Corporate Credit Spreads under Parameter Uncertainty

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Abstract

In this paper we examine the impact of parameter uncertainty on corporate bond credit spreads. Using data for 5,300 firm-years between 1994 and 2008, we find that parameter uncertainty explains up to 40% of the credit spread that is typically attributed to liquidity, taxes and jump risk, without significantly raising bankruptcy probabilities. We find that parameter uncertainty affects firms with large intangible assets and volatile earnings growth the most. Moreover, uncertainty varies over time and increases significantly during periods of market stress: the credit crisis of 2008 is characterized by high uncertainty about asset valuations, and parameter uncertainty alone raised spreads by 50 basis points.

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In this paper we examine the impact of parameter uncertainty on corporate bond credit spreads. Using data on $5,300$ firm-years over the period 1994 to 2008, we find that parameter uncertainty varies substantially across firms and over time. For example, uncertainty was particularly high during the credit crisis of 2008 and we estimate that parameter uncertainty alone raised corporate credit spreads by as much as 50 basis points.

Including parameter uncertainty in corporate bond valuation helps explain the *credit spread puzzle*: short maturity, investment-grade corporate bonds have credit spreads that are too large to be explained by existing structural models of the firm (see, e.g. Eom, Helwege, Huang, 2004). Researchers have tried to explain the credit spread puzzle by including factors such as taxes (Liu et al., 2007), jumps in asset values (Delianedis and Geske, 2001), and liquidity effects (Huang and Huang, 2003). We find that these factors may be less important than initially thought by showing that in an existing simple structural model, parameter uncertainty can explain up to 40% of the credit spread that is typically attributed to liquidity, taxes and jump risk. This increase in spreads is not associated with a significant increase in bankruptcy probabilities, even when taking into account that the default boundary is not known with certainty (Davydenko, 2007). Surprisingly, for long maturity bonds with high default risk, parameter uncertainty lowers bankruptcy probabilities while raising model-implied spreads.

We document a number of new results regarding the time series of uncertainty. Parameter uncertainty varies considerably over time, is counter-cyclical, and is greater during 1998-2000 and the credit crisis of 2008. Whereas the 1998-2000 period is characterized by high uncertainty about asset volatility, the credit crisis is characterized by high uncertainty about asset valuations. Parameter uncertainty is highly correlated with the VIX, which suggests that the VIX is a proxy for aggregate uncertainty.

On the theoretical side, we show that the effect of parameter uncertainty is driven by the concavity of debt prices in model parameters such as asset value and volatility, which reduces the value of such securities. Since corporate debt is, in essence, a risk-free loan combined with a short put on the assets of the firm, its value is concave in the underlying asset value and volatility. There are direct implications for derivatives pricing in general. In particular, parameter uncertainty cannot be diversified away in portfolios of derivatives because a portfolio of options cannot be treated as an option on a portfolio. This suggests that parameter uncertainty is relevant for a vast array of derivative and structured finance products such as CDOs and mortgage-backed securities.

Our analysis has a number of empirical predictions. First, parameter uncertainty is highest for young and small firms with volatile earnings and large intangible assets, so these firms will experience the largest increase in spreads. Second, debt values are the most concave for moderately risky firms, especially at the short end of the term structure of maturities, and parameter uncertainty will therefore increase their spreads the most. Third, parameter uncertainty is higher during periods of market stress, increasing credit spreads for all firms. To our knowledge the latter two predictions are new and have not been tested before. We find that all three predictions are largely borne out by the data.

A natural approach to estimate the impact of parameter uncertainty within structural models of the firm is to use Markov Chain Monte Carlo (MCMC) methods (Johannes and Polson, 2009) to uncover the posterior distribution of a firm's asset value and volatility given equity market valuations and accounting data. We show that the posterior standard deviations of asset value and volatility (our measures of parameter uncertainty) are related to a range of uncertainty proxies such as firm age, size, intangibles, earnings growth volatility and analyst EPS forecast dispersion. We find that there is a large component to parameter uncertainty that cannot be explained by these proxies, highlighting the importance of our approach of working with direct estimates of parameter uncertainty. To alleviate concerns that the residual component of parameter uncertainty simply mimics jump or liquidity effects, we show that parameter uncertainty has distinct effects on fitted bankruptcy probabilities compared to jumps and liquidity, bringing longer-run estimated bankruptcy probabilities closer to observed rates. This shows that parameter uncertainty is a separate channel that has different model fit implications compared to structural models with jumps and liquidity that ignore parameter uncertainty.

Duffie and Lando (2001) provide theoretical motivation for incorporating parameter uncertainty in corporate bond pricing. They show that incomplete accounting information and the updating of market beliefs lead to models in which investors are unsure about asset values. Pastor and Veronesi (2003) provide a related motivation in a standard present value setting, where uncertainty about dividend-growth results in a range of possible firm valuations where investors are unsure about which is the "true" value. Investors have to take account of this parameter uncertainty by solving a filtering problem and calculating the posterior distribution of beliefs over the unobserved asset value and parameters, given their current information set.

In practice, investors may have more information than the econometrician, or they are able to identify and measure further significant risk factors (Duffie et al., 2008). The importance of parameter uncertainty may therefore be smaller than we report. On the other hand, the majority of trading in corporate bonds and the calculation of default probabilities is model-driven and standard practice is to calibrate these models using only equity values and some basic accounting information (for example, Moody's KMV). Our results indicate that parameter uncertainty should also be accounted for.

Our framework does not incorporate aversion to estimation risk. When investors do not know the exact model parameters, the standard no-arbitrage dynamic hedging argument no longer applies, leaving investors exposed to the systematic risk of the underlying security. Although this may be a second-order effect that raises credit spreads, we consider the impact of parameter uncertainty on hedge ratios an important avenue for future research that is beyond the scope of this paper.

The impact of parameter uncertainty in portfolios of stocks is well known (e.g. Klein and Bawa, 1976, Barry and Brown, 1985, Barberis, 2000, Kumar et al., 2008, and Johannes, Korteweg and Polson, 2010). However, little research has been done on parameter uncertainty and corporate bonds. David (2008) extends the Merton (1974) model to a regime-switching setting, and documents a convexity effect on corporate credit spreads due to time-variation in the solvency ratio over the business cycle. Froot and Posner (2002) look at the pricing of event risks (such as catastrophe bonds) with uncertainty about event probabilities. Heitfield (2009) examines the role of parameter uncertainty in copula models of collateralized debt obligations. The closest work to ours is by Cremers and Yan (2009) who extend the Pastor-Veronesi (2003) intuition, developing a Merton (1974) type model that explicitly models the effect of uncertainty about profitability on both stocks and bonds. They find a counter-intuitive effect of firm age on corporate credit spreads. They also use our measures of uncertainty about asset value and volatility and conclude that they are more effective at capturing the effect of uncertainty on bonds. In addition, they find that stock valuations increase with our measures, as predicted by their model.

To our knowledge, ours is the first paper to directly estimate and quantify the effect of uncertainty on credit spreads within a structural model, without relying on simple linear regressions of credit spreads on proxies for uncertainty. Our analysis shows that the relation between uncertainty and credit spreads is highly non-linear, depending on maturity and the level of credit risk. Our empirical method can be directly applied to existing models of credit risk, and uses direct estimates of parameter uncertainty that increase the power of tests. This is important as return volatility is typically used as an important control in regressions, but proxies for parameter uncertainty as well (e.g. Campbell and Taksler, 2003, Yu, 2005, Güntay and Hackbarth, 2007, Buraschi et al., 2009, and Cremers and Yan, 2009).

The rest of the paper is outlined as follows. Section I introduces bond pricing using structural models of the firm. Section II describes the effect of parameter uncertainty on credit spreads and bankruptcy probabilities. Section III describes the MCMC approach to estimation. Section IV presents the data. We discuss our results on credit spreads in Section V, and on bankruptcy probabilities in Section VI. Section VII shows robustness and Section VIII concludes.

I Structural Models of Bond Pricing

Starting with Merton (1974), structural models of the firm treat a company's outstanding securities as derivatives on the market value of the firm's assets, V_t. The value of the firm's assets changes over time and the standard assumption is that V_t follows an exogenously specified geometric Brownian Motion:

$$\frac{dV_t}{V_t} = (\mu_{\mathbb{P}} - \delta)dt + \sigma dB_t^{\mathbb{P}} \tag{1}$$

where δ is the (constant) continuous payout rate and $dB_t^{\mathbb{P}}$ is a standard Wiener process under risk-natural $\mathbb{P}$-dynamics. For risk-neutral $\mathbb{Q}$-pricing we use $dV_t/V_t = (r-\delta)dt + \sigma dB_t^{\mathbb{Q}}$ with risk-free rate r.

Investors in the firm receive a cash flow at time t of $F(V^t, \Theta)$, where V^t denotes the history of firm values up to time t and Θ denotes the model parameters, including the parameters in equation (1). Each type of claim, for example debt or equity, has its own payoff function $F(\cdot)$. Using risk-neutral pricing and given (V_t, Θ), the value of a security at time t, $P_t^{V_t,\Theta}$, is given by

$$\begin{aligned} P_t^{V_t,\Theta} &= \int_t^\infty \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(s-t)}F(V^s,\Theta)|V_t,\Theta\right] ds \\ &= \int_t^\infty \int_0^\infty e^{-r(s-t)}F(V^s,\Theta)p(V^s|V_t,\Theta)dV^s ds \\ &= \mathcal{P}(V_t,\Theta) \end{aligned} \tag{2}$$

namely, the sum of expected future cash flows discounted at the risk-free rate, r, assumed to be constant for simplicity. The distribution of future asset values, $p(V^s|V_t,\Theta)$, is with respect to the risk-neutral $\mathbb{Q}$-dynamics.

The workhorse model in this paper is Leland's (1994b) generalization of the well-known Leland (1994a) model that allows for finite maturity debt. At each point in time the firm has debt with constant principal P that pays a continuous coupon at the rate C. The firm continuously rolls over its debt by retiring a fraction m and replacing it with newly issued debt of equal coupon, principal and seniority. This implies that the average debt maturity equals $M = 1/m$. The company files for bankruptcy when V_t hits a boundary denoted by V_B. At that point debtholders recover a fraction $1-\alpha$ of the remaining firm value, and cash flows are zero afterward. The cash flow function for corporate debt is then:

$$\begin{aligned} F(V^s,\Theta) &= C + mP \quad \text{if} \quad \min_{\tau=t,\ldots,s}(V_\tau) > V_B \\ &= (1-\alpha)V_B \quad \text{if} \quad V_s = V_B \\ &= 0 \quad \text{if} \quad \min_{\tau=t,\ldots,s}(V_\tau) < V_B \end{aligned}$$

Equity is valued similarly by changing the payout function $F(\cdot)$. This results in simple closed-form solutions for the value of debt, D_t, and equity, E_t:[1]

$$
\begin{aligned}
D_t &= \frac{C+mP}{r+m}\left[1-\left(\frac{V_t}{V_B}\right)^{-y}\right]+(1-\alpha)V_B\left(\frac{V_t}{V_B}\right)^{-y}, && (3)\\
E_t &= V_t+\frac{\tau C}{r}\left[1-\left(\frac{V_t}{V_B}\right)^{-x}\right]-\alpha V_B\left(\frac{V_t}{V_B}\right)^{-x}-D_t, && (4)
\end{aligned}
$$

where

$$
\begin{aligned}
x &= \frac{(r-\delta-1/2\sigma^2)+\sqrt{(r-\delta-1/2\sigma^2)^2+2r\sigma^2}}{\sigma^2}, && (5)\\
y &= \frac{(r-\delta-1/2\sigma^2)+\sqrt{(r-\delta-1/2\sigma^2)^2+2(m+r)\sigma^2}}{\sigma^2}. && (6)
\end{aligned}
$$

The corporate tax rate τ is assumed constant. Finally, the bankruptcy threshold, V_B, is derived endogenously:

$$
V_B = \frac{\left[\frac{(C+mP)y}{r+m}-\frac{\tau C x}{r}\right]}{1+\alpha x+(1-\alpha)y}. \tag{7}
$$

Note that the pricing functions for debt and equity only depend on V_t and parameters, $\Theta = (\alpha, \delta, \sigma, \tau, r, C, P, M)$.

II Bond Valuation under Parameter Uncertainty

The pricing equation (2) states that if investors know the current firm value, V_t and the model parameters, Θ, they can price the debt and equity of the firm.

The standard approach to empirical research is *calibration*, where the parameters and current firm value are fit to observed quantities. Plugging the point estimates $(\widehat{V}_t, \widehat{\Theta})$ into the pricing function then gives the price of a security:

$$
P_t^{\widehat{V}_t,\widehat{\Theta}} = \mathcal{P}(\widehat{V}_t, \widehat{\Theta}) \tag{8}
$$

For example, a popular method proposed by Jones et al. (1984) calibrates $\widehat{V}_t$ and $\widehat{\sigma}$ to the observed market value of equity, E_t, and its historical return volatility σ_t^E, using

[1] Leland (1994) uses a replicating portfolio strategy to price debt and equity. Another way to obtain these results is to break up (2) and use the distribution of the first-passage time of V_t to V_B, as shown in Leland and Toft (1996).

the pricing equation for equity. This method is similar to using implied volatility in option pricing and is used extensively in the empirical literature (Ronn and Verma, 1986, Ogden, 1987, Delianedis and Geske, 1999, Teixeira, 2007). A related calibration method is used by Vassalou and Xing (2004) and Bharath and Shumway (2008).

In practice, however, investors do not know the true value of parameters. Instead, investors use available data such as annual reports and the market value of the firm's equity to estimate the parameters. A natural way to capture investors' uncertainty about unobserved asset value and model parameters is through the posterior distribution $p(V_t, \Theta|\mathcal{F}_t)$, conditional on information known at time t, $\mathcal{F}_t$.[2]

We now consider the properties of prices that fully reflect this uncertainty using this posterior distribution for unobservables, namely:

$$P_t^{\mathcal{F}_t} = \mathbb{E}_{V_t,\Theta|\mathcal{F}_t}\left[\mathcal{P}(V_t, \Theta)\right] \tag{9}$$

where expectations are taken with respect to the posterior distribution $p(V_t, \Theta|\mathcal{F}_t)$. This can be seen as follows:

$$\begin{aligned} P_t^{\mathcal{F}_t} &= \int_t^\infty \left[\int e^{-r(s-t)} F(V^s, \Theta) p(V^s, V_t, \Theta|\mathcal{F}_t) d(V^s, V_t, \Theta)\right] ds \\ &= \int_t^\infty \int \left[\int e^{-r(s-t)} F(V^s, \Theta) p(V^s|V_t, \Theta) dV^s\right] p(V_t, \Theta|\mathcal{F}_t) d(V_t, \Theta) ds \\ &= \int \left[\int_t^\infty \int e^{-r(s-t)} F(V^s, \Theta) p(V^s|V_t, \Theta) dV^s ds\right] p(V_t, \Theta|\mathcal{F}_t) d(V_t, \Theta) \\ &= \mathbb{E}_{V_t,\Theta|\mathcal{F}_t}\left[\mathcal{P}(V_t, \Theta)\right] \end{aligned}$$

The key implication is that if the pricing function $\mathcal{P}(V_t, \Theta)$ is concave, then the price under parameter uncertainty is lower than the price without uncertainty: this can be shown by noting that under Jensen's inequality

$$\mathbb{E}_{V_t,\Theta|\mathcal{F}_t}\left[\mathcal{P}(V_t, \Theta)\right] \leq \mathcal{P}(\mathbb{E}(V_t|\mathcal{F}_t), \mathbb{E}(\Theta|\mathcal{F}_t)) \tag{10}$$

or, equivalently,

$$P_t^{\mathcal{F}_t} \leq P_t^{\hat{V}_t, \hat{\Theta}} \tag{11}$$

[2]If some parameters, such as spot interest rates, are known, the posterior is degenerate in these dimensions. This does not affect the formulas derived below.

where $(\hat{V}_t, \hat{\Theta})$ is the expected value of the posterior distribution $p(V_t, \Theta|\mathcal{F}_t)$.

In structural models of the firm, debt value is generally concave in the key parameters, firm value and volatility.[3] Figure 1 illustrates this relation for Merton's (1974) model, in which bondholders own a risk-free bond but have shorted a European put option to shareholders, valued by the Black-Scholes formula. This relation is robust across models. Since debt values are lower, credit spreads widen when accounting for parameter uncertainty.

A simple numerical example illustrates the effect of parameter uncertainty. Suppose investors use Merton's model with the parameters of Figure 1: the firm has a 10-year zero-coupon bond with face value of 50, and the risk-free rate is 5%. Suppose investors know that asset value is 100, but they are uncertain about asset volatility, and believe it is either 20% or 30% with equal probability. If they ignore parameter uncertainty and use the average volatility of 25%, they find that bond value is 29.15, implying a credit spread of 41.83 basis points. With parameter uncertainty, bond value is 28.93, the average of bond values at 20% and 30% volatility. The credit spread is then 49.66 basis points, which is 7.83 basis points, or 19%, higher than the spread without parameter uncertainty. It is straightforward to generate larger differences in spreads, for example by lowering bond maturity or decreasing asset value, or by allowing for uncertainty over asset values in addition to uncertainty over volatility.

In general, parameter uncertainty has the largest impact on credit spreads when: i) the debt pricing function is highly concave in the region of parameters considered, and; ii) investors are highly uncertain about the parameters of the pricing function. Figure 2 illustrates these effects by plotting model-implied credit spreads against firm value. Since the face value of debt is kept constant this is analogous to plotting credit spreads against leverage, or against default risk. Short, 1-year maturity bonds are nearly linear in firm value for very safe bonds (region C) and very risky bonds (region A), but highly convex for bonds of intermediate risk (region B). We should therefore

[3] For extremely high levels of asset volatility, the debt pricing function becomes a convex function of volatility, reversing the impact of parameter uncertainty. Our empirical results fully reflect the impact of convex regions of the parameter space. We thank David Lando for pointing this out to us.

find that for short-maturity bonds, parameter uncertainty matters primarily for those bonds that are moderately risky. Conversely, credit spreads on 10-year bonds are moderately convex over the entire range of firm values, so we expect the uncertainty effect to be present for levels of credit risk.

Parameter uncertainty is also more important when the degree of parameter uncertainty is larger, i.e. when investors consider a larger range of parameters on the horizontal axis of Figure 2. The degree of parameter uncertainty is related to the amount of information investors have about a firm, and to uncertainty about the firm's future. For example, one would expect that young and small companies, and firms with volatile earnings and large intangible assets have large parameter uncertainty.

The concavity of the debt pricing function is the key feature that raises estimated credit spreads in the presence of parameter uncertainty. We do not separately introduce aversion to estimation risk. To be precise, the underlying structural model prices corporate bonds by dynamically hedging the underlying firm risk. This works just fine if parameters are known exactly (in addition to the usual assumptions of perfect markets and continuous trading). However, uncertainty about parameters means uncertainty about hedge ratios, which may leave the investor exposed to the systematic risk of the underlying firm, even if parameter uncertainty itself is idiosyncratic. The investor should be compensated for this "hedging risk", and by ignoring it, we understate the effect of parameter uncertainty on credit spreads. We show that even without this risk premium, the parameter measurement problem has an important effect on pricing and credit spreads and should not be ignored.

It may seem odd that investors consider a range of current firm valuations, V_t, since we would only see one value if the firm were a traded asset.[4] Duffie and Lando (2001) show that investors are unsure about firm value when accounting reports are noisy

[4]One could argue that the underlying firm value is observed if we know the market value of debt and equity. In reality we do not observe prices of all debt securities (e.g. bank debt, trade credit). Moreover, V_t represents the value of the *unlevered* firm, which is generally different from the value of the *levered* firm in models that incorporate tax shields and bankruptcy costs (e.g. Leland, 1994, and Leland and Toft, 1996).

or delayed, or when there are other barriers to monitoring. Alternatively, treating firm value as arising from a discounted cash flow analysis, Pastor and Veronesi (2003) show that uncertainty about earnings growth leads to a probability distribution of possible firm values. The traded (observed) value is the expected value over all possible earnings growth rates. We can therefore think of the uncertainty about V_t as arising from a more basic model of the firm with parameter uncertainty fitting naturally within existing structural models of the firm, such as the model developed in Cremers and Yan (2009).[5]

Finally, we assume that management knows the underlying parameters but cannot credibly convey them to bond investors. If management were uncertain about model parameters then this may change the bankruptcy boundary (7) due to the increased option value of equity (Childs, Ott and Riddiough, 2004, illustrate this effect for a lease-encumbered real asset). The effect of parameter uncertainty on managerial decisions is a potentially important avenue of future research that is outside the scope of this paper.

III Estimation

In this section we explain our Markov Chain Monte Carlo (MCMC) approach to estimating the posterior distribution of parameters and firm values given observed prices, namely $p(V_t, \Theta|\mathcal{F}_t)$. We generically represent the structural model of the firm

[5] We have developed an EBIT-based model with finite maturity debt. This model is observationally equivalent to Leland (1994b). As in Pastor and Veronesi (2003), uncertainty about EBIT growth results in uncertainty about V_t. Most importantly, debt value is concave in earnings growth so uncertainty about earnings growth raises credit spreads, analogous to uncertainty about V_t. We use the standard modeling approach with V_t (not EBIT) as the underlying state variable since the connection between EBIT growth uncertainty and asset value has already been studied by Pastor and Veronesi and the EBIT-based model does not provide any new insights over and above Leland (1994b). The EBIT model addendum is available from the authors on request.

as a state-space model:

$$P_t = \mathcal{P}(X_t, \Theta) \quad : \quad \text{Observation Equation} \tag{12}$$

$$dX_t = \mu(X_t, \Theta)dt + \sigma(X_t, \Theta)dB_t \quad : \quad \text{State Evolution} \tag{13}$$

The vector of prices, P_t, consist of security prices of the firm at time t, for example equity values, options, credit default swaps, and various bond prices. The state vector, X_t, consists of the model's latent variables, such as firm value or a stochastic default boundary.

For a given firm we observe a time series of prices, $P^T = \{P_t\}_{t=1}^T$, and we want to know the joint posterior distribution of latent states and parameters, $p(X^T, \Theta | P^T)$, needed to value the debt using equation (9). The intuition behind MCMC is to break up this posterior distribution into a set of complete conditional distributions, from which it is relatively easy to sample. The conditional distribution of parameters given the state vector and prices, $p(\Theta | X^T, P^T)$, is typically a standard Bayesian regression. We use standard filtering techniques to draw from the distribution $p(X^T | \Theta, P^T)$. If we keep sampling from these distributions until convergence, the Clifford-Hammersley theorem (Johannes and Polson, 2009) assures us that we obtain a random sample of draws from the joint posterior distribution. Armed with a sample of draws from the joint posterior distribution, the expectation in (9) becomes a simple summation over model-implied debt prices at each sample point. The appendix describes our algorithm in detail.

In the Leland (1994b) model described in Section II, the state vector is the natural logarithm of firm value, $X_t = \ln(V_t)$. We discretize the diffusion for X_t in equation (1), and estimate the model using a time-series of observed market values of equity, $P^T = E^T$. We assume that equity investors, like bondholders, do not exactly know the value of the firm. They use the right value on average, but may be off by a pricing error, $\epsilon_t \sim \mathcal{N}(0, \nu^2)$. This assumption also breaks the stochastic singularity problem of having a model that has fewer parameters than there are observations and therefore cannot fit all observations perfectly. To summarize, for each company we estimate the empirical model:

$$\ln\left[\mathcal{P}_E^{-1}(E_t,\Theta)\right] = X_t + \epsilon_t \tag{14}$$

$$X_t = X_{t-1} + \left(\mu_{\mathbb{P}} - \delta - \frac{1}{2}\sigma^2\right) + \sigma \cdot \eta_t \tag{15}$$

where $\mathcal{P}_E^{-1}$ is the inverse pricing function (4) that yields the model-implied firm value, V_t, given an equity value, E_t. The innovation in log-firm values, η_t, is distributed $\mathcal{N}(0,1)$, and ϵ_t and η_t are assumed i.i.d. and independent of each other.[6] We discard the first 100 draws to allow the algorithm to converge to the true posterior distribution. We then draw 1,000 samples from which we compute the market value of debt, and credit spreads.

IV Data

We construct a sample of monthly debt and equity values for non-financial firms in the National Association of Insurance Commissioners (NAIC) database between 1994 and 2008. Insurance companies are required to file all their trades in corporate bonds with the NAIC, which makes these records available in electronic form. Hong and Warga (2000) report that insurance companies account for about 40% of all trades in the investment grade bond market, and 25% of trades in the market for non-investment grade bonds, allowing us to observe a large spectrum of credit ratings.

From the NAIC transactions data we compute end-of-year bond values for each outstanding bond issue of every firm. Since not all bonds are traded every month, it is not always possible to aggregate the individual bond values to obtain the market value of all publicly traded debt. To mitigate this missing data problem we group together bonds of the same firm of equal security and seniority, and maturity within two years of one another. Assuming these bonds have the same interest rate and credit risk, missing values are calculated from contemporaneous market-to-book values of bond values in the same group that are observed at the end of the year. Finally, we add

[6] We assume in equation (14) that equityholders get the *logarithm* of firm value right, on average. Making the $(1/2)\nu^2$ adjustment to get an unbiased estimate of firm value has little effect on the empirical results. See Huang and Yu (2008) for an alternative specification.

the book value of the unobserved portion of debt (bank debt and capitalized leases) and subtract the firm's cash balance.

We augment our sample with accounting data from Compustat and monthly share price data from CRSP. We measure firm age as the number of years since first incorporation, from Mergent Online and Moody's Corporate Manuals, and dispersion of analyst long-term earnings-per-share forecasts from I/B/E/S. These variables are used to proxy for parameter uncertainty, and will be discussed in more detail in the next section. The final sample comprises 5,300 firm-years for 988 unique companies. Table I reports summary statistics on firm size, leverage, the proxies for parameter uncertainty and corporate bond variables.

The standard procedure in the empirical literature is to fix all model parameters, except asset volatility, σ (Jones et al, 1984, Lyden and Saraniti, 2000, Eom et al., 2004, Teixeira, 2007). Following this practice, we assume a loss-given-default rate $\alpha = 51.31\%$ and a corporate tax rate $\tau = 35\%$, as in Eom et al (2004) and Teixeira (2007). In this case the loss-given-default is a deadweight loss to bondholders as a fraction of the face value of debt, not the costs of financial distress that the entire firm faces (Korteweg, 2010). The total principal (P) and coupon payment (C) are observed, as is the payout ratio δ, calculated as the median of interest plus dividends divided by book debt plus market equity over the previous five years. Debt maturity (M) is calculated as the average maturity of the outstanding bonds weighted by face value. Since changes in interest rates affect debt pricing, we choose the risk-free rate (r) in each month such that the model-implied debt value equals the risk-free debt value if the corporate debt is truly riskless (as in Teixeira, 2007). We find the risk-free debt value by discounting the interest and principal payments of all outstanding bond issues of the firm, using zero yields from Gürkaynak, Sack and Wright (2006), available online from the Federal Reserve. These zero yields are based on the Svensson (1994) extension of the Nelson-Siegel model.

V Results

We estimate Leland's (1994b) model for each firm-year, using the time series of monthly equity values over the year. This model is a simple extension of the seminal Leland (1994a) model that inspired a wave of new structural models of the firm (e.g. Leland and Toft, 1996, Fan and Sundaresan, 2000, Goldstein, Ju, and Leland, 2001, Collin-Dufresne and Goldstein, 2001). The model allows for finite maturity debt, interest payments, corporate taxes, and deadweight costs of default.[7]

Table II shows the posterior means and standard deviations for the estimated parameters, broken down by credit rating. We estimate that asset volatility, σ, is 26% per year on average across firm-years, and is virtually the same across credit ratings. For the moderately risky ratings classes the expected return on assets, $\mu_{\mathbb{P}}$, averages 8% per year. The two safest ratings classes, AAA and AA+, have a slightly higher expected return of about 12% and the two riskiest classes, B- and CCC+, have lower expected return of 4% and -1%, respectively.

The standard deviation of the equity pricing error, ν, is 0.05 on average, which translates to a 95% confidence interval on firm value that spans roughly 20% of the posterior mean V_t.

The quantity V_t/V_B is the value of the firm at year-end, scaled by the default boundary, V_B, which is a function of estimated parameters, given in (7). For AAA rated firms the average V_t/V_B is 73, indicating that these firms are far away from their default boundary. The ratio goes down almost monotonically with credit rating, to 1.8 for CCC rated firms.

A Credit Spreads by Rating

We first compute credit spreads without the effect of parameter uncertainty by calculating debt values at year-end using the posterior means of asset value and volatility as point estimates, as in equation (8). We focus our analysis on the *residual* spread,

[7]A previous version of this paper used Leland's (1994a) paper. The results were qualitatively the same as the results presented here. We chose Leland's (1994b) model because it allows for finite-maturity debt, which is important for the concavity of debt values, as shown in Section II.

defined as the observed minus the model-implied credit spread. This residual spread reveals by how much the model misprices debt relative to the market.

The top plot in Figure 3 shows the median observed and residual credit spreads by credit rating. Investment grade firms have observed credit spreads of 50 to 200 basis points. Non investment grade firms have observed spreads of 250 to 650 basis points. Ignoring parameter uncertainty, the solid line shows that the residual spread is about half the observed spread, or 50 basis points, for high quality bonds. The residual spread grows in absolute value as we move down the credit ratings, to about 150 basis points for BB-rated firms, but shrinks again for the lower rated firms. The hump-shaped form of residual spreads across credit ratings is consistent with the intuition that the concavity of the debt pricing function is largest in the middle range of credit ratings, leading to higher mispricing for the bonds of intermediate riskiness.

We calculate credit spreads with parameter uncertainty from equation (9), using the posterior distribution of the model parameters, V_t and σ.[8] Figure 3 shows that parameter uncertainty raises credit spreads, as residual spreads are lower for virtually all credit ratings. The bottom plot in Figure 3 reveals that the median residual spread of investment grade firms decreases by about 10%, or 5 bps, when parameter uncertainty is accounted for. This may not seem all that impressive, but there are a large number of firm-years for which the reduction is much higher. For 5% of the firm-years the reduction in residual spreads is 30% or more. Moreover, the effect of parameter uncertainty is larger for non-investment grade bonds. We see the largest reduction for B-rated bonds, with a median reduction in residual spreads of 40%, or 40 basis points. For the riskiest bonds the reduction is lower. This hump-shaped reduction in residual spreads is consistent with debt values being more concave for the moderately risky bonds, leading to a larger impact of parameter uncertainty for these bonds.

[8]Debt value is linear in loss-given-default, α, so allowing for uncertainty about loss-given-default will not affect credit spreads. We do not use the distribution of $\mu_{\mathbb{P}}$ here since debt value is not a function of $\mu_{\mathbb{P}}$. It does play a role when we calculate bankruptcy probabilities below.

B Maturity

In Section II we showed that the concavity of the bond pricing function varies across maturities. In Figure 4 we cut our sample into maturity quintiles and plot the residual spread across credit ratings for the highest and lowest maturity quintiles. The top plot of Figure 4 reveals a pattern that is consistent with the effects of parameter uncertainty: long maturity bonds have modest mispricing across all credit ratings, whereas short maturity bonds have large residual spreads in the middle region but low mispricing for the safest and riskiest bonds.

We find that the pattern of residual spreads across maturities is not driven by systematic differences in within-quintile maturities across credit ratings, and is not particular to our use of Leland's model. Results in Huang and Huang (2003) exhibit the same pattern in residual spreads across maturities as we see in Figure 4 for all four models that they estimate (including jump models).

The reduction in residual spreads due to parameter uncertainty follows the expected pattern. The reduction is largest for BB-rated short maturity bonds, at about 50-70 bps or 30% of the residual spread. Standard t-tests of the reduction in spreads of the upper versus the lower maturity quintile (not reported) show that the reduction in spreads is significantly higher (at the 1% level) for the upper maturity quintile for bonds rated A and higher, but is significantly lower for bonds rated BB+ to B+. For bonds rated B the difference is insignificant. These findings are consistent with the notion that short bonds of medium riskiness have the largest price concavity, and hence the largest reduction in residual spreads, while short maturity bonds of low and high risk have the lowest concavity.

C Parameter Uncertainty Proxies

We expect residual spreads to be larger for firms with a larger degree of parameter uncertainty, especially in the region of the parameter space where debt values are most concave. We use six proxies for parameter uncertainty, with descriptive statistics reported in Table I. First, firms with many tangible assets (property, plant and equipment or PPE) relative to total book assets are arguably easier to value, leading to lower parameter uncertainty. Moreover, firms with more tangible assets tend to be more transparent from an accounting perspective (e.g. Yu, 2005). Second, a higher volatility of EBIT growth rates ($EBITgvol$) over the past 5 years makes it more difficult to assess value and volatility. Third, the standard deviation of analyst long-term earnings-per-share forecasts scaled by the absolute mean estimate ($EPSfcdisp$) measures the disagreement about the inputs to the model. Fourth, the Age of the firm since first incorporation measures the existing quantity of information that helps investors determine what parameters to use. Fifth, the market-to-book ratio (M/B) proxies for the growth opportunities of the firm, which are inherently more difficult to value than the assets-in-place, and is larger when uncertainty about growth rates is larger (Pastor and Veronesi, 2003). Finally, firms' book value of assets ($Size$) proxies for the amount of information and press coverage that is readily available to investors to assess the company, and is also correlated with accounting transparency (Yu, 2005).

Figure 5 plots the residual spread against credit ratings for the lowest and highest quintiles of each proxy. The pattern is generally consistent with the hypothesized effects of parameter uncertainty for PPE and $EBITgvol$. When uncertainty is large, the residual spreads are large, and the effect tends to be stronger in the middle region of credit ratings, where pricing concavity is largest. The reduction in spreads is also larger for the high uncertainty quintile, compared to the lower quintile.

Residual spreads tend to be higher for high forecast dispersion, $EPSfcdisp$. Güntay and Hackbarth, (2007) and Buraschi et al. (2009) also document a positive relation between analyst earnings forecast dispersion and credit spreads. The pattern for M/B is weak but broadly consistent with parameter uncertainty, particularly for non-investment grade debt. We find no consistent pattern in the relation between residual spreads and Age. In contrast, Cremers and Yan (2009) find a puzzlingly positive relation between Age and credit spreads. Although this is the uncertainty measure proposed by Pastor and Veronesi (2003), it is not intuitively obvious why investors should necessarily be less uncertain about the valuation of older firms. After all, companies have to adapt to a constantly changing economic environment and with change comes uncertainty, no matter how long a company has been in business. Turning to *Size*, we find that smaller firms tend to experience higher residual spreads and a larger impact of parameter uncertainty. This result is consistent with small firms being more opaque (e.g. due to less analyst coverage or media attention) and therefore surrounded by more parameter uncertainty than large firms. However, for credit ratings of BB- and below, larger firms have larger residual spreads.

To gain more insight into the relation between parameter uncertainty and the proxies we use, we regress the posterior standard deviations of asset volatility, σ, and asset value, V_t, on the six parameter uncertainty proxies. Since larger firms will naturally have larger posterior standard deviations of V_t, we scale this measure by its posterior mean. We allow for time-fixed effects and cluster errors by firm to account for unobserved firm effects. Table III shows that firms with many tangible assets and low earnings growth volatility have small posterior standard deviations of volatility. In other words, there is less uncertainty about σ for these firms. The other proxies are insignificant or not robust across specifications, but their signs are as expected, except for $EPSfcdisp$ and $Size$. The uncertainty about V_t is more difficult to explain using our proxies, with no proxy having consistently significant loadings across specifications. We attribute the weak results for $EPSfcdisp$, M/B, Age and $Size$ in Figure 5 to their weak relation to parameter uncertainty as documented in Table III.

We also include the signal-to-noise ratio σ/ν in the regressions. A high σ relative to ν means that the observation errors in the state space are small relative to changes in the latent variable, V_t. In other words, a high signal-to-noise ratio means that it is easy to filter the latent variable, V_t, from the observed equity values. Table III shows that both the posterior standard deviation on V_t and σ are lower when the signal-to-noise ratio is larger.

Finally, one might expect that parameter uncertainty is larger for firms with lower credit ratings, but Table II does not confirm this intuition.

We conclude that the proxies capture some component of parameter uncertainty, but that a large part of the uncertainty is not explained with the proxies we use. Working with direct estimates of parameter uncertainty naturally avoids the problem of poor proxies.

D Time Series of Uncertainty and Credit Spreads

There is substantial time variation in the degree of uncertainty in financial markets. Figure 6 plots the median posterior standard deviation across firm-years of σ and V_t (the latter normalized by its posterior mean) over time. Parameter uncertainty was high during 1998-2000, the period of LTCM, the Russian and Argentina debt crises and the bursting of the internet bubble, and during the credit crisis of 2008. This pattern is consistent across credit ratings and is therefore not driven by the distribution of credit ratings over time.

Although both measures of uncertainty increased, the credit crisis saw more of an increase in the uncertainty about valuations than volatility, whereas the 1998-2000 period saw roughly equal increases in both.

The VIX index is often interpreted as a measure of uncertainty. Figure 6 shows that the level of the VIX is indeed highly correlated with the posterior standard deviations of σ and V_t. It appears that the uncertainty about σ leads the VIX whereas the uncertainty about V_t moves contemporaneously, but we have too short of a time series to make any strong claims.

Figure 7 shows the time series of credit spreads for bonds rated A, BBB and BB. These are the rating categories with the most observations available to construct a time series. Consistent with the time series of uncertainty in Figure 6, the effect of parameter uncertainty is largest in 1998-2000 and in particular in 2008. The widening of spreads due to parameter uncertainty is about 50 basis points in 2008.

E Pricing Errors

Figures 3 through 5 clearly show that there remains unexplained pricing error on corporate bonds, even after controlling for parameter uncertainty. The residual spreads in Figure 3 are in line with liquidity premia in DeJong and Driessen (2005), who regress corporate bond returns on a liquidity risk factor. They find that investment grade bonds have a liquidity premium of about 40bps, compared to 100bps for non-investment grade bonds. Other unmodeled features that may cause pricing errors are jumps in asset values, bond convertibility, and other embedded call or put options.

In Table IV we regress the pricing error on proxies for liquidity and bond-specific features, by firm-year. We use two alternative measures of pricing error: (i) the ratio, and (ii) the difference between the model-implied spread (with parameter uncertainty) and the observed credit spread. Since model-implied spreads are lower than observed spreads, a positive coefficient in either regression means lower pricing error.

Rather surprisingly, liquidity in the stock market, as measured by the average number of shares traded divided by the total number of shares outstanding, is negatively related to the difference in model-implied and observed spreads: bonds of firms with more liquid stocks have higher pricing errors. Using firm size itself as a proxy for liquidity, also has a negative and significant effect. We do find that older bond issues, which tend to be less liquid (Ericsson and Renault, 2006), have higher pricing errors based on a significantly negative coefficient on the face-value weighted average age of a firm's bonds in Table IV. Measures of liquidity based on credit quality (financial leverage and a dummy for non-investment grade firms) are not statistically related to pricing errors after accounting for parameter uncertainty.

Recent papers (e.g. Huang and Huang, 2003, Cremers et al., 2008) look at jumps as a possible explanation for pricing errors. Jumps raise the credit spreads by raising the probability of default. By lack of a better measure of jump intensity, we use the posterior mean estimated asset volatility. Firms that experience more frequent and larger jumps will have higher estimated asset volatility in the absence of a modeled jump component. We do find a significant negative relation between σ and pricing error i.e. firms with higher volatility experience higher pricing errors.

Finally, unmodeled bond features such as convertibility of bonds and embedded call and put options could also drive the pricing error. Convertibility of bonds and embedded put options increase observed bond values (lower spreads) relative to the model and therefore decrease the pricing error. Callable bonds have lower prices and therefore increase the pricing error. Note also that the lower effective maturity of callable bonds may increase the importance of parameter uncertainty for moderately risky bonds. We explore the robustness of our results to derivative features in more detail in Section VII.

Controlling for year fixed effects does not materially change the results.

Ultimately, it is difficult to find a good proxy for bond liquidity (see e.g. Bao and Pan (2008) for a different measure), and it is not clear how to interpret the coefficients on the above liquidity measures as they may capture a number of other phenomena. At the minimum we can say that the pricing error that remains after controlling for parameter uncertainty appears to be related to liquidity, but also other unmodeled bond features such as jumps and embedded options.

VI Bankruptcy Probabilities

In this section we look at model-implied bankruptcy probabilities with and without parameter uncertainty and compare them to historical default probabilities across credit ratings. A good model of the firm not only explains the credit spread on corporate debt, but also provides good predictions for bankruptcy.[9]

Leland and Toft (1996) show that at time t, given the process for firm value in equation (1), the probability of bankruptcy in the next τ years is:

$$p_{t,\tau} = \Phi\left(\frac{-\ln\left(\frac{V_t}{V_B}\right) - \lambda\tau}{\sigma\sqrt{\tau}}\right) + \left(\frac{V_t}{V_B}\right)^{-2\lambda/\sigma^2} \cdot \Phi\left(\frac{-\ln\left(\frac{V_t}{V_B}\right) + \lambda \cdot \tau}{\sigma\sqrt{\tau}}\right), \qquad (16)$$

where $\lambda = \mu_{\mathbb{P}} - \delta - 1/2\sigma^2$.

The first term in (16) is the probability that asset value, V_t, is below the default boundary, V_B, at time $t + \tau$ (e.g. Cram et al., 2004). The second term accounts for the probability that V_t may drop below V_B at some time during the next τ years, but still end up above V_B at time $t + \tau$.

[9] We use 'default' and 'bankruptcy' interchangeably since the model does not draw a distinction between the two events.

The probability of bankruptcy ignoring parameter uncertainty, $\widehat{p_{t,\tau}}$, is calculated by plugging the point estimates $\widehat{V_t}$, $\widehat{\sigma}$, and $\widehat{\mu_{\mathbb{P}}}$ into (16). With parameter uncertainty, investors consider the average probability over all possible combinations of parameters i.e. $\mathbb{E}_{V_t,\Theta|\mathcal{F}_t}(p_{t,\tau})$. Note that this takes into account that parameter uncertainty implies uncertainty about the endogenous default boundary, V_B.

The difference between $\mathbb{E}_{V_t,\Theta|\mathcal{F}_t}(p_{t,\tau})$ and $\widehat{p_{t,\tau}}$ depends on the relation between bankruptcy probabilities and asset value and volatility, analogous to the result on debt values above. Figure 8 shows how bankruptcy probabilities vary with asset value and volatility in Merton's (1974) model. The relation between 5 and 10-year bankruptcy probabilities and asset value and volatility is close to linear, showing slight concavity in σ. The 1-year bankruptcy probability is close to linear in asset volatility, but exhibits some convexity for low values of V_t. This relation is robust across models, as the assumption that V_t follows a geometric Brownian motion is common to most structural models (except for the few models that incorporate jumps).

Table V shows the bankruptcy probabilities across credit ratings, using firm values at year-end. We report the median across firm-years for each rating category, and compare with historic default probabilities from Fons and Kimball (1991) and De Jong and Driessen (2005). The model tends to underpredict bankruptcy rates for investment grade firms, and overpredict bankruptcy for non-investment grade firms. Note that the probability of bankruptcy is never truly zero, and it is higher under risk-neutral probabilities. This explains why, for example, AA rated bonds still have a positive credit spread in the model.

The impact of parameter uncertainty on bankruptcy probabilities is small for most credit ratings. Consistent with the convexity result from Figure 8, the effect is largest for non-investment grade firms, and at short horizons. For example, investors think that B-rated bonds have a 1.1% higher chance of bankruptcy within the next year due to parameter uncertainty, relative to an observed 8.1% default probability. However, most bonds in our sample have maturities that are longer than one year. Table I shows that the median time-to-maturity is 8 years, and the lowest quintile is 5.03 years. In Table V the effect of parameter uncertainty on default probabilities within the next five years is smaller. Moreover, the effect goes in the opposite direction for non-investment grade firms, lowering the 5-year default probability relative to point estimates even though credit spreads widen as a result of parameter uncertainty. This counter-intuitive result is due to the concavity of bankruptcy probabilities as seen in the bottom plot of Figure 8.[10] This result demonstrates that the effect of parameter uncertainty on credit spreads of the previous Section is not due to simply raising the probability of default. For the bonds where the effect on spreads is largest (rated B and BB), parameter uncertainty actually lowers default probabilities, bringing them closer in line with historical probabilities.

In the previous section we showed that the portion of credit spreads unexplained by our model is consistent with liquidity and jump effects. Still, the fact that a large portion of our parameter uncertainty measure cannot be explained by the observed proxies raises the concern that this component mimics the unmodeled jump and liquidity effects identified in the literature. Cremers et al. (2008) show that jump models raise the default probability of bonds relative to standard diffusion models such as the one we use in this paper, especially for longer maturity and lower rated bonds. Ericsson and Renault (2006) argue that liquidity has no effect on the bankruptcy decision. In contrast to liquidity and jump effects, Table V reveals that bankruptcy probabilities are lower for non-investment grade firms at longer horizons, bringing them closer to observed frequencies. This suggests that parameter uncertainty is a different channel that changes the model fit compared to structural models that incorporate jumps and liquidity.

Excluding 2007 and 2008 from the sample lowers bankruptcy probabilities somewhat: accounting for parameter uncertainty, the model-implied 5-year bankruptcy probability for a CCC+ rated firm before 2007 is 48.25%, compared to 54.19% for the full sample. For a BBB rated firm, the 5-year probability is 1.71% without 2007-

[10]Another way to see this result is to consider a simple one-period model with p the (risk-neutral) probability of default and ϕ the recovery-given-default, so that $D = (1-p) + \phi p$. The (continuously compounded) credit spread equals $-\log((1-p)+\phi p) - r^f$, and is clearly convex in p. By Jensen's Inequality it is therefore possible to have a posterior spread in p that lowers $\mathbb{E}(p)$ relative to the point estimate $\widehat{p}$ but at the same time raises the credit spread.

08, compared to 1.81% in Table V.

To summarize, since credit spreads are more highly non-linear in asset value and volatility than bankruptcy probabilities, we find that the impact of parameter uncertainty on credit spreads is large relative to the impact on bankruptcy probabilities.

VII Robustness

We perform several robustness tests exploring the effects of derivative features in corporate bonds, capital structure complexity, the credit crisis, using a longer time-series for estimation, and accounting for the impact of uncertainty on equity values.

Corporate bonds often have some derivative features such as embedded call and put options, or they may be converted into stock at some point in the future. Since our pricing model does not account for embedded options, we check robustness to these features by limiting the sample to the 2,748 firm-years with no convertible or puttable bond issues. Our results are qualitatively the same as reported above. If in addition we limit the sample to firms with 2 or fewer bonds outstanding, to control for capital structure complexities that the pricing model does not incorporate, the sample size reduces to 1,011 firm-years but the results in fact appear more consistent with the effects of parameter uncertainty, particularly the findings across maturities.

We also ran the results without the 2007 and 2008 credit crisis years and find that this does not change the conclusions.

In the previous section we estimated the model for each firm-year separately. To check robustness with respect to the length of the estimation period, we re-estimate the model using the full time series of equity values starting in 1994. This reduces the uncertainty about σ, as measured by its posterior standard deviation, to 0.04 (from 0.05 when estimating the model by firm-year), and consequently slightly lowers the impact of parameter uncertainty, but the main insights still hold.

Given the impact of parameter uncertainty on debt prices, it stands to reason that equity values are also affected, but in opposite direction. Intuitively, equity is a long call option on the firm's assets, and its value therefore increases in the uncertainty about asset value and volatility. One concern is that we may overvalue equity when pricing it using our posterior distribution. We already partly account for uncertainty in equity values through our pricing error on log equity values in equation (14), but fully pricing equity with the posterior distribution within the MCMC estimation is extremely difficult. Instead we determine by how much we need to "shift" the entire posterior distribution of σ or V_t in order to correctly price equity. It turns out that we need to lower the distribution of σ by 0.003 on average across firm-years, which lowers the median credit spread both with and without parameter uncertainty by 0.4 basis points. Alternatively, lowering the distribution of V_t by 0.44% (as a percentage of V_t) raises spreads by 0.63 basis points. We conjecture that the changes are small because equity, unlike shorter maturity bonds, is a long-lived security and therefore has low convexity.

VIII Conclusions

We quantify the impact of parameter uncertainty on corporate bond pricing and document a number of new results regarding: i) the relation between parameter uncertainty, firm and bond characteristics and credit spreads; ii) the time series of uncertainty, and; iii) bankruptcy probabilities. Standard structural models of the firm have trouble generating credit spreads that are large enough to match the empirical data. The difference between observed and model-implied spreads is usually attributed to jumps in asset values or to non-credit phenomena such as taxes and liquidity. While we do not deny the existence of such factors, we find that parameter uncertainty explains up to 40% of the credit spread that is otherwise attributed to these factors. We show that firms with large intangible assets and volatile earnings growth experience high parameter uncertainty, and this uncertainty materially affects credit spreads on their debt. We find that parameter uncertainty exhibits counter-cyclical behavior and was particularly high in the period 1998 to 2000 and during the credit crisis of 2008, raising credit spreads by as much as 50 basis points. We conclude that it is important to account for parameter uncertainty, and that a simple model of credit risk can explain a significantly larger fraction of the corporate credit spread than is usually thought.

Theoretically, the effect of parameter uncertainty is driven by the concavity of debt prices as a function of the model's parameters, most importantly asset value and volatility. The prior literature has relied on (linear) regressions to study the relation between parameter uncertainty proxies and credit spreads. We show that it is important to take into account that this relation is highly non-linear, and that parameter uncertainty is most important for firms of moderate credit risk (with a "B" rating) and at short maturity.

We find that a large component of our estimates of parameter uncertainty is not explained by a set of observable proxies for uncertainty (such as the level of intangibles and earnings growth forecast dispersion). This naturally raises concerns that this unexplained portion simply mimics unmodeled jump and liquidity effects that have been incorporated in a number of existing structural models. However, we show that effect of parameter uncertainty on bankruptcy probabilities is distinct from the effect of jumps and liquidity, bringing the fitted probabilities closer to observed frequencies.

In some respect, the effect of parameter uncertainty may be larger than we estimate here. Uncertainty about parameters implies uncertainty about hedge ratios, leaving hedgers exposed to the underlying if we are not using exactly the right parameters. This may give rise to a risk-premium that we do not measure. On the other hand, investors may have more information than we use here, reducing the uncertainty about parameters. We conjecture that the latter effect may be relatively small given that the application of structural or reduced-form models to price corporate debt and estimate default probabilities with the same information as we use, is wide-spread.

Parameter uncertainty also matters for *portfolios* of corporate bonds. In other words, the concavity effect cannot be diversified away. As such, parameter uncertainty should matter for a large array of derivatives and structured finance products, including credit default swaps, collateralized debt obligations, and mortgage-backed securities.

Parameter uncertainty is closely related to model uncertainty, which we do not specifically address in this paper. For models that are nested, the different models are simply parameter restrictions on one general model. For example, the Leland (1994) model is a special case of Fan and Sundaresan (2000). By estimating a general model and pricing debt using our approach, uncertainty over nested sub-models is taken into account. For a related discussion, see Johannes, Korteweg and Polson (2010).

Our Bayesian estimation framework is a natural setting for estimation and pricing under parameter uncertainty. Simulation-based MCMC methods provide a flexible estimation tool for complicated corporate debt pricing models with non-linearities, and we show that pricing under parameter uncertainty is straightforward to apply without the need to write down new models. The filtering methodology allows for multiple dynamic and possibly latent state variables such as unlevered firm value, stochastic interest rates (Longstaff and Schwartz, 1995) and default barriers (Collin-Dufresne and Goldstein, 2001, Davydenko, 2007). Our framework can also accommodate other important extensions such as stochastic volatility and jumps in asset values. Such extensions are common in standard derivative pricing problems (see for example Bates, 1996, Pan, 2002, Polson and Stroud, 2003, and Eraker, Johannes and Polson, 2003), but so far have been under-explored in models of credit risk.

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Appendix: MCMC Algorithm

We need to sample from four conditionals:

$$
\begin{aligned}
\text{Log Asset Values} &: \quad p(X|\mu_{\mathbb{P}}, \sigma^2, \nu^2, P^T) \\
\text{Asset Return Volatility} &: \quad p(\sigma^2|X, \mu_{\mathbb{P}}, \nu^2, P^T) \\
\text{Asset Return Drift} &: \quad p(\mu^*_{\mathbb{P}}|X, \sigma^2, \nu^2, P^T) \\
\text{Equity Pricing Error Variance} &: \quad p(\nu^2|X, \mu_{\mathbb{P}}, \sigma^2, P^T)
\end{aligned}
$$

The asset values are sampled using the Kalman filter and FFBS (filter forward and backwards sample, see Carter and Kohn, 1994). This allows us to get a full draw of the vector of unobserved asset values given observed prices and parameters. Volatility and drift of asset returns are drawn using the fact that, conditional on the state variable, the state equation is a simple regression. It is easier to estimate the drift of log-asset returns, which is related to the drift of arithmetic returns in equation (1) by a Jensen term, $\mu^*_{\mathbb{P}} = \mu_{\mathbb{P}} - 1/2\sigma^2$. Finally, the pricing error variance ν^2 is drawn from the posterior distribution of the error variance of the observation equation. We describe these steps in detail.

1. Asset Values $p(X|\mu^*_{\mathbb{P}}, \sigma^2, \nu^2, P^T)$

   First the algorithm uses the Kalman Filter to find filtered forward estimates of the expectation $E(X_t^{(g+1)}|P^t, \Theta^{(g)})$ and variance $V(X_t^{(g+1)}|P^t, \Theta^{(g)})$ for all t, given the parameter values from the previous draw g. Second, it simulates backwards using the identity:

$$p(X|\Theta^{(g)}, P^T) = p(X_T|P^T, \Theta^{(g)}) \prod_{t=1}^{T-1} p(X_t|P^t, X_{t+1}, \dots, X_T, \Theta^{(g)})$$

   The intuition behind this identity is that, given future values of X, future prices carry no information on X_t. The first component samples the filtered distribution of X_T using the Kalman filter moments. Carter and Kohn (1994) find the moments of the normal distributions for the remaining terms. In the backward sampling procedure we account for the fact that the company has not gone bankrupt in our sample period i.e. X_t cannot be below $\ln(V_B)$ in the sample. We incorporate this information by drawing the state vector from the normal distribution from the Kalman filter, truncated at $\ln(V_B)$. For simplicity we limit ourselves to homoscedastic, independent error terms, although it is straightforward to have heteroscedastic error terms by using a mixture of normals. Auto-correlated error terms can also be implemented with little trouble, as well as cross-correlation between debt and equity errors.

2. Asset Volatility $p(\sigma^2|X, \mu_{\mathbb{P}}^*, \nu^2, P^T)$

   The conditional distribution of asset return volatility σ^2 is the most difficult to draw from. The parameter σ^2 appears in both the state variable dynamics and the pricing equation. Using Bayes' Law, we can write:

$$p(\sigma^2|\mu_{\mathbb{P}}^*, \nu^2, X, P^T) \propto p(P^T|\mu_{\mathbb{P}}^*, \nu, X, \sigma^2)p(\sigma^2|\mu_{\mathbb{P}}^*, \nu^2, X)$$

   This is a non-standard distribution. The second part of the posterior distribution is an inverse gamma distribution, given an inverse gamma prior: $p(\sigma^2) \sim IG(a, b)$. From standard Bayesian regression techniques we have

$$\begin{aligned} &p(\sigma^2|\mu_{\mathbb{P}}^{*,(g)}, \left(\nu^2\right)^{(g)}, X^{(g+1)}) \sim \\ &\qquad IG\left(a + \frac{T}{2}, b + \frac{1}{2}\sum_{t=2}^{T}\left(X_{t+1}^{(g+1)} - X_t^{(g+1)} - \mu_{\mathbb{P}}^{*,(g)}\right)^2\right) \end{aligned}$$

   We draw a proposal from this distribution, with a diffuse prior $a = 2.1$ and $b = 200$. Then we use the Metropolis algorithm to adjust the draws by accepting with probability $\min\left(L^{(g+1)}/L^{(g)}, 1\right)$ where

$$L^{(g+1)} = \frac{p(\sigma^{(g+1)}|\mu_{\mathbb{P}}^{*,(g)}, \left(\nu^2\right)^{(g)}, X^{(g+1)}, P^T)}{p(\sigma^{(g+1)}|\mu_{\mathbb{P}}^{*,(g)}, \left(\nu^2\right)^{(g)}, X^{(g+1)})}$$

   The distribution in the denominator is the proposal Inverse Gamma, so the acceptance probability is:

$$\min\left(1, \frac{p(P^T|\mu_{\mathbb{P}}^{*,(g)}, \left(\nu^2\right)^{(g)}, X^{(g+1)}, \left(\sigma^2\right)^{(g+1)})}{p(P^T|\mu_{\mathbb{P}}^{*,(g)}, \left(\nu^2\right)^{(g)}, X^{(g+1)}, \left(\sigma^2\right)^{(g)})}\right)$$

   which is easily evaluated from the observation equation. The mean (median) acceptance probability is 80% (94%).

3. Drift of Log Asset Value $p(\mu^*_{\mathbb{P}}|X, \sigma^2, \nu^2, P^T)$

   Next, we draw the parameter $\mu^*_{\mathbb{P}}$ given the state vector X, prices P^T, and the parameters σ^2 and ν^2. First we realize that observed prices carry no information on $\mu^*_{\mathbb{P}}$ so we can ignore the conditioning on P^T. Given the states, the conditional of μ^*_P is a Bayesian regression. We adopt the prior belief that $\mu^*_{\mathbb{P}}|\sigma^2 \sim N(b, B^{-1}\sigma^2)$, with prior mean $b = 15\%$ and $B = 5$. The posterior distribution from which we sample is then:

$$p(\mu^{*,g+1}_{\mathbb{P}}|X^{(g+1)}, \left(\sigma^2\right)^{(g+1)}, \left(\nu^2\right)^{(g)}, P^T) \sim \\ N\left((B^*)^{-1}\left(B \cdot b + \frac{1}{2}\left(X^{(g)}_T - X^{(g)}_1\right)\right), (B^*)^{-1} \cdot (\sigma^2)^{(g+1)}\right)$$

   where $B^* = B + (T-1)/2$.

4. Pricing Error Variance $p(\nu^2|X, \mu^*_{\mathbb{P}}, \sigma^2, P^T)$

   We obtain a draw from the conditional distribution of ν^2, the equity pricing error variance, by drawing from the posterior distribution of the error variance from the Bayesian regression that is the pricing equation. We sample from the posterior inverse gamma distribution with a prior $IG(a, b)$:

$$p(\nu^2|\mu^{*,(g+1)}_{\mathbb{P}}, \left(\sigma^2\right)^{(g+1)}, X^{(g+1)}, P^T) \sim \\ IG\left(a + \frac{T}{2}, b + \frac{1}{2}\sum_{t=1}^{T}\left(\ln\left(\mathcal{P}^{-1}_E(P_t, \Theta^{(g)})\right) - X^{(g+1)}_t\right)^2\right)$$

   where $\mathcal{P}^{-1}_E$ is the inverted equity pricing formula that yields the implied asset value given a market value of equity and parameters Θ.

   We discard the first 100 draws to allow the algorithm to converge to the true posterior distribution. We then draw 1,000 samples from which we compute the market value of debt, and credit spreads.

Table I

Summary Statistics

Panel A reports summary statistics over 5,300 firm-years between 1994 and 2008. Size is the book value of assets at fiscal year-end as reported in Compustat (item 6). Debt/Assets is the book value of interest-bearing debt (item 9 + item 34) divided by the book value of assets. PPE is property, plant and equipment (item 8) scaled by book value of assets. M/B is the year-end market value of equity (from CRSP) divided by the book value of equity (item 6 minus item 181). EBITgvol is the standard deviation of EBIT/book assets growth over the last 5 years. EPSfcdisp is the standard deviation of analysts' long-term EPS forecasts scaled by the absolute mean EPS estimate (from I/B/E/S). Age is the number of years since first incorporation (from Moody's Corporate Manuals). Non-Inv is a dummy variable that equals one when the firm is rated below investment-grade. Issues/Firm is the number of bond issues per firm (from FISD). Coupon and Maturity are the face-value weighted coupon rate and time to maturity, respectively, for the firm's outstanding bonds (from FISD). Conv, Call and Put are dummy variables that equal one if at least one of the firm's bonds is convertible, callable or puttable, respectively. Panel B shows correlations between our proxies for parameter uncertainty, across firm-years.

Panel A: Summary Statistics

Variable	mean	s.d.	quintiles		
			1	median	5
Corporate variables					
Size ($b)	9.382	19.834	1.108	3.347	12.057
Debt / Assets	0.334	0.199	0.179	0.306	0.461
PPE	0.340	0.254	0.100	0.289	0.586
M/B	2.956	2.521	1.227	2.127	3.876
EBITgvol	0.369	0.323	0.111	0.238	0.661
EPSfcdisp	0.209	1.035	0.023	0.051	0.156
Age (years)	47.97	32.46	16	39	81
Non-Inv = 1	0.377	-	-	-	-
Bond-related variables					
Issues / Firm	13.308	9.180	4	8	18
Maturity (in yrs)	9.233	5.599	5.033	8.000	12.461
Coupon (in %)	7.076	2.440	5.625	7.125	8.950
Conv = 1	0.38	-	-	-	-
Call = 1	0.91	-	-	-	-
Put = 1	0.36	-	-	-	-

Panel B: Correlations between Parameter Uncertainty Proxies

	PPE	M/B	EBITgvol	EPSfcdisp	Age
Size	-0.139	-0.031	-0.055	-0.051	0.051
PPE		-0.103	0.141	0.066	-0.126
M/B			-0.176	-0.081	0.147
EBITgvol				0.161	-0.154
EPSfcdisp					-0.025

Table II

Parameter Estimates

MCMC estimates of the parameters of Leland's (1994b) model for 5,300 firm-years between 1994 and 2008, by credit rating. Parameters are estimated for each firm-year, and the posterior mean and standard deviation are reported, averaged over all firm-years within a given credit rating. N is the number of firm-years. σ is the annualized volatility of asset returns. $\mu_{\mathbb{P}}$ is the expected return on assets. ν is the standard deviation of the equity pricing error, and V_t/V_B is the value of the firm at year-end, scaled by the default boundary, V_B.

Credit	N	σ		$\mu_{\mathbb{P}}$		ν		V_t/V_B	
rating		mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
AAA	27	0.24	0.05	0.13	0.08	0.04	0.03	72.95	2.57
AA+	7	0.24	0.05	0.12	0.08	0.04	0.03	38.45	1.37
AA	93	0.25	0.05	0.09	0.08	0.05	0.03	24.92	8.86
AA-	123	0.25	0.05	0.10	0.09	0.05	0.03	24.45	0.93
A+	270	0.25	0.05	0.09	0.09	0.04	0.03	36.38	2.86
A	569	0.25	0.05	0.09	0.09	0.05	0.03	15.25	0.66
A-	429	0.26	0.05	0.08	0.09	0.05	0.04	13.65	0.56
BBB+	561	0.26	0.05	0.09	0.09	0.05	0.04	8.67	0.38
BBB	689	0.26	0.05	0.09	0.09	0.04	0.03	7.14	0.30
BBB-	548	0.26	0.05	0.08	0.09	0.05	0.03	5.44	0.24
BB+	359	0.26	0.05	0.09	0.09	0.05	0.03	4.26	0.20
BB	494	0.26	0.05	0.08	0.09	0.05	0.04	3.24	0.17
BB-	517	0.25	0.05	0.08	0.09	0.05	0.04	2.96	0.15
B+	376	0.25	0.05	0.07	0.09	0.05	0.04	2.54	0.13
B	159	0.25	0.05	0.08	0.09	0.05	0.04	2.38	0.14
B-	62	0.26	0.05	0.04	0.10	0.06	0.05	2.43	0.16
CCC+	17	0.25	0.05	-0.01	0.09	0.05	0.04	1.82	0.10
Overall	5,300	0.26	0.05	0.08	0.09	0.05	0.04	4.45	1.55

Table III

Parameter Uncertainty Regressions

Regressions of the posterior standard deviations of asset volatility, σ (in %), and the posterior standard deviation (in %) of asset value, V_t, scaled by its posterior mean. The signal-to-noise ratio σ/ν is calculated using posterior means of the MCMC estimation. The other explanatory variable are as defined in Table I. Standard errors are clustered by firm, and reported in parentheses. N is the number of firm-years. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively.

	posterior s.d. (σ)		posterior s.d. (V_t) / mean(V_t)	
	I	II	I	II
PPE	-0.425	-0.382	0.215	0.548
	(0.100)***	(0.097)***	(0.480)	(0.580)
EBITgvol	0.346	0.306	0.533	0.215
	(0.095)***	(0.093)***	(0.151)***	(0.240)
EPSfcdisp	-0.165	-0.202	-0.007	-0.300
	(0.134)	(0.130)	(0.461)	(0.529)
Log(Age)	-0.054	-0.039	-0.205	-0.088
	(0.036)	(0.035)	(0.116)*	(0.086)
M/B	0.020	0.013	0.065	0.010
	(0.011)*	(0.011)	(0.060)	(0.040)
Log(Size)	0.059	0.047	0.122	0.029
	(0.022)***	(0.021)	(0.131)	(0.094)
σ / ν	.	-0.245	.	-1.943
	.	(0.041)***	.	(0.709)***
Year fixed effects	Y	Y	Y	Y
R^2	0.134	0.153	0.012	0.069
F	27.61	27.89	27.83	27.26
p	0.000	0.000	0.000	0.000
N	4,283	4,283	4,283	4,283

Table IV

Pricing Error Regressions

This table reports regressions of pricing errors on proxies for liquidity and bond-specific features. Pricing error is defined as either the ratio (in the left two columns) or the difference (in the right two columns) between the fitted credit spread under parameter uncertainty ($cs^{\mathcal{F}_t}$) and the observed credit spread (cs^{obs}), for firm i in year t. Share Liquidity is the average number of shares traded divided by the total number of shares outstanding. Bond Age is the number of years is the number of years since bond issuance, face value weighted over the firm's outstanding bonds. σ is the posterior mean estimate of asset volatility. All other variables are as defined in Table I. Standard errors (in parentheses) are clustered by firm. N is the number of firm years. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively.

Dependent:	$cs^{\mathcal{F}_t}/cs^{obs}$		$cs^{\mathcal{F}_t} - cs^{obs}$ (in %)	
	I	II	I	II
Share Liquidity	0.093	0.123	-1.809	-2.327
	(0.208)	(0.207)	(0.805)**	(0.815)***
Log(Size)	-0.094	-0.094	-0.176	-0.888
	(0.014)***	(0.014)***	(0.043)***	(0.042)**
Non-Inv = 1	0.027	.	-0.446	.
	(0.044)	.	(0.142)***	.
Debt / Assets	.	0.084	.	0.463
	.	(0.102)	.	(0.407)
Bond Age	-0.030	-0.030	-0.127	-0.115
	(0.006)***	(0.006)***	(0.020)***	(0.021)***
Time-to-Maturity	-0.004	-0.004	0.016	0.021
	(0.003)	(0.003)	(0.007)**	(0.007)***
σ	-2.587	-2.545	-13.176	-12.758
	(0.381)***	(0.381)***	(1.545)***	(1.437)***
Conv = 1	0.289	0.294	-0.014	-0.032
	(0.041)***	(0.041)***	(0.112)	(0.117)
Put = 1	0.148	0.145	-0.049	-0.0414
	(0.044)***	(0.044)***	(0.112)	(0.112)
Call = 1	-0.194	-0.196	-0.117	-0.234
	(0.085)**	(0.085)**	(0.177)	(0.184)
Intercept	2.497	2.462	4.435	3.411
	(0.179)***	(0.187)***	(0.647)***	(0.608)***
adjusted R^2	0.081	0.082	0.073	0.070
F	26.53	26.61	24.25	20.96
p	0.000	0.000	0.000	0.000
N	4,519	4,518	4,519	4,518

Table V

Bankruptcy Probabilities

This table reports bankruptcy probabilities, calculated as the probability that the firm's asset value is below the default boundary after 1 and 5 years. For each firm-year, we calculate the τ-year bankruptcy probability as:

$$p_{t,\tau} = \Phi\left(\frac{-\ln\left(\frac{V_t}{V_B}\right) - \lambda\tau}{\sigma\sqrt{\tau}}\right) + \left(\frac{V_t}{V_B}\right)^{-2\lambda/\sigma^2} \cdot \Phi\left(\frac{-\ln\left(\frac{V_t}{V_B}\right) + \lambda \cdot \tau}{\sigma\sqrt{\tau}}\right),$$

where $\lambda = \mu_{\mathbb{P}} - \delta - 1/2\sigma^2$ and using firm value at year-end. We define $\widehat{p}$ as the bankruptcy probability calculated using point estimates (posterior means) of the parameters. $\mathbb{E}(p)$ is the posterior mean bankruptcy probability accounting for parameter uncertainty. For comparison, historical 1 and 5-year cumulative bankruptcy probabilities (from Fons and Kimball (1991) and De Jong and Driessen (2005)) are also reported. Bankruptcy probabilities are expressed as percentages, and we report median values over firm-years within each rating class.

Credit	1-Year bankruptcy prob.			5-Year bankruptcy prob.			$\mathbb{E}(p) - \widehat{p}$	
rating	$\widehat{p}$	$\mathbb{E}(p)$	Historic	$\widehat{p}$	$\mathbb{E}(p)$	Historic	1-Year	5-Year
AAA	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00
AA+	0.00	0.00	-	0.00	0.00	-	0.00	0.00
AA	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.00
AA-	0.00	0.00	-	0.00	0.01	-	0.00	0.01
A+	0.00	0.00	-	0.01	0.07	-	0.00	0.06
A	0.00	0.00	0.10	0.02	0.11	0.65	0.00	0.09
A-	0.00	0.00	-	0.11	0.36	-	0.00	0.25
BBB+	0.00	0.00	-	0.30	0.67	-	0.00	0.37
BBB	0.00	0.00	0.20	1.13	1.81	3.41	0.00	0.67
BBB-	0.00	0.00	-	2.55	3.36	-	0.00	0.80
BB+	0.00	0.04	-	7.08	7.98	-	0.04	0.89
BB	0.12	0.30	1.80	15.13	15.41	12.38	0.18	0.28
BB-	0.45	0.83	-	20.92	20.88	-	0.38	-0.04
B+	1.98	2.81	-	30.23	29.88	-	0.84	-0.35
B	8.54	9.66	8.10	44.76	44.07	26.82	1.12	-0.69
B-	7.21	8.60	-	42.81	41.83	-	1.39	-0.98
CCC+	17.71	18.70	-	54.74	54.19	-	0.99	-0.55

Figure 1

Debt Value as a Function of Value and Volatility of Assets

This figure shows the theoretical relation between debt value and firm asset value (V_t) in the top graph, and asset volatility (σ) in the bottom graph, for Merton's (1974) model. We set the face value of debt to 50, time-to-maturity to 10 years and risk-free rate $r = 5\%$. In the top plot we set $\sigma = 20\%$ and in the bottom graph we fix $V = 100$.

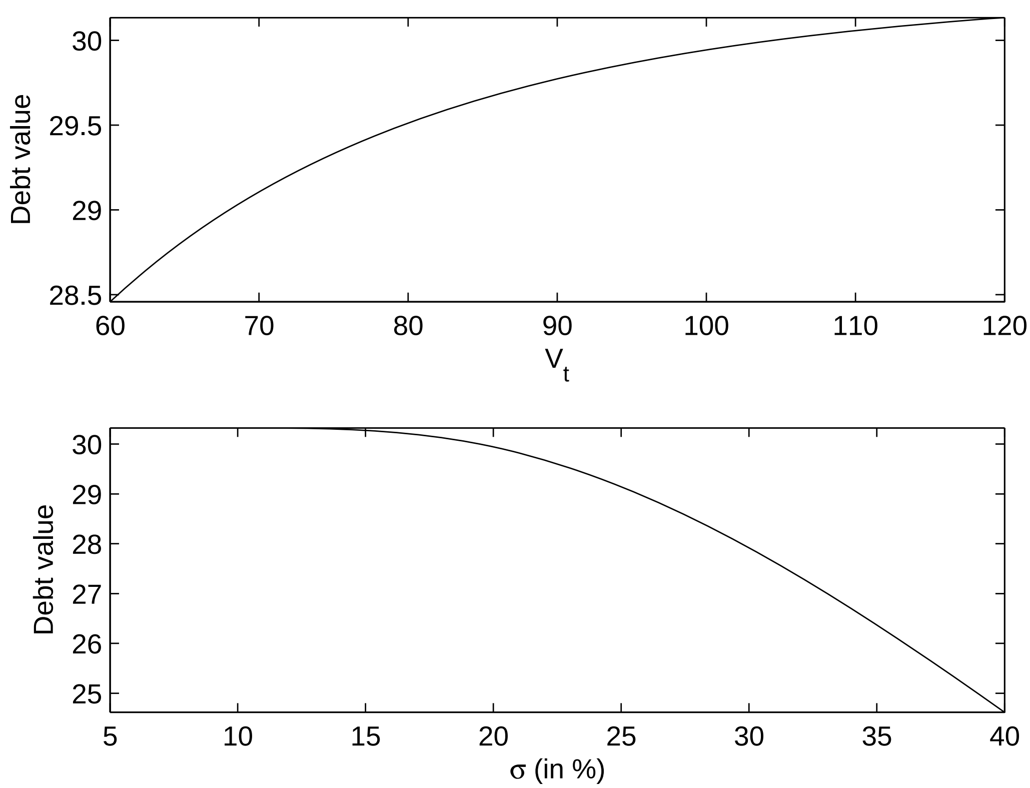

Figure 2

Credit Spreads across Firm Value and Debt Maturity

This figure shows the theoretical relation between credit spreads and asset value (V) for a 1-year and a 10-year bond, using Merton's (1974) model with face value of debt of 50, risk-free rate $r = 5\%$, and asset volatility $\sigma = 20\%$.

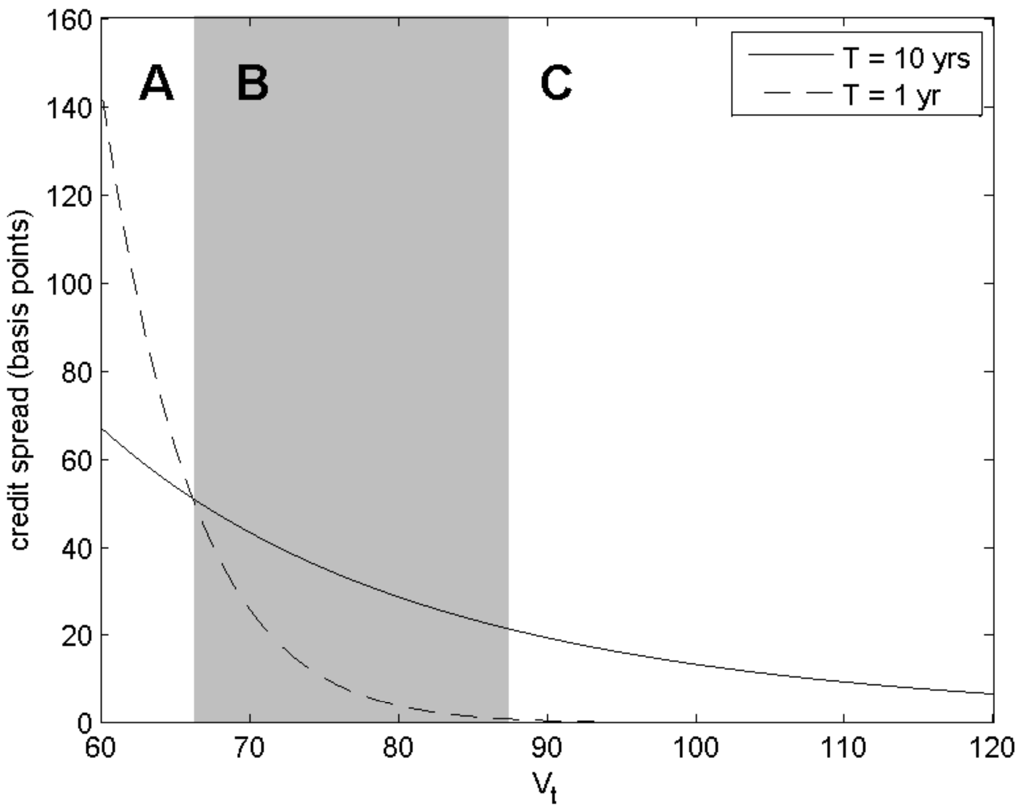

Figure 3

Observed and Residual Credit Spreads

Plot of the median observed and residual credit spread across credit ratings. The residual credit spread is calculated as the difference between the observed spread and the spread implied by Leland's (1994b) model, with and without accounting for parameter uncertainty. The bottom plot shows the median percentage decrease in the residual credit spread that results from taking parameter uncertainty into account. The gray area shows the (5,95)% firm-year percentiles.

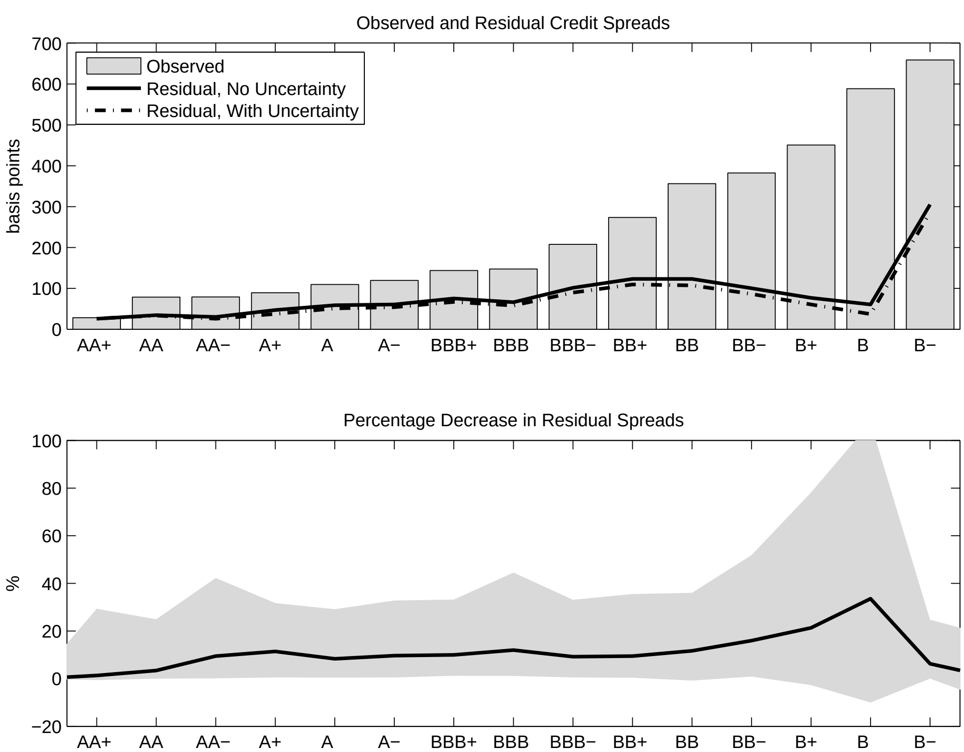

Figure 4

Residual Credit Spreads across Maturities

Plot of the median residual credit spread for the lowest and highest quintiles of debt maturity, by credit rating. The residual spreads with and without the effect of parameter uncertainty are calculated as in Figure 3. The bottom plot shows the median percentage decrease in the residual credit spread that results from taking parameter uncertainty into account.

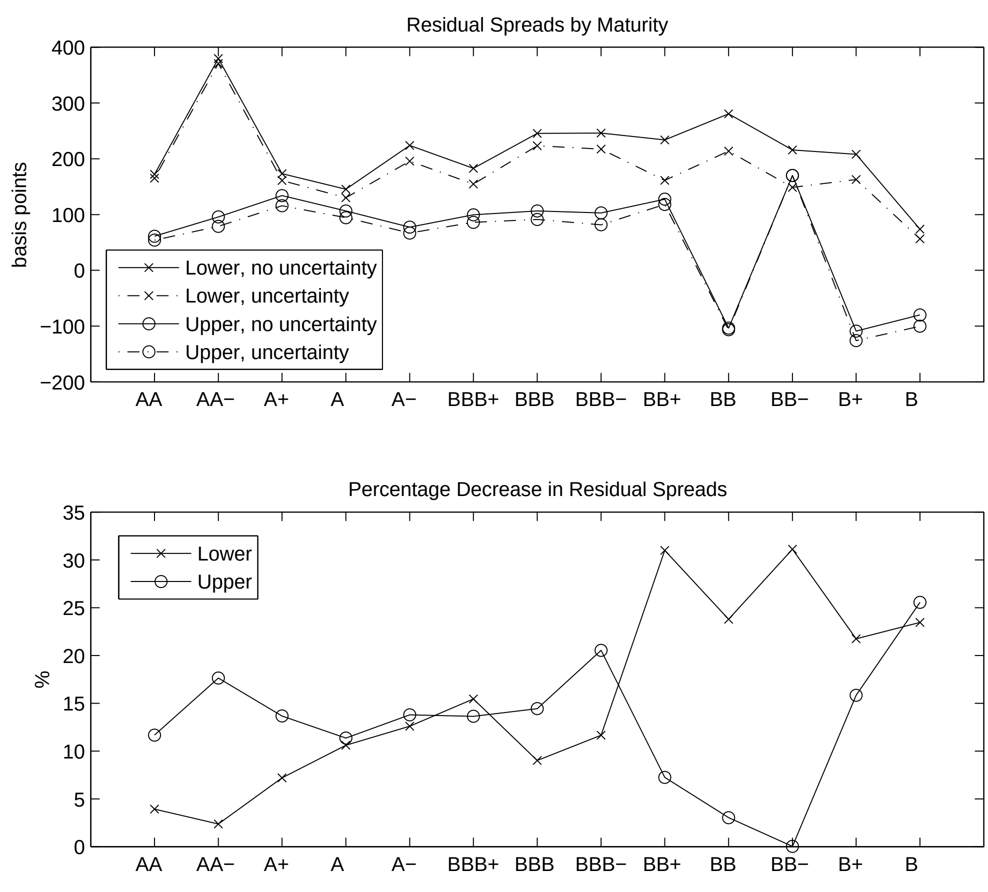

Figure 5

Residual Credit Spreads across Parameter Uncertainty Proxies

Plot of the median residual credit spread for the lowest and highest quintiles of parameter uncertainty proxies with and without the effect of parameter uncertainty. The proxies are as explained in Table I, and the residual spreads are calculated as in Figure 3.

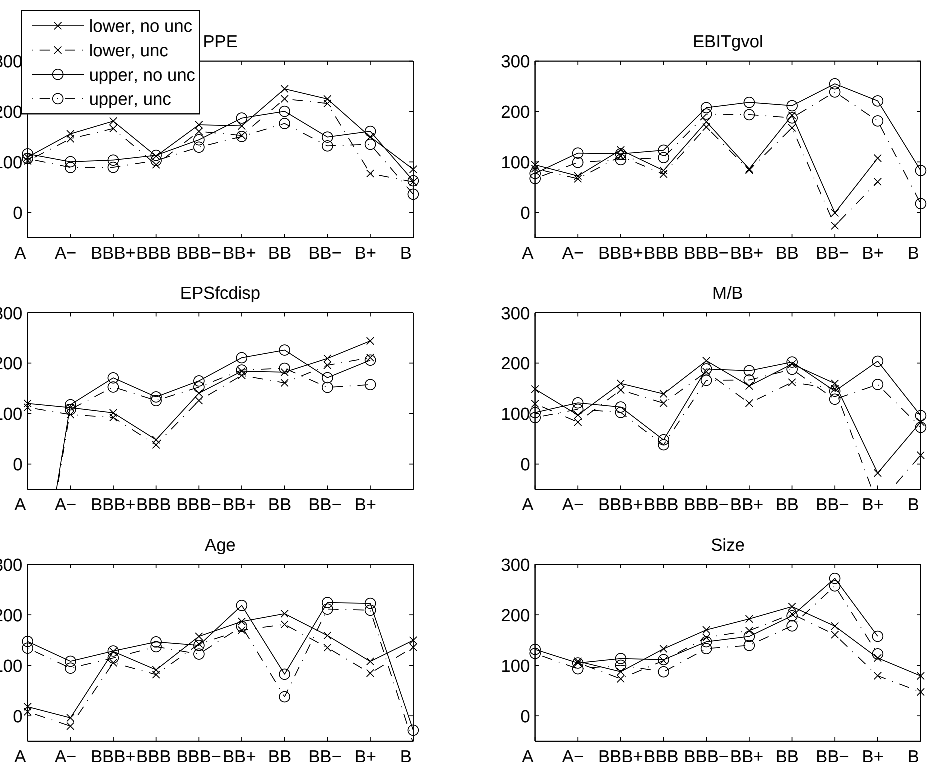

Figure 6

Time-series of Parameter Uncertainty Measures

This figure shows the time series of the posterior standard deviation of asset volatility ($s.d.(\sigma)$) and the posterior standard deviation of asset value scaled by its posterior mean ($s.d.(V_t)/E(V_t)$). Both standard deviations are medians across firm-years, and plotted on the left axis. The average level of the VIX in each year is plotted on the right axis.

Figure 7

Time-series of Credit Spreads

This figure shows the time series of median credit spreads for bonds rated A, BBB and BB, with and without the effect of parameter uncertainty.

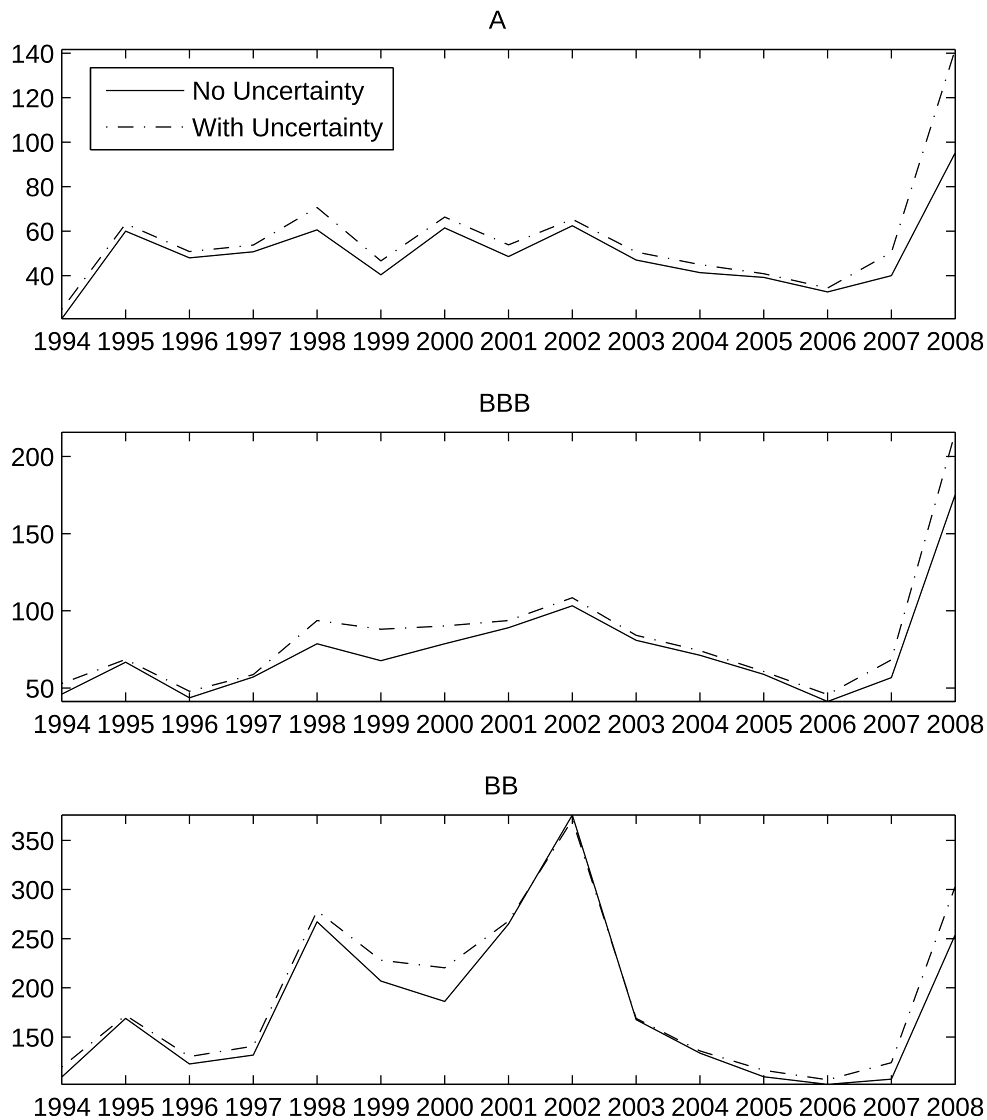

Figure 8

Bankruptcy Probability as a Function of Value and Volatility of Assets

This figure shows the theoretical relation between bankruptcy probability and firm asset value (V_t) in the top graph, and asset volatility (σ) in the bottom graph, for Merton's (1974) model. The parameters are the same as in Figure 1.