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Heterogeneity, Job Creation and Unemployment Volatility

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Abstract

In this paper, I explore the impact of match-specific heterogeneity at the job creation margin on business cycle fluctuations. I show that this form of heterogeneity alone does not help to amplify labor market volatility, either under full or under asymmetric information. First, I show analytically that, under full information, heterogeneity has no first-order effect on the response of unemployment and job creation to productivity, and actually tends to dampen the response of market tightness. Then, in a series of calibrations, I show that with both full and asymmetric information, the model delivers labor market volatilities close to the representative-agent, full-information benchmark.

Keywords: Unemployment; job creation; market tightness; heterogeneity; asymmetric information

JEL classification: *E*24; *E*32; *J*41; *J*63; *J*64

I. Introduction

Whether search models can match the observed cyclical behavior of unemployment and vacancies in the U.S. economy is an issue that has recently received new attention. Shimer (2005) and Hall (2005a) opened the debate by showing that the conventional search model *à la* Mortensen and Pissarides cannot account for the high responsiveness of unemployment to productivity shocks; see Mortensen and Pissarides (1994) and Pissarides (2000). The main reason is that, in such a model, wages absorb the shocks, thereby reducing the response of firms' profits and, hence, of job creation.

Workers' heterogeneity in terms of skills and ability is a pervasive element of labor markets. In particular, at the moment of hiring, a firm has to assess whether the worker is a good fit for the job. In this paper, I focus on this form of heterogeneity, which is relevant at the job creation margin, and I ask whether it can help to explain the cyclical behavior of labor markets.

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Given my interest in the job creation margin, I make the simplifying assumption that workers only differ in terms of effort costs they have to incur at the beginning of the employment relationship. These costs capture both time and energy spent in training activities, and the disutility from moving to the new location and adapting to the new environment. I could also include the effort spent in production as long as there are substantial sunk costs at the beginning of the match. This assumption, together with the absence of on-the-job mobility, ensures that heterogeneity does not affect the destruction margin. Although both endogenous separation and on-the-job mobility are relevant ingredients of the labor market,[1] I abstract from these issues here to investigate, in isolation, the role of an endogenous hiring margin.

In this paper, I analyze both the case of full information and the case in which workers privately observe their own effort costs. The main difference from the standard model is that the job creation rate can now be decomposed into two margins: the standard matching margin and an endogenous hiring margin generated by the heterogeneity element. When a good productivity shock hits the economy, the hiring margin becomes looser, in the sense that workers with higher costs are hired. On the one hand, this implies that both the expected cost of a matched worker and wages increase. On the other hand, firms' profits can potentially increase because output is higher for each worker hired. When information is asymmetric, there is an additional potential source of volatility. The firm has to pay the workers some rents in order to give them an incentive to reveal their information. Then, when there is a good productivity shock, there is more surplus to cover the rents and the distortion becomes smaller, thus generating a boost in job creation. Nevertheless, when the hiring margin increases, the rents that need to be paid to the workers become higher and asymmetric information could dampen the responsiveness of vacancies. Therefore, on theoretical grounds, heterogeneity together with asymmetric information may or may not amplify the response of the economy to productivity shocks. Under realistic parametric assumptions, however, amplification is absent or quantitatively not significant.

I set up a competitive search model,[2] where workers are hit by match-specific idiosyncratic shocks. Employers and workers are both risk neutral and *ex ante* homogeneous. Employers post contracts and workers direct their search towards them. When a match is formed, the worker's type is drawn randomly and can be either public information or private information

[1] See Davis, Faberman and Haltiwanger (2006) for the role of job destruction in explaining unemployment fluctuations. See Fallick and Fleischman (2004) and Nagypál (2005) for the role of job-to-job transitions.

[2] See Shimer (1996), Moen (1997) and Acemoglu and Shimer (1999).

of the worker. An employment contract is a mechanism that is incentive compatible when the information is private, and satisfies a participation constraint on the worker's side. A worker cannot be forced to participate in the employment relationship; he can always quit and join the ranks of the unemployed.

First, I introduce the model with full information and derive the main analytical result: in steady state, heterogeneity does not amplify the response of the unemployment rate to changes in productivity. On the contrary, it tends to dampen the response of market tightness to productivity. Next, I turn to the model with asymmetric information and show that in steady state, there is an additional effect on the response of market tightness to productivity, coming from the binding incentive-compatibility constraint, which makes a comparison with the standard model ambiguous.

Finally, I report the results of a series of calibrations. I describe how I parametrized the model to match the facts of the U.S. labor market documented in Shimer (2005). This guarantees that the version of the model with degenerate distribution of idiosyncratic shocks corresponds to the standard model calibrated in Shimer (2005). Then, I explore the model with more general distributions for the idiosyncratic shocks, under both full and asymmetric information. Using different families of distributions, I show that, also under asymmetric information, heterogeneity at the job creation margin alone cannot help in amplifying the responsiveness of market tightness and unemployment to productivity.

A growing number of papers react to Shimer (2005) by attempting to generate a higher volatility for the unemployment rate.[3] My paper contributes to this literature by showing that workers' heterogeneity at the creation margin alone cannot help in explaining business cycle fluctuations. This does not mean that heterogeneity cannot be part of a more complex explanation. Some papers explore the role of other sources of volatility and the interaction of heterogeneity with other crucial ingredients of the labor market. Among others, Hall (2005b) constructs a model exhibiting wage rigidity, Menzio (2004) assumes employers have private information about productivity with a distribution of types that varies over the cycle, and Nagypál (2004) combines workers' heterogeneity, asymmetric information and on-the-job search. The paper by Brugemann and Moscarini (2007) is the closest in spirit to mine. They investigate alternative wage determination structures in search models and their ability to match the response of the unemployment rate to productivity shocks. In particular, they explore wage-bargaining models with asymmetric information and construct theoretical bounds on the level of market tightness elasticity with respect to

[3] Mortensen and Nagypál (2005) offer a comprehensive review of this literature together with an alternative calibration proposal.

the productivity that these models can generate. My analysis is complementary to their model in that I assume wage posting, focus on the role of heterogeneity, with both full and asymmetric information, and analyze the equilibrium dynamics using numerical exercises, while they perform a comparative-static analysis. My results are consistent with their claim that asymmetric information cannot generate realistic levels of volatility.[4]

Hagedorn and Manovskii (2007) show a calibration where the value of non-market activity is much higher than in Shimer (2005),[5] thereby generating a higher elasticity of market tightness to productivity. In my calibration I am careful to keep the value of non-market activity at less than half of labor income, in order to preclude that explanation and highlight the role of heterogeneity and information.

From a theoretical point of view, the model in this paper is related to Shimer and Wright (2004) and Moen and Rosén (2006) who consider asymmetric information in a competitive search model. More specifically, my model is the stochastic version of the dynamic model in Guerrieri (2007), where I explore the dynamic efficiency properties of competitive search models under asymmetric information. As explained in Section II, apart from introducing an aggregate productivity shock into the basic environment, there is another modeling difference for the sake of analytical tractability. Here, the worker's type represents a sunk cost that the worker has to pay at the moment of hiring, such as a training or a mobility cost, instead of a flow cost, such as work effort. This change makes the model with aggregate shocks more tractable because it then becomes possible to avoid endogenous separation, which would otherwise emerge once a good shock hits the economy.

This paper is organized as follows. In Section II, I introduce the model with *ex post* heterogeneous workers and full information, as well as define and characterize the competitive search equilibrium. Moreover, I illustrate some comparative-static results in the extreme case of no aggregate shock. In Section III, I extend the model to the case of asymmetric information, and define and characterize the competitive search equilibrium in this environment. In Section IV, I describe the calibration exercises and report the results of the main specifications. Section V concludes.

II. The Model with Full Information

I begin by exploring a stochastic directed search model with match-specific heterogeneity and full information. I show that when the distribution of the

[4] In Section III, I show that their weak bound applies to the steady-state version of my model.
[5] They calibrate the worker's bargaining power and the non-market activity to match elasticity of wages and average firms' profits.

idiosyncratic shocks is degenerate, the model boils down to the discrete version of Shimer (2005), where the Hosios condition is satisfied. When, instead, the distribution of idiosyncratic shocks is non-degenerate, the reaction of market tightness to productivity shocks can in fact be dampened.

Environment

Consider an economy with infinite horizon and discrete time, populated by a continuum of measure 1 of workers and a large continuum of potential employers. Both workers and employers are *ex ante* homogeneous and have linear preferences with discount factor β. Workers can search freely, while employers need to pay an entry cost c to post a vacancy. When a match is formed, the worker has to face some training cost x, where x is drawn randomly from the cumulative distribution function $\Phi(\cdot)$, with support $X \equiv [\underline{x}, \overline{x}]$. I assume that the cumulative distribution function $\Phi(\cdot)$ is differentiable, with $\varphi(\cdot)$ denoting the associated density function. Any worker–employer match produces p_t units of output at any time t in which it is productive. Productivity p_t follows a first-order Markov process in discrete time,[6] according to some distribution $G(p', p) = \Pr(p_{t+1} \leq p' | p_t = p)$ with finite support $P \equiv [p_1, \ldots, p_N]$. The net surplus of the match is given by $p_t - x$ at time t, when the match is created, and p_τ at any future time $\tau \geq t$ when the match is still productive. Both x and p are common knowledge.

At the beginning of each period t, the aggregate productivity shock p_t is realized. Employers can be either productive or not, and workers can be either employed or unemployed. Non-productive employers can open a vacancy at a cost c, which entitles them to post an employment contract contingent on the aggregate shock. A contract is a revelation mechanism, that is, a map $\mathcal{C} : X \times P \mapsto [0, 1] \times \mathbb{R}_+$, specifying for each worker of type x matched when productivity is p_t, the hiring probability $e(x, p_t) \in [0, 1]$ and the expected net present value of wages $\omega(x, p_t) \in \mathbb{R}_+$. Each worker observes the contracts posted by active firms, the set $\mathbb{C}^P \subset \mathbb{C}$, and chooses to search for a specific contract $\mathcal{C} \in \mathbb{C}^P$. Then, matching takes place and, for each match, the draw x is realized. Conditional on the aggregate shock p_t and on his match-specific shock x, the worker decides whether to participate in the employment relationship or not. If the worker does not meet an employer or is not hired, he remains unemployed, gets a non-transferable utility from leisure z and looks for another match next period. If the worker is hired, the parties are productive until separation, which occurs according to a Poisson process with parameter s.

[6] Note that productivity p_t is common to all the matches existing at time t even though they are created at different times $t - \tau$, for $\tau = 0, 1, \ldots, t$.

Apart from the stochastic aggregate shock, the main difference from the environment in Guerrieri (2007) concerns the idiosyncratic shock. In this paper, x is a sunk cost that the worker suffers at the moment of the match, which I interpret as cost of training. In Guerrieri (2007), x represents the expected value of the disutility that the worker suffers each period in which he is productive, which I interpret as work effort. Once the aggregate shock is introduced, this change is crucial for making the model tractable, given that it ensures that at any point in time $\tau > t$, it is never optimal to end the match created at time t, as long as $p_\tau > 0$. If x were per-period work effort, then during bad times the firm would like to fire workers with a high enough x, thereby generating an additional endogenous separation mechanism.

Trading frictions in the labor market are modeled through random matching. Employers and workers know that their matching probabilities will depend on the contract that they post and seek, respectively. Each type of contract $\mathcal{C}$ is associated with a labor submarket, where a mass $v(\mathcal{C})$ of employers posts contracts of type $\mathcal{C}$ and a mass $u(\mathcal{C})$ of unemployed workers applies for jobs at firms offering that type of contract. I assume that each submarket is characterized by a constant returns to scale matching function $m(v(\mathcal{C}), u(\mathcal{C}))$ and by an associated "tightness" $\theta(\mathcal{C}) = v(\mathcal{C})/u(\mathcal{C})$.[7] Hence, for each contract $\mathcal{C}$, I can define the function $f(\theta) \equiv m(\theta, 1)$, which represents the matching probability of a worker applying for $\mathcal{C}$. On the other hand, the matching probability for a firm posting $\mathcal{C}$ is represented by the non-increasing function $f(\theta)/\theta$. The function $f(\theta) : [0, \infty) \mapsto [0, 1]$ satisfies standard conditions: (i) $f(\theta) \leq \min\{\theta, 1\}$,[8] (ii) $f(\theta)$ is twice differentiable with $f'(\theta) > 0$ and $f''(\theta) < 0$.

Bellman Values

Linear preferences imply that the wage profile over the lifetime of the relationship is irrelevant for the analysis. This makes it possible to specify the contract in terms of the net present value of wages. The utility of a worker of type x who meets an employer when productivity is p_t, is given by

$$v(x, p_t) = e(x, p_t)[\omega(x, p_t) - x + V(p_t)] + [1 - e(x, p_t)]U(p_t),$$

where $V(p_t)$ represents the continuation value of employed workers *net of wages and training cost* when the aggregate shock is p_t (hereafter simply the continuation value of employed workers) and $U(p_t)$ represents the continuation value of unemployed workers at the same point in time.

[7] In order to simplify the notation, from now on I drop the dependence of u, v and θ on the contract $\mathcal{C}$, whenever this does not cause any confusion.

[8] With discrete time, this condition ensures that both $f(\theta)$ and $f(\theta)/\theta$ are proper probabilities.

The continuation value of employed workers, $V(p_t)$, represents the discounted expected value of being separated and becoming unemployed, that is,

$$V(p_t) = \beta E\left[sU(p_{t+1}) + (1-s)V(p_{t+1})\,|\,p_t\right]. \tag{1}$$

Moreover the continuation value of unemployed workers, $U(p_t)$, is given by

$$\begin{aligned} U(p_t) = \beta E\Bigg[& f(\theta(p_{t+1}))\int_{\underline{x}}^{\overline{x}} e(x, p_{t+1})\left[\omega(x, p_{t+1})\right. \\ & \left. - x + V(p_{t+1})\right] d\Phi(x)\,|\,p_t\Bigg] \\ & + z + \beta E\left[\left(1 - \int_{\underline{x}}^{\overline{x}} e(x, p_{t+1})\,d\Phi(x)\right)U(p_{t+1})\,|\,p_t\right]. \end{aligned} \tag{2}$$

Note that, in order for every type x to participate, it must be that

$$v(x, p_t) \geq U(p_t) \quad \text{for any } x, p_t. \tag{3}$$

The large number of potential firms ensures free entry and implies that the value of an open vacancy will be zero at each point in time, that is,

$$\beta\frac{f(\theta(p_t))}{\theta(p_t)}\int_{\underline{x}}^{\overline{x}} e(x, p_t)\left[T(p_t) - \omega(x, p_t)\right] d\Phi(x) = c, \tag{4}$$

where

$$T(p_t) = p_t + \beta(1-s)\int T(p_{t+1})\,dG(p_{t+1}|p_t). \tag{5}$$

Competitive Search Equilibrium

I now generalize the standard definition of the competitive search equilibrium to an environment with an aggregate shock. For simplicity, I adopt a definition in recursive terms.[9]

It can be shown that a recursive competitive search equilibrium takes the following simple form. It is a set of contracts $\mathbb{C}^*(p)$ contingent only on the aggregate shock, a tightness function $\Theta^*(p, \mathcal{C}(p))$ which, for any p and $\mathcal{C}(p) \in \mathbb{C}(p)$ gives a market tightness $\theta(p) \in \mathbb{R}_+ \cup \infty$, and a pair of continuation utility functions $\{U^*(p), V^*(p)\}$ such that, given p, employers maximize profits and workers apply optimally for jobs, taking as given the future values of being employed and unemployed and aware of the market

[9] The definition and characterization of the competitive search equilibrium in both the full information and the asymmetric information case follow the analysis in Guerrieri (2007). The proof of Proposition 1 follows steps similar to Acemoglu and Shimer (1999) and Guerrieri (2007) and is therefore omitted.

tightness associated with each contract, even if not offered in equilibrium. Moreover, expected profits are driven to zero by free entry.

The following proposition characterizes the stochastic symmetric competitive search equilibrium in recursive terms. In the remainder of the analysis, I adopt a recursive notation by dropping the t whenever this causes no confusion, and denoting a variable at time $t+1$ with a prime.

Proposition 1. *If* $\{\mathbb{C}^*(p), \Theta^*(p), U^*(p), V^*(p)\}$ *is a recursive competitive search equilibrium, then any pair* $(\mathcal{C}^*(p), \theta^*(p))$ *with* $\mathcal{C}^*(p) \in \mathbb{C}^*(p)$ *and* $\theta^*(p) = \Theta^*(p, \mathcal{C}^*(p))$ *satisfies the following*:

(*i*) *for a given pair of functions* $\{U^*(p), V^*(p)\}, \mathcal{C}^*(p) = [e^*(x, p), \omega^*(x, p)]x \in \Theta$ *and* $\theta^*(p)$ *solve*

$$W(U^*(p), V^*(p), p) = \max_{\mathcal{C}(p), \theta(p)} \beta f(\theta(p)) \int_{\underline{x}}^{\overline{x}} e(x, p) \times [\omega(x, p) - x + V^*(p)]\, d\Phi(x) + \beta\left[1 - f(\theta(p)) \int_{\underline{x}}^{\overline{x}} e(x, p) d\Phi(x)\right] U^*(p), \tag{P1}$$

subject to $e(x,p) \in [0, 1]$, *the individual rationality constraints (3) and the free-entry condition (4);*

(*ii*) *for a given pair* $\{\mathcal{C}^*(p), \theta^*(p)\}$, *the pair of functions* $\{U^*(p), V^*(p)\}$ *evolves according to*

$$U^*(p) = z + \int W(U^*(p'), V^*(p'), p') dG(p'|p)$$

and

$$V^*(p) = \beta \int \left[sU^*(p') + (1-s)V^*(p')\right] dG(p'|p).$$

Conversely, if a pair of functions $\{\mathcal{C}^*(p), \theta^*(p)\}$ *solves program (P1), then there exists an equilibrium* $\{\mathbb{C}^*(p), \Theta^*(p, \cdot), U^*(p), V^*(p)\}$ *such that* $\mathcal{C}^*(p) \in \mathbb{C}^*(p)$ *and* $\theta^*(p) = \Theta^*(p, \mathcal{C}^*(p))$.

Using pointwise maximization with respect to $e(x)$, the trading area can be fully described by a cut-off value $x(p)$ such that $e(x, p) = 1$ if $x \leq x(p)$ and $e(x,p) = 0$, otherwise. Then, the equilibrium can be characterized for given p and given continuation utilities $U(p)$ and $V(p)$, by a pair of functions $\hat{x}(p)$ and $\theta(p)$ satisfying the first-order conditions of the maximization problem (P1), that is,

$$T(p) - \hat{x}(p) - [U(p) - V(p)] = 0 \tag{6}$$

$$\beta f'(\theta(p)) \int_{\underline{x}}^{\hat{x}(p)} [T(p) - x - [U(p) - V(p)]] \, d\Phi(x) = c. \tag{7}$$

Comparative Statics

Here, I look at some comparative statics to get a sense of the volatility of job creation implied by the model. Following Shimer (2005), I derive the elasticity of market tightness with respect to productivity, when there are no aggregate shocks. I first consider the case of a degenerate distribution for the idiosyncratic shocks and show that, in this case, my model is isomorphic to the one analyzed in Shimer (2005) with random matching and Nash bargaining, where the Hosios condition is met. Next, I introduce a non-degenerate distribution of idiosyncratic shocks and show that effective job creation depends not only on the equilibrium matching probability $f(\theta(p))$, but also on the equilibrium hiring decision, reflected in $\Phi(\hat{x}(p))$, which may amplify the impact of productivity shocks on market tightness and unemployment rate. However, I show that when the parameters of the model are properly re-calibrated, the presence of heterogeneity tends to dampen the response of market tightness to productivity and does not alter the response of job creation and, hence, unemployment.

Assume that the productivity p is fixed and that the matching function takes a Cobb–Douglas form, so that $f(\theta) = \mu\theta^{1-\alpha}$. Then, from the fixed-point problem described in the second part of Proposition 1, we get that in equilibrium

$$U(p) - V(p) = \frac{\delta\beta f(\theta(p)) \int_{\underline{x}}^{\hat{x}(p)} [\delta p - x] \, d\Phi(x) + \delta z - \delta\theta(p)c}{1 + \delta\beta f(\theta(p)) \, \Phi(\hat{x}(p))}, \tag{8}$$

where $\delta = (1 - \beta(1 - s))^{-1}$ and the equilibrium $\hat{x}(p)$ and $\theta(p)$ solve problem (P1). Combining equation (8) with the first-order conditions (6) and (7), after some algebra, the equilibrium can be characterized by the following two conditions:

$$\hat{x}(p) = \delta(p - z) - \delta\left(\frac{\alpha}{1-\alpha}\right)\theta(p)c, \tag{9}$$

$$\frac{1}{\delta\beta\Phi(\hat{x}) f(\theta)/\theta} + \alpha\theta = (1-\alpha) \frac{\int_0^{\hat{x}(p)} [p - x/\delta - z] \, d\Phi(x)}{c\Phi(\hat{x})}. \tag{10}$$

Note that if I assume that the distribution of the idiosyncratic shocks is degenerate and, for simplicity $x=0$,[10] then all matches are productive and the equilibrium can be characterized simply by

$$\frac{1}{\delta\beta f(\theta)/\theta}+\alpha\theta=(1-\alpha)\frac{p-z}{c}.$$

This expression is exactly the discrete time version of the equilibrium equation of the baseline model in Shimer (2005), given that the firm's matching probability $f(\theta)/\theta$ is equal to the queue length $q(\theta)$, and $(1-\beta(1-s))/\beta\approx r+s$, where r is the discount rate in the continuous time version. Then, the elasticity of market tightness with respect to productivity, $\varepsilon_{\theta p}$, is equal to

$$\varepsilon_{\theta p}=\frac{p}{p-z}\frac{1-\beta(1-s)+\alpha\beta f(\theta)}{\alpha[1-\beta(1-s)+\beta f(\theta)]}.$$

Can heterogeneity increase this elasticity? I now show that when the distribution of the idiosyncratic shocks is non-degenerate, there is an extra term in the expression for $\varepsilon_{\theta p}$, coming from the responsiveness of the hiring margin to productivity. However, I also show that when the model parameters are properly re-calibrated, this will not help to increase the equilibrium value of $\varepsilon_{\theta p}$.

Let subscript D denote the model with a degenerate distribution of idiosyncratic shocks (and $x=0$) and set the parameters μ_D, z_D, α_D, c_D and p, β, s at the corresponding values in Shimer (2005). In the degenerate case, the job creation rate j_D is simply equal to the probability for a firm to find a worker, that is, $j_D(p)=f_D(\theta(p))=\mu_D\theta(p)^{1-\alpha_D}$. When agents are heterogeneous, the job creation rate is obtained from the combination of the standard finding probability and the additional hiring margin, that is, $j(p)=\mu\theta(p)^{\alpha}\Phi(\hat{x}(p))$, where no subscript is used for this general case. Moreover, the steady-state level of unemployment is given by

$$u(p)=\frac{s}{s+j(p)}. \tag{11}$$

Next, note that the job creation rate and the unemployment rate in the model with heterogeneity are identical to those in the degenerate case, when p,β,s are the same in both models and the parameters μ,z,α and c satisfy:

$$\mu\Phi(\hat{x})=\mu_D, \tag{12}$$

$$E\left[x/\delta+z|x\leq\hat{x}\right]=z_D, \tag{13}$$

$$\alpha=\alpha_D, \tag{14}$$

[10] The equivalence would work for any x small enough, where the effective outside option for the worker is $z+x/\delta$, instead of z.

$$c = c_D. \tag{15}$$

In the Appendix, it is shown that, when (12)–(15) hold, the derivative of the market tightness with respect to productivity in the model with idiosyncratic shocks coincides with the degenerate case.

Proposition 2. *Suppose conditions (12)–(15) are satisfied, then* $\theta'(p) = \theta_D'(p)$.

Proof: See the Appendix. ■

Proposition 2 shows that when the elasticity of the matching function, $1-\alpha$, is kept constant, the response of the hiring margin to productivity does not change the response of market tightness. On the one hand, when the hiring margin increases, firms have an incentive to post more vacancies because the return per vacancy posted is higher. On the other hand, firms face a higher cost for a posted vacancy because of two effects: a direct one, as the expected training cost for a matched worker has increased, and an indirect one, as the outside option of the matched worker has also increased. Proposition 2 shows that in equilibrium, the increase in the benefit of a posted vacancy coincides exactly with the increase in its cost, and the response of market tightness to productivity is not affected by the adjustment in the hiring margin.

Under the parameterization proposed, the model matches the job creation rate in the baseline model (and, hence, in the data). However, the model does not match the elasticity of job creation to market tightness. As Shimer (2005) shows, there is an approximate log-linear relationship between market tightness and the job creation rate in the data. In the degenerate model, where $j(p) = \mu\theta(p)^{1-\alpha}$, the elasticity of the job creation rate to market tightness, $d\log j/d\log\theta$, is simply equal to $1-\alpha$ and can be estimated directly by regressing $\log j$ on $\log\theta$. However, in the model with a non-degenerate distribution of idiosyncratic shocks $j(p) = \mu\theta(p)^{1-\alpha}\Phi(\hat{x}(p))$, which gives

$$\frac{d\log j}{d\log p} = (1-\alpha)\frac{d\log\theta(p)}{d\log p} + \frac{\varphi(\hat{x}(p))}{\Phi(\hat{x}(p))}\frac{d\log\hat{x}(p)}{d\log p}\hat{x}(p).$$

By dividing this expression by $d\log\theta/d\log p$, it follows that the elasticity of the job creation rate to market tightness corresponds to

$$\frac{d\log j}{d\log\theta} = 1-\alpha + \frac{\varphi(\hat{x}(p))}{\Phi(\hat{x}(p))}\frac{\hat{x}'(p)}{\theta'(p)}\theta. \tag{16}$$

This suggests that parameters μ, z, α and c should be chosen such that

$$\mu\theta^{1-\alpha}\Phi(\hat{x}) = \mu_D\theta^{1-\alpha_D}, \tag{17}$$

$$E[x/\delta + z | x \leq \hat{x}] = z_D, \tag{18}$$

$$\alpha - \frac{\varphi(\hat{x})}{\Phi(\hat{x})} \frac{\hat{x}'(p)}{\theta'(p)} \hat{x}(p) = \alpha_D, \tag{19}$$

$$c \frac{1 + \alpha\beta\delta\mu\theta^{1-\alpha}\Phi(\hat{x})}{1-\alpha} = c_D \frac{1 + \alpha_D\beta\delta\mu_D\theta^{1-\alpha_D}}{1-\alpha_D}. \tag{20}$$

With this parameterization, the model can match not only the job creation rate and the unemployment rate, but also the elasticity of job creation to market tightness of the baseline economy.

Proposition 3. *Suppose conditions (17)–(20) are satisfied, then* $\theta'(p) < \theta'_D(p)$.

Proof: See the Appendix. ■

Proposition 3 shows that once the model is properly parameterized, the responsiveness of market tightness to productivity not only is not amplified, but it is actually dampened.

Finally, I look at the impact of heterogeneity on the responsiveness of job creation and the unemployment rate to a productivity shock. First, note that when the distribution of the idiosyncratic shocks x is non-degenerate, then the response to productivity of the cut-off $\hat{x}(p)$, that is, the hiring margin, is always positive. By differentiating condition (9) and using the expression for $\theta'(p)$ derived in the Appendix (see proof of Proposition 3), it follows that

$$\hat{x}'(p) = [1 - \beta(1-s) + \beta f(\theta)\Phi(\hat{x})]^{-1} > 0. \tag{21}$$

Both of the parameterizations above ensure that the job creation rate and, thus, the unemployment rate, are the same as in the degenerate model. However, the job creation rate can be matched in two ways. In the first parameterization, I just shift the job creation function by the constant factor μ in order to match the equilibrium level. In the second parameterization, instead, I shift both μ and the elasticity α of the matching function, in order to match both the equilibrium level and the equilibrium elasticity of job creation. Then Proposition 4 shows that the first parameterization amplifies the response of job creation to productivity and, consequently, also amplifies the response of unemployment. However, under the proper parameterization (the second one), these responses both remain unchanged, once heterogeneity enters into the picture.

Proposition 4. *Suppose conditions (12)–(15) hold, then* $j'(p) > j'_D(p)$ *and* $u'(p) < u'_D(p)$. *If, instead, conditions (17)–(20) are satisfied, then* $j'(p) = j'_D(p)$ *and* $u'(p) = u'_D(p)$.

Proof: See the Appendix. ■

The results in this section highlight the importance of choosing the correct parameterization to evaluate the impact of heterogeneity on job creation. It will be shown in Section IV that the dynamics of the stochastic model closely follow these analytical steady-state results.

III. Asymmetric Information

Asymmetric information is now introduced by assuming that the worker privately observes his type. I show that in steady state there is an additional effect on the response of market tightness to productivity due to the informational problem, which makes the comparison with the standard model ambiguous. In Section IV, I show that asymmetric information can generate amplification in the fluctuations of the labor market, depending on the distribution of the idiosyncratic shocks. However, these effects are typically small.

Environment and Bellman Values

The economy is the same as described in Section II, except that the training cost x is now private information of the worker and $\varphi(x)$ now satisfies the monotone hazard rate condition, that is, $d[\Phi(x)/\varphi(x)]/dx > 0$. Given that x is private information, a contract is now a revelation mechanism, that is, a map $\mathcal{C} : \Theta \times P \mapsto [0, 1] \times \mathbb{R}_+$, specifying for each matched worker who reports type $\tilde{x}$ when productivity is p_t, the hiring probability $e(\tilde{x}, p_t) \in [0, 1]$ and the expected net present value of wages $\omega(\tilde{x}, p_t) \in \mathbb{R}_+$.[11] Without loss of generality, by invoking the revelation principle, I can focus on the set $\mathbb{C}$ of incentive-compatible and individually rational direct revelation mechanisms. Note that I can restrict attention to this set of contracts owing to the assumption that unemployed workers are anonymous. All the unemployed workers searching for a job cannot be distinguished, so that contracts cannot be conditioned on past employment history. Moreover, as in the full information case, linear preferences imply that the wage profile over the lifetime of the relationship is irrelevant for the analysis.

The employment contract $\mathcal{C}$ must be incentive compatible and individually rational, that is, it has to ensure that the worker reveals his type truthfully and chooses to participate in the employment relationship after the draw has been realized. The expected utility of a worker of type x

[11] In Guerrieri (2007), I show that this contract is without loss of generality, given that the firm would make a zero transfer to workers whom it decides not to hire.

reporting type $\tilde{x}$ when productivity is p_t, is given by

$$v(x, \tilde{x}, p_t) = e(\tilde{x}, p_t)[\omega(\tilde{x}, p_t) - x + V(p_t)] + [1 - e(\tilde{x}, p_t)]U(p_t),$$

where $V(p_t)$ and $U(p_t)$, as in the case of full information, satisfy equations (1) and (2), and denote, respectively, the continuation value of employed workers *net of wages and training cost* and the continuation value of unemployed workers when productivity is p_t. Given p_t, an employment contract is *incentive compatible* (*IC*) whenever it satisfies

$$v(x, x, p_t) \geq v(x, \tilde{x}, p_t) \quad \text{for all } x, \tilde{x} \in \Theta$$

and *individually rational (IR)* whenever

$$v(x, x, p_t) \geq U(p_t) \quad \text{for all } x \in \Theta.$$

Following Myerson (1981), I can reduce IC and IR to a monotonicity condition $e_x(x, p_t) < 0$ for all x, the IR binding for the worse type

$$v(\overline{x}, \overline{x}, p_t) \geq U(p_t), \tag{22}$$

and the following condition

$$v(x, x, p_t) = v(\overline{x}, \overline{x}, p_t) + \int_x^{\overline{x}} e(y, p_t)\, dy \quad \text{for all } x \in \Theta. \tag{23}$$

The large number of potential firms ensures free entry and implies that the value of an open vacancy will be zero at each point in time, that is,

$$\beta \frac{f(\theta(p_t))}{\theta(p_t)} \int_{\underline{x}}^{\overline{x}} [e(x, p_t)\, T(p_t) - \omega(x, p_t)]\, d\Phi(x) = c, \tag{24}$$

where $T(p_t)$ is defined in (5).

Competitive Search Equilibrium

The definition of competitive search equilibrium introduced in Section II can now be generalized to an environment with asymmetric information. It is possible to show that a recursive competitive search equilibrium takes the same simple form as the full information case, except that now the equilibrium set of contracts contingent on the aggregate shock, $\mathbb{C}^*(p)$, is a set of incentive-compatible and individually rational direct mechanisms.

The next proposition is a natural extension of Proposition 1 and states the characterization of a stochastic symmetric competitive search equilibrium under asymmetric information in recursive terms.[12]

[12] Propositions 5 and 6 are natural extensions of the analysis in Guerrieri (2007) and their proofs are therefore omitted here.

Proposition 5. *If* $\{\mathbb{C}^*(p), \Theta^*(p), U^*(p), V^*(p)\}$ *is a recursive competitive search equilibrium, then any pair* $(\mathcal{C}^*(p), \theta^*(p))$ *with* $\mathcal{C}^*(p) \in \mathbb{C}^*(p)$ *and* $\theta^*(p) = \Theta^*(p, \mathcal{C}^*(p))$ *satisfies the following:*

(*i*) *for a given pair of functions* $\{U^*(p), V^*(p)\}$, $\mathcal{C}^*(p) = [e^*(x, p), \omega^*(x, p)]_{x \in \Theta}$ *and* $\theta^*(p)$ *solve*

$$W(U^*(p), V^*(p), p) = \max_{\substack{e(x,p),\omega(x,p)\\ \theta(p)}} \beta f(\theta(p)) \times \int_{\underline{x}}^{\overline{x}} e(x, p)\left[\omega(x, p) - x + V^*(p)\right] d\Phi(x) + \beta\left[1 - f(\theta(p)) \int_{\underline{x}}^{\overline{x}} e(x, p) d\Phi(x)\right] U^*(p), \tag{P2}$$

subject to $e(x, p) \in [0, 1]$, *the free-entry condition (24), the constraints IC and IR reduced to the conditions (22), (23) and* $e_x(x, p) < 0$;

(*ii*) *for a given pair* $\{\mathcal{C}^*(p), \theta^*(p)\}$, *the pair of functions* $\{U^*(p), V^*(p)\}$ *evolves according to*

$$U^*(p) = z + \int W(U^*(p'), V^*(p'), p') dG(p'|p)$$

and

$$V^*(p) = \beta \int [sU^*(p') + (1 - s)V^*(p')] dG(p'|p).$$

Conversely, if a pair of functions $\{\mathcal{C}^*(p), \theta^*(p)\}$ *solves program (P4), then there exists an equilibrium* $\{\mathbb{C}^*(p), \Theta^*(p, \cdot), U^*(p), V^*(p)\}$ *such that* $\mathcal{C}^*(p) \in \mathbb{C}^*(p)$ *and* $\theta^*(p) = \Gamma^*(p, \mathcal{C}^*(p))$.

Proposition 5 shows that for given $U(p)$ and $V(p)$, a recursive symmetric equilibrium incentive-compatible and individually rational contract $\mathcal{C}(p)$ and tightness $\theta(p)$ must solve problem (P2). The next proposition shows that the equilibrium can be equivalently described by a hiring function $e(x,p)$ and tightness $\theta(p)$ that solve a simplified program (P3). Given $e(x,p)$ and $\theta(p)$, an associated wage function $\omega(x,p)$ can be constructed so that the incentive-compatibility and participation constraints are satisfied.

Proposition 6. *For given* $U(p)$ *and* $V(p)$, *any couple of functions* $e(x,p)$ *and* $\theta(p)$ *which solve problem (P2), also solve*

$$W(U(p), V(p), p) = \max_{e(x,p),\theta(p)} \beta f(\theta(p)) \times \int_{\underline{x}}^{\overline{x}} e(x,p)[T(p) - x + V(p) - U(p)]\, d\Phi(x) + \beta U(p) - \theta(p)c, \tag{P3}$$

subject to

$$\beta f(\theta(p)) \int_{\underline{x}}^{\overline{x}} e(x,p)\left[T(p) - x - \frac{\Phi(x)}{\varphi(x)} + V(p) - U(p)\right] d\Phi(x) \geq \theta(p)c. \tag{25}$$

Furthermore, for any pair of functions $e(x,p)$ *and* $\theta(p)$ *which solves problem (P3), there exists a function* $\omega(x,p)$ *such that the contract* $\mathcal{C}(p) = [e(x,p), \omega(x,p)]_{x\in\Theta, p\in Y}$ *and* $\theta(p)$ *solve problem (P2).*

In order to characterize the equilibrium, I proceed by studying the relaxed problem without the monotonicity assumption on $e(x,p)$. Then, using pointwise maximization with respect to $e(x,p)$, the trading area can be fully described by a cut-off value $\hat{x}(p)$, as in the full information case, implying that the optimal $e(x,p)$ is effectively non-increasing in x. When the constraint of problem (P3) is binding,[13] the equilibrium can be characterized, for given $U(p) - V(p)$, by a set of functions $\hat{x}(p)$, $\theta(p)$ and $\lambda(p)$ satisfying the first-order conditions

$$T(p) - \hat{x}(p) - \lambda(p)\frac{\Phi(\hat{x}(p))}{\varphi(\hat{x}(p))} - (U(p) - V(p)) = 0, \tag{26}$$

$$\beta f'(\theta(p)) \int_{\underline{x}}^{\hat{x}(p)} \left[T(p) - x - \lambda(p)\frac{\Phi(x)}{\varphi(x)} - (U(p) - V(p))\right] d\Phi(x) = c \tag{27}$$

and the binding constraint (25). The variable $\lambda(p)$ represents a normalized version of the shadow value of the *informational rents*.[14] Note that when $\lambda(p) = 0$, the constraint is slack and the full information allocation described in Section II is achieved. Clearly, for given $U(p) - V(p)$, asymmetric information reduces job creation, as the surplus of the economy must cover

[13] A generalization of Lemma 2 in Guerrieri (2007) gives that the constraint is binding if and only if the cost of posting a vacancy c is strictly positive.

[14] Note that $\lambda \equiv \hat{\lambda}/(1+\hat{\lambda})$, where $\hat{\lambda}$ is the Lagrangian multiplier attached to the constraint of problem (P3). As in Guerrieri (2007), define $v(x,x,p) - v(\overline{x},\overline{x},p)$ as the *informational rent* of a worker of type $x \leq \overline{x}$, that is, the additional utility that such a worker must receive in order to reveal his own type.

not only the cost of vacancy creation but also the rents needed to extract information from the workers. As intuition suggests, $\hat{x}(p)$ decreases with $\lambda(p)$.

Equilibrium with No Aggregate Shocks

In order to provide some intuition for the role of asymmetric information, let me fix p and consider the version of the model with no aggregate shocks.

From the fixed-point problem described in the second part of Proposition 5, I obtain that in steady state $U-V$ satisfies expression (8), where the equilibrium $\hat{x}$ and θ now solve problem (P3). Combining equation (8) with the binding constraint (25) and the first-order conditions (26) and (27), after some algebra, the equilibrium can be characterized by the following three conditions:

$$p-\frac{\hat{x}}{\delta}-\frac{\lambda}{\delta}\frac{\Phi(\hat{x})}{\varphi(\hat{x})}-z+\theta c=\beta f(\theta)\int_{\underline{x}}^{\hat{x}}\left(\hat{x}-x+\lambda\frac{\Phi(\hat{x})}{\varphi(\hat{x})}\right)d\Phi(x),$$

$$\beta(1-\alpha)f(\theta)\int_{\underline{x}}^{\hat{x}}\left[\hat{x}-x+\lambda\left(\frac{\Phi(\hat{x})}{\varphi(\hat{x})}-\frac{\Phi(x)}{\varphi(x)}\right)\right]d\Phi(x)=\theta c,$$

$$\lambda=1-\left[(1-\alpha)\beta\frac{f(\theta)}{\theta}\int_{\underline{x}}^{\hat{x}}\frac{\Phi(x)}{\varphi(x)}d\Phi(x)\right]^{-1}\alpha c.$$

Note that when $\lambda=0$, this system boils down to (6) and (7), as expected. The third equation shows that at a first-order approximation, λ decreases with θ, since the more vacancies that are posted, the higher the expected surplus created per vacancy which covers the workers' rents and increases with $\hat{x}$. In other words, as the hiring margin increases, more rents need to be paid to the workers and the distortion worsens. On the other hand, the first two equations show that as λ changes, market tightness and the hiring margin change endogenously. Hence, the responsiveness of market tightness to productivity seems ambiguous. In any case, it is shown in Section IV shows that the impact of asymmetric information on volatility is not quantitatively significant.

Before turning to the calibration exercises, it is useful to note that this model of wage determination is a special discrete-time case of the general one considered in Brugemann and Moscarini (2007).[15] It is easy to show that the weak upper bound on the elasticity of market tightness with respect

[15] I thank one of the referees for pointing this out.

to productivity derived in Brugemann and Moscarini (2007) applies.[16] This comparative-static analysis suggests that market tightness volatility may not be too high. Several numerical exercises below confirm that this is indeed the case, also once aggregate shocks are explicitly introduced in the model.

IV. Unemployment Volatility

I now analyze the cyclical behavior of unemployment and market tightness generated by the model, under full and asymmetric information, and compare this to the benchmark model with no idiosyncratic shocks. After describing how I calibrated the different versions of the model, I report the results.

Calibration

In order to investigate the role of heterogeneity and asymmetric information in explaining U.S. unemployment rate volatility, I parameterized the model to match the facts of the U.S. labor market documented in Shimer (2005).

I normalized a time period to be one quarter and then set the discount factor β to 0.988, consistent with an annual discount factor of 0.953. I set the separation rate s equal to 0.1, consistent with jobs lasting about 2.5 years on average. The stochastic process for labor productivity p was chosen following a discrete-time version of the strategy in Shimer (2005). An underlying stochastic variable y was defined such that $p_t = z + e^{y_t}(p^* - z)$, where $y_{t+1} = \rho y_t + \varepsilon_t$ with $\rho \in (0, 1)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. I approximated the AR(1) process for y with a 35-state Markov chain[17] and normalized p^* to 1. I then calibrated ρ and σ_ε^2 in order to match the

[16] The trading probability coincides with the job creation probability, the worker rents are

$$\int_x^{\hat{x}} \Phi(x)dx,$$

and the firm rents are

$$\int_x^{\hat{x}} \left[T(p) - x - \frac{\Phi(x)}{\varphi(x)} - (U(p) - V(p)) \right] d\Phi(x).$$

In steady state $T(p) - (U(p) - V(p)) = \alpha[p - (1 - \beta)U(p)]$. Substituting this into (26) and (27) and into the binding constraint (25), it is immediate that the trading probability, $f(\theta)\Phi(\hat{x}(p))$ depends on p and $(1 - \beta)U(p)$ only through the expression $\alpha[p - (1 - \beta) \times U(p)]$. Moreover, applying the implicit function theorem, it is easy to show that $\hat{x}(p)$ is increasing in p, which together with the monotone hazard rate implies that worker rents are increasing in p, that worker rents are increasing in the sense of Brugemann and Moscarini (2007), and that firms' rents are non-increasing in $(1 - \beta)U(p)$.

[17] I chose a 35-state Markov process as in Hagedorn and Manovskii (2007).

volatility and autocorrelation of the resulting process for $\log p$ with the correspondent values in the data, reported in Shimer (2005), that is, 0.020 and 0.878, respectively.

I set the instantaneous utility of unemployed workers z such that $z + E[x/\alpha | x \leq \hat{x}(p)]$ is approximately equal to 0.4 of labor income. In my model, the effective opportunity cost of being employed for a matched worker of type x is $z + x/\alpha$, so that to calibrate z equal to 0.4 of labor income would be equivalent to choosing an effective higher value of leisure, in the direction of Hagedorn and Manovskii (2007). My objective is to preclude their mechanism so as to highlight the role of heterogeneity and information.

For the job creation rate, j_t, I used the series constructed by Shimer (2005).[18] His series for the job creation rate, 1951–2003, is based on the series for the behavior of the unemployment level and the short-term unemployment level constructed by the Bureau of Labor Statistics from the Current Population Survey. In particular, j_t is such that the number of unemployed next month is equal to the number of unemployed last month who did not find a job, that is, $(1 - j_t)\, u_t$, and workers who have lost their job between the beginning and end of this month, denoted by u^s_{t+1},[19] that is,

$$j_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t}.$$

Using this series, the monthly job creation probability is 0.45, corresponding to a quarterly job creation probability of 0.8336. As explained in Section II, in the model with idiosyncratic shocks, the job creation probability corresponds to $f(\theta)\Phi(\hat{x})$ because in order to get a job, unemployed workers have to both find an employer and be hired by him.[20] Hence, I matched

$$E\,[f(\theta)\Phi(\hat{x})] = 0.8336. \tag{28}$$

I used the standard Cobb–Douglas matching function and set $f(\theta) = \mu\theta^{1-\alpha}$. Shimer (2005) estimates the elasticity parameter $1 - \alpha = 0.28$ with a first-order autoregressive residual using detrended data on the matching rate and the v/u ratio for the U.S. between 1951 and 2003. When the distribution of the idiosyncratic shocks is degenerate, the job creation probability coincides with the finding probability and the elasticity of job creation to

[18] I thank Robert Shimer for making the series available. Note that what he calls the finding rate, I call the job creation rate to clarify that in my model, finding a match does not necessarily translate into being hired.

[19] Note that u^s_{t+1} represents the number of workers unemployed for less than a month.

[20] The hiring margin is trivial in the standard model because it is optimal to hire any matched worker and the job creation rate simply reduces to $f(\theta)$.

market tightness is simply equal to $1-\alpha$. However, when the distribution of the idiosyncratic shocks is not degenerate and job creation is equal to $j_t = f(\theta_t)\Phi(\hat{x}_t)$, then the elasticity of job creation to market tightness is given in equation (16) and takes into account the fact that the hiring margin reacts to changes in market tightness. I performed two different types of exercises: *quasi-calibration* and *proper calibration*. In the quasi-calibration, I kept $1-\alpha$ equal to 0.28, and calibrated μ in order to match expression (28). In the proper calibration, instead, I calibrated α and μ in order to match simultaneously expression (28) and the elasticity of job creation to market tightness, using the simulated series for market tightness and job creation generated by the model, that is,

$$\frac{\mathrm{Cov}(\log j_t, \log \theta_t)}{\mathrm{Var}(\log \theta_t)} = 0.28. \tag{29}$$

As in Shimer (2005), if c is doubled and μ is multiplied by $2^{1-\alpha}$, then θ is reduced by 1/2 and the rate at which firms find workers is doubled, but the worker's finding rate is not affected, that is, the scale of θ is meaningless in the model. I set $c = 0.2$ in order to normalize the average θ to 1 and make the calibration of the degenerate version of the model identical to the calibration of the discrete version of Shimer (2005).

My results are relatively not sensitive to the choice of the distribution for idiosyncratic shocks. I report the results using three families of distributions: Pareto, uniform and log-normal.

Results

The calibrated parameter values were used to simulate the model and create artificial time series of unemployment and vacancy rates, market tightness and the finding rate. I then compared the volatility of the simulated series obtained with the degenerate model to those obtained with the model with heterogeneity, under both full and asymmetric information. I report the results for different distributions of the idiosyncratic shock (Pareto, uniform and log-normal). Overall, I find that heterogeneity does not help much in amplifying the volatility of unemployment and vacancy rates in comparison with the benchmark model, even in the case of asymmetric information.

As regards the results for the degenerate version of the model that match the results in Shimer (2005), Table 1 reports the standard deviation for unemployment, vacancy, market tightness, the job creation rate and productivity in detrended logs.

Next, I compare these results with the model with heterogeneity when the distribution of the idiosyncratic shocks is Pareto. Table 2 reports the same statistics under full and asymmetric information, for both the quasi-calibration and the full-calibration exercise.

Table 1. *Degenerate model*

	u	v	θ	j	p
Standard deviation	0.009	0.027	0.035	0.010	0.020

Notes: Results from simulating the model with degenerate distribution of idiosyncratic shocks. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 2. *Pareto distribution of x*

		u	v	θ	j	p
Full information	Quasi-calibration	0.010	0.027	0.035	0.011	0.020
	Proper calibration	0.009	0.027	0.034	0.010	0.020
Asymmetric information	Quasi-calibration	0.010	0.027	0.036	0.012	0.020
	Proper calibration	0.009	0.028	0.035	0.010	0.020

Notes: Results from simulating the model with Pareto distribution of idiosyncratic shocks with parameters 0.4 and 3. In the full information case, the quasi-calibration requires $b = 0.327$ and $\mu = 0.851$, and the proper calibration requires $b = 0.329, \mu = 0.887$ and $1 - \alpha = 0.249$. In the asymmetric information case, the quasi-calibration requires $b = 0.330$ and $\mu = 0.874$, and the proper calibration requires $b = 0.332, \mu = 0.924$ and $1 - \alpha = 0.230$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

In the quasi-calibration exercise for the full information case, I kept $1 - \alpha = 0.28$ and set $\mu = 0.851$ in order to match (28). Note that μ has to increase in comparison with the benchmark $\mu = 0.834$ because when workers are heterogeneous, a match does not automatically translate into job creation. Table 2 shows that both unemployment and job creation are slightly more volatile in comparison with the benchmark, as Proposition 4 suggests. This is due to the endogenous movements in the hiring margin which responds to changes in productivity. Moreover, as is consistent with Proposition 2, market tightness volatility is approximately equal to the benchmark. However, the movements in unemployment and job creation are minuscule and, once I properly calibrated the model by also changing α in order to match condition (29), even these small volatility gains approximately disappear. Moreover, as Proposition 3 suggests, market tightness volatility actually decreases slightly.

When asymmetric information enters the picture, there is an extra source of volatility, given that the distortion generated by the informational rents that employers have to pay to the workers is also responsive to productivity. Using the quasi-calibration, unemployment, market tightness and job creation exhibit higher volatility, although the increase in volatility is minuscule. Moreover, in the proper calibration exercise, once again, the increase in unemployment and job creation volatility virtually disappears and only the vacancy rate seems to be slightly more volatile than in the benchmark. Unemployment appears more volatile only at the fourth digit; unemployment goes from 0.0086 in the degenerate case to 0.0087 in the proper

Table 3. *Log-normal distribution of x*

		u	v	θ	j	p
Full information	Quasi-calibration	0.009	0.027	0.035	0.010	0.020
	Proper calibration	0.009	0.027	0.035	0.010	0.020
Asymmetric information	Quasi-calibration	0.011	0.028	0.036	0.012	0.020
	Proper calibration	0.009	0.028	0.035	0.010	0.020

Notes: Results from simulating the model with log-normal distribution of idiosyncratic shocks with parameters 0.2 and 0.2. In the full information case, the quasi-calibration requires $b = 0.252$ and $\mu = 0.836$, and the proper calibration requires $b = 0.252, \mu = 0.852$ and $1 - \alpha = 0.265$. In the asymmetric information case, the quasi-calibration requires $b = 0.253$ and $\mu = 0.856$, and the proper calibration requires $b = 0.256, \mu = 0.913$ and $1 - \alpha = 0.220$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

calibration for the case of asymmetric information. Job creation only goes from 0.346 to 0.353. It follows also that asymmetric information, once calibrated properly, does not seem to have any relevant impact on volatility of unemployment, market tightness or job creation.

Table 3 reports similar results for the case of a log-normal distribution for the idiosyncratic shocks. In this case as well, heterogeneity does not seem to have any significant impact on labor market fluctuations, with or without full information. The table shows that, under full information, heterogeneity seems to have virtually no impact on the volatility of unemployment, job creation or market tightness (this actually drops slightly, from 0.0346 to 0.0345). As in the case of a Pareto distribution, under asymmetric information a quasi-calibration would deliver slight increases in volatilities. However, using the proper calibration, these increases almost disappear.

Finally, I checked whether similar results would be obtained using the uniform distribution. It is interesting to note that a proper calibration exercise imposes some restrictions on the parameters of the uniform distribution. In this paper, the distribution of the idiosyncratic shocks is not pinned down by the data. However, this example shows that not all of the distributions are consistent with the model. In fact, the range of the uniform distribution imposes a lower bound on the response of the job creation rate to business cycle fluctuations, that is, on the matching parameter $1 - \alpha^D$ in the case of a degenerate distribution. Given that the data require $1 - \alpha^D = 0.28$, the set of possible distributions consistent with the model is restricted. When I set $\underline{x} = 0$ for the model with asymmetric information, a proper calibration imposes an upper bound $\overline{x}$ of 1.25. Table 4 reports the results for both full and asymmetric information for $\overline{x} = 1.25$.

The table shows that asymmetric information generates small increases in volatilities of unemployment, market tightness and job creation. As in the previous cases, these increases in volatility are even smaller with a proper

Table 4. *Uniform distribution of x*

		u	v	θ	j	p
Full information	Quasi-calibration	0.009	0.027	0.035	0.010	0.020
	Proper calibration	0.009	0.027	0.035	0.010	0.020
Asymmetric information	Quasi-calibration	0.015	0.026	0.037	0.017	0.020
	Proper calibration	0.010	0.029	0.036	0.011	0.020

Notes: Results from simulating the model with uniform distribution of idiosyncratic shocks with parameters 0 and 1.25. In the full information case, the quasi-calibration coincides with the proper calibration and requires $b = 0.321$ and $\mu = \mu^D$. In the asymmetric information case, the quasi-calibration requires $b = 0.323$ and $\mu = 0.882$, and the proper calibration requires $b = 0.328, \mu = 0.956$ and $1 - \alpha = 0.134$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 5. *Uniform distribution of x*

		u	v	θ	j	p
Full information	Quasi-calibration	0.009	0.027	0.035	0.010	0.020
	Proper calibration	0.009	0.027	0.035	0.010	0.020

Notes: Results from simulating the model with uniform distribution of idiosyncratic shocks with parameters 0 and 1.62. In the quasi-calibration $b = 0.301$ and $\mu = 0.833$. In the proper calibration $b = 0.301$, $\mu = 0.837$ and $\eta = 0.276$. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

calibration. Moreover, in this case, the results under full information are exactly the same as for the degenerate case because the equilibrium hiring margin is at a corner, that is, $\hat{x} = \overline{x}$.

Table 5 reports the results for the full information case with $\overline{x} = 1.62$. Once again, the results show that heterogeneity under full information has virtually no impact on the volatility of unemployment, market tightness and job creation.

These calibration exercises suggest that the type of heterogeneity introduced here in the standard search model cannot explain the high observed volatility of market tightness, unemployment and job creation.

V. Conclusions

In this paper I have examined the role of heterogeneity and asymmetric information in explaining the cyclical behavior and the volatility of unemployment and vacancies in search models.

First, I derived some steady-state comparative statics. I then showed that, when information is symmetric, heterogeneity dampens the response of market tightness to productivity shocks and does not affect the response of job creation and, hence, of the unemployment rate. Next, I performed some calibration exercises for the case of both symmetric and asymmetric information. The parameter values were chosen in order to match some basic statistics for the postwar U.S. economy. I considered idiosyncratic shocks distributed according to uniform, Pareto and log-normal distributions, and showed that in all cases there is no significant amplification in the volatility of unemployment and vacancies. Attempts to explicitly calibrate the distribution of idiosyncratic shocks are left for future research.

The main result of the paper is negative: match-specific heterogeneity that affects the job creation margin does not significantly amplify the response of the unemployment rate and vacancies to productivity shocks in comparison with the standard model. This is true whether or not asymmetric information is introduced.

Appendix

Proof of Proposition 2

First, rewrite equation (10) as the following implicit function:

$$h(\hat{x}(p), \theta(p), p) \equiv \frac{\int_{\underline{x}}^{\hat{x}(p)} [p - x/\delta - z]\, d\Phi(x)}{\Phi(\hat{x})} - \left[\frac{1-\beta(1-s)}{(1-\alpha)\beta f(\theta)\Phi(\hat{x})} + \frac{\alpha}{1-\alpha}\right] c\theta = 0.$$

By totally differentiating it, I get

$$h_{\hat{x}}(\hat{x}(p), \theta(p), p)\hat{x}'(p) + h_\theta(\hat{x}(p), \theta(p), p)\theta'(p) + h_p(\hat{x}(p), \theta(p), p) = 0.$$

Then,

$$\begin{aligned}\theta'(p) &= -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)} - \frac{h_{\hat{x}}(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)}\hat{x}'(p) \\ &= -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)}\left[1 + \frac{h_{\hat{x}}(\hat{x}(p), \theta(p), p)}{h_p(\hat{x}(p), \theta(p), p)}\hat{x}'(p)\right],\end{aligned}$$

where

$$h_{\hat{x}}(\hat{x}(p), \theta(p), p) = \left[p - \frac{\hat{x}(p)}{\delta} - z - E\left[p - \frac{x}{\delta} - z | x \le \hat{x}\right] + \frac{1-\beta(1-s)}{(1-\alpha)\beta f(\theta)\Phi(\hat{x})} c\theta\right] \frac{\varphi(\hat{x}(p))}{\Phi(\hat{x})}.$$

Using equation (10) I can rewrite

$$h_{\hat{x}}(\hat{x}(p), \theta(p), p) = p - \frac{\hat{x}(p)}{\delta} - z - \left(\frac{\alpha}{1-\alpha}\right) c\theta,$$

which is equal to zero from the first equilibrium condition equation (9), implying that

$$\theta'(p) = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)}.$$

Next, consider the case of a fixed $\hat{x} = 0$. The equilibrium condition then becomes

$$h^D(\theta(p), p) \equiv p - z_D - \left[\frac{1-\beta(1-s)}{(1-\alpha)\beta f_D(\theta)} + \frac{\alpha}{1-\alpha}\right] c\theta = 0.$$

By totally differentiating it, I obtain

$$h_\theta^D(\theta(p), p)\theta'(p) + h_p^D(\theta(p), p) = 0,$$

so that

$$\theta_D'(p) = -\frac{h_p^D(\theta(p), p)}{h_\theta^D(\theta(p), p)}.$$

Conditions (12)–(15) give that

$$\begin{aligned} h_p(\hat{x}(p), \theta(p), p) &= h_p^D(\theta(p), p) = 1, \\ h_\theta(\hat{x}(p), \theta(p), p) &= h_\theta^D(\theta(p), p) \\ &= -\frac{[1-\beta(1-s)+\beta f_D(\theta)]\,\alpha}{1-\beta(1-s)+\beta f_D(\theta)\alpha}\left(\frac{p-z_D}{\theta}\right), \end{aligned}$$

and, hence,

$$\frac{\theta'(p)}{\theta_D'(p)} = 1,$$

thereby completing the proof. ■

Proof of Proposition 3

Following the first steps of the previous proof, I can show that

$$\theta'(p) = -\frac{h_p(\hat{x}(p), \theta(p), p)}{h_\theta(\hat{x}(p), \theta(p), p)}$$

and

$$\theta_D'(p) = -\frac{h_p^D(\theta(p), p)}{h_\theta^D(\theta(p), p)}.$$

Now, conditions (17)–(20) give that

$$h(\hat{x}(p), \theta(p), p) = p - z_D - \left[\frac{1-\beta(1-s)}{(1-\alpha)\beta f_D(\theta)} + \frac{\alpha}{1-\alpha}\right] c\theta = 0.$$

It follows that

$$h_\theta(\hat{x}(p), \theta(p), p) = -\frac{[1-\beta(1-s)+\beta f_D(\theta)]\,\alpha}{(1-\alpha)\beta f_D(\theta)} c,$$

and, using the first-order condition (10), I obtain

$$h_p(\hat{x}(p), \theta(p), p) = h_p^D(\theta(p), p) = 1,$$
$$h_\theta(\hat{x}(p), \theta(p), p) = -\frac{[1-\beta(1-s)+\beta f_D(\theta)]\,\alpha}{1-\beta(1-s)+\beta f_D(\theta)\alpha}\frac{p-z_D}{\theta},$$
$$h_\theta^D(\theta(p), p) = -\frac{[1-\beta(1-s)+\beta f_D(\theta)]\,\alpha_D}{1-\beta(1-s)+\beta f_D(\theta)\alpha_D}\frac{p-z_D}{\theta}.$$

After some algebra, this implies that

$$\theta'(p)-\theta_D'(p) = \left[\frac{1}{\alpha}-\frac{1}{\alpha_D}\right]\frac{[1-\beta(1-s)]\,\theta}{[1-\beta(1-s)+\beta f_D(\theta)]\,(p-z_D)}.$$

Given that $\hat{x}'(p)$ and $\theta'(p)$ are positive, from (18), it follows that $\alpha > \alpha_D$ and then

$$\theta'(p) < \theta_D'(p),$$

thus completing the proof. ■

Proof of Proposition 4

From the definition of $j(p)$ and $j_D(p)$, it follows that

$$j'(p)-j_D'(p) = f_D(\theta(p))\frac{\varphi(\hat{x}(p))}{\Phi(\hat{x}(p))}\hat{x}'(p) - (\alpha-\alpha_D)\,f_D(\theta(p))\,\theta(p)^{-1}\,\theta'(p).$$

If condition (14) is satisfied, then $\alpha_D = \alpha$ and equation (21) implies that

$$j'(p)-j_D'(p) = f_D(\theta(p))\frac{\varphi(\hat{x}(p))}{\Phi(\hat{x}(p))}\hat{x}'(p) > 0.$$

If, instead, condition (19) holds, then

$$\alpha-\alpha_D = \frac{\varphi(\hat{x})}{\Phi(\hat{x})}\frac{d\hat{x}}{d\theta}\theta,$$

and $j'(p) = j_D'(p)$. Differentiating equation (11) gives

$$u'(p) = -\frac{u(p)}{s+j(p)}j'(p).$$

It follows immediately that, under conditions (12)–(15),

$$u'(p) = -j'(p)\frac{u(p)}{s+j(p)} < -j_D'(p)\frac{u_D(p)}{s+j_D(p)} = u_D'(p),$$

while under conditions (17)–(20), $u'(p) = u_D' u\, p)$, thereby completing the proof. ■

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