Stock-Based Compensation and CEO (Dis)Incentives *

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Abstract

The use of stock-based compensation, motivated as the solution to agency problems between shareholders and managers, has increased dramatically since the early 1990’s. We show that in a dynamic rational expectations model with asymmetric information, stock-based compensation not only induces managers to exert costly effort, but it also induces them to conceal bad news about future growth options, and choose sub-optimal investment policies to support the pretense. This leads to a severe overvaluation and a subsequent crash in the stock price. Our model produces many predictions that are consistent with the empirical evidence, and are relevant to understanding the current crisis.

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1 Introduction

While a large theoretical literature views stock-based compensation as a solution to an agency problem between shareholders and managers, there is a growing body of empirical evidence that shows it may lead to earnings management, misreporting, and outright fraudulent behavior. Does stock-based compensation amplify the tension between the incentives of managers and shareholders instead of aligning them?

The ongoing global financial crisis has brought forth renewed concerns about the adverse incentives that stock-based compensation may encourage. Many managers of the recently troubled financial institutions were amongst the highest-paid executives in the U.S., with huge equity-based personal profits realized at the time when their firms’ stock prices were high.1 While the subsequent sharp decline of their firms’ stock prices may be due to exogenous systemic shocks to the economy, it is an important open question whether the size of their stock-based compensation may have induced CEOs to willingly drive prices up in full awareness of the impending crash. Indeed, similar concerns about these possible perverse effects of stock-based compensations on CEOs’ behavior were raised after the burst of the Dot.Com bubble. As governments across the globe are preparing a new wave of sweeping regulation, it is important to study the incentives induced by stock-based compensation, as well as the trade-offs involved in any decision that may affect the stock component in executives’ compensation packages.2

In this paper, we show formally that while stock-based compensation induces managers to exert costly effort to increase the firms’ investment opportunities, it also induces incentives for sub-optimal investment policies designed to hide bad news about the firm long term growth. We analyze a dynamic rational expectations equilibrium model, and identify conditions under which stock-based executive compensation leads to misreporting, suboptimal investment, run-up and a subsequent sharp decline in equity prices.

More specifically, we study a hidden action model of a firm that is run by a CEO,  

\footnote{For instance, Richard Fuld, the former CEO of bankrupt Lehman Brothers was the 14th highest-paid CEO in 2007, with $71.92 million in compensation including more than $40 million from realized stock options. Angelo Mozilo, the CEO of Countrywide Financial Corp was the 3rd highest-paid CEO in 2007, making $125 million. Mozilo’s total compensation from July 1, 2003 to June 30, 2008 mounted to $470 million, out of which $378 million were proceeds from stock sales. Similarly, during the same period, James E. Cayne, the former CEO and Chairman of the Board of Bear Stearns took home $163 million with $111 million as proceeds from exercising options and selling stocks.}

\footnote{For instance, in January 2009 the U.S. Government has imposed further restrictions on the non-performance-related component of the compensation packages. In light of our results, it seems that the administration is moving in the wrong direction.}
whose compensation is stock-based. The firm initially experiences high growth in investment opportunities and the CEO must invest intensively to exploit the growth options. The key feature of our model is that at a random point in time the growth of the firm’s investment opportunities slows down. The CEO is able to postpone the expected time of this decline by exercising costly effort. But when the investment opportunities growth does inevitably slow down, the investment policy of the firm should change appropriately. We assume that while the CEO privately observes the slowdown in the growth rate, shareholders are oblivious to it. Moreover, they do not observe investments, but base their valuation only on dividend payouts. When investment opportunities decline, the CEO has two options: revealing the decline in investment opportunities to shareholders, or behaving as if nothing had happened. Revealing the decline to shareholders leads to an immediate decline in the stock price. If the CEO chooses not to report the change in the business environment of the firm, the stock price does not fall, as the outside investors have no way of deducing this event, and equity becomes overvalued. In order to behave as if nothing has changed the CEO must design a sub-optimal investment strategy to maintain the pretense. We assume that as long as the reported dividends over time are consistent with the high growth rate, the CEO keeps her job. Any deviation that is not at the time of a declared drop of the growth rate leads to the CEO’s dismissal.

We show that when a CEO is compensated based on stock, and, the range of possible growth rates is large, there is a pooling Nash equilibrium for most parameter values. In this equilibrium, the CEO of a firm that experienced a decline in the growth rate of investment opportunities follows a suboptimal investment policy designed to maintain the pretense that investment opportunities are still strong. We are able to solve for the dynamic pooling equilibrium in closed form and fully characterize the CEO’s investment strategy. In particular, since the CEO is interested in keeping a high growth profile for as long as possible, initially he invests in negative NPV projects as storage of cash, and later on foregoes positive NPV projects in order to meet rapidly-growing demand for dividends. In both cases, he destroys value. Since this strategy cannot be kept forever, at some point the firm experiences a cash shortfall, the true state is revealed and the stock price sharply declines as the firm needs to recapitalize.

Our model highlights the tension that stock-based compensation creates. While the common wisdom of hidden action models is to align the manager’s incentive with those of investors by tying her compensation to the stock price, stock-price-based compensation may lead the manager to invest suboptimally and destroy value. The trade off is made apparent by the fact that for most reasonable parameter values, and especially for medium to
high growth companies, stock-based compensation indeed induces an equilibrium with high effort but also leads to suboptimal investment strategy. That is, the cost of inducing high managerial effort ex-ante comes from the suboptimal investment policy after the slowdown in investment opportunities.

While our analysis focuses on a linear stock-based compensation contract, we also consider alternative compensation schemes that are widely used in the industry. We analyze: (i) flat wage contracts; (ii) deferred compensation; (iii) option-based compensation; (iv) bonuses, and (v) claw back clauses. We discuss the pros and cons of each of these contracts and show that none of these commonly-used compensation schemes is efficient both \textit{ex-ante} in inducing managerial effort, and \textit{ex-post} in forcing the manager to reveal the true state of the firm.

We then analyze and propose an optimal managerial compensation contract. We show that this double incentive problem (i.e. inducing high effort and revelation) can often be overcome by a firm-specific compensation scheme characterized by a combination of stock-based compensation and a bonus awarded to the CEO upon revelation of the bad news about long term growth. Indeed, while stock-based compensation is necessary to induce the manager to exert costly effort and increase investment opportunities, it also implicitly punishes the CEO for truth telling, as the stock price would sharply decline. The bonus is necessary then to compensate the loss in the CEO stock holdings. However, we also show that only a bonus contract would not work ex ante, because while it induces truth telling, it also provides an incentive not to exert effort, as this behavior anticipates the time of the bonus. The stock-based component ensures high effort.

An important implication of the model, however, is that different types of firms need to put in place different levels of stocks in the compensation package. Specifically, we find that the CEO's compensation package of growth firms, that is, those with high investment opportunities growth and high return on capital, should have only little stock-price sensitivity. Indeed, a calibration of the model shows that for most firms the stock-based compensation component should never be above 50% of the total CEO compensation in order to induce truth revelation and optimal investments. Similarly, for most firms with medium-high return on investment the stock-based compensation component should be strictly positive to induce high effort. These results suggest that policymakers and firms' boards of directors should be careful with both an outright ban of stock-based compensation as well as with too much reliance on it.

Our model's predictions are consistent with the empirical evidence documenting that
stock-based executive compensation is associated with earnings management, misreporting and restatements of financial reports, and outright fraudulent accounting (e.g. Healy (1985), Beneish (1999), Bergstresser and Philippon (2006), Ke (2005), Burns and Kedia (2006), Kedia and Philippon (2006), and Johnson et al (2009).) In fact, our model’s predictions go beyond the issue of earnings manipulation and restatements, as we focus on the entire investment behavior of the firm over the long haul. Similarly, our model’s predictions are consistent with the survey results of Graham et al. (2004), according to which most managers state that they would forego a positive NPV project if it causes them to miss the earnings target, with high tech firms much more likely to do so. High tech firms are also much more likely to cut R&D and other discretionary spending to meet the target. On the same note, our model’s prediction are also consistent with Skinner and Sloan (2002), who show that the decline in firm value following a failure to meet the analysts’ forecasts is more pronounced in high growth firms.

While our paper is related to the literature on managerial “short-termism” and myopic corporate behavior (e.g. Stein (1989), Bebchuk and Stole (1993), Jensen (2005), Aghion and Stein (2007)) our results do not rely on behavioral biases, and apply to a wider range of firms. In terms of assumptions, our paper bears some similarities to Miller and Rock (1985) who study the effects of dividends announcements on the value of the firm. Similarly to Eisfeldt and Rampini (2007) and Inderst and Mueller (2006), we also assume that the CEO has a significant informational advantage over investors, but differently from them we focus on investors’ beliefs about future growth rates and their effect on the incentives of the managers. Our paper is also related to Bolton, Scheinkman and Xiong (2006), Goldman and Slezak (2006), and Kumar and Langberg (2007), but it differs from them as we emphasize the importance of firms’ long term growth options, which have a strong “multiplier” impact on the stock price and thus on CEO incentives to hide any worsening of investment opportunity growth. Overall, these papers complement each other, and conclude that contrary to the traditional prescriptions, providing managers with large high-powered short-run incentives based on the stock price may be dangerous, because the stock price accumulates the beliefs about the uncertain future. The manager can use deceptive or even fraudulent practices that destroy value to maintain the pretense of a bright tomorrow.

Finally, our paper is also related to the recent literature on dynamic contracting under asymmetric information (e.g. Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007)). These papers focus on the properties of the optimal contract that induces full revelation, such that there is no information asymmetry in equilibrium. While we also find the contract that induces full revelation and the first best, the main focus of our
paper is to study the properties of the dynamic pooling equilibrium in which the manager does not reveal the true state, which we believe to be widespread. This analysis is complicated by the feedback effect that the equilibrium price dynamics exerts on the CEO compensation and thus on his optimal intertemporal investment strategy, which in turn affects the equilibrium price dynamics itself through shareholders beliefs. The solution of this fixed point problem is absent in other dynamic contracting models, but is at the heart of our paper.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 presents the disincentives that stock-based compensation create when there is information asymmetry. Section 4 considers alternative compensation schemes. Section 5 provides quantitative implications of our model. We discuss the broader implications of our results on Section 6.

2 The Model

We consider a firm run by a manager who a) chooses an unobservable effort level that affects the growth opportunities of the firm; b) privately observes the realization of the growth opportunities, and decides whether to report them to the public; and c) chooses the investment strategy of the firm that is consistent with his public announcement. Our analysis focuses on the manager’s tradeoff between incentives to exert costly effort to maintain a high growth of investment opportunities, and his incentives to reveal to shareholders when investment opportunities growth slows down.

We start by defining firm’s investment opportunities that are described by the following production technology: given the stock of capital $K_t$, firm’s operating profit (output) $Y_t$ is:

$$
Y_t = \begin{cases} 
zK_t & \text{if } K_t \leq J_t 
zJ_t & \text{if } K_t > J_t 
\end{cases}
$$

(1)

where $z$ is the rate of return on capital, and $J_t$ defines an upper bound on the amount of deployable productive capital that depends on the technology, operating costs, demand and so on. The Leontief technology specification (1) implies constant return to scale up to the upper bound $J_t$, and then zero return for $K_t > J_t$. This simple specification of a decreasing return to scale technology allows us to conveniently model the evolution of the growth rate in profitable investment opportunities, which serves as the driving force of our model. The existing stock of capital depreciates at the rate of $\delta$. 

5
We assume that the upper bound \( J_t \) in (1) grows according to
\[
\frac{dJ_t}{dt} = \tilde{g}J_t
\] (2)
where \( \tilde{g} \) is a stochastic variable described below. The combination of (1) and (2) yields growing investment opportunities of the firm. Since the technology displays constant returns to scale up to \( J_t \), it is optimal to keep the capital at the level \( J_t \) if these investments are profitable, which we assume throughout. Figure 1 illustrates investment opportunities growth.

We set time \( t = 0 \) to be the point when shareholders know firm’s capital \( K_0 \), as well as the current growth rate of investment opportunities \( \tilde{g} = G \). One can think of \( t = 0 \) as the time of the firm’s IPO, SEO or of a major corporate event, such as a reorganization, that has elicited much information about the state of the firm. This is mostly a technical simplifying assumption, as we believe that all the insights would remain if the market has a system of beliefs over the initial capital and the growth rate.

Firms tend to experience infrequent changes in their growth rates. We are interested in the declines of the growth rate, as this is the time when the manager faces a hard decision of whether to reveal the bad news to the public. Any firm may experience such a decline, thus our analysis apply to a wide variety of scenarios. We model the stochastic decline in investment opportunities growth as a discrete shift from the high growth regime, \( \tilde{g} = G \), to a low growth regime, \( \tilde{g} = g(<G) \), that occurs at a random time \( \tau^* \). Formally,
\[
\tilde{g} = \begin{cases} 
G & \text{for } t < \tau^* \\
g & \text{for } t \geq \tau^* 
\end{cases}
\] (3)

We assume that \( \tau^* \) is exponentially distributed with parameter \( \lambda \):
\[
f(\tau^*) = \lambda e^{-\lambda \tau^*}.
\]

At every instant \( dt \), there is a constant probability \( \lambda dt \) that a shift from \( G \) to \( g \) occurs.

We assume that manager’s actions affect the time at which the decline occurs. After all, CEOs must actively search for investment opportunities, monitor markets and internal developments, all of which require time and effort. In our model, higher effort translates into a smaller probability to shift to a lower growth. More specifically, the manager can choose to exert high or low effort, \( e^H > e^L \). Choosing higher effort increases the expected time \( \tau^* \) at which the investment opportunities growth decline. Formally:
\[
\lambda^H \equiv \lambda(e^H) < \lambda(e^L) \equiv \lambda^L \iff E[\tau^*|e^H] > E[\tau^*|e^L]
\]
The cost of high effort is positive, whereas the cost of low effort is normalized to zero:

\[ c(e) \in \{c^H, c^L\}, \text{ s.t. } c^H > c^L = 0. \]

To keep the analysis simple, we assume linear preferences of the manager:

\[ U_t = E_t \left[ \int_t^T e^{-\beta(u-t)} w_u \left[ 1 - c(e) \right] du \right], \tag{4} \]

where \( w_u \) is the periodic wage of the CEO, and \( \beta \) is his discount rate.\(^3\) We specify a cost of effort in a multiplicative fashion, which allows us to preserve scale invariance. Economically, this assumption implies a complementarity between the wage and “leisure” \([1 - c(e)]\), a relatively standard assumption in macroeconomics. That is, effort is costly exactly because it does not allow the CEO to enjoy his pay \( w_t \) as much as possible.

In (4), \( T \) is the time at which the manager leaves the firm, possibly \( T = \infty \).\(^4\) However, the departing date \( T \) may occur earlier, as the manager may be fired if the shareholders learn that he has followed a suboptimal investment strategy.

We must make several technical assumptions to keep the model from diverging, degenerating or becoming intractable. First, we assume for tractability that manager’s decisions are firm-specific, and thus do not affect the systematic risk of the stock and its cost of capital, which we denote \( r \). Then we must assume that

\[ z > r + \delta, \tag{5} \]

that is, the return on capital \( z \) is sufficiently high to compensate for the cost of capital \( r \) and depreciation \( \delta \). This assumption implies that it is economically optimal for investors to provide capital to the company and invest up to its fullest potential, as determined by the Leontief technology described in (1).

To ensure a finite value of the firm’s stock price, we must assume that

\[ r > G - \lambda^H \text{ and } r > g. \]

We further assume that \( \beta > G \), which is required to keep the total utility of the manager finite. We also assume that \( \beta \geq r \), i.e. the manager has a higher discount rate than fully diversified investors.\(^5\)

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\(^3\)Our results also hold with risk averse managers, although the analytical tractability is lost.

\(^4\)\( T = \infty \) also corresponds to the case in which there is a constant probability that the manager leaves the firm or dies, whose intensity is then included in the discount rate \( \beta \), as in Blanchard (1985).

\(^5\)It is intuitive that the discount rate of an individual, \( \beta \), is larger than the discount rate of shareholders: for instance, a manager may be less diversified than the market, or \( \beta \) may reflects some probability of leaving the firm early or death, or simply a shorter horizon than the market itself.
While we assume that the market does not observe the investments and the capital stock, there is a limit to what the firm can conceal. We model this by assuming that to remain productive, the firm must maintain a minimum level of capital $K_t \geq K*$, where $K*$ is exogenously specified, and for simplicity it depends on the optimal size of the firm:

$$K_t \geq K* = \xi J_t \quad \text{for} \quad 0 \leq \xi < 1$$

(6)

where $J_t$ is defined in (2). This is a purely technical assumption and $\xi$ is a free parameter.

Finally, we assume for simplicity that the firm does not retain earnings, thus the dividend rate equals its operating profit $Y_t$ derived from its stock of capital, $K_t$, less the investment it chooses to make, $I_t$. Given the technology in (1), the dividend rate is

$$D_t = z \min(K_t, J_t) - I_t.$$  

(7)

2.1 Investments and Stock Prices in the First Best

To build intuition, it is useful to derive first the optimal investment and the stock price dynamics under the first best. To maximize the firm value the manager must invest to its fullest potential, that is, to keep $K_t = J_t$ for all $t$. We solve for the investment rate $I_t$ that ensures this constraint is satisfied, and we obtain the following:

**Proposition 1:** The first-best optimal investment policy given $\lambda$ is

$$I_t = \begin{cases} 
(G + \delta)e^{Gt} & \text{for} \quad t < \tau* \\
(g + \delta)e^{Gr^*+g(t-\tau^*)} & \text{for} \quad t \geq \tau* 
\end{cases}$$

(8)

The dividend stream of a firm that fully invests is given by:

$$D_t = zK_t - I_t = \begin{cases} 
D^G_t = (z - G - \delta)e^{Gt} & \text{for} \quad t < \tau^* \\
D^\theta_t = (z - g - \delta)e^{Gr^*+g(t-\tau^*)} & \text{for} \quad t \geq \tau^* .
\end{cases}$$

(9)

The top panel of Figure 2 plots the dynamics of the optimal dividend path for a firm with a high growth in investment opportunities until $\tau^*$, and a low growth afterwards. As the figure shows, the slowdown in the investment opportunities requires a decline in the investment rate, which initially increases the dividend payout rate: $D^\theta_\tau - D^G_\tau = (G - g)e^{G\tau^*}$.

Given the above assumptions, the dividend rate is always positive. Moreover, from (9) the dividend growth rate equals the growth rate of investment opportunities, $\tilde{g}$. Given the dividend profile, the price of the stock follows:
Proposition 2: Given \( \lambda \), under symmetric information the value of the firm is:

\[
P_{\text{after}}^{\text{fi},t} = \int_t^\infty e^{-r(s-t)} D_s^G ds = \left( \frac{z-g-\delta}{r-g} \right) e^{G_{t^*} + g(t-\tau^*)} \quad \text{for} \quad t \geq \tau^*,
\]

\[
P_{\text{before}}^{\text{fi},t} = E_t \left[ \int_t^{\tau^*} e^{-r(s-t)} D_s^G ds + e^{-r(\tau^*-t)} P_{\text{after}}^{\text{fi},\tau^*} \right] = e^{Gt} A_{\lambda}^{fi} \quad \text{for} \quad t < \tau^*
\]

where

\[A_{\lambda}^{fi} = \left( \frac{z-G-\delta}{r+\lambda-G} \right) + \lambda \left( \frac{z-g-\delta}{(r-g)(r+\lambda-G)} \right).\]  

(12)

In the pricing functions (10) and (11) the subscript "fi" stands for "full information" and superscripts "after" and "before" are for \( t \geq \tau^* \) and \( t < \tau^* \), respectively. Under full information, the share price drops at time \( \tau^* \) by:

\[
P_{\text{after}}^{\text{fi},\tau^*} - P_{\text{before}}^{\text{fi},\tau^*} = -e^{G_{\tau^*}} \frac{(z-r-\delta)(G-g)}{(r-g)(r+\lambda-G)},
\]

which increases in \( \tau^* \). The bottom panel of Figure 2 plots the price path in the benchmark case corresponding to the dividend path in the top panel.

We finally show that all else equal, shareholders prefer the manager to exert high effort, as the choice of \( e^H \) maximizes firm value.

**Corollary 1.** The firm value under \( e^H \) is always higher than under \( e^L \), that is

\[
P_{\text{before}}^{\text{fi},t} (\lambda^H) > P_{\text{before}}^{\text{fi},t} (\lambda^L).
\]

(13)

By simple substitution in Proposition 2, it is easy to see that (13) holds if and only if

\[z - r - \delta > 0,
\]

(14)

which is always satisfied (see condition (5)).

It is intuitive that without a proper incentive scheme, the CEO won’t exert high effort in this environment, even if \( \tau^* \) was observable, because of the cost of effort. In our setting, shareholders cannot solve this incentive problem by simply “selling the firm to the manager”:

**Corollary 2.** The manager’s personal valuation of the firm before \( \tau^* \) is as in (11), but with \( \beta \) substituted for \( r \). Thus, a manager/owner exerts high effort, \( e^H \), iff

\[
\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{1 + \lambda^L H^{\text{Div}}}{1 - e^H + \lambda^H H^{\text{Div}}};
\]

(15)

where

\[H^{\text{Div}} = \frac{(z - g - \delta)}{(z - G - \delta)(\beta - g)}.\]

(16)
Condition (15) is intuitive. First, when effort does not produce much increase in the expected \( \tau^* \), i.e. when \( \lambda^H \approx \lambda^L \), then the condition is never satisfied with \( c^H > 0 \), and therefore the manager does not exert high effort. Second, and less intuitively, even when effort is costless \( (c^H = 0) \) the manager may still not choose high effort, even if he owns the firm. In this case, condition (15) is satisfied if and only if:

\[
z - \beta - \delta > 0. \tag{17}
\]

which is similar to (14), but with the manager discount \( \beta \) taking place of the shareholders’ discount \( r \). That is, if the manager is impatient, and the return on capital is low, so that \( \beta > z - \delta \), then the manager/owner won’t exert high effort. Because of this, the manager’s personal valuation of the firm is much lower than shareholders’ valuation, a difference that goes beyond the simple difference due to discounting.

In line with previous work, stock-based compensation provides the incentive for the manager to exert costly effort. For simplicity, we focus first on the simplest compensation program, in which the manager receives shares at the constant rate \( \eta \) per period. Because of linear preferences, it is always optimal for the manager to sell the shares immediately,\(^7\) thereby his effective compensation at time \( t \) is

\[
w_t = \eta P_t \tag{18}
\]

**Proposition 3.** In equilibrium with consistent beliefs where \( \lambda \) is the current intensity of \( \tau^* \), and the stock price is \( P_{fi,t}^{before} = e^{Gt} A_{\lambda}^{fi} \), where \( A_{\lambda}^{fi} \) is in (12), the manager exerts high effort, \( e^H \), under stock-based compensation (18), if and only if:

\[
\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{A_{\lambda}^{fi} + \lambda^L H^{Stock}}{A_{\lambda}^{fi}(1 - c^H) + \lambda^H H^{Stock}}, \tag{19}
\]

where

\[
H^{Stock} = \frac{z - g - \delta}{(r - g) (\beta - g)},
\]

It follows that a high effort Nash equilibrium occurs iff (19) is satisfied for \( A_{\lambda}^{fi} \). A low effort Nash equilibrium occurs if and only if (19) is not satisfied for \( A_{\lambda}^{fi} \).

We note that if the cost of effort is zero, \( c^H = 0 \), then condition (19) is always satisfied, as the price multiplier of the stock, largely due future investment opportunities, is capitalized in the current salary, providing the proper incentive for the manager to exert high effort.

\(^6\)This condition is obtained by substituting \( c^H = 0 \) and the value of \( H\text{Div} \) into (15) and rearranging terms.

\(^7\) This is true in the full information equilibrium, as there is no signaling role of consumption. It is also true in a pooling asymmetric equilibrium, discussed next, as any deviation from this rule would reveal the manager’s information, and destroy the equilibrium.
3 The (Dis)Incentives of Stock-Based Compensation

The previous section shows that when $\tau^*$ is observable, then a simple stock-based compensation would resolve shareholders’ incentive problem. Clearly the manager has much more information than the investors regarding the future growth opportunities of the firm, as well as about its actual investments. Is this compensation type still optimal when $\tau^*$ is private information of the manager?

To build intuition, consider now time $\tau^*$ in the bottom panel of Figure 2, and assume that $\tau^*$ is private information of the manager. In this case, if the manager discloses the information, his wage would drop from $w_t = \eta P_t^{before}$ to $w_t = \eta P_t^{after}$, depicted in the figure by the relatively sharp drop in price. That is, a pure form of stock-based compensation effectively implies that shareholders severely punish the manager for revealing bad news about the growth prospects of the firm. It is important to note that the bad news is only about the growth prospects – which we refer to as growth options, in line with the asset pricing terminology – and not about the return on assets in place, which we assume constant and equal to $z$. The manager conceals not the ability of the firm to produce returns now, but his knowledge that the firm will not be able to produce cash flows in the future. Moreover, notice that this event occurs by chance, even though the manager determines its probability ex ante.

Given an opportunity, the manager will try to conceal this information. However, this conceal strategy is harder to implement than it seems at first, even if shareholders have no information about the firms’ investment and capital dynamics. In reality, shareholders have only imprecise signals about the amount of economic capital and investment undertaken by the corporation. This is especially true for those industries characterized by high R&D expenditures, intellectual property, high degree of opacity in their operation (e.g. financial institutions), and rapidly growing new industries, as the market does not know how to distinguish between investments and costs.

For tractability reasons, we assume that signals about the level of capital and investments have in fact infinite noise, and thus shareholders form beliefs about the manager’s actions only by observing realized dividend payouts. While this assumption is extreme, it reflects the lack of randomness of the return on capital $z$, that we assume. A more realistic model would have both $z$ stochastic and informative signals about capital and investments. Such a model is substantially more challenging to analyze, not only because shareholders’ beliefs dynamics, which affect prices, are more complicated, but also because the manager’s optimal
investment strategy would be extremely complex, as he would have to balance out the amount of information to reveal with his need to conceal the bad news for as long as possible. We note that while some information about investments and capital would make it easier for the shareholders to monitor the manager, the presence of random return on capital $z$ would also make it easier for the manager to conceal bad news for longer, as he could blame low dividend payouts to temporary negative shocks on $z$ rather than a permanent decline in investment opportunities. It is thus not obvious that our assumption of non-stochastic $z$ but unobservable $K$ and $I$ make it easier for the manager to conceal $\tau^*$ than a more realistic setting.

Shareholders know that at $t = 0$ the firm has a given $K_0$ of capital and high growth rate $G$ of investment opportunities. As long as the firm is of type $G$, they expect a dividend $D_t^G$ as described in (9). 8 We assume that whenever the dividend deviates from the path of a $G$ firm, shareholders perform an internal investigation in which the whole history of investments is made public. 9

### 3.1 Investment under Conceal Strategy

If the CEO decides to conceal the truth, he must design an investment strategy that enables the firm to continue paying the high growth dividend stream $D_t^G$ in (9). Intuitively, such strategy cannot be held forever, as it will require more cash than the firm produces. We denote by $T^{**}$ the time at which the firm experiences a cash shortfall and must disclose the truth to investors. Since the firm’s stock price will decline at that time, and the manager will lose his job, it is intuitive that the best strategy for the CEO is to design an investment strategy that maximizes $T^{**}$, as established in the following Lemma:

**Lemma 1:** Conditional on the decision to conceal the true state at $\tau^*$, the manager’s optimal investment policy is to maximize the time until the cash shortfall $T^{**}$.

Next proposition characterize the investment strategy that maximizes $T^{**}$:

**Proposition 4:** Let $K_{\tau^*}$ denote the capital accumulated in the firm by time $\tau^*$. If the CEO chooses to conceal the decline in growth opportunities at $\tau^*$, then:

1. He employs all the existing capital stock: $K_{\tau^*} = K_{\tau^*-}$.

8The assumption that dividends can be used to reduce agency costs and monitor managers has been suggested by Easterbrook (1984).

9For instance, a substantial change in the firm’s dividend policy may act as a coordination device across dispersed shareholders, who may then call for an internal investigation on the firm.
2. His investment strategy for \( t > \tau^* \) is:

\[
I_t = z \, \min(K_t, J_t) - (z - G - \delta)e^{Gt}.
\]

(20)

3. The firm's capital dynamics is characterized as follows: Let \( h^* \) and \( h^{**} \) be the two constants defined in (52) and (53) in the Appendix, with \( T^{**} = \tau^* + h^{**} \). Then:

(a) For \( t \in [\tau^*, \tau^* + h^*] \) firm's capital \( K_t \) exceeds its optimal level \( J_t \).

(b) For \( t \in [\tau^* + h^*, T^{**}] \), firm's capital \( K_t \) is below its optimal level \( J_t \).

Point 1 of Proposition 4 shows that in order to maximize the time of cash shortfall \( T^{**} \), the manager must invest all of its existing capital in the suboptimal investment strategy. This suboptimal investment strategy, in (20), ensures that dividends are equal to the higher growth profile \( D_t^G = (z - G - \delta)e^{Gt} \) (see (9)) for as long as possible. The extent of the suboptimality of this investment strategy is laid out in point 3 of Proposition 4. In particular, the CEO initially amasses an amount of capital that is above its optimal level \( J_t \) (for \( t < \tau^* + h^* \)), while eventually the capital stock must fall short of \( J_t \) (for \( t \in [\tau^* + h^*, T^{**}] \)).

These dynamics are illustrated in Figure 3 for a parametric example. Panel A shows that the optimal capital stock initially exceeds the upper bound on the employable capital, \( K_t > J_t \). This implies that the pretending firm must initially invest in negative NPV projects, as shown in Panel B. Indeed, while the excess capital stock \( K_t - J_t \) has a zero return by assumption, it does depreciate at the rate \( \delta \). Intuitively, when investment opportunities slow down, the CEO is supposed to return capital to the shareholders (see Figure 2). Instead, if the CEO pretends that nothing has happened, he will invest this extra cash in negative NPV projects as a storage of value to delay \( T^{**} \) as much as possible. Panel B of Figure 3 shows that as the time goes by the pretending firm engages in disinvestment to raise cash for the larger dividends of the growing firm. The firm can do this as long as its capital \( K_t \) is above the minimal capital \( \overline{K}_t \). Indeed, \( T^{**} \) is determined by the condition \( K_T^{**} = \overline{K}_T^{**} \).10

In a conceal Nash equilibrium, rational investors anticipate the behavior of the managers, and price the stock accordingly. We derive the pricing function next.

### 3.2 Pricing Functions under Asymmetric Information

At time \( T^{**} \) the pretending firm experiences a cash shortfall and is not able to pay its dividends \( D_t^G \). At this time, there is full information revelation and thus the valuation of the

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10 Therefore the technical assumption of a minimal capital stock in equation (6) affects the time at which the firm can no longer conceal the decline in its stock of capital.

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13
firm becomes straightforward. The only difference from the symmetric information case is
that the firm now does not have sufficient capital to employ to its full potential, thus it needs
to re-capitalize. Since at $T^{**}$ the firm’s capital equals the minimum employable capital level,
$K_{T^{**}} = K_{T^{**}}$, while the optimal capital should be $J_{T^{**}}$, the firm must raise $J_{T^{**}} - K_{T^{**}}$
From assumption (6), $K_{T^{**}} = \xi J_{T^{**}}$, which yields the pricing function:

$$P_{ai,T^{**}} = \int_{T^{**}}^{\infty} D_t^G e^{-r(t-T^{**})} dt - J_{T^{**}}(1 - \xi) = e^{(G-g)\tau^{**}+gT^{**}} \left( \frac{z - r - \delta}{r - g} + \xi \right)$$

(21)

The pricing formula for $t < T^{**}$ is then

$$P_{ai,t} = E_t \left[ \int_t^{T^{**}} e^{-r(s-t)} D_s^G ds + e^{-r(T^{**}-t)} P_{ai,T^{**}}^L \right]$$

(22)

The subscript “$ai$” in (22) stands for “Asymmetric Information”. Expression (22) can be
compared with the analogous pricing formula under full information (11): the only difference
is that the switch time $\tau^{*}$ is replaced by the (later) $T^{**}$, and the price $P_{fi,\tau^{*}}$ is replaced with
the much lower price $P_{ai,T^{**}}^L$. We are able to obtain an analytical solutions:

**Proposition 5:** Let shareholders believe that $\lambda$ is the current intensity of $\tau^{*}$. Under asym-
metric information and conceal strategy equilibrium, the value of the stock for $t \geq h^{**}$ is:

$$P_{ai,t} = e^{Gt} A_{\lambda}^{ai}$$

(23)

where

$$A_{\lambda}^{ai} = \frac{(z - G - \delta)}{(r + \lambda - G)} + \lambda e^{-(G-g)h^{**}} \left( \frac{z - r - \delta + (r - g)\xi}{(r - g)(r + \lambda - G)} \right)$$

(24)

Comparing the pricing formulas under asymmetric and symmetric information, (23) and
(11), we observe that the first term in the constants $A_{\lambda}^{fi}$ and $A_{\lambda}^{ai}$ is identical. However,
the second term is smaller in the case of asymmetric information: the reason is that under
asymmetric information, rational investors take into account two additional effects. First,
even if the switch time $\tau^{*}$ has not been declared yet, it may be possible that it has already
taken place and the true investment opportunities are growing at a lower rate $g$ for a while
(up to $h^{**}$). The adjustment $e^{-(G-g)h^{**}} < 1$ takes into account this possibility. Second, at
time $T^{**}$ the firm must re-capitalize to resume operations, which is manifested by the smaller
numerator of the second term, compared to the equivalent expression in (11).

The top panel of Figure 4 illustrates the value loss associated with the conceal strategy.
Since the manager’s compensation is not coming out of the firm’s funds, the value loss is

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11The case of $t < h^{**}$ does not yield additional intuition relative to the case of $t \geq h^{**}$, yet it is much
more complex to analyze. For this reason, we leave it to the appendix.
equal to the loss of the shareholders relative to what they would have got under the reveal strategy (full information). These costs can be measured by the present value (as of $\tau^*$) of the difference in the dividends paid out to the shareholders under the two equilibria. Relative to the reveal strategy, the conceal strategy pays lower dividends for a while, as the manager pretends to actively invest, and then must pay higher dividends, that arise from allegedly high cash flow. These higher dividend payouts come at the expense of investment, thus are essentially borrowed from the future dividends. The lower the minimum employable capital $K_t$ (i.e. lower $\xi$ in (6)), the longer the CEO can keep the pretense, and thus the higher the required recapitalization that is necessary when the firm experience a cash shortfall. This also implies lower dividends forever after the $T^{**}$.

How does the information asymmetry affect the price level? The bottom panel of Figure 4 plots price dynamics under the conceal equilibrium and compares them to prices under the reveal equilibrium. Rational investors initially reduce prices in the conceal equilibrium, as they correctly anticipate the suboptimal manager’s behavior after $\tau^*$. The stock price however at some point exceeds the full information price, as the firm’s cash payouts increase (see top panel). The price finally drops at $T^{**}$ when the firm experiences a severe cash shortfall, and needs to recapitalize. The exact size of underpricing and price drop depends on parameter values, as further discussed in Section 5.

The conceal equilibrium discussed in this section also provides CEOs a motive to “meet analysts’ earnings expectations,” a widespread managerial behavior, as recently documented by Graham et al. (2005). Indeed, the stock behavior at $T^{**}$ is consistent with the large empirical evidence documenting sharp price reductions following failures to meet earnings expectations, even by a small amount (see e.g. Skinner and Sloan (2002)).

### 3.3 Equilibrium Strategy at $t = \tau^*$

Now that we computed the equilibrium pricing function under conceal equilibrium, we can consider the manager’s incentives at time $\tau^*$ to conceal or reveal the true growth rate. Since after $\tau^*$ there is nothing the manager can do to restore high growth $G$ the choice is driven solely by the comparison between the present value of the infinite compensation stream under the reveal strategy, and the finite stream under the conceal strategy. Recall also that after $\tau^*$ the manager no longer faces any uncertainty (even $T^{**}$ is known to him), and thus the two utility levels can be computed exactly.

The rational expectations pure strategies Nash equilibrium must take into account in-
vestors’ beliefs about the manager strategy at time $\tau^*$, since they determine the price function. There are three intertemporal utility levels to be computed at $\tau^*$ depending on the equilibrium. In a *Reveal Equilibrium*, the manager’s utility is determined by $P_{fi,t}^{after}$ in equation (10) if at $\tau^*$ the manager decides to reveal. In contrast, if the manager decides to conceal, his utility is determined by the price function $P_{fi,t}^{before}$ in equation (11). In a *Conceal Equilibrium*, if the manager follows the Nash equilibrium strategy (conceal at $\tau^*$), then the price function must be the asymmetric information price function $P_{ai,t}$ in equation (23). If instead, the manager reveals at $\tau^*$ the true state of the firm, the price function reverts back to the full information price $P_{fi,t}^{after}$ in equation (10). These three levels of utility are given by:

12

$U_{Stock,\tau^*}^{reveal} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta P_{fi,t}^{after})dt = \frac{\eta e^{G\tau^*}}{(\beta - g)} \left( \frac{z - g - \delta}{r - g} \right)$ (25)

$U_{Stock,\tau^*}^{conceal,ai} = \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta P_{ai,t}^{after})dt = \eta A_{ai}^{\lambda} e^{G\tau^*} \left( \frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G} \right)$ (26)

$U_{Stock,\tau^*}^{conceal,fi} = \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta P_{fi,t}^{before})dt = \eta A_{fi}^{\lambda} e^{G\tau^*} \left( \frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G} \right)$ (27)

In the next proposition, we obtain the resulting conditions for a reveal and conceal equilibrium under stock-based compensation:

**Proposition 6:** Let $\tau^* \geq h^{**}$. A necessary and sufficient condition for a *conceal equilibrium* under stock-based compensation is

$$\frac{A_{ai}^{\lambda}}{(\beta - G)} \left( 1 - e^{-(\beta - G)h^{**}} \right) > \frac{(z - g - \delta)}{(r - g)(\beta - g)}$$ (28)

where the constant $A_{ai}^{\lambda}$ is given in equation (24). Similarly, a necessary and sufficient condition for a *reveal equilibrium* under stock-based compensation is

$$\frac{A_{fi}^{\lambda}}{(\beta - G)} \left( 1 - e^{-(\beta - G)h^{**}} \right) < \frac{(z - g - \delta)}{(r - g)(\beta - g)}$$ (29)

where the constant $A_{fi}^{\lambda}$ is given in equation (12).

Intuitively, the right-hand-side of both conditions (28) and (29) is the discounted utility under reveal strategy. Since the compensation is stock-based, the stock multiplier “$1/(r - g)$” enters the formula. The left-hand-side of both conditions is the discounted utility value under

12 The “Stock” subscript indicates pure stock based compensation to distinguish it from alternatives discussed below.

13 The solution for the case $\tau^* < h^{**}$ is cumbersome, thus delegated to the Appendix.
the conceal strategy. In particular, now the stock multiplier $A_{\lambda}^{ai}$ appears under the conceal equilibrium, while the stock multiplier $A_{\lambda}^{fi}$ appears under the reveal equilibrium. Since $A_{\lambda}^{fi} > A_{\lambda}^{ai}$, conditions (28) and (29) imply that the two equilibria in pure strategies are mutually exclusive, and thus it is not possible to find parameters for which both equilibria can exist at the same time. However, it may happen that for some parameter combination, no pure strategy Nash equilibrium exists.

### 3.4 Rational Expectations Equilibrium with the Choice of Effort

We now move back to $t < \tau^*$ and obtain conditions for Nash equilibrium that includes the manager’s effort choice. The equilibrium depends on the type of compensation and on the equilibrium at time $\tau^*$. The expected utility for $t < \tau^*$ is given by

$$U_t = E_t \left[ \int_t^{\tau^*} e^{-\beta(u-t)} w_u \left[ 1 - c(e) \right] du + e^{-\beta(\tau^*-t)} U_{\tau^*} \right],$$

where $U_{\tau^*}$ is the manager utility at $\tau^*$, computed in the previous section, whose exact specification depends on the equilibrium itself.

We now derive the conditions under which the stock-based compensation induces high effort. Since “conceal” is the most frequent equilibrium at $t = \tau^*$ (see Section 5), we focus our attention only to this case.

**Proposition 7:** Let $t \geq h^{**}$ and let $\lambda^H$ be such that a conceal equilibrium is obtained at $\tau^*$. Then, high effort $e^H$ is the equilibrium strategy if and only if

$$\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{1 + \lambda^L H_{ai}^{Stock}}{1 - e^{(\beta-G)h^{**}} + \lambda^H H_{ai}^{Stock}}$$

where

$$H_{ai}^{Stock} = \frac{1 - e^{-(\beta-G)h^{**}}}{\beta - G}$$

To see the intuition behind this proposition, note that condition (31) is similar to condition (19) in the benchmark case. Two properties are important. First, if effort is costly and has a low impact on $\lambda$, i.e. $\lambda^H \approx \lambda^L$, then the condition is violated, and the manager never chooses high effort. Second, if effort costs little, $c^H \approx 0$, then the manager always chooses high effort. Intuitively, the benefit for the manager to exert high effort stems from the longer tenure period ($T^{**}$ is pushed forward) while enjoying the long term rewards of his efforts earlier, as they are capitalized in the stock price.
4 Equilibrium under Alternative Simple Compensation Schemes

The previous section focused on a simple, linear stock-based compensation scheme. In this section we consider alternative simple compensation schemes widely used in the industry. The final section also discusses the properties of an optimal contract.

4.1 Flat Wage

Suppose the manager simply gets a wage $w_t$ that is not contingent on anything. For simplicity, assume $w_t = w$, a constant. In this case, it is intuitive that the manager at time $\tau^*$ prefers to reveal the decrease in investment opportunities to shareholders, as he would get $w$ for a longer period. The drawback, of course, is that the manager has no incentive to exert costly effort, because it would bear the effort cost $cH$. Thus, the resulting Nash equilibrium is a (Low Effort, Reveal) equilibrium, as shareholders expect the manager will not exert effort, and prices adjust to $P_{fi,t}^{before} = e^{Gi}A_{fi}^{\tau}$ in (11).

From the shareholders’ point of view, the interesting question is whether it is better to induce a (Low Effort, Reveal) equilibrium through a simple flat wage, or a (High Effort, Conceal) equilibrium through a stock-based compensation. Because each of the two equilibria has one positive and one negative feature, the question is which one is the best. Next corollary answers this question, while Section 5 contains a quantitative assessment of this corollary.

Corollary 3. There are $\Delta^H$ and $\Delta^L$ such that for $\lambda^H < \Delta^H$ and $\lambda^L > \Delta^L$ the value of the firm under (High Effort / Conceal) equilibrium is higher than under (Low Effort / Reveal) equilibrium. That is, $P_{ai,0} > P_{fi,0}^{before}$.

Intuitively, as $\lambda^H \to 0$, the price under asymmetric information converges to the Gordon growth formula with high growth: $P_{ai,0} \to (z - G - \delta)/(r - G)$. Similarly, as $\lambda^L \to \infty$, the price under full information converges to the same model, but with low growth rate $g$, $P_{fi,0}^{before} \to (z - g - \delta)/(r - g)$. Since in our model $z > r + \delta$, in the limit $P_{ai,0} > P_{fi,0}^{before}$.

This corollary implies that if the manager’s effort strongly affects the investment opportunities growth, then shareholders prefer an incentive scheme that induces a conceal strategy as a side effect. They are willing to tolerate the stock price crash at $T^{**}$ and re-capitalization as a delayed cost to provide incentives for longer term growth.
This fact implies that it is not necessarily true that finding ex-post managers that have not been investing optimally during their tenure is in contrast with shareholders’ ex-ante choice. Given the choice between these two equilibria, ex ante shareholders would be happy to induce high growth at the expense of the later cost of a market crash. We believe that this is a new insight in the literature. Section 5 below shows that stock-based compensation is ex-ante optimal for a wide range of reasonable parameters.

4.2 Deferred Compensation: Vesting

A popular incentive scheme is to delay the compensation of managers for a few years. Indeed, there is a conventional wisdom that delayed stock-based compensation provides both the incentives to exert high effort and to reveal any bad news about the company. Unfortunately, this conventional wisdom is not warranted, as we now show.

To see the problem with the argument, consider the case in which the firm pays the managers a rate $\eta_t$ shares per period, which are vested for $k$ years. Because of linear preferences, it is optimal for the manager to sell off all of the shares that are becoming eligible for vesting, and consume out of the proceeds.\(^{14}\) Thus, at time $t$ the manager’s consumption is

$$w_t = \eta_{t-k} P_t$$

As in the previous section, we study the case in which the firm awards always the same number of shares per period, $\eta_t = \eta$, which makes the consumption at time $t$ simply $w_t = \eta P_t$. Assume that if the CEO conceals at $\tau^*$, he will lose all the non-vested shares at the time of the cash shortfall $T^{**}$. It is immediate to see then that if $\tau^* > k$, the intertemporal utilities at $\tau^*$ under both a reveal or a conceal equilibrium are given by expressions (25) - (27). Thus, in this case the incentive problem of the manager is identical to the one examined earlier in Proposition 7. That is, delayed stock-based compensation is completely ineffective in inducing the manager to reveal bad news about growth options. Intuitively, if $\tau^* > k$ then at $\tau^*$ the manager has accumulated enough shares coming eligible for vesting per period that the revelation of bad news about growth options would undermine. The manager will then retain the information.

What if $\tau^* < k$? The only change in the expressions for the intertemporal utilities (25) - (27) is that the integral would start from $k$ instead of $\tau^*$, and thus the expressions have to be modified accordingly. In this case, indeed, for $k$ long enough the intertemporal utility under

---

\(^{14}\)See footnote 7.
conceal strategy always decreases to zero more quickly than the one under reveal strategy and thus a large vesting period \( k \) would provide the correct incentives if \( \tau^* < k \). However, relying on this event \( (\tau^* < k) \) to provide the proper incentives to CEO is misguided, as the lower utility under stock-based compensation only stems from the fact that the CEO has had not enough time to accumulate shares before \( \tau^* \). This logic is problematic for two reasons: First, shareholders actually want \( \tau^* \) to occur as far in the future as possible, and thus hoping to also have \( \tau^* < k \) is against their desire to push the manager to exert high effort, unless the delay \( k \) is unreasonably high. Second, the argument relies on \( t = 0 \) being the time at which the CEO starts accumulating shares, which is also problematic. In our model, \( t = 0 \) is any time at which there is full disclosure about both the capital \( K_0 \) and the company growth rate \( \tilde{g} = G \). For instance, if this time reflects the time of an IPO, it is typically the case that the owner selling the firm becomes the CEO of the new company while retaining a large fraction of firm’s ownership initially. Similarly, managers that are promoted from within the firm have already accumulated shares during their time at the firm. Thus, if we assume that at time \( t = 0 \) the manager is already endowed with shares coming eligible for vesting, then \( \tau^* < k \) can never happen, and the equilibrium is effectively identical to the one in the previous section.

We finally note that when the manager decides at \( \tau^* \) to conceal the bad news about future investment opportunities, he does so in the full knowledge that at the later time \( T^{**} \) he will lose all of the non-vested shares \( (\eta k) \), suffering an effective loss at \( T^{**} \) equal to \( \eta k P_{T^{**}} \). This amount can be quite substantial, and thus it may appear at first that a CEO who loses a massive amount of wealth at the time of the firm’s liquidity crisis (i.e. \( T^{**} \)) cannot be responsible for the crisis itself, as he is the first to lose. But in our model this conclusion is not warranted, because the price reached the large level \( P_{T^{**}} \) exactly because of the (misleading) behavior of the CEO. Had the CEO behaved in the best interest of the shareholders, such high value of the stock would have not realized in the first place. This analysis therefore cautions against reaching any type of conclusion on the behavior of CEOs based on his personal losses of wealth at the time of the firm’s liquidity crisis.

4.3 Deferred Compensation: Delayed Payments

The main problem with the vesting argument in the previous section is that at time \( \tau^* \), the manager may have accumulated enough shares not to have an incentive to reveal bad news about long term growth. A popular variation to the deferred compensation is to only delay
the payment to $k$ years in the future, so that the consumption at time $t$ is

$$w_t = \eta_{t-k}P_{t-k}$$

if the manager acts in the best interest of shareholders, or zero after the cash shortfall $T^*$, if it happens. This compensation scheme is equivalent to placing the cash equivalent of the pure stock-based compensation in an escrow account, and pay it $k$ years later if the CEO has not been caught misbehaving. While this scheme indeed induces the manager to reveal at $\tau^*$ when $k$ is sufficiently large, it unfortunately also induces the manager not to exert effort when $c_H$ is sufficiently high, as next proposition shows:

**Proposition 8:** (a) Let $k$ be defined by the following equation

$$A^f_i \left( \frac{1 - e^{-(\beta-G)(h^{**}-k)}}{\beta - G} \right) = \frac{z - g - \delta}{(r - g) (\beta - g)}$$

Then the manager reveals at $\tau^*$ if and only if $k \geq k$.

(b) Let $t \geq k = k$ and let $\tau^*$ not have been realized yet.\(^{15}\) Then, the manager exerts high effort $e^H$ if and only if

$$c^H \left[ \frac{\beta + \lambda^L - G}{\beta + \lambda^H - G} \right] < (\lambda^L - \lambda^H) e^{-(\beta-G)h^{**}}$$

Thus, there exists a constant $\zeta^H$, such that if $c^H < \zeta^H$, then (High Effort/Reveal) is a Nash equilibrium.

This proposition shows that the stock-based, delayed payment compensation scheme may effectively achieve the first best, as the CEO exerts high effort (part (b)) and reveals the true state at $\tau^*$ (part (a)). This is good news, and it matches the intuition that the stock component of the compensation still provides the incentives to work hard, and the delayed payment provides the incentive to reveal any bad news about the long term. However, the proposition also highlights two facts: First, that the delay must be sufficiently high ($k > k$), and second, that the cost of effort must be sufficiently low $c^H < \zeta^H$. Unfortunately, if either of these requirements is not satisfied, the equilibrium breaks down. In our basic calibration, which is better described below, we find that the minimum delay to induce truth revelation is $k \approx 11.5$ years, which we find unrealistically large. In addition, we also find that the lower bound to the managerial cost is $\zeta^H \approx 1.6\%$, which is below the parameter range we use in our calibration. The optimal contract described in Section 4.7 and its implementation through a stocks-plus-bonus compensation scheme in Section 5.5 provides a solution that works across a large range of parameter values.

\(^{15}\)We assume $t \geq k$ to sidestep the issue of having already accumulated shares by $\tau^*$, as discussed in the previous section.
4.4 Option-based Compensation

How does an option-based contract affect the incentive to conceal bad news about growth options? We now show that such a contract amplifies the incentive to conceal. Let $\eta_t$ denote the number of options awarded at time $t$, and let $k$ be their time to maturity. As it is standard practice, we assume these options are issued at-the-money, with strike price $H_t = P_t$. Thus, the consumption of the manager at $t$ is given by

$$w_t = \eta_{t-k} \max (P_t - H_{t-k}, 0) = \eta_{t-k} \max (P_t - P_{t-k}, 0)$$

Consider again time $\tau^*$. In this case, the intertemporal utilities under the reveal and conceal strategy in the reveal Nash equilibria\textsuperscript{16} are given by:

$$U_{\text{Reveal},fi}^{\text{Option,} \tau^*} = \int_{\tau^*}^{\tau^*+\kappa} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{\text{after},fi,t} - P_{\text{after},fi,t-k}, 0 \right) dt$$

$$+ \int_{\tau^*+\kappa}^{\infty} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{\text{after},fi,t} - P_{\text{after},fi,t-k}, 0 \right) dt$$

$$U_{\text{Conceal},fi}^{\text{Option,} \tau^*} = \int_{\tau^*}^{\tau^* + h^{**}} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{\text{before},fi,t} - P_{\text{before},fi,t-k}, 0 \right) dt$$

The intertemporal utilities under these two strategies in the conceal equilibrium are identical, but with the asymmetric information price $P_{ai,t}$ substituted in place of $P_{fi,t}$ in both (34) and (35). The next proposition shows that the leverage implied by option-like contracts in fact makes the conceal strategy more likely.

**Proposition 9:** (a) Let $k_{fi}^*$ be defined by

$$k_{fi}^* = \frac{1}{G} \log \left( \frac{r - g}{z - g - \delta} A_{fi}^\lambda \right)$$

Then for $k < k_{fi}^*$ a Reveal Equilibrium at $\tau^*$ holds if and only if

$$\left( \frac{e^{gk} - 1}{1 - e^{-Gk}} \right) \left( \frac{\beta - G}{\beta - g} \right) > e^{Gk_{fi}} e^{\beta k} \left( 1 - e^{-(\beta-G)h^{**}} \right)$$

(b) Let $k_{ai}^*$ be defined as in (36) but with $A_{ai}^\lambda$ in place of $A_{fi}^\lambda$, and let $\tau^* > h^{**} + k$. Then, for $k < k_{ai}^*$ a conceal Nash equilibrium at $\tau^*$ occurs if

$$\left( \frac{e^{gk} - 1}{1 - e^{-Gk}} \right) \left( \frac{\beta - G}{\beta - g} \right) < e^{Gk_{ai}} e^{\beta k} \left( 1 - e^{-(\beta-G)h^{**}} \right)$$

\textsuperscript{16}See notation and discussion in Section 3.3.
There exist $g > 0$ such that this condition is always satisfied for $g < g$. Thus, a conceal equilibrium can always be supported when $g$ is small.

The intuition behind this proposition is straightforward. Consider the case in which $g = 0$. In this case, the pricing formula in (11) shows that upon revealing the information, the price drops to $P_{fi,t}^{after}$ which is a constant. The value of $k_{fi}^*$ in (36) is the time lag that ensures the price at revelation $P_{fi,\tau^*}^{after}$ equals the price before revelation while it was still increasing, $P_{fi,\tau^*}^{before} = P_{fi,\tau^*}^{after}$. Clearly, for $k < k_{fi}^*$ it follows that the price after revelation is always smaller than the strike price $H_{r*} - k = P_{fi,\tau^*}^{before}$, pushing the option out of the money. The case in which $g = 0$ also implies that the manager cannot expect that his future options will ever be in the money. Thus, in this case, by revealing the CEO gets an intertemporal utility equal to zero. By concealing, in contrast, he always receive positive utility. It follows from this argument that option-like payoffs tend to increase the incentive to conceal bad news compared to the case in which the manager has a linear contract. If $k > k_{fi}^*$ calculations are less straightforward, but since the simple stock-based compensation can be considered a special type of option-based contract with strike price equal to $H = P_0$, it follows that increasing the strike price only decreases the payoff if the manager reveals information, decreasing his incentive to reveal.

4.5 Cashflow-Based (Bonus) Compensation

One alternative to stock-based compensation is a profit-based compensation contract. In the spirit of our simple neoclassical model with no frictions, we assume the firm pays out its output net of investments in the form of dividends. From an accounting standard, these should be considered the firm’s free cash flow, which coincides with dividends and earnings in our model, but that in reality they are different, and subject to different degrees of manipulation. Free cash flows are arguably harder to manipulate and thus we consider a simple compensation defined on cash flows.

Let then the compensation be given by $w_t = \eta_d D_t$. In this case, we obtain the following proposition:

**Proposition 10.** Under cash-flow based compensation, a necessary and sufficient condition for a “reveal” equilibrium at $t = \tau^*$ is:

$$\left(\frac{z - G - \delta}{\beta - G}\right) (1 - e^{-(\beta - G)h^{**}}) < \left(\frac{z - g - \delta}{\beta - g}\right).$$

\(^{17}\)We thank Ray Ball for pointing this out.
In addition, the manager exerts high effort, \( e^H \), iff

\[
\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{1 + \lambda^L H^{Div}}{1 - c^H + \lambda^H H^{Div}},
\]

(39)

where

\[
H^{Div} = \frac{(z - g - \delta)}{(z - G - \delta)(\beta - g)}.
\]

(40)

A Nash equilibrium with high (low) effort obtains iff (39) is (is not) satisfied.

This compensation strategy can achieve first best under some parameterization. In particular, we find that condition (38) is satisfied for most parameter configurations. This result is in fact intuitive, and leads us to the optimal compensation discussed later. Referring to the top panel of Figure 2 we see that when the manager optimally reveals, he has to increase the payout to shareholders, as there are no longer any investment opportunities available. Effectively, by doing so, the manager also increases his own compensation. That is, this cashflow-based compensation resembles a “bonus” contract in which the revelation of bad news leads to a higher cash payment and thus higher utility.

Of course, a drawback of this compensation scheme is that it provides the incentives to payout too much dividends, and thus sacrifice investments. In fact, because the manager is impatient \( \beta > r \), he prefers to have \( \tau^* \) occurring as soon as possible, thus decreasing his incentive to exert high effort. Indeed, we find that condition (39) is satisfied only under some extreme parameterization, in which both the return on capital \( z \) and the growth rate \( G \) are large. In this case, the higher discount \( \beta \) is compensated by (much) larger cash flows in the future if the manager invests heavily, i.e., if he exerts high effort. In all the other cases, the cash-flow based compensation leads to low effort and revelation, thereby generating the same type of conundrum already discussed in Section 4.1.

### 4.6 Claw back Clauses

One final popular incentive scheme is to insert claw back clauses in the CEO compensation package. Such clauses establishes that the CEO has to return part or all of the compensation he received during a given time if he is found guilty of misconduct. In our model, in a conceal equilibrium the CEO is not disclosing all the information to shareholders, which can be considered reasonable cause for shareholders or regulators to proceed against the CEO. Clearly, if the difference between \( \tau^* \) and \( T^{**} \) is verifiable in court, then by imposing a penalty at \( T^{**} \) sufficiently large we can always ensure that the manager discloses. The claw back clause is just such a penalty.
Our model, however, suggests that shareholders and regulators have to be careful even with claw back clauses. For instance, suppose that the distinction between $\tau^*$ and $T^{**}$ is observable but not verifiable, meaning that it would be hard to prove in court that effectively the manager has misbehaved. While in our stylized model it is simple to detect a misbehavior of the CEO, in reality the non-optimal investment strategy of the CEO is much harder to detect, let alone to prove in court. In this case, one may decide to make the claw back clause contingent on some measure of performance. Consider, for instance, that shareholders move against the manager to claw back the salary paid when the price drops. In this case it is intuitive that the manager has no incentive to reveal his information at time $\tau^*$, as it would induce a price decline. By concealing, the manager can push the price decline further back in the future, and thus maximize his utility.

Another possibility is to set up a claw back clause contingent on a (large) recapitalization, which is the main difference between $\tau^*$ and $T^{**}$. This clause would indeed solve the problem within our model, although it relies once again on the fact that in our model the firm never needs to go to the capital markets. In an extension of the model in which the firm may need to raise more capital for investment purposes, for instance to open a new identical firm with the same technology that allows to increase the size, then again there is the risk that by putting a claw back clause the shareholders would not be inducing the optimal CEO behavior.

### 4.7 Optimal Contract

The previous sections considered relatively standard simple compensation packages, adapted to our stylized model, and discussed their pros and cons. In this final section we briefly discuss the characteristics of the optimal contract, and compare them to the previous contracts. As in our dynamic model, all quantities increase at an exponential rate, we restrict our attention to contracts of the following form:

$$
\begin{align*}
  w_t &= \begin{cases} 
  w^b_t &= A_b e^{B_b t} & \text{if } t < \tau^* \\
  w^a_t &= A_a e^{B_a t + C_a \tau^*} & \text{if } t \geq \tau^*
  \end{cases}
\end{align*}
$$

where the subscript $a$ stands for “after $\tau^*$” and subscript $b$ stands for “before $\tau^*$”. We assume for simplicity that although $\tau^*$ is not ex-ante observable by shareholders, they are able to observe whether $\tau^*$ has been realized or not once the announcement is made. As the claw back clause must require the CEO to return the actual compensation received. As shown in Section 4.2, only losing the shares $\eta k$ not yet vested at $T^{**}$, for instance, would not alleviate the incentive to conceal the bad news at $\tau^*$.
discussed earlier, the manager may produce convincing evidence that investment opportunities deteriorated at $\tau^*$, while he may refrain from producing this information in a conceal equilibrium. This simplifying assumption allows us to make the contract’s payoff contingent on the announcement itself.\footnote{For simplicity, we sidestep here the issue of truthfully revelation, that is, the incentive to have the manager announce $\tau^*$ when it actually happens.} All bargaining power is with the firm, but the manager has an outside option. Since this is a growing firm, we assume that outside option is also growing over time $U^O_t = A_O e^{B_O t}$.

Since in our model the resources to pay the manager are outside the model (do not come from dividends themselves), effectively the firm solves

$$\min E \left[ \int_{0}^{\infty} e^{-rt} w_t dt \right]$$

conditional on the following incentive compatibility constraints:

\begin{align}
\text{(Reveal at $\tau^*$)} & \quad U^\text{Reveal}_{\tau^*} \geq U^\text{Conceal}_{\tau^*} \quad \text{for all } \tau^* \\
\text{(High effort before $\tau^*$)} & \quad U^H_t \geq U^L_t \quad \text{for all } t \leq \tau^* \\
\text{(Outside Option)} & \quad U_t \geq U^O_t \quad \text{for all } t 
\end{align}

where

\begin{align*}
U^\text{Reveal}_{\tau^*} &= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)} w_t^a dt; \\
U^\text{Conceal}_{\tau^*} &= \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)} w_t^b dt \\
U^i_t &= E_t \left[ \int_t^{\tau^*} e^{-\beta(s-t)} w_s^b (1 - c^i_s) ds + e^{-\beta(\tau^*-t)} U^\text{Reveal}_{\tau^*} \bigg| i \right] \quad \text{for } i = H, L
\end{align*}

Note that we assume that by concealing, the manager loses the outside option. That is, there is a serious penalty from concealing: This is realistic. Adding back the outside option after concealing is possible at the cost of additional complications, but without much change in intuition. This assumption skews the manager against concealing. Since we find that with stock-based compensation concealing is widespread, adding the outside option even after (being caught) concealing would just make it even more frequent.

Finally, to ensure $E \left[ \int_{0}^{\infty} e^{-rt} w_t dt \right]$ is finite, we must assume

$$r + \lambda > B^b; \text{ and } r > B^a$$

That is, the compensation does not grow at a faster rate than the cost of capital.
Proposition 11. The incentive compatibility constraints are satisfied if and only if the following constraints are:

(Reveal at $\tau^*$) \[ A_a \geq A_b \frac{(\beta - B_a)}{(\beta - B_b)} \left(1 - e^{-(\beta - B_b)h^{**}}\right); \quad (B_a + C_a) \geq B_b \] (44)

(High effort before $\tau^*$) \[ A_a \leq A_b \frac{(\beta - B_a)}{(\beta - B_b)} \left(1 - e^{H(\beta+\lambda^L-B_b)}\right); \quad B_b \geq (C_a + B_a) \] (45)

(Outside Option) \[ A_b \left(1 - c^H\right) + \frac{\lambda^H A_a}{(\beta - B_a)} \geq A_O; \quad B_b \geq B_O; \] (46)

\[ \frac{A_a}{\beta - B_a} \geq A^O; \quad C_a \geq 0; \quad B_a \geq B_O \] (47)

Subject to these constraints, the firm then minimizes

\[ V = E \left[ \int_0^\infty e^{-rt}w_t dt \right] = \left[ A_b + \frac{\lambda^H A_a}{(r - B_a)} \right] \left[ \frac{1}{(r + \lambda^H - B_b)} \right] \]

Constraint (44) shows that after $\tau^*$, the level $A_a$ of the compensation has to be above some value to induce the manager to reveal the bad news. Similarly, constraint (45) shows that the level of compensation cannot be too high after $\tau^*$, otherwise the manager prefers not to exert effort and increase his payoff sooner. These two constraints combined imply

\[ B_a + C_a = B_b \]

and

\[ A_b \frac{(\beta - B_a)}{(\beta - B_b)} \left(1 - c^H(\beta+\lambda^L - B_b)\right) \geq A_a \geq A_b \frac{(\beta - B_a)}{(\beta - B_b)} \left(1 - e^{-(\beta - B_b)h^{**}}\right); \]

This last constraint determines a feasibility region for $A_a$. This region is not empty if and only if the cost to exert high effort is below a threshold:

\[ c^H \leq \frac{\left[\lambda^L - \lambda^H\right] e^{-(\beta - B_b)h^{**}}}{(\beta + \lambda^L - B_b)} \]

Because the right-hand-side is increasing in $B_b$, this constraint implies a lower bound on the growth rate of the CEO compensation before $\tau^*$. We further discuss the properties and the intuition of the optimal contract in our calibration analysis in Sections 5.4 and 5.5. There we also compare the stock-based compensation to the optimal contract, and illustrate how the optimal contract can be approximated by using an appropriate stock-plus-bonus compensation. It is worth emphasizing immediately, however, that because of the simplicity of the model, we are able here to make the contract only contingent on time $t$ and $\tau^*$. This is useful to gauge the characteristics of the contract. In its implementation, however, one must use a better proxy than $t$ for the firm’s growth. We return on this issue below.
5 Quantitative Implications

Although our model is stylized, its dynamic properties still allow us for a reasonable calibration of its parameters to derive quantitative implications. In particular, we show that for most plausible parameter values, pure stock-based compensation leads to a conceal equilibrium. In spite of this, however, we also find that a (High-Effort, Conceal) equilibrium induced by a pure stock-based compensation is preferable to a (Low-Effort, Reveal) equilibrium induced by a constant or cashflow-based compensation, as it still maximizes firm value, since it provides an incentive to exert effort. In this exercise, we also consider the potential costs to the firm to provide incentives through stocks or dividends. Finally, we characterize the optimal weight on stock in the combined compensation package.

5.1 Stock-Based Compensation and Conceal Equilibrium at $t = \tau^*$

Given the effort choice $e$ and thus $\lambda$, when is it optimal to conceal the change in the growth rate of investment opportunities? Figure 5 plots the areas in which pure strategy Nash conceal or reveal equilibria obtain under stock-based compensation. The base numerical values of the parameters are in Table 1. In all panels, the $x$-axis reports the initial high growth rate, $G$, ranging between 0 and 14%, while the $y$-axis represents a different variable in each panel: in the top panel it is low growth rate $g$; in the middle panel, it is the return on investment $z$, which ranges between 12% and 25%, and in the bottom panel, it is the expected time of maturity of the firm $E[\tau^*] = 1/\lambda$, that ranges between 3 and 25 years. As the top panel indicates, under stock-based compensation even a 5% difference between $G$ and $g$ is sufficient to induce a conceal Nash equilibrium, in which the manager chooses the conceal strategy and investors rationally anticipate this behavior. There is no pure strategy reveal equilibrium for any combination of $G$ and $g$. The intuition is as follows: if investors think that the manager follows a reveal strategy, the pricing function would reflect this belief and it is then given by the perfect information pricing formula (11). However, given these high prices, it is optimal for the manager to deviate and conceal the shift in investment opportunities. If in contrast shareholders think that the manager conceals, the pricing function is given by (23), and induces the manager to reveal.

The middle panel of Figure 5 plots the areas of conceal and reveal equilibrium under stock-based compensation in the $(z, G)$ space, where $z$ is the return on capital. We see that the conceal strategy choice is, once again, a pervasive equilibrium outcome. In contrast with the top panel, there is a small region in which a reveal Nash equilibrium obtains under
stock-based compensation. This is the area in the top-left corner, in which $G$ is small and the return on capital $z$ is extremely high. The intuition is that if growth is low, and return on capital high, there is little gain from concealing the change in investment opportunities ($G$ is low anyway) and the cost of future repercussion is high, as the higher profitability of investments implies higher future prices, and thus a higher utility of the manager.

Finally, the bottom panel reports the conceal and reveal strategy areas under stock-based compensation in the space $(E[\tau^*], G)$. The outcome is once again the same: the conceal equilibrium prevails for most parameters, and especially for high growth $G$ and high expected maturity time $\tau^*$ (or low $\lambda$).

We mentioned in Section 4.5, after Proposition 10, that a cashflow-based, bonus type of compensation always induces truth revelation at $\tau^*$. Indeed, for all parameter combinations shown in the three panels of Figure 5, the manager compensated with a cashflow based, bonus type of contract always reveals his information. These examples point to a broad dichotomy: the stock-based compensation induces concealing strategy, while the cashflow-based, bonus compensation yields truth revelation. Unfortunately, the latter compensation typically does not induce the manager to exert high effort, as we show next.

### 5.2 Choice of Effort before $\tau^*$

The previous subsection considers the parameter regions that determine a reveal or conceal equilibrium depending on the compensation type. We now study the parameter combinations that induce an effort level given the compensation type.

Figure 6 shows the partition of the parameter space of $(z, G)$ into regions corresponding to various equilibria. In the top-right area the manager chooses high effort regardless of the compensation mode. Consequently, in this region compensating the manager based only on a cashflow (bonus) type of contract achieves the first best, as in this case he also reveals the bad news to investors, and maximizes firm value. This region consists of firms characterized by high returns on investment, $z$, and high growth $G$ of investment opportunities. Such firms do not have to use stock-based compensation to induce high effort.

The region below and to the left of the top-right area is where the cashflow-based, bonus

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20 In fact the space is $(z, G - g)$ as we assume $g = 0$ in these numerical calculations.

21 In a model in which the growth in investment opportunities can go up as well, it is also possible that under cashflow-based compensation, the manager will be reluctant to reveal good news and hence will not exert effort ex-ante or reveal the truth ex-post. Stock-based compensation instead would lead the CEO to reveal good news.
compensation no longer induces high effort, while the stock-based compensation does, although in a conceal equilibrium. This is indeed the most interesting region, where we observe a trade-off between the effort inducement and truth telling inducement - one cannot obtain both. Firms with reasonably high growth rates and return on investment are in that region. Finally, the region below and to the left from there is where we no longer have a pure strategy equilibrium under stock-based compensation, while cashflow-based, bonus compensation still induces a low effort equilibrium. This is a region where a stock-compensated manager prefers to conceal if he chooses high effort, but would no longer choose high effort, if conceals. Part of that region corresponds to the conceal-low effort equilibrium (the worst possible scenario), whenever it exists. The existence depends on $\lambda^L$: it does not exist for high levels of $\lambda^L$. The remainder of the region corresponds to equilibria in mixed strategies. Solving for those is complicated, as the dynamic updating of investors’ beliefs becomes very tedious. They are not likely to provide new intuitions, thus we ignore them. In fact we find the region above that, where the real trade-off takes place, of most interest.

5.3 High effort or truthful revelation?

The previous section shows a large area in the parameter space in which both a (High effort/Conceal) equilibrium and (Low effort/Reveal) equilibrium may co-exist. Are shareholders better off with low effort and an optimal investment strategy, or high effort and a suboptimal investment strategy? Corollary 3 shows that the choice depends on the difference between $\lambda^L$ and $\lambda^H$. This section provides a quantitative illustration of the trade off.

To illustrate the trade off, Figure 7 plots the hypothetical price and dividend paths under the (High Effort/Conceal) and (Low Effort/Reveal) equilibrium. For comparison, it also reports the first best, featuring high effort and the optimal investment after $\tau^*$. As shown in Corollary 3, the (Low Effort/Reveal) equilibrium induced for instance by a flat wage or a cashflow based (bonus) compensation may induce too low an effort, and this loss outweighs the benefits of the optimal investment behavior at $\tau^*$. The (High Effort/Conceal) equilibrium, induced by stock-based compensation, in contrast, gets closer to the first best, yet also leads to suboptimal investment behavior, which generates the bubble-like pattern in dividend growth and prices.

In order to gauge the size of the trade-off between the two equilibria under various parameter choices, Table 2 reports the firm value at time $t = 0$, $P_{ar,0}$ and $P_{bef,0}^{before}$, under the two equilibria (columns 2 and 4), and the average decline in price when the true growth rate of investment is revealed, at $T^{**}$ in the (High Effort/Conceal) equilibrium (column 3) and at
\( \tau^* \) in the (Low Effort/Reveal) equilibrium (column 5). The appendix contains closed form formulas to compute the average decline (see Corollary A1). The first column reports the parameter that we vary compared to the benchmark case in Table 1. The last two columns report the value and the expected decline in the first best case.

Panel A of Table 2 shows that even for a low growth of investment opportunities \( G = 5\% \), the (High Effort/Conceal) equilibrium achieves a higher firm value \( (P_{ai,0} = 2.25) \) than the (Low Effort/Reveal) equilibrium \( (P_{fi,0}(\lambda^L) = 1.96) \), even though the former equilibrium induces a substantial expected market crash \( E \left[ \frac{P_{T^*/P^*}}{P_{T^*/P^*}} - 1 \right] = -53.53\% \) at \( T^{**} \), against a milder decline of only \( E \left[ \frac{P_{\tau^*}/P_{\tau^*} - 1} \right] = -4.22\% \) in the latter case. The last two columns show that the first best achieves an even higher firm value, \( P_{fi,0}(\lambda^H) = 2.33 \), although this value is not so much higher than the one under asymmetric information. Note that at revelation, even in the first best case there is a market decline (-19.44%), although far smaller than in the asymmetric information case.

The remainder of Table 2 (Panels B to F) shows that a similar pattern realizes for a large range of parameter choices.\(^{22}\) For instance, in the base case low effort induces an expected time of investment growth \( E[\tau^*|e^L] = 2 \) years. However, Panel B shows that even if low effort induces a longer expected time \( E[\tau^*|e^L] \), in fact up to 8 years, a similar result applies. In this case, the distance between the asymmetric information price \( P_{ai,0} \) and \( P_{fi,0}(\lambda^L) \) declines, but the latter price is still lower. Panel C shows that higher return on investments \( z \) leads to an increase in prices (across equilibria) and a mild decline in size of the crash at \( T^{**} \) for the asymmetric information case. Panel D shows that higher cost of capital \( r \) reduces both the prices across the equilibria and the decline at revelation, although the impact on the asymmetric information case is smaller than in the symmetric information case. The last two panels show the results for various depreciation rates \( \delta \) and minimum employable capital \( \xi \). In particular, the smaller is the minimum capital requirement \( \xi \) and the higher is the size of the crash at \( T^{**} \), as the firm can pretend for longer and will need an even larger recapitalization at \( T^{**} \).\(^{23}\)

To conclude, this section shows that if shareholders’ choice is between a contract that induces a (Low Effort/Reveal) equilibrium versus one that induces a (High Effort/Conceal)

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\(^{22}\)In Panels B, E, and F we set the cost \( c_H = 2\% \) instead of \( c_H = 5\% \) assumed throughout to ensure that all three equilibria existed under the parameter choices in Column 1.

\(^{23}\)We note that some parameters do not affect the comparative statics: for instance, the manager discount rate \( \beta \) or the cost of effort \( e^H \) only affect whether a conceal equilibrium or reveal equilibrium is obtained. But since the CEO strategy conditional on concealing is just to push the time of cash shortfall \( T^{**} \) as far in the future as possible, the latter only depends on the technological parameters, and not on preferences. Thus, both the value of the firm and the size of the crash at \( T^{**} \) are independent of these preference parameters.
equilibrium, they should choose the latter, as the value of the firm is much larger in this case and in fact rather close to the first best (High Effort/Reveal) equilibrium. This finding may also explain why stock-based compensation is so widespread in the real world: If there is any difficulty in implementing an optimal contract that induces the first best, then a simple stock-based compensation indeed achieves a second best that is not too far from the first best, as it can be seen by comparing the value of the assets in Column 2 and 6 of Table 2. Erring on the side of inducing truth revelation but low effort, instead, has a much worse impact on the value of the firm than erring on the side of providing incentives to increase investment opportunities.

5.4 The Optimal Contract

How does the optimal contract in fact look like? Figure 8 compares the optimal contract established in Section 4.7 to the stock-based compensation, for a hypothetical realization of $\tau^*$, under the assumption that at $\tau^*$ there is full revelation in both cases. This is the relevant scenario to understand why the CEO is reluctant to reveal information under stock-based compensation. The figure contains six panels: The left-hand-side panels plot the optimal contract (dashed line) and the stock-based contract. Each of the three panels corresponds to a different level of the cost of effort $c_H$. In all cases, the payoffs have been scaled to ensure that the CEO receives the same expected utility at time 0. Of course, the optimal contract is cheaper for the firm. Finally, we assume the outside option is constant at $A_O = 1$, which reflects the fact that in our baseline case in Table 1 we assume that at $\tau^*$ the growth of the firm is $g = 0$.

It is convenient to start from the middle panel (Panel C) which contains our base case $c_H = 5\%$ (see Table 1). We see two noteworthy features of the optimal contract: First, the payoff to the manager in increasing over time, and it jumps up at $\tau^*$ upon revelation. The latter discrete increase is a bonus compensation that the manager must receive to provide the incentive to reveal the bad news about investment opportunities. Because the manager must be compensated for telling when $\tau^*$ occurs, the impatient manager has an incentive to work little in order to anticipate $\tau^*$ (which is ex-post observable) as much as possible. An increasing payoff provides the incentive to the manager to push $\tau^*$ into the future.

This characteristics of the optimal contract, namely, an increasing pattern plus bonus to reveal bad news, strongly contrasts with the payoff implicit in the stock-based compensation, which is depicted by the solid line. As the plot shows, the stock based compensation provides a fast growing payoff, which provides the incentive to work hard and increase investment
opportunities. At the same time, however, the payment to the CEO drops substantially at $\tau^*$ if he reveals the bad news about growth opportunities. That is, a stock-based contract implicitly punish the CEO for good behavior of truth revelation. It follows that this implicit punishment provides an incentive to conceal the bad news as discussed in Section 3.1

We see an additional interesting characteristics of the optimal contract by comparing Panel C with Panels A and E, which differ from Panel C in their effort cost $c_H$: The higher the effort cost $c_H$ the higher must be the increasing pattern in the optimal contract to ensure that the manager is willing to exert high effort for longer, and thus the higher must be the bonus when he reveals the bad news. Indeed, when $c_H$ is small (top panel), then the overall compensation of the manager indeed drops at $\tau^*$. Still, even in this case, the drop is far smaller than the one implied by the stock based compensation.

5.5 The Stock-plus-Bonus Contract

This discussion shows that indeed an optimal compensation is in general rising over time to provide the incentive for high effort and it has a bonus component to reveal bad news. While the stock-based compensation implies a drop at $\tau^*$, we recall from Section 4.5 that a compensation $w_t = \eta_d D_t$ generates a bonus type of compensation at time $\tau^*$. However, we also recall that this compensation has a bonus that is too generous, and thus the manager has an incentive to work less to anticipate its payment. A possibility is to combine a stock based compensation with the cashflow based compensation to mimic the optimal contract, using variables that are observable in the firm.

The right-hand-side panels of Figure 8 report the contract obtained by combining the stock based compensation $w_t = \eta P_t$ with the cashflow based compensation $w_t = \eta_d D_t$, where we normalize $\eta_d = 1/(r - g)$, a value that ensures that the expected utility under stock-based and cashflow-based compensation are the same. We emphasize that we use dividend $D_t$ to be consistent with the model, but the interpretation of $D_t$ could also be as a cash payment over time with the same characteristics as dividends in our model, in particular, with a bonus component at revelation. We let $\omega$ be the weight on the stock-based compensation, so that the combined compensation is

$$w_t = \omega \eta P_t + (1 - \omega) \eta_d D_t.$$  \hspace{1cm} (48)

The compensation is only linked to variables in the firm, and they do not depend anymore on time $t$, nor on $\tau^*$ itself. We must choose $\omega$ to ensure that under compensation (48)
the manager has both an incentive to exert high effort and reveal at $\tau^*$. The following proposition is useful:

**Proposition 12:** Let $\omega^* \in [0, 1]$ be such that

\[
L_2 > L_1(\omega^*) \left( \frac{1 - e^{-(\beta - G)h^*}}{\beta - G} \right)
\]

(49)

\[
\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{L_1(\omega^*) + \lambda^L L_2}{(1 - c H) L_1(\omega^*) + \lambda^H L_2}
\]

(50)

where $L_1(\omega)$ and $L_2$ are in the Appendix. Then, the combined compensation $w = \omega^* \eta_p P_t + (1 - \omega^*) \eta_d D_t$ achieves the first best (High Effort / Reveal) equilibrium.

Condition (49) guarantees that “Reveal” is optimal at time $\tau^*$, conditional the full information pricing function (11) with $\lambda = \lambda^H$. The second condition (50) guarantees that “High Effort” is optimal at $t < \tau^*$, conditional on reveal being optimal at time $\tau^*$.

We find that $\omega = 30\%$ works for all three right-hand-side panels in Figure 8.\(^{24}\) Consider first the base case in Panel D. In this case, the combined compensation is similar to the optimal contract, in that it implies both a rising compensation over time and a bonus at $\tau^*$. When the cost of effort is higher, as $c_H = 7\%$ in Panel F, the match between the optimal contract and the combined stock-plus-bonus contract is very accurate. Instead, the combined compensation is not close to the optimal in the case in which the cost is small, such as $c_H = 3\%$ as in Panel B.

The stock-plus-bonus contract resembles real contracts that include golden parachutes and generous severance packages as a form of compensating the manager for revealing the bad news at time $\tau^*$.

An important question pertains to the weight to give to stock in a combined compensation to ensure the manager does not retain important information about the company (recall that this discussion applies also to the deferred compensation, see Section 4.2). Figure 9 shows the range of the weight $\omega$ in the compensation package that can induce the first best outcome as a function of $G$. That is, those $\omega$’s that satisfy both conditions (49) and (50). The three panels assumes that the return on investment is 16% (top), 18% (middle) and 20% (bottom). In each panel, the upper line indicates the $\omega$ at which the manager is indifferent between conceal and reveal strategies under the high effort choice. For values of $\omega$ below that the manager prefers to reveal, which is what long term shareholders would like him to do. The

\(^{24}\)Again, we rescaled $\eta$ to ensure that in all cases, the utility at time 0 from the optimal contract and from the combined compensation is the same.
lower line represents the level at which the manager is indifferent between choosing high and low effort, when he is in a reveal equilibrium. For $\omega$ above that the manager prefers to exert high effort. Thus the area between the two lines represents all possible weights $\omega$ on stocks in the combined compensation package that support the first best.

Notice that when the lower line reaches zero, the cashflow-based compensation alone is enough to induce high effort (recall the top-right area in Figure 6). This does not mean that a little stock-based component would necessarily ruin the incentives to reveal, but aggressive stock compensation (or high proportion of ownership) will. Indeed, across the three panels, we see that the maximal weight on stock-based compensation never reaches 50%. That is, it is never optimal to provide more than 50% of the total compensation to CEOs in stocks, however delivered. Still, across panels we see that the lower line is in general above zero, especially if the return on capital $z$ is low, implying that a zero weight to stocks in the CEO compensation package is also suboptimal. Moreover, the first best equilibrium obtains for a larger set of parameters than the conceal equilibrium. Indeed, returning to Figure 6, while the stock-based compensation only induced high effort for a sufficiently high return on investment $z$ and growth rate $G$, and no pure strategy equilibrium exists for lower values of both, the combined compensation package achieves a (High Effort / Reveal) equilibrium for all of the parameter combinations depicted in the figure.\(^25\)

Our analysis has abstracted from the costs that different incentive schemes impose on the firm itself.\(^26\) Endogenizing the compensation costs to the firm in our dynamic model, however, is quite hard, as dividend flows have to be adjusted depending on the equilibrium price, and the fixed point that sustains the Nash equilibrium is hard to obtain. Nevertheless, we can approximate the size of these costs in the various equilibria by taking their present value at the cost of capital of the firm, and comparing the value of the firm net of these costs across the various incentive schemes we study. Our analysis (see the Appendix), although approximate, shows that indeed, the combined optimal compensation plan discussed earlier achieves the first best without imposing too high a burden on the company.

In conclusion, this section shows that the choice of a compensation package should be firm-specific and depends on firm’s characteristics. As a consequence, the exact compensation package that induces the manager to act in the interest of the shareholders in all stages of the life cycle of the firm has to be chosen carefully. In particular, excessive stock compensation

\(^{25}\)This is not a generic statement though, as first best cannot always be achieved for all parameter combinations. For instance, a higher cost of effort reduces the area $(z, G)$ in which first best holds, although the area is still larger than the conceal equilibrium.

\(^{26}\)For instance, inducing high effort by using the combined compensation package may be too costly, and thus the firm could be better off with a low effort equilibrium.
or too little stock compensation are clearly suboptimal choices for most cases. This implies, for instance, that executives’ bonuses that depend exclusively on either earnings or stocks performance are not advisable. For the same reasons, situations in which the CEO owns a large packet of shares may lead to suboptimal investment.

6 Discussion and Conclusions

Our paper contributes to the debate on executive compensation. On the one hand, advocates of stock-based compensation highlight the importance of aligning shareholder objectives with managers’ and argue that compensating managers with stocks achieves the goal. Detractors argue that stock-based compensation instead gives managers the incentives to misreport the true state of the firm, and in fact even engage at times in outright fraudulent behavior. This paper sheds new light on the debate by analyzing both the ex ante incentive problem to induce managers to exert costly effort to maximize the firms’ investment opportunities, and simultaneously to induce the manager to reveal the true outlook for the firm and follow an optimal investment rule.

We show that a combined compensation package that uses both stock-based performance and a cashflow-based, bonus type of compensation reaches the first best, inducing the manager to exert costly effort and reveal any worsening of the investment opportunities, if it happens. Firm value is then maximized in this case. Each component (stocks and cashflows) in the combined compensation package serves a different purpose and thus they are both necessary “ingredients”: the stock-based component increases the manager’s effort to expand the growth options of the firm, while compensating managers also proportionally to cash flows or dividends significantly reduces his incentives to engage in value destroying activities to support the inflated expectations. It is crucial to realize, though, that the weight on stocks in the combined compensation package is not identical across firms: for instance, high growth firms should not make much use of stocks in their compensation package, while the opposite is true for low growth firms. That is, there is no fixed rule that work for every type of firm. As a consequence, generalized regulatory decisions that ban stock-based compensation, for instance, or board of directors’ decisions on CEO compensation that are based on some “conventional wisdom” are particularly dangerous, as they do not consider that each firm necessitates a different incentive scheme.

See Murphy (1999), Hall and Murphy (2003), Bebchuk and Fried (2004), Edmans and Gabaix (2009), and Gabaix and Landier (2007) for recent discussions.
Our model also sheds light on the incentives and disincentives of the CEO when his compensation is too heavily tilted towards stocks. Indeed, while we believe that the problem with too much stock-based compensation is widespread in general, the 1990’s Hi-Tech boom and collapse as well as the 2007 - 2008 financial crisis offer interesting examples of the mechanism discussed in our model. The 1990’s Hi-Tech boom was characterized by expectations of high growth rates and high uncertainty, coupled with high-powered stock-based executive compensation. Firms with perceived high growth options were priced much higher than firms with similar operating performance, but with lower perceived growth options. We argue that because of their high-powered incentives, executives had an incentive to present theirs as high growth firms, even when the prospects of high future growth faded at the end of the 1990s. Our analysis suggests that high-powered incentives induce the pretense of high growth firms and lead eventually to the crash of the stock price.

Similarly, the source of the banking crisis of 2007 - 2008 may also be partially understood through the mechanism discussed in the paper, as banks also share some of the key characteristics assumed in the model. In particular, there is a serious lack of transparency of banks investment behavior (e.g. complicated derivative structures) as well as of the available investment opportunities. Banks, and especially investment banks, employ high-powered, stock-based incentives. Consider for instance the growth in the mortgage market. It is reasonable to argue that banks’ CEOs observed a slowdown in the growth rate of the prime mortgage market. When investment opportunities decline, the first best action is to disclose the news to investor, return some of the capital to shareholders, and suffer a capital loss on the stock market. However, if a CEO wants to conceal the decline in investment opportunities’ growth, then our model implies that in order to maintain the pretense that nothing happened, the bank’s manager has to first invest in negative NPV projects, such as possibly the subprime mortgages, if the mortgage rate charged does not correspond to the riskiness of the loan.

Moreover, in order to keep the pretense for as long as possible the manager has also to disinvest and pass on positive NPV projects. According to the model, the outcome of the suboptimal investment program is a market crash of the stock price, and the need for a large recapitalization of the firm. As the debate about optimal CEO compensation is evolving, our model shows that too much stock sensitivity is “bad,” as it induces this perverse effect on manager’s investment ex-post. Nevertheless, too little stock sensitivity has also an adverse effect providing the CEO no incentives to search for good investment opportunities.

\footnote{Laeven and Levine (2008) provide empirical evidence that bank risk taking is positively correlated with ownership concentration. Based on evidence on CEOs personal losses during the crisis, Fahlenbrach and Stulz (2009) conclude that bankers’ incentives “cannot be blamed for the credit crisis or for the performance of banks during the crisis.” (Fahlenbrach and Stulz p. 18.)}
Appendix

This appendix contains only sketches of the proofs of the propositions. Details can be found in a separate technical appendix available on the authors’ web pages.

Proof of Proposition 1. The capital evolution equation is given by

\[ \frac{dK_t}{dt} = I_t - \delta K_t. \]  

From (2), the target level of capital, \( J_t \), is given by \( J_t = e^{Gt} \) for \( t < \tau^* \) and \( J_t = e^{G\tau^* + g(t-\tau^*)} \) for \( t \geq \tau^* \). Imposing \( K_t = J_t \) for every \( t \) and using (51) the optimal investment policy is given by (8). From (7), the dividend stream is (9). Q.E.D.

Proof of Proposition 2. For \( t \geq \tau^* \), the \( P_{fi,t}^{after} \) stems from integration of future dividends. For \( t < \tau^* \), the expectation in \( P_{fi,t}^{before} \) can be computed by integration by parts. Q.E.D.

Proof of Corollary 1: \( P_{fi,t}^{before} (L_H) > P_{fi,t}^{before} (L_L) \) iff \( A_{\lambda H}^f > A_{\lambda L}^f \). Substituting, this relation holds iff \( z - r - \delta > 0 \), which is always satisfied. Q.E.D.

Proof of Corollary 2: The first part immediately follows the fact that a manager/owner values the firm as the present value of future dividends discounted at \( \beta \). The second part follows the utility of the manager at \( \tau^* \) and \( t < \tau^* \). First, after \( \tau^* \), there is no benefit from exerting effort. Thus the manager/owner’s utility is:

\[ U_{Div,\tau^*} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)} D_t^d dt = \frac{\eta_d(z-g-\delta)}{\beta-g} e^{G\tau^*} \]

Before \( \tau^* \), the utility of the manager for given effort \( e \) is

\[ U_{Div,t}(e) = E \left[ \int_t^{\tau^*} e^{-\beta(u-t)} \eta_d D_u^G (1-c(e)) du + e^{-\beta(\tau^*-t)} U_{Div,\tau^*} \right] = e^{Gt} \eta_d \frac{(z-G-\delta)}{\beta+\lambda(e)-G} \left[ 1-c(e) + \lambda(e) \frac{(z-g-\delta)}{(z-G-\delta)(\beta-g)} \right] \]

Given \( H_{Div} \) in (16), the condition \( U_{Div,t}(e^H) > U_{Div,t}(e^L) \) translates into (39). Q.E.D.

Proof of Proposition 3: As in Corollary 2, from \( \tau^* \) onward the manager will not exercise high effort, resulting in a utility level at \( \tau^* \) given by

\[ U_{Stock,\tau^*} = \int_{\tau^*}^{\infty} e^{-\beta(s-t)} \left( \eta P_{fi,s}^{after} \right) ds = \frac{\eta(z-g-\delta)}{r-g}(\beta-g) e^{G\tau^*} \]

Thus, for \( t < \tau^* \) we have

\[ U_{Stock,t}(e) = E \left[ \int_t^{\tau^*} e^{-\beta(s-t)} \left( \eta P_{fi,s}^{before} \right) (1-c(e)) ds + e^{-\beta(\tau^*-t)} U_{Stock,\tau^*} \right] = \frac{\eta e^{Gt}}{\beta+\lambda(e)-G} \left[ A_{\lambda}^f (1-c(e)) + \lambda(e) \left( \frac{z-g-\delta}{(r-g)(\beta-g)} \right) \right] . \]
Let $e^H$ be the optimal strategy in equilibrium. The price function is then $P_{fi,t}^{before}$ with $A_{\lambda t}^{fi}$. We then obtain the condition $U_{Stock,t}(e^H) > U_{Stock,t}(e^L)$ iff (19) holds. The Nash equilibrium follows. Similarly, if $e^L$ is the optimal strategy in equilibrium, then the price function is $P_{fi,t}^{before}$ with $A_{\lambda t}^{fi}$. Thus, $U_{Stock,t}(e^H) < U_{Stock,t}(e^L)$ iff (19) does not hold. Q.E.D.

**Proof of Lemma 1.** Conditional on the decision to conceal $g$, the manager must provide a dividend stream $D_t^G$, as any deviation make her lose her job. Since she cannot affect the stock price, after $\tau^*$ her utility only depends on the length of her tenure. Since we normalize the manager’s outside options to zero, her optimal choice is to maximize $T^{**}$. Q.E.D.

**Proof of Proposition 4:** (a) The manager must mimic $D_t^G$ for as long as possible. This target determines the investments $I_t$ and thus the evolution of capital $\frac{dK_t}{dt} = I_t - \delta K_t$ for given initial condition $\hat{K}_{\tau^*}$. From the monotonicity properties of differential equations in their initial value and the definition of $T^{**}$ as the time at which $K_{T^{**}} = K_{\tau^*} = \xi J_{T^{**}}$, $T^{**}$ must be increasing with $\hat{K}_{\tau^*}$. The claim follows from Lemma 1.

(b) At time $\tau^*$ we have $K_{\tau^*} = J_{\tau^*} = e^{Gr^*}$, which implies

$$\frac{dK_t}{dt} \bigg|_{\tau^*} = z e^{Gr^*} - \delta e^{Gr^*} - (z - G - \delta) e^{Gr^*} = G e^{Gr^*}$$

This implies that $dK_t / dt > dJ_t / dt$ after the switch, and thus $K_{t+dt} > J_{t+dt}$. The trajectory of capital at $\tau^*$ is then above $J_t$. By continuity, there is a period $[0, t_1]$ in which $K_t > J_t$. During this period, the ODE for capital accumulation becomes:

$$\frac{dK_t}{dt} = z e^{Gr^*+g(t-\tau^*)} - \delta K_t - (z - G - \delta) e^{Gt}$$

Given the initial condition $K_{\tau^*} = J_{\tau^*} = e^{Gr^*}$, the ODE solution implies the excess capital:

$$K_t - J_t = e^{Gr^*} \left[ \left( \frac{z - G - \delta}{\delta + g} \right) \left( e^{g(t-\tau^*)} - e^{-\delta(t-\tau^*)} \right) - \frac{z - G - \delta}{\delta + G} \left( e^{G(t-\tau^*)} + e^{-\delta(t-\tau^*)} \right) \right]$$

As $t$ increases, $K_t - J_t \rightarrow -\infty$. Since $K_{\tau^*} - J_{\tau^*} > 0$, there must be a $t_1$ at which $K_{t_1} - J_{t_1} = 0$. Since $t_1 > \tau^*$, we can define $h^* \equiv t_1 - \tau^*$. Substituting in $K_{t_1} - J_{t_1} = 0$, $h^*$ must satisfy:

$$0 = \left( \frac{z - G - \delta}{\delta + g} \right) e^{-Gh^*} \left( e^{gh^*} - e^{-\delta h^*} \right) + \left( e^{-(\delta+G)h^*} - 1 \right) \left[ \frac{z - G - \delta}{\delta + G} \right]$$

(52)

For $t > t_1$, $K_t < J_t$, and thus the ODE switches to

$$\frac{dK_t}{dt} = (z - \delta) K_t - (z - G - \delta) e^{Gt}$$

Given the initial condition $K_{t_1} = J_{t_1}$, the ODE solution yields

$$K_t - J_t = e^{Gr^*} e^{G(t-t_1+h^*)} \left[ 1 + (e^{-(G-g)h^*} - 1) e^{(z-G)(t-t_1)} - e^{-G(g)(t-t_1+h^*)} \right]$$

which again diverges to $-\infty$ as $t \rightarrow \infty$. From the condition $K_{T^{**}} - J_{T^{**}} = 0$, and defining $h^{**} \equiv T^{**} - \tau^*$, we obtain the equation defining $h^{**}$:

$$0 = 1 + (e^{-(G-g)h^*} - 1) e^{(z-G)(h^{**}-h^*)} - e^{-(G-g)h^{**}}$$

(53)
Q.E.D.

**Proof of Proposition 5:** Let \( t > h^* \). If a cash shortfall has not been observed by \( t \), then a shift cannot have occurred before \( t > h^* \). Bayes formula implies that time \( T^* = \tau^* + h^* \) conditional on not observing a cash shortfall by time \( t \) has the following conditional distribution

\[
F_{T^*}(t'|T^* > t) = \Pr(\tau^* < t' - h^*|\tau^* > t - h^*) = \frac{e^{-\lambda(t-h^*)} - e^{-\lambda(t'-h^*)}}{e^{-\lambda(t-h^*)}} = 1 - e^{-\lambda(t'-t)}
\]

That is, \( T^* \) has the exponential distribution \( f(T^*|no \ cash \ shortfall \ by \ t) = \lambda e^{-\lambda(T^*-t)} \). The value of \( P_{ai,t} \) for \( t > h^* \) in (23) then follows from the pricing formula and integration by parts.

Let \( t < h^* \), then the conditional distribution of \( T^* \) is zero in the range \([t, h^*] \), as even a shift at 0 would only be revealed at \( h^* \). The density is then \( f(T^*) = \lambda e^{-\lambda(T^*-h^*)}1_{(T^*>h^*)} \). Using this density to compute the expectation, we find

\[
P_{ai,t} = (z - G - \delta)e^{Gr} \frac{1 - e^{-(r-G)(h^*-t)}}{(r-G)} + e^{rt}e^{(G-r)h^*}A^a_i
\]

Q.E.D

**Proof of Proposition 6:** Let \( \tau^* > h^* \). There are two equilibria to consider: a reveal equilibrium and a conceal equilibrium. In both equilibria, if the manager reveals at \( \tau^* \), then her utility depends \( P_{f_i,t}^{after} \) in equation (11). In contrast, the price path is different under the conceal strategy, depending on the equilibrium: In a conceal equilibrium, investors’ expect the manager to conceal and thus her utility depends on \( P_{ai,t} \) in (23) until \( T^* \). In a reveal equilibrium, then investors expect the manager to reveal and thus her utility under the conceal strategy depends on \( P_{f_i,t}^{before} \) in (11). We then obtain the utility functions in (25), (26), and (27). A reveal equilibrium is obtained if \( U_{Stock,\tau^*}^{reveal} > U_{Stock,\tau^*}^{conceal,fi} \) and a conceal equilibrium obtains if \( U_{Stock,\tau^*}^{conceal,ai} > U_{Stock,\tau^*}^{reveal} \). The conditions in the claim are obtained by simple substitution.

Finally, if \( \tau^* < h^* \), then \( U_{Stock,\tau^*}^{conceal,ai} \) depends on the price in (54). Details are left to the technical appendix. Q.E.D.

**Proof of Proposition 7:** Let \( t > h^* \). In a conceal equilibrium with high effort, \( P_{ai,t} \) in (23) with \( A^a_i \) determines the wage \( w_t = \eta P_{ai,t} \). The expected utility under effort \( e \) is:

\[
U_{Stock,t}(e) = E \left[ e^{-\beta(s-t)}w_s(1 - c(e))ds + e^{-\beta(\tau^*-t)}U_{Stock,\tau^*}^{conceal,ai} \right]
\]

\[
= e^{Gr} \eta A^a_i \frac{[1 - c(e) + \lambda(e)H_{Stock}]}{\beta + \lambda(e) - G}
\]

where \( \lambda(e) \) and \( c(e) \) are the intensity and the cost of effort under effort choice \( e \). The condition in the Proposition follows from the maximization condition \( U_{Stock,t}(e^H) > U_{Stock,t}(e^L) \). Finally, given \( e^H \) chosen by the manager, then indeed \( \lambda^H \) applies in equilibrium, a conceal equilibrium obtains at \( \tau^* \) and thus the price function is \( P_{ai,t} \) in (23), concluding the proof.
A similar proof holds for $t < h^*$. The expressions are in the technical appendix. Q.E.D.

**Proof of Proposition 8:** (a) If $t \geq k$, then $\tau^* \geq k$. Thus, $w_t = \eta P_{t-k}$ implies the utility from revealing and concealing under the reveal equilibrium are

$$
U_{\text{Reveal}}^{\tau^*, t} = \int_{\tau^*}^{t} e^{-\beta(t-\tau^*)} w_{t} dt = \int_{\tau^*}^{t} e^{-\beta(t-\tau^*)} \eta P_{t-k}^{\text{before}} dt + \int_{\tau^*}^{t} e^{-\beta(t-\tau^*)} \eta P_{t-k}^{\text{after}} dt
$$

$$
= \eta e^{\theta \tau^* - k} \left( A_1 \frac{1 - e^{\theta G}}{\beta - G} + \left( \frac{z - g - \delta}{\tau - g} \right) e^{\theta G} \right)
$$

$$
U_{\text{Conceal}}^{\tau^*, t} = \int_{\tau^*}^{t} e^{-\beta(t-\tau^*)} w_{t} dt = \int_{\tau^*}^{t} e^{-\beta(t-\tau^*)} \eta P_{t-k}^{\text{before}} dt
$$

$$
= \eta e^{\theta \tau^* - k} A_1 \frac{1 - e^{\theta G}}{\beta - G}
$$

Thus, algebra shows that $U_{\text{Reveal}}^{\tau^*, t} > U_{\text{Conceal}}^{\tau^*, t}$ iff

$$
A_1 \frac{1 - e^{\theta G}}{\beta - G} < \left( \frac{z - g - \delta}{\tau - g} \right) e^{\theta G}
$$

The claim follows from the definition of $k$ in (32).

(b) Moving to $k < t < \tau^*$, we have for $i = H, L$, the ex-ante utility is

$$
U_{t}^i = E \left[ \int_{t}^{\tau^*} e^{-\beta(s-t)} \eta P_{s-k}^{\text{before}} (1 - c_i) ds + e^{-\beta(t-\tau^*)} U_{\text{Reveal}}^{\tau^*, t} \right]
$$

$$
= A_1 \frac{1 - c_i e^{\theta G}}{\beta + \lambda - G} + \eta \left( \frac{\lambda e^{\theta G}}{\beta + \lambda - G} A_1 \left( \frac{1 - e^{\theta G}}{\beta - G} + \left( \frac{z - g - \delta}{\tau - g} \right) e^{\theta G} \right) \right)
$$

Using the definition of $k$ in (32), and substituting for the last term in the parenthesis, tedious algebra shows that $U_{t}^H > U_{t}^L$ iff

$$
e^{H} \left[ \frac{\beta + \lambda - G}{\beta + \lambda - G} \right] < \left( \frac{\lambda e^{\theta G}}{\beta + \lambda - G} \right) e^{-(\beta - G)(h^* - k + k)}
$$

Choosing $k$ as small as possible, we obtain condition (33). Q.E.D.

**Proof of Proposition 9.** We start with the reveal equilibrium. The proof follows from comparing the utility functions (34) and (35). Consider the first term in (34). For $t \in [\tau^*, \tau^* + k]$ we have

$$
P_{f_{i,t}}^{\text{after}} - P_{f_{i,t-k}}^{\text{before}} = \left( \frac{z - g - \delta}{\tau - g} \right) e^{\theta G + g(t-\tau^*)} - A_1 \frac{1 - e^{\theta G}}{\beta - G}
$$

Exploiting the definition of $k_{f_{i,t}}^*$, algebra shows that if $k < k_{f_{i,t}}^*$ then $P_{f_{i,t}}^{\text{after}} - P_{f_{i,t-k}}^{\text{before}} < 0$ for all $t \in [\tau^*, \tau^* + k]$. It follows that the first term in (34) is always zero for $k < k_{f_{i,t}}^*$. Turning to the second term of (34), for $t > \tau^* + k$, we have

$$
\int_{\tau^* + k}^{\infty} e^{-\beta(t-\tau^*)} \eta \max \left( P_{f_{i,t}}^{\text{after}} - P_{f_{i,t-k}}^{\text{after}}, 0 \right) dt = \eta \left( \frac{z - g - \delta}{\tau - g} \right) e^{(\beta + G - g) t} \int_{\tau^* + k}^{\infty} e^{-\beta t} (e^{g t} - e^{g(t-k)}) dt
$$

$$
= \eta \left( \frac{z - g - \delta}{\tau - g} \right) e^{G \tau^*} \left( 1 - e^{-\beta g k} \right) \frac{e^{-(\beta - G) k}}{\beta - G}
$$

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Consider now the utility under a conceal strategy (35), which is a deviation under the reveal equilibrium. If the CEO conceals, his utility is

\[
U_{\text{Conceal, } fi}^{\text{Option, } \tau^*} = \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)} \eta \max \left( P_{fi,t}^{\text{after}} - P_{fi,t-k}^{\text{before}}, 0 \right) dt \\
= \eta A_{\lambda}^{fi} \left( 1 - e^{-Gk} \right) e^{G_{\tau^*}} \frac{1 - e^{-\left(\beta-G\right)h^{**}}}{\beta - G}
\]

The second equality stems from noticing that \( P_{fi,t}^{\text{before}} - P_{fi,t-k}^{\text{before}} = A_{\lambda}^{fi} \left( e^{Gt} - e^{G(t-k)} \right) = A_{\lambda}^{fi} e^{Gt} \left( 1 - e^{-Gk} \right) > 0 \), which implies that the utility (35) is the same as with stock-based compensation, but with the adjustment \( 1 - e^{-Gk} \). Finally, for a reveal Nash equilibrium to be supported we must have \( U_{\text{Reveal, } fi}^{\text{Option, } \tau^*} > U_{\text{Conceal, } fi}^{\text{Option, } \tau^*} \), which yields

\[
\left( z - g - \delta \right) (1 - e^{\beta g}) \frac{\beta - G}{\beta - g} > A_{\lambda}^{fi} \left( 1 - e^{-Gk} \right) 1 - e^{-\left(\beta-G\right)h^{**}}
\]

Using the definition of \( k_{fi}^{*} \), some algebra shows that this condition is equivalent to

\[
\frac{\left( e^{gk} - 1 \right)}{(1 - e^{-Gk})} \left( \frac{\beta - G}{\beta - g} \right) > e^{G k_{fi}^{*}} e^{\beta k} \left( 1 - e^{-\left(\beta-G\right)h^{**}} \right)
\]

For any \( k \), as \( g \to 0 \), the left-hand-side converges to 0, while the right-hand-side is independent of \( g \). The claim follows.

A similar proof holds for the conceal equilibrium. The two utilities at \( \tau^* \) are

\[
U_{\text{Reveal, } ai}^{\text{Option, } \tau^*} = \int_{\tau^*}^{\tau^*+k} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{t}^{\text{after}} - P_{ai,t-k}^{\text{after}}, 0 \right) dt \\
+ \int_{\tau^*+k}^{\infty} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{t}^{\text{after}} - P_{t-k}^{\text{before}}, 0 \right) dt \\
= \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)} \eta_{t-k} \max \left( P_{ai,t} - P_{ai,t-k}, 0 \right) dt
\]

In particular, the formula under reveal strategy (a deviation in this equilibrium) is the same as in the previous part, but with \( A_{\lambda}^{ai} \) in place of \( A_{\lambda}^{fi} \) in the definition of \( k_{ai}^{*} \). The utility under the (equilibrium) conceal strategy instead is

\[
U_{\text{Conceal, } ai}^{\text{Option, } \tau^*} = \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)} \eta P_{ai,t} - \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)} \eta P_{ai,t-k} dt
\]

Thus, for \( \tau^* > h^{**} + k \), the formula is the same as with stock-based compensation, but with the adjustment \( 1 - e^{-Gk} \). It then follows that a conceal strategy is optimal if and only if

\[
\frac{\left( e^{gk} - 1 \right)}{(1 - e^{-Gk})} \left( \frac{\beta - G}{\beta - g} \right) < e^{G k_{ai}^{*}} e^{\beta k} \left( 1 - e^{-\left(\beta-G\right)h^{**}} \right)
\]

which is always satisfied if \( g \) is small enough. The claim follows. Q.E.D.
Proof of Proposition 10: Under cash-flow based compensation, if the manager decides to reveal (resp. conceal) $\tilde{g}_{t^*} = g$, the dividend path is given by $D^9_t$ (resp. $D^G_t$ until $T^{**}$). The expected utilities are, respectively:

$$U_{Div,t^*}^{reveal} = \int_{t^*}^{\infty} e^{-\beta(s-\tau^*)} (\eta_d D^9_s) \, ds = \eta_d e^{Gr^*} \left( \frac{z - g - \delta}{\beta - g} \right)$$ (57)

$$U_{Div,t^*}^{conceal} = \int_{t^*}^{T^{**}} e^{-\beta(s-\tau^*)} (\eta_d D^G_s) \, ds = \eta_d (z - G - \delta) e^{Gr^*} \left( \frac{1 - e^{-(\beta-G)h^{**}}}{\beta - G} \right)$$ (58)

A conceal equilibrium obtains if $U_{Div,t^*}^{reveal} < U_{Div,t^*}^{conceal}$, otherwise a reveal equilibrium obtains. Condition (38) follows from $U_{Div,t^*}^{reveal} > U_{Div,t^*}^{conceal}$ by rearranging terms.

Let parameters be such that a reveal equilibrium obtains at $\tau^*$. Then, for $t < \tau^*$, the manager with $w_t = \eta_d D_t$ has the same utility function as the manager/owner in Corollary 2, but scaled by $\eta_d$. It follows that the same incentives go through in this case, and the same result as in Corollary 2 applies. Q.E.D.

Proof of Proposition 11. (a) Truthful revelation constraint. The two utilities under reveal and conceal are

$$U_{t^*}^{Reveal} = \int_{t^*}^{\infty} e^{-\beta(s-t^*)} w_s^0 \, ds = A_a e^{(B_a + C_a)\tau^*}$$ (59)

$$U_{t^*}^{Conceal} = \int_{t^*}^{T^{**}} e^{-\beta(s-t^*)} w_s^b \, ds = A_b e^{B_b \tau^*} \frac{1 - e^{-(\beta-B_b)h^{**}}}{(\beta - B_b)}$$ (60)

It follows $U_{t^*}^{Reveal} \geq U_{t^*}^{Conceal}$ for all $\tau^*$ iff conditions (44) are satisfied.

(b) High effort constraint. For $i = H, L$ and $t < \tau^*$ we can compute

$$U_t^i = E_t \left[ \int_t^{\infty} e^{-\beta(s-t)} w_s (1 - c_s^i) \, ds | i \right]$$

$$= E_t \left[ \int_t^{\tau^*} e^{-\beta(s-t)} w_s^b (1 - c_s^i) \, ds + \int_{\tau^*}^{\infty} e^{-\beta(s-t)} w_s^b \, ds | i \right]$$

We obtain

$$U_t^i = \frac{A_b e^{B_b t} (1 - c^i)}{(\beta + \lambda^i - B_b)} + \frac{\lambda^i A_a e^{(C_a + B_a)t}}{(\beta - B_a)}$$

Thus, the high effort constraint $U_t^H \geq U_t^L$ translates into

$$\frac{A_b e^{B_b t} (1 - c^H)}{(\beta + \lambda^H - B_b)} + \frac{\lambda^H A_a e^{(C_a + B_a)t}}{(\beta - B_a)(\beta + \lambda^H - B_b)} \geq \frac{A_b e^{B_b t}}{(\beta - B_a)(\beta + \lambda^L - B_b)} + \frac{\lambda^L A_a e^{(C_a + B_a)t}}{(\beta - B_a)(\beta + \lambda^L - B_b)}$$

Tedious algebra shows that this constraint is satisfied for all $t$ iff conditions (45) are satisfied.

(c) Outside option constraint. For all $t$ and $\tau^*$ we must have $U_t \geq U_t^O$. For $t < \tau^*$, the earlier constraints obtained above imply $U_t^H$ is the relevant utility, which can be written as

$$U_t^H = \left[ \frac{A_b (1 - c^H)}{(\beta + \lambda^H - B_b)} + \frac{\lambda^H A_a}{(\beta - B_a)(\beta + \lambda^H - B_b)} \right] e^{B_b t}$$

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where we impose the earlier restriction that \( B_b = C_a + B_a \). We must have that for all \( t \), \( U_t^H \geq A O e^{B_0 t} \), which implies

\[
A_b (1 - e^H) + \frac{\lambda^H A_a}{(\beta - B_a)} \geq A_0 e^{B_0 t}
\]

This condition is satisfied for all \( t \) if and only if conditions in (46) are satisfied.

Similarly, for \( t \geq \tau^* \) the constraints above imply that \( U_t = U_t^{\text{Reveal}} \), given by (59). Thus, the outside option condition is

\[
\frac{A_a}{\beta - B_a} e^{C_a \tau^*} e^{B_a t} \geq A_0 e^{B_0 t}
\]

which is satisfied for all \( t \) and \( \tau^* \) if and only if conditions (46) are satisfied. Q.E.D.

**Proof of Proposition 12**: First, we need to compute the condition that guarantees a reveal strategy at time \( \tau^* \). The equilibrium price function to use in this calculation is \( P_{f,i,t}^{\text{before}}(\lambda^H) \) if conceal (i.e. the manager deviates), and \( P_{f,i,t}^{\text{after}} \) if it reveals. We obtain

\[
U_{\text{comb,}\tau^*}^{\text{reveal}} = \left( \omega \eta + \frac{1}{r - g} (1 - \omega) \eta_d \right) e^{G\tau^*} \left( \frac{z - g - \delta}{\beta - g} \right)
\]

\[
U_{\text{comb,}\tau^*}^{\text{conceal}} = \left( \omega \eta A_{\lambda}^f + (1 - \omega) \eta_d (z - G - \delta) \right) e^{G\tau^*} \left( \frac{1 - e^{-(\beta - G)h^*}}{\beta - G} \right)
\]

Let \( \eta_d = \eta/(r - g) \). At \( \tau^* \), (49) follows from \( U_{\text{comb,}\tau^*}^{\text{reveal}} > U_{\text{comb,}\tau^*}^{\text{conceal}} \), where

\[
\mathcal{L}_2 = \frac{z - g - \delta}{(\beta - g)(r - g)}
\]

Before \( \tau^* \), the expected utility under the combined package depends on both a dividend component and a stock component. For given effort \( e \), the dividend-based component is

\[
U_{\text{Div},t}(e) = E \left[ \int_t^{\tau^*} e^{-\beta(u-t)} (w_u (1 - c(e_u))) du + e^{-\beta(\tau^*-t)} U_{\text{comb,}\tau^*}^{\text{reveal}} \right]
\]

\[
= \frac{e^{Gt}}{\beta + \lambda (e) - G} \eta_d \left( (z - G - \delta) (1 - c(e_u)) + \lambda (e_u) \left( \frac{z - g - \delta}{\beta - g} \right) \right)
\]

The stock-based component, conditional on a reveal equilibrium and thus price \( P_{f,i,t}^{\text{before}}(\lambda^H) \):

\[
U_{\text{Stock},t}(e) = E \left[ \int_t^{\tau^*} e^{-\beta(u-t)} (w_u (1 - c(e_u))) du + e^{-\beta(\tau^*-t)} U_{\text{comb,}\tau^*}^{\text{reveal}} \right]
\]

\[
= \frac{e^{Gt}}{\beta + \lambda - G} \eta \left( A_{\lambda}^f (1 - c(e)) + \frac{\lambda}{r - g} \left( \frac{z - g - \delta}{\beta - g} \right) \right)
\]

Thus, the total combined utility before \( \tau^* \) is \( U_{\text{Comb},t}(e) = \omega U_{\text{Stock},t} + (1 - \omega) U_{\text{Div},t} \). Tidious computations show that (50) follows from \( U_{\text{Comb}}(e^H) > U_{\text{Comb}}(e^L) \), where

\[
\mathcal{L}_1 (\omega) = \omega A_{\lambda}^f + (1 - \omega) \left( \frac{z - G - \delta}{r - g} \right)
\]

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and $A_{λH}^f$ is in (12). Finally, given the behavior of the manager (High Effort/Reveal), the price function is $P_{fi,t}^{before}$ for $t < τ^*$ and $P_{fi,t}^{after}$ for $t ≥ τ^*$. Q.E.D.

**Corollary A1:** Let $λ$ be the equilibrium intensity. Then:

1. Let $C^{fi} = \left(\frac{z - g - δ}{r - g}\right)$. Under perfect information the average market decline at $τ^*$ is

   \[ E_0 \left[ \frac{P_{τ^*} - P_{τ^* -}}{P_{τ^* -}} \right] = \frac{(r - G)}{(z - G - δ)} C^{fi} - 1 \frac{λ}{(z - G - δ)} C^{fi} + 1 \]

2. Under asymmetric information the average market decline at $T^{**}$ is

   \[ E_0 \left[ \frac{P_{T^{**}} - P_{T^{**} -}}{P_{T^{**} -}} \right] = \frac{(r - G)}{(z - G - δ)} C^{ai} - 1 \frac{λ}{(z - G - δ)} C^{ai} + 1 \]

   where $C^{ai} = e^{-(G - g)t^{**}} C^{fi} - (1 - ξ) e^{-(G - g)t^{**}} < C^{fi}$

**Proof of Corollary A1:** See technical appendix. Q.E.D.

**Cost of CEO’s Compensation**

We approximate the total costs paid to the CEO under the various cases by computing:

\[ V_0 = E \left[ \int_0^∞ e^{-rt} w_t dt \right] \tag{61} \]

where $w_t$ is the CEO compensation. We obtain closed form formulas for the three types of compensation, stock-based, cashflow-based, and combined, separately below. The key insight is that we can choose the key parameters to make the present value of the total compensation for the manager practically identical under every model. Given $V_0$ for each case, we can compute the quantity $P_0 - V_0$, that is, the firm value net of payments to the CEO.

More specifically, we start by setting $η_d = 5\%$ as our benchmark value under the dividend-based compensation in the (Low Effort/Reveal) equilibrium. In this case, the CEO’s compensation equals 5% of firm value (see (62) in the appendix). Denote this compensation cost $V_{Div}^{reveal,L}$. Next, we compute the value of $η$ to make the compensation cost under stock-based compensation (High Effort/Conceal) equilibrium, $V_{Stock}^{Conceal,H}$, equal to the dividend-based (Low Effort/Reveal) equilibrium compensation cost $V_{Div}^{reveal,L}$. That is, such that $V_{Stock}^{Conceal,H} = V_{Div}^{reveal,L}$. Finally, given these two values for $η_d$ and $η$, we compute the cost for the combined compensation, denoted $V_{Comb}^{Reveal,H}$. In this latter case, we need to choose the weight $ω$ from the range of possible values (see Figure 9). We choose the minimum $ω$ that induces the CEO to exert costly effort, as in this case, when $G$ and $z$ are high, the combined compensation boils down to a dividend-based compensation with high effort (see bottom panel of Figure 9.)
Given the compensation costs, we then compute the net firm values in the three cases, namely, \( P_{\text{before,}fi,\lambda L,0} - V_{\text{reveal,}H} \) for dividend-based compensation, \( P_{\text{ai,}0} - V_{\text{Conceal,}H} \) for stock-based compensation, and \( P_{\text{before,}fi,0} - V_{\text{Reveal,}Comb} \) for the combined compensation. These quantities are plotted in Figure 10 as functions of the high growth rate \( G \) and for three values of return on investments \( z \). In all panels, the solid line corresponds to the combined compensation case, the dotted line to the stock-based (High Effort/Conceal) equilibrium, and the dashed line to the dividend-based (Low Effort/Reveal) equilibrium. The figure makes apparent two facts: First, inducing high effort increases the net firm value, especially for high growth companies. This is true for both the stock-based compensation, which has the conceal strategy behavior as a side effect, and the combined compensation. Second, the combined compensation equilibrium leads to a higher net firm value compared to both the other equilibria.

**Formulas for Cost of CEO Compensation:** We report here the expected discounted value of the compensation costs. The derivations are left to the technical appendix. Consider first the pure dividend-based compensation under full revelation and low effort. In this case, \( w_t = \eta_d D_t \) and therefore

\[
V_{\text{Reveal,}L}^{\text{Div}} = E \left[ \int_0^\infty e^{-rt} \eta_d D_t dt \right] = \eta_d A_{\lambda L}^{fi} \quad (62)
\]

where \( A_{\lambda L}^{fi} \) is given in (12). Similarly, the present value of all payments to the CEO under the high-effort, stock-based compensation and conceal equilibrium, requires \( w_t = \eta P_{\text{ai,t}} \), where \( P_{\text{ai,}t} \) is given by (23). We obtain

\[
V_{\text{Conceal,}H}^{\text{Stock}} = \eta (z - G - \delta) \left( \frac{1 - e^{-(r-G)h^{**}}}{r-G} \right)^2 + \eta A_{\lambda H}^{ai} e^{-(r-G)h^{**}} \left( h^{**} + \frac{z - G - \delta}{r+\lambda-G} \right) \quad (63)
\]

where \( A_{\lambda H}^{ai} \) is given in (24). Finally, the total costs under the full revelation / high effort equilibrium obtained from the combined compensation is given by:

\[
V_{\text{Reveal,}Comb}^{\text{Reveal,}H} = \omega \eta \left( A_{\lambda H}^{fi} + \lambda H \frac{z - g - \delta}{(r - g)^2} \right) \frac{1}{(r - G + \lambda H)} + (1 - \omega) \eta_d A_{\lambda L}^{fi} \quad (64)
\]

where \( A_{\lambda H}^{fi} \) is given in (12).
References


Table 1: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of capital ( r )</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Return on capital ( z )</td>
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<td>16%</td>
</tr>
<tr>
<td>Depreciation rate ( \delta )</td>
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<td></td>
</tr>
<tr>
<td>CEO discount rate ( \beta )</td>
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</tr>
<tr>
<td>Low growth rate ( g )</td>
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<td></td>
</tr>
<tr>
<td>Expected ( \tau^* ) (high effort)</td>
<td>( E[\tau^*</td>
<td>e^H] = 1/\lambda^H )</td>
</tr>
<tr>
<td>High growth rate ( G )</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>Expected ( \tau^* ) (low effort)</td>
<td>( E[\tau^*</td>
<td>e^L] = 1/\lambda^L )</td>
</tr>
<tr>
<td>Minimal capital level ( \xi )</td>
<td>80%</td>
<td></td>
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<tr>
<td>CEO Cost of Effort ( e^H )</td>
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<td>5%</td>
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</table>
Table 2: High Effort or Truthful Revelation?

<table>
<thead>
<tr>
<th>Panel A: Investment Opportunities Growth</th>
<th>Panel B: Expected ( \tau_L ) under Low Effort</th>
<th>Panel C: Return on Investment</th>
<th>Panel D: Shareholders Discount Rate</th>
<th>Panel E: Depreciation Rate ( \delta )</th>
<th>Panel F: Minimum Capital Requirement ( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>( P_{a1,0} )</td>
<td>( E \left[ \frac{P_{r+\tau}}{P_{r+\tau}'} - 1 \right] )</td>
<td>( E_{\text{before}}(\lambda^L) )</td>
<td>( E \left[ \frac{P_{r+\tau}}{P_{r+\tau}'} - 1 \right] )</td>
<td>( E_{\text{before}}(\lambda^H) )</td>
</tr>
<tr>
<td>5.00%</td>
<td>2.25</td>
<td>-53.53%</td>
<td>1.96</td>
<td>-4.22%</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Notes: Column 1 reports the value of the parameter that is changed from its base value in Table 1. Columns 2 and 3 report the firm value at \( t = 0 \) and the average stock decline at \( T^{**} \), respectively, under the (High Effort/Conceal) equilibrium induced by stock-based compensation. Column 4 and 5 report the firm value at \( t = 0 \) and the average stock decline at \( \tau^* \), respectively, under the (Low Effort/Reveal) equilibrium induced, e.g., by a flat wage compensation, or a cashflow-based (bonus) compensation. The last two columns report the same quantities under the (High Effort/Reveal) equilibrium induced by the optimal contract. Panels B, E, and F use \( c_H = 2\% \) instead of \( c_H = 5\% \) to ensure that all three types of equilibria exist under the parameters in Column 1.
Figure 1: Growth in Investment Opportunities. This figure reproduces the earnings profile $Y_t$ as a function of capital $K_t$, for three different time periods $\tau = 0$, $\tau = 1$ and $\tau = 4$. 
Figure 2: A dividend path (top panel) and a price path (bottom panel) under perfect information. We use the following parameters: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$, $\delta = 1\%$. 
Figure 3: The dynamics of capital and investments under reveal and conceal equilibrium after $\tau^*$ (normalized to 0 in this figure). This figure shows the capital dynamics (top panel) and investment dynamics (bottom panel) for a $g$ firm pretending to be a $G$ firm (dashed line), relative to the revealing strategy (solid line). The vertical dotted line denotes “default” time $T^{**}$. The following parameters are used: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$, $\delta = 1\%$, $\lambda = 1/15$, $\xi = .8$. 
Figure 4: Dividend dynamics and price dynamics in reveal and conceal equilibria. The vertical dotted line denotes time $\tau^*$ of the growth change from $G$ to $g$. The following parameters are used: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$, $\delta = 1\%$, $\xi = .8$, $\lambda = 1/15$. 
Figure 5: Conceal equilibrium under stock compensation. The figure reports the conceal and reveal equilibria areas under stock compensation. In all figures, the $x$–axis reports the initial high growth $G$. In the top panel, the $y$–axis is the low growth $g$, in the middle panel, the $y$–axis is the return on capital $z$; and in the bottom panel, the $y$–axis is given by the expected time to maturity $E[\tau^*] = 1/\lambda$. The base parameters are in Table 1.
Figure 6: Equilibrium Areas under Stock-Based Compensation and Cashflow-Based Compensation. In the \((z, G)\) space, the figure shows the areas in which the following equilibria are defined: (a) the high effort / revealing equilibrium under dividend-based compensation; (b) the low effort / revealing equilibrium under dividends based compensation; (c) the high effort / conceal equilibrium under stock-based compensation. For all combination of parameters, dividend compensation generates a reveal equilibrium. \(z\) ranges between 12\% and 30\%, while \(G\) ranges between 6\% and 16\%. The remaining parameters are in Table 1.
Figure 7: Dividend and Price Paths in Three Equilibria. The Figure plots hypothetical dividend (top panel) and price (bottom panel) paths under the case of “Stock-Based Compensation” (solid line); “dividend-based Compensation” (dotted line); and the first best Benchmark Case with Symmetric Information and Optimal Investment (dashed line). The parameters are in Table 1.
Figure 8: Optimal Contract versus Stock-based Compensation. The Figure plots compensation paths under the optimal contract and a simple stock-based compensation (left panels) and a combined compensation with both stocks and cashflow (bonus) components (right panels). The vertical dotted bar corresponds to the an hypothetical occurrence of $\tau^*$. The figure reports three sets of panels, each pair corresponding to a different effort cost $c_H$. The remaining parameters are in Table 1. The stock-based compensation and combined compensation are normalized to provide the same intertemporal utility to the CEO as the optimal contract at time 0, conditional on full revelation.
Figure 9: Optimal Weight $\omega$ on Stocks in Compensation Package. This figure reports the range of weights on the stock component of the combined compensation package that induces the first best for shareholders, that is, the high effort / reveal equilibrium. In each panel, which only differ for the level of return on capital $z$, the top line is the maximum $\omega$ that still induces the manager to reveal the shift in investment opportunities, while the bottom line is the minimum $\omega$ that induces the manager to exert high effort. The remaining parameters are in Table 1.
Figure 10: Firm Value Net of CEO’s Incentive Contract Cost. This figure compares the firm value net of the CEO incentive contract costs in the first best equilibrium under the combined compensation package (solid line) to the firm value under (a) dividend compensation when CEO exerts low effort (dashed line), and (b) stock-based compensation when CEO exerts high effort but conceals the worsening of investment opportunities at $\tau^*$ (dotted line). Each panel corresponds to a different return on capital $z$. The combined package in each panel is the one corresponding to the minimum weight $\omega$ to stock that still induces the CEO to exerts high effort. $\eta_d = 5\%$ while for each panel $\eta_p$ is chosen so that the cost to the firm under case (a) and (b) is the same, and thus differs across $G$ and $z$ cases. The remaining parameters are in Table 1.