Lecture 3A

Behavior Under Uncertainty:
The Economics of Imperfect Information

To this point our discussion of demand has assumed that information is perfect, so an individual makes decisions among goods with known qualities. Of course information is not typically perfect—do we really know what that banana will taste like or whether that book is really a good read?—but uncertainty in these cases is not central to the analysis. So we ignore it. But there are many market outcomes in which uncertainty about outcomes is the most important feature. Why else would there be a market for insurance, which is nothing more than a contract that promises to pay off if certain events occur—like if your house burns down—and to pay nothing otherwise?

This lecture develops tools for tackling problems in which imperfect information is central to the analysis. We begin with the concept of *risk aversion*, which means that people often prefer certain outcomes to uncertain ones with the same expected value. We use this tool to understand the demand for insurance, portfolio diversification, and other aspects of risk avoidance. We then extend the analysis to allow for *moral hazard*, which means that a person with insurance may not take sufficient care to avoid a loss. This helps us to understand deductibles in insurance policies and certain forms of compensation in organizations, such as incentive pay. Finally we analyze issues of *asymmetric information*, which means that one party to a trade may have superior knowledge about characteristics of the good being traded.

I. The Saint Petersburg Paradox

Consider a simple lottery that pays $150,000 with probability $\frac{1}{2}$ and $50,000$ with probability $\frac{1}{2}$. The expected income from the lottery is $E_y=\frac{1}{2}($150,000$)+\frac{1}{2}($50,000$) =$100,000. What is the largest amount that a typical (rich) person would be willing to pay for a lottery ticket—that is, for the right to play?

Some would answer “$100,000” because that is the expected income from the game. This suggests a sort of rule that we might use as a model for how people value uncertain prospects, such as a lottery, a career, a business venture or a share in a publicly traded
corporation. Compute the expected income from the prospect, and that is what people will pay for it. Let’s call this idea “The Expected Income Hypothesis” (EIH). If you find that an attractive model, an old mathematician name Daniel Bernoulli would like to sell you something.

Bernoulli was court mathematician to the Czar of Russia, located in St. Petersburg. (Then as now, academics were sort of like court jesters). Lacking anything useful to do, he devised a game that went as follows: Flip a coin. If it comes up heads, which happens with probability $\frac{1}{2}$, the payoff is $2. If it comes up tails, flip again. If it then comes up heads—the probability of the sequence (T,H) being $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$—the payoff will be $4. If the first two flips are tails, flip again, and if it comes up heads on the third flip—the probability of (T,T,H) is $\frac{1}{8}$—then they payoff is $8$. And so on. The expected income from this game is

\[
Ey = \frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \frac{1}{16} \times 16 + \ldots
\]

which is a big number.

Bernoulli tried to shop his game around, hoping some adherent of the EIH would pay him an infinite amount to play. But even the Czar, who had lots of money, demurred. Bernoulli concluded that people did not behave in a way that is consistent with the EIH, so he had to come up with something better. As he reached this conclusion in St. Petersburg, the game described above came to be known as the “St. Petersburg Paradox.”

II. Attitudes toward Risk: The Expected Utility Hypothesis

Bernoulli’s resolution of this paradox was to posit that people did not value an uncertain outcome in terms of the income, $y$, received if the outcome occurred. Instead, they cared about the “utility” $u(y)$, that they derived from the income. If this utility was bounded above by some number, $\bar{U}$, like the concave $u(y)$ in Figure 1, then Bernoulli’s game turns out to have finite value. So Bernoulli posited that people care about the expected utility that they derive from a risky prospect, not the expected income of the prospect. Bernoulli is credited with the first statement of what came to be known as the Expected Utility Hypothesis, which is an extraordinarily useful tool. Here is what you have to know about it.
Suppose that a person receives an income of $y$. We denote the happiness, or utility that the person receives from this income by $u(y)$, and we assume that if $y_1 > y_2$ then $u(y_1) > u(y_2)$: more money makes people happier because they can consume more. Now imagine a “lottery” that pays $y_1$ with probability $p$ ($0 < p < 1$) and $y_2 < y_1$ with probability $1 - p$. If the “good” outcome occurs, the person receives $y_1$ and his utility is $u(y_1)$. If the “bad” outcome occurs he receives $y_2$ and his utility is $u(y_2)$. His expected utility is the utility he gets on average, or

$$ Eu = pu(y_1) + (1 - p)u(y_2). $$

According to the expected utility hypothesis, equation (1) tells us how the person “values” a risky prospect that pays $y_1$ with probability $p$ and $y_2$ with probability $1 - p$. More generally:

**The Expected Utility Hypothesis:** Suppose an individual faces a prospect that pays uncertain rewards $y_1$ with probability $p_1$, $y_2$ with probability $p_2$, ..., $y_n$ with probability $p_n$. Then the expected utility that the individual receives from the prospect is

$$ Eu = p_1u(y_1) + p_2u(y_2) + \ldots + p_nu(y_n) $$

In choosing between two prospects, the expected utility hypothesis says that the one with the larger expected utility is preferred.

For example, if an alternative prospect, call it A, offered the same payoffs, $y_1 > y_2$, but the probability of receiving $y_1$ was $p_A > p$, then prospect A would be preferred because it offers
larger expected utility (Why?). Notice that this is a hypothesis about the way people value uncertain things – it is useful to the extent that it explains the behavior we are interested in.

**Risk Aversion: Diminishing Marginal Utility of Income**

In the 1940s the economists John von Neumann and Oskar Morgenstern built on Bernoulli’s ideas to study behavior toward risk. Attitudes toward risk depend on the form of the utility function, \( u(y) \). Consider Wilma’s utility function shown in Figure 2. The horizontal axis in the figure measures amounts of income, \( y \), and the vertical axis measures the utility derived from each level of income, \( u(y) \). The curve labeled \( u(y) \) indicates that greater income yields greater utility. But the declining slope of \( u(y) \) indicates that each additional dollar of income yields smaller marginal utility than the dollar before it. This is called **diminishing marginal utility of income**.\(^1\)

![Figure 2. Diminishing Marginal Utility of Income: \( u'(y_1) > u'(y_2) \)](image)

Now let’s consider the implications of diminishing marginal utility for attitudes toward risk. Suppose that Wilma has a riskless income of \( \bar{y} = $50,000 \) per year. Her utility is then \( U = u(\bar{y}) \), corresponding to point \( D \) on \( u(y) \). Now Barney offers Wilma a simple gamble. Barney will flip a coin, and if it comes up heads (the “good” state of nature) Wilma will win \( w = $10,000 \), in which case her consumable income is \( y^g = $60,000 \). If the coin comes up tails (the “bad” state of nature) Wilma will lose \( l = $10,000 \) and her consumable income is \( y^b = $40,000 \). The coin is fair, so the probability of the good state is \( p_g = 1/2 \). Wilma’s expected monetary gain from playing the game is

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\(^1\)Mathematically, this means that the second derivative of \( u(y) \) is negative: \( u''(y) < 0 \)
We say that Wilma faces a “fair” gamble because her expected winnings are equal to her expected losses.

**Definition: Fair Gamble**

A gamble is called “fair” if expected winnings are equal to expected losses.

This means that Wilma’s expected income if she takes the bet is exactly $50,000:

\[
Ey = p_g (\bar{y} + w) + (1 - p_g)(\bar{y} - l) \\
= \bar{y} + p_g w - p_g l \\
= \bar{y} + 1/2(10,000) - 1/2(10,000) \\
= \bar{y}
\]

The question is: Will Wilma take a fair bet? In other words, which does Wilma prefer:

a. A riskless prospect in which she can consume $\bar{y}=$50,000 for sure, or
b. A risky prospect in which her expected income is $Ey = \bar{y}=$50,000.

With diminishing marginal utility of income Wilma always prefers (a), the riskless prospect, because the risky prospect offers lower expected utility. This fact is demonstrated in **Figure 3**.

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**Figure 3**

Risk Aversion: With DMU, $u(Ey) > Eu(y)$
Figure 3 shows the utilities derived from the potential rewards $y^0_b$ and $y^0_g$, labeled $u(y^0_g)$ and $u(y^0_b)$. According to the expected utility hypothesis, Wilma’s expected utility is:

$$U^0 = Eu = p_g u(y^0_g) + p_b u(y^0_b)$$

This is just a probability-weighted-average of the two utilities. We can find this value graphically as follows. First, find point A on $u(y)$ directly above $y^0_b$, and point B directly above $y^0_g$. The height of $u(y)$ at these points represents utility at each potential level of income. Next, draw a straight line connecting points A and B, as shown in the figure. Since expected utility is the probability-weighted average of points A and B, $Eu$ is exactly $p_g$ percent of the way from to A from B along the connecting line AB. This is shown as point C, which is halfway in our example because $p_g = 1/2$. It also must be directly above $Ey$ on the horizontal axis, which is halfway between $y^0_b$ and $y^0_g$.

Height D corresponds to $u(Ey) = u($50,000). Height C corresponds to $U^0 = Eu$. By inspection, $u(Ey) > Eu(y)$—the utility from expected income is greater than expected utility from the fair gamble, so Wilma will never take the fair bet. Why won’t she take the bet? Since Wilma has diminishing marginal utility of income, the extra utility she gains when she wins $10,000 is smaller than her reduction in utility when she loses $10,000. So even though the risky prospect left her expected income unchanged, it reduced her expected utility because Wilma has diminishing marginal utility of income. This is a general result:

**Fact: Diminishing Marginal Utility and the Expected Utility of a Fair Gamble**

Consider a risky prospect with expected income $Ey = p_g y^0_g + p_b y^0_b$ and expected utility $Eu = p_g u(y^0_g) + p_b u(y^0_b)$. If $u(y)$ has diminishing marginal utility of income like in Figure 1, then

$$u(Ey) > Eu(y)$$

The utility of expected income is bigger than the expected utility of income.

When offered a choice between $Ey$ as a sure thing and a gamble that pays $Ey$ on average, a person with diminishing marginal utility of income will always take the sure thing. Because of this, we say that a person with diminishing marginal utility of income is risk averse.

**Definition: Risk Aversion**

A person with diminishing marginal utility of income is said to be risk averse. A risk averse individual will not accept a fair bet.

**Question 3.A.1:** Suppose that Barney increases the stakes, so that $w=l=$ $20,000. How does this affect the variance of income in the gamble? How does it affect Wilma’s expected utility from
the gamble? What do you conclude from your answer?

**Question 3.A.2:** Suppose that Wilma is offered a choice between: (a) \( y^g = $60,000 \) and \( y^b = $40,000 \), each with probability \( .5 \); or (b) a certain income of \( $47,000 \). Which will she prefer? Graphically, show the lowest certain income, \( y^C \), that Wilma would prefer to the gamble in (a).

**Question 3.A.3:** We have demonstrated that Wilma will never accept a fair bet if she is risk averse. But Wilma accompanies Fred to Las Vegas and plays the slot machines—surely an unfair bet if the casino expects to make money. Why would risk averse Wilma do this?

The idea that people are risk averse helps us understand why people buy insurance. To see how this works, let’s change some of the labels we put on the outcomes in Figure 3. Suppose now that Wilma faces a particular risk—say the theft of her car. In the “good” state of nature her car is not stolen and she has income \( y^g = $60,000 \) that can be spent on goods and services. If her car is stolen—the “bad” state of nature—she incurs a loss of \( L = $20,000 \) and her spendable income is \( y^b = y^g - L = $40,000 \). If the probability of a loss is \( p_b = 1 - p_g = \frac{1}{2} \) (Wilma lives in Hyde Park) then Wilma’s expected income is simply

\[
Ey = p_g y^g + p_b y^b = y^g - p_b L
\]

\[
= $60,000 - 0.5($20,000) = $40,000
\]

so all the payoffs are exactly as above. Graphically, \( Ey \) is shown as a point between \( y^g \) and \( y^b \) on the horizontal axis of Figure 3. For example, if \( p_b = .5 \) then \( Ey \) is halfway between the two potential payoffs. If \( p_b = .75 \) it is one-fourth of the way to \( y^g \) from \( y^b \), and so on. \( Ey \) in the figure is drawn under the assumption that \( p_b = .5 \).

Now suppose that Wilma is offered an “insurance policy” with the following properties. First, she must pay a premium equal to her expected loss: \( \Pi = y^g - Ey = p_b L = $10,000 \). Then if her car is not stolen—the good state—her spendable income is \( y^g = y^g - \Pi = Ey = $50,000 \). If her car is stolen—the bad state occurs—the policy pays a benefit of \( B = L \); that is, her car is replaced. Wilma’s spendable income when her car is stolen is

\[
y^b = y^g - L - \Pi + L = Ey = $50,000.
\]

So if Wilma buys the policy her spendable income is \( Ey = y^g - p_b L \) whether her car is stolen or not. Such an insurance policy is called “actuarially fair” because the premium collected by the insurance company is equal to the expected benefit that it must pay: \( \Pi = p_b L \).

**Definition: Actuarially Fair Insurance**

An insurance policy is called *actuarially fair* if the premium is equal to the expected benefit.
If Wilma’s behavior is governed by the expected utility hypotheses, will she buy the actuarially fair insurance policy? She will. If she does not buy the policy, her expected utility is $U^0$, which is the height of point $C$ in the figure. If she buys the policy her consumption is $Ey$ for certain, so her certain utility is $U^1 = u(Ey)$, shown as height $D$. As $u(Ey) > Eu$, Wilma would rather purchase the actuarially fair insurance policy than bear the risk of having her car stolen. Therefore, a risk averse person will always purchase actuarially fair insurance.

The utility function in Figures 2 & 3 is the identifying characteristic of risk aversion. So remember: A person with diminishing marginal utility of income is risk averse. Intuitively, a risk averse person gains less happiness from a $100 increase in his income than he loses from a $100 reduction in his income. Because of this, he avoids risks in which the expected monetary gain if he wins is exactly the same as the monetary loss if he loses. This means that “a risk averse person will never accept a fair bet.”

**Risk Neutrality: Constant Marginal Utility of Income**

Of course an individual does not have to be risk averse. A person who is risk neutral doesn't care about risk. Such a person is depicted in Figure 4, where the utility function is the straight line $u(y) = y$: each additional dollar of income gives constant amount of additional happiness. Intuitively, a risk neutral person gains just as much happiness from a $100 increase in his income as he loses from a $100 reduction in his income. Because of this, a risk neutral person is indifferent between the uncertain prospect that pays $Ey$ on average and the alternative scheme that pays $S = Ey$ with certainty.

**Risk Neutrality:** If offered a choice between a gamble with expected payoff $Ey$ and a “sure thing” that pays $\overline{y} = Ey$ with certainty, a risk neutral person will be indifferent.
This means that a risk neutral person only cares about his expected income; risk doesn't matter. Remember this: a risk neutral person has constant marginal utility of income.

**Question 3.A.5:** What happens in the previous analysis if Wilma’s marginal utility of income is
increasing, as in Figure 4b? Would Wilma accept a fair bet? Why?

III. The Demand for Insurance

A Graphical Analysis Using Indifference Curves and Budget Constraints

I told you earlier that one “trick” to doing economics is to make the problem you are working on look like something you already know how to do. Doing that here can help us understand more about the role of risk aversion and the market for insurance.

Let’s go back to risk-averse Wilma and the problem of theft insurance for her car. Wilma’s income is $y_g = $60,000. With probability $p_b$ her car is stolen with a resulting loss of $L = $20,000, so her income in the bad state of nature is $y^0_b = y^0_g - L = $40,000. Her expected income is a probability-weighted average of $y^0_g$ and $y^0_b$:

$$\bar{y} = p_g y^0_g + p_b y^0_b = y^0_g - p_b L$$

Now suppose Wilma is offered actuarially fair insurance. The nature of insurance is that for every dollar of benefits (an increase in income) in the bad state, $\Delta y_b = $1, Wilma must pay an insurance “premium” in the good state (a reduction in income), $\Delta y_g < 0$. The premium (the price of insurance) is *actuarially fair* if the expected premium she pays for each increment of coverage is equal to the expected benefit she receives:

$$p_g \Delta y_g = -p_b \Delta y_b$$

$$\pi_b^F = \frac{-\Delta y_g}{\Delta y_b} = \frac{p_b}{p_g} = \frac{p_b}{1 - p_b}$$

So we can think of “income in the bad state” as a good that Wilma purchases on the insurance market for an actuarially fair price. For every dollar of bad state income that Wilma wants to purchase, she must sacrifice $\frac{p_b}{1 - p_b}$ dollars worth good state income, so $\pi_b^F = \frac{p_b}{1 - p_b}$ is the fair “price” of insurance, or the actuarially fair premium per dollar of coverage: the greater the probability of a loss, $p_b$, the higher will be the price of insurance. At this price, how much insurance will Wilma purchase?

Look at Figure 5. In the figure, $y_g$ and $y_b$ are two different goods. Absent any insurance, Wilma must consume at point $A = (y^0_b, y^0_g)$ with expected utility:
The combinations of good-state income $y_g$ and bad-state income $y_b$ that yield expected utility $U^0$ lie along an indifference curve labeled $U^0$ passing through point $A$. We know that the slope of this indifference curve at $A$ measures willingness to pay for the good along the horizontal axis ($y_b$) in terms of the good on the vertical axis ($y_g$). That is, it measures Wilma’s marginal value of an additional dollar of insurance coverage. To find the slope, we increase $y_b$ by a small amount $\Delta y_b > 0$, and solve for the offsetting $\Delta y_g < 0$ that leaves expected utility unchanged. Denoting the marginal utility of income by $u'(y)$, we have:

$$\Delta Eu = p_g u'(y_g^0) \Delta y_g + p_b u'(y_b^0) \Delta y_b = 0$$

or

$$MV_b = -\left.\frac{\Delta y_g}{\Delta y_b}\right|_{y_b^0} = \frac{p_b u'(y_b^0)}{1 - p_b u'(y_g^0)}$$

Wilma’s marginal value of coverage is equal to the odds of a loss times the ratio of the marginal utility of a dollar in the bad state to the marginal utility of a dollar in the good state. If $y_b < y_g$, we know that $u'(y_b) > u'(y_g)$ because Wilma has diminishing marginal utility of income. So:

$$U^0 = p_g u(y_g^0) + p_b u(y_b^0)$$
Wilma’s marginal value of additional coverage is larger than its (fair) price so long as \( y_b < y_g \).

Faced with an actuarially fair price of insurance, Wilma will continue to buy coverage until 
\[ y_b = y_g = Ey \]; that is until she no longer faces any risk. So, risk averse Wilma will purchase full coverage if the price of insurance is actuarially fair.

**The Demand for Actuarially Fair Insurance**

If the price of insurance is actuarially fair, a risk averse person will purchase full coverage. That is, she will insure away all risk.

**Question 3.A.6:** An insurer that offers actuarially fair insurance is acting as if it is risk-neutral—all it cares about is expected premiums and expected benefits. But insurance companies are owned by people who insure their cars, houses, and lives, among other things. In other words, they are risk averse just like you and me. So why can insurance companies offer nearly-fair insurance?

Actuarially fair insurance policies earn zero profit on the policy itself, which leaves nothing for the insurer to cover other costs of operation. So the price of insurance has to be slightly unfair, \( \pi_b > \pi_b^F \), as shown in Figure 6. Then a risk averse individual will purchase “incomplete coverage”—she will still bear some risk because \( y_g^* > y_b^* \). We might interpret the difference between \( y_b^* \) and \( y_g^* \) as a “deductible”—so this is one reason why insurance policies contain deductibles: the price is high enough to cover costs. Of course, with competition insurance premiums will be driven down to the level that is just sufficient to cover costs, so they may be quite close to the actuarially fair rate.
IV. Asymmetric Information: Moral Hazard, Adverse Selection, and Signaling

“Asymmetric information” means that one party to a transaction has relevant information that is not available to the other party. There are two effects of asymmetric information that are relevant to our discussion. One is called moral hazard – where actions by one party are unobserved by the other – and the other is called adverse selection – where characteristics known by one party are unknown by the other. The 2002 Nobel Prize was given to three economists (Joe Stiglitz, George Akerlof, and Michael Spence) for their work on the economics of asymmetric information.

Moral Hazard

“Moral hazard” means that an insured person has some control over the probability of a loss, and may not take proper care to avoid a loss. For example, we found that if the price of insurance is actuarially fair, Wilma will buy an insurance policy that replaces the full value of her car if it is stolen. Then she has little incentive to lock the car, or to park it in a garage where it won’t be stolen. These actions are not directly observed by the insurance company, though the insurance company recognizes that rational people will act that way. Then the insurance company would like to design policies that mitigate the incentive for moral hazard.

Some incentive to avoid a loss can be restored if the insurance company offers policies with a deductible minimum. That is, the insurance policy will replace the full value of the car above some minimum amount, say $1000. The insured person still bears some risk. If the car is...
stolen the insured individual is out $1000, which gives her some incentive to lock it and park it in safe places. In this way, the design of the policy indirectly serves to control the behavior of the insured person. This is one reason why homeowner’s insurance, health insurance, accident insurance, and fire insurance (among nearly all others) all contain deductibles.

Moral hazard comes up in a number of business contexts, which are typically gathered under the heading of principal-agent problems, or simply agency problems. An agency problem occurs when one party, called the principal, delegates authority to another party, called the agent, to take actions that affect the principal’s welfare. The agent’s actions, however, cannot be directly observed by the principal, so there is asymmetric information—the agent knows what his actions and opportunities are, but the principal does not. The insurance problem just discussed is one example of an agency problem: the insurance company (the principal) must design a policy for an insured person (the agent) that takes account of the fact that only the agent knows her true actions. Here are some other examples:

1. An employer (the principal) delegates authority to a risk-averse salesman (the agent) to make sales calls on clients. Sales are more likely when the salesman works hard, but sales are always uncertain. The salesman’s effort in making a sale is unobservable, so there is asymmetric information. If the employer pays the salesman a salary, independent of whether a sale is made, then the salesman bears no risk of failure but also has no incentive to expend sales effort—there is moral hazard. If the employer pays the salesman a commission based on the amount of sales that he makes the salesman must bear some risk. So a higher commission rate increases incentives (a benefit) but also increases risks for risk-averse salesmen (a cost). The optimal commission rate balances the benefits of greater incentives against the costs of greater risks.

2. Shareholders (the principals) delegate authority to a risk-averse CEO (the agent) to run their company. The value of the company depends on the CEO’s actions—which are partly unobserved—and on risky market circumstances. If the CEO is paid a salary, then he has little incentive to maximize the value of the company; there is moral hazard. If he is paid entirely in company stock, he bears substantial risk. The optimal compensation package balances the benefits of ownership (strong incentives) against the risks imposed on the risk-averse CEO.

3. Automobile manufacturers like Toyota (the principal) often allow small independent (risk averse) parts suppliers (the agents) to pass through some of their cost overruns in making parts that are specific to Toyota. If Toyota did not allow some pass-through, then suppliers would face substantial risks. If Toyota allowed all costs to be passed on, suppliers would have no risk but also little incentive to keep costs down (moral hazard). The optimal supply contract balances these forces.

**Adverse Selection**

Another effect of asymmetric information is adverse selection. This means that an offer to trade under particular terms, such as price, attracts exactly the people that you don’t want to trade with. This can occur when one party to a trade – either buyer or seller – knows something
about the good being traded that the other party does not.

The effect of adverse selection in insurance markets is best explained by an example. Consider the market for health insurance, which pays peoples’ medical bills. When a person takes ill, he incurs medical costs of $L = 10,000$ (the loss). Suppose that there are two types of people who might buy insurance: Sick (S) people have probability $p_S = .4$ of incurring a loss, while healthy (H) people have probability $p_H = .1$. Finally, assume that half of the population is type S, and half is type H.

If insurance companies could tell the difference between Hs and Ss (say with a health exam) then they could offer fair insurance to each group with premiums $P_H = p_H \times 10,000 = 1000$ for the type H people and $P_S = p_S \times 10,000 = 4000$ for the type S people. (Some of this type of screening obviously occurs). Type S’s would pay more because they are more likely to generate a loss. But suppose that (a) the insurance company cannot determine who is an S and who is an H; and (b) the individuals themselves know their type. This is a case of asymmetric information in which the buyers of insurance have better information than sellers.

Since buyers cannot be told apart an insurance company that offered a policy that would break even on the H’s will lose money—all the sick people would buy the policy. An alternative possibility is that insurers offer policies that would break even if the population of Hs and Ss both bought policies. Since half are of each type, the break even premium would be $P = \frac{1}{2} (4000) + \frac{1}{2} (1000) = 2500$. But this premium is more than double the expected loss of type H people. Unless they are extremely risk averse, they will not buy. This means that the actual buyers of the policy will be entirely type S people, with expected loss equal to $4000$. Since the premium is smaller than the expected loss, this type of policy cannot survive in a private insurance market; insurers lose money. *Adverse selection*—the policy attracted only sick people—doomed the policy. In fact, it may be the case that the only possible policy to offer is one that insures the type S’s, at a premium of $4000$, driving the type H’s out of the insurance market.

Though it’s beyond the scope of our analysis, asymmetric information and adverse selection can also generate more sophisticated premium structures that cause people to self-select to particular types of policies. Often insurance companies offer policies that have a low premium for small amounts of coverage, with rising premiums per dollar of coverage as people buy more. Why? Insurance companies know that people who want more coverage are more likely to be sick, so they demand a higher premium. The outcome is that healthy people buy low-premium policies with a big deductible (coverage), while sick people buy high-premium policies with a small deductible (greater coverage). The pricing policy causes people to self-select based on their type. (Notice that even here, adverse selection means that healthy people cannot buy full coverage at an actuarially fair rate. Why?)

**Signaling**

A final example of asymmetric information, adverse selection and sorting has to do with why people invest in *signals*. A signal is an acquired thing— for example a characteristic or
credential – that tells other people something about the person with the signal. The need for the signal is caused by asymmetric information: I know something about myself that I want others to know. For example, a person may wear a Rolex watch in order to tell others that he can afford such things (“Look at me. I’m rich and tasteful.”). A key point, however, is that the signal is useless if everyone can acquire it. If Rolexes were cheap, then they would be useless as a signal of wealth.

Education is often alleged to have important signaling value. Why are you studying for an MBA? One view is that you are obtaining new skills that will raise your productivity with future employers. But another, perhaps more cynical view is that nothing you learn at Booth will do anything for your productivity – education does not raise productivity. People who can earn an MBA from the Booth School would have been more productive anyway, and employers use the degree as a “signal” of who is productive. Since productive people get paid more, you have an incentive to acquire the MBA signal – it doesn’t raise your productivity, but it tells employers that you will be more productive than other people. Then Booth’s role is to certify the most productive people by making them perform useless tasks that others can’t do.

To see what is necessary to get this view of education (or other types of signals) to “work,” let’s do an example. Suppose there are 2 types of people, Quicks (Q) and Slows (S). As shown in Table 1, all Q’s have a marginal product of 2, and all slows have a marginal product of 1. If types were known to employers, then Q’s would get a wage of W=2, and S’s would get a wage of W = 1. But we assume that employers cannot tell the Q’s from the S’s ex-ante. There is asymmetric information: the Q’s and S’s know their own types, but potential employers do not.

<table>
<thead>
<tr>
<th>Worker Type</th>
<th>Population Share</th>
<th>Marginal Product</th>
<th>Cost per unit of Signal</th>
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<tr>
<td>S</td>
<td>( \lambda )</td>
<td>1</td>
<td>( Y )</td>
</tr>
<tr>
<td>Q</td>
<td>1-( \lambda )</td>
<td>2</td>
<td>( \frac{y}{2} )</td>
</tr>
</tbody>
</table>

The proportion of S’s in the population is \( \lambda \). Absent any method of distinguishing Q’s from S’s, firms would pay workers their expected marginal product, \( EMP = (1-\lambda)2 + \lambda 1 < 2 \). Of course Q’s don’t like this. If they could just tell employers (truthfully) who they are, then they could get a wage of 2. To do this, they need a signal.

Suppose such a signal exists, and suppose that employers believe that anyone with \( y^* \) units of the signal (an MBA) is a type Q worker, with MP = 2. Now, if everyone has the same cost of getting the signal, and if people with the signal get paid 2, then everyone will get it, including the S’s. Then employers’ original belief (only Q’s have the signal) is wrong, and the signal doesn’t work. For the signal to work, it must be the case that only the Q’s choose to obtain it. For this to be the case, the cost of obtaining the signal must be lower for them. In the
In the table, we assume that the cost of obtaining y units of the signal for the S’s is y, but the cost for the Q’s is y/2. So Q’s are better at obtaining the signal.

The sequence of events is

Beliefs → Wage Offers → Actions → Outcome → Beliefs

Here “Beliefs” refers to what employers believe about the productivity of persons with the signal, for example: “Only type Q people have an MBA.” These beliefs then determine “Wage Offers” that employers are willing to make. Based on how much employers pay to persons with the signal, Q’s and S’s undertake an “Action” i.e. whether to get the signal or not. Then the “Outcome” is data on who has the signal. If in fact it is the Q’s who have it, and the S’s do not, then the original beliefs have been confirmed. Then we have a signaling “equilibrium” in the sense that employers have no reason to change their beliefs.

Let’s see what happens:

### Beliefs

- If $y < y^*$ then type S
- If $y > y^*$ then type Q

### Actions

**Type S:**

- If obtain $y = y^*$ then income $Y = 2 - y^*$
- If obtain $y = 0$ then income $Y = 1$
- Choose $y = 0$ if $y^* > 1$

**Type Q:**

- If obtain $y = y^*$ then income $Y = 2 - y^* / 2$
- If obtain $y = 0$ then income $Y = 1$
- Choose $y = y^*$ if $y^* < 2$

So if $1 < y^* < 2$ the type Q workers set $y = y^*$ and the type S workers set $y = 0$ – just as employers expected. Only the type Q workers obtain the signal. Beliefs are confirmed, and we get an equilibrium because there is no reason for employers to change their beliefs.

For our purposes, there are two things to note about this example:
1. The signal only works because it is cheaper for the type Q’s to obtain it. The point of the signal is to distinguish among types, and it only works as a sorting device if the Q’s get it but the S’s don’t. In the case of schooling, it is easier (less costly) for a smart person to get it, so employers may use schooling as a signal of ability.

2. Socially inefficient investment in the signal is privately optimal. No one’s productivity was affected by the signal. In fact, the total value of social output was unchanged, because all productivities were unchanged. This means that signal is a social waste, even though it is privately rational to obtain it, because society’s resources were devoted to producing it. This is actually the view of some critics of education – it allows the elite to distinguish themselves, raising their incomes and reducing the incomes of others, but it has no social value.

The signaling model just outlined is a good explanation for why people do certain things, like wear Rolex watches or get tattoos. But it is surely not the main reason that people obtain schooling and advanced degrees. Indeed, empirical tests show that persons with more schooling earn more even when there is no opportunity to signal – more educated farmers are more productive, as are self-employed persons with more schooling. Further, we expect that employers would learn a person’s true productivity as the person participates year after year in the labor market. This ability to learn surely eliminates the value of education as a signal after some, probably short, period of time. (And empirical evidence indicates just that.) In other words, the main reason that people invest in education is that it raises wages by raising productivity. Education produces human capital. Even so, there is probably a grain (but just a grain) of truth to the signaling model of education, at least early on in a career. Then an MBA may have some aspect of a “credential,” quite apart from what you learn.

Problems
Lecture 3A

1. If people are risk averse, why do they buy risky stocks?

2. The salespeople at Marshall Fields are risk averse. In spite of their efforts, some days they sell lots of stuff, and other days they sell hardly anything.
   a. If the activities of Fields’ salespeople are perfectly observable and easily monitored by management, how should they be paid: by a salary or by commissions? Why?
   b. If the activities of Fields’ salespeople cannot be easily observed or monitored by management, but they amount they sell is known, how should they be paid: by a salary or by commissions? Why?

3. Suppose that “Healthys” (H) exercise regularly while “Sloths” (S) do not. Hs and Ss have identical preferences $u(y)$ and identical incomes, $y^0$, differing only in their probabilities of becoming ill: $p_H < p_L$. If either type becomes ill he incurs medical
expenses of $C. Using a figure or figures like Figure 3.:

a. On the same figure, draw the indifference curves of Healthy and Sloth individuals passing through the combination \((y^0 - C, y^0)\). Do the indifference curves have the same shape? Why? How can they differ if Healthy and Sloths have the same utility function?

b. If insurers are able to identify the Healthy and the Sloths before selling them policies—say by taking their blood pressure—show the budget constraints corresponding to fair insurance offered to each type. Do both types buy insurance? How much do they buy? Is one group better off than the other? Why?

c. In part (b), if the Sloths could purchase the policy offered to the Healthy, would they? Why?

d. Now suppose insurers cannot tell Sloths from Healthy. A clever insurer comes up with the following idea. “I will offer an \(S\)-policy with a premium equal to the fair price for Sloths, \(\pi_S\). I will offer an \(H\)-policy with a premium equal to the fair price for the Healthy, \(\pi_H\). As \(\pi_H < \pi_S\), I know that the Sloths will prefer the \(H\)-policy, so I will impose a limit (a deductible) on the amount of insurance that anyone can buy at the lower price, \(\pi_H\). That way the Sloths will buy the \(S\)-policy and the Healthy will buy the \(H\)-policy.” Might this work? Can you draw a picture showing what the insurer is trying to do? Compared to your answer in part (b), is anyone harmed by the inability to distinguish Sloths and Healthy ex-ante?

e. The University of Chicago offers a choice of two health care plans to its employees. Under the University of Chicago Health Plan (UCHP) participants must be treated by approved doctors, and typically in University of Chicago clinics. Under the Maroon Plan, participants may go to the doctor of their choice and there are fewer limitations on allowable care. Premiums are higher under the Maroon Plan. Explain this University policy in the context of your answer to (d). Who joins the Maroon Plan? Why? What other characteristics does your analysis predict will differ between the UCHP and Maroon plans? Are employees better off with a choice of plans, or would a single plan be better?

4. White collar criminals are risk averse. Suppose that criminals earn an income of \(y^0\) from their illegal activities. If they are caught, which occurs with probability \(p\), they are required to pay a fine, \(F\), and then they are sent on their merry way.

a. Draw a picture with income on one axis and utility on the other that shows the criminal’s expected utility and expected income from criminal activities.

b. In a cost cutting frenzy, the authorities decide to reduce the number of personnel devoted to detecting white-collar crime. This reduces \(p\). Show that this raises the expected income and expected utility from crime.

c. To compensate for the effects in (b), the authorities decide to increase punishments \(F\) by just enough so that the expected income from criminal activities is the same as it was before the reduction in \(p\). Compared to the pre-cost-cutting outcome, is the expected utility of criminals higher or lower than
before? Why? What will happen to the amount of crime?

5. Betty, who is risk averse, works as a stone cutter. She can choose her own hours, and she now works 30 hours per week at a wage of $40/hour. Suppose she is offered a job by a rival firm that differs from her old one only in that wages are variable. With probability .5 her wage in any given week will be $50, and with probability .5 her wage will be $30. Should she accept the new job? Why or why not?