# Capital Deepening and Non-Balanced Economic Growth*

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**Abstract**

We present a model of non-balanced growth based on differences in factor proportions and capital deepening. Capital deepening increases the relative output of the more capital-intensive sector, but simultaneously induces a reallocation of capital and labor away from that sector. Using a two-sector general equilibrium model, we show that non-balanced growth is consistent with an asymptotic equilibrium with constant interest rate and capital share in national income. For plausible parameter values, the model generates dynamics consistent with US data, in particular, faster growth of employment and slower growth of output in less capital-intensive sectors, and aggregate behavior consistent with the Kaldor facts.

**Keywords:** capital deepening, multi-sector growth, non-balanced economic growth.
**JEL Classification:** O40, O41, O30.

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# 1 Introduction

Most models of economic growth strive to be consistent with the "Kaldor facts", i.e., the relative constancy of the growth rate, the capital-output ratio, the share of capital income in GDP and the real interest rate (see Kaldor, 1963, and also Denison, 1974, Homer and Sylla, 1991, Barro and Sala-i-Martin, 2004). Beneath this balanced picture, however, there are systematic changes in the relative importance of various sectors (see Kuznets, 1957, 1973, Chenery, 1960, Kongsamut, Rebelo and Xie, 2001). A recent literature develops models of economic growth and development that are consistent with such structural changes, while still remaining approximately consistent with the Kaldor facts.[1] This literature typically starts by positing non-homothetic preferences consistent with Engel's law and thus emphasizes the *demand-side* reasons for non-balanced growth; the marginal rate of substitution between different goods changes as an economy grows, directly leading to a pattern of uneven growth between sectors. An alternative thesis, first proposed by Baumol (1967), emphasizes the potential non-balanced nature of economic growth resulting from differential productivity growth across sectors, but has received less attention in the literature.[2]

In this paper, we present a two-sector model that highlights a natural *supply-side* reason for non-balanced growth related to Baumol's (1967) thesis. Differences in factor proportions across sectors (i.e., different shares of capital) combined with capital deepening lead to non-balanced growth because an increase in capital-labor ratio raises output more in sectors with greater capital intensity. We illustrate this economic mechanism using an economy with a constant elasticity of substitution between two sectors and Cobb-Douglas production functions within each sector. We show that the equilibrium (and the Pareto optimal) allocations feature non-balanced growth at the sectoral level but are consistent with the Kaldor facts in the long run.

[1] See, for example, Matsuyama (1992), Echevarria (1997), Laitner (2000), Kongsamut, Rebelo and Xie (2001), Caselli and Coleman (2001), Gollin, Parente and Rogerson (2002). See also the interesting papers by Stokey (1988), Matsuyama (2002), Foellmi and Zweimuller (2002), and Buera and Kaboski (2006), which derive non-homothetiticites from the presence of a "hierarchy of needs" or "hierarchy of qualities". Finally, Hall and Jones (2006) point out that there are natural reasons for health care to be a superior good (because expected life expectancy multiplies utility) and show how this can account for the increase in health care spending. Matsuyama (2005) presents an excellent overview of this literature.

[2] Two exceptions are the recent independent papers by Ngai and Pissarides (2006) and Zuleta and Young (2006). Ngai and Pissarides (2006) construct a model of multi-sector economic growth inspired by Baumol. In Ngai and Pissarides's model, there are exogenous Total Factor Productivity differences across sectors, but all sectors have identical Cobb-Douglas production functions. While both of these papers are potentially consistent with the Kuznets and Kaldor facts, they do not contain the main contribution of our paper: non-balanced growth resulting from factor proportion differences and capital deepening.

In the empirically relevant case where the elasticity of substitution between the two sectors is less than one, one of the sectors (typically the more capital-intensive one) grows *faster* than the rest of the economy, but because the relative prices move against this sector, its (price-weighted) value grows at a *slower* rate than the rest of the economy. Moreover, we show that capital and labor are continuously reallocated away from the more rapidly growing sector.[3]

Figure 1 shows that the distinctive qualitative implications of our model are consistent with the broad patterns in the US data over the past 60 years. Motivated by our theory, this figure divides US industries into two groups according to their capital intensity and shows that there is more rapid growth of fixed-price quantity indices (corresponding to real output) in the more capital-intensive sectors, while (price-weighted) value of output and employment grow more in the less capital-intensive sectors. The opposite movements of quantity and employment (or value) between sectors with high and low capital intensities is a distinctive feature of our approach (for the theoretically and empirically relevant case of the elasticity of substitution less than one).[4]

Finally, we present a simple calibration of our model to investigate whether its quantitative as well as its qualitative predictions are broadly consistent with US data. Even though the model does not feature the demand-side factors that are undoubtedly important for non-balanced growth, it generates relative growth rates of capital-intensive sectors that are consistent with the US data over the past 60 years. For example, our calibration generates increases in the relative output of the more capital-intensive industries that are consistent with the changes in US data between 1948 and 2004 and accounts for one sixth to one third of the increase in the relative employment of the less capital-intensive industries. Our calibration also shows that convergence to the asymptotic (equilibrium) allocation is very slow, and consistent with the Kaldor facts, along this transition path the share of capital in national income and

[3] As we will see below, in our economy the elasticity of substitution between products will be less than one if and only if the (short-run) elasticity of substitution between labor and capital is less than one. The time-series and cross-industry evidence suggests that the short-run elasticity of substitution between labor and capital is indeed less than one. See, for example, the surveys by Hamermesh (1993) and Nadiri (1970), which show that the great majority of the estimates are less than one. Recent works by Krusell, Ohanian, Rios-Rull, and Violante (2000) and Antras (2001) also report estimates of the elasticity that are less than 1. Finally, estimates implied by the response of investment to the user cost of capital also typically imply an elasticity of substitution between capital and labor significantly less than 1 (see, e.g., Chirinko, 1993, Chirinko, Fazzari and Mayer, 1999, Mairesse, Hall and Mulkay, 1999).

[4] It should be noted that the differential evolutions of high and low capital intensity sectors shown in Figure 1 is distinct from the major structural changes associated with changes in the share of agriculture, manufacturing and services. Our model does not attempt to account for these structural changes.

the interest rate are approximately constant.

The rest of the paper is organized as follows. Section 2 presents our model of non-balanced growth, characterizes the full dynamic equilibrium of this economy, and shows how the model generates non-balanced sectoral growth while remaining consistent with the Kaldor facts. Section 3 undertakes a simple calibration of our benchmark economy to investigate whether the dynamics generated by the model are consistent with the changes in the relative output and employment of capital-intensive sectors and the Kaldor facts. Section 4 concludes. Appendix A contains additional theoretical results, while Appendix B, which is available on the Web, provides further details on the NIPA data and the sectoral classifications used in Figure 1 and in Section 3, and additional evidence consistent with the patterns shown in Figure 1.

# 2 A Model of Non-Balanced Growth

In this section, we present the environment, which is a two-sector model with exogenous technological change. The working paper version, Acemoglu and Guerrieri (2006), presents results on non-balanced growth in a more general setting as well as an extension of the model that incorporates endogenous technological change.

## 2.1 Demographics, Preferences and Technology

The economy admits a representative household with the standard preferences

$$\int_0^{\infty} \exp\left(-\left(\rho - n\right)t\right) \frac{\tilde{c}(t)^{1-\theta} - 1}{1-\theta} dt, \tag{1}$$

where $\tilde{c}\left(t\right)$ is consumption per capita at time $t$, $\rho$ is the rate of time preferences and $\theta \geq 0$ is the inverse of the intertemporal elasticity of substitution (or the coefficient of relative risk aversion). Labor is supplied inelastically and is equal to population $L\left(t\right)$ at time $t$, which grows at the exponential rate $n \in [0, \rho)$, so that

$$L\left(t\right) = \exp\left(nt\right) L\left(0\right). \tag{2}$$

The unique final good is produced competitively by combining the output (intermediate goods) of two sectors with an elasticity of substitution $\varepsilon \in [0, \infty)$:

$$\begin{aligned} Y\left(t\right) &= F\left[Y_1\left(t\right), Y_2\left(t\right)\right] \\ &= \left[\gamma Y_1\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\gamma\right) Y_2\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned} \tag{3}$$

where $\gamma \in (0,1)$. Both sectors use labor, $L$, and capital, $K$. Capital depreciates at the rate $\delta \geq 0$.

The aggregate resource constraint, which is equivalent to be the budget constraint of the representative household, requires consumption and investment to be less than the output of the final good,

$$\dot{K}(t) + \delta K(t) + C(t) \leq Y(t), \tag{4}$$

where $C(t) \equiv \tilde{c}(t) L(t)$ is total consumption and investment consists of new capital, $\dot{K}(t)$, and replenishment of depreciated capital, $\delta K(t)$.

The two goods $Y_1$ and $Y_2$ are produced competitively with production functions

$$Y_1(t) = M_1(t) L_1(t)^{\alpha_1} K_1(t)^{1-\alpha_1} \text{ and } Y_2(t) = M_2(t) L_2(t)^{\alpha_2} K_2(t)^{1-\alpha_2}, \tag{5}$$

where $K_1, L_1, K_2$ and $L_2$ are the levels of capital and labor used in the two sectors.

If $\alpha_1 = \alpha_2$, the production function of the final good takes the standard Cobb-Douglas form. Hence, we restrict attention to the case where $\alpha_1 \neq \alpha_2$. Without loss of generality, we assume that sector 1 is more labor intensive (or less capital intensive), that is,

$$\Delta \equiv \alpha_1 - \alpha_2 > 0. \tag{A1}$$

Technological progress in both sectors is exogenous and takes the form

$$\frac{\dot{M}_1(t)}{M_1(t)} = m_1 > 0 \text{ and } \frac{\dot{M}_2(t)}{M_2(t)} = m_2 > 0. \tag{6}$$

Capital and labor market clearing require that at each date

$$K_1(t) + K_2(t) \leq K(t), \tag{7}$$

and

$$L_1(t) + L_2(t) \leq L(t), \tag{8}$$

where $K$ denotes the aggregate capital stock and $L$ is total population. The restriction that each of $L_1$, $L_2$, $K_1$, and $K_2$ has to be nonnegative is left implicit throughout.

## 2.2 The Competitive Equilibrium and the Social Planner's Problem

Let us denote the rental price of capital and the wage rate by $R$ and $w$ and the interest rate by $r$. Also let $p_1$ and $p_2$ be the prices of the $Y_1$ and $Y_2$ goods. We normalize the price of the

final good, $P$, to one at all points, so that

$$1 \equiv P(t) = \left[\gamma^{\varepsilon} p_1(t)^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_2(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{9}$$

A competitive equilibrium is defined in the usual way as paths for factor and intermediate goods prices $[r(t), w(t), \ p_1(t), p_2(t)]_{t\geq 0}$, employment and capital allocations $[L_1(t), L_2(t), K_1(t), K_2(t)]_{t\geq 0}$ such that firms maximize profits, and markets clear, and consumption and savings decisions $[c(t), \dot{K}(t)]_{t\geq 0}$ which maximize the utility of the representative household.

Since markets are complete and competitive, we can appeal to the Second Welfare Theorem and characterize the competitive equilibrium by solving the social planner's problem of maximizing the utility of the representative household.[5] This problem takes the form:

$$\max_{[L_1(t),L_2(t),K_1(t),K_2(t),K(t),\tilde{c}(t)]_{t\geq 0}} \int_0^{\infty} \exp\left(-(\rho - n)t\right) \frac{\tilde{c}(t)^{1-\theta} - 1}{1-\theta} dt \tag{SP}$$

subject to (2), (6), (7), (8), and the resource constraint

$$\begin{aligned} \dot{K}(t) + \delta K(t) + \tilde{c}(t) L(t) &\leq Y(t) \\ &= F\left[M_1(t) L_1(t)^{\alpha_1} K_1(t)^{1-\alpha_1}, M_2(t) L_2(t)^{\alpha_2} K_2(t)^{1-\alpha_2}\right], \end{aligned} \tag{10}$$

together with the initial conditions $K(0) > 0$, $L(0) > 0$, $M_1(0) > 0$ and $M_2(0) > 0$. The objective function in this program is continuous and strictly concave, while the constraint set forms a convex-valued continuous correspondence, thus the social planner's problem has a unique solution, and this solution corresponds to the unique competitive equilibrium.

Once this solution is characterized, the appropriate multipliers give the competitive prices. For example, given the normalization in (9), which is equivalent to the multiplier on (10) being normalized to one, the multipliers associated with (7) and (8) at time $t$ give the rental rate, $R(t) \equiv r(t) + \delta$, and the wage rate, $w(t)$, at time $t$. The prices of the intermediate goods are then obtained as

$$p_1(t) = \gamma \left(\frac{Y_1(t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}} \quad \text{and} \quad p_2(t) = (1-\gamma)\left(\frac{Y_2(t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}}. \tag{11}$$

## 2.3 The Static Equilibrium

Inspection of (SP) shows that the maximization problem can be broken into two parts. First, given the state variables, $K(t)$, $L(t)$, $M_1(t)$ and $M_2(t)$, the allocation of factors across sectors,

[5] See Acemoglu and Guerrieri (2006) for an explicit characterization of the equilibrium.

$L_1(t), L_2(t), K_1(t)$ and $K_2(t)$, is chosen to maximize output $Y(t) = F[Y_1(t), Y_2(t)]$ so as to achieve the largest possible set of allocations that satisfy the constraint set. Second, given this choice of factor allocations at each date, the time path of $K(t)$ and $\tilde{c}(t)$ can be chosen to maximize the value of the objective function. These two parts correspond to the characterization of the *static* and *dynamic* optimal allocations, which are equivalent to the static and dynamic equilibrium allocations. We first characterize the static equilibrium and the implied competitive prices $p_1(t)$, $p_2(t)$, $r(t)$ and $w(t)$, and then turn to equilibrium dynamics.

Let us define the *maximized value* of current output given capital stock $K(t)$ at time $t$ as

$$\begin{aligned} \Phi(K(t), t) \quad = \quad & \max_{L_1(t), L_2(t), K_1(t), K_2(t)} F[Y_1(t), Y_2(t)] \\ & \text{subject to (5), (6), (7), (8),} \\ & \text{and given } K(t) > 0,\ L(t) > 0,\ M_1(t) > 0,\ M_2(t) > 0. \end{aligned} \tag{12}$$

It is straightforward to see that this will involve the equalization of the marginal products of capital and labor in the two sectors, which can be written as

$$\gamma\alpha_1\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{L_1(t)} = (1-\gamma)\,\alpha_2\left(\frac{Y(t)}{Y_2(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_2(t)}{L_2(t)}, \tag{13}$$

and

$$\gamma(1-\alpha_1)\left(\frac{Y(t)}{Y_1(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_1(t)}{K_1(t)} = (1-\gamma)(1-\alpha_2)\left(\frac{Y(t)}{Y_2(t)}\right)^{\frac{1}{\varepsilon}}\frac{Y_2(t)}{K_2(t)}. \tag{14}$$

Since the key static decision involves the allocation of labor and capital between the two sectors, we define the shares of capital and labor allocated to the labor-intensive sector (sector 1) as

$$\kappa(t) \equiv \frac{K_1(t)}{K(t)} \text{ and } \lambda(t) \equiv \frac{L_1(t)}{L(t)}.$$

Clearly, we also have $1-\kappa(t) \equiv K_2(t)/K(t)$ and $1-\lambda(t) \equiv L_2(t)/L(t)$. Combining (13) and (14), we obtain

$$\kappa(t) = \left[1 + \left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{Y_1(t)}{Y_2(t)}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1} \tag{15}$$

and

$$\lambda(t) = \left[1 + \left(\frac{1-\alpha_1}{1-\alpha_2}\right)\left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{1-\kappa(t)}{\kappa(t)}\right)\right]^{-1}. \tag{16}$$

Equation (16) shows that, at each time $t$, the share of labor in sector 1, $\lambda(t)$, is (strictly) increasing in $\kappa(t)$. We next determine how these two shares change with capital accumulation and technological change.

**Proposition 1** In the competitive equilibrium,

$$\frac{d\ln\kappa(t)}{d\ln K(t)} = -\frac{d\ln\kappa(t)}{d\ln L(t)} = \frac{(1-\varepsilon)\Delta(1-\kappa(t))}{1+(1-\varepsilon)\Delta(\kappa(t)-\lambda(t))} > 0 \Leftrightarrow \Delta(1-\varepsilon) > 0; \text{ and} \tag{17}$$

$$\frac{d\ln\kappa(t)}{d\ln M_2(t)} = -\frac{d\ln\kappa(t)}{d\ln M_1(t)} = \frac{(1-\varepsilon)(1-\kappa(t))}{1+(1-\varepsilon)\Delta(\kappa(t)-\lambda(t))} > 0 \Leftrightarrow \varepsilon < 1. \tag{18}$$

**Proof.** To derive these expressions, rewrite (15) as

$$\phi(\kappa, L, K, M_1, M_2) \equiv \kappa - \left[1 + \left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{Y_1}{Y_2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1} = 0,$$

where, from (5),

$$\frac{Y_1}{Y_2} = \lambda^{\alpha_1}(1-\lambda)^{-\alpha_2}\kappa^{1-\alpha_1}(1-\kappa)^{-(1-\alpha_2)}\left(\frac{L}{K}\right)^{\Delta}\frac{M_1}{M_2},$$

with $\lambda$ given as in (16). Applying the Implicit Function Theorem to $\phi(\kappa, L, K, M_1, M_2)$ and using the expression for $Y_1/Y_2$ given here, we obtain (17) and (18). ■

Equation (17) states that the fraction of capital allocated to the labor-intensive sector increases with the stock of capital if $\varepsilon < 1$ and it increases if $\varepsilon > 1$. To obtain the intuition for this comparative static, which is useful for understanding many of the results that will follow, note that if $K$ increased and $\kappa$ remained constant, then the capital-intensive sector, sector 2, would grow by *more* than sector 1—because an equi-proportionate increase in capital raises the output of the more capital-intensive sector by more. The prices of intermediate goods given in (11) then imply that when $\varepsilon < 1$, the relative price of the capital-intensive sector will fall more than proportionately, inducing a greater fraction of capital to be allocated to the less capital-intensive sector 1. The intuition for the converse result when $\varepsilon > 1$ is straightforward. An important implication of this proposition is that as long as $\varepsilon \neq 1$ and there is *capital deepening* (i.e., as long as $K/L$ is increasing over time), growth will be *non-balanced* and capital will be allocated unequally between the two sectors. This is the basis of non-balanced growth in our model.[6] The rest of our analysis will show that non-balanced growth in this model is consistent with the Kaldor facts asymptotically and can approximate the Kaldor facts even along transitional dynamics.

Equation (18) also implies that when the elasticity of substitution, $\varepsilon$, is less than one, an improvement in the technology of a sector causes the share of capital allocated to that sector

[6] As a corollary, note that with $\varepsilon = 1$, output levels in the two sectors could grow at different rates, but there would be no reallocation of capital and labor between the two sectors, and $\kappa$ and $\lambda$ would remain constant.

to fall. The intuition is again the same: when $\varepsilon < 1$, increased production in a sector causes a more than proportional decline in its relative price, inducing a reallocation of capital away from it towards the other sector (again the converse results and intuition apply when $\varepsilon > 1$).

In view of equation (16), the results in Proposition 1 also apply to $\lambda$. In particular, we have that $d \ln \lambda(t) / d \ln K(t) = -d \ln \lambda(t) / d \ln L(t) > 0$ if and only if $\Delta(1-\varepsilon) > 0$.

Next, since equilibrium factor prices, $R$ and $w$, correspond to the multipliers on the constraints (7) and (8), we also obtain

$$w(t) = \gamma \alpha_1 \left( \frac{Y(t)}{Y_1(t)} \right)^{\frac{1}{\varepsilon}} \frac{Y_1(t)}{L_1(t)}, \tag{19}$$

and

$$R(t) = \Phi_K(K(t), t) = \gamma(1-\alpha_1) \left( \frac{Y(t)}{Y_1(t)} \right)^{\frac{1}{\varepsilon}} \frac{Y_1(t)}{K_1(t)}, \tag{20}$$

where $\Phi_K(K(t), t)$ is the derivative of the maximized output function, $\Phi(K(t), t)$, with respect to capital. Equilibrium factor prices take the familiar forms and are equal to the (values of) marginal products from the derived production function in (10). To obtain an intuition for the economic forces, we next analyze how changes in the state variables, $L$, $K$, $M_1$ and $M_2$, impact on these factor prices. Combining (19) and (20), relative factor prices are obtained as

$$\frac{w(t)}{R(t)} = \frac{\alpha_1}{1-\alpha_1} \left( \frac{\kappa(t) K(t)}{\lambda(t) L(t)} \right), \tag{21}$$

and the capital share in aggregate income is

$$\sigma_K(t) \equiv \frac{R(t) K(t)}{Y(t)} = \gamma(1-\alpha_1) \left( \frac{Y(t)}{Y_1(t)} \right)^{\frac{1-\varepsilon}{\varepsilon}} \kappa(t)^{-1}. \tag{22}$$

**Proposition 2** In the competitive equilibrium,

$$\frac{d \ln(w(t)/R(t))}{d \ln K(t)} = -\frac{d \ln(w(t)/R(t))}{d \ln L(t)} = \frac{1}{1 + (1-\varepsilon)\Delta(\kappa(t) - \lambda(t))} > 0;$$

$$\begin{aligned} \frac{d \ln(w(t)/R(t))}{d \ln M_2(t)} &= -\frac{d \ln(w(t)/R(t))}{d \ln M_1(t)} \\ &= -\frac{(1-\varepsilon)(\kappa(t) - \lambda(t))}{1 + (1-\varepsilon)\Delta(\kappa(t) - \lambda(t))} < 0 \Leftrightarrow \Delta(1-\varepsilon) > 0; \end{aligned}$$

$$\frac{d \ln \sigma_K(t)}{d \ln K(t)} = -\frac{(1-\varepsilon)\Delta^2(1-\kappa(t))\kappa(t)}{[1-\alpha_1 + \Delta\kappa(t)][1 + (1-\varepsilon)\Delta(\kappa(t) - \lambda(t))]} < 0 \Leftrightarrow \varepsilon < 1; \text{ and} \tag{23}$$

$$\begin{aligned}\frac{d\ln\sigma_K(t)}{d\ln M_2(t)} &= -\frac{d\ln\sigma_K(t)}{d\ln M_1(t)} \\ &= \frac{\Delta(1-\varepsilon)(1-\kappa(t))\kappa(t)}{[1-\alpha_1+\Delta\kappa(t)][1+(1-\varepsilon)\Delta(\kappa(t)-\lambda(t))]} < 0 \\ &\qquad \Leftrightarrow \Delta(1-\varepsilon) > 0.\end{aligned} \tag{24}$$

**Proof.** The first two expressions follow from differentiating equation (21) and Proposition 1. To prove (23) and (24), note that from (3) and (15), we have

$$\left(\frac{Y_1}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \gamma^{-1}\left(1+\left(\frac{1-\alpha_1}{1-\alpha_2}\right)\left(\frac{1-\kappa}{\kappa}\right)\right)^{-1}.$$

Then, (23) and (24) follow by differentiating $\sigma_K$ as given in (22) with respect to $L$, $K$, $M_1$ and $M_2$, and using the results in Proposition 1. ■

The most important result in this proposition is (23), which links the impact of the capital stock on the capital share in national income to the elasticity of substitution between the two sectors, $\varepsilon$. Since a negative relationship between the share of capital in national income and the capital stock is equivalent to an elasticity of substitution between aggregate labor and capital that is less than one, this result also implies that, as claimed in footnote 3, the elasticity of substitution between capital and labor is less than one if and only if $\varepsilon$ is less than one. Intuitively, an increase in the capital stock of the economy causes the output of the more capital-intensive sector, sector 2, to increase relative to the output in the less capital-intensive sector (despite the fact that the share of capital allocated to the less-capital intensive sector increases as shown in equation (17)). This then increases the production of the more capital-intensive sector, and when $\varepsilon < 1$, it reduces the relative price of capital more than proportionately; consequently, the share of capital in national income declines. The converse result applies when $\varepsilon > 1$.

Moreover, when $\varepsilon < 1$, (24) implies that an increase in $M_1$ is "capital biased" and an increase in $M_2$ is "labor biased". The intuition for why an increase in the productivity of the sector that is intensive in capital is biased toward labor (and vice versa) is once again similar: when the elasticity of substitution between the two sectors, $\varepsilon$, is less than one, an increase in the output of a sector (this time driven by a change in technology) decreases its price more than proportionately, thus reducing the relative compensation of the factor used more intensively in that sector (see Acemoglu, 2002). When $\varepsilon > 1$, we have the converse pattern, and an increase in $M_1$ is "labor biased," while an increase in $M_2$ is "capital biased".

### 2.4 Equilibrium Dynamics

We now characterize the dynamic competitive equilibrium allocations. We again use the social planner's problem (SP) introduced above. The previous subsection characterized the static optimal (equilibrium) allocation of resources and the resulting maximized value of output $\Phi(K(t), t)$ for given values of $K(t)$, $L(t)$, $M_1(t)$, and $M_2(t)$. Given $\Phi(K(t), t)$, problem (SP) can be written as

$$\max_{[K(t),\tilde{c}(t)]_{t\geq 0}} \int_0^\infty \exp(-(\rho-n)t) \frac{\tilde{c}(t)^{1-\theta}-1}{1-\theta} dt \tag{SP$'$}$$

subject to

$$\dot{K}(t) = \Phi(K(t), t) - \delta K(t) - \exp(nt) L(0) \tilde{c}(t), \tag{25}$$

and the initial condition $K(0) > 0$. The constraint (25) is written as an equality since it cannot hold as a strict inequality in an optimal allocation (otherwise, consumption would be raised, yielding a higher objective value). The other initial conditions of the original problem (SP) are incorporated into the maximized value of output $\Phi(K(t), t)$ in constraint (25). This maximization problem is simpler than (SP), though it is still not equivalent to the standard problems encountered in growth models because constraint (25) is not an autonomous differential equation. To further simplify the characterization of equilibrium dynamics, we will work with transformed variables. For this purpose, we first assume:

$$\text{either (i) } m_1/\alpha_1 < m_2/\alpha_2 \text{ and } \varepsilon < 1; \text{ or (ii) } m_1/\alpha_1 > m_2/\alpha_2 \text{ and } \varepsilon > 1, \tag{A2}$$

which ensures that the *asymptotically dominant sector* will be the labor-intensive sector, sector 1. The asymptotically dominant sector is the sector that determines the long-run growth rate of the economy. Observe that this condition compares not the exogenous rates of technological progress, $m_1$ and $m_2$, but $m_1/\alpha_1$ and $m_2/\alpha_2$, which we refer to as the *augmented* rates of technological progress. This is because the two sectors differ in terms of their capital intensities, and technological change will be augmented by the differential rates of capital accumulation in the two sectors. For example, with equal rates of Hicks-neutral technological progress in the two sectors, the adjustment of the capital stock to technological change implies that the labor-intensive sector 1 will have a lower augmented rate of technological progress than sector 2.

Notice that when $\varepsilon < 1$, the sector with the lower rate of augmented technological progress will be the asymptotically dominant sector. This is because, when $\varepsilon < 1$, the output of the two

sectors are highly complementary and the slower growing sector will determine the asymptotic growth rate of the economy. When $\varepsilon > 1$, the converse happens and the more rapidly growing sector determines the asymptotic growth rate of the economy and is the asymptotically dominant sector. In this light, Assumption (A2) implies that sector 1 is the asymptotically dominant sector. Appendix A shows that parallel results apply when the converse of (A2) holds and sector 2 is asymptotically dominant. The empirically relevant case is part (i) of Assumption (A2); as already argued, $\varepsilon < 1$ provides a good approximation to the data, and $\alpha_1 > \alpha_2$ from (A1). Therefore, as long as $m_1$ and $m_2$ are close to each other, the economy will be in case (i) of (A2).

Let us next introduce the following normalized variables,

$$c(t) \equiv \frac{\tilde{c}(t)}{M_1(t)^{1/\alpha_1}} \quad \text{and } \chi(t) \equiv \frac{K(t)}{L(t)\, M_1(t)^{1/\alpha_1}}, \tag{26}$$

which represent consumption and capital per capita normalized by the *augmented technology* of the *asymptotically dominant sector*, which, in view of Assumption (A2), is sector 1 (and thus the corresponding augmented technology is $M_1(t)^{1/\alpha_1}$). Next proposition shows that the solution to (SP)—and thus the dynamic equilibrium—can be expressed in terms of three autonomous differential equations in $c$, $\chi$ and $\kappa$.

**Proposition 3** Suppose that (A1) and (A2) hold. Then, a competitive equilibrium satisfies the following three differential equations

$$\begin{aligned}
\frac{\dot{c}(t)}{c(t)} &= \frac{1}{\theta}\left[(1-\alpha_1)\,\gamma\eta(t)^{1/\varepsilon}\,\lambda(t)^{\alpha_1}\,\kappa(t)^{-\alpha_1}\,\chi(t)^{-\alpha_1} - \delta - \rho\right] - \frac{m_1}{\alpha_1}, \\
\frac{\dot{\chi}(t)}{\chi(t)} &= \lambda(t)^{\alpha_1}\,\kappa(t)^{1-\alpha_1}\,\chi(t)^{-\alpha_1}\,\eta(t) - \chi(t)^{-1}\,c(t) - \delta - n - \frac{m_1}{\alpha_1}, \\
\frac{\dot{\kappa}(t)}{\kappa(t)} &= \frac{(1-\kappa(t))\left[\Delta\frac{\dot{\chi}(t)}{\chi(t)} + m_2 - \frac{\alpha_2}{\alpha_1}m_1\right]}{(1-\varepsilon)^{-1} + \Delta\left(\kappa(t) - \lambda(t)\right)},
\end{aligned} \tag{27}$$

where

$$\eta(t) \equiv \gamma^{\frac{\varepsilon}{\varepsilon-1}}\left[1 + \left(\frac{1-\alpha_1}{1-\alpha_2}\right)\left(\frac{1-\kappa(t)}{\kappa(t)}\right)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{28}$$

with initial conditions $\chi(0)$ and $\kappa(0)$, and also satisfies the transversality condition

$$\lim_{t\to\infty} \exp\left(-\left(\rho - \frac{(1-\theta)\,m_1}{\alpha_1} - n\right)t\right)\chi(t) = 0. \tag{29}$$

Moreover, any allocation that satisfies (27)-(29) is a competitive equilibrium.

**Proof.** See Appendix A. ■

The first equation in (27) is the standard Euler equation, written in terms of the normalized variables. The first term in square brackets, $(1-\alpha_1)\gamma\eta(t)^{1/\varepsilon}\lambda(t)^{\alpha_1}\kappa(t)^{-\alpha_1}\chi(t)^{-\alpha_1}$, is the marginal product of capital. The second equation in (27) is the law of motion of the normalized capital stock, $\chi(t)$. The third equation, in turn, specifies the evolution of the share of capital between the two sectors. We also impose the following parameter condition, which ensures that the transversality condition (29) holds:

$$\rho - n \geq (1-\theta)\frac{m_1}{\alpha_1}. \tag{A3}$$

Our next task is to use Proposition 3 to provide a tighter characterization of the dynamic equilibrium allocation. For this purpose, let us first define a *Constant Growth Path* (CGP) as a dynamic competitive equilibrium that features constant aggregate consumption growth.[7] The next theorem will show that there exists a unique CGP that is a solution to the social planner's problem (SP) and will provide closed-form solutions for the growth rates of different aggregates in this equilibrium. The notable feature of the CGP will be that despite the constant growth rate of aggregate consumption, growth will be *non-balanced* because output, capital and employment in the two sectors will grow at *different rates*. Let us define:

$$\frac{\dot{L}_s(t)}{L_s(t)} \equiv n_s(t), \frac{\dot{K}_s(t)}{K_s(t)} \equiv z_s(t), \frac{\dot{Y}_s(t)}{Y_s(t)} \equiv g_s(t) \text{ for } s=1,2, \text{ and } \frac{\dot{K}(t)}{K(t)} \equiv z(t), \frac{\dot{Y}(t)}{Y(t)} \equiv g(t),$$

so that $n_s$ and $z_s$ denote the growth rates of labor and capital stock, $m_s$ denotes the growth rate of technology, and $g_s$ denotes the growth rate of output in sector $s$. Moreover, whenever they exist, we denote the corresponding asymptotic growth rates by asterisks, so that $n_s^* = \lim_{t\to\infty} n_s(t)$, $z_s^* = \lim_{t\to\infty} z_s(t)$, and $g_s^* = \lim_{t\to\infty} g_s(t)$. Similarly, let us denote the asymptotic capital and labor allocation decisions by asterisks, that is,

$$\kappa^* = \lim_{t\to\infty}\kappa(t) \text{ and } \lambda^* = \lim_{t\to\infty}\lambda(t).$$

Then we have the following characterization of the unique CGP.

**Theorem 1** Suppose that (A1)-(A3) hold. Then, there exists a unique CGP where consumption per capita grows at the rate $g_c^* = m_1/\alpha_1$, and $\kappa^* = 1$,

$$\chi^* = \left[\frac{(\theta m_1/\alpha_1 + \rho + \delta)}{\gamma^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha_1)}\right]^{-\frac{1}{\alpha_1}}, \tag{30}$$

[7] Kongsamut, Rebelo and Xie (2001) refer to this as a "Generalized Balanced Growth Path".

and

$$c^* = \gamma^{\frac{\varepsilon}{\varepsilon-1}} \left(\chi^*\right)^{1-\alpha_1} - \chi^* \left(\delta + n + \frac{m_1}{\alpha_1}\right). \tag{31}$$

Moreover, the growth rates of output, capital and employment in the two sectors are

$$g^* = g_1^* = z^* = z_1^* = n + \frac{m_1}{\alpha_1}, \tag{32}$$

$$z_2^* = g^* - (1-\varepsilon)\,\omega, \tag{33}$$

$$g_2^* = g^* + \varepsilon\omega, \tag{34}$$

$$n_1^* = n \text{ and } n_2^* = n - (1-\varepsilon)\,\omega, \tag{35}$$

where

$$\omega \equiv m_2 - \frac{\alpha_2}{\alpha_1} m_1.$$

**Proof.** From Proposition 3, a competitive equilibrium satisfies (27)-(29). A CGP requires that $\lim_{t\to\infty} \dot{c}(t)/c(t)$ is constant, thus from the first equation of (27), $\eta(t)^{1/\varepsilon} \lambda(t)^{\alpha_1} \kappa(t)^{-\alpha_1} \chi(t)^{-\alpha_1}$ must be constant as $t \to \infty$. Consequently, as $t \to \infty$,

$$\frac{1}{\varepsilon}\frac{\dot{\eta}(t)}{\eta(t)} + \alpha_1 \frac{\dot{\lambda}(t)}{\lambda(t)} - \alpha_1 \frac{\dot{\kappa}(t)}{\kappa(t)} - \alpha_1 \frac{\dot{\chi}(t)}{\chi(t)} = 0. \tag{36}$$

Differentiating (16) and (28) with respect to time and using the third equation of the system (27), we obtain expressions for $\dot{\eta}(t)/\eta(t)$, $\dot{\lambda}(t)/\lambda(t)$, and $\dot{\chi}(t)/\chi(t)$ in terms of $\dot{\kappa}(t)/\kappa(t)$. Substituting these into (36) and rearranging, we can express (36) as an autonomous first-order differential equation in $\kappa(t)$ as

$$\frac{\dot{\kappa}(t)}{\kappa(t)} = G(\kappa(t))\,\alpha_1\alpha_2(1-\varepsilon)\left(\frac{m_2}{\alpha_2} - \frac{m_1}{\alpha_1}\right), \tag{37}$$

where

$$G(\kappa(t)) = \frac{\left[\Delta\kappa(t) + (1-\alpha_1)\right](1-\kappa(t))}{\Delta\kappa(t) + \alpha_2(1-\alpha_1)}.$$

Clearly, $G(0) = 1/\alpha_2$, $G(1) = 0$ and $G'(\kappa) < 0$ for any $\kappa$. These observations together with Assumption (A2) imply that (37) has a unique solution with $\kappa(t) \to \kappa^* = 1$. Next, $\dot{\kappa}(t)/\kappa(t) = 0$ combined with (16) and (28) implies that $\dot{\eta}(t)/\eta(t) = \dot{\lambda}(t)/\lambda(t) = 0$. Moreover, given $\kappa(t) \to \kappa^* = 1$, (16) implies $\lambda(t) \to \lambda^* = 1$ and (28) implies $\eta(t) \to \eta^* = \gamma^{\varepsilon/(\varepsilon-1)}$. Equation (36) then implies that $\lim_{t\to\infty} \chi(t) = \chi^* \in (0,\infty)$ must also exist, thus $\dot{\chi}(t)/\chi(t) \to 0$. Now setting the first two equations in (27) equal to zero and using the fact that $\kappa^* = \lambda^* = 1$ and

$\eta^* = \gamma^{\varepsilon/(\varepsilon-1)}$, we obtain (30) and (31). By construction, there are no other allocations with constant $\dot{c}(t)/c(t)$.

To derive equations (32)-(35), combine (26) with the result that $\dot{\chi}(t)/\chi(t) \to 0$, which implies $z^* = n + m_1/\alpha_1$. Moreover, $\kappa^* = \lambda^* = 1$ together with the market clearing conditions (7) and (8)—holding as equalities—give $z_1^* = z^*$ and $n_1^* = n$. Finally, differentiating (3), (5), (13) and (14), and using the preceding results, we obtain (32)-(35) as unique solutions.

To complete the proof that this allocation is the unique CGP we need to establish that it satisfies the transversality condition (29). This follows immediately from the fact that $\lim_{t\to\infty} \chi(t) = \chi^*$ exists and is finite combined with Assumption (A3), which implies that $\lim_{t\to\infty} \exp(-(\rho - (1-\theta) m_1/\alpha_1 - n) t) = 0$. ■

There are a number of important implications of this theorem. First, growth is non-balanced, in the sense that the two sectors grow at different asymptotic rates (i.e., at different rates even as $t \to \infty$). The intuition for this result is more general than the specific parametrization of the model and is driven by the juxtaposition of *factor proportion differences* between sectors and *capital deepening*. In particular, suppose that there is capital deepening (which here is due to technological progress). Now, if both capital and labor were allocated to the two sectors in constant proportions, the more capital-intensive sector, sector 2, would grow faster than sector 1. The faster growth in sector 2 would reduce the price of sector 2, leading to a reallocation of capital and labor towards sector 1. However, this reallocation *cannot* entirely offset the greater increase in the output of sector 2, since, if it did, the change in prices that stimulated the reallocation would not take place. Therefore, growth must be non-balanced. In particular, if $\varepsilon < 1$, capital and labor will be reallocated away from the more rapidly growing sector towards the more slowly growing sector. In this case, the more slowly growing sector, sector 1, becomes *the asymptotically dominant* sector and determines the growth rate of aggregate output as shown in equation (32). Note that sector 1 is the one growing more slowly because Assumption (A2), together with $\varepsilon < 1$, implies $m_1/\alpha_1 < m_2/\alpha_2$. Hence, $Y_1/Y_2 \to 0$. Appendix A shows that similar results apply when the converse of (A2) holds.

Second, the theorem shows that in the CGP the shares of capital and labor allocated to sector 1 tend to one (i.e., $\kappa^* = \lambda^* = 1$). Nevertheless, at all points in time both sectors produce *positive* amounts and both sectors grow at rates *greater* than the rate of population growth (so this limit point is never reached). Moreover, in the more interesting case where $\varepsilon < 1$, equation (34) implies $g_2^* > g^* = g_1^*$, so that the sector that is shedding capital and labor (sector 2) is

growing faster than the rest of the economy, even asymptotically. Therefore, the rate at which capital and labor are allocated away from this sector is determined in equilibrium to be *exactly* such that this sector still grows faster than the rest of the economy. This is the sense in which non-balanced growth is not a trivial outcome in this economy (with one of the sectors shutting down), but results from the positive and differential growth of the two sectors.

Finally, it can be verified that the share of capital in national income and the interest rate are constant in the CGP. For example, under (A2), we have $\sigma_K^* = 1 - \alpha_1$. This implies that the asymptotic capital share in national income will reflect the capital share of the dominant sector. Also, again under (A2), the asymptotic interest rate is

$$r^* = (1 - \alpha_1)\, \gamma^{\frac{\varepsilon}{\varepsilon - 1}} \left(\chi^*\right)^{-\alpha_1} - \delta.$$

These results are the basis of the claim in the Introduction that this economy features both non-balanced growth at the sectoral level and aggregate growth consistent with the Kaldor facts. In particular, the CGP matches both the Kaldor facts and generates unequal growth between the two sectors. However, at the CGP one of the sectors has already become (vanishingly) small relative to the other. Therefore, this theorem does not answer the question of whether we can have a situation in which both sectors have non-trivial employment levels and the capital share in national income and the interest rate are approximately constant. This question is investigated quantitatively in the next section. Before doing so, we establish the stability of the CGP.

## 2.5 Dynamics and Stability

The previous subsection demonstrated that there exists a unique CGP with non-balanced sectoral growth—that is, there is aggregate output growth at a constant rate together with differential sectoral growth and reallocation of factors of production across sectors. We now investigate whether the competitive equilibrium will approach the CGP. We focus on allocations in the neighborhood of the CGP, thus only investigating local (saddle-path) stability. Because there are two pre-determined (state) variables, $\chi$ and $\kappa$, with initial values $\chi(0)$ and $\kappa(0)$, this type of stability requires the linearized system in the neighborhood of the asymptotic path to have a (unique) two-dimensional manifold of solutions converging to $c^*$, $\chi^*$ and $\kappa^*$. The next theorem states that this is the case.

**Theorem 2** Suppose that (A1)-(A3) hold. Then, the competitive equilibrium, given by (27), is locally (saddle-path) stable, in the sense that in the neighborhood of $c^*$, $\chi^*$ and $\kappa^*$, there is a unique two-dimensional manifold of solutions that converge to $c^*$, $\chi^*$ and $\kappa^*$.

**Proof.** Let us rewrite the system (27) in a more compact form as

$$\dot{x} = f(x), \tag{38}$$

where $x \equiv \begin{pmatrix} c & \chi & \kappa \end{pmatrix}'$ is the transpose of the row vector $\begin{pmatrix} c & \chi & \kappa \end{pmatrix}$. To investigate the dynamics of the system (38) in the neighborhood of the steady state, consider the linear system

$$\dot{z} = J(x^*) z,$$

where $z \equiv x - x^*$ and $x^*$ is such that $f(x^*) = 0$, where $J(x^*)$ is the Jacobian of $f(x)$ evaluated at $x^*$. Differentiation and some algebra enable us to write this Jacobian matrix as

$$J(x^*) = \begin{pmatrix} 0 & a_{c\chi} & a_{c\kappa} \\ -1 & a_{\chi\chi} & a_{\chi\kappa} \\ 0 & 0 & a_{\kappa\kappa} \end{pmatrix}.$$

Hence, the determinant of the Jacobian is $\det J(x^*) = -a_{\kappa\kappa} a_{c\chi}$, where

$$\begin{aligned} a_{\kappa\kappa} &= -(1-\varepsilon)\left(m_2 - \frac{\alpha_2}{\alpha_1} m_1\right), \\ a_{c\chi} &= -\gamma^{\frac{\varepsilon}{\varepsilon-1}} (\chi^*)^{-\alpha_1 - 1} \frac{\alpha_1 (1-\alpha_1)}{\theta}. \end{aligned}$$

The above expressions show that both $a_{\kappa\kappa}$ and $a_{c\chi}$ are (strictly) negative, since, in view of (A2), $\varepsilon \lesseqgtr 1 \Leftrightarrow m_2/\alpha_2 \gtreqless m_1/\alpha_1$. This establishes that $\det J(x^*) > 0$, so the steady state corresponding to the CGP is hyperbolic (that is, all eigenvalues of the Jacobian evaluated at $x^*$ have nonzero real parts) and thus the dynamics of the linearized system represent the local dynamics of the nonlinear system (see, e.g., Acemoglu, 2008, Theorem B.7). Moreover, either all the eigenvalues are positive or two of them are negative and one is positive. To determine which one of these two possibilities is the case, we look at the characteristic equation given by $\det(J(x^*) - \mathbf{v}I) = 0$, where $\mathbf{v}$ denotes the vector of the eigenvalues. This equation can be expressed as the following cubic in $v$, with roots corresponding to the eigenvalues:

$$(a_{\kappa\kappa} - v)\left[v(a_{\chi\chi} - v) + a_{\chi c} a_{c\chi}\right] = 0.$$

This expression implies that one of the eigenvalue is equal to $a_{\kappa\kappa}$ and thus negative, so there must be two negative eigenvalues. This establishes the existence of a unique two-dimensional manifold of solutions in the neighborhood of this CGP, converging to it. ■

This theorem establishes that the CGP is locally (saddle-path) stable, and when the initial values of capital, labor and technology are not too far from the constant growth path, the competitive equilibrium converges to this CGP, with non-balanced growth at the sectoral level and constant capital share and interest rate at the aggregate.

# 3 A Simple Calibration

We now undertake an illustrative calibration to investigate whether the equilibrium dynamics generated by our model economy are broadly consistent with the patterns in the US data. For this exercise, we use data from the National Income and Product Accounts (NIPA) between 1948 and 2005. Industries are classified according to North American Industrial Classification System (NAICS) at the 22-industry level of detail.[8] We use industry-level data for current-price value added (which we refer to as *value*), chain-type fixed-price quantity indices for value added (which we refer to as *quantity*), total number of employees,[9] total employee compensation, and fixed assets. To map our model to data, we classify industries into low and high capital intensity "sectors", each comprising approximately 50% of total employment. Table 1 below shows the average capital share for each industry and the sector classification.[10]

Figure 1 in the introduction depicts the evolution of relative quantity, value, and employment in these sectors and shows the more rapid growth of quantity in the capital-intensive sectors and the more rapid growth of value and employment in the less capital-intensive sectors.[11]

For our calibration, we take the initial year, $t = 0$, to correspond to the first year for

[8] Throughout, we exclude the Government and Private household sectors and Agriculture, forestry, fishing, and hunting and Real estate and rental.

[9] In particular, we use full-time and part-time employees, since this is the only measure for which we have consistent NAICS data going back to 1948. The alternative classification system, SIC, does not enable us to extend data to 2005 and also reports the quantity indices for value added only back to 1977.

[10] NAICS data on compensation of employees are only available between 1987 and 2005 (all the other variables are available between 1948 and 2005). We therefore compute the capital share of each industry as the average capital share between 1987 and 2005. The average capital shares in the two sectors are relatively stable over time. In particular, the average capital share of sector 1 is 0.288 in 1987 and 0.290 in 2005, while the average capital share of sector 2 is 0.466 in 1987 and 0.499 in 2005.

[11] Appendix B provides more details on data sources and constructions. It also shows that the general patterns in the US data, in particular those plotted in Figure 1, are robust to changing in the cutoff between high and low capital-intensity industries.

which we have NIPA data for our sectors, 1948. In our model, $L(t)$ corresponds to total employment at time $t$, $K(t)$ to fixed assets, $Y_j(t)$ to the quantity of output in sector $j$, and $Y_j^N(t) \equiv p_j(t) Y_j(t)$ to the value of output in sector $j$.

Our model economy is fully characterized by ten parameters $\rho$, $\delta$, $\theta$, $\gamma$, $\varepsilon$, $\alpha_1$, $\alpha_2$, $n$, $m_1$ and $m_2$, and five initial values, $L(0)$, $K(0)$, $M_1(0)$, $M_2(0)$ and $\kappa(0)$. We choose these parameters and initial values as follows. First, we adopt the standard parameter values for the annual discount rate, $\rho = 0.02$, the annual depreciation rate, $\delta = 0.05$, and the annual (asymptotic) interest rate, $r^* = 0.08$.[12] We take the annual population growth rate $n = 0.018$ from the NIPA data on employment growth for 1948-2005 and choose the asymptotic growth rate to ensure that our calibration matches total output growth between 1948 and 2005 in the NIPA, which is 3.4%. This implies an asymptotic growth rate of $g^* = 0.033$. The initial values $L(0) = 40,3360,000$ and $K(0) = 244,900$ (in millions of dollars) are also directly from the NIPA data for 1948. Next, our classification of industries leads to two "aggregate sectors" with average shares of labor in value added of 0.72 and 0.52. In terms of our model this implies $\alpha_1 = 0.72$ and $\alpha_2 = 0.52$.

An important parameter for our calibration is the elasticity of substitution between the two sectors. Although we do not have independent information on this variable, our model suggests a way of evaluating this elasticity. In particular, equation (11) implies the following relationship between value and quantity ratios in the two sectors:

$$\log\left(\frac{Y_1^N(t)}{Y_2^N(t)}\right) = \log\left(\frac{\gamma}{1-\gamma}\right) + \frac{\varepsilon - 1}{\varepsilon}\log\left(\frac{Y_1(t)}{Y_2(t)}\right). \tag{39}$$

We can therefore estimate $(\varepsilon - 1)/\varepsilon$ by regressing the log of the ratio of the nominal value added between the two sectors on the log of the ratio of the real value added. Since our focus is on medium-run frequencies (rather than business cycle fluctuations), we use the Hodrick and Prescott filter to smooth both the dependent and the independent variables (with smoothing weight 1600) and use the smoothed variables to estimate (39). This simple regression yields an estimate of $\varepsilon \simeq 0.76$ (and a two standard error confidence interval of $[0.73, 0.79]$). We therefore choose $\varepsilon = 0.76$ for our benchmark calibration. We also choose the parameter $\gamma$ to ensure that equation (39) holds at $t = 0$.

Throughout, motivated by our estimate of $\varepsilon$ reported in the previous paragraph, the existing evidence discussed in footnote 3, and the pattern shown in Figure 1 indicating that employment

[12] These numbers are the same as those used by Barro and Sala-i-Martin (2004) in their calibration of the baseline neoclassical model.

and value grow more in the less capital-intensive sector, we focus on the case in which $\varepsilon < 1$ and $m_1/\alpha_1 < m_2/\alpha_2$ (case (i) of Assumption (A2)). In particular, for our benchmark calibration we set $m_1 = m_2$ (even though, as we will see below, higher values of $m_2$ improve the fit of the model to US data). Since in this case sector 1 is the asymptotically dominant sector, the asymptotic growth rate of output is $g^* = n + m_1/\alpha_1$. The above-mentioned values of $g^*$, $n$ and $\alpha_1$ imply $m_1 = m_2 = 0.0108$. The growth rate of output also pins down the intertemporal elasticity of substitution. In particular, the Euler equation (27) together with (26) yields $g^* = (r^* - \rho)/\theta + n$, which implies $\theta = 4$ and results in a reasonable elasticity of intertemporal substitution of 0.25.

This leaves us with the initial values for $\kappa$, $M_1$ and $M_2$. First, notice that equation (15) at time 0 can be rewritten as:

$$\kappa(0) = \left[1 + \left(\frac{1-\alpha_2}{1-\alpha_1}\right)\frac{Y_2^N(0)}{Y_1^N(0)}\right]^{-1}. \tag{40}$$

This equation together with the 1948 levels of values in the two sectors from the NIPA data, $Y_1^N(0)$ and $Y_2^N(0)$, gives $\kappa(0) = 0.32$. Equation (16) then gives the initial value of relative employment as $\lambda(0) = 0.52$.[13] It is also worth noting that in addition to the parameters and the initial values necessary for our calibration, the numbers we are using also pin down the initial interest rate and the capital share in national income as $r(0) = 0.095$ and $\sigma_K(0) = 0.39$.[14] Moreover, since sector 1 is the asymptotically dominant sector, the asymptotic capital share in national income and the interest rate are determined as $\sigma_K^* = 1 - \alpha_1 = 0.28$ and $r^* = 0.08$. This implies that, by construction, both the interest rate and the capital share must decline at some point along the transition path. A key question concerns the speed of these declines. If this happened at the same frequency as the change in the composition of employment and capital across the two sectors, then the model would not generate a pattern that is simultaneously consistent with non-balanced sectoral growth and the aggregate Kaldor facts. We will see that this is not the case and that the model's implications are broadly consistent with both sets of facts.

[13] We can also compute the empirical counterparts of $\lambda(0)$ and $\kappa(0)$ from the NIPA data using employment and capital in the two sector aggregates. These give the values of 0.49 and 0.50, which are similar, though clearly not identical, to the implied values we use. The fact that the theoretically-implied values of $\lambda(0)$ and $\kappa(0)$ differ from their empirical counterparts is not surprising, since we are assuming that the US economy can be represented by two sectors with Cobb-Douglas production functions and with their output being combined with a constant elasticity of substitution. Naturally, this is at best an approximation, and in the data, sectoral factor intensities are not constant over time.

[14] This is because the capital share and interest rate are functions of $\kappa$ only (just combine (20) and (22) with (3) and (15)).

Finally, given $\kappa(0) = 0.32$ and $\lambda(0) = 0.52$, the NIPA data imply values for $L_1(0)$, $L_2(0)$, $K_1(0)$ and $K_2(0)$, and we also have the values for $Y_1(0)$ and $Y_2(0)$ directly from the NIPA. We then obtain the remaining initial values, $M_1(0)$ and $M_2(0)$, from equation (5). This completes the determination of all the parameters and initial conditions of our model. We then compute the time path of all the variables in our model using two different numerical procedures (both giving equivalent results).[15]

Figure 2 shows the results of our benchmark calibration with the parameter values described above. In particular, in this benchmark we have $\varepsilon = 0.76$ and $m_1 = m_2 = 0.0108$. The four panels depict relative employment in sector 1 ($\lambda$), relative capital in sector 1 ($\kappa$), the interest rate ($r$) and the capital share in national income ($\sigma_K$) for the first 150 years (in terms of data, corresponding to 1948 to 2098).

A number of features are worth noting. First, for the first 150 years, there is significant reallocation of capital and labor away from the more capital-intensive sector towards the less capital-intensive sector, sector 1, and the economy is far from the asymptotic equilibrium with $\kappa = \lambda = 1$. In fact, the economy takes a very long time, over 5000 years, to reach the asymptotic equilibrium. This illustrates that our model economy generates interesting and relatively slow dynamics, with a significant amount of structural change. Second, while there is non-balanced growth at the sectoral level, the interest rate and the capital share remain approximately constant. The interest rate shows an early decline from about 9.5% to 9%, which largely reflects the initial consumption dynamics. It then remains around 9%. The capital share shows a very slight decline over the 150 years. This relative constancy of the interest-rate and the capital share is particularly interesting, since as noted above, we know that both variables have to decline at some point to achieve their asymptotic values of $r^* = 0.08$ and $\sigma_K^* = 0.28$. Nevertheless, our model calibration implies more rapid structural change than

[15] In particular, we first return to the two-dimensional non-autonomous system of equations in $c$ and $\chi$ (rather than the three-dimensional representation used for theoretical analysis in Proposition 3). This two dimensional system has one state and one control variable. Following Judd (1998, Chapter 10), we discretize these differential equations using the Euler method to obtain a system of first-order difference equations in $c(t)$ and $\chi(t)$. This system can in turn be transformed into a second-order non-autonomous system in $\chi(t)$, which is easier to work with. We then compute the numerical solution to this second-order difference equation using either a shooting algorithm or by minimizing the squared residuals. Our main numerical method is to use a basic shooting algorithm starting with initial value of $\chi(0)$, and then guess and adjust the value for $\chi(1)$ to ensure convergence to the asymptotic CGP. The second numerical procedure is to choose a polynomial form for the normalized capital stock as $\chi(t) = \Gamma(t; \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ represents the parameters of the polynomial. We estimate $\boldsymbol{\theta}$ by minimizing the squared residuals of the difference equations using nonlinear least squares (see, e.g., Judd, 1998, Chapter 4) and then compute $c(t)$ and $\kappa(t)$. The two procedures give almost identical results, and throughout we report the results from the shooting algorithm.

the speed of these aggregate changes, and thus over the horizon of about 150 years, there is little change in the interest rate and the capital share, while there is significant reallocation of labor and capital across sectors.

These patterns are further illustrated in Table 2, which shows the US data and the numbers implied by our benchmark calibration between 1948 and 2005 for the relative quantity and relative employment of the high versus low capital intensity sectors as well as the aggregate capital share in national income (in terms of the model, $t = 0$ is taken to correspond to 1948, thus $t = 57$ gives the values for 2005). The first two columns of this table confirm the patterns shown in Figure 1 in the Introduction—quantity grows faster in high capital intensity industries, while employment grows faster in low capital intensity industries. The next two columns show that the benchmark calibration is broadly consistent with this pattern. In particular, while in the data, $Y_2/Y_1$ increases by about 19% between 1948 and 2004, the model leads to an increase of about 17%. In the data, $L_2/L_1$ declines by about 33%.[16] In the model, the implied decline is in the same direction, but considerably smaller, about 5%.[17] However, the evolutions of the capital share in the data and in the model are very similar. In the data, the capital share declines from 0.398 to 0.396, whereas in the model it declines from 0.392 to 0.389.[18]

Table 3 and Table 4 show alternative calibrations of our model economy. In Table 3, we consider different values for the elasticity of substitution, $\varepsilon$, while Table 4 considers the implications of different growth rates of the capital-intensive sector, $m_2$.

The results for different values of $\varepsilon$ in Table 3 are generally similar to the benchmark model. The most notable feature is that when $\varepsilon$ is smaller, for example, $\varepsilon = 0.56$ or $\varepsilon = 0.66$ instead of the benchmark value of $\varepsilon = 0.76$, there are greater changes in relative employments. With $\varepsilon = 0.56$, the decline in the relative employment of the capital-intensive sector is approximately 9% instead of 5% in the benchmark. The opposite happens when $\varepsilon$ is larger and there is even less change in relative employment. This is not surprising in view of the fact that, as noted in

[16] Equivalently, in the data the share of sector 1 in total employment, $L_1/L$ increases from 0.49 in 1948 to 0.56 in 2005. The corresponding increase in our benchmark calibration is from 0.52 to 0.54.

[17] Note that the initial values of $L_2/L_1$ is not the same in the data and in the benchmark model, since, as remarked above, we chose the sectoral allocation of labor implied by the model given the relative values of output in the two sectors (recall equation (40)).

[18] One reason why our model accounts only for a fraction of the structural change in the US economy may be that it focuses on a specific dimension of structural change: the reallocation of output between sectors with different capital intensities. In practice, a significant component of structural change is associated with the reallocation of output across agriculture, manufacturing and services. In addition, our model does not allow for changes in sectoral factor intensities over time.

footnote 6, with $\varepsilon = 1$ there would be no reallocation of capital and labor.

The broad patterns implied by different values of $m_2$ in Table 4 are also similar to the results of the benchmark calibration. It is noteworthy, however, that if $m_2$ is taken to be greater, for example, $m_2 = 0.0128$, the fit of the model to the data is improved. For example, in this case, there is a somewhat larger change in relative employment levels and also a larger decline in the relative quantity of the capital-intensive sector. In contrast, when $m_2$ is smaller than the benchmark, the changes in $Y_2/Y_1$ and $L_2/L_1$ are somewhat less pronounced.

Overall, our calibration exercises indicate that the mechanism proposed in this paper can generate changes in the sectoral composition of output that are broadly comparable with the changes we observe in the US data and changes in relative employment levels that are in the same direction as in the data, though quantitatively smaller. Notably, during this process of structural change the capital share in national income remains approximately constant.

# 4 Conclusion

We proposed a model in which the combination of factor proportion differences across sectors and capital deepening leads to a non-balanced pattern of economic growth. We illustrated the main economic forces using a tractable two-sector growth model, where there is a constant elasticity of substitution between the two sectors and Cobb-Douglas production technologies in each sector. We characterized the constant growth path and equilibrium (Pareto optimal) dynamics in the neighborhood of this growth path. We showed that even though sectoral growth is non-balanced, the behavior of the interest rate and the capital share in national income are consistent with the Kaldor facts. In particular, asymptotically the two sectors still grow at different rates, while the interest rate and the capital share are constant.

The main contribution of the paper is theoretical, demonstrating that the interaction between capital deepening and factor proportion differences across sectors will lead to non-balanced growth, while being still consistent with the aggregate Kaldor facts. We also presented a simple calibration of our baseline model, which showed that the equilibrium path exhibits sectoral employment and output shares changing significantly, while the aggregate capital share and the interest rate remain approximately constant. Moreover, the magnitudes implied by this simple calibration are comparable to, though somewhat smaller than, the sectoral changes observed in the postwar US data. A full investigation of whether the mechanism

suggested in this paper plays a first-order role in non-balanced growth in practice is an empirical question left for future research. It would be particularly useful to combine the mechanism proposed in this paper with non-homothetic preferences and estimate a structural version of the model with multiple sectors using US or OECD data.

# Appendix A

## Proof of Proposition 3

The equivalence of the solutions to (SP) and (SP$'$) follows from the discussion in the text, while the equivalence between (SP) and competitive equilibria follows from the First and Second Welfare Theorems. Next, consider the maximization (SP$'$). This corresponds to a standard optimal control problem. Moreover, the objective function is strictly concave, the constraint set is convex and the state variable, $K(t)$, is non-negative, so that the Arrow Sufficiency Theorem (e.g., Acemoglu, 2008, Theorem 7.14) implies that an allocation that satisfies the Pontryagin Maximum Principle and the transversality condition (29) uniquely achieves the maximum of (SP$'$). Thus we only need to show that equations (27)-(29) are equivalent to the Maximum Principle and the transversality condition. The Hamiltonian for (SP$'$) takes the form

$$H(\tilde{c}, K, \mu) = \exp(-(\rho - n)t)\frac{\tilde{c}(t)^{1-\theta} - 1}{1-\theta} + \mu(t)\left[\Phi(K(t), t) - \delta K(t) - \exp(nt) L(0)\tilde{c}(t)\right],$$

with $\mu(t)$ denoting the co-state variable. Inspection of (SP$'$) shows that paths that reach zero consumption or zero capital stock at any finite $t$ cannot be optimal, thus we can focus on interior solutions and write the Maximum Principle as

$$\begin{aligned} H_{\tilde{c}}(\tilde{c}, K, \mu) &= \exp(-(\rho - n)t)\tilde{c}(t)^{-\theta} - \mu(t)\exp(nt)L(0) = 0 \\ H_K(\tilde{c}, K, \mu) &= \mu(t)(\Phi_K(K(t), t) - \delta) = -\dot{\mu}(t), \end{aligned} \tag{41}$$

whenever the optimal control $\tilde{c}(t)$ is continuous. Combining these two equations, we obtain the Euler equation for consumption growth as

$$\frac{d\tilde{c}(t)/dt}{\tilde{c}(t)} = \frac{1}{\theta}\left[\Phi_K(K(t), t) - \delta - \rho\right]. \tag{42}$$

Moreover, equations (13) and (14) imply

$$\Phi(K(t), t) = \gamma\eta(t)^{1/\varepsilon} M_1(t) L(t)^{\alpha_1} \lambda(t)^{\alpha_1} K(t)^{1-\alpha_1} \kappa(t)^{1-\alpha_1}, \text{ and} \tag{43}$$

$$\Phi_K(K(t), t) = (1-\alpha_1)\gamma\eta(t)^{1/\varepsilon} M_1(t) L(t)^{\alpha_1} \lambda(t)^{\alpha_1} K(t)^{-\alpha_1} \kappa(t)^{-\alpha_1}. \tag{44}$$

The law of motion of technology in (6) together with the normalization in (26) implies $\dot{c}(t)/c(t) = (d\tilde{c}(t)/dt)/\tilde{c}(t) - m_1/\alpha_1$. Also from (26), we have $\chi(t)^{-\alpha_1} \equiv M_1(t) L(t)^{\alpha_1} K(t)^{-\alpha_1}$. Using

the previous two expressions and substituting (44) into (42), we obtain the first equation in (27). Next, again using (26) to write

$$\frac{\dot{\chi}(t)}{\chi(t)} = \frac{\dot{K}(t)}{K(t)} - n - \frac{m_1}{\alpha_1}$$

and substituting for $\dot{K}(t)$ from (25) and for $\Phi(K(t), t)$ from (43), we obtain the second equation in (27). Notice also that both of these equations depend on $\kappa(t)$. To obtain the law of motion of $\kappa(t)$, differentiate (15) and then use (5) and (16). Here $\kappa(0)$ is also taken as given because for given $K(0)$, (15) uniquely pins down $\kappa(0)$.

Finally, the transversality condition of (SP′) requires

$$\lim_{t\to\infty} \left[\exp\left(-(\rho - n)t\right)\mu(t) K(t)\right] = 0.$$

Combining (26) with (41) shows that this condition is equivalent to (29).

### Results with the Converse of Assumption (A2)

In the text, we stated and proved Proposition 3, Theorems 1 and 2 under Assumption (A2). This assumption was imposed only to reduce notation. When it is relaxed (and its converse holds), sector 1 is no longer the asymptotically dominant sector and a different type of normalization than that in (26) is necessary. In particular, the converse of Assumption (A2) is

$$\text{either (i) } m_1/\alpha_1 > m_2/\alpha_2 \text{ and } \varepsilon < 1\text{; or (ii) } m_1/\alpha_1 < m_2/\alpha_2 \text{ and } \varepsilon > 1. \tag{A2′}$$

It is straightforward to see that in this case sector 2 will be the asymptotically dominant sector. We therefore adopt a parallel normalization with

$$c(t) \equiv \frac{\tilde{c}(t)}{M_2(t)^{1/\alpha_2}} \quad \text{and } \chi(t) \equiv \frac{K(t)}{L(t) M_2(t)^{1/\alpha_2}}. \tag{45}$$

Given this normalization, it is straightforward to generalize Proposition 3, Theorems 1 and 2.

**Proposition 4** Suppose that (A1) and (A2′) hold. Then, a competitive equilibrium satisfies the following three differential equations

$$\begin{aligned}
\frac{\dot{c}(t)}{c(t)} &= \frac{1}{\theta}\left[(1-\alpha_2)\gamma\eta(t)^{1/\varepsilon}\lambda(t)^{\alpha_2}\kappa(t)^{-\alpha_2}\chi(t)^{-\alpha_2} - \delta - \rho\right] - \frac{m_2}{\alpha_2}, \\
\frac{\dot{\chi}(t)}{\chi(t)} &= \lambda(t)^{\alpha_2}\kappa(t)^{1-\alpha_2}\chi(t)^{-\alpha_2}\eta(t) - \chi(t)^{-1}c(t) - \delta - n - \frac{m_2}{\alpha_2}, \\
\frac{\dot{\kappa}(t)}{\kappa(t)} &= \frac{(1-\kappa(t))\left[-\Delta\frac{\dot{\chi}(t)}{\chi(t)} + m_1 - \frac{\alpha_1}{\alpha_2}m_2\right]}{(1-\varepsilon)^{-1} - \Delta(\kappa(t) - \lambda(t))}, \text{ where}
\end{aligned} \tag{46}$$

$$\eta(t) \equiv \gamma^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1-\kappa(t)}{\kappa(t)}\right)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{47}$$

with initial conditions $\chi(0)$ and $\kappa(0)$, and also satisfies the transversality condition

$$\lim_{t\to\infty} \exp\left(-\left(\rho - \frac{(1-\theta)\,m_2}{\alpha_2} - n\right)t\right)\chi(t) = 0. \tag{48}$$

Moreover, any allocation that satisfies these conditions is a competitive equilibrium.

**Proof.** The proof is analog to that of Proposition 3 and is omitted. ■

**Theorem 3** Suppose that (A1), (A2′) and (A3) hold. Then, there exists a unique CGP where consumption per capita grows at the rate $g_c^* = m_2/\alpha_2$, and $\kappa^* = 0$,

$$\chi^* = \left[\frac{(\theta m_2/\alpha_2 + \rho + \delta)}{(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha_2)}\right]^{-\frac{1}{\alpha_2}},$$

and $c^* = (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(\chi^*)^{1-\alpha_2} - \chi^*\left(\delta + n + \frac{m_2}{\alpha_2}\right)$. Moreover, the growth rates of output, capital and employment in the different sectors are given by

$$g^* = g_2^* = z_2^* = n + \frac{m_2}{\alpha_2},\; z_1^* = g^* - (1-\varepsilon)\,\tilde{\omega}$$

$$g_1^* = g^* + \varepsilon\tilde{\omega},\; n_2^* = n, \text{ and } \; n_1^* = n - (1-\varepsilon)\,\tilde{\omega},$$

*where*

$$\tilde{\omega} \equiv m_1 - \alpha_1 \frac{m_2}{\alpha_2} > 0.$$

**Proof.** The proof is analog to that of Theorem 1 and is omitted. ■

It can also be easily verified that in this case $\sigma_K^* = 1-\alpha_2$ and $r^* = (1-\alpha_2)\,\gamma^{\frac{\varepsilon}{\varepsilon-1}}(\chi^*)^{-\alpha_2} - \delta$.

**Theorem 4** Suppose that (A1), (A2′) and (A3) hold. Then, the non-linear system (27) is locally (saddle-path) stable, in the sense that in the neighborhood of $c^*$, $\chi^*$ and $\kappa^*$, there is a unique two-dimensional manifold of solutions that converge to $c^*$, $\chi^*$ and $\kappa^*$.

**Proof.** The proof follows that of Theorem 2. Once again linearizing the dynamics around the CGP, we obtain $\dot{z} = J(x^*)\,z$, with $z \equiv x - x^*$ and $x^*$ such that $f(x^*) = 0$, where

$J(x^*)$ is the Jacobian of $f(x)$ evaluated at $x^*$. The determinant of the Jacobian is again $\det J(x^*) = -a_{\kappa\kappa} a_{c\chi}$, where

$$\begin{aligned} a_{\kappa\kappa} &= -(1-\varepsilon)\left(m_1 - \alpha_1 \frac{m_2}{\alpha_2}\right) \\ a_{c\chi} &= -(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}} (\chi^*)^{-\alpha_2 - 1} \frac{\alpha_2 (1-\alpha_2)}{\theta}. \end{aligned}$$

Once again $a_{\kappa\kappa}$ and $a_{c\chi}$ are strictly negative, since, under Assumption (A2′), $\varepsilon \gtrless 1 \Leftrightarrow m_2/\alpha_2 \gtrless m_1/\alpha_1$. Therefore, as in the proof of Theorem 2, $\det J(x^*) > 0$ and the steady state is hyperbolic. The same argument as in that proof shows that there must be two negative eigenvalues and establishes the result. ■

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**Table 1: Industry Capital Shares**

| INDUSTRY | SECTOR | CAPITAL SHARE |
|---|---|---|
| Educational services | 1 | 0.10 |
| Management of companies and enterprises | 1 | 0.20 |
| Health care and social assistance | 1 | 0.22 |
| Durable goods | 1 | 0.27 |
| Administrative and waste management services | 1 | 0.28 |
| Construction | 1 | 0.32 |
| Other services, except government | 1 | 0.33 |
| Professional, scientific, and technical services | 1 | 0.34 |
| Transportation and warehousing | 1 | 0.35 |
| Accommodation and food services | 2 | 0.36 |
| Retail trade | 2 | 0.42 |
| Arts, entertainment, and recreation | 2 | 0.42 |
| Finance and insurance | 2 | 0.45 |
| Wholesale trade | 2 | 0.46 |
| Nondurable goods | 2 | 0.47 |
| Information | 2 | 0.53 |
| Mining | 2 | 0.66 |
| Utilities | 2 | 0.77 |

Note: US data from NIPA. Sector 1 comprises the low capital-intensity industries, while sector 2 comprises the high capital-intensity industries. The capital intensity of each industry is the average capital share between 1987 and 2005, where capital share is computed as value added minus total compensation divided by value added.

**Table 2: Data and Model Calibration, 1948-2005**

| | *US Data* | | *Benchmark Calibration* $\varepsilon = 0.76$ , $m_2 = 0.0108$ | |
|---|---|---|---|---|
| | 1948 | 2005 | 1948 | 2005 |
| $Y_2/Y_1$ | 0.85 | 1.01 | 0.85 | 1.00 |
| $L_2/L_1$ | 1.03 | 0.80 | 0.91 | 0.87 |
| $\sigma_k$ | 0.40 | 0.40 | 0.39 | 0.39 |

Note: US data from NIPA. Classifications and calibration described in the text.

**Table 3: Data and Model Calibration, 1948-2005 (Robustness I)**

| | *US Data* | | *Model* $\varepsilon = 0.56,\ m_2 = 0.0108$ | | *Model* $\varepsilon = 0.66,\ m_2 = 0.0108$ | | *Model* $\varepsilon = 0.86,\ m_2 = 0.0108$ | |
|---|---|---|---|---|---|---|---|---|
| | 1948 | 2005 | 1948 | 2004 | 1948 | 2004 | 1948 | 2004 |
| $Y_2/Y_1$ | 0.85 | 1.01 | 0.85 | 0.96 | 0.85 | 0.98 | 0.85 | 1.02 |
| $L_2/L_1$ | 1.03 | 0.80 | 0.91 | 0.83 | 0.91 | 0.85 | 0.91 | 0.89 |
| $\sigma_k$ | 0.40 | 0.40 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 |

Note: US data from NIPA. Classifications and calibration described in the text.

**Table 4: Data and Model Calibration, 1948-2005 (Robustness II)**

| | *US Data* | | *Model* $\varepsilon = 0.76,\ m_2 = 0.0098$ | | *Model* $\varepsilon = 0.76,\ m_2 = 0.0118$ | | *Model* $\varepsilon = 0.76,\ m_2 = 0.0128$ | |
|---|---|---|---|---|---|---|---|---|
| | 1948 | 2005 | 1948 | 2004 | 1948 | 2004 | 1948 | 2004 |
| $Y_2/Y_1$ | 0.85 | 1.01 | 0.85 | 0.96 | 0.85 | 1.05 | 0.85 | 1.09 |
| $L_2/L_1$ | 1.03 | 0.80 | 0.91 | 0.88 | 0.91 | 0.86 | 0.91 | 0.85 |
| $\sigma_k$ | 0.40 | 0.40 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 |

Note: US data from NIPA. Classifications and calibration described in the text.

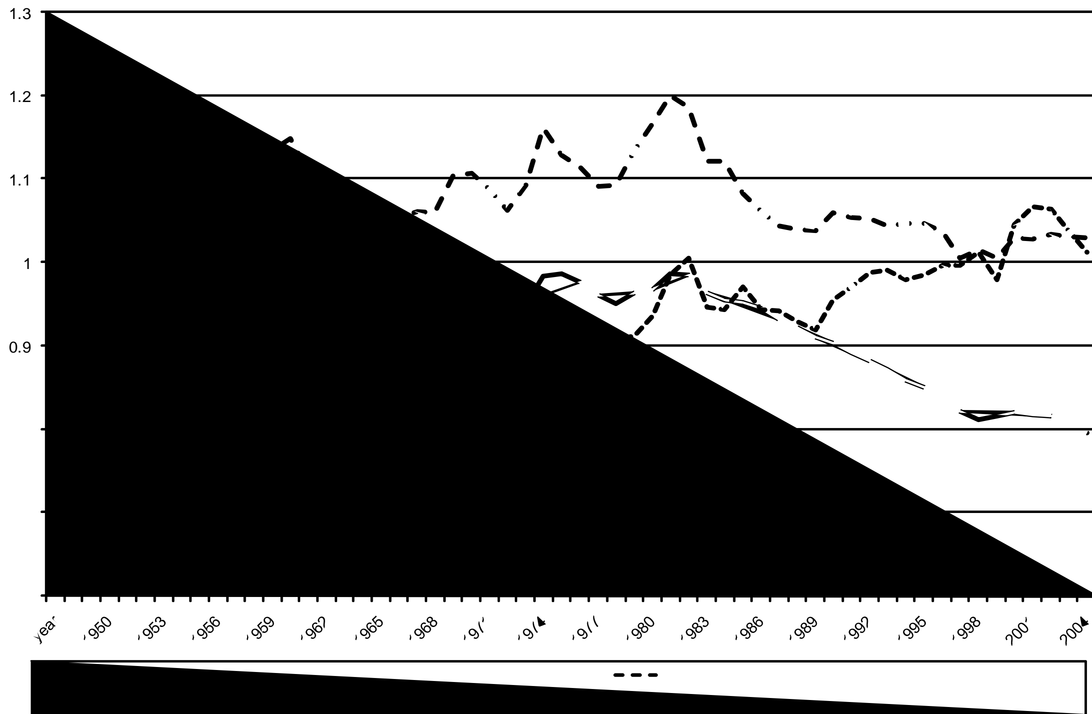


Figure 1: Employment, price-weighted value of output, and fixed-price quantity indices in high capital intensity sectors relative to low capital intensity sectors, 1948-2005. See Section 3 for industry classifications. Data from the National Income and Product Accounts (NIPA).

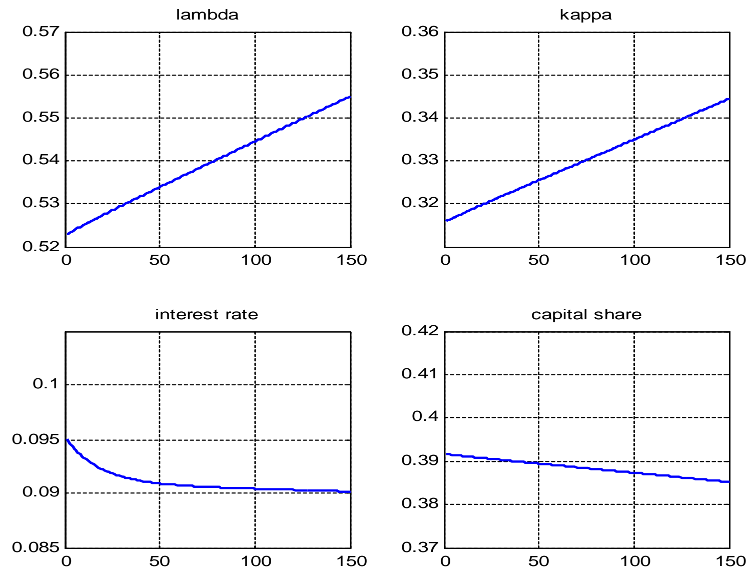


Figure 2: Behavior of $\kappa$, $\lambda$, $r$ and $\sigma_K$ in the benchmark calibration with $\varepsilon = 0.76$ and $m_2 = 0.0108$. See text for further details.

# Appendix B for Acemoglu-Guerrieri "Capital Deepening and Non-Balanced Economic Growth" (Not for Publication)

## National Income Product Accounts Data

All the data used in the paper refer to US data and are from the Gross Domestic Product by Industry Data of the National Income and Product Accounts (NIPA). Industries are classified according to the North American Industrial Classification System (NAICS). Throughout, we use the 22-industry level of detail data. This level of aggregation enables us to extend the sample back to 1948. We exclude the Government and Private household sectors as well as Agriculture, forestry, fishing, and hunting and Real estate and rental. Real estate is excluded since it has a very high capital share due to the value of assets in this sector, which does not reflect the share of capital in the production function of the sector.

Employment is total full-time and part-time employment (FTPT), in thousands of employees, in the indicated industries. We use this measure of employment because it is the only one for which the Bureau of Economic Analysis (BEA) released estimates based on NAICS classification going back to 1948. All other employment measures are calculated using SIC up to 1997. The quantity in a sector is equal to the expenditure-weighted sum of industry-level quantities. More specifically, let $\mathcal{S}$ denote a subset of the industries. Then quantity in this subset of sectors at time $t$ is calculated as:

$$Q_t^{\mathcal{S}} = \frac{\sum_{j \in S} E_{j,t} \cdot Q_{j,t}}{\sum_{j \in S} E_{j,t}},$$

where $Q_{j,t}$ is fixed-price quantity index in sector $j$ at time $t$ in 2000 dollars, calculated as the product of the chain-type quantity index for value added (VAQI), with 2000 as base year, and the year 2000 current-dollar value added of the corresponding series (VA) divided by 100, and $E_{j,t}$ is expenditure on sector $j$ at time $t$, approximated by value added.

The share of capital in national income is computed as value added minus total compensation to employees over value added, i.e.,

$$\text{capital share}_t = \frac{E_t - W_t}{E_t}, \tag{B1}$$

where $E_t$ is total value added (VA) at time $t$ and $W_t$ is total compensation to employees (COMP) at time $t$.

The value of the initial capital stock is the initial value of private fixed assets in current dollars.

As discussed in the text, in Figure 1, and in Section 3, sectors are classified according to their capital intensity. Capital intensity in each sector is calculated as:

$$\text{capital share}_{j,t} = \frac{E_{j,t} - W_{j,t}}{E_{j,t}}, \tag{B2}$$

where $E_{j,t}$ is value added in sector $j$ at time $t$ and $W_{j,t}$ is compensation to employees in sector $j$ at time $t$. Because of the change in the classification of industries before and after 1987, we compute the capital share of each industry as the average between 1987 and 2005, which enables us to use the consistent NAICS classification for the compensation series (before 1987 this variable is only available with the SIC classification). Industries are then ranked according to their average capital intensity as shown in Table 1 in the text, and the capital share cutoff level for separating industries into high and low capital intensity industries is chosen to create two groups of industries with approximately 50% of employment each. According to this ranking, *low capital intensity industries* are: Construction; Durable goods; Transportation and Warehousing; Professional, scientific, and technical services; Management of companies and enterprises; Administrative and waste management services; Educational services; Health care and social assistance; Other services, except government. *High capital-intensive industries* are: Mining; Utilities; Non-durable goods; Wholesale; Retail trade; Information; Finance and Insurance; Arts, Entertainment, and Recreation; Accommodation and food services.

One might worry that the capital intensity of different industries may change over our sample period because of technological reasons or because of changes in relative factor prices. The next two tables show that the classification of industries according to capital industry is highly stable over time. These two tables show the Spearman's rank correlation matrix among alternative rankings of industries based on average capital share in different time periods and using different industry classifications in order to document this point. In these two tables, we denote the ranking based on average capital share between dates $t$ and $t'$ by $\sigma_{t,t'}$. As noted in the text, the NAICS classification starts in 1987, so we can only compare the stability of capital intensity ranking between 1987 and 2005 using the NAICS data. To supplement this, we compute capital shares choosing data for SIC industries matched to the NAICS classification (using the correspondence tables constructed by the US Census Bureau). This enables us both to show the similarity of the ranking according to NAICS and SIC classifications and also to

extend the comparison of the classifications to 1948 (though ending in the year 2000, since this is the last available year for the SIC classification). We denote the rankings between dates $t$ and $t'$ that use data based on SIC by $\hat{\sigma}_{t,t'}$.

Table B1 reports the Spearman's rank correlation matrix among our benchmark ranking, $\sigma_{87,05}$, that averages capital share over all the years for which the NAICS data are available, rankings that use the same data for 5 year subperiods (in particular, between 1987 and 1991; 1992 and 1997; and so on), and the ranking using the SIC data for the period 1948-2000. This table shows a high degree of correlation between the NAICS rankings across different subperiods. In particular, all of the correlation indices are greater than 0.94. The last row of the table also shows that there is a significant correlation between the ranking based on SIC classification using data between 1948 and 2000 and the one based on NAICS classification, though now the correlation is somewhat lower, around 0.75.

**Table B1: Spearman Correlation Matrix for Capital-Intensity Rankings (NAICS)**

| | $\sigma_{87,05}$ | $\sigma_{87,91}$ | $\sigma_{92,96}$ | $\sigma_{97,01}$ | $\sigma_{02,05}$ | $\hat{\sigma}_{48,00}$ |
|---|---|---|---|---|---|---|
| $\sigma_{87,05}$ | 1.00 | | | | | |
| $\sigma_{87,91}$ | 0.96 | 1.00 | | | | |
| $\sigma_{92,96}$ | 0.99 | 0.98 | 1.00 | | | |
| $\sigma_{97,01}$ | 1.00 | 0.95 | 0.98 | 1.00 | | |
| $\sigma_{02,05}$ | 0.99 | 0.94 | 0.97 | 0.99 | 1.00 | |
| $\hat{\sigma}_{48,00}$ | 0.73 | 0.75 | 0.72 | 0.76 | 0.74 | 1.00 |

Note: This table reports the Spearman correlation matrix for different capital intensity rankings of the NAICS at the 22-industry level of detail, measuring each industry's capital intensity as the average value over the whole sample available with the NAICS data, 1987-2005 ($\sigma_{87,05}$), and over 5 year subperiods, 1987-1991 ($\sigma_{87,91}$), 1992-1996 ($\sigma_{92,96}$), and so on. The last row ($\hat{\sigma}_{48,00}$) is obtained using SIC data for the whole sample available with SIC data, 1948-2000, and matching the industries based on the SIC classification to the NAICS classification using the correspondence tables constructed by the US Census Bureau. See text for details.

Table B2 reports the Spearman's rank correlation matrix among rankings that use SIC data only, again averaging capital shares over 5 years periods. This table also shows a considerable amount of stability in rankings based on capital intensity. Most correlation coefficients are above 0.8, though when we compare the rankings in the late 1990s to those in the early 1950s, the correlation becomes as low as 0.50.

**Table B2: Spearman Correlation Matrix for Capital-Intensity Rankings (SIC)**

| | $\hat{\sigma}_{48,00}$ | $\hat{\sigma}_{48,52}$ | $\hat{\sigma}_{53,57}$ | $\hat{\sigma}_{58,62}$ | $\hat{\sigma}_{63,67}$ | $\hat{\sigma}_{68,72}$ | $\hat{\sigma}_{73,77}$ | $\hat{\sigma}_{78,82}$ | $\hat{\sigma}_{83,87}$ | $\hat{\sigma}_{88,92}$ | $\hat{\sigma}_{93,97}$ | $\hat{\sigma}_{98,00}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\sigma}_{48,00}$ | 1.00 | | | | | | | | | | | |
| $\hat{\sigma}_{48,52}$ | 0.60 | 1.00 | | | | | | | | | | |
| $\hat{\sigma}_{53,57}$ | 0.77 | 0.95 | 1.00 | | | | | | | | | |
| $\hat{\sigma}_{58,62}$ | 0.77 | 0.94 | 0.99 | 1.00 | | | | | | | | |
| $\hat{\sigma}_{63,67}$ | 0.77 | 0.92 | 0.96 | 0.98 | 1.00 | | | | | | | |
| $\hat{\sigma}_{68,72}$ | 0.86 | 0.88 | 0.96 | 0.97 | 0.98 | 1.00 | | | | | | |
| $\hat{\sigma}_{73,77}$ | 0.88 | 0.83 | 0.94 | 0.95 | 0.93 | 0.97 | 1.00 | | | | | |
| $\hat{\sigma}_{78,82}$ | 0.94 | 0.76 | 0.90 | 0.90 | 0.88 | 0.94 | 0.98 | 1.00 | | | | |
| $\hat{\sigma}_{83,87}$ | 0.97 | 0.73 | 0.88 | 0.87 | 0.86 | 0.92 | 0.94 | 0.98 | 1.00 | | | |
| $\hat{\sigma}_{88,92}$ | 0.96 | 0.56 | 0.74 | 0.74 | 0.71 | 0.82 | 0.85 | 0.90 | 0.94 | 1.00 | | |
| $\hat{\sigma}_{93,97}$ | 0.99 | 0.50 | 0.69 | 0.69 | 0.69 | 0.80 | 0.82 | 0.89 | 0.93 | 0.96 | 1.00 | |
| $\hat{\sigma}_{98,00}$ | 0.98 | 0.50 | 0.69 | 0.69 | 0.71 | 0.81 | 0.81 | 0.88 | 0.92 | 0.92 | 0.99 | 1.00 |

Note: This table reports the Spearman correlation matrix for different capital intensity rankings of the NAICS at the 22-industry level of detail, measuring each industry's using SIC data over different horizons. We compare rankings obtained averaging industries' capital share over the whole sample available with SIC data, 1948-2000 ($\hat{\sigma}_{48,00}$), and over 5 year subperiods, 1948-1952 ($\hat{\sigma}_{48,52}$), 1953-1957 ($\hat{\sigma}_{53,57}$) and so on. Industries based on the SIC have been matched to the industries based on the NAICS using the correspondence tables constructed by the US Census Bureau. See text for further details.

We next check the robustness of the pattern shown in Figure 1 in the Introduction. Recall that this pattern, which involves more rapid growth of quantity in the high capital-intensive sector and more rapid growth of value and employment in the low capital-intensive sector, is a distinctive prediction of our model. In particular, we would like to ensure that this pattern is not an artifact of the exact cutoff we use in the classification (which also determines the industries that are in the different groups), the specific source of data, and sample period. Table B3 documents the robustness of this pattern by showing the relative growth rate of quantity, employment and value in the high capital intensity industries (compared to low capital intensity industries) in each case with a different classification. The first column corresponds to our benchmark classification and shows that between 1948 and 2005, quantity in high capital intensity industries has grown by 19% more than in low capital intensity industries. In the meantime, employment in these high capital intensity industries has declined by 22% and value has declined by 18% relative to low capital intensity industries. Recall that our benchmark classification placed approximately 50% of employment in each of the two sectors.

The second and third columns report the same statistics, but using alternative cutoffs so that 60% (column 2) and 40% (column 3) of total employment is in the industry grouping with low capital intensity. In the former case, Accommodation and food services move into the low capital intensity group. In the latter, Transport and warehousing move into the high capital intensity group. In both cases, the general patterns remain very similar to the benchmark. For example, with the 60% cutoff, there is a 21% increase in the relative quantity of high capital intensity industries, a 36% decline in their relative employment, and a 19% decline in their value. With the 40% cutoff, the relative increase in quantity is 44%, the relative decline in employment is 19%, and the relative decline in value is 9%. Columns 4 and 5 show similar results with the SIC data for the periods 1948-2000 and 1977-2000. We do not have quantity data before 1977 with the SIC classification, so the first row is missing in the fourth column. The remaining rows in these columns are again similar to the benchmark, though when focusing on the narrower sample of 1977-2000, the relative increase in quantity is only 2%. Finally, in column 6 we report the results for the longer horizon 1929-2000. The only measure of value added by industry available starting in 1929 is data on income by industry from the *Historical Statistics of the United States* (the original source for these data is the BEA). The series of full-time and part-time employees by industry in the NIPA tables also extends back to 1929. As in column 4, we do not have quantity data, so that the first row of this column is missing. The other two rows show a pattern consistent with the rest of the table, with a more rapid growth of employment and value in the less capital-intensive industries than in more capital-intensive industries.

**Table B3: US Data robustness checks**

| | (1) *Benchmark* *(1948-2005)* | (2) *Cutoff 60%* *(1948-2005)* | (3) *Cutoff 40%* *(1948-2005)* | (4) *SIC Data* *(1948-2000)* | (5) *SIC Data* *(1977-2000)* | (6) *SIC Data* *(1929-2000)* |
|---|---|---|---|---|---|---|
| growth % $Y_2/Y_1$ | 19% | 21% | 44% | *n.a.* | 2% | *n.a* |
| growth % $L_2/L_1$ | −22% | −36% | −19% | −27% | −20% | −39% |
| growth % $Y_2^N/Y_1^N$ | −18% | −19% | −9% | −24% | −9% | −37% |

Note: Column 1 in Table B4 reports the growth rate of quantity (quantity index for value added), employment (full-time and part-time employees) and value (value added) in the high capital intensity sector relative to the low capital intensity sector, constructed according to our benchmark classification shown in Table B1. Columns 2 and 3 report the same statistics using alternative sectoral classifications, where the capital share cutoff for dividing industries into high and low capital intensity sectors is chosen to create a group of high-capital intensity industries including approximately 60% and 40% of total employment, instead of 50% as in the benchmark. The last three columns report the same statistics for the benchmark sectoral classification using the SIC data for different sample periods. See text for further details.

Finally, we have also looked for other sources of data to go back further than 1929. *The Historical Statistics of the United States* provides a number of employment series, though in most cases data are only available at a higher level of aggregation than the NIPA and the related data we have used so far. The series with the finest industry classification in the *Historical Statistics of the United States* is based on Sobek (2001) and covers only the time period 1910-1990, which is only slightly earlier than the data we have reported in Table B3. These data are available in 10 year intervals and use the 1950 Census industry classification. Matching these to the NAICS classification, we computed employment growth in the high and low capital intensity sectors.[19] The results from this exercise are consistent with those in Table B3 and show that employment in high capital intensity industries declines by about 10% relative to employment in low capital intensity industries between 1910 and 1990. The remaining sources of data on employment by industry have higher levels of aggregation, making the type of exercise we are performing here more difficult. One source of data on employment by industry is the Bureau of Labor Statistics, which covers the period 1919-1999. However, a significant fraction of the industries are missing before 1939. We matched the industry classification of this dataset to the NAICS classification,[20] and computed employment growth in high and low capital intensity

[19] For this exercise we dropped one of the 1950 census industry groups, "Transportation and Communication," because it combines two industries that are separated according to the NAICS classification.

[20] With this classification, for the same reason as with the Sobek data, we had to drop the composite industry "Transportation, Communication and Utilities".

industries. The results show that employment in high capital intensity industries declines by about 70% relative to employment in low capital intensity industries between 1939 and 1999. The *Historical Statistics of the United States* also provides two other series, one from Lebergott (1964) and Weiss (1992) covering the period 1800-1960, and the other from Lebergott (1984) for 1900-1940. Unfortunately, given the more crude industry classifications, we were not able to create a consistent match with to the NAICS classifications, especially with the former dataset. It is also possible that the ranking of industries according to capital share, which appears to be fairly stable in the postwar era, may be quite different when we go back before 1900. In any case, using these data we were not able to obtain consistent patterns on relative employment growth across high and low capital intensity industries. We believe that this reflects the less fine industry aggregation in these data and the possible changes in the ranking according to capital intensity, though it may also be due to data quality or because the patterns we have documented for the past 80 years do not apply to the period before.

## Additional References for Appendix B

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