Smart Money, Dumb Money, and Learning Type from Price

J. B. Heaton
Bartlit Beck Herman Palenchar and Scott LLP
Chicago

N. G. Polson
Booth School of Business
University of Chicago

This Draft: June 2012

Abstract

We present a simple model of smart money and dumb money. Dumb money tries to learn from market prices whether or not it is dumb. Dumb money's ability to learn depends on its openness to the idea that it may be the dumb money and on its ability to assess the total amount of dumb money in the market. Neither requirement may be met easily in the real world.

Key words: smart money, dumb money, noise traders, limits of arbitrage, learning, heterogeneous beliefs

"Men, it has been well said, think in herds, it will be seen that they go mad in herds, while they only recover their senses slowly, and one-by-one."

-Charles MacKay, Extraordinary Popular Delusions And The Madness Of Crowds (1841)

1 Introduction

We study dumb money learning in a simple parimutuel betting model. Dumb money and smart money bet on the occurrence of two exhaustive and mutually exclusive states of the world. Smart money knows the actual probabilities of the two states. Dumb money holds incorrect beliefs about the probabilities of the two states. Betting between smart money and dumb money generates equilibrium state prices for the two states of the world. Dumb money affects these equilibrium state prices even in the presence of smart money. Dumb money earns negative abnormal returns in that equilibrium, while smart money earns positive abnormal returns.

Our goal is to explore the possibility that dumb money may be able to learn its way out of its bad beliefs using information in the observable state prices and its prior beliefs. Dumb money in our model is exogenously endowed with a non-zero prior probability that it is the dumb money and a prior joint probability distribution on the proportion of dumb money in the market and the size of dumb money valuation errors. Dumb money learning requires that dumb money can assess somewhat accurately the amount of dumb money in the market and that dumb money not believe too strongly that it is the smart money.

Dumb money learning is not guaranteed, however. It may be very difficult to determine what is the proportion of dumb money in the market, a key determinant of learning in our model. Even identifying the possible candidates to take the title of "dumb money" can be a challenge. In one form or another, people often ask of a group of investors, "Are they smart money or dumb money?"[1] It is unpersuasive, for example, to assert that all institutional

[1]See, for example, the question as asked in an article on sovereign wealth funds: Holman W. Jenkins, Jr.,

investors are smart money, while all individual investors are dumb money. Many times the question is *which* institutional investor is the smart money and which is the dumb money.[2] Among finance researchers there is no agreement as to who constitutes the smart money. As DeBondt and Thaler (1990, p. 56) state, "an interesting empirical question is whether the presumed smart money segment actually can be identified." Lakonishok, Shleifer and Vishny (1992) make a compelling case that much of the institutional money management industry acts like dumb money. And there is no evidence that all individual investors are dumb money.[3] Dumb money can be seemingly on the right side of the bet for quite some time.[4] In some cases, such as when there is *complete* uncertainty about the proportions of dumb money and smart money in the market, or when the proportion of dumb money and smart money is equal, our model predicts that learning from prices that one is the dumb money is impossible.

Dumb money learning also requires that dumb money be sufficiently open *a priori* to the possibility of being the dumb money. But being open to the possibility of being the dumb money requires a psychologically difficult self-evaluation that may be unlikely to occur very often in the real world. Indeed, there is a sense in which the *essence* of being the dumb money is thinking too strongly that one is the smart money. Additionally, the dumb money is unlikely to be monolithic in openness to the possibility of being the dumb money. Dumb money that is less open to the possibility of being the dumb money will find it harder to

---

"Sovereign Wimp Funds," Wall Street Journal, January 23, 2008, p. A.24.

[2]In a recent article on a very large trade in silver ETF put options, one commodity trader was quoted,"When you see a trade that size you're left scratching your head– is it smart money or is it dumb money...." Tatyana Shumsky and Brendan Conway, "Silver's Rally Spurs Betting by Doubters," Wall Street Journal (Online), Apr 21, 2011.

[3]For example, Kelly (1997) identifies higher-income households as smart money in a test of a smart money-noise trader model. See also Keswani and Stolin (2008).

[4]Consider, example, the following quote from the Wall Street Journal during the Internet boom: "But so far, the dumb money has been laughing all the way to the bank. It is the skeptical – smart money-investment pros – that has been hurting." E.S. Browning and Aaron Lucchetti, "The New Chips: Conservative Investors Finally Are Saying: Maybe Tech Isn't a Fad — Nasdaq Powers Past 5000 As Cisco, Oracle and Such Become 'Core' Stocks — Is the Shift Itself Bearish?" p. A.1. The NASDAQ peaked the same day.

learn than dumb money more open to the possibility.

Ours is not a model of heterogeneous information. We assume that smart money and dumb money in our model have access to what Harrison and Kreps (1978) call "the same substantive economic information." Disagreement between smart money and dumb money in our model is non-informational, attributable to the different implications that the same information can have for different individuals, or to the inability of dumb money to notice that it is trading on noise, not information [Black (1986)]. At the same, while our model is one of heterogeneous beliefs, it differs from many such models where traders with different beliefs cling inflexibly to their views, even where their views are demonstrably inferior to the views of other traders [see, for example, Harrison and Kreps (1978), Varian (1985), Harris and Raviv (1993), Kandel and Pearson (1995), and Scheinkman and Xiong (2003)].[5] Indeed, one of our goals is to test the robustness of such inflexibility, asking how and when dumb money might learn to give up its bad investment strategy. In general, the potential difficulty with learning that we find in our simple model provides some support to models that assume away the problem of dumb money learning.

Overall, our model suggests that the problem of learning one's *type* in financial markets – whether one is smart money or dumb money – may be as important as learning about "noisy signals" of value, but that learning one's type may be quite difficult. Learning one's type is a "first order" problem. Learning that one is the dumb money may do far more than cause one to shift one's bet a little in one direction or the other; learning one's type may lead dumb money to change the *direction* of its bet, from going long to going short, from betting on no default to betting on default, and so on. Our model also suggests which markets may be more efficient and which less efficient based on the ease or difficulty of dumb money learning. The most efficient markets in our framework will be those with less dumb

[5] Formally, traders' measures typically are not absolutely continuous with respect to one another, meaning that one may assign a zero value to some values that others assign positive value. See, for example, Harrison and Kreps (1978).

money and smaller dumb money valuation errors, which is obvious, but also those where dumb money is more self-aware of the possibility of being the dumb money and where dumb money makes relatively more accurate estimates of the amount of dumb money in the market, which may be less obvious. Less efficient markets will be those with more dumb money that makes bigger dumb money errors and is less self-aware of the possibility of being the dumb money, and where dumb money does a poor job of estimating the amount of dumb money in the market. In our framework, for example, a market with more institutional investors may be more efficient not because institutional investors are "smart money" – though some surely are – but because institutional investors may be more willing to entertain the possibility of being the dumb money than individual investors. Some markets, such as certain fixed income markets, may be more efficient not only because valuation errors are smaller when made, but because it may be easier to identify the proportion of dumb money in the market. Markets with equal proportions of dumb money and smart money may be inefficient, and remain so, since dumb money in such markets cannot as easily determine whether or not it is the dumb money.

The paper continues as follows. Section 2 presents our simple model. Section 3 presents the no-learning equilibrium. Section 4 examines the relative returns of smart money and dumb money in the no learning equilibrium. Section 5 explores the possibility of dumb money learning. Section 6 concludes.

2 The basic set-up

We consider a simple parimutuel betting market. In a parimutuel (from the French for "mutual wager") betting market, the winners split the losers stakes. Parimutuel betting has a natural interpretation as a market in states of the world, where traders place bets on the subsequent existence of states of the world and those betting on the states that actually occur receive the payoffs of all the bets in proportion to the amounts they bet, while those betting on states that do not occur receive nothing. Since the total stakes always can be normalized at 1 unit, and since all the betting proceeds on the the mutually exclusive event are paid to the winners who pick the winning event, the proportion of money bet on a given state is interpretable as a "market probability" or equilibrium state price, that is, the equilibrium probability that clears a market of traders betting on that state of the world. The parimutuel betting system has been widely studied as an information aggregation market mechanism in the context of horse racing, and the results support the assumption that the parimutuel method is an effective method of aggregating beliefs.[6]

Two types of risk neutral traders bet on the outcome of a state-space. One type is the smart money. The other type is the dumb money. Each type bets on σ_G or σ_B, a state of the world that will obtain at a future date. These are exhaustive and mutually exclusive states of the world, such as "up" or "down," "no default" or "default," and so on. Each bettor of a given type shares a probability belief for the states, where $\bar{\theta}$ is the smart money probability of σ_G, so that the smart money probability of σ_B is $(1 - \bar{\theta})$, and $\underline{\theta}$ is the dumb money probability of σ_G, so that the dumb money probability of σ_B is $(1 - \underline{\theta})$. Because the dumb money misjudges the probability of the underlying mutually exclusive states, and because the probability of one state therefore is overestimated as a result, the type of calibration error we explore is easily interpreted as a form of optimism bias where the dumb money is optimistic about the state of the world to which it attaches too much probability [see, for example, Malmendier and Tate (2005), Heaton (2002)]. Dumb money might also be what Geanakopolos (2010) calls the *natural buyer*, a trader with a strong preference for taking one

[6]See, for example, Plott, Wit and Yang (2003): "Broadly summarized, this empirical literature has established the existence of a clear monotone relationship between prices (odds) and observed relative frequencies of winning. High priced bets, the favorites that pay low odds, win regularly whereas low priced bets, the long shots that pay high odds, win occasionally. The information found revealed in naturally occurring examples is not perfect. A 'long shot bias' exists; the odds on long shots tend to be too high, but the bias is not 'large.'" On the long shot bias, see Thaler and Ziemba (1988).

or another side of the market. We assume that the smart money probability, $\bar{\theta}$, is the "true" probability of σ_G, though what is important in our analysis is only that the smart money is more closely calibrated than the dumb money to the true probability so that we can make statements about actual relative expected returns. We assume that smart money and dumb money understand that the smart money probability is the true probability.

We define $\delta \in [0, 1]$ as a "dumbness parameter," where $|\bar{\theta} - \underline{\theta}| = \delta$. The dumbness parameter measures the absolute size of the dumb money's valuation (probability) error relative to the smart money probability. When $\delta = 0$, $\bar{\theta} = \underline{\theta}$ and the dumb money is just as smart as the smart money, that is, there is no valuation error. When $\delta = 1$, then one of $\bar{\theta}$ or $\underline{\theta}$ is 0 and the other is 1. This is as far apart as it is possible for dumb money and smart money to be. The intermediate cases follow. The dumbness parameter allows us the flexibility to consider different markets where the average size of dumb money valuation errors may be different depending on the asset and the available asset valuation technology.

We assume that smart money knows with certainty that it is smart money, but that dumb money does not know whether it is smart money or dumb money. We assume that dumb money does not know that a characteristic of smart money is certainty about its smart money status. Otherwise, mere uncertainty about one's status would indicate dumb money status. An obvious extension of our approach is to introduce uncertainty among the smart money as to whether it is smart or not. The dumb money knows its own probability, which we denote $\theta^{\star}$, but does not not know whether $\theta^{\star} = \bar{\theta}$ (it is the smart money) or $\theta^{\star} = \underline{\theta}$ (it is the dumb money). Dumb money traders share a common prior belief as to the probability of being the dumb money. We denote this prior probability as $p(\theta^{\star} = \underline{\theta})$. The dumb money's prior probability that it is the smart money is then $p(\theta^{\star} = \bar{\theta}) = 1 - p(\theta^{\star} = \underline{\theta})$. Before we turn to the possibility of learning, we assume that dumb money is dogmatic about its view, that is, dumb money bets $\theta^{\star} = \underline{\theta}$ and not a weighted average reflecting prior beliefs. When we discuss learning, below, we present the case where dumb money bets a weighted average belief, given its posterior probabilities on being the dumb or smart money.

The actual proportion of dumb money in the market is ω, so the proportion of smart money is $1-\omega$. We normalize the amount bet to $\omega + (1-\omega) = 1$. Traders do not know the amount of dumb money in the market or the size of the valuation error that dumb money makes. Dumb money's belief about valuation errors and the proportion of dumb money is reflected in a joint probability density, $f(\delta, \omega)$. The dumb money's marginal distribution for the valuation errors is then $f(\delta)$, while the marginal distribution for the proportion of dumb money is $f(\omega)$. This framework allows us to accommodate a wide variety of markets where beliefs about the proportion of dumb money in the market, and about the size of dumb money valuation errors, may differ considerably. For example, many market participants consider fixed income markets to have both a higher percentage of smart money and lower valuation errors than do equity markets. With certain kinds of markets, equity markets for example, the proportion of dumb money and the size of dumb money valuation errors may differ as well depending on the segment of the market. The joint probability density $f(\delta, \omega)$ allows us to model these sorts of difference across and within markets.

3 The No-Learning Equilibrium

We first characterize the equilibrium that obtains before any learning might occur. In a classic paper, Eisenberg and Gale (1959) demonstrate that there exists a unique equilibrium of bets and market probabilities that clear a parimutuel betting market of this type.[7] Recall that $\bar{\theta}$ is the smart money probability of σ_G, while $\underline{\theta}$ is the dumb money probability of σ_G. Smart money and dumb money will bet to maximize the subjective expected values of their bets. We consider the situation where the equilibrium is that the dumb money bets on σ_G and the smart money bets on σ_G. If that is to be an equilibrium, then it must be the case

[7] Brown and Lin (2003) explore the dynamics of this sort of equilibrium. A dynamic version of the model presented here is possible along the same lines.

that both dumb money and smart money are happy with their bets and have no reason to move off of them. Thus, the following conditions hold in the equilibrium where dumb money bets on σ_G and smart money bets on σ_B:

$$(dumb\, money): \;\; max\{\underline{\theta}/\omega,\, (1-\underline{\theta})/(1-\omega)\} = \underline{\theta}/\omega$$
$$(smart\, money): \;\; max\{(1-\bar{\theta})/(1-\omega),\, \bar{\theta}/\omega\} = (1-\bar{\theta})/(1-\omega)$$

Numerators are personal probabilities of dumb money and smart money. Numerators of the form θ are bets on the occurrence of σ_G. Numerators of the form $1-\theta$ are bets on the occurrence of σ_B. Denominators are proportions bet on the events. Denominators of the form ω reflect the total bets of the dumb money, because dumb money exists in the market in proportion ω. Denominators of the form $1-\omega$ reflect the bets of the smart money, because smart money exists in the market in proportion $1-\omega$.

In equilibrium, both types of traders "bet the odds," betting on the state of the world where the ratio of their personal probability (their θ) to the equilibrium state price (that is, the proportion of money bet on that state) is the highest. This maximizes perceived expected return. When dumb money bets on σ_G and smart money bets on σ_B, the proportion of money bet on σ_G is the proportion, ω, of dumb money in the market, while the proportion of money bet on σ_B is the proportion, $1-\omega$, of smart money in the market. These proportions are then the market probabilities. The ratios $\underline{\theta}/\omega$ and $(1-\underline{\theta})/(1-\omega)$ are the subjective expectations of the dumb money of betting on σ_G and σ_B, respectively. For this to be an equilibrium where the dumb money bets on σ_G but not σ_B, the ratio $\underline{\theta}/\omega$ must be greater than the ratio $(1-\underline{\theta})/(1-\omega)$. Otherwise, the dumb money would prefer to allocate money to the available bet on σ_B at its current price. At the same prices, however, the smart money ratio $(1-\bar{\theta})/(1-\omega)$ must be greater than the smart money ratio $\bar{\theta}/\omega$. Otherwise, the smart money would prefer to allocate money to the available bet on σ_G at its current price.

Note that the equilibrium state prices – that is, the proportions bet on the two states – are *not* the smart money (true) probabilities. In this sense, the market is inefficient. State prices are not even weighted averages of dumb money and smart money beliefs; state prices are the *weights* themselves. This is a characteristic of parimutuel markets. These weights – and thus, the equilibrium state prices – clearly can differ significantly from the underlying true probability. In this sense, as in other models, dumb money traders may drive prices far from "fundamentals" (here, fundamentals are the true probabilities) in ways that capital-constrained arbitrageurs cannot correct.

Still, there is no way for smart money to further arbitrage the dumb money, because there is not enough smart money in the market to do so. In this sense, our model incorporates the concept of capital-constrained arbitrage in Shleifer and Vishny (1997). The collapse of the value of subprime mortgage-backed securities presents a potential case study in support of this result. As popular books like "The Big Short" describe [Lewis (2010)], a very small number of smart money hedge funds were able to place bets on the collapse of subprime mortgage-backed securities prices (and associated derivatives) at extremely attractive "odds" against dumb money financial institutions: "Supply [of credit default swaps on subprime mortgage securities], thanks to AIG, was virtually unlimited. The problem was demand: investors who wanted to do [the] trade. Incredibly, at this critical juncture in financial history, after which so much changed so quickly, the only constraint in the subprime mortgage market was a shortage of people willing to bet against it." [Lewis (2010, p. 80)] The credit default swaps that formed the "bets" that the hedge funds made bear a resemblance to the parimutuel betting model we study here.[8] At least *ex post*, there was too little smart money in the market for credit default swaps on subprime mortgage-backed securities.[9]

[8]On default probabilities inherent in credit default swaps, see, for example, Berndt, *et al.* (2005).

[9]A similar circumstance occurs when smart money must short sell stock to speculate against mispricing at times when an inadequate number of shares are available for shorting. See, for example, Shiller (2003, p. 97) ("The smart money who know that the stock is priced ridiculously high may well use up all the easily available shortable shares and then will be standing on the sidelines, unable to short more shares and profit

4 Relative expected returns of smart money and dumb money

Recall that dumb money (incorrectly) perceives odds on its bet on σ_G of $\underline{\theta}/\omega$, while smart money (correctly) perceives odds on its bet of $(1-\bar{\theta})/(1-\omega)$. The equilibrium conditions necessary for dumb money to bet on σ_G while smart money bets on σ_B imply that the proportion of dumb money traders is greater than the smart money probability but less than the dumb money probability: $\bar{\theta} < \omega < \underline{\theta}$. This relationship helps us describe relative expected returns of smart money and dumb money in the no-learning equilibrium.

The dumb money expects to receive $\underline{\theta}$ for total bets of ω. Since $\omega < \underline{\theta}$, this is a perceived positive expected value bet. But dumb money actually receives $(\underline{\theta} - \delta) = \bar{\theta}$ for total bets of ω, where $(\underline{\theta} - \delta) = \bar{\theta} < \omega$, a negative expected value bet. The bet transfers wealth in expectation to the smart money. Smart money expects to receive $(1-\bar{\theta})$ for a bet of $1-\omega$, where $1-\omega < (1-\bar{\theta})$, a positive expected value bet. Thus, the resulting prices are bad for dumb money and good for smart money.

If the state prices fully reflected the smart money probability, $\bar{\theta}$, then the market odds would be $1/\bar{\theta}$ and $1/(1-\bar{\theta})$ on σ_G and σ_B, respectively. Because the actual market odds on σ_G and σ_B are $1/\omega$ and $1/(1-\omega)$, respectively, the odds on σ_G (the dumb money bet) are too low, since $\bar{\theta} < \omega$, while the odds on σ_B (the smart money bet) are too high, since $(1-\omega) < (1-\bar{\theta})$. This is implied by $\bar{\theta} < \omega$, with $\bar{\theta}$ a probability and ω a proportion. The higher is the proportion of dumb money, the worse it is for the dumb money and the better it is for the smart money, holding constant $\bar{\theta} = \underline{\theta} - \delta$, the smart money probability. The larger is δ, the difference between the smart money probability and the dumb money probability, the worse it is for the dumb money and the better it is for the smart money, holding constant ω. As we might expect, the gains to being the smart money are larger the

from their knowledge.")

more dumb money is in the market and the bigger are the mistakes that the dumb money makes.[10]

Consider a simple example. Let the dumb money probability $\underline{\theta} = 0.6$, the smart money probability $\bar{\theta} = 0.3$, the proportion of dumb money in the market $\omega = 0.45$. Dumb money bets only on the state σ_G while smart money bets only on the state σ_B. This is an equilibrium because the dumb money prefers betting on σ_G since $0.6/0.45 > 0.4/0.55$, while the smart money prefers betting on σ_B since $0.3/0.45 < 0.7/0.55$. Both types of traders believe that they are making positive expected value bets, and that the opposite bet is negative expected value. Only the smart money's belief is correct, however. The dumb money expects to receive $\underline{\theta} = 0.6$ for total bets on σ_G of $\omega = 0.45$, a perceived positive expected value bet. But dumb money actually receives $\bar{\theta} = \underline{\theta} - \delta = 0.3$ for a bet of 0.45. Betting on σ_G at the market probabilities is a negative expected value bet. The bet transfers wealth in expectation to the smart money. Smart money expects to receive $1 - \bar{\theta} = 0.7$ for a bet on σ_B of $1 - \omega = 0.55$, a positive expected value bet. The dumb money's loss of 0.15 is smart money's gain. If the proportion of dumb money increased to $\omega = 0.5$, holding constant $\delta = 0.3$ $(\underline{\theta} - \bar{\theta})$, then dumb money would expect to receive $\underline{\theta} = 0.6$ for total bets on σ_G of $\omega = 0.5$, but would in fact receive -0.2, the smart money's gain betting 0.5 for an expectation of 0.2. If the proportion of dumb money is held constant at 0.45 while the smart money probability decreases to 0.25 (that is, δ increases from 0.3 to 0.35), dumb money's expected gain is 0.15, but its actual expected loss is $0.25 - 0.45 = -0.20$.

[10]For another discussion of the effect of changing ratios of smart money to dumb money, see, for example, DeLong, Shleifer, Summers and (1989, p. 691).

5 Dumb money learning

"The old-fashioned way of measuring a persons beliefs is to propose a bet and see what are the lowest odds which he will accept. This method I regard as fundamentally sound. ... [But] the proposal of a bet may inevitably alter his state of opinion."

-Frank P. Ramsey, Truth and Probability (1926).

We turn now to the possibility that dumb money might be able to learn that it is dumb money – that is, learn its type – by considering the observable state prices, knowing that those prices reflect the interaction of dumb money and smart money beliefs. Our approach is similar in spirit to Lindley (1982), who shows how one might improve on one's own probability judgments by using information inherent in the probability judgments of others. Here, the dumb money tries to improve its own probability judgment by using information inherent in the market probability. Let m be the observable proportion of money bet on σ_G and $1-m$ be the observable proportion of money bet on σ_B. Recall that dumb money does not know if it is the smart money or the dumb money. Dumb money knows, however, that the observable proportion m is one of two values depending on whether it is the dumb money or the smart money. The proportion m is ω if the dumb money is betting on σ_G, but it is $1-\omega$ if the smart money is betting on σ_G.

The possibility of learning arises because dumb money can ask whether, given m, it is more likely that $\omega = m$ (that is, more likely that it is the dumb money betting on σ_G and the smart money betting on σ_B) or (noting that $m = 1-\omega \implies \omega = 1-m$) more likely that $\omega = 1-m$ (that is, more likely that it is the smart money betting on σ_G and the dumb money betting on σ_B). Essentially, the dumb money can observe the proportion of investors betting in the direction that it is betting. The dumb money understands that one side of the betting is dumb money, while the other side of the betting is smart money. This is the case since the odds are not even, which they would be if $\delta = 0$, that is, if there was no dumb money. The question for the dumb money is whether it is the dumb money or the smart money. The answer to that question must depend on the dumb money's beliefs about those proportions. Thus, learning depends on having a good sense of how much dumb money is in the market as a proportion of all money in the market. This is a type of learning different from, for example, learning from subsequent returns, a form of learning that may take a very long time and require a relatively stationary environment.

The dumb money's Bayesian posterior odds are a natural expression of learning in this sense. Posterior odds follow directly from personal probabilistic belief. If someone's prior odds (odds prior to observing additional data, here m) in favor of some hypothesis are 1.5, that means the person believes the probability of the hypothesis to be 1.5 times the probability of a mutually exclusive alternative hypothesis under consideration prior to seeing the data. If the likelihood ratio of the data is, for example, 2, then the posterior odds are $2 \times 1.5 = 3$. That is, after seeing the data, the person now believes the probability of the hypothesis to be 3 times the probability of the mutually exclusive alternative hypothesis. Here, the mutually exclusive hypotheses are being dumb money or smart money, respectively, and the data is the market price, m, on σ_G. The dumb money's posterior odds that it is the dumb money are thus:

$$\frac{p(\theta^\star = \underline{\theta}|m)}{p(\theta^\star = \bar{\theta}|m)} = \frac{p(m|\theta^\star = \underline{\theta})}{p(m|\theta^\star = \bar{\theta})} \frac{p(\theta^\star = \underline{\theta})}{p(\theta^\star = \bar{\theta})} = \frac{p(\omega = m)}{p(\omega = 1 - m)} \frac{p(\theta^\star = \underline{\theta})}{p(\theta^\star = \bar{\theta})}$$

The Bayesian posterior odds are the likelihood ratio times the prior odds. The likelihood ratio is the probability that $\omega = m$ divided by the probability that $\omega = 1 - m$, where $f(\omega)$, the marginal distribution for ω from $f(\delta, \omega)$. This is the case because the probability of observing proportion m *given that the trader is the dumb money* is the probability that $\omega = m$, while the probability of observing proportion m *given that the trader is the smart money* is the probability that $\omega = 1 - m$. The Bayesian posterior odds of being the dumb money are higher when it is more probable, under $f(\omega)$, that $\omega = m$ than that $\omega = 1 - m$. The prior odds are simply the prior probability (in the mind of the dumb money trader) that it is the dumb money divided by the prior probability (in the mind of the same dumb money trader) that it is the smart money. The prior odds measure the willingness of the dumb money to believe, *a priori*, that it is the dumb money and not the smart money.

The posterior odds allow the following set of possible inferences:

$$\frac{p(\theta^\star = \underline{\theta}|m)}{p(\theta^\star = \bar{\theta}|m)} > 1 \quad \textit{learning (I am probably dumb money)}$$

$$\frac{p(\theta^\star = \underline{\theta}|m)}{p(\theta^\star = \bar{\theta}|m)} \leq 1 \quad \textit{no learning (I am probably smart money)}$$

If the posterior odds on being the dumb money are greater than 1, then the dumb money concludes it is more likely to be dumb money. The posterior odds are higher the more likely it seems to the dumb money that the observed proportion that includes its bet is the proportion of dumb money in the market, and the more likely that the dumb money believes itself likely to be the dumb money. If the posterior odds on being the dumb money are less than 1, the dumb money concludes it is more likely to be smart money. Returning to our example where $\underline{\theta} = 0.6$, $\bar{\theta} = 0.3$, and $\omega = 0.45$, suppose that under the the dumb money's prior distribution for ω, $f(\omega)$, values of $\omega < 0.50$ are twice as likely as values of $\omega > 0.50$, that is, the dumb money believes that a little dumb money in the market is more likely than a lot of dumb money. Suppose further that the dumb money has even prior odds (that is, 50/50) on being the dumb money. In this case, the dumb money will calculate

$$\frac{p(\theta^\star = \underline{\theta}|0.45)}{p(\theta^\star = \bar{\theta}|0.45)} = 2 > 1 \;\; \textit{learning (I am probably dumb money)}$$

We can see that there are two determinants of the ease or difficulty of dumb money learning in this simple model. First, dumb money learning is harder when the probability that $\omega = m$ is close to the probability that $\omega = 1 - m$. That is, it is hard for dumb money to infer that the proportion it is betting with is the dumb money if the proportion and its complement are both more or less equally likely. In general, this will depend on dumb money's prior beliefs about the proportion of dumb money in the market, but an extreme example that does not depend on the shape of the $f(\omega)$ distribution is when $m = 1 - m = 0.5$, that is, when there an equal proportion of dumb money and smart money. In that case, dumb money cannot learn from the market price, because, whatever is the shape of $f(\omega)$, $p(\omega = m = 0.5) = p(\omega = 1 - m = 0.5)$. While we do not model it here, this fact suggests the interesting possibility – appropriate for future research – that an increase in the proportion of smart money toward 0.5, if accurately perceived by the dumb money, might actually make learning more difficult. While the increase in smart money would, in general, increase the quality of the price without learning, it remains an interesting question whether the effect of the increase toward 0.5 on making learning harder would leave a market such as we study here better off or worse off on balance.

Second, dumb money learning is harder when dumb money is tilted too far in its prior beliefs in favor of being the smart money. Being open to the possibility of being the dumb money requires a psychologically difficult self-evaluation. Even where the dumb money believes that there is a preponderance of dumb money in the market, dumb money may ignore such "base rates" in forming their prior beliefs [see Kahneman and Tversky (1974)], continuing to believe that they probably are the smart money. Even a uniform prior on being dumb money might be hard to muster, psychologically speaking. Few people may be able to believe that it is as likely as not that they are the dumb money. Past trading success may exacerbate the problem, even creating dumb money: Gervais and Odean (2001) show how trading successes can lead to overconfidence in one's ability. Nevertheless, it may be easier to muster this sort of self-criticism in some markets rather than others. In markets, such as derivatives markets and certain fixed income markets, where pricing is known to be highly quantitative, those lacking such skills may find it easier to assume that they may be the dumb money in those markets. By contrast, in markets less subject to highly quantitative, arbitrage-based pricing, such as in equity markets, it may be more difficult for dumb money to believe that its views are any less well-shaped than others, especially since reliable modeling is more difficult in such environments and those without highly quantitative strategies often do better than those employing them.

Third, it is not enough that the dumb money may determine that it is more likely dumb money than smart money. Dumb money learning must be strong enough to allow an updating of dumb money's beliefs (of $\theta^\star$ given m) to a value that causes the dumb money to wish to revise its current bet. The dumb money can update $\theta^\star$ given m to

$$\theta^\star|m = p(\theta^\star = \bar{\theta}|m)\theta^\star + p(\theta^\star = \underline{\theta}|m)(\theta^\star - \delta^\star)$$

The updated $\theta^\star$ is a weighted average of two possibilities: the trader is the smart money or the trader is the dumb money. If the trader is the smart money, then $\theta^\star = \bar{\theta}$. The probability that the trader is the smart money, given m, is $p(\theta^\star = \bar{\theta}|m)$. If the trader is the dumb money, then $\theta^\star = \underline{\theta}$. The probability that the trader is the dumb money, given m, is $p(\theta^\star = \underline{\theta}|m)$. If the trader is the dumb money, and $\theta^\star = \underline{\theta}$, the the trader knows that his probability is incorrect by amount δ. The trader does not know δ, but does have a marginal distribution, $f(\delta)$, and can estimate $\bar{\theta}$ by $\theta^\star - \delta^\star$, where $\delta^\star$, where $\delta^\star$ is the dumb money's estimate of δ given its probability distribution, $f(\delta)$. Recall that the equilibrium conditions when dumb money bets on σ_G and the smart money bets on σ_B imply that the proportion of dumb money traders is greater than the smart money probability but less than the probability of the dumb money probability: $\bar{\theta} < \omega < \underline{\theta}$. If $\bar{\theta} < \omega < \theta^\star | m$, the dumb money will not want to change its bet despite believing that it probably is the dumb money. The dumb money is more likely to want to change its bet the larger is $p(\theta^\star = \underline{\theta} | m)$ and $\delta^\star$, that is, the more likely it is, given m, that it is the dumb money, and the larger is the expected valuation error given that one is the dumb money.

What happens if dumb money does decide to abandon its beliefs? In order for a price impact to occur, there must be additional betting. Recall that the initial equilibrium probabilities are ω on σ_G and $1-\omega$ on σ_B, respectively. At these market probabilities, the dumb money (inaccurately) perceived odds $\underline{\theta}/\omega > (1-\underline{\theta})/(1-\omega)$, while smart money (accurately) perceived odds $(1-\bar{\theta})/(1-\omega) > \bar{\theta}/\omega$. When dumb money learns fully, it knows it is misperceiving the market probabilities, that $\underline{\theta} = \theta^\star$, and that $\bar{\theta} = \theta^\star - \delta$. Dumb money understands that if all traders held the correct belief, then the market odds must be such as to leave the bettors (all of whom then share the same beliefs) indifferent between the bets on σ_G and σ_B. Dumb money knows that it has not correctly estimated δ unless and until it is indifferent between σ_G and σ_B. Dumb money's knowledge that it has not correctly estimated δ unless and until it is indifferent between the available bets acts as a sort of "reverse lemons" problem where only high quality beliefs survive [compare Akerlof (1970)]. However, this happens only when dumb money has learned sufficiently to want to change its bet.

6 Conclusion

We explore a model of smart money and dumb money where dumb money tries to learn its type from equilibrium state prices. Our model is related in spirit to an early literature that studied whether and how prices reflect the different information or different "signals" of an asset's value that different investors may have. Implicit in the structure of those early models was the assumption that investors understand that each has access to only some of the valuation-relevant data. Each is willing to learn what information other investors are holding. Depending on the model, the price may allow them to do so, revealing all [see, for example, Grossman (1976, 1978)] or some [see, for example, Figlewski (1978)] of the valuation-relevant data that investors collectively hold. Yet, so long as poorly informed investors continue to exist in the market in sufficient proportion, prices may continue to reflect their poor beliefs [Feiger (1978)]. Our model considers the inverse problem of identifying beliefs reflected in the equilibrium. Price can reveal something about whether one's beliefs are more likely smart money beliefs or dumb money beliefs. In our model, dumb money traders know this, and that makes learning possible, if not guaranteed.

Our paper contributes to the literature on the possibility that irrational traders may affect asset prices.[11] There are two serious objections to the assertion that irrational traders can influence prices persistently. First, there is the objection that rational traders will exploit irrational traders and drive irrationality-induced price deviations to zero (see Friedman, 1953; Fama, 1965) or near-zero (see Grossman and Stiglitz, 1980) levels. We may call this the "arbitrage objection." To rebut the arbitrage objection, researchers have argued that real-world arbitrage is more difficult than researchers recognize [Shleifer and Vishny (1997), Barberis and Thaler (2003)]. In our model, as in Shleifer and Vishny (1997), a limited supply

[11] A comprehensive early review is Barberis and Thaler (2003). Seminal papers in the literature include DeLong, Shleifer, Summers and Waldmann (1989, 1990a, 1990b, 1991), Campbell and Kyle (1993), Shleifer and Vishny (1997), Daniel, Hirshleifer, and Subrahmanyam (1998), Odean (1998), Hong and Stein (1998), Barberis, Shleifer and Vishny (1998).

of smart money limits arbitrage. Second, there is the objection that irrational traders can learn that they are being irrational and change their behavior. As DeLong, et al. (1990a) observe, "An important objection to this approach is that [irrational traders] are really dumb: they do not realize how much money they lose.... Why don't [irrational] traders...learn that they are making mistakes...?" We may call this the "learning objection." While researchers have paid considerable attention to rebutting the arbitrage objection, the learning objection remains largely unexplored. Nevertheless, it remains a significant hurdle to the viability of predictions that irrational traders may affect asset prices. Our paper addresses an aspect of the learning objection by exploring the problem of dumb money learning.

In an episode of the sitcom, "Seinfeld" titled "The Opposite," George Costanza complains, "My life is the opposite of everything I want it to be. Every instinct I have, in every aspect of life, be it something to wear, something to eat ... It's all been wrong." Jerry Seinfeld convinces George to do the opposite of everything he's been doing, and, for the rest of the episode, George enjoys great success. Dumb money is the George Costanza of the capital market. Occasionally, however, even if only for an episode or two, it may learn to "do the opposite" and behave like smart money.

7 References

Akerlof, G.A. (1970). The Market for "Lemons": Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, 84, 488-500.

Barberis, N., A. Sheifler and R. Vishny. (1998). A Model of Investor Sentiment. *Journal of Financial Economics*, 49, 307-343.

Barberis, N. and R. Thaler. (2003). A survey of behavioral finance, in: G. M. Constantinides, M. Harris, and R. M. Stulz (eds.), Handbook of the Economics of Finance, Elsevier, First edition, Volume 1, Chapter 18, pp. 1053-1128.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz. (2005). Measuring Default Risk Premia from Default Swap Rates and EDFs. *Bank for International Settlements, BIS Working Paper No. 173*, (March).

Black, F. (1986). Noise. *Journal of Finance*, 41, 529-543.

Brown, L.D. and Y.Lin. (2003). Racetrack Betting and Consensus of Subjective Probabilities. *Statistics & Probability Letters*, 62, 175-87.

J. Campbell and A. Kyle. (1993). Smart Money, Noise Trading and Stock Price Behavior. *The Review of Economic Studies*, 60, 1-34.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. (1998). Investor Psychology and Security Market Under- and Overreactions. *Journal of Finance*, 53(6), 1839-1885.

DeBondt, W. and R. Thaler. (1990). Do Security Analysts Overreact? *The American Economic Review*, 80(2), 52-57

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. (1989). The Size and Incidence of the Losses from Noise Trading. *Journal of Finance*, 44(3), 681-696.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. (1990a). Positive feedback investment strategies and destabilizing rational speculation. *Journal of Finance,* 45(2), 379-395.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. (1990b). Noise trader risk in financial markets. *Journal of Political Economy*, 98, 703-738.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. (1991). The survival of noise traders in financial markets. *Journal of Business*, 64, 1-19.

Eisenberg, E. and D. Gale. (1959). Consensus of Subjective Probabilities: The Pari-mutuel Method. *Annals of Mathematical Statistics*, 30, 165-168.

Fama, E. F. (1965). The behavior of stock market prices. *Journal of Business*, 38, 34-105.

Feiger, George M. (1978). Divergent Rational Expectations Equilibrium in a Dynamic Model of a Futures Market. *Journal of Economic Theory*, 17, 164-178.

Friedman, M. (1953). The case for flexible exchange rates. In: *Essays in Positive Economics*, University of Chicago Press, Chicago.

Geanakoplos, J. (2010). The Leverage Cycle. *Cowles Foundation Paper No. 1304*, working paper.

Gervais, S. and T. Odean. (2001). Learning to be overconfident. *The Review of Financial Studies*, 14(1), 1-27.

Grossman, S. (1976). On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information. *Journal of Finance*, 31, 573-85.

Grossman, S. (1978). Further Results on the Informational Efficiency of Competitive Stock Markets. *Journal of Economic Theory*, 18, 81-101.

Grossman, S. J. and J.E. Stiglitz. (1980). On the impossibility of informationally efficient markets. *American Economic Review*, 70, 393-408.

Harris, M. and A. Raviv. (1993). Differences of Opinion make a Horse Race. *Review of Financial Studies*, 6(3), 473-506.

Harrison, J.M and D. Kreps. (1978). Speculative Behaviour in a Stock market with Heterogeneous Expectations. *Quarterly Journal of Economics*, 92(2), 323-336.

Heaton, J.B. (2002). Managerial Optimism and Corporate Finance. *Financial Management.*

Hellwig, M. F. (1980). On the Aggregation of Information in Competitive Markets. *Journal of Economic Theory*, 22, 477-98.

Hong, H., and J. C. Stein. (1999). A Unified Theory of Underreaction, Momentum Trading and Overreaction in Asset Markets. *Journal of Finance*, 54(6), 2143-84.

Hong, H., and J. C. Stein. (2003). Differences of Opinion, Short-Sales Constraints, and Market Crashes. *Review of Financial Studies*, 16, 487-525.

Kandel, E., and N. D. Pearson. (1995). Differential Interpretation of Public Signals and Trade in Speculative Markets. *Journal of Political Economy*, 103, 831-872.

M. Kelly. (1997). Do Noise Traders Influence Stock Prices? textitJournal of Money, Credit and Banking], 29, 351-363.

Keswani, A. and D. Stolin. (2008). Which Money is Smart? Mutual Fund Buys and Sells of Individual and Institutional Investors. *Journal of Finance*, 63, 85-118.

Kyle, A. S. and F. A. Wang. (1997). Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test? *Journal of Finance*, 52(5), 2073-90.

Lakonishok, J., A. Shleifer and R. Vishny. (1992). The Structure and Performance of the Money Management Industry. *Brookings Papers on Economic Activity: Microeconomics*, 339-391.

Lindley, D.V. (1982). The Improvement of Probability Judgments. *Journal of Royal Statistical Society, A*, 145(1), 117-26.

Malmendier, U. and G. Tate. (2005). CEO Overconfidence and Corporate Investment. *Journal of Finance*, 60, 2661-2700.

Plott, C.R., J.Wit and W.C. Yang. (2003). Parimutuel Betting Markets as Information Aggregation Devices: Experimental Results. *Economic Theory*, 22, 311-351.

Ramsey, F.P. (1926). Truth and Probability. Reprinted in *Studies in Subjective Probability*, Kyburg and Smokler (1980) (eds).

Scheinkman, J.A. and W. Xiong. (2003). Overconfidence and Speculative Bubbles. *The Journal of Political Economy*, 111(6), 1183-1220.

Shiller, R. (2003.) From Efficient Markets Theory to Behavioral Finance. *The Journal of Economic Perspectives*, 17, 83-104.

Shleifer, A. and Robert W. Vishny. (1997). The Limits of Arbitrage. *Journal of Finance*, 52(1), 35-55.

Thaler, R. and W. Ziemba. (1988). Anomalies-Parimutuel Betting Markets: Racetracks and Lotteries. *The Journal of Economic Perspectives*, 2, 161-74.

Varian, Hal R. (1985). Divergence of Opinion in Complete Markets: A Note. *Journal of Finance*, 40(1), 309-317.