Inefficient Unemployment Dynamics under Asymmetric Information

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I explore the efficiency properties of a competitive search model with match-specific private information and limited commitment on the workers' side. In a static setting the competitive search equilibrium is constrained efficient, whereas in a dynamic setting it is constrained inefficient whenever the initial unemployment rate is different from its steady-state level. Inefficiency arises because the workers' outside option becomes endogenous and affects the severity of the distortion due to the informational friction. This generates a novel externality: firms offering contracts at a given time do not internalize their effect on the outside option of workers hired in previous periods.

I. Introduction

It has long been recognized that trade in labor markets is costly and subject to frictions. Firms need to post vacancies, workers spend time

For countless discussions and support, I thank Daron Acemoglu, Mike Golosov, Guido Lorenzoni, and, last only for alphabetical order, Iván Werning. This paper was improved significantly thanks to the suggestions of Robert Shimer, an anonymous second editor, and two anonymous referees. I am very grateful to all of them. I also thank, for very helpful comments, Manuel Amador, Pol Antras, Gadi Barlevy, Olivier Blanchard, V. V. Chari, Emmanuel Farhi, Francesco Giavazzi, Robert Hall, William Hawkins, Erik Hurst, Pat Kehoe, Narayana Kocherlakota, Christopher Pissarides, Richard Rogerson, Aleh Tsyvinski, Marcelo Veracierto, Randall Wright, and seminar participants at the Chicago Graduate School of Business, Federal Reserve Bank of Chicago, Federal Reserve Bank of Cleveland, Columbia University, Centre de Recerca en Economia Internacional and Pompeu Fabra, Institute for International Economic Studies, University of Iowa, Kellogg Managerial Economics and Decision Sciences Department, University of Minnesota, Massachusetts Institute of Technology, Northwestern University, Federal Reserve Bank of New York, New York University Stern School of Business, University of Rochester, Stanford University, University of Pennsylvania, National Bureau of Economic Research Summer Institute, and the North American Econometric Society summer meeting.

[*Journal of Political Economy,* 2008, vol. 116, no. 4]

searching for jobs, and wages are determined by decentralized contracting. This imposes a departure from the frictionless Walrasian paradigm. A classic question arises: To what extent do decentralized labor markets achieve efficiency? In particular, do they reach an efficient level of unemployment and job creation?

Search theory offers a natural environment to represent decentralized markets with trading frictions. In addressing the efficiency issue, the literature has focused on the bargaining side of the model—that is, on the way in which the worker and the firm split the surplus of the employment relationship. Different assumptions on wage determination can drive different implications in terms of efficiency. On the one hand, the classic Mortensen-Pissarides model shows that random matching combined with Nash bargaining introduces a search externality that generically generates inefficiency.[1] On the other hand, Shimer (1996) and Moen (1997) show that efficiency can be restored once an appropriate notion of competition is introduced—that is, when firms post wages and workers direct their search toward the most attractive ones. This form of competition is known as *competitive search*.

However, the contracting problem of workers and employers is not only about how to divide the surplus generated by the match. A crucial additional problem is that the contracting parties typically have private information necessary to evaluate this surplus, and such information is needed to decide whether starting an employment relationship is profitable. In this paper, I focus on this informational problem and on its impact on efficiency. Specifically, I propose a competitive search model with private information and limited commitment on the workers' side. My main result is that the dynamic competitive search equilibrium is generically constrained inefficient. In particular, the unemployment rate reacts suboptimally to initial shocks. This implies a potential role for government intervention in the presence of labor market fluctuations.

I consider an economy in which employers and workers are both risk neutral and ex ante homogeneous. Employers post contracts and workers direct their search toward them. When a match is formed, the disutility of labor is drawn randomly and observed privately by the worker. Moreover, there is limited commitment on the worker's side, in the sense that the worker cannot be forced to work and is always free to walk away and join the ranks of the unemployed. An employment contract is an incentive-compatible and individually rational mechanism that ensures that the worker truthfully reveals his information and participates voluntarily in the employment relationship. I begin by char-

[1] The conventional model is built on Diamond (1982), Mortensen (1982*a*, 1982*b*), Pissarides (1984, 1985), and Mortensen and Pissarides (1994). Hosios (1990) shows that this model is constrained inefficient, except for a specific bargaining power division. See Pissarides (2000) for an overview.

acterizing the competitive search equilibrium, and I show that the equilibrium contract is equivalent to a wage contract, in which the firm offers a flat wage that the worker can accept or reject. Then, I turn to study *constrained efficiency*. I define a social planner who faces the same informational and limited commitment constraints of the market economy. Given these and the aggregate resource constraint, the planner decides how many vacancies to open and how to allocate consumption to employed and unemployed workers. In both the competitive equilibrium and the planner problems, I assume that workers who reject the job cannot be distinguished from other unemployed workers.

In a static setting, I show that the competitive search equilibrium is constrained efficient. By contrast, once I turn to a dynamic setting, I show that the competitive search equilibrium is generically constrained inefficient. The crucial difference between the static and the dynamic environments is that the workers' outside option is exogenously given in the former, whereas in the latter it is endogenously determined as the continuation utility of unemployed workers. When workers have private information, they can appropriate a fraction of the net surplus created in a match, which I refer to as *informational rents*. The workers' outside option affects the size of these rents and, hence, the distortion driven by the informational problem. Firms who post contracts at time $t+1$ affect the workers' outside option at time t, but they do not take into account the informational cost that they impose on contracts designed by other firms at time t. This externality is not internalized by competitive search and is the source of constrained inefficiency. The social planner takes into account the impact of unemployed workers' continuation utility on current contracts and is able to improve upon the equilibrium allocation.

The inefficiency result holds whenever the economy starts at an unemployment rate level different from its steady state. If the initial unemployment rate is above the steady-state level, the mass of unemployed workers who can meet a firm and obtain private information is higher today than tomorrow. Hence, the average informational distortion is also higher today. It follows that the planner would like to reduce job creation tomorrow, in order to reduce the continuation utility of unemployed workers today and increase current job creation. The opposite happens when the initial unemployment rate is below the steady-state level.

The inefficiency in my model is driven neither by the search externality arising in the standard Mortensen-Pissarides model nor by suboptimality in private contracting. On the one hand, my model retains the Walrasian spirit of competitive search to abstract from inefficiencies associated with ex post bargaining. On the other hand, I allow for general employment contracts under asymmetric information. Prescott and

Townsend (1984) show that, in the presence of private information, competitive markets can decentralize the constrained efficient mechanism. In their paper, however, agents can enter exclusive contracts. In this paper, instead, when workers enter unemployment they become anonymous and free to enter a new contractual relationship. This matching environment is a natural way of introducing dynamic competition among contracts and introduces a novel externality. Such an externality is akin to the pecuniary externalities explored by Arnott and Stiglitz (1987) and Golosov and Tsyvinski (2006) in models of insurance in which side trades are feasible.

This paper is related to the vast literature on search theoretic models of the labor market (see the survey by Rogerson, Shimer, and Wright [2005]) and, in particular, to models using competitive search, such as Shimer (1996), Moen (1997), and Acemoglu and Shimer (1999*a*). A series of papers highlight the robustness of the efficiency properties of competitive search in an environment with full information.[2]

My paper is also related to a growing body of literature on asymmetric information in search environments. In particular, Shimer and Wright (2004) and Moen and Rosen (2007) analyze labor markets in which trading frictions interact with asymmetric information, using competitive search. However, they neither focus on efficiency nor analyze the transitional dynamics of the equilibrium. Faig and Jerez (2005) propose a theory of commerce in which buyers have private information about their willingness to pay for a product. They define a notion of constrained efficiency similar to the one in this paper, but they focus on the static version of the model, hence obtaining an efficiency result. In a similar spirit, Wolinsky (2005) analyzes the efficiency properties of a sequential procurement model with lack of commitment on the buyer's side and finds inefficient equilibria. However, in his model the inefficiency arises because of contracting restrictions. The fact that the seller's effort is not contractible distorts the buyer's search intensity. In my paper, private contracts are unrestricted, and the inefficiency comes only from a general equilibrium effect.

Finally, from a methodological standpoint, my paper is related to the vast literature on mechanism design with asymmetric information, which goes back to Mirrlees (1971), Laffont and Maskin (1980), Myerson (1981), and Myerson and Satterthwaite (1981). The novelty of this paper is that it embeds a classic contracting problem with asymmetric information into a search environment. This generates a form of competition

[2] For example, Acemoglu and Shimer (1999*b*) show that competitive search is efficient even with ex ante investments, and Mortensen and Wright (2002) generalize results on price determination and show how competitive search achieves efficiency by exploiting all gains from trade. Hawkins (2006) shows that even when a firm can hire more workers, competitive search is efficient when firms post contracts that are general enough.

among contracts, in the sense that when an informed agent rejects a contract offered by a given principal, he is free to search for a new contract. I assume that the shock that is privately observed by the agents (their type) is match specific and that, after rejecting a contract, agents are anonymous. These two assumptions greatly simplify the analysis by allowing for a recursive representation of the optimal contracting problem and by making the agents' outside option independent of their match-specific type. This differentiates my approach from the literature on agency problems with countervailing incentives (Lewis and Sappington 1989), in which the agent's type affects his outside option, giving him countervailing incentives to reveal his information. This mechanism is absent in my model because the agent's type is match specific instead of being fixed over time.

This paper is organized as follows. In Section II, I analyze the static version of the economy. In Section III, I describe the dynamic environment and characterize the dynamic competitive search equilibrium. In Section IV, I describe the welfare properties of the dynamic model and derive the main inefficiency result. In Section V, I explore an economy in which workers have full commitment. Section VI concludes. Finally, the Appendix contains all the proofs that are not present in the text.

II. Static Economy

In this section, I introduce the static version of the economy, define and characterize the competitive search equilibrium, and analyze its efficiency properties.

Environment.—The economy is populated by a continuum of measure 1 of workers and a large continuum of employers. Both workers and employers are risk neutral and ex ante homogeneous. For simplicity, assume that all the workers are initially unemployed. Workers can search freely, whereas employers need to pay a positive entry cost k to post a vacancy. When an employer hires a worker, the match produces y. The value of y is common to all the matches and is exogenously given. However, workers suffer a match-specific disutility from labor θ. When a match is formed, θ is randomly drawn from the cumulative distribution function $F(\cdot)$, with full support on $\Theta \equiv [\underline{\theta}, \overline{\theta}]$, and is observed privately by the worker.[3] The cumulative distribution function $F(\cdot)$ is differentiable, with $f(\cdot)$ denoting the associated density function, and satisfies the monotone hazard rate condition, $d[F(\theta)/f(\theta)]/d\theta > 0$.

At the beginning of the period, employers choose whether to post an employment contract $\mathcal{C}$ in the space of the feasible contracts. Each

[3] The value θ can also be interpreted as the cost of the effort that the worker has to exert to make the match productive.

worker observes the contracts posted by active firms and chooses to search for a specific contract. Then, matching takes place and, for each match, the draw θ is realized and privately observed by the worker. Next, the worker can decide either to participate in the employment relationship or to walk away and stay unemployed. Unemployed workers obtain utility b. Assume that $y - \underline{\theta} > b$, in order to make the problem interesting.[4]

Matching frictions.—Trading frictions in the labor market are modeled through random matching.[5] Employers and workers know that their matching probabilities depend on the contract that they, respectively, post and seek. For each contract $\mathcal{C}$, let $v(\mathcal{C})$ denote the mass of employers offering $\mathcal{C}$ and $u(\mathcal{C})$ the mass of unemployed workers searching for $\mathcal{C}$. The mass of matches created is given by a constant returns-to-scale matching function, $m(v(\mathcal{C}),\ u(\mathcal{C}))$. Let $\Gamma(\mathcal{C}) \equiv v(\mathcal{C})/u(\mathcal{C})$ denote the "tightness" of the market for the contract $\mathcal{C}$ and define the function $\mu(\gamma) \equiv m(\gamma, 1)$. Then, $\mu(\Gamma(\mathcal{C}))$ represents the probability that a worker applying for $\mathcal{C}$ finds an employer, and $\mu(\Gamma(\mathcal{C}))/\Gamma(\mathcal{C})$ denotes the probability that a firm posting $\mathcal{C}$ finds a worker. The function $\mu(\gamma) : [0, \infty) \mapsto [0, 1]$ satisfies the two standard conditions: (i) $\mu(\gamma) \leq \min\{\gamma, 1\}$,[6] and (ii) $\mu(\gamma)$ is continuous and twice differentiable with $\mu'(\gamma) > 0$ and $\mu''(\gamma) < 0$ for any $\gamma \in [0, \infty)$.[7]

Employment contracts.—Firms are allowed to post general contracts. Thanks to the revelation principle, a contract can be specified as an incentive-compatible and individually rational direct revelation mechanism. The worker reports his type θ, and, conditional on this report, the firm hires him with a certain probability and pays him a certain transfer. Incentive compatibility ensures that the worker has the incentive to truthfully reveal his type, and individual rationality guarantees that the worker participates voluntarily in the employment relationship after θ has been realized. Individual rationality reflects the limited commitment on the worker's side, corresponding to the typical "at-will" employment contracts enforced in the United States. In contrast, firms can fully commit to the posted contract.

Without loss of generality, I can restrict attention to *wage contracts*, given by a flat wage $w \in \mathbb{R}_+$, which the worker decides either to accept

[4] If $y < b + \underline{\theta}$, the equilibrium would be characterized by zero trade for any θ.

[5] Random matching can be interpreted as the result of coordination frictions, as in Burdett, Shi, and Wright (2001).

[6] With discrete time, this condition ensures that both $\mu(\gamma)$ and $\mu(\gamma)/\gamma$ are proper probabilities.

[7] The exponential function $\mu(\gamma) = 1 - \exp(-\gamma)$ satisfies these conditions. One can relax them to include functions with one or two kinks, such as a modified Cobb Douglas of the form $\mu(\gamma) = \min\{\eta\gamma^{\alpha}, \gamma, 1\}$. See Guerrieri (2006).

or reject. A worker of type θ will get utility $w - \theta$ if he accepts and b if he rejects. Hence, he accepts the job if and only if $\theta \leq \hat{\theta}$, where

$$\hat{\theta} = w - b. \tag{1}$$

This implies that as firms choose the wage posted, they choose at the same time the hiring cutoff $\hat{\theta}$. The higher the posted wage, the greater the chance that a matched worker will accept the job.

It is immediate to see that a wage contract is equivalent to a general contract in which workers reveal θ and firms hire only workers with $\theta \leq \hat{\theta}$ and give a flat positive transfer w to all hired workers and a zero transfer to all the workers who are not hired. On the one hand, it is straightforward to see that such a contract is incentive compatible and individually rational. All the hired workers obtain the same wage and do not have an incentive to lie, and, by construction, the workers who are hired prefer working over staying unemployed and receiving b. On the other hand, in Section III, I show that any incentive-compatible and individually rational contract that is traded in equilibrium is equivalent to a wage contract. Let me go through the logic of the argument. First, if two types of hired workers could obtain different transfers, the type getting the lowest would pretend to be the other type, violating incentive compatibility. Hence, there must be a flat transfer to all the hired workers. Second, workers' lack of commitment ensures that the transfer to workers who are not hired must be nonnegative. In equilibrium it will actually be zero, given that reducing it does not affect the surplus created by the match, while it relaxes the incentive-compatibility constraint. Finally, notice that the marginal hired worker must be indifferent between being hired and receiving his outside option. This indifference condition pins down the cutoff $\hat{\theta}$ such that all the workers with $\theta \leq \hat{\theta}$ are hired. The monotone hazard rate condition on $F(\cdot)$ ensures that firms never use probabilistic hiring. The formal argument is presented in Section III. In this section, I simply assume that firms can only offer wage contracts.

A. *Competitive Search Equilibrium*

I now define a competitive search equilibrium in which firms can only post wage contracts.

DEFINITION 1. In a static economy in which firms can only post wage contracts, a competitive search equilibrium is a set of wages $\mathcal{W}^{\text{CE}}$ together with a function $\hat{\Theta}^{\text{CE}} : \mathbb{R}_+ \mapsto \mathbb{R}$, a function $\Gamma^{\text{CE}} : \mathbb{R}_+ \mapsto \mathbb{R}_+ \cup \infty$, and a utility level $U^{\text{CE}} \in \mathbb{R}_+$, satisfying

i) employers' profit maximization and free entry: for all w,

$$\frac{\mu(\Gamma^{CE}(w))}{\Gamma^{CE}(w)}F(\hat{\Theta}^{CE}(w))(y-w)-k\leq 0,$$

with equality if $w\in\mathcal{W}^{CE}$;

ii) workers' optimal job application: for all w,

$$U^{CE}\geq\mu(\Gamma^{CE}(w))\int_{\underline{\theta}}^{\hat{\Theta}^{CE}(w)}(w-\theta-b)dF(\theta)+b,$$

and $\mu(\Gamma^{CE}(w))\leq 1$ with complementary slackness, where U^{CE} is given by

$$U^{CE}=\max_{w'\in\mathcal{W}^{CE}}\mu(\Gamma^{CE}(w'))\int_{\underline{\theta}}^{\hat{\Theta}^{CE}(w')}(w'-\theta-b)dF(\theta)+b,$$

or $U^{CE}=b$ if $\mathcal{W}^{CE}$ is empty; and

iii) workers' optimal job acceptance: for all w,

$$\hat{\Theta}^{CE}(w)=w-b.$$

In equilibrium, both firms and workers know the market tightness associated with each wage; that is, they know the tightness function $\Gamma^{CE}(w)$. Employers also know the hiring cutoff function $\hat{\Theta}^{CE}(w)$. Notice that both these functions are defined for any wage $w\in\mathbb{R}_+$, even if not offered in equilibrium. Given these functions, firms post wages that maximize their ex ante profits, and free entry drives these profits to zero. Moreover, optimal job application ensures that workers look only for wages that maximize their ex ante utility, and optimal job acceptance ensures that, after meeting an employer, a worker will choose to work only if it is better than remaining unemployed. Notice that in equilibrium firms will never post wages that do not guarantee U^{CE} to workers, because they anticipate that they would not otherwise be able to attract any worker.

Generalizing the standard result in the search literature (see Shimer 1996; Moen 1997; Acemoglu and Shimer 1999*a*), the competitive search equilibrium is such that the expected utility of the unemployed workers is maximized subject to the zero profit condition for the employer.

Proposition 1. There exists a unique competitive search equilibrium $\{\mathcal{W}^{CE},\hat{\Theta}^{CE},\Gamma^{CE},U^{CE}\}$, where $\mathcal{W}^{CE}=\{w^{CE}\}$. Let $\hat{\theta}^{CE}=\hat{\Theta}^{CE}(w^{CE})$ and $\gamma^{CE}=\Gamma^{CE}(w^{CE})$. The pair $(\hat{\theta}^{CE},\gamma^{CE})$ solves

$$\max_{\hat{\theta},\gamma}\ \mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}(y-\theta-b)dF(\theta)+b-\gamma k \tag{P1}$$

subject to the free-entry condition

$$\frac{\mu(\gamma)}{\gamma} F(\hat{\theta})(y - \hat{\theta} - b) = k. \tag{2}$$

Moreover, $w^{\mathrm{CE}} = \hat{\theta}^{\mathrm{CE}} + b$.

Equilibrium characterization.—There are two crucial frictions in the model: asymmetric information between workers and employers and workers' limited commitment. The first implies that firms cannot price discriminate among workers with different disutility θ. The second implies that workers cannot commit to make payments to the firm if they are not hired. The combination of these two frictions implies that I can restrict attention to wage contracts and that $w = \hat{\theta} + b$. Then, a worker of type $\hat{\theta}$ is exactly indifferent between working and remaining unemployed, whereas all the inframarginal workers with $\theta < \hat{\theta}$ strictly prefer to work and obtain a positive net surplus $\hat{\theta} - \theta$. It follows that, ex ante, workers expect to appropriate an average net surplus from the match equal to

$$\mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta} - \theta) dF(\theta) = \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \frac{F(\theta)}{f(\theta)} dF(\theta).$$

I will refer to this expression as workers' expected *informational rents.* Using this expression, constraint (2) can be rewritten as

$$\mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} (y - \theta - b) dF(\theta) = \gamma k + \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \frac{F(\theta)}{f(\theta)} dF(\theta). \tag{3}$$

This equation shows that the expected net surplus of a match must cover not only the vacancy creation cost, γk, but also the workers' informational rents.

The equilibrium $\hat{\theta}^{\mathrm{CE}}$ and γ^{CE} solve problem (P1). After replacing constraint (2) with (3), I obtain the first-order conditions

$$\hat{\theta}^{\mathrm{CE}} = y - b - \frac{\lambda}{1 + \lambda} \frac{F(\hat{\theta}^{\mathrm{CE}})}{f(\hat{\theta}^{\mathrm{CE}})} \tag{4}$$

and

$$\mu'(\gamma^{\mathrm{CE}}) \int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{CE}}} \left[y - \theta - b - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k, \tag{5}$$

where λ is the multiplier attached to constraint (3). The presence of the terms multiplied by the factor $\lambda/(1 + \lambda)$ in the expressions above is due to the workers' informational rents and generates a distortion in the equilibrium allocation. In particular, the presence of the information constraint (3) introduces a wedge between the equilibrium values $\theta^{\rm CE}$ and $\gamma^{\rm CE}$ and the first-best values $\hat{\theta}^{\rm FB}$ and $\gamma^{\rm FB}$ that maximize the ex ante net surplus of the economy

$$\mu(\gamma)\int_{\underline{\theta}}^{y}(y-\theta-b)dF(\theta)+b-\gamma k.$$

It is immediate that $\hat{\theta}^{\rm FB}$ and $\gamma^{\rm FB}$ satisfy conditions (4) and (5) with $\lambda = 0$.[8] Hence, the equilibrium with asymmetric information would achieve the first best only if $\lambda = 0$. However, the next lemma shows that this is impossible. In particular, it shows that both $\hat{\theta}^{\rm CE}$ and $\gamma^{\rm CE}$ are lower than their first-best counterparts. In this economy, job creation is equal to $\mu(\gamma)F(\hat{\theta})$ and depends positively both on the matching probability $\mu(\gamma)$ and on the hiring margin $\hat{\theta}$. Therefore, asymmetric information unambiguously reduces job creation; that is, $\mu(\gamma^{\rm CE})F(\hat{\theta}^{\rm CE}) < \mu(\gamma^{\rm FB})F(\hat{\theta}^{\rm FB})$.

Lemma 1. In the static economy, the competitive search equilibrium does not achieve the first-best allocation. Moreover, equilibrium job creation is lower than in the first best.

The distortion comes from the fact that when workers have some informational advantage over the employers, all hired workers need to be paid a flat wage. Hence, to implement the first-best hiring cutoff, the wage should be equal to y. This would imply that the net revenues of any employer after hiring a worker, $y - w$, would be zero. Given that employers have to pay ex ante the cost k to open a vacancy, this contradicts the zero profit condition. Lemma 1 highlights the tension between ex ante and ex post efficiency, which keeps the economy away from the first best. Ex post allocative distortions are necessary to induce employers to open vacancies ex ante.

B. *Constrained Efficiency*

I now define the social planner problem. The planner faces the same frictions present in the market economy: asymmetric information and limited commitment on the workers' side. He does not observe the disutility of the matched workers and cannot force workers to accept a job. If a worker rejects the job, he joins the pool of workers who never received a job offer. Workers who remain unemployed are anonymous; that is, the planner cannot distinguish between workers who have never matched with a firm and workers who have rejected a job. Moreover,

[8] It is possible to show that the first-best allocation can be achieved by the competitive search equilibrium under perfect information—i.e., when employers can observe the type of the workers they meet. In this case, in equilibrium wages are contingent on θ.

unemployed workers cannot commit to make any transfer to the planner. Given these constraints and the aggregate resource constraint, the social planner decides how many vacancies to open at the beginning of the period and chooses how to allocate consumption to employed and unemployed workers. Similarly to the equilibrium analysis, to simplify the exposition, I restrict the planner to offer a flat consumption level to all employed workers, regardless of their type θ, and to use a cutoff rule for hiring. In Section IV, I show that this is without loss of generality.

An *allocation* is described by a consumption level c for employed workers, a consumption level C^U for unemployed workers, a hiring cutoff $\hat{\theta}$, and a tightness of the market γ. The utility of an employed worker of type θ is $c - \theta$, whereas that of an unemployed worker is just C^U. After being matched and observing his type, a worker can decide whether to accept the job offer and be hired or to reject it and stay unemployed. Hence, after the match, a worker accepts to work if and only if $c - \theta \geq C^U$—that is, $\theta \leq \hat{\theta}$, where

$$\hat{\theta} = c - C^U. \tag{6}$$

Limited commitment implies that an unemployed worker can always choose to keep b, which requires that $C^U \geq b$. Finally, the resource constraint for the static economy ensures that aggregate consumption is covered by aggregate net resources—that is,

$$\mu(\gamma)F(\hat{\theta})(c - C^U) + C^U \leq \mu(\gamma)F(\hat{\theta})(y - b) + b - \gamma k. \tag{7}$$

I can now define a constrained efficient allocation. Given that all the workers are initially unemployed, the social welfare coincides with the ex ante value of being unemployed.

Definition 2. A constrained efficient allocation maximizes the workers' ex ante utility

$$\mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}(c - \theta)dF(\theta) + [1 - \mu(\gamma)F(\hat{\theta})]C^U \tag{8}$$

subject to the optimal participation constraint (6), the resource constraint (7), and the limited commitment constraint $C^U \geq b$.

It is straightforward to show that the resource constraint holds with equality. After substituting it into the objective (8), the planner problem can then be rewritten as

$$\max_{c, C^U, \hat{\theta}, \gamma} \; \mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}(y - \theta - b)dF(\theta) + b - \gamma k \tag{P2}$$

subject to

$$\mu(\gamma)F(\hat{\theta})(y - c) = \gamma k + (C^U - b)[1 - \mu(\gamma)F(\hat{\theta})], \tag{9}$$

$\hat{\theta} = c - C^U$, and $C^U \geq b$. The only difference with problem (P1) is that the planner can potentially transfer resources to the unemployed workers and make their consumption level higher than b. However, given that C^U does not appear in the objective function, the planner will choose to keep it at the minimum feasible level b in order to relax as much as possible constraint (9). This proves the following proposition.

Proposition 2. In the static economy, a competitive search equilibrium is constrained efficient.

C. *Money Burning*

I now propose a simple exercise to introduce the mechanism that will lead to dynamic inefficiency. Suppose that b can be destroyed. I now show that destroying b, which I call *money burning*, can increase the workers' ex ante utility.

Proposition 3. There exists an open set of the parameter space $(k, y, F(\cdot))$ for which the workers' ex ante utility is decreasing in b.

Notice that b represents the workers' outside option, which is exogenous in the static setting. The proof in the Appendix shows that the workers' ex ante utility is decreasing in b whenever $1 - \mu(\gamma^{\text{CE}}) F(\hat{\theta}^{\text{CE}}) (1 + \lambda) < 0$. This expression represents the effect of the workers' outside option on welfare. There is a direct positive effect coming from the fact that as the outside option is higher, workers who end up unemployed are better off. This is captured by $1 - \mu(\gamma^{\text{CE}}) F(\hat{\theta}^{\text{CE}})$, which represents the ex ante probability of being unemployed at the end of the period. However, there is a negative indirect effect coming from the fact that as b increases, the wage has to increase for all workers, tightening the information constraint (3). This effect is captured by $\lambda\mu(\gamma^{\text{CE}}) F(\hat{\theta}^{\text{CE}})$. When $1 - \mu(\gamma^{\text{CE}}) F(\hat{\theta}^{\text{CE}}) < \lambda\mu(\gamma^{\text{CE}}) F(\hat{\theta}^{\text{CE}})$, the indirect effect dominates and the workers' ex ante utility is decreasing in b.

This result suggests that a planner who could affect the workers' outside option could improve upon the competitive equilibrium allocation. This will be crucial in the welfare analysis of the dynamic model, in which the workers' outside option will become an endogenous object.

III. Dynamic Economy

Environment.—Consider an economy with infinite horizon and discrete time. Both workers and employers have linear preferences and discount factor β. The search and production technologies are natural generalizations of the static setting. Each match lasts until separation, which happens according to a Poisson process with parameter s, while y and θ now denote the expected present value of output and disutility at the moment of the match.[9] At the beginning of each time t, workers can be either employed or unemployed and employers can be either active or inactive. Inactive employers can open a vacancy at a cost k, which entitles them to post an employment contract. Invoking the revelation principle, without loss of generality I can restrict attention to the set Ω_t of incentive-compatible and individually rational direct revelation mechanisms at time t.[10] A contract posted at time t is a map $\mathcal{C}_t$: $\Theta \mapsto [0, 1] \times \mathbb{R}_+$, specifying the hiring probability $e_t(\tilde{\theta}) \in [0, 1]$ and the expected present value of transfers $\omega_t(\tilde{\theta}) \in \mathbb{R}_+$ from the employer to the worker for each matched worker at time t who reports type $\tilde{\theta}$.[11] Notice that the transfer profile over the life of the relationship is irrelevant for the analysis, given that workers are risk neutral, types are fixed over time within a match, and there is no commitment problem after the match is implemented. Notice that contracts cannot be conditioned on the past employment history, since I assume that unemployed workers are anonymous.

Let $\mathbb{C}_t \subset \Omega_t$ be the set of contracts posted by active firms. Each unemployed worker observes $\mathbb{C}_t$ and applies for a contract $\mathcal{C}_t \in \mathbb{C}_t$. Similarly to the static setting, each contract $\mathcal{C}_t$ is associated with a specific γ_t so that employers and workers know that their matching probabilities will depend on the contract that they, respectively, post and seek. When matching takes place, the draw θ is realized and is observed by the worker. Then, the worker chooses a report $\tilde{\theta}$ and whether to participate in the employment relationship. If he walks away or is not matched, he enters an anonymous pool of unemployed workers, gets b, and searches for a job in the next period. If the worker is hired, the match is productive until separation. Notice that, in order to ensure nonzero job creation, I assume that $(y - \underline{\theta})\,[1 - \beta\,(1 - s)] - b > 0$, where $(y - \underline{\theta})\,[1 - \beta\,(1 - s)]$ represents the per-period net surplus of a match under the best possible realization of disutility.

Bellman values.—Let $v_t(\theta, \tilde{\theta})$ denote the expected utility for a worker

[9] Let the instantaneous output and disutility be $\tilde{y}$ and $\tilde{\theta}$, which are both constant for the duration of the match. Then, $y \equiv \tilde{y}[1 - \beta\,(1 - s)]^{-1}$ and $\theta \equiv \tilde{\theta}[1 - \beta\,(1 - s)]^{-1}$.

[10] The set Ω_t is time varying because the outside option for unemployed workers is potentially changing over time.

[11] More generally, $\omega_t(\tilde{\theta})$ could depend not only on the report $\tilde{\theta}$ but also on whether or not the worker is hired. However, because of risk neutrality, this would have no effect on the results.

of type θ, matched at time t, and reporting type $\tilde{\theta}$—that is,

$$v_t(\theta, \tilde{\theta}) \equiv [\omega_t(\tilde{\theta}) - e_t(\tilde{\theta})\theta] + e_t(\tilde{\theta})\beta V_{t+1} + [1 - e_t(\tilde{\theta})](b + \beta U_{t+1}). \quad (10)$$

For analytical convenience it is useful to split $v_t(\theta, \tilde{\theta})$ into three components as follows: (i) the worker receives the discounted present value of wages, net of disutility, denoted by $\omega_t(\tilde{\theta}) - e_t(\tilde{\theta})\theta$; (ii) if he is hired, he obtains βV_{t+1}, where V_{t+1} denotes the continuation utility of employed workers, net of wages and disutility; and (iii) if he is not hired, he enjoys b plus βU_{t+1}, where U_{t+1} represents the continuation utility of being unemployed. The value V_t reflects the possibility of being separated and becoming unemployed in future periods, and it satisfies the recursion

$$V_t = s(b + \beta U_{t+1}) + (1 - s)\beta V_{t+1}. \quad (11)$$

Moreover, the value U_t satisfies

$$U_t = \mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} [\omega_t(\theta) - e_t(\theta)(\theta - \beta V_{t+1})]dF(\theta) + \left[1 - \mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)dF(\theta)\right](b + \beta U_{t+1}). \quad (12)$$

An employment contract $\mathcal{C}_t$ is incentive compatible whenever

$$v_t(\theta, \theta) \geq v_t(\theta, \tilde{\theta}) \quad \text{for all } \theta, \tilde{\theta} \in \Theta, \quad \text{(IC)}$$

and individually rational whenever

$$v_t(\theta, \theta) \geq b + \beta U_{t+1} \quad \text{for all } \theta \in \Theta. \quad \text{(IR)}$$

Following a standard result in the mechanism design literature (e.g., see Laffont and Tirole 1993), I can reduce the dimensionality of the constraints. In particular, conditions (IC) and (IR) are equivalent to $e_t(\cdot)$ being nonincreasing together with the following two conditions for $v_t(\theta, \theta)$:

$$v_t(\theta, \theta) = v_t(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} e_t(y)dy \quad \text{for all } \theta \in \Theta \quad \text{(IC}')$$

and

$$v_t(\bar{\theta}, \bar{\theta}) \geq b + \beta U_{t+1}. \quad \text{(IR}')$$

This allows me to separate the problem of finding an optimal hiring schedule $e_t(\cdot)$ from the problem of finding a wage schedule $\omega_t(\cdot)$ that implements it.

A. *Dynamic Competitive Search Equilibrium*

In this section, I define the dynamic version of a competitive search equilibrium, when there are no restrictions on the contracts that firms can post. Such a competitive search equilibrium is a sequence of sets of incentive-compatible and individually rational contracts $\{\mathbb{C}_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ and a sequence of tightness functions $\{\Gamma_t^{\mathrm{CE}}\}_{t=0}^{\infty}$, where $\Gamma_t^{\mathrm{CE}}: \Omega_t \mapsto \mathbb{R}_+ \cup \infty$, such that, at any t, employers maximize profits and workers apply optimally for jobs. At time t, both workers and employers take as given the sequence of tightness functions $\{\Gamma_\tau^{\mathrm{CE}}\}_{\tau=t}^{\infty}$ and of sets of posted contracts $\{\mathbb{C}_\tau^{\mathrm{CE}}\}_{\tau=t}^{\infty}$.

I define the equilibrium in recursive terms. The crucial thing to notice is that the pair of continuation utilities for unemployed and employed workers at time $t+1$, U_{t+1}, and V_{t+1} are sufficient statistics for the future sets of posted contracts $\{\mathbb{C}_\tau^{\mathrm{CE}}\}_{\tau=t+1}^{\infty}$ and tightness functions $\{\Gamma_\tau^{\mathrm{CE}}\}_{\tau=t+1}^{\infty}$. This allows me to use the following definition.

DEFINITION 3. In the dynamic economy, a symmetric competitive search equilibrium is a sequence of sets of incentive-compatible and individually rational contracts $\{\mathbb{C}_t^{\mathrm{CE}}\}_{t=0}^{\infty}$; a sequence of functions $\{\Gamma_t^{\mathrm{CE}}\}_{t=0}^{\infty}$, where $\Gamma_t^{\mathrm{CE}}: \Omega_t \mapsto \mathbb{R}_+ \cup \infty$; and a bounded sequence of continuation utilities $\{U_t^{\mathrm{CE}},\ V_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ that satisfy

i) employers' profit maximization and free entry at each time t: for all $\mathcal{C}_t \equiv \{e_t(\theta),\ \omega_t(\theta)\}_{\theta\in\Theta}$, for given V_{t+1}^{CE} and U_{t+1}^{CE},

$$\frac{\mu(\Gamma_t^{\mathrm{CE}}(\mathcal{C}_t))}{\Gamma_t^{\mathrm{CE}}(\mathcal{C}_t)}\beta\int_{\underline{\theta}}^{\bar{\theta}}[e_t(\theta)y - \omega_t(\theta)]dF(\theta) - k \le 0,$$

with equality if $\mathcal{C}_t \in \mathbb{C}_t^{\mathrm{CE}}$; and

ii) workers' optimal job application at each time t: for all $\mathcal{C}_t \equiv \{e_t(\theta),\ \omega_t(\theta)\}_{\theta\in\Theta}$, for given V_{t+1}^{CE} and U_{t+1}^{CE},

$$U_t^{\mathrm{CE}} \ge \mu(\Gamma_t^{\mathrm{CE}}(\mathcal{C}_t))\int_{\underline{\theta}}^{\bar{\theta}}\{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1}^{\mathrm{CE}} - V_{t+1}^{\mathrm{CE}})]\}dF(\theta) + b + \beta U_{t+1}^{\mathrm{CE}},$$

and $\mu(\Gamma_t^{\mathrm{CE}}(\mathcal{C}_t)) \le 1$ with complementary slackness, where

$$U_t^{\mathrm{CE}} = \max_{\mathcal{C}'_t\in\mathbb{C}_t^{\mathrm{CE}}} \mu(\Gamma_t^{\mathrm{CE}}(\mathcal{C}'_t))\int_{\underline{\theta}}^{\bar{\theta}}\{\omega'_t(\theta) - e'_t(\theta)[\theta + b + \beta(U_{t+1}^{\mathrm{CE}} - V_{t+1}^{\mathrm{CE}})]\}dF(\theta) + b + \beta U_{t+1}^{\mathrm{CE}},$$

or $U_t^{\mathrm{CE}} = b + \beta U_{t+1}^{\mathrm{CE}}$ if $\mathbb{C}_t^{\mathrm{CE}}$ is empty, and

$$V_t^{\mathrm{CE}} = s(b + \beta U_{t+1}^{\mathrm{CE}}) + (1-s)\beta V_{t+1}^{\mathrm{CE}}.$$

In equilibrium, both firms and workers take as given the continuation utilities of employed and unemployed workers and the tightness function that associates a market tightness with each potential contract, including those not offered in equilibrium. Moreover, profits are driven to zero at each point in time by free entry. Similarly to the static environment, at time t firms will never post contracts that do not guarantee U_t^{CE} to the workers, because they anticipate that otherwise they would not be able to attract any worker.

The next proposition gives a characterization of a competitive search equilibrium in recursive terms.

PROPOSITION 4. If $\{\mathbb{C}_t^{\mathrm{CE}}, \Gamma_t^{\mathrm{CE}}, U_t^{\mathrm{CE}}, V_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ is a competitive search equilibrium, then any pair $(\mathcal{C}_t^{\mathrm{CE}}, \gamma_t^{\mathrm{CE}})$ with $\mathcal{C}_t^{\mathrm{CE}} \in \mathbb{C}_t^{\mathrm{CE}}$ and $\gamma_t^{\mathrm{CE}} = \Gamma_t^{\mathrm{CE}}(\mathcal{C}_t^{\mathrm{CE}})$ satisfies the following:

i) at any time t, $\mathcal{C}_t^{\mathrm{CE}} = \{e_t^{\mathrm{CE}}(\theta), \omega_t^{\mathrm{CE}}(\theta)\}_{\theta\in\Theta}$ and γ_t^{CE} solve

$$\max_{e_t(\cdot),\omega_t(\cdot),\gamma_t} \mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} \{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1} - V_{t+1})]\}dF(\theta) + b + \beta U_{t+1} \tag{P3}$$

subject to $e_t(\theta) \in [0, 1]$, the constraints (IC′) and (IR′), together with the monotonicity assumption on $e_t(\theta)$, and the free-entry condition

$$\frac{\mu(\gamma_t)}{\gamma_t}\int_{\underline{\theta}}^{\bar{\theta}} [e_t(\theta)y - \omega_t(\theta)]dF(\theta) = k, \tag{13}$$

for $U_{t+1} = U_{t+1}^{\mathrm{CE}}$ and $V_{t+1} = V_{t+1}^{\mathrm{CE}}$;

ii) the sequences $\{\mathcal{C}_t^{\mathrm{CE}}, \gamma_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ and $\{U_t^{\mathrm{CE}}, V_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ satisfy equations (11) and (12).

Conversely, if a sequence $\{\mathcal{C}_t^{\mathrm{CE}}, \gamma_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ solves problem (P3) at any t and $\{U_t^{\mathrm{CE}}, V_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ satisfy (11) and (12), then there exists an equilibrium $\{\mathbb{C}_t^{\mathrm{CE}}, \Gamma_t^{\mathrm{CE}}, U_t^{\mathrm{CE}}, V_t^{\mathrm{CE}}\}_{t=0}^{\infty}$ such that $\mathbb{C}_t^{\mathrm{CE}} = \{\mathcal{C}_t^{\mathrm{CE}}\}$ and $\Gamma_t^{\mathrm{CE}}(\mathcal{C}_t^{\mathrm{CE}}) = \gamma_t^{\mathrm{CE}}$.

The next proposition shows that the analysis of the competitive search equilibrium can be substantially simplified, given that it is possible to restrict attention to wage contracts without loss of generality, as I did in the static economy. Recall that a wage contract is equivalent to a direct revelation mechanism characterized by a hiring cutoff rule, a flat transfer paid to the hired workers, and a zero transfer for all the workers who are not hired.

PROPOSITION 5. Take any $\mathcal{C}_t$ and γ_t that solve problem (P3) for given U_{t+1} and V_{t+1}. The contract $\mathcal{C}_t = \{e_t(\theta), \omega_t(\theta)\}_{\theta\in\Theta}$ takes the form of a wage contract; that is,

$$e_t(\theta) = \begin{cases} 1 & \text{if } \theta \leq \hat{\theta}_t \\ 0 & \text{if } \theta > \hat{\theta}_t \end{cases} \quad \text{and} \quad \omega_t(\theta) = \begin{cases} w_t & \text{if } \theta \leq \hat{\theta}_t \\ 0 & \text{if } \theta > \hat{\theta}_t \end{cases},$$

where $w_t = \hat{\theta}_t + b + \beta\,(U_{t+1} - V_{t+1})$ and the pair $(\hat{\theta}_t, \gamma_t)$ is the unique solution to the following problem:

$$\Phi(U_{t+1} - V_{t+1}) \equiv \max_{\hat{\theta}_t, \gamma_t} \; \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} [y - \theta - b - \beta(U_{t+1} - V_{t+1})] dF(\theta) - \gamma_t k \tag{P3$'$}$$

subject to

$$\frac{\mu(\gamma_t)}{\gamma_t} F(\hat{\theta}_t)[y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1})] = k. \tag{14}$$

Equilibrium characterization.—The previous proposition shows that without loss of generality I can restrict attention to wage contracts. In particular, notice that in the dynamic setting the characterization of the equilibrium allocation at time t for given $U_{t+1} - V_{t+1}$ is analogous to the static one, where b is replaced by $b + \beta\,(U_{t+1} - V_{t+1})$, given that the value of remaining unemployed is now $b + \beta U_{t+1}$ and that an employed worker gets, in addition to the wage net of disutility, the discounted continuation utility βV_{t+1}. Similarly to the static setting, constraint (14) can be rewritten as

$$\mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} [y - \theta - b - \beta(U_{t+1} - V_{t+1})] dF(\theta) = \gamma_t k + \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} \frac{F(\theta)}{f(\theta)}\, dF(\theta). \tag{15}$$

This constraint is the analog of the information constraint (3) and requires that the expected net surplus of a match created at time t covers both the vacancy creation cost $\gamma_t k$ and the workers' expected informational rents $\mu\,(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} [F(\theta)/f(\theta)] dF(\theta)$.

For given $U_{t+1}^{\mathrm{CE}} - V_{t+1}^{\mathrm{CE}}$, the equilibrium values $\hat{\theta}^{\mathrm{CE}}$ and γ^{CE} solve problem (P3$'$), and the analysis is similar to the static case. After replacing constraint (14) with (15), I obtain the first-order conditions

$$\hat{\theta}_t^{\mathrm{CE}} = y - b - \beta(U_{t+1}^{\mathrm{CE}} - V_{t+1}^{\mathrm{CE}}) - \frac{\lambda_t}{1 + \lambda_t} \frac{F(\hat{\theta}_t^{\mathrm{CE}})}{f(\hat{\theta}_t^{\mathrm{CE}})} \tag{16}$$

and

$$\mu'(\gamma_t^{\mathrm{CE}})\int_{\underline{\theta}}^{\hat{\theta}_t^{\mathrm{CE}}}\left[y-\theta-b-\beta(U_{t+1}^{\mathrm{CE}}-V_{t+1}^{\mathrm{CE}})-\frac{\lambda_t}{1+\lambda_t}\frac{F(\theta)}{f(\theta)}\right]dF(\theta) = k, \quad (17)$$

where λ_t is the Lagrange multiplier attached to constraint (15). The analog to lemma 1 can be proved in the dynamic setting to show that $\lambda_t > 0$ and that the equilibrium is away from the first-best allocation.

In a dynamic economy, competition among firms posting contracts at time t leads to an allocation that is analogous to the static one, except that now the workers' outside option is an equilibrium object. This outside option captures an additional channel of competition among contracts posted at different points in time: a worker who rejects a contract offered by a given firm at time t is free to search for a new contract at time $t+1$. Firms and workers at time t take as given the continuation utilities U_{t+1} and V_{t+1} that summarize the effect of the equilibrium contracts offered in all future periods. The function $\Phi\,(U_{t+1}-V_{t+1})$, defined in proposition 5, denotes the maximized net surplus of a match at time t for given $U_{t+1}-V_{t+1}$, which represents the net outside option of the unemployed workers. Using the laws of motion (11) and (12) and the function $\Phi\,(\cdot)$, I can then define a pair of difference equations for $\{U_t, V_t\}$. To complete the equilibrium characterization, it is sufficient to find a bounded solution to these difference equations.

The next proposition shows that such a solution always exists and gives an equilibrium characterized by constant w^{CE}, $\hat{\theta}^{\mathrm{CE}}$, γ^{CE}, U^{CE}, and V^{CE}.[12] Moreover, under mild conditions, this equilibrium is unique.

Proposition 6. In the dynamic economy, there exists a competitive search equilibrium, in which $\hat{\theta}_t$, w_t, γ_t, U_t, and V_t are constant and independent of the initial condition u_0. If

$$\Phi'(x) + \beta(1-s) \geq -\beta \quad \text{for all } x \geq 0, \quad (18)$$

where the function $\Phi\,(\cdot)$ is defined in proposition 5, then the equilibrium is unique.

I have imposed condition (18) in order to rule out the possibility of cycles. This condition essentially rules out situations in which the distortion generated by asymmetric information is too large. More specifically, the left-hand side of condition (18) represents the effect of a change in the expected utility of future unemployed on the expected utility of current unemployed. Notice that if this effect is negative and strong enough, it is possible to have cycles in which periods with low

[12] Note that in equilibrium the continuation utility of the employed workers turns out to be smaller than the continuation utility of the unemployed. This is natural once I define the continuation value of the employed, net of wage and disutility.

expected utility for the unemployed are followed by periods of high expected utility. The term $\Phi'(x)$ is equal to $-\beta\mu(\gamma_t)F(\hat{\theta}_t)(1+\lambda_t)$, where λ_t is the Lagrange multiplier attached to the information constraint (14) and represents the impact of an increase in the outside option of the unemployed on the maximized net surplus of a current match. This impact is negative because of two effects: the relative advantage of creating a match is lower, and the unemployed workers can extract larger informational rents; that is, the information constraint is tighter. Condition (18) can be rewritten as $1-\mu(\gamma_t)F(\hat{\theta}_t)(1+\lambda_t)-s\geq -1$. It is straightforward to see that if there is no informational problem and $\lambda_t=0$, condition (18) is always satisfied and cycles are not possible. Notice that as long as $1-\mu(\gamma_t)F(\hat{\theta}_t)(1+\lambda_t)>0$, condition (18) is satisfied even when $\lambda_t>0$. However, the money-burning result in Section II.C shows that this may not be the case if the informational distortion is sufficiently strong. Hence, condition (18) imposes a bound on the strength of the informational distortion. One can show that condition (18) is satisfied for a wide range of plausible parameterizations. In the rest of the paper I will assume that (18) holds.

The characterization of the competitive search equilibrium immediately implies that the only nontrivial transitional dynamics in the economy are those of the unemployment rate:

$$u_{t+1}=u_t[1-\mu(\gamma^{\text{CE}})F(\hat{\theta}^{\text{CE}})]+(1-u_t)s. \tag{19}$$

In the steady state, not only are w, $\hat{\theta}$, γ, V, and U constant, but so is the unemployment rate. The steady-state unemployment rate is given by

$$u^{\text{SS}}=\frac{s}{s+\mu(\gamma^{\text{CE}})F(\hat{\theta}^{\text{CE}})}. \tag{20}$$

IV. Dynamic Efficiency

In this section, I explore the efficiency properties of the dynamic competitive search equilibrium. I characterize the social planning problem and show the main result of the paper: the competitive search equilibrium is constrained inefficient whenever the initial unemployment rate is different from its steady-state level.

A. *Social Planning Problem*

As in the static setting, the social planner does not observe the disutilities of the matched workers and has to induce them to truthfully reveal them. Moreover, workers have limited commitment, in the sense that they can always decide to remain in the anonymous pool of the unemployed and enjoy b. The planner faces the same anonymity restriction present in the decentralized economy: if a worker enters the unemployment pool, his history is indistinguishable from that of any other unemployed worker. Given these constraints, together with the resource constraint of the economy, the social planner decides how many vacancies to open at the beginning of each period, which jobs to create, and how to intertemporally allocate consumption to employed and unemployed workers.

An allocation is a sequence of functions $\{e_t(\tilde{\theta})\}_{\tilde{\theta}\in\Theta}$ representing the hiring decision of a worker who meets an employer at time t and reports type $\tilde{\theta}$, a sequence of functions $\{c_t(\tilde{\theta})\}_{\tilde{\theta}\in\Theta}$ denoting the expected present value of the consumption of the same worker, a sequence C_t^U of consumption values for unemployed workers, and a sequence of tightness values γ_t. Notice that, as in the equilibrium, the consumption profile over the employment relationship is irrelevant for the analysis, given that agents have linear utility, types are fixed over time within a match, and there is no commitment problem after the match is implemented.

Once a worker is matched at time t and observes his disutility θ, he decides the report $\tilde{\theta}$ and expects utility

$$v_t(\theta,\tilde{\theta}) = [c_t(\tilde{\theta}) - e_t(\tilde{\theta})\theta] + e_t(\tilde{\theta})\beta V_{t+1} + [1 - e_t(\tilde{\theta})](C_t^U + \beta U_{t+1}) \quad \text{for all } \tilde{\theta},\ \theta \in \Theta. \tag{21}$$

First, the worker receives the discounted present value of consumption, net of disutility from working, $c_t(\tilde{\theta}) - e_t(\tilde{\theta})\theta$. Moreover, if he is hired, he enjoys βV_{t+1}, where V_{t+1} denotes the continuation value of being employed, net of consumption and disutility, and represents the expected present value of being separated and becoming unemployed in the future. It satisfies the recursion

$$V_t = s(C_t^U + \beta U_{t+1}) + (1-s)\beta V_{t+1}. \tag{22}$$

Finally, if the worker is not hired, he gets the unemployment transfer C_t^U and enjoys βU_{t+1}, where U_{t+1} represents the continuation utility of remaining unemployed and satisfies

$$U_t = \mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} [c_t(\theta) - e_t(\theta)(\theta - \beta V_{t+1})]dF(\theta) + \left[1 - \mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)dF(\theta)\right](C_t^U + \beta U_{t+1}). \tag{23}$$

As in the equilibrium analysis, an allocation is incentive compatible when $e_t(\cdot)$ is nonincreasing and

$$v_t(\theta,\ \theta) \ = \ v_t(\bar{\theta},\ \bar{\theta}) + \int_{\theta}^{\bar{\theta}} e_t(y)dy \quad \text{for all } \theta \in \Theta. \tag{24}$$

Workers' limited commitment imposes that matched workers always prefer to participate in the employment relationship rather than staying unemployed—that is,

$$v_t(\bar{\theta},\ \bar{\theta}) \geq C_t^U + \beta U_{t+1}. \tag{25}$$

Moreover, it requires that unemployed workers can always choose to enjoy b—that is, $C_t^U \geq b$.

The social planner can transfer resources intertemporally at the interest rate $\beta^{-1} - 1$. Then, the intertemporal resource constraint ensures that the expected present value of aggregate consumption is covered by the expected present value of aggregate output and takes the form

$$P_0 \equiv \sum_{t=0}^{\infty} \beta^t \Bigg(u_t \Bigg\{ \mu(\gamma_t) \int_{\underline{\theta}}^{\bar{\theta}} [e_t(\theta)(y - b + C_t^U) - c_t(\theta)] dF(\theta) \tag{26}$$

$$+ b - C_t^U - \gamma_t k \Bigg\} + (1 - u_t)s(b - C_t^U) \Bigg) \geqslant 0,$$

where u_t follows the law of motion

$$u_{t+1} \ = \ u_t \left[1 - \mu(\gamma_t) \int_{\underline{\theta}}^{\bar{\theta}} e_t(y) dF(\theta) \right] + (1 - u_t)s. \tag{27}$$

DEFINITION 4. An allocation $\{e_t(\cdot),\ c_t(\cdot),\ C_t^U,\ \gamma_t\}$ is feasible if there exists a bounded sequence $\{U_t,\ V_t\}$ such that the following are satisfied for all t: (i) the incentive-compatibility constraints, summarized by (24) and $e_t(\cdot)$ nonincreasing; (ii) the participation constraints, (25) and $C_t^U \geq b$; (iii) the resource constraint, (26); and (iv) the law of motion for the unemployment rate, (27).

For each feasible allocation and a given initial rate of unemployment u_0, the pair $(U_0,\ V_0)$ represents the expected utility of unemployed and employed workers at time 0. The social planner will choose a point on the Pareto frontier of the set of feasible pairs $(U_0,\ V_0)$.

DEFINITION 5. For given u_0, an allocation is constrained efficient if it maximizes U_0 subject to feasibility and $V_0 \geq \bar{V}$.

B. General Characterization: Dual Problem

In order to analyze the constrained efficient allocations, it is convenient to approach the social planner problem from a dual perspective—that is, to maximize the net resources P_0 subject to $U_0 \geq \bar{U}$ and $V_0 \geq \bar{V}$ for given $\bar{U}$ and $\bar{V}$. This problem can be characterized in recursive terms. The planner's Bellman equation, at time t, is a function of three state variables: the promised utility to employed workers, V_t; the promised utility to unemployed workers, U_t; and the unemployment rate, u_t. The planning problem can be written as

$$P(V_t, U_t, u_t) = \max_{e_t(\cdot), c_t(\cdot), C_t^U, \gamma_t, V_{t+1}, U_{t+1}, u_{t+1}} u_t\left\{\mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} [e_t(\theta)(y - b + C_t^U) - c_t(\theta)]dF(\theta) + b - C_t^U - \gamma_t k\right\} + (1 - u_t)s(b - C_t^U) + \beta P(V_{t+1}, U_{t+1}, u_{t+1}) \tag{P4}$$

subject to the promise-keeping constraints for V_t and U_t, (22) and (23); the law of motion for u_t, (27); the incentive-compatibility constraints, summarized by (24) and $e_t(\cdot)$ nonincreasing; and the participation constraints, (25) and $C_t^U \geq b$. Given the value function $P(\cdot, \cdot, u_0)$ for a given u_0, if $(\bar{U}, \bar{V})$ is on the Pareto frontier, it must be that $P(\bar{V}, \bar{U}, u_0) = 0$ and that $P(\cdot, \cdot, \cdot)$ is monotone decreasing in the first two arguments at $(\bar{V}, \bar{U}, u_0)$.

PROPOSITION 7. The constrained efficient allocation $\{e_t(\cdot), c_t(\cdot), C_t^U, \gamma_t\}$ is characterized by

$$e_t(\theta) = \begin{cases} 1 & \text{if } \theta \leq \hat{\theta}_t \\ 0 & \text{if } \theta > \hat{\theta}_t \end{cases} \quad \text{and} \quad c_t(\theta) = \begin{cases} c_t & \text{if } \theta \leq \hat{\theta}_t \\ 0 & \text{if } \theta > \hat{\theta}_t \end{cases},$$

where $c_t = \hat{\theta}_t + C_t^U + \beta(U_{t+1} - V_{t+1})$ and $\hat{\theta}_t$, γ_t, and C_t^U solve the problem

$$P(V_t, U_t, u_t) = \max_{C_t^U, \hat{\theta}_t, \gamma_t, u_{t+1}, V_{t+1}, U_{t+1}} u_t\{\mu(\gamma_t)F(\hat{\theta}_t)[y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1})] + b - C_t^U - \gamma_t k\} - (1 - u_t)s(C_t^U - b) + \beta P(V_{t+1}, U_{t+1}, u_{t+1}) \tag{P4$'$}$$

subject to (22), (27), $C_t^U \geq b$, and

$$U_t \geq \mu(\gamma_t)\int_{\underline{\theta}}^{\hat{\theta}_t} (\hat{\theta}_t - \theta)dF(\theta) + C_t^U + \beta U_{t+1}. \tag{28}$$

Proposition 7 shows that asymmetric information induces the planner to offer a flat consumption level to all the employed workers, regardless of their type θ. This implies, similarly to the equilibrium analysis, that workers with disutility θ lower than $\hat{\theta}_t$ appropriate a positive net surplus equal to $\hat{\theta}_t - \theta$ when they are hired at time t. Equation (28) comes from the combination of the incentive compatibility and the participation constraints. It shows that the continuation utility of unemployed workers depends on their expected informational rents, $\mu(\gamma_t)\int_{\underline{\theta}}^{\hat{\theta}_t}(\hat{\theta}_t - \theta)dF(\theta)$, which are increasing with job creation, both at the matching and at the hiring margin.

Next, I show the main result of this paper: the competitive search equilibrium is constrained inefficient whenever the initial unemployment rate is different from its steady-state level.

PROPOSITION 8. In the dynamic economy, if $u_0 \neq u^{SS}$, then the competitive search equilibrium is constrained inefficient.

The crucial difference between the static and the dynamic environment is that the workers' outside option is exogenously given in the former, whereas it is endogenously determined in the latter. Inefficiency arises because firms do not internalize the fact that the workers' outside option affects the workers' informational rents for other firms. The social planner can internalize this informational externality and thus achieve a Pareto improvement.

Let me sketch the mechanism behind this result, leaving the general proof to the Appendix. Let $u_t\eta_t$ be the Lagrange multiplier associated with constraint (28). Notice that the planner promises the same value U_t to a mass u_t of unemployed workers. Hence, η_t represents the shadow cost of increasing U_t for each unemployed worker. It is possible to show that when information is asymmetric, η_t is smaller than one. On the one hand, when U_t increases by one unit, the planner has to give one unit more to each unemployed worker. On the other hand, by constraint (28), this allows the planner to increase the informational rents of the workers at time t, increasing job creation and hence increasing the net social surplus. The difference $1 - \eta_t$ reflects the benefit coming from this second effect and can be interpreted as the shadow value per worker of relaxing the informational distortion.

Now, consider a planner at date 0 who is choosing U_1 optimally and suppose, for simplicity, that all workers start unemployed—that is, $u_0 = 1$. If the planner increases U_1, this affects the promise-keeping constraint (28) for the workers hired both at time 0 and at time 1, given that U_1 appears on the right-hand side of the constraint in period 0 and on the left-hand side of the same constraint in period 1. In particular, the expected informational rents of the workers and, hence, job creation have to decrease at time 0, whereas they can increase at time 1. This implies that the planner sustains the informational cost $1 - \eta_0$

per worker unemployed at time 0 and enjoys the informational benefit $1 - \eta_1$ per worker unemployed at time 1. The planner's first-order condition with respect to U_1 reduces to

$$\frac{\partial \mathcal{L}}{\partial U_1} = -\beta u_0(1 - \eta_0) + \beta u_1(1 - \eta_1), \tag{29}$$

where $\mathcal{L}$ denotes the Lagrangian associated with the planner's problem.[13] Now suppose by contradiction that the competitive equilibrium solves the planner problem. Recall that the equilibrium allocation, except for the unemployment rate, is constant over time. Using the first-order conditions with respect to $\hat{\theta}_t$ and γ_t, one can show that η_t should be constant over time as well; that is, $\eta_t = \eta$ for all t. Then, from equation (29), it would follow that $\partial\mathcal{L}/\partial U_1 = -\beta\,(u_0 - u_1)\,(1 - \eta)$. This means that the planner could locally improve upon the competitive equilibrium by decreasing U_1, given that $u_0 > u_1$.

The argument is more general than this specific example. A perturbation argument shows that whenever $u_t > u_{t+1}$, the planner can improve upon the equilibrium by reducing U_{t+1}.[14] The intuition is that, in this case, a reduction in U_{t+1} leads to an increase in the informational rents for a mass u_t of unemployed workers at time t, which is larger relative to the mass of unemployed workers at time $t + 1$, whose informational rents are depressed by a reduction in U_{t+1}. Since at the competitive equilibrium the shadow value of information per worker is constant, the total benefit at time t of reducing U_{t+1} is higher than the total cost experienced at time t. Symmetrically, when $u_t < u_{t+1}$, there is a gain from increasing U_{t+1}.

This suggests that the direction of the inefficiency depends on the initial conditions of the economy. In particular, when the initial unemployment rate is above its steady-state level, then equation (27) implies that the unemployment rate is decreasing over time. Then, at any time t, the planner has an incentive to reduce U_{t+1}, thus increasing job creation and speeding up the convergence of the unemployment rate to the steady state. On the other hand, when the initial unemployment rate is below its steady-state level, the reverse applies.

Finally, if the initial unemployment rate is at its steady-state level, then the competitive search equilibrium satisfies the necessary conditions of

[13] To derive expression (29), use the first-order condition and the envelope condition with respect to U_1, together with the law of motion (27).

[14] Note that if $u_t < 1$, when U_{t+1} changes, V_{t+1} must also adjust to satisfy the promise-keeping constraint for V_t. In turn, this requires U_{t+2} to change as well. This is why it is easier to make the perturbation argument at $t = 0$ with $u_0 = 1$. The perturbation argument for the general case is available on request.

the social planning problem. In this case, the mass of unemployed workers is constant over time, and the externality described above is muted.[15]

V. Full Commitment

An essential ingredient for the inefficiency result discussed above is the assumption of limited commitment on the workers' side. To illustrate the role of this condition, let me now consider an environment with full commitment. The money-burning exercise in Section II.C shows that if the planner can wastefully destroy b, in some cases he can obtain a Pareto improvement. When there is no commitment problem, both the planner and the private economy can do better than that, since they can take resources away from workers who do not work and redistribute them. Hence, they can reduce the workers' outside option without wasting aggregate resources.

When there is no problem of commitment, workers can fully commit to pay b to firms before observing their type. Hence, the individual rationality constraint (IR′) becomes

$$v(\bar{\theta}, \bar{\theta}) \geq \beta U_{t+1}.$$

Hence, the optimal wage schedule takes the form of a flat wage for hired workers and a flat application fee z for all the matched workers—that is,

$$\omega^{\text{CE}}(\theta) = \begin{cases} w - z & \text{if } \theta \leq \hat{\theta}^{\text{CE}} \\ -z & \text{if } \theta > \hat{\theta}^{\text{CE}} \end{cases},$$

where $w = \hat{\theta}^{\text{CE}} + \beta(U^{\text{CE}} - V^{\text{CE}})$. Feasibility imposes $z \leq b$. After the match, the firm asks the worker to pay an application fee z before he observes the realization of the shock. If the worker is hired, he will receive the wage, net of the fee—that is, $w - z$—whereas if he is not hired, he will just pay the fee z. This implies that if the unemployed workers have enough resources, they can commit ex ante to pay for the option of seeing their realization θ and, hence, can subsidize the informational rents that the employers will have to pay to the workers whom they effectively hire. In the next proposition, I show that when b is sufficiently large, the competitive search equilibrium can achieve the first-best allocation.

PROPOSITION 9. Suppose that there is full commitment on the work-

[15] Unfortunately, problem (P4′) is not concave, and I cannot conclude that at the steady state the equilibrium is constrained efficient, although it seems a reasonable conjecture.

ers' side and that the following inequality holds:

$$b \geq \frac{k\gamma^{\mathrm{FI}}}{\mu(\gamma^{\mathrm{FI}})}.$$

Then the competitive search equilibrium achieves the first-best allocation.

A fortiori, the social planner can restore the first-best allocation. Actually, the planner can do so for a larger set of parameters than the market economy can. The difference comes from the fact that firms cannot extract resources from workers they do not meet, whereas the social planner can impose a tax on all the workers, even those not matched.[16] Formally, under full commitment, C^U can be smaller than b, as long as $C^U \geq 0$.

Proposition 10. Suppose that there is full commitment on the workers' side and the following inequality holds:

$$b \geq \gamma^{\mathrm{FI}}k.$$

Then the first-best allocation can be decentralized by subsidizing job creation with a lump-sum tax on workers, both employed and unemployed.[17]

When there are enough resources in the economy, full commitment restores efficiency, in the specific sense that both the equilibrium and the planner achieve the first-best allocation. However, as soon as the equilibrium is away from the first-best allocation, the constrained inefficiency result applies.

VI. Conclusions

In this paper, I have explored the efficiency properties of decentralized labor markets characterized by bilateral contracting, asymmetric information, and workers' limited commitment. I have shown that the equilibrium unemployment dynamics are typically constrained inefficient and that there is a role for the government to improve upon the equilibrium.

A natural extension of the model would be to add aggregate shocks. A business-cycle interpretation of the results would suggest that decentralized economies react inefficiently to recessions and booms. In par-

[16] Allowing for a broader interpretation of competitive search, I could think of market makers who impose an application fee on all the workers who search for a match. This delivers a problem that is equivalent to the one of the social planner. In this case, the competitive search equilibrium will be able to restore the full information allocation exactly for the same set of parameters as those of the social planner.

[17] An isomorphic policy to implement the full information allocation would be to transfer resources directly from unemployed to employed workers.

ticular, the analysis seems to indicate that there is insufficient creation when the economy recovers from a recession and excessive creation when the economy slows down after a boom. Imagine that the economy is at the steady state and is hit by a temporary negative shock, which pushes the unemployment rate above its steady-state level. After the shock, the planner would like to speed up the convergence toward the original steady state. In terms of policy, this would mean that countercyclical subsidies to job creation could be an optimal response to temporary cyclical shocks. An interesting area for future research is to introduce explicitly aggregate shocks in the model and to study its implications for optimal policy over the business cycle.

The matching environment, together with the anonymity assumption for unemployed workers, provides a useful setting to study competition among nonexclusive contracts. In this context, the inefficiency result is driven by the endogenous nature of the workers' outside option. Firms offering contracts in the future do not internalize the fact that they affect the outside option of unemployed workers who meet other firms today. I believe a similar externality can arise in other models of decentralized contracting. It would be interesting to explore its effects in alternative applications, such as financial markets or monetary economies.

Appendix

Proof of Proposition 1

The proof proceeds in three steps: step 1 shows that any equilibrium corresponds to a solution of problem (P1), step 2 shows that for any solution to (P1) I can construct an equilibrium, and step 3 shows that there exists a unique solution to problem (P1). The first two steps closely follow Acemoglu and Shimer (1999*a*).

Step 1: Let $\{\mathcal{W}^{CE}, \hat{\Theta}^{CE}, \Gamma^{CE}, U^{CE}\}$ be an equilibrium with $w^{CE} \in \mathcal{W}^{CE}$, $\hat{\theta}^{CE} = \hat{\Theta}^{CE}(w^{CE})$, and $\gamma^{CE} = \Gamma^{CE}(w^{CE})$. I show that $(\hat{\theta}^{CE}, \gamma^{CE})$ solves (P1) and $w^{CE} = \hat{\theta}^{CE} + b$. First, profit maximization and optimal job acceptance ensures that the pair $(\hat{\theta}^{CE}, \gamma^{CE})$ satisfies constraint (2). Now consider another triple $\{w, \hat{\theta}, \gamma\}$ that satisfies optimal job acceptance—that is, $\hat{\theta} = \hat{\Theta}^{CE}(w)$—but achieves a higher value of the objective, that is,

$$\mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}(w-\theta-b)dF(\theta) + b > U^{CE}. \tag{A1}$$

I show that $\{w, \hat{\theta}, \gamma\}$ must violate constraint (2). Since $\{\mathcal{W}^{CE}, \hat{\Theta}^{CE}, \Gamma^{CE}, U^{CE}\}$ is an equilibrium, optimal job application implies that

$$\mu(\Gamma^{CE}(w))\int_{\underline{\theta}}^{\hat{\theta}}(w-\theta-b)dF(\theta) + b \leq U^{CE},$$

and, given (A1), it follows that $\mu(\Gamma^{CE}(w)) < \mu(\gamma)$, and so $\Gamma^{CE}(w) < \gamma$. Then, combining this with profit maximization and optimal job acceptance, it follows that

$$\frac{\mu(\gamma)}{\gamma}F(\hat{\theta})(y-w) - k < \frac{\mu(\Gamma^{CE}(w))}{\Gamma^{CE}(w)}F(\hat{\theta})(y-w) - k \leq 0.$$

This implies that $\{w, \hat{\theta}, \gamma\}$ violates (2), completing the first step of the proof.

Step 2: This step shows that for any $\{w^{CE}, \hat{\theta}^{CE}, \gamma^{CE}\}$, such that $(\hat{\theta}^{CE}, \gamma^{CE})$ solves problem (P1) and $w^{CE} = \hat{\theta}^{CE} + b$, there is an equilibrium $\{\mathcal{W}^{CE}, \hat{\Theta}^{CE}, \Gamma^{CE}, U^{CE}\}$ with $\mathcal{W}^{CE} = \{w^{CE}\}$, $\hat{\Theta}^{CE}(w^{CE}) = \hat{\theta}^{CE}$, and $\Gamma^{CE}(w^{CE}) = \gamma^{CE}$. Let $\hat{\Theta}^{CE}(w) = w - b$. Moreover, set

$$U^{CE} = \mu(\gamma^{CE})\int_{\underline{\theta}}^{\hat{\theta}^{CE}}(w^{CE} - \theta - b)dF(\theta) + b,$$

and let $\Gamma^{CE}(w)$ satisfy

$$U^{CE} = \mu(\Gamma^{CE}(w))\int_{\underline{\theta}}^{\hat{\Theta}^{CE}(w)}(w - \theta - b)dF(\theta) + b,$$

or $\Gamma^{CE}(w) = 0$ if there is no solution to the equation, which happens if $w < b + \underline{\theta}$. It follows that the array $\{\mathcal{W}^{CE}, \hat{\Theta}^{CE}, \Gamma^{CE}, U^{CE}\}$ satisfies the optimal application for jobs and the optimal job acceptance decision.

To complete the proof, I now show that it also satisfies the firms' profit maximization. Suppose by contradiction that some triple $\{w, \hat{\Theta}^{CE}(w), \Gamma^{CE}(w)\}$ violates profit maximization, that is,

$$\frac{\mu(\Gamma^{CE}(w))}{\Gamma^{CE}(w)}F(\hat{\Theta}^{CE}(w))(y-w) - k > 0.$$

Then, I can choose $\gamma > \Gamma^{CE}(w)$ such that

$$\frac{\mu(\gamma)}{\gamma}F(\hat{\Theta}^{CE}(w))(y-w) - k = 0.$$

By construction, $\gamma > \Gamma^{CE}(w)$ and $w \geq b + \underline{\theta}$ imply that

$$U^{CE} < \mu(\gamma)\int_{\underline{\theta}}^{\hat{\Theta}^{CE}(w)}(w - \theta - b)dF(\theta) + b,$$

so that the triple $\{w, \hat{\Theta}^{CE}(w), \gamma\}$ satisfies all the constraints but generates a higher value for the objective function, giving a contradiction.

Step 3: In this step, I show that there exists a unique solution to problem (P1). This completes the proof, given that, from the previous two steps, it implies that the unique solution to (P1) together with equation (1) characterizes the unique competitive search equilibrium.

First, let me show that there exists a solution to problem (P1). It is straightforward to see that the objective function of problem (P1) is continuous in $\hat{\theta}$ and γ and that the constraint set is compact, since $(\mu(\gamma)/\gamma)(y - \hat{\theta} - b)F(\hat{\theta}) - k$

is continuous in both its arguments and is not empty, given that, for example, $\gamma = 0$ and any $\hat{\theta}$ satisfies it. Existence follows directly.

Next, let me prove that this solution is unique. A solution to problem (P1) is an array $(\hat{\theta}^{CE}, \gamma^{CE}, \lambda)$ that satisfies the necessary conditions (3), (4), and (5). Notice that equation (4) implicitly defines $\hat{\theta}$ as a function of λ with $\partial\hat{\theta}/\partial\lambda < 0$. This implicit function $\hat{\theta}(\lambda)$ can be substituted into equations (3) and (5), giving two equations in two unknowns, γ and λ:

$$f_1(\gamma, \lambda) = \frac{\mu(\gamma)}{\gamma}\int_{\underline{\theta}}^{\hat{\theta}(\lambda)}\left[y - \theta - b - \frac{F(\theta)}{f(\theta)}\right] dF(\theta) - k = 0$$

and

$$f_2(\gamma, \lambda) = \mu'(\gamma)\int_{\underline{\theta}}^{\hat{\theta}(\lambda)}\left[y - \theta - b - \lambda\frac{F(\theta)}{f(\theta)}\right] dF(\theta) - k = 0.$$

After solving for γ and λ, $\hat{\theta}$ can be derived using (4). Notice that $f_1(\gamma, \lambda)$ and $f_2(\gamma, \lambda)$ implicitly define two functions, which I name $\gamma_1(\lambda)$ and $\gamma_2(\lambda)$. Then, using the implicit function theorem, it follows that at the equilibrium

$$\left|\frac{d\gamma_1(\lambda)}{d\lambda}\right|_{CE} = -\frac{\partial f_1(\gamma, \lambda)/\partial\lambda}{\partial f_1(\gamma, \lambda)/\partial\gamma} > 0 \text{ and } \left|\frac{d\gamma_2(\lambda)}{d\lambda}\right|_{CE} = -\frac{\partial f_2(\gamma, \lambda)/\partial\lambda}{\partial f_2(\gamma, \lambda)/\partial\gamma} < 0,$$

given that $y - b - \hat{\theta}^{CE} - F(\hat{\theta}^{CE})/f(\hat{\theta}^{CE}) < y - b - \hat{\theta}^{CE} - \lambda F(\hat{\theta}^{CE})/f(\hat{\theta}^{CE}) = 0$. It follows that the two curves must intersect at most once. Moreover, given that I have already proved existence, they must intersect exactly at one point, proving that there exists a unique pair $(\hat{\theta}^{CE}, \gamma^{CE})$ that solves problem (P1). Hence, there exists a unique $w^{CE} = \hat{\theta}^{CE} + b$, and, given the previous steps, there exists a unique competitive search equilibrium, completing the proof. QED

Proof of Lemma 1

First, let me show that problem (P1) is equivalent to the same problem in which constraint (2) is replaced by

$$\frac{\mu(\gamma)}{\gamma}F(\hat{\theta})(y - \hat{\theta} - b) \geq k. \tag{A2}$$

Let $\lambda \geq 0$ be the multiplier attached to this constraint. By contradiction, assume that the solution to this problem is a pair $(\hat{\theta}, \gamma)$ with $\lambda = 0$. Then, the necessary first-order condition with respect to $\hat{\theta}$ takes the same form as in problem (P1), equation (4), with $\lambda = 0$—that is, $\hat{\theta} = y - b$. Substituting this into constraint (A2) gives a contradiction, since $k > 0$. This implies that $\lambda > 0$ and, hence, that (A2) must be binding. Given that, by proposition 1 the solution to problem (P1) is unique. It follows that the two problems are equivalent and that the solution to (P1) is fully characterized by (2), (4), and (5) with $\lambda > 0$. The first statement of the lemma follows immediately.

To prove the second statement, notice that $\lambda > 0$ immediately implies that

$\hat{\theta}^{\mathrm{FB}} > \hat{\theta}^{\mathrm{CE}}$ by equation (4). Moreover, this implies that

$$\int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{CE}}}\left[y - \theta - b - \frac{\lambda}{1+\lambda}\frac{F(\theta)}{f(\theta)}\right]dF(\theta) \leq$$

$$\int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{CE}}}(y - \theta - b)dF(\theta) \leq \int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{FB}}}(y - \theta - b)dF(\theta),$$

where the second inequality follows because $y - \theta - b \geq 0$ for all $\theta \in [\hat{\theta}^{\mathrm{CE}}, \hat{\theta}^{\mathrm{FB}}]$. Then, from equation (5) and the strict concavity of $\mu(\cdot)$, it follows that $\gamma^{\mathrm{FB}} > \gamma^{\mathrm{CE}}$. Hence, $\mu(\gamma^{\mathrm{FB}})F(\hat{\theta}^{\mathrm{FB}}) > \mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}})$, completing the proof. QED

Proof of Proposition 3

First, using the envelope condition, notice that the workers' ex ante utility at the competitive search equilibrium is decreasing in b whenever $1 - \mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}})(1+\lambda) < 0$. For convenience, define $\rho \equiv \lambda/(1+\lambda)$. Hence, to complete the proof, I need to show that $1 - \mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}}) < \rho$ for an open set of the parameter space $(k, y, F(\cdot))$. To define a metric on the space of cumulative density functions $F(\cdot)$ continuously differentiable on Θ, I use the norm $\|F\| = \sup_{\theta\in\Theta}\{|F(\theta)| + |F'(\theta)|\}$.

Recall that for any given y and k, the equilibrium values $\hat{\theta}^{\mathrm{CE}}$, γ^{CE}, and ρ must satisfy equations (3), (4), and (5), where $\rho = \lambda/(1+\lambda)$. Using integration by parts, I can rewrite equations (3) and (5) as

$$\rho\frac{\mu(\gamma^{\mathrm{CE}})}{\gamma^{\mathrm{CE}}}\frac{[F(\hat{\theta}^{\mathrm{CE}})]^2}{f(\hat{\theta}^{\mathrm{CE}})} = k \tag{A3}$$

and

$$\frac{\mu(\gamma^{\mathrm{CE}})}{\gamma^{\mathrm{CE}}\mu'(\gamma^{\mathrm{CE}})} - 1 = \frac{\rho}{1-\rho}\frac{f(\hat{\theta}^{\mathrm{CE}})}{[F(\hat{\theta}^{\mathrm{CE}})]^2}\int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{CE}}}F(\theta)d\theta. \tag{A4}$$

For a given $F(\cdot)$, consider a family of economies parameterized by (ε, δ), with (ε, δ) belonging to a (one-sided) neighborhood of $(0, 0)$, $\mathcal{I} \equiv (0, \bar{\varepsilon}) \times (0, \bar{\delta})$, with $\bar{\varepsilon}$ and $\bar{\delta}$ strictly positive. For each pair (ε, δ), set the parameters k and y such that

$$k(\varepsilon, \delta) = \rho(\varepsilon, \delta)\frac{\mu(\varepsilon)}{\varepsilon}\frac{[F(\bar{\theta} - \delta)]^2}{f(\bar{\theta} - \delta)}, \tag{A5}$$

$$y(\varepsilon, \delta) = b + \bar{\theta} - \delta + \rho(\varepsilon, \delta)\frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)}, \tag{A6}$$

where

$$\rho(\varepsilon,\ \delta)\ =\frac{\eta(\varepsilon)D(\delta)}{1-\eta(\varepsilon)[1-D(\delta)]},$$

$$D(\delta)\ \equiv f(\overline{\theta}-\delta)F(\overline{\theta}-\delta)^{-2}\int_{\underline{\theta}}^{\overline{\theta}-\delta}F(\theta)d\theta, \tag{A7}$$

and $\eta(\varepsilon)\ =\ \varepsilon\mu'(\varepsilon)/\mu(\varepsilon)$. Given these parameters, I can construct an equilibrium with $\gamma\ =\ \varepsilon$ and $\hat{\theta}\ =\ \overline{\theta}-\delta$. Notice that $\rho(\varepsilon,\ \delta)<1$ for any pair $(\varepsilon,\ \delta)$.

Next, I show that equations (A5) and (A6) define a continuous and invertible mapping between the space of pairs $(\varepsilon,\ \delta)$ and that of pairs $(k,\ y)$. The determinant of the Jacobian of the function $f(\varepsilon,\ \delta):\mathbb{R}^2\rightarrow\mathbb{R}^2$, with $f_1\ \equiv\ k(\varepsilon,\ \delta)$ and $f_2\equiv\ y(\varepsilon,\ \delta)$, is

$$\begin{aligned}\det J(\varepsilon,\ \delta)\ =\ &-\rho(\varepsilon,\ \delta)\frac{1}{\varepsilon}\left[\mu'(\varepsilon)-\frac{\mu(\varepsilon)}{\varepsilon}\right]\frac{[F(\overline{\theta}-\delta)]^2}{f(\overline{\theta}-\delta)}\\ &\times\left[1+\rho(\varepsilon,\ \delta)\partial\frac{F(\overline{\theta}-\delta)}{f(\overline{\theta}-\delta)}\Big/\partial(\overline{\theta}-\delta)-\frac{\partial\rho(\varepsilon,\ \delta)}{\partial\delta}\frac{F(\overline{\theta}-\delta)}{f(\overline{\theta}-\delta)}\right]\\ &-\frac{\mu(\varepsilon)}{\varepsilon}\frac{\partial\rho(\varepsilon,\ \delta)}{\partial\varepsilon}\frac{[F(\overline{\theta}-\delta)]^2}{f(\overline{\theta}-\delta)}[1-\rho(\varepsilon,\ \delta)],\end{aligned}$$

where, from (A7),

$$\frac{\partial\rho(\varepsilon,\ \delta)}{\partial\delta}=\frac{\eta(\varepsilon)[1-\eta(\varepsilon)]D'(\delta)}{\{1-\eta(\varepsilon)[1-D(\delta)]\}^2}$$

and

$$\frac{\partial\rho(\varepsilon,\ \delta)}{\partial\varepsilon}=\frac{\eta'(\varepsilon)D(\delta)}{\{1-\eta(\varepsilon)[1-D(\delta)]\}^2}.$$

Notice that the strict concavity of $\mu(\cdot)$ implies that $\mu'(\varepsilon)<\mu(\varepsilon)/\varepsilon$. In turn, this can be used to show that $\eta'(\varepsilon)<0$ for any $\varepsilon>0$ and, hence, that $\partial\rho(\varepsilon,\ \delta)/\partial\varepsilon<0$. Given that $\rho(\varepsilon,\ \delta)<1$ for any pair $(\varepsilon,\ \delta)$, this directly implies that the second term of $\det J(\varepsilon,\ \delta)$ is strictly positive. Moreover, the assumption of monotone hazard rate implies that $D(\delta)<1$ for any δ, which, after some algebra, also implies that the first term of $\det J(\varepsilon,\ \delta)$ is strictly positive. It follows that $\det J(\varepsilon,\ \delta)>0$ for any $(\varepsilon,\ \delta)\in\mathcal{I}$ where $\mathcal{I}$ is small enough. This completes the proof that $f(\varepsilon,\ \delta)$ is invertible.

Next, define the function $g(\varepsilon,\ \delta)$; that is,

$$g(\varepsilon,\ \delta)\ \equiv\ 1-\mu(\varepsilon)F(\overline{\theta}-\delta)-\rho(\varepsilon,\ \delta).$$

If, for a small enough neighborhood $\mathcal{I}$, $g(\varepsilon,\ \delta)<0$ for any $(\varepsilon,\ \delta)\in\mathcal{I}$ and $F(\cdot)$ such that $f(\overline{\theta})\int_{\underline{\theta}}^{\overline{\theta}}F(\theta)\,d\theta>-\mu''(0)/[\mu'(0)^2]$, then there exists an open set of the space $(k,\ y)$ for which $g<0$.

First, I show that $\lim_{\varepsilon\rightarrow0}g(\varepsilon,\ \delta)\ =\ 0$ for any $\delta<\overline{\theta}-\underline{\theta}$. Given that $\mu(0)\ =\ 0$ and $\mu(\gamma)$ is everywhere differentiable, it follows that $\lim_{\varepsilon\rightarrow0}\eta(\varepsilon)\ =\ 1$. Then, equation

(A7) yields $\lim_{\varepsilon\to 0}\rho(\varepsilon,\delta) = 1$ given that $D(\delta) > 0$ if $\delta < \bar{\theta} - \underline{\theta}$. It follows that

$$\lim_{\varepsilon\to 0} g(\varepsilon,\delta) = 0 \quad \forall\delta < \bar{\theta} - \underline{\theta}.$$

Then, notice that equation (A4) yields

$$\lim_{(\varepsilon,\delta)\to(0,0)} \left| \frac{\partial g(\varepsilon,\delta)}{\partial\varepsilon} \right|_{(\varepsilon,\delta)\in\mathcal{I}} = -\lim_{(\varepsilon,\delta)\to(0,0)} \mu'(\varepsilon) - \lim_{\varepsilon\to 0}\frac{\eta'(\varepsilon)}{f(\bar{\theta})\int_{\underline{\theta}}^{\bar{\theta}} F(\theta)d\theta}.$$

By assumption, $\lim_{\varepsilon\to 0}\mu'(\varepsilon) = \mu'(0) > 0$ and $|\mu''(0)|/[\mu'(0)^2] < f(\bar{\theta})\int_{\underline{\theta}}^{\bar{\theta}} F(\theta)\,d\theta$. It follows that I can choose $\bar{\varepsilon}$ and $\bar{\delta}$ small enough such that $|\partial g(\varepsilon,\delta)/\partial\varepsilon|_{(\varepsilon,\delta)\in\mathcal{I}} < 0$. This argument can be extended to an open set of $(k, y, F(\cdot))$, completing the proof. QED

Proof of Proposition 4

This proof is similar to the first part of the proof of proposition 1 and proceeds in two steps: step 1 shows that any contract traded in equilibrium corresponds to a solution of problem (P3), and step 2 shows that for any solution to (P3) I can construct an equilibrium.

Step 1: Let $\{\mathbb{C}_t^{\text{CE}}, \Gamma_t^{\text{CE}}, U_t^{\text{CE}}, V_t^{\text{CE}}\}_{t=0}^{\infty}$ be an equilibrium. First, notice that $\{U_t^{\text{CE}}, V_t^{\text{CE}}\}_{t=0}^{\infty}$ satisfy (11) and (12) by definition. Take a time t and any pair $(\mathcal{C}_t^{\text{CE}}, \gamma_t^{\text{CE}})$, where $\mathcal{C}_t^{\text{CE}} = \{e_t^{\text{CE}}(\theta), \omega_t^{\text{CE}}(\theta)\}_{\theta\in\Theta} \in \mathbb{C}_t^{\text{CE}}$ and $\gamma_t^{\text{CE}} = \Gamma_t^{\text{CE}}(\mathcal{C}_t^{\text{CE}})$. Here, I show that the pair $(\mathcal{C}_t^{\text{CE}}, \gamma_t^{\text{CE}})$ solves (P3) for given $(U_{t+1}^{\text{CE}}, V_{t+1}^{\text{CE}})$. Profit maximization ensures that the pair $(\mathcal{C}_t^{\text{CE}}, \gamma_t^{\text{CE}})$ satisfies constraint (13). Suppose now that another pair $(\mathcal{C}_t, \gamma_t)$, with $\mathcal{C}_t \in \mathbb{C}_t$, achieves a higher value of the objective—that is,

$$\mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}}\{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1}^{\text{CE}} - V_{t+1}^{\text{CE}})]\}dF(\theta) + b + \beta U_{t+1}^{\text{CE}} > U_t^{\text{CE}}. \quad \text{(A8)}$$

I show that $(\mathcal{C}_t, \gamma_t)$ must violate constraint (13). Since $\{\mathbb{C}_t^{\text{CE}}, \Gamma_t^{\text{CE}}, U_t^{\text{CE}}, V_t^{\text{CE}}\}_{t=0}^{\infty}$ is an equilibrium, optimal job application implies that

$$\mu(\Gamma_t^{\text{CE}}(\mathcal{C}_t))\int_{\underline{\theta}}^{\bar{\theta}}\{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1}^{\text{CE}} - V_{t+1}^{\text{CE}})]\}dF(\theta) + b + \beta U_{t+1}^{\text{CE}} \le U_t^{\text{CE}},$$

and, given (A8), it follows that $\mu(\Gamma_t^{\text{CE}}(\mathcal{C}_t)) < \mu(\gamma_t)$, and so $\Gamma_t^{\text{CE}}(\mathcal{C}_t) < \gamma_t$. Then, combining this with profit maximization, it follows that

$$\frac{\mu(\gamma_t)}{\gamma_t}\int_{\underline{\theta}}^{\bar{\theta}}[e_t(\theta)y - \omega_t(\theta)]dF(\theta) - k < \frac{\mu(\Gamma^{\text{CE}}(\mathcal{C}_t))}{\Gamma^{\text{CE}}(\mathcal{C}_t)}\int_{\underline{\theta}}^{\bar{\theta}}[e_t(\theta)y - \omega_t(\theta)]dF(\theta) - k \le 0.$$

This implies that $\{\mathcal{C}_t, \gamma_t\}$ violates (13), completing the first step of the proof.

Step 2: This step shows that for any sequence $\{\mathcal{C}_t^{\text{CE}}, \gamma_t^{\text{CE}}\}_{t=0}^{\infty}$ that solves problem (P3) at any t and $\{U_t^{\text{CE}}, V_t^{\text{CE}}\}_{t=0}^{\infty}$ that satisfy (11) and (12), I can construct an equilibrium $\{\mathbb{C}_t^{\text{CE}}, \Gamma_t^{\text{CE}}, U_t^{\text{CE}}, V_t^{\text{CE}}\}_{t=0}^{\infty}$ with $\mathbb{C}_t^{\text{CE}} = \{\mathcal{C}_t^{\text{CE}}\}$ and $\Gamma_t^{\text{CE}}(\mathcal{C}_t^{\text{CE}}) = \gamma_t^{\text{CE}}$ for any

t. Let $\Gamma_t^{CE}(\mathcal{C}_t)$ satisfy

$$U_t^{CE} = \mu(\Gamma_t^{CE}(\mathcal{C}_t)) \int_{\underline{\theta}}^{\bar{\theta}} \{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1}^{CE} - V_{t+1}^{CE})]\} dF(\theta) + b + \beta U_{t+1}^{CE},$$

or $\Gamma_t^{CE}(\mathcal{C}_t) = 0$ if there is no solution to the equation. It follows that $\{\mathbb{C}_t^{CE}, \Gamma_t^{CE}, U_t^{CE}, V_t^{CE}\}_{t=0}^{\infty}$ satisfies the optimal application for jobs at any t.

To complete the proof, I now show that it also satisfies the firms' profit maximization. Suppose by contradiction that a pair $(\mathcal{C}_t, \Gamma_t^{CE}(\mathcal{C}_t))$, with $\mathcal{C}_t \in \mathbb{C}_t$, violates profit maximization—that is,

$$\frac{\mu(\Gamma_t^{CE}(\mathcal{C}_t))}{\Gamma_t^{CE}(\mathcal{C}_t)} \beta \int_{\underline{\theta}}^{\bar{\theta}} [e_t(\theta)y - \omega_t(\theta)] dF(\theta) - k > 0.$$

Then, I can choose $\gamma_t > \Gamma_t^{CE}(\mathcal{C}_t)$ such that

$$\frac{\mu(\gamma_t)}{\gamma_t} \int_{\underline{\theta}}^{\bar{\theta}} [e_t(\theta)y - \omega_t(\theta)] dF(\theta) - k = 0.$$

Then, by the construction of Γ_t^{CE}, $\gamma_t > \Gamma_t^{CE}(\mathcal{C}_t)$ implies that

$$U_t^{CE} < \mu(\gamma_t^{CE}) \int_{\underline{\theta}}^{\bar{\theta}} \{\omega_t(\theta) - e_t(\theta)[\theta + b + \beta(U_{t+1}^{CE} - V_{t+1}^{CE})]\} dF(\theta) + b + \beta U_{t+1}^{CE},$$

so that the pair $(\mathcal{C}_t, \gamma_t)$ satisfies all the constraints but generates a higher value for the objective function, yielding a contradiction and completing the proof. QED

Proof of Proposition 5

Consider problem (P3). First, integrating both sides of constraint (IC′) and using integration by parts, I obtain

$$\int_{\underline{\theta}}^{\bar{\theta}} v_t(\theta, \theta) dF(\theta) = v_t(\bar{\theta}, \bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta) F(\theta) d\theta. \tag{A9}$$

Then, substituting for (10) and using the constraint (IR′) gives

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \omega_t(\theta) - e_t(\theta) \left[\theta + b + \beta(U_{t+1} - V_{t+1}) + \frac{F(\theta)}{f(\theta)} \right] \right\} dF(\theta) \geq 0. \tag{A10}$$

By using the free-entry condition (13), one can substitute for the wage, and problem (P3) can be rewritten as

$$\max_{e_t(\cdot), \gamma_t} \mu(\gamma_t) \int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)[y - \theta - b - \beta(U_{t+1} - V_{t+1})] dF(\theta) + b - \gamma_t k + \beta U_{t+1} \tag{P3″}$$

subject to

$$\mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)\left[y-\theta-b-\beta(U_{t+1}-V_{t+1})-\frac{F(\theta)}{f(\theta)}\right]dF(\theta)\geq \gamma_t k, \tag{A11}$$

and $e(\cdot)$ nonincreasing. For given U_{t+1} and V_{t+1}, any function $\{e_t(\theta)\}_{\theta\in\Theta}$ and γ_t that solve problem (P3) also solve problem (P3″). Furthermore, for any function $\{e_t(\theta)\}_{\theta\in\Theta}$ and γ_t that solve problem (P3″), (IC′) and (10) can be used to recover the function $\{\omega_t(\theta)\}_{\theta\in\Theta}$ such that the contract $\mathcal{C}_t = \{e_t(\theta), \omega_t(\theta)\}_{\theta\in\Theta}$ and γ_t solve problem (P3).

Let me now characterize the equilibrium contract $\{e_t^{\mathrm{CE}}(\theta), \omega_t^{\mathrm{CE}}(\theta)\}_{\theta\in\Theta}$. Consider a relaxed version of problem (P3″), without imposing the monotonicity of $e_t(\cdot)$. Pointwise maximization, together with the monotone hazard rate assumption, implies that there exists a threshold $\hat{\theta}_t^{\mathrm{CE}}$ such that $e_t^{\mathrm{CE}}(\theta) = 1$ if $\theta < \hat{\theta}_t^{\mathrm{CE}}$ and $e_t^{\mathrm{CE}}(\theta) = 0$ otherwise, with

$$\hat{\theta}_t^{\mathrm{CE}} = y-b-\beta(U_{t+1}-V_{t+1})-\frac{\lambda}{1+\lambda}\frac{F(\hat{\theta}_t^{\mathrm{CE}})}{f(\hat{\theta}_t^{\mathrm{CE}})}, \tag{A12}$$

where λ is the multiplier associated with constraint (A11). This directly shows that $e_t^{\mathrm{CE}}(\cdot)$ is in fact nonincreasing and that a solution to the relaxed version of problem (P3″) is also a solution to the original problem. Then, from (10), one obtains

$$v_t^{\mathrm{CE}}(\theta,\theta) = \begin{cases}\omega_t(\theta)-\theta+\beta V_{t+1} & \text{if } \theta\leq\hat{\theta}_t^{\mathrm{CE}}\\ \omega_t(\theta)+b+\beta U_{t+1} & \text{if } \theta>\hat{\theta}_t^{\mathrm{CE}}\end{cases},$$

and by using (IC′) one can construct the equilibrium wage schedule

$$\omega_t^{\mathrm{CE}}(\theta) = \begin{cases}\omega_t^{\mathrm{CE}}(\bar{\theta})+\hat{\theta}_t^{\mathrm{CE}}+b+\beta(U_{t+1}-V_{t+1}) & \text{if } \theta\leq\hat{\theta}_t^{\mathrm{CE}}\\ \omega_t^{\mathrm{CE}}(\bar{\theta}) & \text{if } \theta>\hat{\theta}_t^{\mathrm{CE}}\end{cases}.$$

Finally, notice that $\omega_t^{\mathrm{CE}}(\bar{\theta}) = 0$ if (IR′) is binding, and (IR′) is binding if and only if constraint (A11) is binding. Assume by contradiction that $\lambda = 0$. Using (A12), I obtain

$$\frac{\mu(\gamma_t)}{\gamma_t}\int_{\underline{\theta}}^{\hat{\theta}_t}\left[\hat{\theta}_t-\theta-\frac{F(\theta)}{f(\theta)}\right]dF(\theta) = k,$$

where integration by parts implies that $\int_{\underline{\theta}}^{\hat{\theta}_t}(\hat{\theta}_t-\theta)dF(\theta) = \int_{\underline{\theta}}^{\hat{\theta}_t}F(\theta)d\theta$. Given that $k>0$, this yields a contradiction and implies that $\omega^{\mathrm{CE}}(\bar{\theta}) = 0$, and, hence, $\omega_t^{\mathrm{CE}}(\theta)$ takes the form stated in the proposition. In order to complete the proof, notice that problem (P3′) reduces exactly to problem (P3″) if one imposes that $e(\cdot)$ has a cutoff form as it does in equilibrium.

Proof of Proposition 6

Step 1: Existence.

First, notice that with the use of propositions 4 and 5 the dynamic competitive search equilibrium can be equivalently characterized as follows:

i) For given $D_{t+1} \equiv U_{t+1} - V_{t+1}$, $\hat{\theta}_t$ and γ_t solve the following problem:

$$\Phi(D_{t+1}) = \max_{\hat{\theta}_t, \gamma_t} \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} (y - \theta - b - \beta D_{t+1}) dF(\theta) - \gamma_t k \tag{P3*}$$

$$\text{subject to } \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} \left[y - \theta - b - \beta D_{t+1} - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \geq \gamma_t k.$$

ii) For given $\{\hat{\theta}_t, \gamma_t\}_{t=1}^{\infty}$, the sequence $\{D_t\}_{t=1}^{\infty}$ must satisfy

$$D_t = \Phi(D_{t+1}) + (1 - s)(b + \beta D_{t+1}). \tag{A13}$$

Then I can find the optimal sequence $\{V_t, U_t\}_{t=1}^{\infty}$ using

$$V_t = s(b + \beta D_{t+1}) + \beta V_{t+1}, \tag{A14}$$

$$U_t = \Phi(D_{t+1}) + b + \beta U_{t+1}, \tag{A15}$$

and the wage $w_t = \hat{\theta}_t + b + \beta(U_{t+1} - V_{t+1})$. For a given D_{t+1}, problem (P3*) has a unique solution, as follows straight from proposition 1. Define the function $H(\cdot)$ as follows:

$$H(D) \equiv \Phi(D) + (1 - s)(b + \beta D).$$

To complete the existence proof, I can invoke the intermediate value theorem to show that there exists a fixed point D^* for $H(\cdot)$, such that the sequence $\{D_t\}$ with $D_t = D^*$ for all t satisfies equation (A13). Notice that

$$H(0) = \Phi(0) + (1 - s)b > 0,$$

and

$$H\left(\frac{y - \underline{\theta} - b}{\beta}\right) - \frac{y - \underline{\theta} - b}{\beta} = (1 - s)(y - \underline{\theta}) - \frac{y - \underline{\theta} - b}{\beta} < 0,$$

given that $(y - \underline{\theta})[1 - \beta(1 - s)] - b > 0$. This implies that there exists a D^* such that $H(D^*) - D^* = 0$. Given D^*, I can find constant values for V_t and U_t satisfying (A14) and (A15), which after some algebra are

$$U^{\text{CE}} = (1 - \beta)^{-1} \left[\mu(\gamma^{\text{CE}}) \int_{\underline{\theta}}^{\hat{\theta}^{\text{CE}}} F(\hat{\theta}) d\theta + b \right] \tag{A16}$$

and

$$V^{\text{CE}} = \frac{\beta s}{1 - \beta(1 - s)} U^{\text{CE}}. \tag{A17}$$

Since I have analyzed the equilibrium problem in a recursive form in order to verify that workers' behavior is optimal, I still need to check that the values U_t and V_t satisfy the boundary conditions $\lim_{t\to\infty} \beta^t U_t < \infty$ and $\lim_{t\to\infty} \beta^t V_t < \infty$. Given that U_t and V_t are constant, these conditions are immediately satisfied, completing the first step of the proof.

Step 2: Uniqueness.

First, notice that by definition V_t and U_t represent the continuation utility of employed and unemployed workers. This immediately implies that they are nonnegative. Moreover, in any equilibrium, the per-period wage $[1-\beta(1-s)]w_t$ must be bounded above by the firm's per-period output $[1-\beta(1-s)]y$. The maximum expected utility that a worker can obtain at any point in time t is then bounded above by $M \equiv \sum_{\tau=t}^{\infty}\beta^{\tau}\{[1-\beta(1-s)]\,(y-\underline{\theta})\}$ (recall that I assume that $(y-\underline{\theta})\,[1-\beta(1-s)]-b>0$). Therefore, M is an upper bound for both V_t and U_t, and, hence, D_t needs to be bounded as well. Notice that the equilibrium characterized above satisfies these bounds.

Next, it is easy to see that a sufficient condition for H to be a contraction with modulus β is that $|H'(D)| \leq \beta$. Notice that $H'(D) = \Phi'(D) + (1-s)\beta$, where, from problem (P3*),

$$\Phi'(D) = -\beta\mu(\gamma)F(\hat{\theta})(1+\lambda) < 0,$$

with $\lambda \in [0,\infty)$ being the multiplier attached to the constraint of the problem. Given that $s<1$, this immediately implies that $H'(D)<\beta$. Moreover, the assumption that $\Phi'(D)+(1-s)\beta \geq -\beta$ ensures that $H'(D) \geq -\beta$, completing the proof that H is a contraction. Hence, H has a unique fixed point D^*. If D_t is constant, equations (A14) and (A15) show that the only nonexplosive equilibrium values for U_t and V_t are constant. Therefore, there exists a unique equilibrium with D_t constant. Now, assume by contradiction that there exists an equilibrium where D_t is time varying. Since $D_t = H(D_{t+1})$, then $D_t - D^* = H(D_{t+1}) - H(D^*)$, and, by the fact that H is a contraction, $\|D_t - D^*\| \leq \beta\|D_{t+1} - D^*\|$. If the equilibrium D_t is not constant, then there exists $\varepsilon>0$ such that $\|D_t - D^*\| \geq \varepsilon$ for any t. Hence, $\|D_{t+T} - D^*\| \geq \beta^{-T}\varepsilon \to \infty$, which implies that the sequence $\{D_t\}$ is explosive and, thus, that this cannot be an equilibrium. This completes the proof. QED

Proof of Proposition 7

Consider problem (P4). Integrating both sides of the constraints (IC′) and using the constraint (IR′) gives

$$\int_{\underline{\theta}}^{\bar{\theta}}\left\{c_t(\theta) - e_t(\theta)\left[\theta + \frac{F(\theta)}{f(\theta)} + \beta(U_{t+1} - V_{t+1}) + C_t^U\right]\right\}dF(\theta) \geq 0.$$

By using constraint (23), one can substitute for $\int_{\underline{\theta}}^{\bar{\theta}} c_t(\theta)\,dF(\theta)$, and problem (P4) can be rewritten as

$$P(V_t, U_t, u_t) = \max_{C_t^U,\hat{\theta}_t,\gamma_t,u_{t+1},V_{t+1},U_{t+1}} u_t\left\{\mu(\gamma_t)\int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)[y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1})]dF(\theta) + b - C_t^U - \gamma_t k\right\} - (1-u_t)s(C_t^U - b) + \beta P(V_{t+1}, U_{t+1}, u_{t+1}) \qquad \text{(P4″)}$$

subject to (22), (27), $C_t^U \geq b$,

$$U_t \geq \mu(\gamma_t) \int_{\underline{\theta}}^{\bar{\theta}} e_t(\theta)(\hat{\theta}_t - \theta) dF(\theta) + C_t^U + \beta U_{t+1},$$

and $e(\cdot)$ nonincreasing. For given U_{t+1} and V_{t+1}, any function $e_t(\cdot)$ and γ_t that solve problem (P4) also solve problem (P4″). Furthermore, for any function $e_t(\cdot)$ and γ_t that solve problem (P4″), equations (21) and (24) can be used to recover the function $c_t(\cdot)$ such that the allocation $\{e_t(\cdot), c_t(\cdot), C_t^U, \gamma_t\}$ solves problem (P4).

Let me now characterize the constrained efficient allocation $\{e_t(\cdot), c_t(\cdot), C_t^U, \gamma_t\}$. Consider a relaxed version of problem (P4″), without imposing the monotonicity of $e_t(\cdot)$. Pointwise maximization, as in the competitive equilibrium analysis, implies that there exists a threshold $\hat{\theta}_t$ such that $e_t(\theta) = 1$ if $\theta \leq \hat{\theta}_t$, and $e_t(\theta) = 0$ otherwise. This shows that the solution to the relaxed problem is characterized by a monotone $e_t(.)$, so that it also solves the full planning problem. Moreover, from equation (21), I obtain

$$v_t(\theta, \theta) = \begin{cases} c_t(\theta) - \theta + \beta V_{t+1} & \text{if } \theta \leq \hat{\theta}_t \\ c_t(\theta) + C_t^U + \beta U_{t+1} & \text{if } \theta > \hat{\theta}_t \end{cases},$$

and, with the use of (24),

$$c_t(\theta) = \begin{cases} \hat{\theta}_t + C_t^U + \beta(U_{t+1} - V_{t+1}) & \text{if } \theta \leq \hat{\theta}_t \\ 0 & \text{if } \theta > \hat{\theta}_t \end{cases}.$$

To complete the proof, notice that problem (P4′) reduces exactly to problem (P4″) if one imposes that $e(\cdot)$ has a cutoff form as it does at the optimum. QED

Proof of Proposition 8

First, let me rewrite the social planner problem (P4′):

$$P(V_t, U_t, u_t) =$$

$$\max_{C_t^U, \hat{\theta}_t, \gamma_t, u_{t+1}, V_{t+1}, U_{t+1}} u_t\{\mu(\gamma_t)F(\hat{\theta}_t)[y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1})]$$

$$+ b - C_t^U - \gamma_t k\} + (1 - u_t)s(b - C_t^U) + \beta P(V_{t+1}, U_{t+1}, u_{t+1})$$

subject to

$$[(1 - u_t)\nu_t] \quad V_t = s(C_t^U + \beta U_{t+1}) + (1 - s)\beta V_{t+1}, \tag{A18}$$

$$[u_t\eta_t] \quad U_t = \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} \frac{F(\theta)}{f(\theta)} dF(\theta) + C_t^U + \beta U_{t+1}, \tag{A19}$$

$$[\pi_t] \quad u_{t+1} = u_t[1 - \mu(\gamma_t)F(\hat{\theta}_t)] + (1 - u_t)s, \tag{A20}$$

and

$$[\chi_t] \quad C_t^U \geq b. \tag{A21}$$

Notice that a solution to the social planner problem must satisfy the following first-order conditions:

$$y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1}) - (1 - \eta_t)\frac{F(\hat{\theta}_t)}{f(\hat{\theta}_t)} - \pi_t = 0, \tag{A22}$$

$$\mu'(\gamma_t)\int_{\underline{\theta}}^{\hat{\theta}_t}\left[y - \theta - b - \beta(U_{t+1} - V_{t+1}) - (1 - \eta_t)\frac{F(\theta)}{f(\theta)} - \pi_t\right]dF(\theta) = k, \tag{A23}$$

$$\chi_t = u_t(1 - \eta_t) + (1 - u_t)(1 - \nu_t)s, \tag{A24}$$

$$P_V(V_{t+1}, U_{t+1}, u_{t+1}) + u_t\mu(\gamma_t)F(\hat{\theta}_t) + (1 - u_t)(1 - s)\nu_t = 0, \tag{A25}$$

$$P_U(V_{t+1}, U_{t+1}, u_{t+1}) - u_t\mu(\gamma_t)F(\hat{\theta}_t) + (1 - u_t)s\nu_t + u_t\eta_t = 0, \tag{A26}$$

and

$$P_u(V_{t+1}, U_{t+1}, u_{t+1}) = \pi_t, \tag{A27}$$

where the multipliers ν_t, η_t, π_t, and χ_t must be such that the constraints (A18)–(A21) hold with complementary slackness. Finally, the envelope conditions are

$$P_U(V_t, U_t, u_t) = -u_t\eta_t, \tag{A28}$$

$$P_V(V_t, U_t, u_t) = -(1 - u_t)\nu_t, \tag{A29}$$

and

$$P_u(V_t, U_t, u_t) = \mu(\gamma_t)F(\hat{\theta}_t)[y - \hat{\theta}_t - b - \beta(U_{t+1} - V_{t+1})] + b - C_t^U - \gamma_t k + s(C_t^U - b) + \pi_t[1 - \mu(\gamma_t)F(\hat{\theta}_t) - s]. \tag{A30}$$

Proceeding by contradiction, suppose that the competitive equilibrium allocation is constrained efficient for a given initial value of unemployment rate u_0; that is, $\hat{\theta}_t = \hat{\theta}^{CE}$, $\gamma_t = \gamma^{CE}$, $C_t^U = b$, $u_{t+1} = u_{t+1}^{CE}$, $V_{t+1} = V^{CE}$, and $U_{t+1} = U^{CE}$ solve the planner problem, and $P(V^{CE}, U^{CE}, u_0) = 0$.

Recall that $\hat{\theta}^{CE}$, γ^{CE}, U^{CE}, V^{CE}, u_{t+1}^{CE}, and the normalized multiplier λ must satisfy by definition the binding constraint (14) and equations (16), (17), (19), (A16), and (A17).

First, combining equations (16) and (17) with equations (A22) and (A23), it follows that

$$(1-\eta_t-\lambda)\frac{F(\hat{\theta}_t^{\mathrm{CE}})}{f(\hat{\theta}_t^{\mathrm{CE}})}+\pi_t = 0,$$

$$(1-\eta_t-\lambda)\int_{\underline{\theta}}^{\hat{\theta}_t^{\mathrm{CE}}}\frac{F(\theta)}{f(\theta)}dF(\theta)+\pi_t\int_{\underline{\theta}}^{\hat{\theta}_t^{\mathrm{CE}}}dF(\theta) = 0.$$

Given that

$$\frac{F(\hat{\theta}_t^{\mathrm{CE}})}{f(\hat{\theta}_t^{\mathrm{CE}})}\left\{\frac{\int_{\underline{\theta}}^{\hat{\theta}_t^{\mathrm{CE}}}[F(\theta)/f(\theta)]dF(\theta)}{\int_{\underline{\theta}}^{\hat{\theta}_t^{\mathrm{CE}}}dF(\theta)}\right\}^{-1}>1,$$

it must be that

$$\eta_t = 1-\lambda \quad \text{and} \quad \pi_t = 0 \quad \text{for any } t.$$

Notice that, using the envelope conditions (A28) and (A29), I can rewrite equations (A25) and (A26) as

$$(1-\eta_{t+1})u_{t+1} = (1-\eta_t)u_t+(1-\nu_t)(1-u_t)s \tag{A31}$$

and

$$\nu_{t+1}(1-u_{t+1}) = u_t\mu(\gamma_t)F(\hat{\theta}_t)+(1-u_t)(1-s)\nu_t\,. \tag{A32}$$

When $\gamma_t = \gamma^{\mathrm{CE}}$, $\hat{\theta}_t = \hat{\theta}^{\mathrm{CE}}$, and $1-\eta_{t+1} = 1-\eta_t = \lambda$, equation (A31) yields an expression for ν_t at any t—that is,

$$\nu_t(1-u_{t-1}) = (1-u_{t-1})-\frac{\lambda}{s}(u_t^{\mathrm{CE}}-u_{t-1}^{\mathrm{CE}}).$$

Notice that $\lambda>0$ implies that $\eta_t<1$ and $\nu_t<1$ and, hence, $\chi_t>0$, so that $C_t^U = b$, as we assumed. Substituting for ν_t and ν_{t+1} into (A32) and using the law of motion for u_{t+1}^{CE} gives

$$\{s-u_t[\mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}})+s]\}[\mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}})+s] = 0.$$

A contradiction follows immediately as long as $u_t \neq u^{\mathrm{SS}} = s[\mu(\gamma^{\mathrm{CE}})F(\hat{\theta}^{\mathrm{CE}})+s]^{-1}$. This implies that the competitive search equilibrium is constrained inefficient whenever $u_t \neq u^{\mathrm{SS}}$, completing the proof. QED

Proof of Proposition 9

The first-best allocation is characterized by $\hat{\theta}^{\mathrm{FB}}$, γ^{FB}, and $U^{\mathrm{FB}}-V^{\mathrm{FB}}$ such that

$$\hat{\theta}^{\mathrm{FB}} = y-b-\beta(U^{\mathrm{FB}}-V^{\mathrm{FB}}), \tag{A33}$$

$$\mu'(\gamma^{\mathrm{FB}}) \int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{FB}}} [y - \theta - b - \beta(U^{\mathrm{FB}} - V^{\mathrm{FB}})] dF(\theta) = k, \tag{A34}$$

and

$$U^{\mathrm{FB}} - V^{\mathrm{FB}} = \frac{\mu(\gamma^{\mathrm{FB}}) \int_{\underline{\theta}}^{\hat{\theta}^{\mathrm{FB}}} F(\theta) d\theta + b}{1 - \beta(1-s)}.$$

When workers can fully commit, the competitive search equilibrium can be defined as in proposition 4, except that the participation constraint (IR′) now becomes

$$v_t(\bar{\theta}, \bar{\theta}) \geq \beta U_{t+1}.$$

For given U_{t+1} and V_{t+1}, the equilibrium problem can be written as

$$\max_{\hat{\theta}_t, \gamma_t} \ \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} [y - \theta - b - \beta(U_{t+1} - V_{t+1})] dF(\theta) - \gamma_t k + b + \beta U_{t+1}$$

$$\text{subject to } \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} \left[y - \theta - b - \beta(U_{t+1} - V_{t+1}) - \frac{F(\theta)}{f(\theta)} \right] dF(\theta)$$

$$+ \mu(\gamma_t) b \geq \gamma_t k. \tag{A35}$$

Suppose that $U_{t+1} - V_{t+1} = U^{\mathrm{FB}} - V^{\mathrm{FB}}$. Then I show that $\hat{\theta}^{\mathrm{FB}}$, γ^{FB}, and $\lambda = 0$ (where λ is the Lagrange multiplier associated with [A35]) are a solution when $b \geq \gamma^{\mathrm{FI}} k / \mu(\gamma^{\mathrm{FI}})$. By substituting $\hat{\theta}^{\mathrm{FB}}$ and γ^{FB} into (A35), I obtain

$$\mu(\gamma^{\mathrm{FB}}) \int_{\underline{\theta}}^{\hat{\theta}_t} \left[\hat{\theta}^{\mathrm{FB}} - \theta - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) + \mu(\gamma^{\mathrm{FB}}) b \geq \gamma^{\mathrm{FB}} k.$$

Integration by parts implies that whenever $b \geq \gamma^{\mathrm{FI}} k / \mu(\gamma^{\mathrm{FI}})$, then $\lambda = 0$ and the first-order conditions with respect to $\hat{\theta}_t$ and γ_t are equivalent to equations (A33) and (A34), confirming that $\hat{\theta}_t^{\mathrm{CE}} = \hat{\theta}^{\mathrm{FB}}$ and $\gamma_t^{\mathrm{CE}} = \gamma^{\mathrm{FB}}$ for any t. Then, the law of motion for U_t and V_t verify that $U_t^{\mathrm{CE}} = U^{\mathrm{FB}}$ and $V_t^{\mathrm{CE}} = V^{\mathrm{FB}}$, completing the proof. QED

Proof of Proposition 10

When the workers can fully commit, the planner can seize b. Assume that the government gives a subsidy τ_t to firms hiring at time t by taxing lump-sum employed and unemployed workers. Then, the subsidy τ_t is covered by a tax

T_t equal to $\tau_t \mu(\gamma_t) F(\hat{\theta}_t)$, so that the budget is balanced at each time t. Feasibility imposes that $T_t \leq b$. In this case, the equilibrium $\hat{\theta}_t$ and γ_t satisfy

$$\max_{\hat{\theta}_t, \gamma_t} \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} [y - \theta - b - \beta(U_{t+1} - V_{t+1})] dF(\theta) - \gamma_t k + b + \beta U_{t+1}$$

$$\text{subject to} \quad \mu(\gamma_t) \int_{\underline{\theta}}^{\hat{\theta}_t} \left[y + \tau_t - \theta - b - \beta(U_{t+1} - V_{t+1}) - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \geq \gamma_t k \,, \qquad \text{(A36)}$$

where τ_t must be such that $\tau_t \mu(\gamma_t) F(\hat{\theta}_t) \leq b$. Suppose that $U_t^{\text{CE}} - V_t^{\text{CE}} = U^{\text{FB}} - V^{\text{FB}}$. Then I guess and verify that the equilibrium is characterized by $\hat{\theta}^{\text{FB}}$, γ^{FB}, and $\lambda = 0$, where λ is the multiplier associated with (A36). The social planner can choose the minimum transfer τ_t that makes the constraint satisfied at the full information allocation—that is, τ_t such that

$$\mu(\gamma^{\text{FB}}) \int_{\underline{\theta}}^{\hat{\theta}^{\text{FB}}} \left[y + \tau_t - \theta - b - \beta(U_{t+1}^{\text{FB}} - V_{t+1}^{\text{FB}}) - \frac{F(\theta)}{f(\theta)} \right] = \gamma^{\text{FB}} k. \qquad \text{(A37)}$$

In this case, the first-order conditions with respect to $\hat{\theta}_t$ and γ_t are equivalent to equations (A33) and (A34), yielding $\lambda = 0$, $\hat{\theta}_t^{\text{CE}} = \hat{\theta}^{\text{FB}}$, and $\gamma_t^{\text{CE}} = \gamma^{\text{FB}}$. Then, the law of motion for U_t and V_t verify that $U_t^{\text{CE}} = U^{\text{FB}}$ and $V_t^{\text{CE}} = V^{\text{FB}}$. Finally, notice that τ_t is feasible only when $b \geq \tau_t \mu(\gamma_t) F(\hat{\theta}_t)$—that is, when the unemployed have enough resources to cover the informational rents. Substituting (A33) into (A37) gives $\tau_t = k\gamma^{\text{FI}}/\mu(\gamma^{\text{FI}}) F(\hat{\theta}^{\text{FI}})$. It follows that full information is achieved when $b \geq \gamma^{\text{FI}} k$, completing the proof. QED

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