rogoff (1986) propose that the central bank commit itself to peg the value of the monetary unit in terms of some real commodity by standing ready to exchange the commodity for money in event that the real value of the total money supply ever shrinks to a certain very low level. If it is assumed that this level of real balances is one that would never be reached except in the case of a self-fulfilling inflation, the commitment has no effect except to exclude such paths as possible equilibria. ...

...[This proposal could] well be added as a hyperinflation provision in a regime that otherwise follows a Taylor rule.

(Obstfeld and Rogoff study models with a money growth target, not an interest rate target, so I defer a detailed description of their proposal to the next chapter.)

A backup commodity standard would certainly *stop* a large inflation. But again, stopping the inflation does not rule out the inflationary equilibrium path. That commitment alone would not “exclude such paths as possible equilibria.” The key in Woodford’s quote must therefore be “otherwise follows a Taylor rule.” If a government continues to follow the Taylor rule (Taylor principle, really) requiring high nominal interest rates, even after it has switched to a commodity standard that requires low nominal interest rates, then, yes, no equilibrium can form. But all of the above problems apply. How could a government both “stand ready to exchange the
commodity for money,” at a fixed rate, while also following a Taylor rule that targets the interest rate by providing whatever money people want at that rate? And our central banks do not make such a commitment. Reversion to a gold standard (the only commodity standard imaginable) is not on the agenda. So it is at best a proposal for future central banks, not a proposal one can appeal to in the analysis of current data or policies.

Atkeson, Chari, and Kehoe (2010), Minford and Srinivasan (2011), and Christiano and Takahashi (2018) give more explicit examples. In these papers, the central bank follows an active interest rate target, \( i_t = \pi_t \) until inflation exceeds bounds \( [\pi_L, \pi_U] \). When inflation exceeds those bounds, the government reverts to a money growth rule. They model an economy with constant velocity and hence money demand \( M_tV = PY \). The central bank operates by setting the money supply in both interest-rate target and money-growth regimes.

Now, how does that policy configuration rule out multiple equilibria, rather than just stop, and thus solve, their inflations? Let period \( T \) be the first period in which inflation exceeds the upper bound \( \pi_U \). During this period, the central bank follows an active interest rate target \( i_T = \phi \pi_T \), until inflation exceeds bounds \( [\pi_L, \pi_U] \). When inflation exceeds those bounds, the government reverts to a money growth rule. They model an economy with constant velocity and hence money demand \( M_tV = PY \). The central bank operates by setting the money supply in both interest-rate target and money-growth regimes.

Well, that is indeed a policy configuration for which no equilibrium can form. One may say “agents cannot satisfy their intertemporal optimization condition,” since a very high interest rate \( i_T = \phi \pi_T \) is inconsistent with a low inflation \( \mu = \pi_{T+1} \) and the Fisher relation \( i_T = r + \pi_{T+1} \), or its generalization in a production economy \( u'(c_T) = E_t[\beta u'(c_{T+1}) (1 + i_T) P_T / P_{T+1}] \). One might equally say that agents satisfy intertemporal optimization, but agents cannot satisfy their money demand equation (cash in advance constraint), or one might say that the economy cannot not satisfy market clearing conditions. In any case, an equilibrium cannot form at period \( T \), and therefore the inflationary path leading to \( T \) is not an equilibrium.

But just how could the central bank do it? How could a central bank, with one instrument, the money supply \( \{M_T, M_{T+1}\} \), simultaneously set \( i_T \) to a large level and \( \pi_{T+1} \) to a low level, in the face of consumers whose first-order conditions demand a high level of \( \pi_{T+1} \) in order that the interest rate \( i_T \) be large?

In addition to the usual complaints, we must add here that velocity is interest-elastic in our world. A constant money growth rule leaves just as many indeterminacies as the interest rate rule, covered in section 17.1 and following. So it doubly cannot apply to our economies.
16.4.2 Fiscal equilibrium trimming

A second group of proposals tries to trim equilibria by fiscal means: helicopter drops of money, deliberately unbacked fiscal expansions, or a contingent switch to fiscal theory. Again, though, if an inflation or deflation breaks out, and is stopped by fiscal means, then the inflation and its aftermath remain valid equilibrium paths. Again, the equilibria are ruled out by a period of inconsistent blow-up-the-economy policy, simultaneously following a high interest rate target and the fiscal policy. Again, the example is revealing of confusion between stabilizing inflation and ruling out multiple equilibria.

Benhabib, Schmitt-Grohe, and Uribe (2002), mirrored in Woodford (2003) section 4.2, try to trim equilibria by adding fiscal commitments to the Taylor rule. Their ideas are aimed at trimming deflationary liquidity trap equilibria, \( \Pi_L \) in Figure 16.1, but the same ideas could apply to inflations as well, since hyperinflations are also stopped by fiscal reforms.

These proposals are inspired by many policy proposals to exit liquidity traps: helicopter drops of money, deliberately unbacked fiscal expansions, i.e. issuing debt that the government commits to inflate rather than repay.

But here too, proposals that fix a liquidity trap do not rule out the trap or the equilibria leading to the trap. If the government successfully exits a liquidity trap, that trap, and the inflation path leading to it, remain a valid equilibrium. The fix makes matters worse, in fact, because now there is less reason to discount the multiple equilibrium. The multiple equilibrium leads back to where it started, so it is now locally bounded. To rule out the trap, and equilibria leading to it, one must specify an inconsistent policy; a policy regime so that no equilibrium can form. It is the inconsistent policy, not the trap-exit policy, that does the work.

Benhabib, Schmitt-Grohe, and Uribe (2002) specify that in low-inflation states, near \( \Pi_L \) of Figure 16.1, the government abandons the passive fiscal assumption. It lowers taxes, real debt grows explosively, the consumer’s transversality condition is violated, and the government debt valuation equation no longer holds at the original low price level. Specifically, [their equations (18)-(20)] in a neighborhood of \( \Pi_L \), the government commits to surpluses \( s_t = \alpha(\Pi_t)(B_{t-1}/P_t) \) with \( \alpha(\Pi_L) < 0 \) in place of a passive rule such as \( s_t = r/(1 + r)B_{t-1}/P_t \). They also suggest a target for the growth rate of nominal liabilities, a “4% rule” for nominal debt. If deflation breaks out with such a commitment, real debt will then explode, violating the consumer’s transversality condition. Woodford suggests this implementation as well (p. 132):
“let total nominal government liabilities \( D_t \) be specified to grow at a constant rate \( \bar{\mu} > 1 \) while monetary policy is described by the Taylor rule ...” “Thus, in the case of an appropriate fiscal policy rule, a deflationary trap is not a possible rational expectations equilibrium.”

These proposals are inspired by sensible and time-honored prescriptions to inflate the economy out of a liquidity trap. Benhabib, Schmitt-Grohé, and Uribe (2002) describe them this way (p. 548):

... this type of policy prescription is what the U.S. Treasury and a large number of academic and professional economists are advocating as a way for Japan to lift itself out of its current deflationary trap...A decline in taxes increases the household’s after-tax wealth, which induces an aggregate excess demand for goods. With aggregate supply fixed, price level must increase in order to reestablish equilibrium in the goods market.

Zero interest rates and $1.5 trillion deficits soon followed in the 2008 recession. This quote is, indeed, how a coordinated fiscal-dominant regime works, it is good intuition for operation of the fiscal theory of the price level, and undoubtedly what real-world proponents of these policies have in mind.

But that’s not their, or Woodford’s, proposal. The proposal does not “lift the economy out of a deflationary trap” back to \( \Pi^* \). Their proposal sits at \( \Pi_L \) with an uncoordinated policy and lets government debt explode. If their proposal did successfully steer the economy back to \( \Pi^* \) then the whole path to \( \Pi_L \) and back would have been an equilibrium. Benhabib, Schmitt-Grohé and Uribe change tax policy while also maintaining the Taylor rule \( \Phi(\Pi) \) and the dynamics of Figure 16.1. The government switches to an active-fiscal regime, which demands higher inflation, while simultaneously keeping the interest rate rule in place, which demands continued low inflation. The transversality condition is a consumer optimality condition. The government is trying to set policy parameters for which consumer optimality conditions cannot hold.

### 16.4.3 Threaten negative nominal rates

Why not just threaten substantially negative nominal rates – remove the lower equilibrium \( \Pi_L \), keep the Taylor rule going throughout the negative interest rate range. That would violate first order conditions – infinite money holdings – and forbid
deflationary equilibrium from forming! This is a logically interesting possibility – why insist that the government accommodate the first order condition of money vs. bonds, but then add specifications that violate first order conditions in other dimensions?

Once we see that central point, that the government or central bank eliminates multiple equilibria by threatening policies for which “no equilibrium can form,” we can think of many monetary-fiscal policies that preclude deflationary equilibria equivalently and more transparently. If inflation gets to an undesired level, tax everything. Burn the money stock. Introduce an arbitrage opportunity.

Cleanest of them all, specify a \( \Phi(\Pi) \) function that includes negative nominal interest rates – just eliminate the \( \Pi_L \) equilibrium in the first place by straightening out the policy rule in Figure 16.1. Bassetto (2004) suggests this option. Since there can be no equilibrium at negative nominal rates, which introduce an arbitrage opportunity between debt and money, such a \( \Phi(\Pi) \) function works exactly the same way to rule out equilibria: In a deflationary state, the government commits to a policy that allows no equilibrium. Negative nominal rates are no more or less possible than letting debt explode, or running a commodity standard or money growth rule with an inconsistent interest rate target. In retrospect, why demand a Ramsey approach in setting up the problem – the policy rule must not prescribe negative nominal rates, because that would violate consumer optimality conditions – and then patch it up with policy prescriptions that deliberately do violate optimality conditions? Why not just commit to negative nominal rates that violate first order conditions in the first place?

Well, it’s fairly clear that the central bank can’t do this. But it is no harder to threaten negative nominal interest rates than it is to threaten the other uncoordinated policies. This policy just does not sneak in on the coattails of a stabilization policy.

\subsection{Weird Taylor rules}

The Fed could threaten to blow up the economy by setting inflation to infinity above some value.

\text{Woodford (2003)} suggests (p.136) a stronger policy rule, that the graph in Figure 16.1 becomes vertical at some finite inflation \( \Pi_U \) above \( \Pi^* \), that the central bank will set an infinite interest rate target in finite time. Similarly, Alstadheim and Henderson
(2006) remove the $\Pi_L$ equilibrium by introducing discontinuous policy rules, or V-shaped rules that only touch the 45° line at the $\Pi^*$ point. Bassetto (2004), mentioned above, suggests that the policy rule ignore the $i \geq 0$ bound and promise negative nominal rates in a deflation.

These proposals blow up the economy directly, and in finite time. And if one grants the idea that the central bank follows $\phi > 1$ and promises ever increasing inflation or deflation as a selection device, they make sense. If $\phi > 1$ isn’t quite enough to eliminate equilibria, then turn up the volume a notch. Hyperinflating away the entire monetary system ($\Phi(\Pi)$ becoming vertical), introducing an arbitrage opportunity (allowing $i < 0$ in the policy rule), and so forth remove these equilibria more effectively than an inflation that slowly gains steam.

But all the problems remain. Just how can a central bank set policy so equilibrium can form, would it do so ex-post, and does anyone believe our central banks do anything like this?

16.4.5 Residual money demand

In monetary economies, the Fed could threaten infinite inflation indirectly, with finite interest rate targets.

Schmitt-Grohé and Uribe (2000) and Benhabib, Schmitt-Grohé, and Uribe (2001) offer a similar way to rule out hyperinflations. They add money, in such a way that the economy explodes to infinite inflation, despite finite interest rates. This idea is also reviewed by Woodford (2003) (p. 137), and has long roots in the literature on hyperinflations with fixed money supply and interest-elastic demand.

The idea is easiest to express with money in the utility function, $u(C_t)$ becomes $u(C_t, M_t/P_t)$. With money and a constant endowment $C_t = Y$, the intertemporal first-order condition becomes:

$$1 + i_t = \Pi_{t+1} \frac{u_c(Y, M_t/P_t)}{\beta u_c(Y, M_{t+1}/P_{t+1})} = \Pi_{t+1} (1 + r_t),$$

(16.26)

where $r_t$ denotes the real interest rate. (This is a perfect foresight model, so the expectation is missing. We’ll see this model in detail in section 17.2) Suppose the policy rule is

$$1 + i_t = \frac{1}{\beta} \Phi(\Pi_t).$$
Substituting $i_t$ from this policy rule into (16.26), and expressing the money $u_m$ vs. consumption $u_c$ first order condition as $M_t/P_t = L(Y, i_t)$, inflation dynamics follow

$$
\Pi_{t+1} = \Phi(\Pi_t) \frac{u_c[Y, L(Y, \Phi(\Pi_{t+1}))]}{u_c[Y, L(Y, \Phi(\Pi_t))]} \tag{16.27}
$$

instead of (16.14),

$$\Pi_{t+1} = \Phi(\Pi_t).$$

The difference equation (16.27) may rise to require $\Pi_{t+1} = \infty$ above some bound $\Pi_U$, even if the policy rule for nominal interest rates $1 + i_t = \Phi(\Pi_t)/\beta$ remains bounded for all $\Pi_t$. Woodford and Schmitt-Grohé and Uribe give examples of specifications of $u(C, M/P)$ for which this situation can happen.

Is this the answer? First, if we do not regard a belief that the central bank will directly blow up the economy ($i_t$ rises to $\infty$) as a reasonable characterization of expectations, is it that much more plausible why would people believe that the central bank will to take the economy to a state in which the economy blows up all on its own? Infinite inflation and finite interest rates mean infinitely negative real rates; a huge monetary distortion. Surely the central bank would notice that real interest rates are approaching negative infinity! And the point is, this should not happen by accident. The central bank should be threatening this outcome as its way to engineer determinacy.

Second, it is delicate. This approach relies on particular behavior of the utility function or the cash-credit goods specification at very low real balances. Are monetary frictions really important enough to rule out inflation above a certain limit, sending real rates to negative infinity, or to rule out deflation below another limit? We have seen some astounding hyperinflations; real rates did not seem all that affected.

Sims (1994) pursues a similar idea. Perhaps there is a lower limit on nominal money demand. Everyone keeps one last dollar bill around, no matter how low the price level. Then real money demand explodes in a deflation violating the transversality condition, and ruling out a perpetual deflation as an equilibrium.

But perhaps not; perhaps the government can print any number it wants on bills, or will run periodic currency reforms; perhaps real money demand is finite for any price level. Perhaps once it becomes worth a billion of today’s dollars, people will indeed try to spend that one last dollar bill.

In sum, these proposals require two things: First, they require expectations that the government will follow the Taylor rule to explosive hyperinflations and deflations.
Second, they require belief in a deep-seated monetary non-neutrality sufficient to send real rates to negative infinity or real money demand to infinity, though such events has never been observed.

16.5 Identification and threats

The parameter $\phi$ in the new-Keynesian model is not identified. It represents an off-equilibrium threat that is never seen in equilibrium. In the equilibrium of the model driven by an AR(1) disturbance, equilibrium inflation and interest rates follow $i_t = r + \rho \pi_t$. A Taylor rule regression recovers $\rho$, not $\pi$. In the rule $i_t = r + \phi \pi_t + v_t$, the right hand variable $\pi$ is perfectly correlated with the error term $v$. $\phi$ does not appear in the likelihood function. Writing the rule in form $i_t = i_t^* + \phi (\pi_t - \pi_t^*)$, in equilibrium $\pi = \pi^*$ there is no movement in the right hand variable needed to measure $\phi$. For theory, lack of identification means that agents in the model cannot observe the central bank’s threat to hyperinflate and thereby rule out equilibria.

The parameter $\phi$ in the new-Keynesian model is not identified. This is an important point for both theory and empirical work. For theory, given that central banks do not make threats to hyperinflate, or set policy so that equilibrium cannot form, one might say that people learn the value of $\phi$ from experience. That $\phi$ is not identified means they cannot learn it. For empirical work, that $\phi$ is not identified clearly dooms efforts to tie different behavior of inflation to different values of $\phi$, or to test for $\phi > 1$.

To see this point, return to the simplest model of section 16.1,

\begin{align}
    i_t &= r + E_t \pi_{t+1} \\  
    i_t &= r + \phi \pi_t + v_t \\  
    v_t &= \rho v_{t-1} + \varepsilon_t.
\end{align}

Suppose the solution

\begin{align}
    \pi_t = \frac{-v_t}{\phi - \rho}
\end{align}

is in fact correct, what are its observable implications? Since inflation $\pi_t$ is proportional to the disturbance $v_t$, the dynamics of equilibrium inflation are simply those of the disturbance $v_t$,

\begin{align}
    \pi_t = \rho \pi_{t-1} + w_t.
\end{align}
The error $w_t$ is the error of a regression of $\pi_t$ on past $\pi$. It is related to the structural interest rate shock by $w_t \equiv -\varepsilon_t/(\phi - \rho)$, but $\varepsilon_t$ is not directly observed, so we can’t infer $\phi$ or $\varepsilon$ from this relation. All we know is that $\pi_t$ follows an AR(1) with coefficient $\rho$.

Using (16.28) or (16.29), we can find the equilibrium interest rate,

$$i_t = r + \rho \pi_t.$$  \hfill (16.32)

A regression of $i_t$ on $\pi_t$ will estimate the disturbance serial correlation parameter $\rho$ rather than the Taylor rule parameter $\phi$.

What happened to the policy rule, (16.29)? The solution (16.30) shows that the right hand variable $\pi_t$ and the disturbance $v_t$ are correlated – perfectly negatively correlated in fact.

Is there nothing clever we can do? Some instrument we can use? No. The equilibrium dynamics of the observable variables $\{i_t, \pi_t\}$ are completely described by equations (16.31) and (16.32). These equilibrium dynamics, and the resulting likelihood function, do not involve $\phi$. $\phi$ is not identified from data on $\{i_t, \pi_t\}$ in the equilibrium of this model.

The identification point is clearest by writing the policy rule in the equivalent form

$$i_t = i_t^* + \phi (\pi_t - \pi_t^*)$$

where $\pi_t^*$ is inflation in the equilibrium the central bank wishes to select, and $i_t^* = v_t + \phi \pi_t^*$ is the equilibrium interest rate. With $\phi > 1$, $i_t = i_t^*$ and $\pi_t = \pi_t^*$ are the observed values. In this form, you see there is no variation in the right hand variable $\pi_t - \pi_t^*$ of the regression that identifies the policy rule. In equilibrium, the threat to deviate from that equilibrium is not observed.

The monetary policy disturbances $v_t$ are also not identified and thus not measurable. If you could learn $\phi$, you could infer $v_t = i_t - \phi \pi_t$. If you could see the disturbance $v_t$, you could measure $\phi$ by $\phi = (i_t - v_t)/\pi_t$. Previous plots of the inflation, output and interest rate response to monetary policy disturbances $v_t$ are plots of responses to an un-measurable quantity. This is one reason I emphasized plotting and thinking about responses to interest rates, rather than responses to monetary policy disturbances.

These are deep and simple economic points, not subtle econometric problems. Inflation is supposed to jump to the one value for which accelerating inflation at rate $\phi$
is not observed. If inflation does jump to that value, there is no way to measure how
fast the inflation would accelerate if it did not jump. We cannot measure, and agents
cannot learn, off-equilibrium threats from data drawn from equilibrium. Once we
rule out inflationary paths, just how fast the inflation comes about on those paths is
irrelevant to anything.

The central bank’s threat to hyperinflate for any but its desired equilibrium is like
a parent’s successful threat that if the children don’t eat spinach, there won’t be
dessert. If it works, we observe (spinach, dessert). We never see off-equilibrium
threats (no spinach, no dessert) from data drawn from equilibrium.

The fact that $\phi$ is not identified bears on theory. It means that agents in the model
have no way of learning $\phi$, any more than we econometricians looking at data and
the model can do. We must imagine that the central bank announces this threat,
and commits to it, and people believe it, despite the fact that executing the threat
ex-post is horribly damaging to the central bank’s objectives. Again, our central
banks announce nothing of the sort.

The fact that $\phi$ is not identified bears on empirical work. Clarida, Galí, and Gertler
(2000) is perhaps the most famous piece of evidence in favor of the new-Keynesian
framework. They estimate policy rules, and find $\phi < 1$ before 1980, and $\phi > 1$
afterwards. They interpret this finding in terms of the new-Keynesian model, so
that $\phi < 1$ means multiple-equilibrium volatility, and $\phi > 1$ means determinacy,
which should reduce the volatility of inflation. Read carefully – this is not the
conventional wisdom that $\phi < 1$ meant instability and $\phi > 1$ restored stability.
That’s old-Keynesian. Indeed, they write (p. 150)

the pre-Volcker rule leaves open the possibility of bursts of inflation
and output that result from self-fulfilling changes in expectations... On
the other hand, self-fulfilling fluctuations cannot occur under the esti-
rated rule for the Volcker-Greenspan era since, within this regime, the
Federal Reserve adjusts interest rates sufficiently to stabilize any changes
in expected inflation.

They regard the lower volatility of inflation after 1980 as confirming evidence for the
model. (The last sentence is revealing. In their model, the Federal Reserve adjusts
interest rates to destabilize expected inflation. “Stable” can be taken to mean “less
volatile.” But it also harks back to old-Keynesian intuition, which does not describe
the model. This is all only clear in retrospect. See Cochrane (2011a) for a detailed
review.)
But $\phi$ is not identified in their model. Their regressions, on artificial data generated by their model, cannot not produce $\phi$. The coefficient $\phi$ represents an off equilibrium threat not seen in equilibrium. This greatest of all estimates does not, in fact, provide evidence in favor of the new-Keynesian model.

Now, identification is a property of a model, not of data. Their regressions measure something. In my example they measure $\rho$. Similar regressions can identify the parameter $\phi$ in old-Keynesian models. One may interpret the \cite{ClaridaGalíGertler2000} estimate in the light of the old-Keynesian model, to say that Fed brought inflation stability from instability, thereby conquering inflation in the 1980s, not determinacy from indeterminacy. Add a slightly different introduction, and all is well. But the regressions do not identify the $\phi$ of the new-Keynesian model, and we cannot take them as evidence for $\phi > 1$ in the later period, which was their objective.

One may object that \cite{ClaridaGalíGertler2000} found an estimate greater than one, where in my example $\rho < 1$. But an estimate greater than one is entirely possible in the full new-Keynesian model, though $\phi$ still does not measure the policy rule parameter. \cite{Cochrane2011a} gives an example.

This lack of identification pervades new-Keynesian empirical work. For example, the \cite{SmetsWouters2007} new-Keynesian model restricts the estimate of $\phi$ a-priori to be greater than one. The prior and posterior for the inflation response of monetary policy $\phi_\pi$ are nearly identical (Figure 1C p. 1147). The estimate is 1.68 relative to a prior mean of 1.70, suggesting that the policy rule parameters are at best weakly identified, even in a local sense.

One can of course identify anything by sufficient assumptions. For example, \cite{GiannoniWoodford2005} identify the policy rule parameters by assuming 1) The monetary policy disturbance $\varepsilon_t$ is i.i.d. and not predictable by any variables at time $t - 1$; 3) The Fed does not react to expected future output, or wage, price inflation, or other state variables; 3) Wages, prices, and output are fixed a period in advance. These are all unrealistic assumptions. Disturbances are persistent. Central banks deviate from rules for years at a time. The Fed reacts to expectations about the future, and wages and prices move within a quarter. More deeply, the logic of the new-Keynesian model is that some state variable must jump with any shock, jumping the economy to the unique equilibrium that now (after the shock) does not explode, just as $\pi_t$ jumps coincident with $v_t$ in the simple model. If inflation $\pi_t$ cannot jump, say if it is fixed one quarter in advance, then some other state variable must jump. \cite{GiannoniWoodford2005} assume that the central bank does not respond to
that state variable.

More generally, one must achieve identification by tying the un-measurable, unobserved behavior $i_t - i^*_t = \phi(\pi_t - \pi^*_t)$ to something observable. We could assume that if our parent has a glass of wine with dinner, then no spinach will be followed by no dessert, and with that assumption make the latter threat measurable. But there is no way to verify the assumption. And since the economic function of the correlation in equilibrium between interest rate and inflation is utterly different from the response of interest rates to inflation used to induce an explosive equilibrium, I can’t think of a reason to make such an assumption. Assumptions about unobservables are difficult.

The parameter $\phi$ is identified if there is no disturbance in the policy rule $i_t = \phi\pi_t$, and if there are shocks to other equations leading to some volatility in the right hand variable $\pi_t$. This assumption ties unobservable behavior to observable behavior, by assuming that the off-equilibrium reaction $(i_t - i^*_t) = \phi(\pi_t - \pi^*_t)$ is the same as the on-equilibrium relation $i^*_t = \phi\pi^*_t$. But there really is no reason to make such an assumption. More importantly, in reality there are always disturbances – no rule fits with 100% $R^2$.

We might then try to assume that suppose that monetary policy disturbances $v_t$ are orthogonal to the other equation’s disturbances, and suppose we could measure the latter. (Giannoni and Woodford (2005) are a case of this general idea.) In $i_t = \phi\pi_t + v_t$, that could give us an instrument, a movement in $\pi_t$ orthogonal to the shock $v_t$. But why should the central bank not respond to other shocks, especially if we and hence they can measure such shocks? Optimal monetary policy (minimizing output and inflation variance) directs the central bank to respond, to set $v_t$ in response to other shocks in the economy. Written in the equivalent form $i_t = i^*_t + \phi(\pi_t - \pi^*_t)$, the “stochastic intercept” of the policy rule should respond to other shocks. In fact central banks clearly describe all of their actions, especially deviations from policy rules, as responses to other shocks. And we are still assuming that the off-equilibrium response is equal to the on-equilibrium correlation.

One may respond that “well, all identification involves assumptions,” which is true. But most of the time in economics we are trying to identify things that are in principle measurable. Identifying a supply curve, for example, is hard because there are both supply and demand shocks. If the supply shocks would be quiet for a minute, or if we could isolate demand shocks that do not move the supply curve, we could measure that curve. Here we are trying to measure something that is inherently unmeasurable. The identifying assumptions must tie it to something that is measurable, which is a
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16.6 Discovery

How can such a large and long literature be confused on such basic points? These points were not so obvious in advance. The distinction between stability and determinacy is subtle. That central banks do not stabilize inflation, but instead destabilize it to fight multiple equilibria is so unlike intuition and central bank statements, that it was hard to recognize in the equations. The new-Keynesian model was developed on the hope that it would deliver ISLM intuition. Recognizing that the equations have totally different intuition is therefore even harder. We can see this tension clearly in the proposals to trim equilibria, which are built on sensible policies to stabilize inflation, rather than make direct and clear blow up the world threats. Active policy itself takes a very sensible policy in old-Keynesian model and turns it into a hyper-inflationary equilibrium-selection threat. That it has this new and different role was not clear. But now that the distinction is clear, we should recognize just how the equations of the new-Keynesian model behave.

This literature on pruning multiple equilibria in monetary models goes back a long way. We will go back to Obstfeld and Rogoff (1983, 1986), and as above it continues to this day.

In retrospect, it seems strange that such a large and long literature could appear so confused on such basic points. This is, however, understandable, and an interesting reflection on the history and philosophy of science. Ideas start confused and get slowly simple over time. It takes a lot of reflection and debate to understand what equations really say.

The equilibrium-pruning debate is a part of the more general 40-year (and counting) process of figuring out that new-Keynesian rational expectation models are, in their equations, quite different from old-Keynesian adaptive expectation models: An interest rate peg is stable but indeterminate, not unstable and determinate; Active interest rate policy $\phi > 1$ makes a stable economy unstable to render it locally determinate, it does not make an unstable economy stable; Active policy $\phi > 1$ threatens severe consequences to rule out multiple equilibria; Active policy $\phi > 1$ does not bring inflation back under control; The central bank’s role is centrally equilibrium...
selection, not inflation stabilization; and so forth. These properties of the models are diametrically opposed to old-Keynesian intuition. The goal of the research program was to write down respectable models (optimizing agents, budget constraints, rational expectations, intertemporal equilibrium) that would produce old-Keynesian intuition. So it is not surprising that it took authors a long time to realize what the equations are really saying.

That process is especially visible in policy prescriptions to trim multiple equilibria. If the point is equilibrium selection, making a blow-up-the-world, equilibrium-can’t form threat to rule out multiple equilibria, why build that threat in a subtle transition period in an otherwise sensible existing policy idea, advocated to cure inflation or deflation? Well, clearly, the distinction between “stabilize inflation,” or “stop a hyperinflation or deflation,” and “rule out an inflationary equilibrium in the first place” was not clear.

The idea that the central bank does not want to cure or to stabilize inflation, but instead set policy so “equilibrium cannot form,” is so foreign, it’s not surprising it took a long time to recognize it in the equations of the model. It is natural for researchers, recognizing that speculative hyperinflations and deflations are a problem in the model, knowing many commonsense ideas such as switching to a money growth rule are standard cures to stop inflations or deflations, to start with those ideas, unwittingly put the one-period overlap of inconsistent policy into the model, notice that the equations rule out the inflation equilibrium in a rational-expectations model, and declare success. The above quote by Benhabib, Schmitt-Grohé, and Uribe (2002), at variance with the equations of their model, provides an excellent example.

Why didn’t these authors follow the much simpler approach as in Atkeson, Chari, and Kehoe (2010)? Why not just specify that if inflation deviates from the desired equilibrium, the central bank immediately blows up the world? Done, equilibrium ruled out. Why did that paper come so late in the game, and why is it so hard too? Well, because that idea is so obviously silly as a description of our world. Central banks don’t do that. They don’t want to blow up the world. They want to stabilize inflation. You can see that only a proposal which seems stabilizing but hides a blow-up-the-world threat deep in the equations where it is hard to see will survive authors’ own searches for a good model, to say nothing of readers’ evaluations.

Similarly, why insist that central banks must respect the zero bound, and then add a different policy specification that means agents can’t be on first order conditions, rather than follow Bassetto (2004), and directly threaten negative interest rates, so equilibrium cannot form? Well, it’s clear central banks can’t do that or threaten
it credibly. But then they can’t make the other blow-up-the-world threat. It only makes sense if the distinction between stopping inflation and ruling out equilibria is still a bit fuzzy.

The active policy itself is part of this discovery process. The active \( i_t = \phi \pi_t, \phi > 1 \) policy was developed in the old-Keynesian context, to stabilize inflation. That rule is the same in new-Keynesian models – it is the change in the rest of the model that alters its role from stabilizing and inflation control to destabilizing and equilibrium selection. It’s natural not to notice that one is assuming radically different central bank behavior, by using the same equation. Playing with new-Keynesian models, one discovers \( \phi > 1 \) leads to explosions in the multiple equilibria. It’s easy to just rule out explosive equilibria without thinking too hard about just why, and presto, the theory is complete. I long regarded transversality conditions and such as pointless technicalities and didn’t pay too much attention as well. I was wrong. It is natural to think \( \phi > 1 \) just expresses “stabilization” in the new model. That \( \phi > 1 \) works exactly the opposite way, to destabilize, to threaten to blow up the world slowly by hyperinflation – a threat that then needs shoring up by really threatening to blow up the world – is so radical compared to intuition, and to the explicit goal of this literature of justifying old-Keynesian intuition, that it is completely understandable that it took quite some time for researchers to realize just what the equations really mean.

Another example shows the discovery process. The original Taylor rule was designed to describe how the central bank behaves, and as such includes output responses,

\[
i_t = \phi \pi_t + \phi_x x_t + v_t^i. \tag{16.33}
\]

Empirical rules, designed to be even more realistic, include inertia and responses to expected values,

\[
i_t = \rho i_{t-1} + \phi \pi_t + \phi_x x_t + \phi_{\pi,1} E_{t-1} \pi_{t+1} + \phi_{x,1} E_{t-1} x_{t+1} + v_t^i. \tag{16.34}
\]

One goes on to model how the disturbance \( v_t^i \) responds to structural shocks in the economy, as optimal policy (Section 16.9.3 below) recommends. These Taylor rules then morphed into a recommendation how the central bank should behave, and these sorts of rules make lots of sense in old-Keynesian, stabilizing models.

In new-Keynesian models, Taylor rules must still make lots of sense as descriptions of the correlations between observed, equilibrium, quantities, say

\[
i_t^* = \phi \pi_t^* + \phi_x x_t^* + v_t^i. 
\]
But there is no reason this last expression should generate explosive eigenvalues and prune deviations from the $\ast$ equilibrium. Writing deviations, or the equilibrium-selection rule as

$$i_t - i_t^* = \tilde{\phi}_\pi (\pi_t - \pi_t^*) + \tilde{\phi}_x (x_t - x_t^*),$$

or, putting it all together,

$$i_t = \left[ \phi_\pi \pi_t^* + \phi_x x_t^* + \nu_t^* \right] + \tilde{\phi}_\pi (\pi_t - \pi_t^*) + \tilde{\phi}_x (x_t - x_t^*),$$

and given that we never observe deviations in equilibrium, there is no reason to choose the equilibrium-selection threat equal to the equilibrium correlation, $\tilde{\phi}_x = \phi_x$. Equilibrium selection policy should be as stark – to “coordinate expectations” – as possible. Well, this takes time. If you plug (16.33) into the model, which usually requires numerical solution, and (of course) rule out explosions, you get a unique solution and pleasant-looking response functions. Why look harder? That complex, old-Keynesian, observed rules morph into complex, new-Keynesian, unobserved off-equilibrium threats shows that it just took a long time to realize what the equations were actually telling us.

(The parameter regions that generate explosive eigenvalues in rules like (16.34) can become quite complex – $\phi_\pi > 1$ is neither necessary nor sufficient. Cochrane (2011c) characterizes a number of such regions. The lesson that $\phi_\pi < 1$ and oscillating inflation rules out equilibria includes many other possibilities.)

So do not read old papers harshly, or my conclusion that they are fundamentally wrong as criticism of the authors. It has taken me twenty years to figure out what I now think these equations are telling us, and you will see many confusions in my early papers too.

But, now we do understand what the equations mean. And I can only conclude that all of these efforts to trim multiple equilibria without active fiscal policy have failed. The natural economic model gives us an equation that determines the price level, the government debt valuation equation. If we throw out that equation by assuming globally passive policy (fiscal responses to off-equilibrium as well as on-equilibrium inflation), that equation can’t be replaced, and we lose the ability to determine one endogenous variable, the price level.
16.7 Response to monetary policy in the simple model

The new-Keynesian, old-Keynesian, and fiscal theory of monetary policy models also give quite different pictures of how the economy responds to monetary policy. The nature of these responses is revealing about the logic of the models. Where observational equivalence theorems apply, the quite different mechanisms new-Keynesian and FTPL models use to produce the same results tell us a lot about how the models work.

I introduced the simple new-Keynesian model monetary policy response in section 4.4. We return here to look at the new-Keynesian model and understand its logic more deeply.

16.7.1 The standard response to AR(1) shocks

The frictionless model with \( i_t = \phi \pi_t + v_t \) and AR(1) process for \( v_t \) produces AR(1) responses,

\[
\pi_t = -\frac{1}{\phi - \rho} v_t; \quad i_t = -\frac{\rho}{\phi - \rho} v_t.
\]

These are plotted in Figure 4.2. A positive shock to \( v \) gives a negative inflation response, but actual interest rates move with inflation. Inflation jumps immediately and contemporaneously with the shock to \( v \). For \( \rho = 1 \), the response is super-neutral. Inflation equals the interest rate immediately. For \( \rho = 0 \) we have an open-mouth policy: Inflation moves with no movement in interest rates at all. The inflation response is centrally about equilibrium selection.

Let us start with the simple linearized frictionless model of section 16.1

\[
i_t = E_t \pi_{t+1} \]
\[
i_t = \phi \pi_t + v_t \]
\[
v_t = \rho v_{t-1} + \varepsilon_t.
\]

(16.35)

Though much too simple, it turns out this model will capture all the intuition of more realistic, but much more algebraically complex models to follow.
From (16.6), with $\phi > 1$, the standard equilibrium choice is

$$\pi_t = -\frac{1}{\phi - \rho} v_t.$$

(16.36)

$$i_t = -\frac{\rho}{\phi - \rho} v_t$$

We met this response in section 4.2, and graphed in the bottom lines of Figure 4.2. Now we take a closer look.

Figure 16.2 graphs this response function, with parameters $\phi = 1.25; \rho = 0.8$.

![Figure 16.2](image)

Figure 16.2: response of the new-Keynesian model to a monetary policy shock $v_t$. Dashed lines give alternative equilibria.

In response to a positive shock to the disturbance $v_t$, that disturbance dies out slowly. Inflation $\pi_t$ declines immediately, and then reverts slowly following (16.36). So far the model looks promising relative to priors that tighter monetary policy should lower inflation.

(An impulse-response function plots the response of $\{(E_1 - E_0) x_t\}$ to a shock such as $(E_1 - E_0) v_1$. It is therefore equivalent to plotting the response of a perfect-foresight version of the model to a single shock $\varepsilon_1$, starting at zero, and with all other shocks...
set to zero. We are then plotting the path of expected variables \(\{E_1 x_t\}\). Since you can’t expect the unexpected, indeterminacy and equilibrium selection only matter for the time 1 response. \(E_1 [(E_2 - E_1) x_t] = 0.\)

The interest rate response is a first sign of trouble. The actual interest rate declines throughout the episode. What kind of monetary policy tightening is this that interest rates go down? The policy rule (16.35), \(i_t = \phi \pi_t + v_t\), explains. Though the disturbance \(v_t\) is positive, inflation \(\pi_t\) declines so much, and with \(\phi > 1\), that the endogenous part of the policy rule \(\phi \pi_t\) overwhelms the disturbance \(v_t\). The episode is a positive monetary policy disturbance because the interest rate \(i_t\) fell less than the rule \(\phi \pi_t\) prescribels.

You have to know \(\phi\) to infer there was a positive shock \(v_t\). You have to see the decline in interest rates is less than the decline in inflation, and you have to know that the rule specifies an interest rate decline greater than the inflation decline, to infer that the central bank is deviating from its rule. Yet \(\phi\) is not identified, and you can’t see inflation until everyone else has acted anyway. So we must imagine that the central bank announces the positive monetary policy disturbance \(v_1\), along with its rule \(\phi\). On hearing the announcement, price-setters drop their prices so that \(\pi_1\) falls.

Inflation jumps down immediately. We cannot interpret the monetary policy shock in the usual old-Keynesian way – the central bank raises interest rates, this rise causes inflation to fall subsequently, and perhaps falling inflation then drags interest rates back down with it through the rule. The model is all about inducing an immediate inflation jump.

Why does inflation jump, and to the specific value \(\pi_1 = -1/(\phi - \rho)\varepsilon_1\)? Because the central bank threatens hyperinflation or deflation for any other value, and we have ruled out such equilibria. To see this point, the dashed lines in Figure 16.2 graph what would happen if inflation \(\pi_1\) jumped to different values, slightly higher or lower. Any of these jumps are consistent with private sector behavior, which only ties down \(E_0 \pi_1 = i_0 = 0\). But following dynamics \(E_1 \pi_{t+1} = \phi E_1 \pi_t + E_1 v_t\) induced by the central bank’s policy rule, these equilibria spiral away. If inflation did not jump at all – if \(\pi_1 = E_0 \pi_1 = 0\) – then the central bank inflates very quickly as shown. These are the threats of inflation and deflation of section 16.3.2.

Equilibrium-selection policy is central to this model. The jump, the negative response of inflation to the monetary policy disturbance, is entirely a result of the central bank’s equilibrium-selection threats in this model, not supply and demand within an equilibrium. This immediate inflation jump is what causes inflation to be perfectly correlated with the disturbance in \(i_t = \phi \pi_t + v_t\), and thus for the policy rule parameter
not to be identified. In more complex models inflation itself may not jump, but some other state variable does. There is always a jump in a state variable in a new-Keynesian model.

If $\rho = 1$, we have

$$\pi_t = i_t = -\frac{1}{\phi - 1} v_t.$$ 

The observable movement in response to a permanent unanticipated interest rate rise is super-Fisherian. Not only does the change in interest rates change expected inflation one for one, $E_1 \pi_2 = i_1$, but inflation on the date of the shock, time 1, jumps immediately to equal the new higher interest rate. We will see this possibility emerge even with more complex models.

If $\rho = 0$, we have

$$\pi_1 = -\frac{1}{\phi} v_1; \pi_t = 0, \ t > 1$$

$$i_t = 0.$$ 

The announcement of the shock produces a one-period jump in inflation – and no movement at all in interest rates!

Dynamics in this model for $\rho > 0$ come from the exogenous dynamics of the forcing process $v_t$, not from a delayed response of the economy to the monetary policy shock. This is an important point going forward – a dynamic response, such as that displayed in Figure 4.2, can come from either source. People often misread impulse-response functions entirely as delayed economic responses to the initial shock, not as a sequence of instantaneous responses to a dynamic disturbance. This model, with exactly the opposite interpretation, reminds us of the other possibility.

Most of all, this is the first example of an “open-mouth” policy, in which the central bank can move inflation by simply making an announcement and taking, in equilibrium, no concrete action. In this example, merely by stating its wish that it be so, the central bank engineers an instant and permanent change in the price level. The possibility of open-mouth policy emphasizes the importance of off-equilibrium threats as the centerpiece of how monetary policy affects the economy in this model. (In an “open mouth” operation, the central bank only makes announcements and takes no concrete action, yet markets move. New Zealand Reserve Bank Governor Donald Brash coined the term, Brash (2002), referring to his apparent ability to move interest rates without taking any concrete action.)
As a reminder of the discussion in section \[4.4\] the government debt valuation equation is still in this model. Fiscal policy is assumed passive, usually that the government will change lump-sum taxes to adapt to any price level, both in the desired equilibrium and alternative equilibria. The jump in inflation $\pi_1$ must give rise to a jump in the present value of surpluses. The fiscal theorist looking at this model would say that the central bank, by its announcements, convinced the Treasury to embark on a contractionary fiscal policy. That contraction provides an aggregate-demand story for the change in inflation $\pi_1$, in place of the new-Keynesian “coordination” on one of many equilibria. Absent the fiscal contraction, inflation would not move. As we saw in section \[4.4\] this new-Keynesian response to a monetary policy shock is observationally equivalent to a fiscal theory of monetary policy response to that fiscal shock.

These results seem extreme and a bit nutty, and one may worry they are peculiar to the extremely simplified model. Surely adding price stickiness will give a more normal response? The answer is no, which we will verify in a bit by looking at those more complex models. This fact makes seeing the strange behavior here, in simplest form, worthwhile.

### 16.7.2 Inflation targets, equilibrium-selection policy, and open-mouth operations

Writing the policy rule as $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ with $i_t^* = E_t\pi_{t+1}^*$ clarifies how the model works. The central bank can achieve any $\{\pi_t^*\}$ it wants. Policy has two distinct parts: interest rate policy $i_t^*$, which determines expected inflation, and equilibrium-selection policy $\phi(\pi_t - \pi_t^*)$ which threatens hyperinflation or deflation to select unexpected inflation. Open-mouth policy here is a one-period movement in the inflation target $\pi_t^*$.

To see the centrality of announcements and equilibrium-selection policy in this model, it is useful again to write the policy rule in the equivalent form

$$i_t = E_t\pi_{t+1}^* + \phi(\pi_t - \pi_t^*) \tag{16.37}$$

or equivalently

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \tag{16.38}$$

with $i_t^* = E_t\pi_{t+1}^*$ and $\phi > 1$. 
The equilibrium condition becomes
\[
E_t (\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t).
\]
Clearly, with \( \phi > 1 \) the only locally bounded equilibrium is \( \pi_t = \pi^*_t \). We can reach this conclusion quickly in this case, for any process of the monetary policy disturbance, not just an AR(1). One can translate between the two representations of the rule, and the choice \( \{v_t\} \) vs. an inflation target \( \{\pi^*_t\} \) by
\[
v_t = E_t \pi^*_{t+1} - \phi \pi^*_t.
\]
With this parameterization, it’s clear that the central bank can achieve any value of inflation it wishes. All the central bank has to do is to announce its inflation target, announce its threat to hyperinflate or deflate for any other value of inflation, and the private sector jumps to the equilibrium \( \pi^*_t \) represented by the central bank’s inflation target.

Here too we can see “open mouth” operations to control inflation without moving interest rates. Start at zero inflation and zero expected inflation. Let the central bank announce at time 1, \( \pi^*_1 = -10\% \), but \( \pi^*_2, \pi^*_3, \ldots \pi^*_t = 0 \). The unique locally bounded equilibrium is \( \pi_1 = -10\%, \pi_2 = \pi_3, \ldots \pi_t = 0 \). The interest rate does not move at all. This parameterization makes the announcement clearer than the \( \rho = 0 \) case above. Rather than announce “we’re following \( i_t = \phi \pi_t + v_t \) with \( \phi > 1 \), and we’re doing a one-time transitory shock \( v_1 \),” the central bank announces “we are going to move our inflation target for this period only, and if the economy does not move to our target we will raise or lower interest rates to cause a hyperinflation or deflation.” By merely announcing its wish, the central bank can engineer an unexpected 10% downward permanent price level jump, and never touch interest rates!

The second expression (16.38) breaks the policy rule into an interest rate policy \( i^*_t \) and a separate equilibrium-selection policy \( \phi(\pi_t - \pi^*_t) \). In equilibrium, we observe \( i_t = i^*_t \), and the interest rate policy sets expected inflation. Then the central bank chooses whatever ex-post inflation it wants in a separate and distinct equilibrium-selection policy \( \phi(\pi_t - \pi^*_t) \). The fiscal theorist could interpret the same data as a time-varying interest rate peg \( i_t = i^*_t \) with fiscal policy in charge of equilibrium selection.

(The inflation target \( \pi^*_t \) and interest rate target \( i^*_t \) must respect private sector equilibrium conditions, here \( i^*_t = E_t \pi^*_{t+1} \). One can think of the central bank determining its full inflation target \( \{\pi^*_t\} \), and then implementing that plan with an interest rate
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...target \( i_t^* = E_t \pi_{t+1}^* \) and an off-equilibrium threat \( \phi (\pi_t - \pi_t^*) \). Alternatively, one can think of the interest rate target \( i_t^* \) coming first, respect of the private sector equilibrium then constrains the expected inflation target, \( E_t \pi_{t+1}^* \), but the central bank chooses unexpected inflation \( \pi_{t+1}^* - E_t \pi_{t+1}^* \) by the equilibrium-selection policy. I prefer the latter interpretation, as it quickly allows us to replace the second step with fiscal theory, but the equations are the same no matter how you read them.

In both parameterizations, you can see that the correlation of \( \pi_t \) and \( v_t \) that troubles estimation of \( \phi \) in \( i_t = \phi \pi_t + v_t \) is not just a nuisance, an accident, or a statistical assumption. It is central to the model. The whole point of the model, the whole way it generates responses to shocks, is that endogenous variables \( \pi_t \) “jump” in response to shocks \( (\varepsilon_t) \), so as to head off expected explosions.

16.7.3 The response of inflation to interest rates

We can characterize the model by the reaction of inflation to interest rates, rather than policy shocks. In the frictionless model, the answer is trivial: From \( i_t = E_t \pi_{t+1}^* \), inflation follows interest rates with a one-period lag. The model does not restrict the unexpected movement in inflation on the date of an announcement. This calculation does not assume \( \phi = 0 \). Rather, it characterizes the relation between \( i \) and \( \pi \) that emerge from any \( \phi \). Many different \( \phi, v \), combinations produce the same \( i, \pi \).

The response of inflation and interest rates to a monetary policy disturbance \( v_t \) of Figure 16.2 mixes the endogenous response of interest rates \( \phi \pi_t \), the dynamics of policy disturbance \( \rho \), and the economic response of the model to interest rates. Why don’t we ask directly for the response of inflation to interest rates in the model? Especially with questionably identified policy rules and disturbances, that seems an easier task.

The answer is simple in this model, but the answer illustrates the question and paves the way for more complex models. From

\[ i_t = E_t \pi_{t+1}, \]

for \( t = 1, 2, \ldots \) the expected inflation path \( E_t \pi_{t+1} \) just follows the interest rate path \( i_t \) with a one period lag. At period 1, \( \pi_1 \) can take any value. After that, it follows the path of interest rates.

By making this calculation, we do not assume \( \phi = 0 \), or a time-varying peg. We can phrase the question as: Suppose the central bank does whatever it needs to do
with $\phi \pi_t$ and $v_t$ to produce a given equilibrium interest rate path $\{i_t^*\}$, what are the corresponding inflation paths? The answer characterizes the economic part of the model. It is completely independent of the monetary policy rule, $\phi$ and $v_t$ or $\pi_t^*$ and $i_t^*$. It holds if the central bank follows a time-varying peg, holding $i_t$ fixed no matter what $\pi_t$, and it holds if the central bank follows a policy rule with Taylor principle, $i_t = \phi \pi_t + v_t$, selecting just the right $\{v_t\}$ to generate the given $\{i_t\}$. Any policy rule and disturbance that produces $\{i_t\}$ will produce a response $\{\pi_t\}$ in which $E_t \pi_{t+1} = E_t i_t$. In Figure 16.2 for $t = 1, 2, \ldots$, one can trace a horizontal line from $i_t$ to $\pi_{t+1}$. This same response, for every date but $t = 1$, could have been produced by a time-varying interest rate peg following the indicated interest rate path, rather than a policy rule with feedback.

The only contribution of the specific monetary policy rule to the response function, then, is equilibrium selection: $\pi_1$, or more precisely $\pi_1 - E_0 \pi_1$, the ex-post inflation on the day of the shock. Here, as the $\phi(\pi_t - \pi_t^*)$ parameterization emphasizes, we can now summarize the new-Keynesian model by saying that this impact reaction is whatever the central bank desires it to be – and can convince people it will enforce by reacting to any different value with hyperinflation. (The fiscal theory of monetary policy says ex post inflation is whatever the revision in present value of future surpluses is, a fact achieved passively in the new-Keynesian view.)

This characterization is closer to the economics of the model. This is, after all, a completely frictionless model. The entire private sector is summarized by $i_t = E_t \pi_{t+1}$. Of course, you might say, in such a model raising interest rates must raise inflation. If you think raising interest rates lowers inflation, you must put in price stickiness or some other ingredient. How did we seem to get the opposite result in Figure 16.2? How did we seem to perform the magic of getting lower inflation in response to a rise in nominal rates, with constant real rates, constant output, completely flexible prices? Only by forcing the economy to jump to another equilibrium at time 1, and here from confusing a positive interest rate disturbance $v_t$ with a positive rise in interest rates $i_t$.

This question is more general and it also offers a convenient way of characterizing and solving more complex models. In fact, we usually want to condition on the path of interest rates, not the path of disturbances. For example, VARs give us a path of inflation and interest rates following a shock in the data. So, having found the set of inflation paths for a given interest rate path, having chosen one of them if there are many, work backwards: For any $\phi$, construct $v_t = i_t - \phi \pi_t$, the monetary policy shock assumption that gives rise to the interest rate and inflation path we’re trying to match. Or, more transparently, for any $\phi$, imagine $\phi(\pi_t - \pi_t^*)$ selects the $\pi_t^*$ you
This procedure avoids having to search for a set of underlying shocks $v_t$ that for given $\phi$ produce the desired interest rate response. Reverse engineering is quicker than searching. More deeply, this simple construction shows that many different policy disturbances and threats $\phi$ correspond to the same interest rate path. (Werning (2012) innovated this clever idea for solving new-Keynesian models. I use it extensively in Cochrane (2018) to search for models in which higher interest rates lead to lower inflation.)

### 16.7.4 Response to anticipated monetary policy

In these models, expected monetary policy – the announcement of future policy disturbances $v$ or a future interest rate path $i^*$ – affects inflation, and later, output today. Such responses are potentially more interesting for historical analysis and policy than are responses to shocks. We focus on the latter only out of habit with VARs, and for comparing models to VARs.

In the frictionless model, Figure 16.3 graphs the response to an anticipated rise in future interest rates. Expected future inflation rises. There is no negative response on the day of the rate rise, only a possible response on announcement, depending on the equilibrium-selection policy. Figure 16.4 graphs the response to an anticipated AR(1) $v_t$ disturbance. Here, inflation moves well ahead of the announced disturbance.

For concreteness, return for the moment to a policy of the form

$$i_t = \phi \pi_t + v_t; \quad v_t = \rho v_{t-1} + \varepsilon_t.$$

To characterize the effects of monetary policy, it has become conventional to plot the response to monetary policy shocks, how other variables are expected to evolve after a unit unexpected shock $\varepsilon_t$, as I did in Figure 16.2 above. Here I calculate responses to anticipated monetary policy movements. At time 1, the central bank announces that at time $T$, there will be a disturbance $v_T, v_{T+1}, \ldots$ or that the path of interest rates will rise starting at time $T$. What happens to inflation (and later, to output?)

The habit of plotting responses to unexpected disturbances, or shocks, derives from comparing the results to vector autoregressions (VARs). VARs want to answer the
causal question, what if the central bank deviates from the policy rule or raises interest rates? To estimate the answer to that question, VARs want to find a movement in interest rates (or other policy control variable) that is not taken in response to changing expectations of future inflation or output growth. If the central bank raises interest rates in response to higher expected inflation, then higher interest rates will precede higher inflation, and a regression of inflation on interest rates would show a positive sign, even if the true structural response of inflation to interest rates is negative. It helps in this quest to find monetary policy disturbances that are unanticipated, to focus on the shocks $\varepsilon_t$ with $E_{t-1}(\varepsilon_t) = 0$, and to find shocks uncorrelated with the other shocks in the economy that might produce or reflect inflation or output variation.

The habit of looking at responses to unexpected shocks also derives from experience with early information-based rational expectations models such as Lucas (1972), in which only unexpected monetary policy shocks have any real output effect. With such a model in mind, it would make sense to find unexpected shocks, and then to measure their output effects.

But those habits are not relevant to our purpose here. We want to understand the workings of a model, and what the model predicts of a deliberate monetary policy tightening. Moreover, truly exogenous and unexpected monetary policy shocks are tiny, if they exist at all, and VAR-identified monetary policy shocks account for small fractions of the variation of interest rates and inflation in most estimates. Our central banks explain every action as a response to events, not as deliberate random experiments.

Most of all, sticky-price models give output responses to expected monetary policy disturbances, further reducing the rationale for studying only unexpected shocks. Moreover, policy makers routinely announce their actions far in advance, and wish to know from models how the economy will respond to such a preannounced action. For all these reasons, it’s interesting to know how the economy reacts to anticipated policy movements.

For understanding the logic of a model, conventional impulse-response functions mix several ingredients. In response to a shock $\varepsilon_t$, there is a persistent disturbance $\{v_{t+j}\}$. Is the response of an endogenous variable, say $\pi_{t+j}$, a structural, economic, lagged response to the shock $\varepsilon_t$, or is it a structural contemporaneous response to the future disturbance $v_{t+j}$? Does the model have interesting dynamics, or are the dynamics all coming from dynamics of the forcing variables $v_t$?

In this model the answers are simple, and I belabor the question largely to tee it up
for models where the answer is not so simple. In this model, we have $i_t = E_t \pi_{t+1}$ and thus $E_t i_{t+j} = E_t \pi_{t+j+1}$. A rise in expected future interest rates $i_T$ at date $T$ and beyond leads to a rise in expected future inflation $\pi_{T+1}$ with a one period lag, at dates $T + 1$ and beyond. There is a potential response $\pi_1$ or $\pi_1 - E_0(\pi_1)$ on the day 1 of the announcement, with no change in interest rate on that date. In this model that is the only possible negative response. The model cannot produce a decline in inflation at or around date $T$ when interest rates actually rise. Figure 16.3 illustrates this response.

Figure 16.3: response of the simple new-Keynesian model to an anticipated interest rate rise. The dashed lines give potential multiple equilibria.

This deeply Fisherian property is a key to understanding the new-Keynesian model. When, following $i_t = \phi \pi_t$, and therefore expected to follow $E_t i_{t+j} = \phi E_t \pi_{t+j}$, the central bank commits to raising future interest rates in response to future inflation, it thereby raises, not lowers, subsequent inflation. The Taylor principle does not work by stabilizing expectations, by committing to reduce future inflation should it break out. The Taylor principle, as we have seen, commits to exploding inflation. And a deeply Fisherian, positive, connection between expected interest rates and expected inflation is key to that counterintuitive, to Keynesian thinking, outcome. This Fisherian property continues in the sticky price models below and is, as it must
be, how a positive response of interest rates to inflation generates an explosive, not stabilizing, set of expectations. The desire to see the central bank as stabilizing inflation – which it does in adaptive expectations models – is behind many of the misinterpretations of this model, as we have seen.

Conversely, by separating in time the potential announcement effect from the actual rise in interest rates, giving another open-mouth result, or in this case a forward-guidance result. The announcement of future policy can move inflation, and later output, today.

In response to an anticipated disturbance to a policy rule \( v_T, v_{T+1}, \) etc. we have from (16.6),

\[
\pi_t = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t(v_{t+j})
\]

To compute an example, let \( v_t \) jump from \( v_t = 0 \) to \( v_t = \rho^{(t-T)} v_T; \ t \geq T, \) i.e. a preannounced AR(1) shock, with the announcement made at time 1. The unique locally-bounded equilibrium response is

\[
\pi_t = \begin{cases} 
0; & t \leq 0 \\
-\frac{1}{\phi^{T-t}} \frac{v_T}{\phi - \rho}; & 1 \leq t \leq T \\
-\frac{\rho^{(t-T)}}{\phi - \rho} v_T; & t \geq T
\end{cases}
\]

and \( i_t = E_t \pi_{t+1} \) or

\[
i_t = \begin{cases} 
0; & t \leq 0 \\
-\frac{\rho}{\phi^{T-t}} \frac{v_T}{\phi - \rho}; & 1 \leq t \leq T - 1 \\
-\frac{\rho^{(t-(T-1))}}{\phi - \rho} v_T; & t \geq T - 1.
\end{cases}
\]

On announcement there is a downward jump in inflation; inflation steadily declines at the rate \( \phi, \) and then returns after the shock date at the rate \( \rho. \) Interest rates also decline throughout, due to the endogenous response to inflation. Figure 16.4 illustrates, using \( \phi = 1.25, \rho = 0.8. \)

These are not different calculations. The inflation response of Figure 16.4 is the same as the \( i_t = E_t \pi_{t+1} \) response to the interest rate path (16.39)-(16.40) graphed in that
Figure 16.4: Response of the simple new-Keynesian model to an anticipated monetary policy disturbance.

As usual, issues are clearer if we write the rule

\[ i_t = i^*_t + \phi(\pi_t - \pi^*_t) \]

with the restriction

\[ i^*_t = E_t \pi^*_{t+1}. \]

Now each of our calculations specifies an \( i^*_t \), and the central bank can pick any of the time-zero responses \( \pi^*_t \) by the equilibrium-selection policy \( \phi(\pi_t - \pi^*_t) \). The fiscal theory of monetary policy gives the same result, with instead fiscal policy choosing the announcement effect.

16.8 Sticky price responses

We study responses to monetary policy in the simple model with price stickiness. Here, we set up the model. With adaptive expectations, the private sector equilibrium condition generalizes to

\[ \pi_t - \pi_{t-1} = -\sigma \kappa (i_t - \pi_{t-1}) . \]
A high nominal rate reduces inflation. The rational expectations model gives
\[ E_t \pi_{t+1} - \pi_t = \sigma \kappa (i_t - E_t \pi_{t+1}) . \]
A high nominal rate increases expected inflation.

The completely frictionless new-Keynesian model is stark, and clearly not one to take seriously to the data or to policy analysis. One may also suspect that many of the strange properties of the last section are a result of oversimplifying, and one may wonder that I treated it at such length. We should verify that my characterization of how the new-Keynesian model operates extends to the full new-Keynesian model. I start here, keeping it analytically simple, by investigating the response of inflation to monetary policy shocks, via the same set of calculations of the last sections, with the simple model of price stickiness introduced in section 16.3, missing the \( E_t x_{t+1} \) term in the IS equation. I follow with the full new-Keynesian model, where we will see the same principles reflected but the algebra obscures the intuition.

What is the response of each model to a given path of interest rates? From (16.22)
\[ \pi_t = -\sigma \kappa i_t + (1 + \sigma \kappa) \pi_t^e . \]
we can characterize directly the new (rational expectation) and old (adaptive) Keynesian models’ response to interest rates. The adaptive expectations model gives
\[ \pi_t = (1 + \sigma \kappa) \pi_{t-1} - \sigma \kappa i_t ; \quad (16.41) \]
equivalently the growth rate of inflation depends negatively on the real rate,
\[ \pi_t - \pi_{t-1} = -\sigma \kappa (i_t - \pi_{t-1}) . \]
The rational expectations model gives
\[ E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t ; \quad (16.42) \]
equivalently the expected growth rate of inflation depends positively on the real rate,
\[ E_t \pi_{t+1} - \pi_t = \sigma \kappa (i_t - E_t \pi_{t+1}) . \]
Each of these in the end just adds dynamics to the Fisher equation \( i_t = \pi_t^e \) which characterized private sector behavior in the constant-real-rate frictionless models of the last section. In each case you can see that \( i = \pi \) remains a steady state, especially in the equivalent forms.
But now there are dynamics. With adaptive expectations, an interest rate higher than inflation – a high real rate – lowers inflation. The Fisher equation with a fixed interest rate is an unstable steady state. With rational expectations, an interest rate higher than inflation raises inflation. The Fisher equation is a stable steady state. Once again, rational expectations is associated with stability, and adaptive expectations with instability. If you drive a car looking in the rear view mirror – adaptive expectations for the road – you will veer off course. If you drive looking through the front – forward-looking, rational expectations – your car will be stable.

**Sticky prices and rational expectations**

We study responses to interest rate changes and AR(1) disturbances for the simple model with price stickiness. The same forces are at work, with more dynamics. The frictionless model does, in fact, capture the essential economics of the new-Keynesian model, even without price stickiness. Equilibrium-selection remains at the heart of monetary policy.

Figure 16.5 plots the response to an unexpected increase in interest rates. Inflation slowly rises to meet the higher rates. Multiple equilibria now show up as multiple stable paths, which the central bank picks by equilibrium selection policy, or we pick by their fiscal underpinnings. A super-Fisherian, instant inflation rise remains possible despite sticky prices.

Figure 16.5 plots the response to AR(1) disturbances $v_t$ in $i_t = \phi \pi_t + v_t$, $\rho = 1$ still produces the super-Fisherian or super-neutral response. Now an open-mouth result, inflation moves with no movement in interest rate, occurs for positive $\rho$, emphasizing the importance of equilibrium selection in these responses. Now a sufficiently low $\phi$ gives, finally, a response in which higher interest rates go with lower inflation.

It appears that transitory shocks give negative impact responses, but this pairing is the result only of the AR(1). Writing the rule as $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ we see that the central bank can select equilibria in which $i_t^*$ and $\pi_t^*$ go in opposite directions for any value of the persistence of $i_t^*$.

The long-run response of inflation to interest rates is always positive. This model does not produce the old-Keynesian (or monetarist) story for the conquest of inflation in the 1980s – that persistently high interest rates, or persistently tight monetary policy slowly brought inflation down.

Expected monetary policy matters. An expected interest rate rise gives the same
no-jump or “inflation \( \pi \)” response of \[16.5\] plus multiple equilibria transients that start at different values of \( \pi_0 \) and decay.

Figure 16.5 presents the response function of the rational expectations sticky price model, \[16.42\], to an unexpected permanent interest rate shock. (The plot uses \( \sigma = 1 \) and \( \kappa = 0.5 \); the “less sticky” line uses \( \kappa = 1 \).) The dynamics are stable – the rise in interest rates eventually brings inflation up to meet it. As we turn down price stickiness, raising the parameter \( \kappa \), the dynamics happen faster, as graphed by the “less sticky” line, just as one might expect. The model smoothly approaches the frictionless result, in which \( \pi_1 = 1 \) and stays there forever.

Figure 16.5: Response of the simple sticky-price model to a permanent unanticipated interest rate rise.

The equilibrium dynamics don’t pin down the initial impact, i.e. unexpected inflation. Figure 16.5 presents several possibilities. As before, if we rule out nominal explosions and add a policy rule of the form

\[
i_t = E_t \pi_{t+1} + \phi (\pi_t - \pi^*_t),
\]

the central bank can achieve any value of unexpected inflation it desires here. (Or, again, we can use an active fiscal policy with \( \pi_{t+1} - E_t \pi_{t+1} = \varepsilon^s_{t+1} \) to the same end.)
We can also work out the response to traditional AR(1) monetary policy shocks. Combining (16.42) with
\[ i_t = \phi \pi_t + v_t, \]
\[ v_t = \rho v_{t-1} + \varepsilon_t, \]
we have
\[ E_t \pi_{t+1} = \frac{1 + \sigma \kappa \phi}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} v_t. \]

With \( \phi > 1 \), we solve forward just as in the frictionless case,
\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} E_t \pi_{t+1} - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} v_t \]
and applying the rule against nominal explosions,
\[ \pi_t = -\frac{\sigma \kappa}{1 + \sigma \kappa \phi} E_t \sum_{j=0}^{\infty} \left( \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \right)^j v_{t+j}. \]
\[ \pi_t = -\frac{\sigma \kappa}{1 + \sigma \kappa \phi} \frac{1}{1 - \rho \frac{1 + \sigma \kappa}{\sigma \kappa} v_t} \]
\[ \pi_t = -\frac{1}{\phi - \rho + \frac{1 - \rho}{\sigma \kappa} v_t}. \] (16.43)

The interest rate follows
\[ i_t = \left( 1 - \frac{\phi}{\phi - \rho + \frac{1 - \rho}{\sigma \kappa}} \right) v_t = \left( \frac{-\rho + \frac{1 - \rho}{\sigma \kappa}}{\phi - \rho + \frac{1 - \rho}{\sigma \kappa}} \right) v_t. \] (16.44)

You can see here a natural generalization of the frictionless AR(1) case, culminating in (16.6), and the frictionless limit \( \kappa \to \infty \) reduces to that case. One major reason we are here is to verify that the logic of the simple frictionless model does indeed apply to new-Keynesian models more generally, and here you see that it does.

The \( \rho = 1 \) case is revealing. It produces
\[ \pi_t = -\frac{1}{\phi - 1} v_t \]
\[ i_t = -\frac{1}{\phi - 1} v_t. \]
or,

\[ \pi_t = i_t. \]

This case is labeled “\( \rho = 1 \)” in Figure [16.5]. The standard new-Keynesian sticky price model produces a *super*-Fisherian result, even with sticky prices. Not only does inflation jump to equal interest rates in the period after the interest rate rise, \( i_t = E_t\pi_{t+1} \), as if real rates were constant (which they are not, here), but inflation rises *immediately*, the very moment interest rates rise!

In standard monetary theory, the “neutrality” of money refers to the proposition that doubling the quantity of money doubles the price level, eventually. “Super-neutrality” is the proposition that doubling the quantity of money instantly doubles the price level. That is usually thought not to happen when prices are sticky, though it happens when there is a currency reform or currency change (Lira to Euros), which tells us something deep about what “sticky” prices must involve. The Fisherian property, that nominal interest rates rise one for one with inflation in the long run, is the corresponding neutrality result for interest rate targeting policies. The new-Keynesian model is super-Fisherian to permanent monetary policy shocks, despite sticky prices. This occurs because the sticky prices are forward looking.

Once again we see the initially strange behavior that the positive interest rate movement is produced by a negative monetary policy shock, because the \( \phi\pi_t \) term is larger than the \( v_t \) term in the policy rule \( i_t = \phi\pi_t + v_t \).

Figure [16.6] presents this and several other interesting cases. The top left panel presents the response in the \( \rho = 1 \) case, just analyzed. Inflation \( \pi_t \) and interest rates \( i_t \) move exactly one for one, in the opposite direction as the shock \( v_t \). The top right panel reduces persistence somewhat. You can see that this model still produces the Fisherian result.

The bottom left panel produces an open-mouth policy. For

\[ \rho = \frac{1}{1 + \sigma\kappa}, \]

equations (16.43) and (16.44) produce

\[ \pi_t = -\frac{1}{\phi} v_t; i_t = 0 \times v_t. \]

Inflation moves on the announcement of the shock, and interest rates never move. We saw that behavior in the frictionless model for \( \rho = 0 \). Here it appears for positive
Figure 16.6: Response to a monetary shock in the simple sticky-price new-Keynesian model.

$\rho$. Open mouth policy was not a particularity of the frictionless model, though it does occur at a different value of $\rho$.

In the bottom right panel of Figure 16.6, we see the standard result in the literature, for a low value of $\rho$. Now a negative shock $v$ causes a negative interest rate movement, and a positive inflation movement. The standard new-Keynesian sticky price model apparently only produces a negative response of inflation to interest rates from a sufficiently transitory shock.

This oft-cited result is not robust, however. This fact is easiest to see by writing the policy rule in the form

$$i_t = i^*_t + \phi(\pi_t - \pi^*_t)$$

The central bank can, by its equilibrium selection policy, produce positive or negative responses $\pi^*_0$ to an announcement at time 0, for any persistence of the interest rate policy $i^*_t$. In Figure 16.5 the permanent $i^*_t$ can come with the lowest path for $\pi^*_t$, which starts with a negative response. Contrary examples are just as straightforward. One can express any of these examples with a conventional disturbance $v^*_t = i^*_t - \phi\pi^*_t$. They just won’t be AR(1)s.
16.8. STICKY PRICE RESPONSES

As in the frictionless case, the inflation target and interest rate target are constrained by the equilibrium conditions of the model. In the frictionless case, we had $i_t^* = E_t \pi_{t+1}^*$. Here we have from (16.42), a natural generalization,

$$i_t^* = \left(1 + \frac{1}{\sigma \kappa}\right) E_t \pi_{t+1}^* - \frac{1}{\sigma \kappa} \pi_t^*.$$ (16.45)

But even within this constraint, as with $i_t^* = E_t \pi_{t+1}^*$, the persistence of movements in $i_t^*$ does not constrain the sign of $\pi_t^*$ on the date 0 of a shock.

As before, the central bank can achieve any $\{\pi_t^*\}$ it wishes. (16.45) then give a recipe for the interest-rate policy required to produce $\{\pi_t^*\}$, and $\phi(\pi_t - \pi_t^*)$ gives the equilibrium selection policy. We can then express the same result as $i_t = \phi \pi_t + v_t$ with $v_t = i_t^* - \phi \pi_t^*$ if we wish.

The long-run response of inflation to interest rates is always positive. This model does not produce the old-Keynesian (or monetarist) story for the conquest of inflation in the 1980s – that persistently high interest rates, or persistently tight monetary policy, slowly drove inflation down. A persistently tight monetary policy still drives inflation up eventually in all equilibria of this model, as Figure 16.5 emphasizes. To fit the 1980s, one has to imagine a sequence of unexpected shocks, each of which has the negative sign.

In this model also, expected monetary policy matters, and now to output as well. From (16.42),

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t,$$

the only difference in the response of inflation to a fully expected rise in interest rates $\{i_t\}$ is that there cannot be an unexpected movement on the day that the interest rate moves. The unexpected movements must come on the day of the announcement. We have

$$\pi_{t+1} = \frac{\sigma \kappa}{1 + \sigma \kappa} \sum_{j=0}^{\infty} \left( \frac{1}{1 + \sigma \kappa} \right)^j i_t + \left( \frac{1}{1 + \sigma \kappa} \right)^t \pi_0$$

In Figure 16.5, we have the response marked “inflation $\pi$” with no jump at time 0. As before, inflation can jump on the day the policy is announced, and we add the decayed response to such shocks to the no-jump equilibrium graphed.
Sticky prices and adaptive expectations

In response to an interest rate rise, inflation in the adaptive-expectations model spirals away as graphed in Figure 16.7. Higher interest rates induce lower inflation.

In response to a positive Taylor rule disturbance $v_t$, interest rates rise, and inflation declines. But interest rates then decline to catch and stabilize inflation, as graphed in Figure 16.8. This graph captures well the standard view of monetary policy.

From (16.22) and (16.41), the adaptive expectations model’s response to interest rates is

$$\pi_t = (1 + \sigma \kappa) \pi_{t-1} - \sigma \kappa i_t.$$  

The dynamics are unstable, but we don not solve this one forward, since there is no expectational error, and no variable that can jump to offset explosions.

Figure 16.7: Response to a persistent interest rate rise in the simple old-Keynesian model.

Figure 16.7 presents the response to a permanent interest rate rise with this model. The model captures traditional old-Keynesian and policy world beliefs about monetary policy. (Of course, those beliefs are as much or more formed by playing with this model as they are from experience, so it’s not that much of an achievement!)
Higher interest rates lower inflation. There is no difference between expected and unexpected inflation in this response, and nobody stays up at night worrying about multiple equilibria and jumps.

The mechanism is traditional. With adaptive expectations, a higher nominal interest rate means a higher real rate \( i_t - \pi_t^e = i_t - \pi_{t-1} \). The higher real rate means lower output via the IS curve, \( x_t = -\sigma (i_t - \pi_t^e) \). In the sticky-price backward-looking Phillips curve, lower output \( x_t \) means declining inflation \( \pi_t = \pi_{t-1} - \kappa x_t \). In this model, persistently high nominal interest rates do drive inflation down.

An interest rate peg generates an inflation or deflation spiral. The Fisher relationship \( i_t = r + \pi_t^e \) is irrelevant to this prediction. Yes, there is a steady state, \( \pi = i \), but it is an unstable steady state.

Why don’t we see such spirals? Because, usually, the central bank is smart enough not to keep the nominal rate pegged while both inflation and real interest rates spiral away. Once deflation gets going, the central bank should move interest rates sharply to stop it. The economy is unstable, like a seal balancing a ball on its nose. The secret to stabilizing the economy is for the seal (the central bank) to move its nose quickly and more than one for one with movements of the ball.

To see this behavior, let us put in again an explicit Taylor rule \( i_t = \phi \pi_t + \nu_t \), with an AR(1) monetary policy shock. From (16.24), we now have

\[
\pi_t = \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} \pi_{t-1} - \frac{1}{1 + \phi \sigma \kappa} \nu_t.
\]

If \( \phi > 1 \), the economy is now stable, as the coefficient on lagged \( \pi \) is less than one. Inflation follows an AR(2),

\[
(1 - \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} L)(1 - \rho L)\pi_t = -\frac{\sigma \kappa}{1 + \phi \sigma \kappa} \varepsilon_t.
\]

In parallel with the last section, Figure 16.8 plots the response to a permanent monetary policy shock, i.e. the case \( \rho = 1 \). Again, the rise in interest rates sets off a disinflation. But now the endogenous response \( \phi \pi_t \) means that the actual interest rate quickly reverses course and keeps the disinflation from spiraling away.

This is pretty much exactly what Milton Friedman ([Friedman (1968)](Friedman1968)) described as the effect of an attempt to peg interest rates at too low a level, with the opposite sign, and without Friedman’s monetary mechanism. He did not describe a spiral. Instead, he wrote that the threat of ever increasing inflation would force the central
bank to give up the peg and (effectively) increase interest rates quickly, so that the attempt to lower rates would in the end result in higher rates and more inflation. This is pretty much the conventional wisdom to account for the emergence of US inflation in the 1970s and the Volcker disinflation of the early 1980s. In the 1970s, the Fed kept interest rates too low, or followed \( \phi < 1 \), so an inflation spiral began. By a switch to \( \phi > 1 \) and a very long lasting monetary policy tightening, the Fed sharply raised nominal rates. As inflation declined, the Fed was able to lower nominal rates, though keeping real rates persistently high, and slowly squeezed inflation out of the economy. (Whether the Fed in the 1980s followed a rule with a higher coefficient \( \phi \) or simply set higher interest rates is debated. But the result is the same in this model – all that matters is the path of interest rates, not how the Fed gets there.)

The spiral reappears if interest rates do not or cannot move, which happens if the interest rate hits zero or the effective lower bound. That these widely-predicted spirals did not happen is important evidence against this model. I survey the zero bound experience below.

Figure 16.8: Response to a monetary policy shock, old-Keynesian model with a Taylor rule
16.9 Full model responses

The models of the last few sections are deliberately over-simplified. Here we meet the full prototype new-Keynesian model, and we verify that its qualitative behavior is described by the toy models of the last few sections.

The model, which we first met in section 7.1 is

\[ x_t = E_t x_{t+1} - \sigma r_t + v_t^x \]  
\[ i_t = r_t + E_t \pi_{t+1} \]  
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + v_t^\pi \]

We can write \( v_t^x = -\sigma v_t^r \) to interpret that disturbance in units of an interest rate distortion.

16.9.1 Interest rates and inflation

Inflation is a two-sided moving average of interest rates in this model. Figure 16.9 plots the response of inflation to a permanent interest rate rise. Now inflation moves ahead of the interest rate rise as well as following it. As usual, in addition to this plot, there can be an inflation jump on announcement, selected by the central bank’s equilibrium-selection policy. Each equilibrium has a fiscal consequence, in the new-Keynesian interpretation, or can be selected by fiscal policy, in the FTMP interpretation. I calculate the fiscal policy change needed for several equilibria. Since real interest rates vary, there is now a discount rate effect, and the equilibrium with no change in fiscal policy has a small jump in inflation.

As in the last section, I start by characterizing the relationship between equilibrium inflation and interest rates. Equations (16.46)-(16.48) take the place of the simple Fisher equation \( i_t = r + E_t \pi_{t+1} \) of the frictionless model. The real interest rate now varies over time which leads to a dynamic relationship. As before, this representation can give better intuition about how the model of the private economy behaves than thinking right away about a rule with shocks \( i_t = \phi \pi_t + v_t \), and it paves the way for a clearer reconciliation between new-Keynesian and fiscal-theory approaches.

Already in equation (16.69) we substituted out \( x^*_t \) to give a relationship between equilibrium interest rates and inflation,

\[ i^*_t = \frac{1}{\sigma \kappa} \left[ -\beta E_t \pi^*_{t+2} + (1 + \beta + \sigma \kappa) E_t \pi^*_{t+1} - \pi^*_t \right]. \] (16.49)
You can see how this equation generalizes the frictionless $i_t = E_t \pi_{t+1}$. We solved the latter equation to $\pi_{t+1} = i_t + \delta_{t+1}$. To find the corresponding solution here, we factor the lag polynomial, and invert, sending the stable root backward and the unstable root forward. I defer the algebra. The result is that inflation is a two-sided geometrically-weighted distributed lag of interest rates,

$$\pi^*_{t+1} = \frac{\sigma \kappa}{\left( \lambda_- - \lambda_+ \right)} \left[ i^*_t + \sum_{j=1}^{\infty} \lambda_j^+ i^*_{t-j} + \sum_{j=1}^{\infty} \lambda_j^- E_{t+1} i^*_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_j^+ \delta_{t+1-j}$$

(16.50)

where

$$\lambda_\pm = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4 \beta}}{2 \beta}.$$ (16.51)

We have $\lambda_+ > 1$ and $\lambda_- < 1$. (These are the same as the eigenvalues (16.58) with $\phi = 0$.) Here, as usual, $\delta_{t+1}$ with $E_t \delta_{t+1} = 0$, is an expectational shock indexing multiple equilibria. Once again, this calculation represents the response of equilibrium inflation to equilibrium interest rates, and holds for any $\phi$.

From inflation, we can also find the output gap,

$$x^*_{t+1} = \frac{\sigma}{\left( \lambda_- - \lambda_+ \right)} \left[ (1 - \beta \lambda_-) \sum_{j=0}^{\infty} \lambda_j^- i^*_{t-j} + (1 - \beta \lambda_+) \sum_{j=1}^{\infty} \lambda_j^+ E_{t+1} i^*_{t+j} \right]$$

$$+ \frac{(1 - \beta \lambda_-)}{\kappa} \sum_{j=0}^{\infty} \lambda_j^+ \delta_{t+1-j}.$$ (16.52)

Figure [16.9] presents the inflation response to a permanent increase in interest rates as given by (16.50). I plot the case with no unexpected shocks $\delta_t = 0$.

The solid line gives the response to a fully expected interest rate rise. Since (16.50) is a two-sided moving average, inflation rises before the interest rates rise. Expected future interest rate increases increase inflation today. Overall, though, price stickiness just smears out the instant rise with a one-period delay that $i_t = E_t \pi_{t+1}$ and hence $\pi_{t+1} = i_t + \delta_{t+1}$ gave for the frictionless model.

The dashed line gives the response to an unexpected interest rate rise. The forward-looking terms are all zero until the day of the announcement. Then inflation joins the path given for the expected interest rate rise. Inflation is thoroughly Fisherian so far – expected interest rate rises raise, not lower, inflation. As before, to get a
negative response we will have to engineer a jump to a different equilibrium on the announcement date.

The solid line gives the response of output. In this model, output is low if inflation is low relative to expected future inflation, i.e. if inflation is increasing. We see that pattern. Fully expected interest rate rises do lower output, contrary to the classic rational expectations information based models such as Lucas (1972). As before, this observation changes how one interprets VAR evidence. Much of an output reaction is not necessarily the economy’s delayed reaction to the initial shock, but its contemporaneous reaction to current expected interest rates, and the dynamic path that interest rates follow a shock.

The classic old-Keynesian intuition that interest rate rises lower inflation is false in this model. However, the intuition that interest rate rises lower output and that there is little difference between announced and surprise interest rate rises, is correct in this model.

What about multiple equilibria $\delta$? Figure 16.10 graphs the response of inflation to an unexpected step function interest rate rise, this time adding several possibilities.
Figure 16.10: Response of the new-Keynesian model to a permanent unexpected interest rise, with multiple equilibria. A, B, etc. identify equilibria in the text. $\Delta s = \delta$ gives the percent increase in fiscal surpluses necessary to validate each equilibrium.

For $\delta_0$ which indexes multiple equilibria. If the rise were announced in advance, these jumps would take place on the announcement date, not the date of the interest rate rise. Equilibrium-selection policy $i_t = i^*_t + \phi(\pi_t - \pi^*_t)$ can force the equilibrium inflation $\pi^*_0$ to jump to any of these values. Some interesting cases follow.

The original $\delta_0 = 0$ equilibrium already had a little jump in inflation.

Equilibrium A has a positive additional inflation shock, $\delta_0 = 1\%$.

Equilibrium B chooses $\delta_0$ to produce 1% inflation at time 0, $\pi_0 = 1\%$. It shows once again that a super-neutral response is possible by selecting the right equilibrium, even though prices are sticky.

Equilibrium D chooses $\delta_0$ to produce no inflation at time 0, $\pi_0 = 0$, to show that is possible.

Equilibrium E chooses $\delta_0 = -1\%$. By mixing a negative inflation jump $\delta$ with the interest rate rise, we obtain a negative response of interest rates to inflation, at least in the short run. This will be an important example to add to the discussion of VAR.
shocks in the last section: There is no logical connection between the persistence of the interest rate shock and the sign of the inflation movement. Here, a persistent interest rate shock gives rise to a negative inflation movement.

Now, if we write a policy rule in the form

\[ i_t = \pi_t^* + \phi(\pi_t - \pi_t^*) \]

with an interest rate rule and an equilibrium selection rule, where \( \pi_t^* \) and \( \phi(\pi_t - \pi_t^*) \) obey the restriction of (16.49), then by choosing \( \pi_t^* \) as one of the plotted paths, the central bank can, following new-Keynesian rules, choose any of these equilibria.

Each of these movements in unexpected inflation requires a fiscal policy reaction. Even in the new-Keynesian passive assumption, where equilibrium is selected by the central bank’s choice of \( \pi_t^* \) among the graphed possibilities and the threat \( \phi(\pi_t - \pi_t^*) \), we may want to check what the assumed “passive” fiscal policy is. It is at least polite to call up the Treasury and let them know. Alternatively, we can use the fiscal theory to select from these equilibria directly. By not including the equilibrium-selection rule in these calculations they can be used immediately in either fiscal or traditional new-Keynesian completion of the model, which is an advantage relative to the standard calculations.

The \( \Delta s \) numbers tell us by what percentage steady state surpluses must change to produce each equilibrium, whether actively or passively accomplished. For example, to produce equilibrium C, which produces a sudden 1% inflation, the government must reduce the value of the debt by 1%, so \( \Delta s = 1.00\% \).

The \( \Delta s = 0 \) equilibrium is not the equilibrium with \( \delta = 0 \) or with \( \pi_0 - E_{-1}\pi_0 = 0 \). In making the calculation, I allow the discount rate in the government debt valuation equation to vary, as it should. Including discount rate variation is one of the major steps we must take to bring fiscal theory to data and to models with real interest rate variation. In the \( \pi_0 = 0 \) equilibrium D, for example, real interest rates rise. That force lowers the right hand side of the government debt valuation formula, which on its own produces inflation. In order to keep inflation from breaking out, the fiscal authorities must raise \( \Delta s = 1.66\% \). Equilibrium C with \( \Delta s = 0 \) has inflation for the same reason: real interest rates rise, that lowers the present value of government debt, so there is a surprise inflation.
Calculations

Here I derive the explicit solutions (16.50)-(16.52), for inflation and output given the equilibrium path of interest rates. We start with (16.49),

$$E_t [\beta L^{-2} - (1 + \beta + \sigma \kappa) L^{-1} + 1] \pi_t^* = -\sigma \kappa \pi_t^*$$

Factor the lag polynomial

$$E_t (1 - \lambda_{-1}^{-1} L^{-1})(1 - \lambda_{+1}^{-1} L^{-1}) \pi_t^* = -\sigma \kappa \pi_t^*$$

where

$$\lambda_{\pm}^{-1} = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2}.$$ 

(Yes, these are the inverses of (16.51).) Since $\lambda_{-1} > 1$ and $\lambda_{+1} < 1$, reexpress the result as

$$E_t [(1 - \lambda_- L)(1 - \lambda_{-1}^{-1} L^{-1}) \lambda_{-1}^{-1} L^{-1} \pi_t^*] = \sigma \kappa \pi_t^*$$

Inverting, the bounded solutions are

$$\pi_{t+1}^* = E_{t+1} \frac{\lambda_-}{(1 - \lambda_- L)(1 - \lambda_{-1}^{-1} L^{-1})} \sigma \kappa \pi_t^* + \frac{1}{(1 - \lambda_- L)} \delta_{t+1}.$$ 

Using a partial fractions decomposition to break up the right hand side,

$$\frac{\lambda_-}{(1 - \lambda_- L)(1 - \lambda_{-1}^{-1} L^{-1})} = \frac{1}{\lambda_{-1}^{-1} - \lambda_{+1}^{-1}} \left(1 + \frac{\lambda_- L}{1 - \lambda_- L} + \frac{\lambda_{+1}^{-1} L^{-1}}{1 - \lambda_{+1}^{-1} L^{-1}}\right),$$

which gives (16.50). We can find output by

$$\kappa x_t = \pi_t - \beta E_t \pi_{t+1}.$$ 

$$\kappa x_{t+1} = E_{t+1} [(1 - \beta L^{-1}) \pi_{t+1}]$$

$$\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_{-1}^{-1} - \lambda_{+1}^{-1}} E_{t+1} \left[(1 - \beta L^{-1}) \left(1 + \frac{\lambda_- L}{1 - \lambda_- L} + \frac{\lambda_{+1}^{-1} L^{-1}}{1 - \lambda_{+1}^{-1} L^{-1}}\right) \pi_t^* \right]$$

$$+ E_{t+1} \frac{(1 - \beta L^{-1})}{(1 - \lambda_- L)} \delta_{t+1}.$$
We can rewrite the polynomials to give
\[
\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_+ - \lambda_- t} E_{t+1} \left[ \left( \frac{1 - \beta \lambda_-}{1 - \lambda_- L} + \frac{(1 - \beta \lambda_+) \left( \lambda_-^{-1} L^{-1} \right)}{1 - \lambda_+ L^{-1}} \right) i_t \right] + \frac{1}{1 - \lambda_- L} \delta_{t+1}.
\]
(In the second term, I use \( E_t [\beta L^{-1} \delta_{t+1}] = 0 \)) which is [16.52].

To make the \( \Delta s \) calculations, I start from a steady state with constant surplus \( s \). I calculate the fractional permanent change in surplus \( \Delta s \), i.e. \( s_t = S^{\Delta s} \), that is required by the government debt valuation equation assuming one-period debt. Linearizing around that steady state, we have
\[
\Delta s \approx -\Delta E_t (\pi_t) + \frac{1 - \beta}{\sigma} \sum_{j=0}^{\infty} \beta^j \Delta E_t (x_{t+j} - x_t)
\]
(16.53)

where \( \Delta E_t \equiv E_t - E_{t-1} \) and \( t \) is the date of the announcement of a new policy. The first term is familiar. It just says that a 1% unexpected inflation devalues government debt by 1% and so requires a 1% increase in surpluses. The second term reflects discount rate variation. The real interest rate in this model is \( r_t = E_t (x_{t+1} - x_t) / \sigma \).

A higher real interest rate discounts the future more heavily, and acts like a decline in surplus. The calculation is described in Cochrane (2017b).

### 16.9.2 Response to AR(1) monetary policy disturbances

We calculate responses to AR(1) monetary policy disturbances in the classic formulation (16.54)-(16.55). I explain the matrix method and method of undertermined coefficients. Figure 16.11 presents responses, including the open-mouth case, and shows that the qualitative features of the simple sticky-price model continue to hold.

We add the usual monetary policy rule
\[
i_t = \phi \pi_t + v_t, \quad (16.54)
\]
\[
v_t = \rho v_{t-1} + \varepsilon_t. \quad (16.55)
\]

There (at least) four ways to approach a model of the form (16.46)-(16.55). First, express it in a standard matrix AR(1) form; eigenvalue decompose the transition matrix; and solve stable roots backwards and unstable roots forwards. This method
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is the easiest to apply to large models as all the work is done by computers, but it often hides intuition. Second, substitute until you have a lag-operator expression for the variable of interest, \( \pi_t \) here. Factor the lag polynomial, solve unstable roots forward and stable roots backward, to express \( \pi_t \) as a two-sided moving average of the forcing variables. This form shows analytically how the variable of interest responds to the shock of interest, so it is useful for intuition. Third, guess that the final answer is a function of state variables, substitute that guess in (16.46)-(16.55) and use the method of undetermined coefficients. This is often the quickest way to get an analytic solution, but it hides the economics and especially how the model gets rid of multiple equilibria. Fourth, rewrite the rule in the form \( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \) and apply the solution of the last section. I’ll use each method according to which makes the particular point clearest.

Following the matrix method, eliminate \( i_t \) and \( r_t \) and rearrange, leaving

\[
\begin{bmatrix}
E_t x_{t+1} \\
E_t \pi_{t+1}
\end{bmatrix} = \frac{1}{\beta} \begin{bmatrix}
\beta + \sigma \kappa & -\sigma (1 - \beta \phi) \\
-\kappa & 1
\end{bmatrix} \begin{bmatrix}
x_t \\
\pi_t
\end{bmatrix} + \frac{1}{\beta} \begin{bmatrix}
-1 & \sigma & \sigma \beta \\
0 & -1 & 0
\end{bmatrix} \begin{bmatrix}
v^x_t \\
v^\pi_t \\
v^i_t
\end{bmatrix}.
\]

(16.56)

This equation is the generalization of the equilibrium condition

\[
E_t \pi_{t+1} = \phi \pi_t + v^i_t
\]

(16.57)

of the frictionless model.

The eigenvalues of the transition matrix in (16.56) are

\[
\lambda = \frac{1}{2\beta} \left( 1 + \beta + \kappa \sigma \pm \sqrt{(1 + \beta + \kappa \sigma)^2 - 4\beta (1 + \sigma \kappa \phi)} \right)
\]

(16.58)

which we can write

\[
\lambda = 1 + \frac{1}{2\beta} \left( (1 - \beta + \sigma \kappa) \pm \sqrt{(1 - \beta + \sigma \kappa)^2 - 4\beta \sigma \kappa (\phi - 1)} \right).
\]

The + eigenvalue is greater than one. But if \( \phi < 1 \) the − eigenvalue is less than one, i.e. stable. Thus, with \( \phi < 1 \), we solve one part of the system backward. Since the left hand side of (16.56) determines only the expectations of future variables, we need two forward-looking roots and a rule against explosions to get rid of multiple equilibria, so with \( \phi < 1 \) we have multiple equilibria. If \( \phi > 1 \) then both eigenvalues are greater than one, and unstable. We solve the system forward and determine uniquely both \( x_t \) and \( \pi_t \) in order to have a locally bounded solution. This is the
generalization of the idea that led to $\phi > 1$ and then solving the frictionless model (16.57) forward.

Explicitly, to apply the matrix method write (16.56) in the form

$$y_{t+1} = Ay_t + Bv_t + \delta_{t+1}$$

where as usual $\delta_{t+1}$ are arbitrary unforecastable expectational errors. Eigenvalue decompose $A$ to

$$y_{t+1} = QA^{-1}y_t + Bv_t + \delta_{t+1}$$

where $\Lambda$ is diagonal. Then write

$$Q^{-1}y_{t+1} = \Lambda Q^{-1}y_t + Q^{-1}Bv_t + Q^{-1}\delta_{t+1}.$$ 

Define

$$z_t = Q^{-1}y_t.$$

Then we have

$$\begin{bmatrix} z_{t+1} \\ z_{t-1} \end{bmatrix} = \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix} \begin{bmatrix} z_{t+1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \bar{v}_{t+1} \\ \bar{v}_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\delta}_{t+1} \\ \tilde{\delta}_{t-1} \end{bmatrix}$$

$$z_{t+1} = \lambda_+ z_t + \bar{v}_{t+1} + \tilde{\delta}_{t+1}$$

$$z_{t-1} = \lambda_- z_t + \bar{v}_{t-1} + \tilde{\delta}_{t-1}$$

By eigenvalue decomposing, we have reduced the matrix problem to two scalar problems involving specific linear combinations of $\{\pi_t, x_t\}$. Now, $\lambda_+ > 1$. The first equation implies

$$E_t z_{t+1} = \lambda_+ z_t + \bar{v}_{t+1}$$

so $E_t z_{t+j}$ generically grows without bound as $j$ grows. If we decide to rule out such paths, we can conclude that $z_t$ or equivalently $\delta_{t+1} must jump to the one initial condition that rules out such explosions. Mechanically, solve forward,

$$z_{t+1} = -\sum_{j=0}^{\infty} \frac{1}{\lambda_+^{j+1}} E_t \bar{v}_{t+j}$$

and similarly if $\lambda_- > 1$,

$$z_{t-1} = -\sum_{j=0}^{\infty} \frac{1}{\lambda_-^{j+1}} E_t \bar{v}_{t-j}.$$
Equivalently, we select equilibria by
\[
\delta_{+t} = -\sum_{j=0}^{\infty} \frac{1}{\lambda_{+j}^{t+1}} (E_t - E_{t-1}) \tilde{v}_{+t+j}
\]
\[
\delta_{-t} = -\sum_{j=0}^{\infty} \frac{1}{\lambda_{-j}^{t+1}} (E_t - E_{t-1}) \tilde{v}_{-t+j}.
\]

If \(\lambda_- < 1\), however, we have to solve backward,
\[
z_{-t+1} = \sum_{j=0}^{t} \lambda_j^t \tilde{v}_{-t-j} + \sum_{j=0}^{t} \lambda_j^t \tilde{\delta}_{-t+1-j} + \lambda_{-1}^{t+1} z_0.
\]

Not only is the unexpected value \((E_{t+1} - E_t) z_{-t+1}\) indeterminate, with this second source of dynamics, \(z_{-t+1}\) includes indeterminacies coming from past multiple-equilibrium shocks.

Once we have the \(z_t\), the original variables follow by
\[
y_t = Q z_t.
\]

The more general condition is, we need the same number of unstable eigenvalues as there are expectational variables, variables in which the system only determines \(E_t x_{t+1}\) not \(x_{t+1}\) itself.

The point: The logic is the same as the frictionless case and the simplified case, though the algebra is considerably worse. Models of this complexity and more are typically solved on a computer, as the formulas for eigenvalues get worse quickly. Cochrane (2011c) contains the most general analytic formulas I know of.

In this case as well, \(\lambda < -1\) or \(\lambda\) complex with modulus greater than one also lead to local determinacy. The oscillating hyperinflation threat is as good – or better, if we wish to “coordinate equilibria” by ruling out unreasonable expectations. Here
\[
\phi < -\left(1 + 2 \frac{1+\beta}{\sigma \kappa}\right)
\]
serves just as well to rule out multiple equilibria. In models with more complex policy rules including responses to output and expected future inflation, complex possibilities emerge. Cochrane (2011c) contains plots of the determinacy regions for
a variety of such models. The lesson here is even clearer: \( \phi_\pi > 1 \) is neither necessary nor sufficient to generate explosive eigenvalues, so this model really does not embody the standard intuition about the Taylor rule.

In this case, the nominal explosions can induce real explosions, \( E_t x_{t+j} \to \infty \), and it can induce explosions faster than the interest rate so \( E_t \beta^j x_{t+j} \to \infty \). One might rejoice that we now can rule out such solutions by appeal to the real transversality condition. However, the model of price stickiness that turns a nominal explosion to a real explosion, and especially its linearization, is not designed to describe extreme inflation and deflation. In actual hyperinflations and deflations, output does not go to infinity or negative infinity. Barter or use of foreign currencies takes over.

To get a solution quickly, you can follow the above approach analytically. You wade through a mountain of algebra, and then notice how that mountain simplifies itself nicely. You get to the same answer more quickly with the method of undetermined coefficients. Specializing to the monetary policy shock only, guess an answer of the form

\[
\pi_t = \alpha_\pi v_t^i \\
x_t = \alpha_x v_t^i.
\]

Substitute this guess into (16.46)-(16.55), giving

\[
\alpha_x v_t^i = \rho \alpha_x v_t^i - \sigma \left( \phi \alpha_\pi v_t^i + v_t^i - \rho \alpha_\pi v_t^i \right) \\
\alpha_\pi v_t^i = \beta \rho \alpha_\pi v_t^i + \kappa \alpha_x v_t^i.
\]

Since these equations must hold for any \( v_t^i \), conclude

\[
\alpha_x = \rho \alpha_x - \sigma \left[ 1 + (\phi - \rho) \alpha_\pi \right] \\
\alpha_\pi = \beta \rho \alpha_\pi + \kappa \alpha_x,
\]

\[
(1 - \rho) \alpha_x = -\sigma \left[ 1 + (\phi - \rho) \alpha_\pi \right] \\
(1 - \beta \rho) \alpha_\pi = \kappa \alpha_x. \tag{16.60}
\]

Eliminating \( \alpha_x \) and solving,

\[
(1 - \beta \rho) (1 - \rho) \alpha_\pi = -\sigma \kappa \left[ 1 + (\phi - \rho) \alpha_\pi \right] \\
[(1 - \beta \rho) (1 - \rho) + \sigma \kappa (\phi - \rho)] \alpha_\pi = -\sigma \kappa
\]
and finally, therefore

\[ \pi_t = -\frac{1}{\phi - \rho + \frac{(1-\beta\rho)(1-\rho)}{\sigma\kappa}} v_t \quad \text{(16.61)} \]

\[ x_t = \frac{1-\beta\rho}{\kappa} \pi_t \quad \text{(16.62)} \]

\[ i_t = \left( \rho - \frac{(1-\beta\rho)(1-\rho)}{\sigma\kappa} \right) \pi_t. \quad \text{(16.63)} \]

I used (16.54) and (16.60) in the latter two equations.

You can see the inflation response (16.63) is a natural generalization of the simple sticky price model (16.43),

\[ \pi_t = -\frac{1}{\phi - \rho + \frac{1}{\sigma\kappa}} v_t, \]

and of the frictionless model (16.36),

\[ \pi_t = -\frac{1}{\phi - \rho} v_t. \]

Figure 16.11 presents the response to monetary policy shocks in this model, for a variety of persistence parameters \(\rho\). Contrast to Figure 16.5 of the simplified new-Keynesian model, or Figure 16.2 of the frictionless model, and you can see the behavior is qualitatively the same.

Again, \(\rho = 1\) gives a super-Fisherian or super-neutral response, even though prices are sticky:

\[ \pi_t = -\frac{1}{\phi - 1} v_t \]

\[ x_t = -\frac{1-\beta}{\kappa} \frac{1}{\phi - 1} v_t \]

\[ i_t = -\frac{1}{\phi - 1} v_t \]

The inflation rate moves immediately and matches the interest rate one for one.

Output, not shown, rises by a small \((1 - \beta\) is small\) amount and stays there. This model features a small permanent inflation-output tradeoff. That vanishes with
\[ \beta = 1 \] and is not considered a serious prediction of the model. As before, negative shocks give rise to positive interest rates, because the \( \phi \pi_t \) term in \( i_t = \phi \pi_t + v_t \) wins.

Again, there is an "open-mouth" value of \( \rho \), for which output and inflation move with no actual movement of interest rates, where \( \phi \pi_t \) and \( v_t \) exactly balance. From (16.63), this situation occurs for \( \rho \) that solves

\[
\rho - \frac{(1 - \beta \rho)(1 - \rho)}{\sigma \kappa} = 0.
\]

The solution of this equation is

\[
\rho = \frac{1}{2\beta} \left( 1 + \beta + \kappa \sigma - \sqrt{(1 + \beta + \kappa \sigma)^2 - 4\beta} \right).
\]

This is the stable eigenvalue (16.58) in the \( \phi = 0 \) case, and the speed at which multiple-equilibrium shocks dissipate in (16.50).

For even more transitory \( \rho \), we obtain the standard result – a negative \( v_t \) shock lowers interest rates \( i_t \) and raises inflation. The standard interpretation of this result
is that the new Keynesian model delivers a negative response for transitory shocks. Since we observe transitory responses to monetary policy shocks in VARs, this is comforting, though the model’s clear prediction that more permanent shocks – such as we seemed to observe in the decade after 1980 and the decade after 2008 – are immediately Fisherian is, let us say, less well popularized.

But even this interpretation is false, as we have seen. Once we get past the AR(1), there is no connection between the persistence of shocks and the sign of the inflation response to monetary policy shocks, as discussed at the end of the last section. The sign of the inflation jump on announcement is a pure equilibrium-selection policy, which can be paired with any persistence of the interest rate policy.

16.9.3 Determinacy, optimal policy, and selection

Writing policy as an interest rate policy plus equilibrium selection policy, \( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \), we find again that the central bank can achieve any inflation process \( \{\pi^*_t\} \) or output process \( \{x^*_t\} \) it wants, including zero inflation or output gap, ex-post. To achieve these results, the central bank must follow a “stochastic intercept” \( i^*_t \) policy, or equivalent choose disturbances \( v_t \) that respond to and thus systematically offset shocks to the economy. Whether the central bank can or should do this in practice is open to debate. For theory, though, these policies highlight that the \( \phi \) equilibrium selection part of the rule is completely irrelevant to stabilization policy. \( \phi \) appears to matter when a stochastic intercept is ruled out, and one ties the reaction of off-equilibrium \( i \) and \( \pi \) to the equilibrium relation between \( i^* \) and \( \pi^* \).

We express the multiple equilibrium analysis, the matrix equivalent of \( E_t \pi_{t+1} = \phi \pi_t \), and the non-identification of \( \phi \) in the context of the full model.

As before, we gain a lot of intuition by expressing the policy rule as [King (2000)] suggests,

\[
i_t = i^*_t + \phi (\pi_t - \pi^*_t)
\]

(16.64)

where \( i^*_t \) and \( \pi^*_t \) represent the equilibrium the central bank wishes to implement. As before \( \phi > 1 \) will threaten sufficient explosions to ensure that \( i^*_t \) and \( \pi^*_t \) are the unique locally bounded solution, we only observe \( \{i^*_t\} \) and \( \{\pi^*_t\} \) and a unique corresponding \( \{x^*_t\} \), and \( \phi \) disappears from equilibrium dynamics.

As before, the central bank can achieve any \( \{\pi^*_t\} \) or \( \{x^*_t\} \) it wishes. We can then calculate the required interest rate policy \( i^*_t \) in [16.64], and the second half of that
equation is the equilibrium selection policy. We can as usual reexpress the answer as
\[ i_t = \phi \pi_t + v_t. \]

Two examples are interesting and instructive: Write (16.46)-(16.48) as
\[ x^*_t = E_t x^*_{t+1} - \sigma \left( i^*_t - E_t \pi^*_{t+1} \right) + v^x_t \]
\[ \pi^*_t = \beta E_t \pi^*_{t+1} + \kappa x^*_t + v^\pi_t. \]

To set \( \pi^*_t = 0 \), we need
\[ x^*_t = \frac{1}{\kappa} v^\pi_t \]
and hence
\[ i^*_t = \frac{1}{\sigma} \left( -E_t v^\pi_{t+1} + v^\pi_t \right) + \frac{1}{\sigma} v^x_t. \]

To set \( x^*_t = 0 \), we need:
\[ \pi^*_t = E_t \sum_{j=0}^{\infty} \beta^j v^\pi_{t+j} \]
and hence
\[ i^*_t = E_t \sum_{j=0}^{\infty} \beta^j v^\pi_{t+1+j} + \frac{1}{\sigma} v^x_t. \]

One often asks optimal policy questions of new-Keynesian models [Woodford (2003), Ch. 6 for example]. Welfare can be reduced to minimizing the sum of output and inflation variation
\[ \min \lambda \text{var}(x^*_t) + (1 - \lambda) \text{var}(\pi^*_t). \]

The \( x^*_t = 0 \) and \( \pi^*_t = 0 \) are simple examples of such policies, and enough to see the basic point of how they work.

More generally, we can compute the interest rate policy \( i^*_t \) that generates any desired inflation path \( \{\pi^*_t\} \). From (16.65)-(16.66)
\[ i^*_t = E_t \pi^*_{t+1} + \frac{1}{\sigma} \left( E_t x^*_{t+1} - x^*_t \right) + \frac{1}{\sigma} v^x_t \]
\[ i^*_t = E_t \pi^*_{t+1} + \frac{1}{\sigma \kappa} \left[ (-\beta E_t \pi^*_{t+2} + E_t \pi^*_{t+1}) - (-\beta E_t \pi^*_{t+1} + \pi^*_t) - E_t v^\pi_{t+1} + v^\pi_t \right] + \frac{1}{\sigma} v^x_t \]
\[ i^*_t = \frac{1}{\sigma \kappa} \left[ -\beta E_t \pi^*_{t+2} + (1 + \beta + \sigma \kappa) E_t \pi^*_{t+1} - \pi^*_t \right] + \frac{1}{\sigma \kappa} \left[ -E_t v^\pi_{t+1} + v^\pi_t \right] + \frac{1}{\sigma} v^x_t \]
\[ (16.69) \]
You can see here the generalization of $i^*_t = E_t \pi^*_{t+1}$, smeared out by dynamics and with the addition of shocks.

Relative to the earlier discussions of this point, I have added disturbances $v^\pi$ and $v^x$, and you see how the equilibrium interest rate target $i^*_t$ reacts to these shocks. We can write the policy rules following from equations (16.67), (16.68) and (16.69) in conventional form as

$$i_t = (i^*_t - \phi \pi^*_t) + \phi \pi_t = \phi \pi_t + (i^*_t - \phi \pi^*_t).$$

The first form thinks about $(i^*_t - \phi \pi^*_t)$ as a stochastic intercept to the rule. The second form thinks about $(i^*_t - \phi \pi^*_t)$ as a monetary policy disturbance that reacts to other shocks.

The intercept or disturbance should react to, and to offset, the other shocks in the economy. In the $\pi^*_t = 0$ case (16.67) gives the term $(i^*_t - \phi \pi^*_t)$ directly. In the $x^*_t = 0$ case,

$$i^*_t - \phi \pi^*_t = (1 - \phi) E_t \sum_{j=0}^{\infty} \beta^j v^\pi_{t+1+j} + \frac{1}{\sigma} v^\pi_t.$$

Good policy, in this model, does not just follow a rule $i_t = \bar{i} + \phi \pi_t$.

Following such a rule isn’t as easy as it sounds, of course, as the $v^\pi$ and $v^x$ shocks are not directly measurable, by us or by central banks. The art of central banking, as currently construed, consists of distinguishing “supply” from “demand” and other shocks (financial) and reacting accordingly, stimulating in response to deficient demand, abstaining when it’s deficient supply. The ensuing debate whether central banks should follow such advice has gone on, rightly, for decades if not centuries.

Milton Friedman argued for a fixed money growth rule, not because he denied that optimal control of a model with shocks resulted in such a rule, but because his deep study of history persuaded him that central bankers, in real time, are not capable of measuring shocks and reacting appropriately. Reacting to shocks that require central bank divination looks a lot like discretion, and raises the whole time-consistency and rules vs. discretion debate. I read in John Taylor’s advocacy of an interest rate rule today much the same mistrust, along with a desire to stabilize expectations. Markets can’t tell easily stochastic-intercept, or shock-response deviations from deviations that are discretionary and unpredictable.

Current discussions of central bank policy might be phrased in terms of a rule

$$i_t = r^*_t + \pi^* + \phi_v (\pi_t - \pi^*) + \phi_x x_t + v^i_t. \quad (16.70)$$
\( \pi^* \) is the central bank’s long-run inflation target, 2%. \( r^* \) is a very long-run slow movement in the “natural rate,” reflecting “global imbalances,” trend growth, and so on. The current active debate concerns whether \( r^* \) has declined from about 2% to 1% or less, and consequently whether nominal interest rates should asymptote to something like 4% or something like 2% or 3%. \( \nu^t \) then consists of short-run responses to other shocks, above the countercyclical movement in response to the output gap \( \phi_x x_t \); financial events such as 2008, 1987, Y2K and so on are examples. This discussion breaks the stochastic intercept debate into three components \( (r^*_t, \phi_x x_t, \nu^t) \) based on frequency and economic mechanism.

This discussion is not tied to new-Keynesian models. The policy discussions are almost completely in the context of old-Keynesian models, so \( \phi \) in (16.70) represents stabilizing rather than equilibrium-selection policy. The question is how the observed, equilibrium interest rate \( i^*_t \) should react to events, so this analysis is the same if fiscal theory selects equilibria rather than a \( \phi(\pi_t - \pi^*_t), \phi > 1 \) threat.

This optimal-policy digression has a larger point for us. In the context of the new-Keynesian model, we learn that the \( \phi \) reaction part of the rule is completely irrelevant to stabilization policy. The stochastic intercept can do any amount of stabilization or optimal policy necessary, to the extreme of setting either inflation or output gap to zero always, even ex-post.

So why is there so much study of optimal \( \phi \)? (For example, Woodford (2003) Chapter 6.) The equilibrium dynamics here are completely unaffected by the value of \( \phi \). When \( \pi_t = \pi^*_t, \phi(\pi_t - \pi^*_t) = 0 \) for any \( \phi \). The answer is, such optimal \( \phi \) calculations rule out the stochastic intercept, and thereby tie equilibrium dynamics to the off-equilibrium threats. But if the central bank contemplates any deviations from a rule, any reaction to temporary disturbances, any variation in the natural rate, any time-varying inflation target, i.e. \( i^*_t \) and \( \pi^*_t \), then these alone are powerful enough to accomplish everything the central bank can do in equilibrium.

Now, let us return to the determinacy issues. To study potential multiple equilibria, define deviations from a given equilibrium, following King (2000). Use tildes to denote deviations of an alternative equilibrium \( x_t \) from the * equilibrium, \( \tilde{x}_t \equiv x_t - x^*_t. \) Subtracting, deviations must follow the same model as (16.65)-(16.66) and (16.64).
but without constants or disturbances.

\[
\begin{align*}
\tilde{i}_t &= \tilde{r}_t + E_t \tilde{\pi}_{t+1} \\
\tilde{x}_t &= E_t \tilde{\pi}_{t+1} - \sigma \tilde{r}_t \\
\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \gamma \tilde{x}_t, \\
\tilde{\pi}_t &= \phi \tilde{\pi}_t
\end{align*}
\]

(16.71) (16.72) (16.73) (16.74)

In matrix notation,

\[
\begin{bmatrix}
E_t \tilde{x}_{t+1} \\
E_t \tilde{\pi}_{t+1}
\end{bmatrix} = \frac{1}{\beta} \begin{bmatrix}
\beta + \sigma \gamma & -\sigma (1 - \beta \phi) \\
-\gamma & 1
\end{bmatrix} \begin{bmatrix}
\tilde{x}_t \\
\tilde{\pi}_t
\end{bmatrix}.
\]

(16.75)

This is the same transition matrix as \([16.56]\) with eigenvalues \([16.58]\). \(\phi > 1\) generates two explosive \((\lambda > 1)\) eigenvalues and \(\|\phi\| < 1\) leaves one stable \((\lambda < 1)\) eigenvalue.

Thus, if the policy rule is sufficiently active, any equilibrium other than \(\tilde{i}_t = \tilde{y}_t = \tilde{\pi}_t = 0\) is explosive. Ruling out such explosions, we now have the unique locally-bounded equilibrium. This is the expression in the full model that the rule \(i_t = \bar{i}_t + \phi (\pi_t - \pi^*_t)\) and \(\phi > 1\) leads to a unique locally bounded equilibrium \(\pi_t = \pi^*_t\).

As before, this expression makes it immediately clear that \(\phi\) does not enter the equilibrium dynamics of the observed equilibrium variables \(\bar{i}_t, \pi^*_t, x^*_t\). It is entirely an threat used to select equilibria. *Interest rate policy* \(\{\bar{i}_t\}\), which may react and correlate with \(\pi^*_t\) and \(x^*_t\), or structural disturbances \(v_t\), in all sorts of interesting ways, including observed Taylor rule regressions, is completely distinct from *equilibrium selection policy*, the reaction \(\phi (\pi_t - \pi^*_t)\), never seen in equilibrium, by which the central bank makes threats to force a single equilibrium to emerge.

As in the simple model, the point of policy is to induce *explosive* dynamics, eigenvalues greater than one, not to “stabilize” so that the economy always reverts after shocks. The Fisherian response to permanent and to expected interest rate rises of this three-equation model is tied to this effort. Expected interest rate rises \(E_t i_{t+j} = \phi E_t \pi_{t+j}\) raise subsequent inflation.

The analysis so far has exactly mirrored my analysis of the simple model of Section [16.1]. So, in fact, that model does capture the determinacy issues, despite its absence of any frictions. Conversely, determinacy in the new-Keynesian model does not fundamentally rely on frictions, the Fed’s ability to control real rates, or a Phillips curve.
16.10. INTEREST RATE TARGETS: A SUMMARY

The two points of this section add up to a nice view of new-Keynesian monetary policy. The expression of the Taylor rule as \( i_t = i_t^* + \phi (\pi_t - \pi_t^*) \) clearly separates interest rate policy from equilibrium selection policy. One can read its instructions as:

First, the central bank should set the equilibrium interest rate, reacting appropriately to shocks in the economy as suggested by the stochastic intercept rules (16.67) and (16.68) – as constrained by the difficulty of measuring shocks and communicating a rule-based rather than discretionary policy allows. Then, the central bank should decide on a separate and distinct equilibrium-selection policy. If it set \( i_t = i_t^* \) as a time-varying state-contingent peg, in this model, there are still multiple equilibria. It needs to make alternative-equilibrium blow-up-the-world threats to enforce its desired unexpected inflation \( \pi_t^* - E_{t-1} \pi_t^* \).

This separation also makes completing the model by fiscal theory easy to contemplate. Keep absolutely everything about \( \{i_t^*, \pi_t^*, x_t^*, v_t\} \). In lieu of selecting equilibria with \( \phi(\pi_t - \pi_t^*) \), or the enhanced threats described above, and counting on a passive fiscal policy to produce the needed innovation in the present value of fiscal surpluses \( \varepsilon_t^* \), specify directly that fiscal innovation.

And in this form we see clearly the lack of identification in the full model context. The parameter \( \phi \) does not enter equilibrium dynamics. If one accepts empirical evidence that interest rates vary more than one for one with inflation, that evidence says that equilibrium interest rates \( i_t^* \) vary more than one for one with equilibrium inflation, \( \pi_t^* \). Such observations tell us nothing about determinacy issues, whether deviations from equilibrium \( i - i^* \) and \( \pi - \pi^* \) follow the same patterns. A more than one-for-one relation between \( i_t^* \) and \( \pi_t^* \) is perfectly consistent with a less than one-for-one relationship \( \phi < 1 \) between deviations \( (i - i^*) \) and \( (\pi - \pi^*) \). An less than one-for-one relationship between \( i_t^* \) and \( \pi_t^* \) may emerge from a locally determinate regime in which the response to alternative equilibria is stronger. Moreover, there is still no way for agents in the model to learn \( \phi \) by running regressions on any data they can observe.

16.10 Interest rate targets: A summary

I conclude that active interest rate targets, with a globally passive fiscal policy, are not, in fact, a coherent alternative theory of inflation or the price level. Replacing \( \phi(\pi_t - \pi_t^*) \) and related blow-up-the-world threats with the government debt valuation equation, and an active fiscal policy that does not react to off-equilibrium price levels can maintain all the good parts of the new-Keynesian structure, selecting equilibria...
in a different way.

It seemed that active $\phi > 1$ interest rate targets, with globally passive fiscal policy, are a new theory that can determine the price level or inflation rate, overcoming the indeterminacy or instability of interest rate targets from classical theory. I conclude after this tour that it is not successful in that endeavor.

Once we really understand the new-Keynesian model, we see that it turns old-Keynesian intuition on its head. The old-Keynesian model is unstable and indeterminate under an interest rate peg. The new-Keynesian model is stable and indeterminate. The Taylor rule $i_t = \phi \pi_t$, $\phi > 1$, that was born to bring stability to old-Keynesian models, instead induces instability to new-Keynesian models, in an attempt to overcome multiple equilibria. But it doesn’t even do that. The multiple equilibria are still there. And central banks simply don’t act this way – they do not deliberately destabilize the economy. Moreover, this reinterpretation of the Taylor rule is not enough – we have to imagine the central bank threatens to blow up the economy in finite time, setting policy so that “no equilibrium can form,” to prune multiple equilibria, though such an action if even possible would be even more of a disaster for the bank’s objectives than the $i_t = \phi \pi_t$ threat. More deeply, monetary policy in the new-Keynesian is not about moving or stabilizing inflation, but about making the economy jump to a different one of many multiple equilibria, and selecting equilibria with unpleasant threats.

New-Keynesian models were developed in the hope that a model that follows the Lucas-critique rules could justify ISLM intuition, at long last providing the “microfoundations” of Keynesian economics that a previous half-century of effort had failed to produce. Given the hope that motivated the literature, you can see why it took so long to realize that its equations were so diametrically opposite. The difference between determinacy and stability, and the difference between stabilizing inflation and selecting multiple equilibria is subtle. Given the multiple equilibria that $i_t = r + E_t \pi_{t+1}$ imply, it was natural to hope that a Taylor rule would some how cure it. Given that failure, it was natural to hope that other classic devices for curing inflation would select equilibria. The continuing presence of so many irrelevant parts of old-Keynesian policy ideas obscuring equilibrium selection is testament to how hard understanding this model has been. It took me a long time too.

But fiscal theory of monetary policy and new-Keynesian models are not that different. 99% of the new-Keynesian structure need not be thrown out with the bathwater. The intertemporally optimizing IS curve is fine. The explicit models of price stickiness are fine. The further elaborations of large scale models are fine. There is one and only
16.10. INTEREST RATE TARGETS: A SUMMARY

one problem, and it’s easy to fix: Equilibrium selection. If, rather than specify that destabilizing threats $\phi(\pi_t - \pi_t^*)$ or blow-up-the-world threats select the equilibrium, and fiscal policy passively adjusts to any price level, including off-equilibrium price levels, we simply restore the government debt valuation equation, specifying that fiscal policy does not adjust to off-equilibrium price levels, we accomplish this last task, equilibrium selection, while keeping the rest of the model intact.

The results, of course, may be different. As we have seen, the new-Keynesian model generates a jump in inflation after a monetary policy shock with a simultaneous, “passively” induced fiscal contraction. To get the same result (the observational equivalence theorem still haunts us) from a fiscal theory of monetary policy model, we have to specify that fiscal shock directly and coincidentally accompanies the monetary policy shock. That may characterize data, but it is a less compelling description of a monetary policy shock.

16.10.1 Adaptive expectations?

Why not just retreat to adaptive expectations? First, that model fails empirically. Most dramatically in recent history, it predicts a deflation spiral at the zero bound which did not happen. Second, while somewhat irrational expectations and price stickiness may be useful additional ingredients to explain postwar time series or to understand the reaction to never-before-seen events, it would be unfortunate to require these ingredients for even the most basic model – to deny that there is any simple supply and demand model of inflation or the price level on which to build models with frictions. Such a mechanistic model cannot maintain the Lucas-critique hope to work once policy makers exploit it and people get used to the results, or to work out of its institutional framework, such as studying large inflations or financial innovations.

Why not just stick with adaptive expectations, one might reasonably ask? It produces a set of equations that embody the late 1970s ISLM intuition expounded by policy makers – higher interest rates lower inflation – and it gives us a model with determinate inflation, if not quite a price level, and none of these multiple equilibrium problems.

The first reason not to follow this path is empirical. This traditional view predicts quite clearly that if the interest rate does not or cannot move more than one for one with inflation, inflation or deflation should be unstable. Fear of a “deflation spiral” was widespread when the US and Europe hit zero interest rates in 2008, and when
Japan did so in 1994. Yet in 8 years at the zero bound in the US, 10 years in Europe, and a quarter century in Japan, inflation stayed remarkably stable, and no spiral emerged. This theory simply failed its second and greatest prediction. (Failing to predict the rise of inflation in the 1970s, and the quick end of inflation in the 1980s was the first grand failure of ISLM modeling.) The most natural interpretation of this episode is that inflation is stable at a peg, as the rational expectations model predicts. (Cochrane (2018) makes this point in detail. Standard new-Keynesian models have almost as much trouble with the zero bound. We return to interpretation of the episode in section 20.)

A second reason is more esthetic or philosophical, but esthetics are important. Somewhat irrational expectations and sticky prices are fine ingredients as icing on a cake, to understand dynamics of small inflations, and to understand how inflation reacts in the aftermath of never-before-seen policies and events. But if we follow the old-Keynesian path above, turning off the forward-looking elements of the rational expectations model, we put irrational expectations and sticky prices square in the foundations of monetary economics. We say that we cannot understand the basics of price level determination, and the basic sign and stability properties of monetary policy without irrational expectations and sticky prices. We say there is no truly economic theory by which the price level in our economy is determined. There is nothing like the simple money supply = money demand story that undergraduate courses start with. It’s all a conjuring trick, clever bureaucrats fooling a naive populace. And if they ever woke up and figured out what’s going on, or if the internet made prices less sticky, the whole edifice falls apart and we have no theory of the price level at all.

ISLM, with forward-looking behavior turned off, isn’t really an economic model at all. It is at best set of equations that captures historical correlations. It is surely not “policy invariant.” It will not survive the Lucas critique (Lucas (1976)), the whole reason for starting the new-Keynesian agenda in the first place. Its parameters will not stay still as they are regularly and systematically exploited for policy. It does not allow us to ask, what if the Fed starts paying interest on reserves? What if people start using a lot of bitcoin? What if the internet starts making prices less sticky? Or, what if the interest rate hits the zero bound? (It makes a prediction there, of a deflation spiral, but a dramatically false one!) We should also want a theory that works beyond the relatively quiet (so far) postwar US time series. A theory of the price level should extend to currency crashes, hyperinflations, currency reforms, and so on.

In sum, it is one thing to say one can construct a plausible model for postwar US time
series with adaptive expectations (somehow excusing the 1980s and the 2008-2016 period), or to add expectations-formation mechanisms or learning to a simple model. It is quite another to say one must do so, one cannot build a foundation for price level determination without irrational expectations and sticky prices. It is surely a worthwhile path to pursue to find out if there is any basic supply and demand model on which to build the edifice of monetary economics.

If the choice were only between the two models outlined here, one might well choose the adaptive expectations model as the lesser of evils. But the rational expectations model enhanced with fiscal theory provides a model that is simple, stable, and consistent with the evidence. That surely is worth exploration before giving up on the “economics” part of “monetary economics,” before giving up hope that like everything else in economics one could start with a coherent supply and demand model and then add frictions.
Chapter 17

Monetarism

The most important and durable alternative to the fiscal theory is based on fiat money. Money is intrinsically worthless and unbacked. There is a special inventory or transactions demand for this money, making people wiling to hold some of it, despite its intrinsic worthlessness and despite a rate of return less than bonds. There is a limited supply of money. Money demand $MV = PY$ intersected with a limited supply $M = M^s$ leads to a determinate $P$. I’ll lump these ideas together under a common term, “monetarism,” despite differences between early theorists such as Irving Fisher, Milton Friedman whose views define classic “monetarism,” and cash-in-advance, money-in-utility, overlapping generations, search-theoretic and related formal theories of money.

This idea is older than interest rate targets, and more durable. In the end, once an interest rate target discussion gets too confusing, many economists will retreat to $MV = PY$ as a foundation for price level determination, regarding the interest rate target as an indirect way of setting the money supply $M$. Monetarist ideas continue to loosely pervade discussions of central bank policy. We still call it “monetary policy” after all! It’s not just advocates of the view that central banks should return to targeting monetary aggregates. Should central banks return to a small amount of reserves that pay no interest, or less interest, and implement interest rate targets by controlling that quantity? Why do they target the level of reserves anyway and worry about the “size of the balance sheet,” rather than run a pure corridor or peg, here is the rate, come and get as much as you want? And at the zero bound, many economists and central bankers opined quickly that the answer could be dropped from helicopters.
Logically and historically, I should have started with money and moved on to interest rate targets. I went in the opposite direction because interest rate targets in new-Keynesian models are now the nearly universal framework for thinking about monetary policy, and monetarist ideas have faded from research and policy analysis.

Monetary - fiscal coordination has always been part of fully-described monetarist ideas. The government can gain seigniorage by printing non-interest-bearing money, so for a fiat-money regime to successfully control inflation, the government must abstain from printing money to finance fiscal deficits. In fact, one may think of the thousand-year history of paper money as, basically, a long voyage of discovery of institutional and legal constraints that keep fiat money from being quickly inflated away by fiscally-pressed governments. But as long as the government has adequate fiscal space, the fiscal part lies in the background of the analysis of modern economies, along with other caveats such as limitations on inside money, i.e. reserve requirements, legal restrictions on the use of foreign money, and so forth.

In section 6.3 we added money demand,

\[ M_t V_t = P_t Y_t, \]  

(17.1)

to the fiscal theory. With money that does not pay interest, and ignoring inside money, we saw that the government debt valuation equation becomes

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} \right) \]

or

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} \right]. \]

We considered a “active-money, passive fiscal” regime, in which a fixed money supply \( M_t = M_t^* \) and money demand (17.1) determine the price level. In that case, the government must adjust surpluses ex-post so that the government debt valuation equation holds – it must follow a “passive” fiscal policy. This is a formal statement of the traditional idea that control of a fiat currency requires fiscal coordination. But, as we noted, seigniorage is small in low-inflation advanced economies, so really this consideration fades in to the background for inflation as a minor technical footnote. Seigniorage and fiscal limits crop up only in describing hyperinflations or currency collapses.
17.1 Interest-elastic money demand and multiple equilibria

With interest-elastic money demand, control of the money supply is not enough to determine the price level. Multiple inflationary or deflationary equilibria can emerge, and sunspots can cause the economy to jump from one to another arbitrarily. Adding the fiscal theory, in a coordinated money-fiscal regime, solves the multiple-equilibrium problem.

\[ MV = PY \]

seems to determine the price level, but in fact money demand is interest-elastic. \( V \) is not a number, but a rising function of the nominal interest rate. We really should write

\[ M_t V(i_t) = P_t Y_t \]

with \( V'(i) > 0 \). When nominal interest rates are higher, the opportunity cost of holding money is larger. People go to the ATM machines more often and hold less money on average. This fact means that even a fixed money supply is not sufficient to determine the price level with passive fiscal policy. \( MV = PY \) suffers the same indeterminacy problems as interest rate pegs suffer.

To exhibit the problem, consider a simple example. Let output be constant, and let money demand be a declining function of interest rates,

\[ M_t = P_t Y V_0^{-\alpha i_t} \] (17.2)

or in logs

\[ m_t - p_t - y = -\alpha i_t v_0. \]
Introduce the Fisher equation

\[ i_t = r + E_t \pi_{t+1} = r + E_t p_{t+1} - p_t. \]

The price level paths \( \{p_t\} \) consistent with money demand and intertemporal optimization are then given by

\[ m_t - p_t - y = -\alpha v_0 (r + E_t p_{t+1} - p_t). \] (17.3)

\[ E_t p_{t+1} = \frac{1 + \alpha v_0}{\alpha v_0} p_t - \frac{1}{\alpha v_0} (m_t - y) - r. \]

Suppose now that money is constant \( m_t = m \). There is a steady state price level

\[ p = m - y + \alpha v_0 r, \] (17.4)

or in levels

\[ P = MV/Y = MV^0_{\alpha r}/Y. \]

The steady-state price level is higher as the real rate, which equals the interest rate, is higher, because money demand is lower.

But there are other equilibria as well. From (17.3), the full set of equilibrium price levels is any sequence with

\[ (E_t p_{t+1} - p) = \theta (p_t - p), \] (17.5)

where

\[ \theta = \frac{1 + \alpha v_0}{\alpha v_0} > 1. \]

There is a whole family of solutions. Writing (17.5) as

\[ (p_{t+1} - p) = \theta (p_t - p) + \delta_{t+1}, \]

the model restricts \( E_0 \delta_{t+1} = 0 \), but it can take any value ex-post. The full set of solutions is

\[ p_t - p = \theta^t (p_0 - p) + \sum_{s=1}^{t} \theta^{t-s} \delta_s. \]

Yes, the alternative solutions are explosive. At any date for \( p_t \neq p \), people expect explosive hyperinflation or hyperdeflation in logs. But nothing in the specification of
the model so far rules out these alternative solutions, just as we could not rule out
nominal explosions \( E_t \pi_{t+1} = \phi \pi_t, \phi > 1 \) in the simple new-Keynesian model.

These multiple paths are often called “speculative hyperinflations.” If one reads
causality from future to past, changing expectations of future price levels will cause
the price level today to jump, and then the hyperinflation can take off on its own
with no external shock.

For a general money process \( \{m_t\} \), we can solve (17.3) forward, to
\[
E_t p_{t+1} = \theta p_t - (\theta - 1) (m_t - y - \alpha v_0 r).
\]
\[
p_t = \left( 1 - \frac{1}{\theta} \right) (m_t - y - \alpha v_0 r) + \frac{1}{\theta} E_t p_{t+1}.
\]
\[
p_t = \left( 1 - \frac{1}{\theta} \right) \sum_{j=0}^{\infty} \frac{1}{\theta^j} m_{t+j} - (y + \alpha v_0 r) + \lim_{T \to \infty} \frac{1}{\theta^T} E_t \left( p_{t+T} \right) \tag{17.6}
\]

It is tempting to set the right hand term to zero and to declare a unique forward-
looking equilibrium. The price level depends beautifully on a forward-looking mov-
ing average of money rather than today’s money alone, just as in the simple new-
Keynesian model, we found inflation depends on an a forward-looking moving average
of monetary policy disturbances. But there is again no reason to set to zero the last
term of (17.6).

As with interest rate targets, many papers simply ignore the problem and pick the
bounded solution. Others assert correctly that the solution without the last term
is the unique bounded solution, or quickly say they “focus attention” on bounded
solutions. But this is an extra criterion, not part of the economic model.

Once again, the fiscal theory solves this multiple-equilibrium problem. The govern-
ment debt valuation equation is a part of this (implicit, here, explicit below) model.
The monetary analysis throws it out by assuming a globally passive fiscal policy: sur-
pluses adjust to whatever price level emerges, not just from the time-varying choices
of the monetary authority, or time-varying taxes and spending, but from whatever
multiple-equilibrium hopscotching the price level happens to do. Well, let us re-
verse that assumption and add an active fiscal policy to the constant money supply
monetary policy.

In the perfect foresight case, we have
\[
\frac{B_{-1}}{F_0} = \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]
This condition picks the one missing element, $P_0$, and we now fully determine the price level. With no news, subsequent versions of the valuation equation hold automatically, determining debt sales $B_t$.

In the stochastic case, similarly,

$$\frac{B_t}{P_t} (E_{t+1} - E_t) \left( \frac{P_t}{P_{t+1}} \right) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}$$

picks the unexpected inflation at that date, and

$$\frac{B_t}{P_t} e_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}$$

with expected inflation picked by (17.5) determines nominal bond sales $B_t$.

The solutions picked in this way will generically be one of the explosive solutions, not the steady state or bounded solution. We don’t routinely see explosive inflation, one might object. But governments are not so pig-headed as to set constant money forever in the face of exploding inflation and (especially) to follow fiscal policies that validate any inflation that comes along. A sensible government will arrange its fiscal affairs and monetary affairs jointly to back up the price level it wants.

In sum, interest-elastic money demand, along with control of the money supply alone, is not enough to determine the price level. If we add fiscal theory, we can solve the indeterminacy problem, and produce a sensible monetary-fiscal regime, as fiscal theory plus interest rate targets did. I don’t pursue this further because few governments these days are controlling monetary aggregates.

This section is based on the famous Cagan (1956) analysis of hyperinflations. Cagan used adaptive expectations. Sargent and Wallace (1973) used rational expectations, which leads to the forward-looking solutions and determinacy problems here.

### 17.2 Money in utility

We examine the classic money in utility function model. This section introduces the utility function and budget constraints, and defines equilibrium.

I review two standard explicit models for producing a money demand and monetary price level determination, the money in the utility function model here and the cash in advance model in the next section.
The models serve an immediate purpose, to examine more carefully the analysis of the last section. Does \( MV = PY \) and passive fiscal policy really not determine the price level, if we spell out a model completely? No, it turns out. Adding fiscal theory, however, the two models are useful workhorses for studying monetary-fiscal policies when there are special liquid assets. They are worth study for that larger purpose.

To set up the simplest monetary model, I introduce money in the utility function. The representative household maximizes

\[
\max E \sum_{t=0}^{\infty} \beta^t u \left( c_t, \frac{M_t}{P_t} \right). \tag{17.7}
\]

We don’t think people literally derive pleasure from money, taking daily baths in it like Scrooge McDuck. Money in the utility function stands in for the way money makes it easier to purchase goods. Models that detail the search, information, or shopping time frictions that really motivate holding liquid assets end up with something like this indirect utility function, or at least so we hope. This is the easiest model, not the one with deep micro-foundations.

The day follows our usual timing. The household holds nominal one-period government bonds \( B_{t-1} \) and government money \( M_{t-1} \) overnight. Then it receives an endowment \( Y_t \), consumes \( c_t \), pays net real taxes \( s_t \) and buys new bonds \( B_t \) at price \( Q_t \). The household’s period budget constraint is

\[
B_{t-1} + M_{t-1} + P_t(Y_t - c_t) = Q_t B_t + M_t + P_t s_t.
\]

The household operates in complete contingent claim markets with state price \( \Lambda_t \). I don’t write such claims in the budget constraint, since with identical representative households they net out in the end. The nominal bond price is as usual

\[
Q_t = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right).
\]

Money and debt holdings must also satisfy a lower bound, \( M_{t-1} + B_{-1} > -B \), and their optimal choices include transversality conditions. Thereby the household must satisfy the present value budget constraints, either

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right). \tag{17.8}
\]
or
\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right]. \tag{17.9}
\]

The government sets a sequence \(\{M^*_t, B^*_t, s_t\}\). The government obeys a flow constraint, that money not soaked up is left over:
\[
B^*_{t-1} + M^*_{t-1} = P_t s_t + Q_t B^*_t + M^*_t.
\]
(As above, we could also consider past due debt that consumers choose not to redeem on the right hand side, I simplify knowing that consumers will redeem all their debt for money.) The government does not need to obey a transversality condition or present value budget constraint. If people wish to paper their caskets with money, and absorb an ever increasing amount of it, no budget constraint stops the government from satisfying this need.

An equilibrium is a set of \(\{M_t, B_t, s_t, c_t, Y_t\}\) that satisfy consumer optimality, the government flow constraint, and equilibrium \(c_t = Y_t, \ M^*_t = M_t, \ B^*_t = B_t\). The eventual government debt valuation equation results from the consumer’s budget constraint, and equilibrium \(c_t = Y_t\).

### 17.2.1 First order conditions, money demand, and equilibrium

The first-order conditions in equilibrium \(c = Y_t\) give the standard condition linking interest rates to marginal utility growth over time,
\[
Q_t = \frac{1}{1+i_t} = E_t \left( \beta \frac{u_c(t+1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right)
\]

and a money demand function,
\[
\frac{u_m(Y_t, M_t/P_t)}{u_c(Y_t, M_t/P_t)} = \frac{i_t}{1+i_t}
\]

or
\[
M_t = P_t L(Y_t, i_t).
\]
With separable power utility

\[ u(Y_t, M_t/P_t) = \frac{Y_t^{1-\gamma}}{1-\gamma} + \theta \left( \frac{M_t/P_t}{1-\gamma} \right) \]

the money demand function is a simple

\[ M_t = P_t Y_t \left( \frac{1}{\theta} \frac{i_t}{1 + i_t} \right)^{-\frac{1}{\gamma}}. \]

If the central bank sets money growth \( M_{t+1}/M_t = 1 + \mu \), the equilibrium follows a difference equation. The difference equation has two steady states. One steady state features inflation at the rate of money growth \( \pi_t = \mu \), and is unstable. The other is deflation, with a zero nominal interest rate, and is stable. Figure 17.1 graphs the equilibrium dynamics. With passive fiscal policy, we have multiple unstable equilibria around the positive steady state, and multiple stable equilibria around the deflationary steady state.

The first order conditions for maximizing (17.7) subject to (17.9) are\(^1\)

\[ \beta^t u_c \left( Y_t, \frac{M_t}{P_t} \right) = \Lambda_t \]

(17.10)

\[ \beta^t u_m \left( Y_t, \frac{M_t}{P_t} \right) = \Lambda_t \frac{i_t}{1 + i_t}. \]

(17.11)

\(^1\)To derive these first order conditions easily, consider each item as a function of state \( x^t \) in the time zero problem, i.e. think of \( c_t \) as \( c_t(x^t) \) and so forth in

\[
\max_{t=0}^{\infty} \sum \beta^t pr(x^t) u (c_t, M_t/P_t) \\
\text{s.t.} \\
\frac{B_{-1} + M_{-1}}{P_0} = \sum_{j=0}^{\infty} pr(x^t) \frac{\Lambda_t}{\Lambda_0} \left[ \frac{i_t}{1 + i_t} \frac{M_t}{P_t} + s_t + c_t - Y_t \right].
\]

Now introduce a Lagrange multiplier \( \lambda \) on the constraint and take the derivative with respect to \( c_t(x^t) \), yielding

\[ \beta^t pr(x^t) u_c (c_t, M_t/P_t) = pr(x^t) \frac{\Lambda_t(x^t)}{\Lambda_0} \lambda. \]

\[ u_c (c_0, M_0/P_0) = \lambda. \]

Since contingent claim prices are only defined as relative prices, we might as well choose numeraire so that \( \lambda = 1 \).
Here, I save a later step, substituting $Y_t = c_t$ to characterize the equilibrium. We can rewrite these in several useful and intuitive ways.

From the consumption condition we have the standard asset pricing formula,

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{u_c(t + 1)}{u_c(t)}$$

where I use the notation $(t) \equiv \left( Y_t, \frac{M_t}{P_t} \right)$. In particular bond prices follow the standard formula,

$$Q_t = \frac{1}{1 + i_t} = E_t \left( \beta \frac{u_c(t + 1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right). \quad (17.12)$$

Dividing the two first-order conditions,

$$\frac{u_m(Y_t, M_t/P_t)}{u_c(Y_t, M_t/P_t)} = \frac{i_t}{1 + i_t} \quad (17.13)$$

We can rewrite this equation as a money demand or “liquidity preference” function, which is typically interest-elastic

$$M_t = P_t L(Y_t, i_t).$$

Intuitively, suppose the consumer holds one more dollar $M_t$, and one fewer bond $B_t$. Total resources $M_t + B_t$ at the beginning of time $t + 1$ are the same, so this change does not affect anything at $t + 1$ and after. The consumer gains utility from the extra money,

$$u_m(t) \frac{1}{P_t}.$$

But the dollars gained by not buying the bond $1/(1 + i_t) \leq 1$ are less than extra dollar held overnight, so the consumer must devote $1 - 1/(1 + i_t) = i_t/(1 + i_t)$ less dollars to consumption, at a loss of

$$\frac{i_t}{1 + i_t} u_c(t) \frac{1}{P_t}$$

utility. At an optimum, the marginal gain must equal the marginal loss, resulting in the first-order condition $(17.13)$. The budget constraint is not the usual two-good constraint, in which income must be split between purchasing two goods. The opportunity cost of holding money is intertemporal, not static.
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We can also write from the first order conditions

\[ 1 = \frac{u_m(t)}{u_c(t)} + E_t \left[ \beta \frac{u_c(t + 1) \ P_t}{u_c(t) \ P_{t+1}} \right]. \]

The real rate of return on money is \( P_t/P_{t+1} \) which is less than that on other assets, and in particular bonds which pay \((1 + i_t) P_t/P_{t+1} \). That deficient rate of return ("rate of return dominance") in the right side, is made up for by an unobserved "dividend" or "convenience yield" of money in the first term. Iterating, we can state an asset pricing view of money

\[ u_c(t) \ \frac{1}{P_t} = E_t \sum_{j=0}^{T} \frac{u_m(t + j)}{P_{t+j}} + E_t \left[ \beta u_c(t + T + 1) \ \frac{1}{P_{t+T+1}} \right]. \]

An additional dollar, held forever, costs \( 1/P_t \) utility. It generates a stream of benefits, though it depreciates (usually) with inflation. Both of these expressions have interesting limits for zero interest rates, where money has no opportunity cost, or where money pays interest, which we’ll study later.

Now, suppose the central bank sets a money growth target rather than an interest rate target, specifying the sequence \( \{ M_t \} \). We want to find the corresponding sequence of equilibrium price levels \( \{ P_t \} \). We merge the two first order conditions to derive a difference equation for prices. Substituting out \( i_t \) from (17.12) and (17.13) as we have many times before,

\[ \frac{u_m(t)}{u_c(t)} = 1 - \frac{1}{1 + i_t} = 1 - E_t \left( \beta \frac{u_c(t + 1) \ P_t}{u_c(t) \ P_{t+1}} \right) \]

Simplify to a separable utility function. Now the presence of money does not affect the intertemporal condition. With a constant output \( c_t = Y \), and perfect foresight,

\[ \frac{1}{1 + i_t} = \beta \frac{P_t}{P_{t+1}}. \]

With separable utility we also have \( u_m(M_t/P_t) \). The difference equation becomes

\[ \frac{u_m(M_t/P_t)}{u_c(Y)} = 1 - \beta \ \frac{P_t}{P_{t+1}} = 1 - \beta \left( \frac{M_{t+1}}{P_{t+1}} \right) / \left( \frac{M_t}{P_t} \right) \ \frac{M_t}{M_{t+1}}. \] (17.14)
There is a steady state of constant real money holdings $M_t/P_t = M/P$,

$$\frac{u_m(M/P)}{u_c(Y)} = 1 - \beta \frac{M_t}{M_{t+1}} \quad \text{(17.15)}$$

$$\frac{M_{t+1}}{M_t} = \frac{P_{t+1}}{P_t}.$$  

Here prices are proportional to money over time, and inflation equals the money growth rate. The proportionality holds even with variable money growth – this equilibrium is super-neutral.

Higher nominal interest rates mean lower real money demand – the constant relating prices to money is higher for higher nominal interest rates, $P_t Y = V(i)M_t$. Seigniorage revenue is

$$\frac{M_{t+1} - M_t}{P_t} = \frac{M_t}{P_t} (1 + \mu) = \frac{M_t}{P_t} (1 + \pi).$$

Depending on $u_m$, as inflation and nominal interest rates rise, it is possible for the decline in the $M/P$ term to be larger than the increase in the $\pi$ term. Inflationary finance can have a Laffer curve, and one definition of “hyperinflation” is the point at which additional money printing produces less revenue.

There is a second deflationary steady state however. We usually think that the marginal utility of money eventually vanishes,

$$\lim_{m \to \infty} u_m(m) = 0.$$  

In fact, it is plausible that there is some finite level of money at which we are satiated, and more money provides no more help with transactions. Once you hold a lifetime’s worth of money, or two lifetime’s worth, holding more money (and, say, less bonds – this is about transactions services not about wealth) does you no good. In that case there is an upper bound, $m_{sat}$ such that

$$u_m(m) = 0, \, m \geq m_{sat}.$$  

For any $m \geq m_{sat}$ money and short-term bonds are perfect substitutes – and therefore must pay the same return. In this case (17.14) becomes

$$\frac{P_{t+1}}{P_t} = \beta$$

$$\pi_{t+1} \approx -\delta; \, i_t = 0.$$
At a zero interest rate, with slight deflation equal to the discount and real interest rate, people will hold arbitrary amounts of money – the money demand curve becomes a correspondence \( m \geq m_{\text{sat}}, \ i = 0 \), because money and bonds are perfect substitutes. Though labeled “liquidity trap” and often disparaged, or subject to efforts to fix it, this outcome is also the “Friedman rule” quantity of money. Money is free for society to produce, so we should be satiated with it.

To calculate an example, I use a simple separable utility function,

\[
u \left( c_t, M_t \right) = c_t^{1-\gamma} + \frac{\theta}{1-\gamma} \left( \frac{M_t}{P_t} \right)^{1-\gamma}
\]

(17.16)

and constant growth rate \( M_{t+1}/M_t = 1 + \mu \). Money demand is

\[
M_t = P_t Y_t \left( \frac{1}{\theta} \frac{i_t}{1 + i_t} \right)^{-\frac{1}{\gamma}}.
\]

(17.17)

Except for the \( i + i_t \) in the denominator, which we will see is an artifact of the discrete-time timing convention, this is the money demand function (17.2) of section 17.1.

The difference equation (17.14) becomes

\[
\theta \left( \frac{M_t}{P_t Y} \right)^{-\gamma} = 1 - \beta \left( \frac{M_{t+1}}{P_{t+1} Y} \right) / \left( \frac{M_t}{P_t Y} \right) \frac{M_t}{M_{t+1}}.
\]

(17.18)

Figure 17.1 presents the dynamics of this system. The solid curved line presents \( P_{t+1} Y/M_{t+1} \) as a function of \( P_t Y/M_t \), as given by (17.18). (I use the simpler form (17.20) given below.) The parameters are \( \delta = \mu = 0.20, \ \gamma = 2, \) and \( \theta = 1/100 \). I picked the parameters to make the graph pretty, not for realism.

The middle steady state of real money holdings, (17.15), is

\[
\theta \left( \frac{PY}{M} \right)^{\gamma} = 1 - \frac{1}{(1 + \delta) (1 + \mu)} \approx \delta + \mu.
\]

(17.19)

Higher inflation, money growth, and nominal interest rates mean less real money holding.

To get some sense of a reasonable \( \theta \), for \( PY/M \approx 1 \), we need \( \theta \approx \delta + \mu \), the sum of discount rate and money growth rate, and thus already a small number on the order
of 0.1. For a more realistic \( PY/M \approx 10 \), we need \( \theta \) a factor of 10 smaller, on the order of 0.01.

We can rewrite the difference equation (17.18) in terms of this steady state, eliminating \( \delta \) and \( \mu \), as

\[
\left( \frac{P_{t+1}Y}{M_{t+1}} \right) = \left( \frac{P_tY}{M_t} \right)^\gamma \frac{[1 - \theta \left(\frac{P_tY}{M_t}\right)^\gamma]}{[1 - \theta \left(\frac{P_tY}{M_t}\right)^\gamma]} .
\]  

(17.20)

If \( P_0Y/M_0 = PY/M \), the economy stays there (or, if the economy is expected to stay there, then \( P_0Y/M_0 = PY/M \)). Economists using this model like to jump to this solution as quickly as possible. Other values of \( P_0Y/M_0 \) lead to additional equilibria, however. The phase diagram cuts from below at the steady state – the derivative of (17.20) is positive at \( PY/M \) – so dynamics are unstable around \( PY/M \). Paths \( P_0Y/M_0 \) that start just above \( PY/M \) keep growing forever. \( PY/M \) is a unique locally bounded equilibrium, but nothing in this model so far rules out explosive solutions.
There is a second steady state at $PY/M = 0$. The economy approaches the zero bound $i = 0$ and steady deflation at the Friedman rule. To see this fact, write (17.20) as

$$
\frac{P_{t+1}}{P_t} = (1 + \mu) \frac{(1 - \theta \left(\frac{PY}{M}\right)^\gamma)}{(1 - \theta \left(\frac{PY}{M}\right)'^\gamma)}.
$$

so, in the limit $P_t/M_t \to 0$, and using (17.19),

$$
\lim_{P_t Y/M_t \to 0} \frac{P_{t+1}}{P_t} = \frac{1}{1 + \delta}.
$$

Inflation approaches the negative of the real interest rate and discount rate. This steady state is stable; multiple equilibria $P_0 Y/M_0$ in this neighborhood stay nearby.

The analysis parallels exactly the situation of Figure 16.1 in Section 16.2 for interest rate targets. Again, the full nonlinear model includes multiple stable and globally bounded equilibria, like the zero bound for Taylor rules; it includes unstable inflationary equilibria, and stable equilibria that approach slow deflation and the zero bound.

If we allow stochastic multiple equilibria, rather than assume perfect foresight, the economy can jump around between these equilibria at every date. $MV = PY$ with interest-elastic demand does not determine the price level. Perhaps the puzzling volatility and unpredictability of inflation, and especially apparently intractable zero rate situations like Japan’s, say this is our world. But even so, we must add something if we want an economic theory that can determine the price level.

As usual, if we add back the government debt valuation equations, (17.8) (17.9), rather than assume fiscal policy adjusts “passively” to make the valuation equations hold for any price level $P_0$, we obtain a determinate price level, and a complete monetary-fiscal policy description. In particular, with interest-paying debt $B_{t-1}$ as well as money $M_{t-1}$, it’s easy to see how a different choice of $\{s_t\}$ can rationalize any $P_0$, by changing the value of outstanding interest-bearing debt.

This observation could be the basis of elaboration, with more detail on money demand, inside and outside money, money supply rules, fiscal responses, long-term debt, and so forth, just as I did with interest rates in the first part of this book. That elaboration would also describe and reexamine many classic doctrines of monetary economics under money supply rules. I do not follow this path because, though a coherent and complete theory, it does not describe anything like current institutions. Our central banks target interest rates, not money supplies, and money demand is evaporating in a sea of interest-paying liquid assets and innovative transactions technologies.
In sum, the model formalizes the analysis of the last section [17.1]. By looking at a money demand function, a Fisher bond-pricing equation and the government debt valuation formulas, we have indeed exhausted the conditions needed to construct an equilibrium. We haven’t left anything out or gotten anything wrong. One can form an equilibrium with a money supply rule and fully passive fiscal policy. But there are multiple such equilibria. To eliminate multiple equilibria we must add an active fiscal policy. As with the new-Keynesian analysis, one might have hoped that the slightly different forms of the equations here would lead to a different conclusion, or that considering the full nonlinear model would do so. As in that case, the full nonlinear model adds a second deflationary steady state, and makes matters even worse.

The multiple equilibria of this model are the subject of an immense literature, or rather an arms-limited hunting expedition, attempting to eliminate them, but without explicit recourse to active fiscal policy. The next several sections cover some of these issues in detail.

17.2.2 Deflationary equilibria and transversality conditions

A transversality condition argument can rule out the deflationary equilibria when money growth is nonnegative $\mu \geq 0$, and for some specifications of the utility function, when there is no debt and money growth is financed by lump-sum taxation. I argue that this is the fiscal theory, in pure form, not an alternative to fiscal theory. The result is also sensitive to assumptions. If there is nominal debt and the central bank controls money by open market operations, it also fails. If money growth is negative $\mu \leq 0$, deflationary equilibria survive as well.

It is usually thought that the deflationary equilibria can be ruled out from the transversality condition. In the deflationary equilibria, as $P_t Y / M_t$ declines to zero, real money holdings $M_t / P_t Y$ rise to infinity. Moreover, for non-negative money growth, $M_t V / P_t Y$ rises at faster than the real interest rate. We can see this by manipulating (17.18) to give

$$\left( \frac{M_{t+1}}{P_{t+1} Y} \right) / \left( \frac{M_t}{P_t Y} \right) = (1 + \delta) (1 + \mu) \left[ 1 - \theta \left( \frac{P_t Y}{M_t} \right) \right]$$

so, as $P_t Y / M \searrow 0$,

$$\left( \frac{M_{t+1}}{P_{t+1} Y} \right) / \left( \frac{M_t}{P_t Y} \right) \nearrow (1 + \delta) (1 + \mu).$$
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Thus, if $\mu \geq 0$, real money holdings violate the transversality condition,

$$\lim_{T \to \infty} E_t \frac{1}{(1 + \delta)^T} \frac{M_{t+T}}{P_{t+T}Y} \neq 0.$$ 

We can therefore rule out these equilibria. Consumers, seeing this rise in wealth, should try to increase consumption, and in the process drive the price level back up and away from the deflationary equilibrium.

It appears, then, that the Friedman-rule, zero-bound, liquidity-trap equilibrium, paths leading to it, and initial price levels $P_0Y/M_0 < PY/M$ are only a problem for negative money growth, $\mu < 0$. That still is a problem: The Friedman rule is optimal, why not recommend that central banks deliberately encourage it, with negative money growth? Well, answers this analysis, multiple equilibrium volatility would break out. (We solve this problem with active fiscal policy so that an interest rate peg at zero, which implies negative money growth, is now a unique equilibrium. But not here.)

For our determinacy quest, the opposite case is more interesting. For non-negative money growth $\mu \geq 0$, at least, it seems we can rule out at least one category of multiple equilibria, using only monetary arguments. If, for other reasons, central banks dislike “liquidity traps,” simply by printing enough money it seems they can avoid the trap.

But this argument is a version of fiscal theory, not an alternative to fiscal theory. In face of a multiple equilibrium deflation, the government’s fiscal policy makes no reaction. The real value of a government liability then grows, violating the consumer’s transversality condition. Consumers react by trying to spend more of this wealth, which keeps the initial price level up, so we don’t start down that path in the first place. This is the fiscal theory, exactly, and exactly as described in the first chapters here! Printing money $\mu > 0$ in a liquidity trap, where money and bonds are perfect substitutes, with no fiscal backing, is precisely an unbacked fiscal expansion!

I defined “passive” fiscal policy as, fiscal policy does what it takes so that the government debt valuation holds, i.e. that the transversality condition is satisfied, for any price level. That is exactly what does not happen here. The question is then one of categorization – do we regard the fiscal-monetary regime that rules out deflationary equilibria for $\mu \geq 0$ as “passive” or “active” fiscal policy? By the definition here, it is clearly “active.” That this might not have been clear decades ago is understandable.
To substantiate this view, we need to look more carefully at the joint monetary-fiscal regime that generates this result. I start with a reminder about budget constraints, transversality conditions, and valuation equations. We can write the iterated budget constraint in this perfect foresight model as

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right] + \lim_{T \to \infty} \left( \frac{\Lambda_{t+T} B_{t+T} + M_{t+T}}{P_{t+T+1}} \right)
\]  

(17.21)

The valuation equation results as an equilibrium relationship: We substitute first order conditions including \(\Lambda_t = \beta' u(c_t)\), market clearing including \(c_t = Y\), \(M_t = M_t^*\), and so on. To conclude that the valuation equation holds for only one initial price level, we have wanted the last term to be zero, to preclude the possibility that \(P_{t+T+1}\) (or other variables) move with \(P_t\) to validate any initial \(P_t\).

So just why is the last term zero? The true budget constraint tells us only that it is non-negative,

\[
\lim_{T \to \infty} \left( \frac{\Lambda_{t+T} B_{t+T} + M_{t+T}}{P_{t+T+1}} \right) \geq 0.
\]

This is often called a “no-Ponzi” condition. It states you cannot borrow, consume, and roll the debt over forever. It is the limit of the requirement in finite time to end the world with no debts.

That this term is not positive,

\[
\lim_{T \to \infty} \left( \frac{\Lambda_{t+T} B_{t+T} + M_{t+T}}{P_{t+T+1}} \right) \leq 0,
\]

however, is not a budget constraint. In a finite-time model, you can leave money, bonds, or capital behind at the end of the world, and you can roll investments over forever. You typically choose not to do so. This is the “transversality condition,” and it is a condition for optimality, not a budget constraint.

The transversality condition in the model with debt \(B_t\) and no money is straightforward. Examine (17.21). If the final term is greater than zero, then the consumer could increase \(c_{t+j}\) along the way, a free lunch. In finite-period models, terminal wealth \(B_{t+T}/P_{T+t+1}\) serves only to finance consumption after time \(t + T\) so has no effect on the objective.
The case with money is more tricky, however, since reducing $M_{t+T}$ has a direct utility cost as well, $u_m(t + T)$. There is nothing wrong with demands for normal goods that explode to infinity as their relative prices decline.

In sum, then, the transversality condition, which together with the no-Ponzi condition tells us that the final term is zero, depends on the objective as well as the budget constraint. (Once again, our valuation equation is not a budget constraint, and specifically not a government budget constraint!) In this application, when it is zero depends on various limiting properties of the money in the utility function, and the policy specification. This fact accounts for the large literature and controversy surrounding this result.

To proceed in the simplest way, as well as because it is more reasonable, I specify that the marginal utility of money is zero past a satiation point,

$$u_m(m) = 0, \ m \geq m_{sat}. \quad (17.22)$$

We’ll come back to the standard case below. We have now experienced interest rates of zero, or, in Europe and Japan, less than zero, for years. Cash demand and reserve demand did not explode to infinity. More deeply, we should expect money demand to become undefined at very low interest rates and very large money holdings. When interest rates decline from one basis point, at which each person goes to the ATM machine every 5 years, to half a basis point, at which each person decides to take out cash for 10 years’ transactions, because the interest costs of doing so now fall below the cost of one extra 10 minute walk to the ATM machine, the artificialities of this model fall apart. Do not expect abstract models to hold exactly for vanishingly small budget or utility costs. (I once wrote a whole paper making this rather obvious point, Cochrane (1989), but it does tend to get forgotten as we push Greek letters around.) Money and bonds will be close to perfect substitutes long before they become exact substitutes, and it is easy to ignore optimal decisions that have tiny costs.

With (17.22), the deflationary limit point makes sense, since it need not have infinite money holdings. Moreover, there is a region $P_t Y/M_t \leq Y/m_{sat}$ where the same dynamics hold as at the limit point. With $u_m(m) = 0$, the equilibrium condition (17.14) becomes simply the condition that we have deflation at the real interest rate, so the nominal rate is zero,

$$1 = \beta \frac{P_t}{P_{t+1}}.$$

Multiplying and dividing again, real money balances grow at the sum of the real
interest rate and real money growth rate.

\[ 1 = \beta \left( \frac{M_{t+1}}{P_{t+1}} \right) / \left( \frac{M_{t}}{P_{t}} \right) \frac{M_{t}}{M_{t+1}} \]

\[ \left( \frac{M_{t+1}}{P_{t+1}} \right) / \left( \frac{M_{t}}{P_{t}} \right) = (1 + \delta)(1 + \mu). \]

If the money growth rate is negative – \( \mu < 0 \) – then real money holdings grow slower than the discount rate. If the money growth rate \( \mu \) is positive, real money holdings grow faster than the discount rate. If \( \mu \geq -\delta \), then once \( M_t/P_t \) exceeds satiation \( m_{sat} \) it will continue to do so forever, so we don’t have to worry about leaving the \( u_m = 0 \) region.

Figure 17.2: Dynamics for the money in utility function model with fixed money growth and money demand with satiation. The solid line gives the model without satiation, and the dashed line gives the model with satiation.

For an easy explicit example satisfying (17.22), extending the previous example, we can write

\[ u_m = \theta \max \left[ \left( \frac{M_{t}}{P_{t}} \right)^{-\gamma} - m_{sat}^{-\gamma}, 0 \right]. \]
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Figure 17.2 contrasts the phase diagram for this model with phase diagram for the model with no satiation. You can see there is no huge qualitative difference in the dynamics.

(To calculate Figure 17.2, equation (17.14) becomes

\[
\frac{M_{t+1}}{P_{t+1}Y} \bigg/ \frac{M_t}{P_tY} = (1 + \delta) (1 + \mu) \left\{ 1 - \theta \left[ \left( \frac{M_t}{P_tY} \right)^{-\gamma} \left( \frac{m_{sat}}{Y} \right)^{-\gamma} \right] \right\}; \quad M_t > m_{sat}
\]

Equation (17.23) is equivalent to

\[
\frac{M_{t+1}}{P_{t+1}} \bigg/ \frac{M_t}{P_t} = \frac{1 - \theta \left[ \left( \frac{M_t}{P_t} \right)^{-\gamma} \left( \frac{m_{sat}}{Y} \right)^{-\gamma} \right]}{1 - \theta \left[ \left( \frac{M_t}{P_t} \right)^{-\gamma} \left( \frac{m_{sat}}{Y} \right)^{-\gamma} \right]}; \quad M_t \geq m_{sat}.
\]

with a steady state defined by

\[
1 - \theta \left[ \left( \frac{M}{PY} \right)^{-\gamma} \left( \frac{m_{sat}}{Y} \right)^{-\gamma} \right] = \frac{1}{(1 + \delta) (1 + \mu)}.
\]

Now, does real money growth faster than the discount rate really violate the transversality condition? Consider the standard specification so far – there is no debt $B_t = 0$, there are no surpluses other than seigniorage $s_t = 0$, meaning that seigniorage from money growth is redistributed by lump-sum transfers and negative seigniorage from any negative money growth corresponds to lump-sum taxation ($s_t$ denotes the real primary surplus not including seigniorage), and the government commits to a constant money growth rate $\mu$ no matter what happens to the price level. (All of these are important.) Using equilibrium intertemporal prices $\Lambda_t = \beta u_c(t)$ and market clearing with a constant endowment $u_c(t) = u_c(Y)$ to eliminate $\Lambda_t$, and the fact that since $u_m = 0$ we must have $i = 0$ in this region, we have left,

\[
\frac{M_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j \left[ s_{t+j} + c_{t+j} - Y_{t+j} \right] + \lim_{T \to \infty} \left( \beta^j \frac{M_{t+T}}{P_{t+T+1}} \right).
\]

But if this last term is not zero, and with $u_m(m_{t+T}) = 0$, the consumer could lower money demand $m_{t+T}$ and raise consumption along the way $c_{t+j}$, improving utility.
Money and bonds are perfect substitutes where $u_m = 0$, so this just a restatement of the earlier result. Yes, in equilibrium the consumer cannot lower money demand, but that’s the point – equilibrium requires that the sequence of price levels adjust so that money demand equals money supply, and this sequence of price levels does not do the job.

Having seen the argument in gory detail, you can appreciate just how fiscal – and how unrealistic – it is. Most governments do not print money and redistribute the results by lump sum transfers. Most governments also have interest-paying debt outstanding. Let’s put debt back in.

$$B_{t-1} + M_{t-1} = \frac{1}{P_t} \sum_{j=0}^{\infty} \beta^j s_{t+j} + \lim_{T \to \infty} \left( \frac{\beta^j B_{t+T} + M_{t+T}}{P_{t+T+1}} \right).$$

Now, suppose the government really wanted to follow a passive fiscal policy, and the central bank was hell-bent on its money growth policy. Suppose the central bank, as usual, did not have access to fiscal policy (lump sum taxes or transfers) and instead had to use open market operations. Every increase in $M$ now requires an equal decrease in $B$, an open market operation. Now the sum of $B$ and $M$ can be constant, or grow at less than the discount rate, while we have steady deflation. The transversality condition in this case says only that total wealth cannot grow faster than the discount rate, and does not apply to each component separately. Our finding that money growth could rule out the deflationary equilibrium relied crucially on money creation financed by fiscal means, rather than open market operations. (In this example, debt eventually becomes negative, with the government lending to the private sector. But sovereign wealth funds exist.)

The standard model essentially rules out independent fiscal and monetary policy. With no debt, no surpluses, and seigniorage always rebated lump sum, money growth is fiscal expansion, and reduced money growth is fiscal contraction. There is simply no way to accomplish a passive fiscal policy, to do what it takes to satisfy the government valuation equation for any price level while letting monetary policy adjust money supply.

### 17.2.3 Deflationary equilibria with infinite money demand

I offer a quick survey of the vast literature on deflationary equilibria with alternative specifications of the money in utility function.

The more standard case with $u_m > 0$, but declining, keeping the rest of this highly
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A striking feature of the current and past macroeconomic literature on deflationary bubbles is the divergence of opinion over the correct specification of both the transversality condition in models where money is the only financial asset and the correct specification of the transversality and long-run solvency, or “no-Ponzi-game,” conditions in models where there are both money and bonds.

A lucid and concise review of widely-varying, and differing published opinions follows, including Brock (1974), Brock (1975), Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986), Ljungqvist and Sargent (2000), Woodford (2003) and many others one might turn to for guidance here. Opinion differs on whether there are two conditions, one for money and one for bonds, or one condition, for aggregate terminal wealth.

Their conclusion mirrors the claim I started with,

We demonstrate that deflationary bubbles cannot occur when money growth is strictly positive ($\mu > 1$). We show, however, that when the money supply is contracting, but at a lower rate than the discount factor ($\beta < \mu < 1$) deflationary bubbles can occur; indeed, any separable utility function satisfying the usual regularity conditions can produce a deflationary bubble.

However, even they do not get the last word. In particular, they write

”deflationary bubbles accompanied by strictly positive money growth in Woodford (2003) and Benhabib et al. (2002a) cannot exist.”
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This statement appears to invalidate my previous analysis, such as Figure 16.1. But that analysis didn’t have any money at all, and it followed an interest rate target in which, if there is money, it can grow at a slow rate. Moreover the point of Benhabib, Schmitt-Grohé, and Uribe (2002) was exactly that by adding unbacked money or debt growth, an essentially fiscal policy, they could escape the deflation. (Woodford (1994)) explores these issues in detail in a cash and credit good cash in advance model, with interest-elastic money demand. His central point is that an interest rate target allows the zero-bound equilibrium, in a way that a money-growth target does not do. The debate will continue.

Even Chiang (1992) writes “their [transversality conditions] validity is sometimes called into question... it is only fair to warn the reader... that there exists a controversy surrounding this aspect of infinite-horizon problems.” (p. 102) “Many writers consider the question of infinite-horizon transversality conditions to be in an unsettled state.” (p. 243.) He goes on to set straight several counterexamples, many from the economics literature.

The point here is only a fair warning. I don’t pursue the issue further, because the model, though a subject of a large literature – constant money growth, no debt, no surpluses, a definite money demand – is not interesting.

17.2.4 Timing conventions in the inflationary equilibria

The inflationary equilibria explode in finite time. They are nonetheless valid equilibria. This behavior is an unrealistic feature of the discrete-time timing conventions, which result in \( u_m/u_c = i/(1 + i) \) rather than \( u_m/u_c = i \) which results from the continuous time model. I introduce a modification of the utility function which gives the latter first order condition, and removes jumps to infinite price level in finite time.

First, let us examine the inflationary equilibria of the difference equation (17.20),

\[
\left( \frac{P_{t+1}Y}{M_{t+1}} \right) = \left( \frac{P_tY}{M_t} \right) \frac{[1 - \theta (P_t/M_t)^\gamma]}{[1 - \theta (P_t/M_t)^\gamma]},
\]

(17.24)

for \( P_0Y/M_0 > PM/M \), eventually explodes to an infinite price level in finite time. The denominator goes to zero or worse. One might hope to eliminate inflationary equilibria on this basis.
There is nothing theoretically wrong with this result. Since money is just an argument of the utility function in an endowment economy, the economy can trundle along with \( c_t = Y \) and no money. A path that starts with little inflation, goes to hyperinflation, and in finite time demonetizes, is a valid equilibrium of the model. First order conditions and budget constraints hold all the way to the infinite price level and beyond.

In fact, this consideration means there is not a continuum of inflationary equilibria, but a countable number, and the denominator of (17.24) goes exactly to zero but not below. If consumers know that the price level will be infinite at time \( T+1 \), then money demand at time \( T \) is

\[
M_t = P_t Y \left( \frac{1}{\theta} \right)^{-\frac{1}{2}}.
\]

so the last period price level is

\[
P_T = \frac{M_T Y}{\theta^{-\frac{1}{2}}}.
\]

People are happy to hold this much money for a day, even knowing money will be useless tomorrow. We work back from this terminal condition to find \( P_0 Y/M_0 \). For each such \( T \) there is a different \( P_0 Y/M_0 \).

This is the specification, with demonetization in finite time, studied in the classic models that attempt to fix these multiple equilibria, Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986), which we will examine below. These authors add additional elements to the policy mix to try to trim the inflationary equilibria, which they would not need to do if the equilibria weren’t valid in the first place.

Nonetheless, demonetization in finite time feels weird, and it is. It results from a pathology of the discrete-time formulation of the model. This is not a good model for studying money demand in high-inflation economies, for this and many other reasons.

In this discrete-time setup, the money demand function is (17.17),

\[
M_t = P_t Y_t \left( \frac{1}{\theta} \frac{i_t}{1 + i_t} \right)^{-\frac{1}{2}}. \tag{17.25}
\]

In the continuous-time version of the model, we have instead

\[
M_t = P_t Y_t \left( \frac{i_t}{\theta} \right)^{-\frac{4}{7}}. \tag{17.26}
\]
For small $i_t$, the difference between $i_t$ and $i_t/(1 + i_t)$ is minor. However, for large $i_t$, it is not minor. As inflation and interest rates approach infinity, real money demand $M_t/P_t$ in (17.25) approaches a constant, while real money demand in (17.26) smoothly approaches zero. In the discrete-time model, it is worth holding money for one day, even if that money will be worthless the next morning. Interest is only paid overnight, so there is no opportunity cost for holding money for one day, and the price level is constant during the day.

This behavior is not realistic. In times of very large inflation, interest is paid even during literal days, to say nothing of the month, quarter, or year periods for which we usually apply these models, and prices rise hour by hour. You cannot hold money for any discrete period of time without an opportunity cost. The continuous time first order conditions (17.26) reflect this fact.

The best approach is to actually use the continuous-time model, in which pathologies due to timing conventions do not arise. We can however derive a money demand (17.26) in this discrete-time model by modifying the utility function (17.17) to

$$u(c_t, M_t / P_t) = c_t^{1-\gamma} + \frac{\theta}{1 - \gamma} \frac{1}{1 + i_t} \left( \frac{M_t}{P_t} \right)^{1-\gamma}.$$ 

Now, the first order condition

$$\frac{u_m(t)}{u_c(t)} = \frac{i_t}{1 + i_t}$$

becomes, in equilibrium $c_t = Y_t$, (17.26).

The $i_t$ vs. $i_t/(1 + i_t)$ really belongs in the budget constraint – the fact that you cannot, in reality, use money without interest cost or inflation during the day. But it’s awkward at this stage to change the budget constraint we have used throughout the book, and discrete time utility doesn’t mix well with a continuous time budget constraint. So, I add the $1/(1 + i_t)$ to the preferences as a shortcut to get the continuous-time first order condition and limiting behavior out of the discrete time model. The preferences are an indirect utility for some unstated transactions model anyway, and if we allow the price level $P_t$ in to preferences, we can’t object to an intertemporal price $i_t$ as well.

Repeating the previous analysis, we obtain almost exactly the same results for small interest rates $i$, but a smooth limit for high interest rates, and in particular a continuum of inflationary equilibria that go on forever, without demonetizing in finite
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time. The difference equation is, in place of (17.18),

$$\theta \left( \frac{M_t}{PY} \right)^{-\gamma} = \frac{1}{\beta} \frac{P_{t+1}}{P_t} - 1 = \frac{1}{\beta} \left( \frac{M_t}{PY} \right) / \left( \frac{M_{t+1}}{P_{t+1}Y} \right) \frac{M_{t+1}}{M_t} - 1.$$  

The steady state for money holdings is

$$\theta \left( \frac{M}{PY} \right)^{-\gamma} = (1 + \delta)(1 + \mu) - 1.$$  

and in place of (17.20),

$$\left( \frac{P_{t+1}Y}{M_{t+1}} \right) = \left( \frac{P_tY}{M_t} \right) \frac{1 + \theta \left( \frac{PY}{M_t} \right)^{\gamma}}{1 + \theta \left( \frac{PY}{M_t} \right)^{\gamma}}$$

Having $1 + \theta \left( \frac{PY}{M_t} \right)^{\gamma}$ in the numerator rather than $1 - \theta \left( \frac{PY}{M_t} \right)^{\gamma}$ in the denominator makes little difference for small values of $PY/M_t$ but means that the price level never goes to infinity in finite time. Figure 17.1 is visually indistinguishable in the plotted range, but no longer spikes up to infinite $P_{t+1}Y/M_{t+1}$ at a finite $P_tY/M_t$.

This seems to me like a better way to put the continuous-time model in discrete time. I present the traditional model more for consistency with other sources, as the point of this section is that the model does not work. But I would use this formulation or the continuous time version if I were to use the model going forward.

17.2.5 The Obstfeld-Rogoff fix for inflationary equilibria

I review the famous Obstfeld and Rogoff (1983) fix for inflationary equilibria. Obstfeld and Rogoff specify that the government stands ready to redeem money for goods (gold) at a very high price level. Crucially, they specify that the government refuses to sell money for goods at the same price. Therefore, the government disallows the recovery in real money holdings that follows the ends of inflations, ruling out an equilibrium in which the equilibrium is stopped.

On deeper analysis, however, I find that the modification does not rule out the original equilibrium in which the price level jumps to infinity.

So, the basic money in the utility function model, with interest-elastic demand, fixed money supply, and passive fiscal policy, leaves multiple equilibria, in manner quite
similar to the active interest rate target. Even if we rule out deflationary equilibria, the inflationary equilibria remain.

The natural way to fix the models is to add back active fiscal policy. Again, a long literature tries to find some other means to rule out multiple equilibria, some modification of the monetary policy regime. Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986) are the most famous paper cited in this effort.

Obstfeld and Rogoff add to the specification of monetary policy regime that at some very high price level the central bank implements a partial commodity standard. Obstfeld and Rogoff (1983) (p. 676) write:

Speculative paths can be eliminated... provided the government fractionally backs the currency by standing ready to redeem each dollar for a small amount of capital.

Obstfeld and Rogoff’s model is based on backing with capital, which is hard to imagine in practice. What are the independent real units of capital? But as they make clear, capital is a stand-in for a commodity or gold standard: Footnote 17: “We analyze capital backing rather than gold backing here in order to avoid modeling the role of gold in consumption and/or production. But our results would clearly carry over to a model in which currency is redeemable in terms of gold.” A foreign exchange peg would work the same way. I study a simplified model in which the government backs the currency with the consumption good, which it obtains by lump-sum taxes.

Based on the section 16.4 analysis of similar devices to stop multiple equilibria in models with interest rate targets, two natural questions or objections arise: First, such a commodity standard or real backing is the fiscal theory, it is an active fiscal policy, not an alternative to fiscal theory and a defense of purely monetary price level determination. To make this commitment, the government has to have the capital, commodity, gold, foreign exchange or the ability to tax to get it, either now or in the credible future. Indeed, Obstfeld and Rogoff write (p. 684):

“Feasibility of the government’s policy requires that the government have access to sufficient reserves of capital to purchase the entire money stock $M$ at the support price $\varepsilon$.”

This observation is not a criticism: Obstfeld and Rogoff wrote a decade before Leeper and the active/passive distinction was even made or regarded as important. They were not trying to rescue price level determination without fiscal underpinnings, and did not claim to do so. They might have been, in 1983, perfectly happy to interpret
their result as a joint monetary-fiscal policy regime, with the fiscal part of the regime important for equilibrium selection. At the time, the important question was whether any regime could determine the price level. The distinction is important here now, however, for us to understand and categorize their result. And we should not cite them for showing something they did not claim to show.

Second, however, the proposal runs afoul of the earlier conundrum – the difference between stopping an inflation and ruling out an equilibrium. An inflation breaks out, and gets worse and worse. At some point – maybe when the dollar is worth one cent of its original value – the commodity standard kicks in. That stops the inflation, and one would suppose the economy continues on that, fiscally-determined, gold-standard enforced, price level. Great, but the inflation, its end, and the new commodity standard would seem to all be part of an equilibrium. Though there is fiscal theory in such a story, there is not enough fiscal theory. One needs active fiscal policy right away to select an initial price level $P_0$, not a backstop commitment that only comes in to play conditionally.

How did they rule out the equilibrium path? There must be a blow-up-the-world threat or inconsistent policy in there somewhere. What is it, and is it credible, either as something our central bank actually does and is believed to do, or might do in the future? Just how did their equations rule out an equilibrium, rather than stop an inflation? Let us look at those equations. (This section simplifies Cochrane (2011a) p. 609 ff. Obstfeld and Rogoff (2017) is a draft response to that paper. I hope this clearer presentation settles the issue, but one never knows.)

Obstfeld and Rogoff use the model we have been studying, with separable utility, the standard discrete-time timing conventions, and constant endowment. The first order conditions, in equilibrium, lead to the same difference equation, (17.14), which they write

$$\frac{u'(Y)}{P_t} - \frac{v'(M/P_t)}{P_t} = \beta \frac{u'_c(Y)}{P_{t+1}}.$$  \hfill (17.27)

(This is their equation (4), p. 678. In case you want to refer to the original, I use their notation, $u'(c)$ and $v'(m)$ in place of my $u_c(c)$ and $u_m(m)$ here.) They specify a constant money supply $M$, and denote the corresponding steady state by $\bar{P}$,

$$\frac{u'(Y)}{\bar{P}} - \frac{v'(M/\bar{P})}{\bar{P}} = \beta \frac{u'(Y)}{\bar{P}}.$$  

Hyperinflationary equilibria occur in finite time, as discussed above. Such an equi-
Equilibrium ends with
\[ P_{T+1} = P_{T+2} = \ldots = \infty. \]
The price level at time \( T \) is then
\[ v' \left( \frac{M}{P_T} \right) = v' \left( \frac{M}{\bar{P}} \right) = u'(y). \]
The second equality defines \( \bar{P} \), the price level if people know money will be worthless next period. (This is \( \bar{P} \) with a short bar on top, where the steady state (17.27) is \( \bar{P} \) with a long bar on top. Again, I use Obstfeld and Rogoff’s notation in case you want to refer to the original.)

We find earlier price levels by working back with (17.27). Each \( T \) generates a different potential value of \( P_0 \) and a different equilibrium path.

Figure 17.3 plots this path, labeled “\( \varepsilon = 0 \)” The figure plots \( m_t = M/P_t \) with \( M = 1 \) for clarity, so the jump to \( P_{T+1} = \infty \) is a jump of \( m_{T+1} = M/P_{T+1} \) to zero.

There is nothing wrong with these equilibria, and that is Obstfeld and Rogoff’s whole point. We need something else to rule them out. Obstfeld and Rogoff make a small change (p. 684):

“the government promises to redeem each dollar bill for \( \varepsilon \) units of capital but does not offer to sell money for capital.”

I specify equivalently that the government promises to redeem each dollar for \( \varepsilon \) units of the consumption good, which it obtains by a lump-sum tax. Therefore, it seems that by arbitrage the equilibrium price level cannot be higher than

\[ \bar{P} = 1/\varepsilon. \]

Here’s Obstfeld and Rogoff’s central claim that with this extra provision, hyperinflationary equilibrium paths are ruled out (p. 685):

Suppose that \( \{P_t\} \) is an equilibrium path with \( P_0 > \bar{P} \). Let \( P_T = \max \left\{ P_t : P_t < \bar{P} \right\} \). By (14) [my (17.27)] \( P_T \) must be below \( \bar{P} \), so that \( u'(Y) - v'(M/P_T) > 0 \) while \( P_{T+1} \) must exceed \( P_T \) and therefore equal \( \bar{P} \). But there is no \( M_{T+1} \leq M \) such that \( u'(Y) - v'(M_{T+1}/\bar{P}) \geq 0 \). Thus there is no price level \( P_{T+2} \) satisfying (14) and \( \{P_t\} \) is not an equilibrium path.
Figure 17.3: Hyperinflations in the Obstfeld-Rogoff model. “\( \varepsilon = 0 \)” gives the hyperinflation equilibrium that we wish to rule out. “\( \varepsilon = 0.5 \)” gives the equilibrium when the government offers to redeem money for \( \varepsilon \) consumption goods. “Redemption value” plots \( M \varepsilon \), the value of money guaranteed by the government’s redemption promise. “Two-way conversion” gives the equilibrium that results if the government offers to buy as well as to sell the commodity. The lower horizontal line indicates \( M/P \), money holdings at the price level where people are willing to hold money for one period though it is useless the next period. \( u'(y) = 1, M = 1, \beta = 1/2, v(m) = m^{-1/2} \).

The line marked “\( \varepsilon = 0.5 \)” in Figure 17.3 plots this path. If there were the final period with \( P_T = \bar{P} \), as previously hypothesized, now people would be able to turn their money in at value \( \varepsilon \) after using it. Money is more valuable. So \( P_T \) must be less than \( P \), and \( M/P_T \) higher than \( P \) as shown. But that is fine, and everything is fine up to and including period \( T \). The issue is (again) just what happens on day \( T + 1 \) when the redemption promise first kicks in.

Now, you might think that after the commodity standard becomes effective, we simply move to a new equilibrium with \( P_t = \bar{P} = 1/\varepsilon \) forever, as graphed in the “two-way conversion” line of Figure 17.3. We switch on a gold standard or foreign exchange peg, and the fiscal resources to 100% back that peg. Inflation stops. This
is how many historic hyperinflations were stopped. But again, stopping the inflation
does not rule out the equilibrium. Quite the opposite: stopping the inflation simply
and transparently makes the equilibrium more reasonable to rationally expect in the
first place.

Here the second part of the p. 684 specification is crucial: the government “does not
offer to sell money for capital.” (My emphasis.) In an inflation, with high nominal
interest rates, real money demand $M/P$ is low. When inflation ends, and interest
rates perforce return to low values, real money demand increases. Governments that
stop inflations can, and do, continue to print a lot of money as real money demand
recovers. And for good reason: They want equilibrium to form, they want first-order
and market-clearing conditions to hold, they want a successful stabilization. They
do not want to set things up so that no equilibrium can form, whatever that means.
Governments on the gold standard or foreign exchange peg sometimes refuse to give
you gold or foreign exchange when you bring in money, but governments on the gold
standard don’t refuse give you money when you bring them gold!

Indeed, if the government offered a conventional, two-way commodity standard, con-
ditional on reaching a high price level $\overline{P}$, we would in fact see $P_t = \overline{P} = 1/\varepsilon$ for
t = $T + 1$, $T + 2$, ... People would bring in as much of the commodity (capital) as
needed to obtain enough money so this price level would be the new steady state,
as graphed. The inflation and its end would unequivocally be an equilibrium. I
emphasize this point because many readers seem to think this is what Obstfeld and
Rogoff do. It is not.

Obstfeld and Rogoff’s government refuses to increase the money stock, despite the
huge seigniorage opportunity, and despite the crying money demand of its citizens,
even if they bring gold to the window. This limitation is the heart of the equilibrium-
selection concept, and it is what rules out the $\overline{P}$ equilibrium and its antecedents.
Well, this government is not trying to stop an inflation. This government is trying
to set policy so that equilibrium cannot form, to blow up the world should the event
happen. More charitably, this government wants to enforce the low steady-state
price level $\overline{P}$, and it is following monetarist advice to refuse accommodative money
printing, to hold the money supply to $M$ and no more in that quest. This government
is searching for a threat it can make ex-ante to rule out the inflationary equilibrium
path. As before, we can question whether such a threat is believable or even possible,
but for now the important point is to understand what it is.

In this case, however, I believe the conclusion is incorrect. The one-way redemption
is not sufficient to rule out any equilibrium after the inflation. Obstfeld and Rogoff
left out the possibility that \( P_{T+1} = \infty \), and people redeem all their money.

As before, the redemption promise makes no difference to the possibility of equilibrium formation for times 0, \( \ldots T \). The issue is only at time \( T + 1 \), and potentially beyond. To understand the issue, it’s worth reviewing why already in the absence of the redemption promise and with with \( M_{T+1} = M, \ P < P_{T+1} < \infty \) cannot be an equilibrium, and why \( P_{T+1} = \infty \) is an equilibrium. Then our job is only to see how the redemption promise modifies the logic. If \( P_{T+1} \leq \bar{P} \), we are looking at the wrong last period. Look at \( P_{T+2} \) instead. If \( \bar{P} < P_{T+1} < 1 \), the realm money holdings \( M/P_{T+1} \) are lower than what people want to hold, \( M/P \), even if they know money will be useless the following day \( T + 2 \). We have \( v'(M/P_{T+1}) > u'(Y) \). In that circumstance, people would try to sell some of their consumption good to obtain more money \( M_{T+1} \). They can’t, in the aggregate. The effort, however, pushes the price level down until once again \( P_{T+1} = \bar{P} \) and now \( T + 1 \) is the second-to-last period. However, \( P_{T+1} = \infty \) is an equilibrium, despite this force; despite people’s crying demand for money, despite \( v'(M/P_{T+1}) >> u'(Y) \), including even an infinite \( v'(M/P_{T+1}) = \infty \) or \( v(M/P_{T+1}) = -\infty \).

Why do people not demand more money? Are they not similarly off the first order condition? It appears so – \( -u'(Y) - v'(M_{T+1}/P_{T+1}) < 0 \), mirroring the above p. 685 quote. The answer is, that this condition does not apply when \( P_{T+1} = \infty \). When \( P_{T+1} < \infty \), an increase in nominal money \( M_{T+1} \) raises real money holdings \( M_{T+1}/P_{T+1} \), and so the consumer can consider trading of some consumption good for some real money, by buying nominal money. But when \( P_{T+1} = \infty \), buying extra nominal money does not give any increase in real money, nor any decrease in the marginal utility of consumption. At \( P_{T+1} = \infty \), there is no available tradeoff between consumption goods and real money holdings. The full first order condition requires \( [u'(Y) - v'(M_{T+1}/P_{T+1})]/P_{T+1} \geq 0 \), and the numerator can be negative when the denominator is infinite. Obstfeld and Rogoff’s \( A(m) \) and \( B(m) \) lines intersect again at \( m = 0 \).

The equilibrium remains, even if \( \lim_{m \to 0} v'(m) = \infty \), and \( \lim_{m \to 0} v(m) = -\infty \). The individual would really like to hold some money, and at any finite price level would want to do so. But when the value of money is exactly zero, there is nothing the individual can do – holding nominal money does not satisfy the real money demand, \( M_{T+1} = 0 \) satisfies consumer optimization at the price \( P_{T+1} = \infty \). Prices are given to consumers who then choose demands. You take price limits first.

Now, let us see how the redemption promise modifies this logic. The issue is again only period \( T + 1 \). Indeed, no finite price level \( P_{T+1} \) is an equilibrium. We cannot
have \( P_{T+1} < \overline{P} = 1/\varepsilon \), because if so we’re looking at the wrong period – \( P_T \) is defined as the last period with \( P_T < \overline{P} \). We also still cannot have \( \overline{P} > P_{T+1} > \overline{P} \) by the previous logic – people want more money.

\( P_{T+1} = \overline{P} \) seems sensible – that’s the redemption price after all. But the same logic rules out \( P_{T+1} = \overline{P} \) as was the case without the redemption promise. People hold money \( M_T = M \) on entering period \( T + 1 \). Real money holdings \( M/\overline{P} \) are so low, that even if \( P_{T+2} = \infty \), people want to hold more real money than \( M/\overline{P} \), even if they can hold it only for one day – we have \( \overline{P} > \overline{P} \), and \( v'(M/\overline{P}) > v'(Y) \). People want to hold more money, not less, so the redemption promise is irrelevant. At \( P_{T+1} = \overline{P} \), a two-way redemption promise would matter. People would, if allowed to so, rush to bring the consumption good in to get more money at rate \( \overline{P} \). But the government does not allow them to buy money, only to sell it. Instead, all people can do is to bid up the value of money, pushing the price level down until \( T + 1 \) is once again the second-to-last period. This pushes all previous price levels up. Going forward, we seem to rule out the inflationary equilibria. If the last period price must be \( \overline{P} \) or nothing, there is no last period and the steady state is the only equilibrium. This is the intuition of Obstfeld and Rogoff’s argument.

We can rule out price levels \( \overline{P} < P_{T+1} < \infty \) as well, in which money less valuable than the backing. We already ruled these out with no redemption promise. The redemption promise only serves to rule them out even more strongly. Before, people simply demanded more money than was supplied, invalidating the assumed price level. With the redemption promise, people run to cash in their money \( M_T \), receiving \( \varepsilon = 1/\overline{P} \) goods for each dollar. They then run back to markets to sell the goods, getting \( P_{T+1} > \overline{P} \) dollars for each good, and thus ending up with \( P_{T+1}/\varepsilon = P_{T+1}/\overline{P} > 1 \) dollars for each dollar they started with. Around and around they go, each individual hoping to achieve infinite consumption or moneyholdings.

But this arbitrage argument fails at \( P_{T+1} = \infty \), for the same reason that the more-money-demand argument failed. At \( P_{T+1} = \infty \), (and \( P_{T+2} \), etc., = \( \infty \)) the optimal thing for consumers to do is to turn in all their money for the redemption value \( M_T/\overline{P} = M/\overline{P} \). There is no point in holding on to worthless money. And there is no point in the second step of the previous arbitrage strategy, bringing the money to the goods market to get more money.

One might argue the latter point – perhaps an infinite price level means one can get an infinite amount of money for a finite amount of the good, and we start a debate about
limits. But if we have accepted, as Obstfeld and Rogoff have, that $P_{T+1} = \infty$ is the correct equilibrium without the redemption option, then the redemption option does not change that fact. Without the redemption option, consumers holding endowment $Y$ and money $M$ would really like to sell some of their endowment to get some additional real money holdings. We decided that at $P_{T+1} = \infty$ they can’t get any additional real money holdings. With the redemption option, consumers first redeem their money, and then show up the goods market with endowment $Y + M/P_T$ to get additional real money. If they couldn’t trade goods for real money before, they can’t do it now.

Put another way, a government promise to exchange one good for another at a set rate only determines the relative price of those goods if the consumer holds an interior amount of the goods. Obstfeld and Rogoff’s specification that the government does not sell money for goods, $M_t \leq M$, means that the price level can be lower than the peg, $P_t < \overline{P}$, if people are at the constraint $M_t = M$. Similarly, however, the limit $M_t \geq 0$ means that the price level can exceed the peg, $P_t > \overline{P}$, if people are at the constraint $M_t = 0$.

In sum, the jump to zero value of money, and infinite price level, in this model, is not removed by the government’s promise to redeem money for a small amount of consumption good (or capital.)

Moving back, the redemption guarantee does affect the price level at time $T$. Previously, people held money at time $T$ for its utility at $T$, even knowing it would be worthless at time $T+1$. Now, they hold money at $T$ for its value at that time period, and also its redemption value at time $T+1$. In the presence of a redemption promise, the first order condition at time $T$, in equilibrium $c_t = Y$ and $M_T = M$, becomes

$$\frac{u_c(Y)}{P_T} - u_m \left( \frac{M}{P_T} \right) \frac{1}{P_T} = \max \left[ \frac{\beta u'(Y)}{\overline{P}}, \frac{\beta u'(Y)}{P_{T+1}} \right].$$

With the latter term zero at time $T$, the redemption value of the former term remains. This effect gives a small decrease in the price level $P_T < \tilde{P}$ and $M/P_T > M/\tilde{P}$.

The dashed line in Figure 17.3 presents the equilibrium with the redemption guarantee, labeled “$\varepsilon = 0.5$.” Time $T+1$ and beyond have price levels $P_t = \infty$. The time $T$ price level is now a little lower, and time $T$ real money $M/P_T$ a little higher than before, because of the redemption value of money at time $T + 1$. The dashed line marked “redemption value” gives the value $M/\overline{P}$ that the consumer receives from the government at time $T + 1$. This is not $M_{T+1}/P_{T+1}$ since that is $0/\infty$. But drawing this redemption value on the graph in place of a market value of money, you can see
how values propagate back in this equilibrium. Obstfeld and Rogoff study this point, with $M_{T+1} = M$ and $P_{T+1} = \bar{P}$ as the last point of their economy. However, they claim that this point is not an equilibrium, and with that claim seek to rule out the path leading to it. My view here is that the point below it is the equilibrium, with $M_{T+1} = 0$ and $P_{T+1} = \infty$, and the path leading to that point remains valid.

A key to my equilibrium is that monetary policy allows de-monetization, for people to cash in their money. We could rule out this equilibrium by having monetary policy also insist that $M_{T+1} = M$. The combination of $M_T = M$ and the redemption guarantee would indeed be a policy setting for which no equilibrium can form, and we saw in section 16.4 several proposals that amount to such “inconsistent” policy. But we dismissed inconsistent policy before, e.g., insisting simultaneously on an interest rate rule $i = \phi \pi$ together with a money growth rule that requires a lower interest rate. In my reading, Obstfeld and Rogoff do not specify an inconsistent policy. They do allow the government to undershoot the money growth target if people want to redeem their money. Alas, by specifying a consistent policy, they do not rule out multiple equilibria.

(In this treatment, I assume that people tender their money $M_{t-1}$ to the government at the beginning of period $t$. Cochran (2011a) makes the opposite timing assumption, which leads to the same answer but in a more, and unneccessarily complex, way.)

17.2.6 Interpreting Obstfeld and Rogoff

Whether or not one accepts my analysis of Obstfeld and Rogoff, it does not achieve a full price level determination with passive policy, ready to use for analyzing data or policy. The fix is fiscal, as it requires the government to have enough commodity on hand, and the government refuses the seigniorage opportunities of money demand after stopping inflation. The fix does not represent a commitment that our or any other government makes or has made, so it represents at best a policy proposal rather than a basis for description of current or historical policy. None of this is criticism, as Obstfeld and Rogoff did not claim otherwise. It is only a warning not to cite them for results they did not claim to offer.

We should not overemphasize the latter minor disagreement. 99% of the importance of Obstfeld and Rogoff’s analysis for our quest does not depend on it, and can grant their view that the inflationary equilibria are ruled out.
The main point: many authors quickly cite Obstfeld and Rogoff as showing that all multiple equilibrium problems are solved, and a small chance that governments would stop inflation by reverting to a gold standard at very high inflation restores price level determinacy by monetary policy alone. This is not what their result, even taken at face value, accomplishes.

First, it is a joint fiscal-monetary theory, not an alternative that rescues monetary price level determination with fully passive policy. The government must have the gold, and must not change the backstop redemption promise in response to observed prices. The government must also refuse the siren song of seigniorage that a two-way gold standard implies. If they are correct, the result challenges the generality of our earlier finding that fiscal backstops with locally passive fiscal policies do not suffice, but it is not a passive policy.

Second, The government’s refusal to take gold in return for new money is the central ingredient for ruling out inflationary equilibrium paths. Obstfeld and Rogoff’s proposal is not a simple reversion to the gold or commodity standard, which applies both ways! After the backstop price level is reached, inflation stops, and real money demand expands. If the government accommodates this demand, allowing people to bring gold in for new money, and thus allowing the steady state to re-form around the new price level $\overline{P}$ then we successfully transition to a steady price level.

Third, this latter feature really makes the suggestion at best a proposal for threats future central banks might make, not a suggestion for how current central banks behave or are expected by anyone to behave. There is not a whiff of this commitment on the Federal Reserve’s website. Many governments have stopped inflations with joint monetary-fiscal reforms. Some of those have even included gold standards or exchange rate pegs. But, as beautifully documented in Sargent (1983), all such governments have allowed and indeed encouraged the natural recovery of nominal and hence real money holdings once inflation has stopped. No central bank has ever announced that it would refuse to take gold in return for new currency in a stabilization!

Fourth, ex-post, a promise not to take gold (or foreign exchange) in return for currency, and therefore to forbid equilibrium from forming, whatever that means, is disastrous for the government and central bank’s objectives. Citizens are clamoring for the central bank to satisfy a money shortage. The treasury eyes a golden opportunity for non-inflationary seigniorage. Would any central bank, ex-post, inflict a non-formation-of-equilibrium on an economy?
For all these reasons, a one-way gold standard, which rules out equilibrium formation, is not a credible specification of what people currently or historically expect of our central banks.

Finally, of course, it specifies a money growth target which our central banks do not follow.

In these ways, if Obstfeld and Rogoff’s claim to rule out inflationary paths is correct, it does not rescue $MV(i) = PY$ with passive fiscal policy as a viable framework for current or historical monetary policy analysis. That too is not a criticism of Obstfeld and Rogoff – they intended it as a piece of pure theory, and did not claim that current central banks follow their policy, or that people expect it. They write (p. 676) “Speculative paths can be eliminated...” Can, not are.

If my reading of their results is right, Obstfeld and Rogoff’s model does not make an unbelievable end-the-world threat, which is good, but it does not remove the inflationary equilibria, so $MV(i) = PY$ even with this proviso leaves indeterminacies.

You may object that we do not see hyperinflations with constant money growth. All hyperinflations occur with immense money growth. Does that not show the inflationary equilibria are invalid? I agree that this observation shows the model that allows inflationary multiple equilibria is wrong, and incomplete. It needs another ingredient. In my view there is a different missing ingredient: Fiscal theory picks the price level path. Observed hyperinflations are all fiscal, that occur when the fiscal backing of the non-inflationary both disappears.

I spent a lot of time on this one paper, because so many authors casually cite Obstfeld and Rogoff as having solved all these problems and rescued $MV(i) = PY$ as a purely monetary price level determination, even with $V(i)$, and in a realistic way that we can use in analysis of actual economies. Even their claimed result does not achieve this goal – nor was it intended to do so. I spend more time because with an unconventional reading of a cited paper I think it’s important to be extra careful – how am I right and the other 461 Google scholar citers wrong? But that criticism just adds to the same bottom line for our purposes here.
Writing the money in utility model in continuous time, we obtain the money demand equation

\[ \frac{v'(m_t)}{u'(c_t)} = i_t \]

and the transversality condition \( \lim_{t \to \infty} e^{-\delta t} u'(c_t) m_t = 0 \)

The next three sections set out some of the more useful variations on the theme of explicit models of money. They don’t advance the main theme in a great way: They don’t advance our understanding of determinacy and stability issues. I include them mostly for pedagogical reasons, to set out some of the more useful formalisms for introducing money into formal models. If we add fiscal underpinnings and interest rate targets each is a useful foundation for adding liquid assets to a fiscal theory of monetary policy analysis of the world.

It’s easy to get hung up on the timing conventions of discrete time models. For that reason, it is usually much simpler in the end to present these models in continuous time. (Obstfeld (1984) presents this model cleanly.)

The objective is

\[ \max \int_0^\infty e^{-\delta t} [u(c_t) + v(m_t)] dt \]

where \( m_t = M_t / P_t \) is real money holdings. The constraint is

\[ \dot{m}_t = (y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t}. \]  \hspace{1cm} (17.28)

(Nominal money piles up when income is greater than consumption less net tax payments.

\[ dM_t = (y_t - c_t - s_t) P_t dt. \]

Use the definition \( m_t \equiv M_t / P_t \) take the time derivative.) The current value Hamiltonian is

\[ H = u(c_t) + v(m_t) + \mu \left[ (y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t} \right] . \]
The first order conditions are therefore
\[
\frac{\partial H}{dc} = 0 : u'(c_t) = \mu_t \\
-\frac{\partial H}{dm} : \dot{\mu}_t - \delta \mu_t = -v'(m_t) + \mu_t \frac{\dot{P}_t}{P_t} \\
\frac{\partial H}{d\mu} = 0 : \dot{m}_t = (y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t}
\]
\[
\lim_{t \to \infty} e^{-\delta t} m_t \mu_t = 0 \tag{17.29}
\]
Substituting out \(\mu\), we can write the familiar money demand conditions.
\[
\frac{\nu'(m_t)}{u'(c_t)} = -\frac{\dot{\mu}_t}{\mu_t} + \delta + \frac{\dot{P}_t}{P_t} = i_t.
\]
When consumption is constant, \(\dot{\mu} = 0\), the risk free rate is \(r = \delta\). If we add a risk free real investment, then \(r^f = \delta - \dot{\mu}_t/\mu_t\). Either way, the right hand side equals the nominal interest rate. \((17.29)\) becomes the transversality condition, which says that the discounted real value of money may not grow faster than the interest rate.
\[
\lim_{t \to \infty} e^{-\delta t} u'(c_t) m_t = 0.
\]

### 17.2.8 Nonseparable utility and more indeterminacy

Utility nonseparable between money and consumption is possible, and plausible. In this case, it is possible that our constant money growth model leads to multiple stable equilibria around the steady state. The phase diagram of Figure 17.1 can cut from above at the steady state.

A CES money in the utility function separates the interest elasticity of money demand from risk aversion. It also parameterizes how money growth distorts the relationship between consumption and interest rates.

When utility is non-separable between consumption and money, with \(u_{mc}(c_t, m_t) \neq 0\), our first order conditions \((17.12)-(17.13)\),
\[
Q_t = \frac{1}{1 + i_t} = E_t \left( \beta \frac{u_c(c_{t+1}, m_{t+1}) P_t}{u_c(c_t, m_t) P_{t+1}} \right). \tag{17.30}
\]
17.2. MONEY IN UTILITY

\[
\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1 + i_t} = 1 - Q_t
\]  

(17.31)

no longer separate so cleanly.

The marginal rate of substitution or discount factor for asset pricing in (17.30) now contains money holdings as well as consumption. The usual approximation, precise in continuous time, gives

\[
\beta \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} \approx 1 - \delta - \gamma \Delta c_{t+1} - \eta \Delta m_{t+1}.
\]

\[
r^f \approx \delta + \gamma E_t (\Delta c_{t+1}) + \eta E_t (\Delta m_{t+1})
\]

\[
\gamma = - \frac{cu_{cc}}{uc}, \eta = - \frac{u_{cm}}{uc}
\]

The relation between interest rates and consumption growth is distorted by money. Covariances with money growth will drive risk premia. Expected money growth will drive a wedge between risk free rates and expected consumption growth.

A nonseparable utility, in which variation in some additional variable moves the discount factor along with consumption growth, is widespread in finance. One can categorize almost all of the innovations in macro-finance, trying to explain the equity premium puzzle, time varying risk premiums, and unconventional sources of risk, as nonseparable utilities in which some variable other than consumption growth affects the discount factor. (Cochrane (2017a)) (Cochrane (2007)) survey the literature via this framework. Habits, housing, durable goods, recursive non-state-separable utility, and many others are of this form.

A nonseparable utility is also considered a realistic specification for money in the utility function. Money should be essential, in some sense, to procuring consumption. The point of money is, again, not to enjoy Scrooge McDuck swims in it, but because money makes purchasing consumption and selling endowments easier. That thought leads to a nonseparable utility. Cash in advance models, in which you must have

\[
\frac{d\Lambda_t}{\Lambda_t} = \frac{d\left[e^{-\delta t}u_c(c_t, m_t)\right]}{e^{-\delta t}u_c(c_t, m_t)}
\]

\[
= -\delta dt + \frac{c_t u_{cc}}{u_c} \frac{dc_t}{c_t} + \frac{m_t u_{cm}}{u_c} \frac{dm_t}{m_t} (+\text{Ito terms})
\]
cash to purchase consumption, typically lead to an equivalent representation as a nonseparable money in the utility function. In order to consume tomorrow, you must hold money overnight from today to tomorrow, and thereby consumption tomorrow gains an extra cost, the foregone interest.

With constant consumption as in our example, nonseparable utility means that changes in real money growth change the nominal interest rate, and thereby change the price level dynamics that result from (17.31). In fact, we can now have multiple stable equilibria around the steady state, so we have indeterminacy even without worrying about transversality conditions or the zero interest rate state. The phase diagram Figure 17.1 can cut from above rather than below. (Obstfeld (1984) makes this point in a delightfully concise 5 page paper. Why don’t journals publish papers like this any more?)

Suppose
\[ u(c_t, m_t) = -c_t^{-a}m_t^{-b} \]

Now,
\[ u_c(c_t, m_t) = ac_t^{-a-1}m_t^b \]
\[ u_m(c_t, m_t) = bc_t^am_t^{-b-1} \]

so the first order conditions give
\[ \frac{c_t b}{m_t a} = 1 - \beta \left( \frac{c_{t+1}}{c_t} \right)^{-a-1} \left( \frac{m_{t+1}}{m_t} \right)^{-b} \frac{P_t}{P_{t+1}}. \]

In equilibrium with money growth \( \mu \) and endowment \( Y = 1 \), then,
\[ \frac{b}{m_t a} = 1 - \beta \left( \frac{m_{t+1}}{m_t} \right)^{-b} \frac{P_t}{P_{t+1}}. \]
\[ \frac{b}{a m_t} = 1 - \frac{1}{(1+\delta)(1+\mu)} \left( \frac{m_{t+1}}{m_t} \right)^{1-b}. \]
\[ \frac{m_{t+1}}{m_t} = \left[ (1+\delta)(1+\mu) \left( 1 - \frac{b}{a m_t} \right) \right]^{\frac{1}{1-b}}. \]

The steady state is
\[ 1 = (1+\delta)(1+\mu) \left( 1 - \frac{b}{a m} \right). \]
In terms of the steady state,

\[
\frac{m_{t+1}}{m_t} = \left[ 1 - \frac{b}{a m_t} \right] \frac{1}{1 - \frac{b}{a m}}.
\]

Near the steady state,

\[
\frac{m_{t+1}}{m_t} \approx 1 + \frac{d}{dm_t} \left\{ \left[ 1 - \frac{b}{a m_t} \right] \frac{1}{1 - \frac{b}{a m}} \right\}_{m_t=m} (m_t - m).
\]

\[
\frac{m_{t+1}}{m_t} \approx 1 + \frac{b}{a} \frac{1}{1 - b} \frac{(m_t - m)}{m^2}.
\]

The coefficient multiplying the last term is negative for

\[ b > 1. \]

In that case, dynamics are stable around the steady state, giving multiple stable equilibria that unquestionably satisfy the transversality condition!

A CES specification is also useful as it lets us separate intertemporal substitution from the interest elasticity of money demand, but maintaining the useful proportionality of money to nominal income. If

\[
u(c_t, M_t/P_t) = \left[ c_t + \theta (M_t/P_t)^\rho \right] \frac{1}{1 - \gamma} - 1
\]

then we have

\[
u(c_t, M_t/P_t) = \left[ Y_t^\rho + \theta (M_t/P_t)^\rho \right] \frac{1-\gamma-\rho}{\rho} Y_t^{\rho-1}
\]

\[
= \left[ 1 + \theta \left( \frac{M_t}{P_t Y_t} \right)^\rho \right] \frac{1-\gamma-\rho}{\rho} Y_t^{-\gamma}
\]

so asset prices are driven by

\[
\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \left[ 1 + \theta \left( \frac{M_{t+1}}{P_{t+1} Y_{t+1}} \right)^\rho \right] \frac{1-\gamma-\rho}{\rho} \left( \frac{Y_{t+1}}{Y_t} \right)^{-\gamma}
\]
A monetary distortion modifies the standard power utility formula. Growth in real money balances accompanies higher real interest rates, as does growth in consumption. With constant real money \( M/(PY) \), we have the usual power utility formula with risk aversion \( \gamma \).

The money first order condition is

\[
\frac{\theta \left( \frac{M_t}{P_t} \right)^{\rho - 1}}{Y_t^{\rho - 1}} = \frac{i_t}{1 + i_t}
\]

so the money demand function (17.13) becomes

\[
M_t = P_t Y_t \left( \frac{1}{\theta} \frac{i_t}{1 + i_t} \right)^{\frac{1}{\rho - 1}}
\]

Here too we have a unit income elasticity and an interest elasticity governed by the separate parameter \( \rho \).

### 17.3 Cash in advance model

The cash in advance model is in part motivated by the artificiality of the money in the utility function. We don’t literally enjoy money. Cash in advance is the simplest and most tractable model that starts a little deeper and gives us a reason to hold money, in order to make transactions.

Money in the utility function models also deliver results that depend sensitively on the properties of the utility function – separable vs. nonseparable, limits as interest rates rise or fall or money holdings go to zero or infinity – as we have seen. Yet money in utility is just a little too abstract from the actual technology by which people make transactions, and more so from the other, vaguer, demands for liquid assets, that are thought to motivate money holding, in order for us really to be sure of which properties to specify. Cash in advance models, though formally equivalent to money in utility models, can effectively suggest some functional forms as more plausible than others, by deriving the “utility function” from a simplified shopping
story. In my evaluation, in the end, the shopping stories end up also being a bit artificial, but one should be aware of the motivation to digest the literature.

More pragmatically, for the immediate purpose, it appears to be a model that gives a money demand with fixed velocity. Now, velocity is not fixed in the real world, but it would be nice to exhibit even in theory a model capable of price level determination from money demand and fixed supply, \( MV = PY \). It turns out the cash in advance model does not answer even this theoretical desire – it too has multiple equilibria under passive policy. Money demand is still a “curve,” rising when interest rates hit zero, so we cannot rule out equilibria that deflate to the zero bound.

The cash in advance model also allows me to include a frictionless model that eliminates the cash in advance constraint so we can consider how the fiscal theory continues to determine the price level in that environment.


### 17.3.1 Setup

In place of money in the utility function, the cash in advance model specifies that money must be used for transactions, \( P_t c_t \leq M_t^d V \). In the standard specification, that money must be held overnight, despite a potential interest cost. In the frictionless variant, money can be returned at the end of the day. I write the model, cash in advance constraint, budget constraint, and define equilibrium

The government chooses a state-contingent sequence for one-period nominal debt, money and primary surpluses, \( \{ B_t^s, M_t^s, s_t \} \). The representative household maximizes a standard utility function,

\[
\max E \sum_{t=0}^{\infty} \beta^t u(c_t).
\]

The household enters period \( t \) with money balances \( M_{t-1} \) and one-period nominal discount bonds with face value \( B_{t-1} \). Any news is revealed. The household then goes to the asset market. The household redeems maturing bonds \( B_{t-1} \), pays net lump-sum taxes \( P_t s_t \), buys new bonds \( B_t \) and leaves with money \( M_t^d \).
Each household receives a nonstorable endowment $Y_t$ in the goods market. The household cannot consume its own endowment, and must therefore buy the endowments of other households. To do so, the household splits up into a worker and a shopper. The shopper takes the money $M_t^d$ and buys goods $c_t$ subject to a cash in advance constraint,

$$P_t c_t \leq M_t^d V.$$  \hfill (17.32)

The story is cleanest when $V = 1$, but it is useful to introduce the parameter $V$ and consider what happens as it changes later. The worker sells the endowment $Y_t$ in return for money, and gets cash $P_t Y_t$ in return.

In the monetary model, the shopper and worker go home and eat $c_t$. They must hold overnight any money $M_t^d - P_t c_t$ left over from the shopper, and the money $P_t Y_t$ earned by the worker. $M_t$, which denotes money held overnight, is

$$M_t = M_t^d + P_t(Y_t - c_t).$$  \hfill (17.33)

The frictionless cash in advance model makes one small change: The securities market reopens at the end of the day. The household can return to the securities market, i.e. the ATM machine is open in the afternoon, and trade any unwanted cash for more bonds. Thus, the household does not face the constraint (17.33); it can use cash during the day without holding it overnight. The absence of the constraint (17.33) is the only difference in the economic setup of the two models.

There is no interest on intraday bond holdings or cash loans. This is, roughly, the current institutional arrangement. No intraday interest also results if we think of the “day” as an arbitrarily short trading interval, say a minute of each hour, on the way to a continuous time model.

The point is worth stressing: the model is “frictionless,” not because nobody holds any money, and not because people do not use money for transactions. The model is frictionless because nobody holds money overnight, or subject to any interest cost.

The household can trade arbitrary contingent claims in the asset market. The price of a 1 period nominal discount bond at time $t$ is

$$Q_t = \frac{1}{1 + i_t} = P_tE_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{P_{t+1}} \right).$$  \hfill (17.34)

Households are forbidden to issue money, to keep them from arbitraging zero interest money against interest-bearing bonds,

$$M_t \geq 0.$$  \hfill (17.35)
The household’s budget constraints are the same as in the money in utility model. The period budget constraint states that the nominal value of money and bonds at the beginning of the period, plus any profits in the goods market, must equal the nominal value of bonds purchased, money held overnight, and net tax payments,

\[ B_{t-1} + M_{t-1} + P_t (Y_t - c_t) = Q_t B_t + M_t + P_t s_t. \]  

The household’s money and debt demands must also obey the transversality conditions

\[ \lim_{T \to \infty} E_t \left( \frac{\Lambda_{t+T} B_{T-1}}{\Lambda_t P_T} \right) = 0 \]  

(17.37)

\[ \lim_{T \to \infty} E_t \left( \frac{\Lambda_{t+T} M_{T-1}}{\Lambda_t P_T} \right) = 0. \]  

(17.38)

These conditions imply the present value budget constraint. As before, we can write it in two ways, treating the inflation tax either as an interest cost or as dilution due to money printing,

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right) \]  

(17.39)

or

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right] \]  

(17.40)

An equilibrium is a set of initial stocks \( B_0, M_0 \), and sequences for quantities \( \{c_t, M^d_t, M_t, B_t, s_t\} \) and prices \( \{\Lambda_t, P_t\} \) such that:

1. (Household optimization) Given prices \( \{P_t, \Lambda_t\} \), initial stocks \( B_{-1}, M_{-1} \), and the tax and endowment streams \( \{s_t, Y_t\} \), the choices \( \{B_t, M^d_t, \alpha_t\} \) maximize expected utility subject to the budget constraints (17.36)-(17.38), the cash in advance constraint (17.32), and the no-printing-money constraint (17.35). In the cash-in-advance model, the household must also meet the constraint (17.33) that money coming from the goods market is held overnight.

2. (Market clearing) \( c_t = Y_t, \ M_t = M^*_t, \ B_t = B^*_t \) at each date and state of nature.
CHAPTER 17. MONETARISM

17.3.2 Monetary model

We characterize the equilibrium of the monetary model. The standard asset pricing equation holds, without monetary distortions. If interest rates are positive, the cash in advance constraint binds. The government debt valuation must hold.

The consumer’s first order conditions, budget constraints, and market-clearing imply the following characterizations:

1. The marginal rate of substitution is equal to the stochastic discount factor,

\[ \beta^j \frac{u'(Y_{t+j})}{u'(Y_t)} = \frac{\Lambda_{t+j}}{\Lambda_t}. \]  \hfill (17.41)

Hence, nominal bond prices are given by

\[ Q_t = \beta E_t \left[ \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} \right]. \]  \hfill (17.42)

If the endowment is constant over time \( Y_t = Y \), then

\[ \frac{\Lambda_{t+j}}{\Lambda_t} = \beta^j; \quad Q = \beta. \]

2. Any equilibrium with positive nominal interest rates, must have a binding cash constraint,

\[ M_t V = P_t c_t = P_t Y_t. \]  \hfill (17.43)

3. The government debt valuation equation holds,

\[ \frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} E_t \left[ \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) \right] \]  \hfill (17.44)

or, equivalently,

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = \sum E_t \left[ \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \right] \]  \hfill (17.45)

Fact 1 follows from the household’s first order conditions for buying one less consumption good, investing in a contingent claim, and then consuming more at \( t + j \).
Following Sargent (1987), there is no asset-pricing distortion with this timing convention. In order to raise consumption $c_t$ the household must also get more money $M_t^{d}$, but cash overnight $M_t$ will be unaffected because $P_t c_t$ changes by the same amount as $M_t^{d}$ changes (see equation (17.33)). With positive nominal interest rates, money is strictly dominated by bonds, so the household will hold as little money as possible overnight when interest rates are positive. In the CIA model, that quantity is $M_t = P_t e/V$; goods market equilibrium gives $e = c_t$, and hence Fact 2. To derive Fact 3, use the bond price definition (17.34), iterate forward the consumer’s period to period budget constraint (17.36), impose the condition (17.38), and impose market clearing ($Y_t = c_t, M_t = M_t^f$). Jr. Sargent (1987) treat existence of equilibrium. It’s easy enough to construct examples with standard utility functions. Our issue is the uniqueness of equilibrium, and we shall see shortly that it is not.

### 17.3.3 The point

Monetary and fiscal policy must be coordinated. We commonly separate active-money, passive-fiscal or active-fiscal passive-money alternatives for this coordination. But the equilibrium is the same, so the two coordination stories, and the infinite number between them, are observationally equivalent.

The pair (17.43) and (17.44) together determine the price level in terms of variables chosen by the government. That, finally, the whole point of the exercise. We’ve been writing down these two equations. Now we have an explicit model to verify that this was the right thing to do.

Looking at the explicit model helps us again to see that the government valuation equation (17.44) results from the consumer’s budget constraint and equilibrium. It is not a “government budget constraint.”

The government has three levers $\{M_t, B_t, s_t\}$, which produce one outcome $\{P_t\}$. Thus, the government must choose its levers in a coordinated way if it wishes to produce an equilibrium.

The standard solution to this model assumes at this point an “active-money, passive fiscal” regime to that result, in the classic terminology of Leeper (1991). The central bank, by controlling $\{M_t\}$, determines the price level. The treasury then must raise surpluses $\{s_t\}$ to validate whatever price level the central bank has chosen. As before, expected changes in the price level can be met by changing $B_{t-1}$ with no change in $\{s_t\}$. Unexpected changes in the price level must come from unexpected changes.
in surpluses \( \{s_t\} \), or the inflation tax of future money creation. If you look closely, all good cash in advance papers have a footnote somewhere to the effect that the government levies lump-sum taxes ex-post so that (17.44) holds.

But we can also solve the same model, and arrive at the same equilibrium, with a “passive money, active fiscal” regime. Here, by choice of \( \{B_t, s_t\} \) the government valuation equation controls the price level. The central bank must then “passively” provide the money \( \{M_t\} \) needed to solve money demand. The equilibrium is the same, so the two ways of achieving coordination are observationally equivalent.

Moreover, we can tell any number of intermediate or other stories. The means by which central bank and treasury come up with a coordinated policy leaves no trace in the data.

The frictionless and fiscal solution of this cash in advance model formalizes the story I told in the very first chapter about a “day,” in which the government prints up cash to pay off bonds, and that cash is then soaked up at the end of the day by selling new bonds.

### 17.3.4 Frictionless model

We characterize the equilibrium of the frictioness model, in which people do not have to hold money overnight. If interest rates are positive, they will hold no money. Nonetheless, there is a well defined equilibrium under an active fiscal policy.

In the frictionless model, the bank reopens at the end of the day. Now

1. The marginal rate of substitution (17.41) is still equal to the stochastic discount factor or contingent claims prices,

\[
\beta^j \frac{u'(Y_{t+j})}{u'(Y_t)} = \frac{\Lambda_{t+j}}{\Lambda_t}.
\]

and with a constant endowment \( \Lambda_{t+j}/\Lambda_t = \beta^j \).

2. Any equilibrium with positive nominal interest rates \( Q_t < 1 \), must have no money

\[
M_t = 0.
\]

No equilibrium may have negative nominal interest rates, \( Q_t > 1 \).
3. The government debt valuation equation holds, now

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}. \] (17.48)

The consumer’s flow budget constraint (17.36) is not changed, so first order condition behind fact 1 is the same. Removing the constraint (17.33) that cash from sales must be held overnight, the minimum cash that the household can hold overnight is zero, so (17.47) replaces the quantity equation (17.43). Equation (17.47) is still a money demand equation, but it now holds for any price level and so does not help in price level determination. A negative nominal interest rate is an arbitrage opportunity, and leads to infinite money and negative infinite bond demand, and so cannot be an equilibrium. Equation (17.48) specializes (17.45). In periods with positive nominal rates \( i_{t+j} > 0 \), we have \( M_{t+j} = 0 \), so the seignorage term drops because \( M \) is missing. In periods with zero nominal rates, \( i_{t+j} = 0 \), seignorage drops because there is no interest differential between money and bonds.

There are specifications of the utility function, endowment processes, and government choices \( \{B_s^t, M_s^t, s_t\} \) that result in equilibria of the frictionless model with determinate, finite price levels. I can prove this statement most transparently by giving a simple example. Suppose \( u(c) = c^{1-\gamma} \), \( Y_t = Y \), \( B_s^t = B \), \( M_s^t = 0 \), \( s_t = s \), all positive and constant over time. Obviously, we must have \( c_t = Y \). From (17.46), the discount factor is constant,

\[ \Lambda_{t+1}/\Lambda_t = \beta. \]

From (17.48), the price level must be constant and positive,

\[ P_t = P = (1 - \beta) \frac{B}{s}. \]

Nominal interest rates are positive, \( Q_t = \beta < 1 \) so money demand equals money supply \( M = 0 \). \( \lim_{T \to \infty} \beta^T B/P = 0 \) so the transversality condition (17.38) is satisfied. The consumer’s first order conditions and transversality conditions are necessary and sufficient for an optimum. Thus, we have found sequences \( \{c_t, M_d^t, M_t, s_t, Q_t, p_t\} \) and \( M_0, B_0 \) that satisfy the definition of an equilibrium. Furthermore, given all the other variables, \( \{P_t\} \) is unique.

Not all specifications of the utility function, endowment process and government choices \( \{B_s^t, M_s^t, s_t\} \) result in equilibria, as pathological utility functions and “uncoordinated” or otherwise nonsensical policy do not lead to equilibria in the monetary
model. Here, I discuss the issues, but I do not attempt a characterization of the weakest possible restrictions on utility functions and exogenous processes that result in an equilibria.

As in all dynamic models, the endowment process and utility function must be such that equilibrium marginal rates of substitution $\frac{\Lambda_{t+j}}{\Lambda_t} = \beta^j u'(Y_{t+j}) / u'(Y_t)$ are defined. For example, we can’t have occasionally negative endowments in a model with power utility.

Equation (17.48) and market clearing ensure a unique, positive, equilibrium price level sequence $\{P_t\}$, if the government always chooses a positive amount of nominal debt at each date, $\infty > B^*_t + M^*_t > 0$ and a surplus whose present value is positive $\infty > E_t(\Lambda_{t+j}/\Lambda_t s_{t+j}) > 0$. It is not necessary that all these sequences are positive. There can be equilibria with negative debt, surpluses or money supplies, but one must rule out $0/0$ problems in (17.48).

One-period bond prices are determined from $Q_t = P_t E_t(\Lambda_{t+1}/\Lambda_t P_{t+1})$. For there to be an equilibrium, the government must choose a price level sequence, via its choices of $\{B^*_t, M^*_t, s_t\}$, so that the expectation exists, and so that the nominal interest rate is nonnegative, $Q_t \geq 1$. If it chooses the price level sequence so that the nominal interest rate is negative, households will try to hold infinite cash and infinite negative amounts of debt.

Finally, and most importantly, the government must produce a coordinated policy configuration $\{B_t, M_t, s_t\}$ so that equilibrium is possible. In this model the government cannot produce that configuration by setting $\{s_t\}$ to mechanically have (17.48) hold for any price level – it may not set a “passive fiscal” policy. If it did so, the price level would be underdetermined. In a one-period version of the model or if no new debt is sold, the valuation equation (17.48) is

$$\frac{B_{t-1}}{P_t} = s_t.$$  

If the government chooses to increase the real surplus one-for-one with the price level, holding $s_t = P_t s_t$ constant, then there is either no equilibrium (if $s_t \neq B_{t-1}$) or the equilibrium is indeterminate (if $s_t = B_{t-1}$).

Thus, the government must also choose an “active-fiscal” policy in order for there to be an equilibrium price level in the frictionless model.
17.3. CASH IN ADVANCE MODEL

17.3.5 Multiple equilibria re-emerge

We characterize the equilibrium of the monetary model.

The cash in advance model appears to formalize the interest-inelastic case, \( M_t V = P_t Y_t \), in which if the government sets a fixed money supply \( M_t^* \), we have a unique price level for fiat currency with a passive fiscal policy. Alas, even this case fails to determine the price level.

The cash in advance constraint only binds if the interest rate is positive. If the interest rate is zero, the cash in advance constraint does not bind. Then, money demand is indeterminate. Money and bonds are perfect substitutes. With money demand indeterminate, so is the price level for a given amount of money.

The money demand function for the cash in advance model is not, in fact, a perfect \( M_t V = P_t Y_t \) with fixed \( V \). At zero interest rate, money demand becomes a correspondence; any \( M_t \geq P_t Y_t / V \) will do. At negative interest rates, money demand becomes infinite. One can think of the cash in advance money demand function as the limit of the usual function, pushed to the axes – the curve becomes an L. But it is an L, not a horizontal line, and this L brings back the whole train of indeterminacy.

One might think that indeterminacy therefore only holds for a low value of money growth, that drive inflation down to the point that the nominal rate is zero. That is true; even the \( M_t V = P_t Y_t \) equilibrium becomes indeterminate when money growth is too low. But there are multiple, zero-interest rate equilibria for any money growth path, just as we found liquidity-trap equilibria in the new-Keynesian model that start from any value below \( \Pi^* \) and converge to \( \Pi_L \).

For example, consider perfect foresight equilibria. Let \( M_{t+1}/M_t = 1 + \mu \) and so \( M_t = M_0(1 + \mu)^t \), and suppose a constant endowment. The usual equilibrium is \( P_t = M_t V / Y = M_0(1 + \mu)^t / Y \). The nominal interest rate is \( (1 + i) = (1 + \delta)P_{t+1}/P_t = (1 + \delta)(1 + \mu) \) where \( 1 + \delta = 1/\beta \). So, so long as \((1 + \mu) > \beta \), the usual equilibrium will have a positive interest rate and the cash in advance constraint binds.

But there can also be equilibria with \( P_{t+1}/P_t = \beta \), a slight deflation. Start with a price level that is too low, \( P_0 < M_0 V / Y \). This does not violate the cash in advance constraint because \( P_1/P_0 = \beta \), so \( i_0 = 0 \). Now, we will also then have \( P_1 < M_1 V / Y \): We have \( P_1 = \beta P_0 \), and \( M_1 = (1 + \mu)M_0 \), and by assumption, \((1 + \mu) > \beta \) – money growth was set in the hope that the interest rate would be positive. Likewise,

\[
P_t = \beta^t P_0 \leq \beta^t M_0 V / Y \leq (1 + \mu)^t M_0 V / Y = M_t V / Y.
\]
In words, we have a too-low initial price level, and slight deflation. The interest rate is zero, and even though money keeps growing, people are happy to have ever larger amounts of money at zero interest rates.

The cash in advance model does not have the speculative inflations described above. But the cash in advance model has the full set of speculative deflations. The indeterminacy of money demand at zero interest rates – the “liquidity trap” that bonds and money become perfect substitutes – mean that for any money growth rate, deflationary self-fulfilling equilibria can break out.

The fiscal theory; a joint monetary-fiscal policy regime with active fiscal policy – puts a quick stop to these equilibria. The government commits to financing the debt at price levels from the desired equilibrium, $P_t = M_t V / Y$, but will not raise surpluses to validate self-fulfilling deflations. It looks passive on the desired equilibrium path, but it is active in not validating alternative equilibria, as we have seen. But with fully passive fiscal policy, multiple equilibria remain even in this most monetarist of models.

Again, this indeterminacy holds for any money growth process. It’s easy to confuse it with the more obvious possibility, that the government sets money growth to such a low level that even the standard equilibrium $M_t V = P_t Y$ grows indeterminacy. Yes, if $M_{t+1} / M_t = \beta$, then $P_t = M_t V / Y$ also has $P_{t+1} / P_t = \beta$ and $1 + i = (P_{t+1} / P_t) / \beta = 1$. Then any $P_0 \leq M_0 V / Y$ is an equilibrium. If we start with such a $P_0$, then $P_t = (1 + \mu)^t P_0$ will also satisfy $P_t \leq M_t V / Y$. But the indeterminacy problem holds for any money growth process, not just the special case that monetary policy tries to implement a zero interest rate.

This problem is sometimes acknowledged if you read cash-in-advance models very carefully. For example, in the classic textbook treatment, Sargent (1987) writes on p. 162, “except on Section 5.5, we will focus on equilibria in which the currency-in-advance restriction $p_t c_t \leq m_t^p$ is met with equality because the risk-free net nominal interest rate is positive... ” “We will focus on” acknowledges that there are other equilibria, which Sargent simply ignores. The same erasure is more implicit in the usual dynamic programming approach. If you insist that the price level must be a time-invariant function of state variables, then you rule out the multiple equilibria.

Sargent’s section 5.5 (p. 177ff) considers the possibility that money, reserves, pays the same interest as other risk free securities, which is the more general case of $i = 0$ induced by low money growth. When reserves pay full interest, (p. 178) “Because currency is not dominated in rate of return, $m_t^p \geq p_t c_t$ will not generally
hold with equality. Instead the household’s demand for real balances of currency is indeterminate...” As a result, p. 180, “...the price level, level of taxes and real balances, are all indeterminate.” As we have seen before, fiscal theory repairs this indeterminacy as well, which is a good thing given that we now live in the world of interest on reserves and the price level has not turned out to be indeterminate. And indeed here Sargent takes care to rule out the fiscal-theoretic repair: p. 180 “...the government levies whatever lump-sum taxes are necessary to finance the interest payments on currency.” But nowhere does Sargent consider the equilibria of the regular model when the cash in advance constraint is not binding. We just “focus” on the desired equilibrium.

This observation should not be taken as criticism of the cash-in-advance literature. I just point out that the cash in advance models, as written, leave open multiple equilibria. We cannot appeal to them as models for determinate price levels without fiscal theory. With fiscal theory, to easily trim the undesired equilibria, we have fully coherent models capable of examining all sorts of monetary affairs. Really, the only substantive change is to add an equation or two to statements like “we focus on” the binding cash constraint equilibrium. No, we enforce it via an active fiscal policy.

The larger criticism of cash in advance and other monetary models is whether substantial liquidity discounts exist, and whether governments deliberately ration liquid assets. Interest rate targets, implemented by interest payments on excess reserves held to satiation seem like a better description of the world since about 2008. But that is a question of empirical relevance, not of theoretical coherence.

17.3.6 The zero bound

17.3.7 An interest rate target implemented by monetary policy

17.4 The vast literature

Woodford (1994) studies a cash in advance economy with cash and credit goods, so it has an interest-elastic money demand. He shows that a constant money growth policy typically leaves multiple equilibria, and always does so if money supports negative inflation at the interest rate and a zero rate, but an interest rate peg can
have a unique equilibrium, even if the rate is pegged at zero. He concludes that the Friedman rule can be supported by an interest rate peg, but not a money growth target.

With the advantages of hindsight, Woodford’s result is deeply fiscal. Woodford’s money growth policy is financed by lump sum taxes or transfers, rebating all seignorage to households. In the money growth model, Woodford specifies that households receive a nominal transfer equal to money printing, \( H_t = M_t - M_{t-1} \) (p. 350). For the interest rate target, Woodford specifies instead that “the government chooses a deterministic [and later “exogenous”] sequence \( \{h_t\} \) for real net transfers to the private sector.” With the advantages of hindsight, we recognize instantly the fiscal theory of monetary policy at work in the latter case, and a passive fiscal policy in the former. Woodford’s core result, then, is that an active fiscal policy can overturn Sargent and Wallace (1975) indeterminacy of interest rate pegs. Indeed, (p. 378):

“...the failure of homogeneity in (3.3) [the result that an interest rate peg leads to determinate inflation] does depend upon the specification of fiscal policy here; in particular, any process \( \{M_t\} \) satisfying (3.1)-(3.2) can be made to be an equilibrium consistent with an interest rate peg if net transfers\( \{H_t\} \) are assumed to vary with the sunspot state in the way necessary to satisfy the intertemporal budget constraint. On the other hand, the kind of fiscal policy specification required to preserve homogeneity is a very special one; the particular case considered here (real net transfers constant over time and unaffected by the path of nominal variables) is simple to analyze but is hardly the only kind of specification for which the intertemporal budget constraint causes the equilibrium conditions to be inhomogeneous.”

and p. 373,

“For the increase or decrease in the money supply that would be necessary to accommodate a given change in the current price level carries with it a change in the net indebtedness of the government to the private sector, which will affect the budget constraints of consumers and so have a real effect on the economy... It is this second source of inhomogeneity that is relevant for the representative consumer economy considered here.”

Followed by footnote 19,

“Leeper (1991) similarly obtains determinacy due to the intertemporal budget constraint in the case of a variety of types of interest rate policies,
in the context of a linear model.”

If this is not terribly clear, keep in mind this paper was written just around the time Leeper’s foundational “active and passive” [Leeper (1991)] was published, and long before Woodford’s “fiscal requirements for price stability,” [Woodford (2001)].

Footnote 1 in [Woodford (1994)] offers a concise survey of multiple equilibrium issues in money-in-utility models, essentially documenting that there is no consensus general statement: “But while several authors have addressed aspects of this problem...no very general treatment exists for that class of models.”

17.4.1 Money in continuous time

The utility function is

\[
\max E \int_{t=0}^{\infty} e^{-\delta t} u(c_t, M_t/P_t) dt.
\]

The present-value budget constraint is

\[
\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[ c_t - y_t + s_t + (i_t - i_m^m) \frac{M_t}{P_t} \right] dt
\]

where

\[
r_t = i_t - \frac{dP_t}{P_t}
\]

and \(s\) denotes real net taxes paid, and thus the real government primary surplus. This budget constraint is the present value form of

\[
d(B_t + M_t) = i_t B_t + i_m^m M_t + P_t(y_t - c_t - s_t).
\]

Introducing a multiplier \(\lambda\) on the present value budget constraint, we have

\[
\frac{\partial}{\partial c_t} : e^{-\delta t} u_c(t) = \lambda e^{-\int_{s=0}^{t} r_s ds},
\]

where \((t)\) means \((c_t, M_t/P_t)\). Differentiating with respect to time,

\[
-\delta e^{-\delta t} u_c(t) + e^{-\delta t} u_{cc}(t) \frac{dc_t}{dt} + e^{-\delta t} u_{cm}(t) \frac{dm_t}{dt} = \lambda r_t e^{-\int_{s=0}^{t} r_s ds}
\]
where \( m_t \equiv M_t/P_t \). Dividing by \( e^{-\delta t}u_c(t) \), we obtain the intertemporal first order condition:

\[
-\frac{c_t u_{cc}(t)}{u_c(t)} \frac{d c_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{d m_t}{m_t} = (r_t - \delta) \ dt.
\]  

The first-order condition with respect to \( M_t \) is

\[
\frac{\partial}{\partial M_t} e^{-\delta t} u_m(t) \frac{1}{P_t} = \lambda e^{-\int_{s_0}^{t} r_s \ ds} (i_t - i_t^m) \frac{1}{P_t}
\]

\[
e^{-\delta t} u_m(t) = e^{-\delta t} u_c(t) (i_t - i_t^m)
\]

\[
\frac{u_m(t)}{u_c(t)} = i_t - i_t^m.
\]  

The last equation is the usual money demand curve.

Thus, an equilibrium \( c_t = y_t \) satisfies

\[
-\frac{c_t u_{cc}(t)}{u_c(t)} \frac{d c_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{d m_t}{m_t} = -\delta dt + \left( i_t - \frac{dP_t}{P_t} \right) dt
\]  

\[
\frac{u_m(t)}{u_c(t)} = i_t - i_t^m
\]  

\[
\int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s \ ds} \left[ s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt
\]  

The last equation combines the consumer’s budget constraint and equilibrium \( c = y \).

I call it the government debt valuation formula.

**CES functional form**

I use a standard money in the utility function specification with a CES functional form,

\[
u(c_t, m_t) = \frac{1}{1-\gamma} \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{1-\gamma}{1-\theta}}.
\]

I use the notation \( m = M/P \), with capital letters for nominal and lowercase letters for real quantities.

This CES functional form nests three important special cases. Perfect substitutes is the case \( \theta = 0 \):

\[
u(c_t, m_t) = \frac{1}{1-\gamma} [c_t + \alpha m_t]^{1-\gamma}.
\]
The Cobb-Douglas case is $\theta \rightarrow 1$:

$$u(c_t, m_t) \rightarrow \frac{1}{1 - \gamma} \left[ \frac{1}{c_t^{1+\alpha} m_t^{1+\alpha}} \right]^{1-\gamma}. \quad (17.54)$$

The monetarist limit is $\theta \rightarrow \infty$:

$$u(c_t, m_t) \rightarrow \frac{1}{1 - \gamma} \left[ \min (c_t, \alpha m_t) \right]^{1-\gamma}.$$  

I call it the monetarist limit because money demand is then $M_t/P_t = c_t/\alpha$, i.e. $\alpha = 1/V$ is constant, and the interest elasticity is zero. The separable case is $\theta = \gamma$:

$$u(c_t, m_t) = \frac{1}{1 - \gamma} \left[ c_t^{1-\gamma} + \alpha m_t^{1-\gamma} \right].$$

In the separable case, $u_c$ is independent of $m$, so money has no effect on the intertemporal substitution relation, and hence on inflation and output dynamics in a new-Keynesian model under an interest rate target. Terms in $(\theta - \gamma)$ or $(\sigma - \xi)$ with $\sigma = 1/\gamma$ and $\xi = 1/\theta$ will characterize deviations from the separable case, how much the marginal utility of consumption is affected by money.

With this functional form, the derivatives are

$$u_c = \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{\theta}{1-\theta}} c_t^{-\theta}$$

$$u_m = \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{\theta}{1-\theta}} \alpha m_t^{-\theta}.$$  

Equilibrium condition (17.52) becomes

$$\frac{u_m(t)}{u_c(t)} = \alpha \left( \frac{m_t}{c_t} \right)^{-\theta} = i_t - \iota_t.$$  

(17.55)

The second derivative with respect to consumption is

$$\frac{u_{cc}}{u_c} = (\theta - \gamma) \frac{1}{\left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]} c_t^{-\theta} - \theta c_t^{-1}$$

$$- \frac{cu_{cc}}{u_c} = - \left( \theta - \gamma \right) c_t^{1-\theta} - \theta \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]$$

$$\quad \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]$$

$$- \frac{cu_{cc}}{u_c} = \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]$$

$$\quad \left[ c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]$$
\[ \frac{\partial \bar{c}}{\partial u_c} = \gamma \left[ 1 + \frac{\theta}{\gamma} \alpha \left( \frac{m_t}{c_t} \right)^{1-\theta} \right] \]

The cross derivative is

\[ \frac{m \partial u_{cm}}{u_c} = (\theta - \gamma) \frac{\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}} \]

\[ = (\theta - \gamma) \frac{\alpha \left( \frac{m_t}{c_t} \right)^{1-\theta}}{1 + \alpha \left( \frac{m_t}{c_t} \right)^{1-\theta}}. \]

or, using (17.55)

\[ \frac{m \partial u_{cm}}{u_c} = (\theta - \gamma) \frac{\left( \frac{m_t}{c_t} \right) (i_t - i_t^m)}{1 + \left( \frac{m_t}{c_t} \right) (i_t - i_t^m)}. \]

Money demand

Money demand (17.55) can be written

\[ \frac{m_t}{c_t} = \left( \frac{1}{\alpha} \right)^{-\xi} (i_t - i_t^m)^{-\xi}. \]  

where \( \xi = 1/\theta \) becomes the interest elasticity of money demand, in log form, and \( \alpha \) governs the overall level of money demand.

The steady state obeys

\[ \frac{m}{c} = \left( \frac{1}{\alpha} \right)^{-\xi} (i - i^m)^{-\xi}. \]  

so we can write money demand (17.56) in terms of steady state real money as

\[ \frac{m_t}{c_t} = \left( \frac{m}{c} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi}, \]

avoiding the parameter \( \alpha \). (Throughout, numbers without time subscripts denote steady state values.)
17.4. THE VAST LITERATURE

The product \( \frac{m}{c} (i - i^m) \), the interest cost of holding money, appears in many subsequent expressions. It is

\[
\frac{m}{c} (i - i^m) = \left( \frac{1}{\alpha} \right)^{-\xi} (i - i^m)^{1-\xi}.
\]

With \( \xi < 1 \), as interest rates go to zero this interest cost goes to zero as well.

**Intertemporal Substitution**

The first order condition for the intertemporal allocation of consumption (17.51) is

\[
-c_t u_{c}(t) \frac{d c_t}{c_t} - \frac{m_t u_{cm}(t)}{u_{c}(t)} \frac{d m_t}{m_t} = -\delta dt + (i_t - \pi_t) dt
\]

where \( \pi_t = dP_t/P_t \) is inflation. This equation shows us how, with nonseparable utility, monetary policy can distort the allocation of consumption over time, in a way not captured by the usual interest rate effect. That is the central goal here.

In the case of complements, \( u_{cm} > 0 \) (more money raises the marginal utility of consumption), larger money growth makes it easier to consume in the future relative to the present, and acts like a higher interest rate, inducing higher consumption growth.

Substituting in the CES derivatives,

\[
\frac{1 + \frac{\theta}{\gamma} \alpha \left( \frac{m}{c_t} \right)^{1-\theta}}{1 + \alpha \left( \frac{m}{c_t} \right)^{1-\theta}} \frac{d c_t}{c_t} - (\theta - \gamma) \frac{\alpha \left( \frac{m}{c_t} \right)^{1-\theta}}{1 + \alpha \left( \frac{m}{c_t} \right)^{1-\theta}} \frac{d m_t}{m_t} = -\delta dt + (i_t - \pi_t) dt
\]

and using (17.55) to eliminate \( \alpha \)

\[
\frac{1 + \frac{\theta}{\gamma} \left( \frac{m}{c_t} \right) (i_t - i_t^m)}{1 + \left( \frac{m}{c_t} \right) (i_t - i_t^m)} \frac{d c_t}{c_t} - (\theta - \gamma) \frac{\left( \frac{m}{c_t} \right) (i_t - i_t^m)}{1 + \left( \frac{m}{c_t} \right) (i_t - i_t^m)} \frac{d m_t}{m_t} = -\delta dt + (i_t - \pi_t) dt
\]

(17.59)

We can make this expression prettier as

\[
\frac{\gamma}{c_t} \frac{d c_t}{c_t} + (\theta - \gamma) \frac{\left( \frac{m}{c_t} \right) (i_t - i_t^m)}{1 + \left( \frac{m}{c_t} \right) (i_t - i_t^m)} \left( \frac{d c_t}{c_t} - \frac{d m_t}{m_t} \right) = -\delta dt + (i_t - \pi_t) dt
\]
Reexpressing in terms of the intertemporal substitution elasticity \( \sigma = 1/\gamma \) and interest elasticity of money demand \( \xi = 1/\theta \), and multiplying by \( \sigma \),

\[
\frac{dc_t}{c_t} + \left( \frac{\sigma - \xi}{\xi} \right) \left( \frac{m_t}{c_t} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right) \left( \frac{dc_t}{c_t} - \frac{dm_t}{m_t} \right) = -\delta \sigma dt + \sigma (i_t - \pi_t) dt. \quad (17.60)
\]

We want to substitute interest rates for money. To that end, differentiate the money demand curve

\[
\frac{m_t}{c_t} = \left( \frac{m}{c} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi}.
\]

\[
\frac{m_t}{c_t} \frac{dm_t}{m_t} - \frac{dc_t}{c_t} = -\xi \left( \frac{m}{c} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}
\]

\[
\left( \frac{dc_t}{c_t} - \frac{dm_t}{m_t} \right) = \xi \left( \frac{m}{c} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}.
\]

Substituting,

\[
\frac{dc_t}{c_t} + \left( \frac{\sigma - \xi}{\xi} \right) \left( \frac{m_t}{c_t} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right) \left( \frac{m}{c} \right) \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m} = -\delta \sigma dt + \sigma (i_t - \pi_t) dt.
\]

\[
\frac{dc_t}{c_t} + (\sigma - \xi) \frac{m}{c} \left( \frac{i_t - i_t^m}{i - i^m} \right)^{-\xi} d(i_t - i_t^m) = -\delta \sigma dt + \sigma (i_t - \pi_t) dt.
\]

With \( x_t = \log c_t \), \( dx_t = dc_t/c_t \), approximating around a steady state, and approximating that the interest cost of holding money is small, \( (\frac{m}{c} \left( i - i^m \right) << 1 \), we obtain the intertemporal substitution condition modified by interest costs,

\[
\frac{dx_t}{dt} + (\sigma - \xi) \frac{m}{c} \frac{d(i_t - i_t^m)}{dt} = \sigma (i_t - \pi_t). \quad (17.61)
\]

In discrete time,

\[
E_t x_{t+1} - x_t + (\sigma - \xi) \left( \frac{m}{c} \right) [E_t (i_{t+1} - i_{t+1}^m) - (i_t - i_t^m)] = \sigma (i_t - E_t \pi_{t+1}).
\]
For models with monetary control, one wants an IS curve expressed in terms of the monetary aggregate. From (17.60), with the same approximations and $\tilde{m} = \log(m)$,

$$\frac{dx_t}{dt} + \left(\frac{\sigma - \xi}{\xi}\right) \left(\frac{m}{c}\right) (i - \tilde{r}^m) \left(\frac{dx_t}{dt} - \frac{d\tilde{m}_t}{dt}\right) = \sigma (i_t - \pi_t) \, dt.$$ \hfill (17.62)

In discrete time,

$$\left(E_t x_{t+1} - x_t\right) + \left(\frac{\sigma - \xi}{\xi}\right) \left(\frac{m}{c}\right) (i - \tilde{r}^m) \left[E_t x_{t+1} - x_t\right] = \sigma (i_t - \pi_t).$$ \hfill (17.63)

Equations in continuous time from EER.

Discrete time – which timing is easier

Is this interest elastic?

17.5 Hyperinflations and unpleasant arithmetic

Seignorage maximum

Sargent ends of hyperinflations

FTPL vs. Sargent ends of hyperinflations - Unpleasant arithmetic vs. fiscal theory

17.5.1 FTPL vs. Sargent and Wallace

In the famous “Unpleasant Monetarist Arithmetic” Sargent and Wallace (1981) link deficits to inflation. They consider a model with money, in which the equilibrium price level is determined by

$$M_t V = P_t Y.$$ \hfill (17.64)

$$b_{t-1} = \sum_{j=0}^{\infty} E_t \left[ \frac{M_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_t} \right) \right].$$ \hfill (17.65)

Sargent and Wallace consider a regime in which $\{s_{t+j}\}$ is fixed, or governments are already maximizing surpluses, but the central bank can still control money $M_t$. By (17.64), the money supply directly controls the price level. Sargent and Wallace
specify indexed debt, so the value on the left hand side of (17.65) is unaffected by the price level. (This specification is explicit below Sargent and Wallace’s equation (4).) Now, if current or future surpluses \( \{s_{t+j}\} \) decline, the central bank must generate some seignorage revenue. The central bank can still decide when it will print more money. Less money printing and less inflation now will require more money printing and more inflation later.

This was their central point. At the time, the US was going through a period of unprecedented deficits, but also a monetary contraction to fight the inflation of the late 1970s. Sargent and Wallace warned that if the US did not fix its fiscal deficits, the monetary tightening would be for naught – a temporary inflation decline would be met by more severe later inflation.

As we have seen, however, seignorage is small component of government revenue for the US as other advanced economies, leading many to discount this parable as an interesting connection between fiscal difficulties and inflation. Their forecast also did not pan out. The US did not return to higher inflation. Sargent and Wallace were not superhumanly clairvoyant to foresee that by the 1990s the US would return to substantial fiscal surpluses.

With Sargent and Wallace’s main point in front of us, we can see that it is justly famous as a pioneering study of fiscal-monetary links. However, it is no insult to point out that 20 subsequent years of fiscal theorizing have produced some novelty. First, rather than \( b_t \) indexed debt, the left hand side may be the real value of nominal debt \( B_{t-1}/P_t \), as in fact debt was in the 1980s. Thus, a rise in the price level \( P_t \) can reduce the value of the debt directly, rather than just through a seignorage channel. With nominal debt, a fall in \( \{s_{t+j}\} \) can generate inflation (a rise in the price level) in the frictionless version of the model, with no money demand or seignorage whatsoever – deleting (17.64) and setting \( M = 0 \) in (17.65) – a possibility not present in Sargent and Wallace’s analysis. In fact, by removing the link to seignorage revenue, these modifications reinforce Sargent’s (1986) basic point of an underlying fiscal cause of hyperinflations and their ends. More deeply, Sargent and Wallace’s indexed debt is debt, and absent default (17.65) is something of a budget constraint forcing some action to fix the overall surplus. With nominal debt (17.65) is a valuation equation, and no fiscal adjustment, direct or via seignorage, need happen. This is the central assumption that lets us study a cashless model.


