Efficiency of Competitive Search under Asymmetric Information*

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Abstract

In this paper, I study the efficiency properties of competitive search equilibria in economies with informational asymmetries. Employers and workers are both risk-neutral and ex-ante homogeneous. I characterize an equilibrium where employers post contracts and workers direct their search towards them. When a match is formed, the disutility of labor is drawn randomly and observed privately by the worker. An employment contract is an incentive-compatible mechanism that satisfies a participation constraint on the worker's side. I first show that in a static setting the competitive search equilibrium is constrained efficient, that is, it cannot be Pareto improved by a Social Planner subject to the same informational and participation constraints faced by the decentralized economy. I then show that in a dynamic setting, on the contrary, the equilibrium can be constrained inefficient. The crucial difference between the static and the dynamic environment is that the worker's outside option is exogenously given in the former, while in the latter it is endogenously determined as the equilibrium continuation utility of unemployed workers. Inefficiency arises because the worker's outside option affects the ex-ante cost of information revelation, generating a novel externality which is not internalized by competitive search.

1 Introduction

The extent to which decentralized labor markets achieve efficiency is a central economic question. In labor markets, trade occurs bilaterally and is typically costly. Firms need to post vacancies

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and workers must spend time searching for jobs. Moreover, employment contracts are commonly characterized by informational imperfections. The surplus produced by a worker-firm match may depend on idiosyncratic features that are private information of the contracting parties. Costly trade and informational imperfections impose a departure from the Walrasian paradigm,[1] but not necessarily from the property of efficiency. Given these frictions, can the price mechanism still achieve a socially optimal allocation of resources?

Search theory typically models labor market imperfections under the heading of *matching frictions,* by assuming an aggregate matching technology. These frictions are meant to capture the general idea that trade is time-consuming and costly both in terms of coordination and of informational incompleteness. The conventional model, built on Diamond (1982), Mortensen (1982a, 1982b) and Pissarides (1984, 1985), combines random matching with a wage determination process based on Nash bargaining. In this context, the equilibrium level of unemployment is generically inefficient. Decentralized markets do not internalize the search externality generated by the matching frictions[2]. However, going back to the Walrasian spirit, a new generation of search models, Shimer (1996), Moen (1997) and Acemoglu and Shimer (1999a), introduces a novel notion of competition in environments with trading frictions, referred to as *competitive search.* In competitive search models firms post wages and workers direct their search towards them. In this environment, decentralized markets internalize the search externality and the resulting equilibrium is efficient. The efficiency property of competitive search has been proven robust in several contexts and sheds light on the power of the price mechanism to induce firms to open the optimal quantity of vacancies.

In this paper, I propose a search model where informational frictions are modeled explicitly. Informational asymmetries seem to be a crucial element of employment relationships. Here, the problem for firms is not only to meet workers, but also to find out the profitability of their match. Employment contracts are designed optimally in order to extract this information and to induce workers to participate in the productive relationship. In this context, a new type of externality arises. The ability of a firm to extract information depends on the worker's outside option. The outside option, in turn, is determined by the contracts offered by other firms in the future. Because of this externality the equilibrium can fail to be constrained efficient. My model retains the Walrasian spirit of competitive search, by allowing firms to post contracts so as to attract workers. Moreover, I allow for general employment contracts. Therefore, the inefficiency does not depend either on the presence of search externalities or on restrictions on the contract space.

I construct a tractable framework for investigating the role of informational asymmetry in search environments. Employers and workers are both risk-neutral and *ex-ante* homogeneous. Employers

[1] In his AEA presidential address, Friedman (1968) highlights how, in a Walrasian world, a market economy cannot be kept away from the unemployment level that "...would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets..."

[2] See Hosios (1990).

post contracts and workers direct their search towards them. When a match is formed, the disutility of labor is drawn randomly and observed privately by the worker. An employment contract is an incentive compatible mechanism that satisfies a participation constraint on the worker's side. The participation constraint can be interpreted as the result of lack of commitment. A worker cannot be forced to work, he can always quit and join the ranks of the unemployed.[3]

I begin by characterizing the competitive search equilibrium. I show that whenever the *ex-ante* cost of posting a vacancy is positive, *ex-post* inefficiency emerges. In particular, some matches that would produce a positive net surplus are not created. The key source of allocative distortion is that the optimal contract has to offer the same wage to all the types that are hired. This comes straight from incentive compatibility. Moreover, the participation constraint implies that the wage has to be equal to the disutility of the marginal worker. To induce the first-best level of job creation, the wage should be set equal to the firm productivity, driving the firm's profits to zero. This is inconsistent with an equilibrium where firms pay a positive vacancy cost *ex-ante*. A trade-off emerges between the two margins of job creation: efficient creation at the hiring stage has to be sacrificed in order to induce vacancy creation *ex-ante*.

Then, I address my central question: is the competitive search equilibrium *constrained efficient*? I define a social planner who faces *the same frictions* of the competitive economy. The social planner controls the matching process by deciding how many vacancies to post at the beginning of each period and allocates consumption among employed and unemployed workers. He does not observe the match-specific disutility of the workers and has to induce them to reveal it. Moreover, he is subject to the same participation constraint on the workers' side. Workers can always quit and enjoy private utility from leisure, which cannot be transferred. Moreover, workers who quit cannot be distinguished from all the other unemployed workers.

First, I show that in a static setting the competitive search equilibrium is constrained efficient. As in the perfect information benchmark, competition among firms induces them to design contracts in order to attract workers' job applications. This implies that vacancy creation and labor contracts are set to maximize workers' utility, subject to a zero profit condition. This ensures that firms correctly internalize the search externality.

By contrast, I show that in a dynamic setting the competitive search equilibrium is constrained inefficient. The crucial difference between the static and the dynamic environments is that the worker's outside option is exogenously given in the former, while in the latter it is endogenously determined as the continuation utility of unemployed workers. When informational asymmetry is present and workers must be induced to participate, the workers' outside option affects the *ex-ante* cost of information revelation. This generates a novel externality which is not internalized by dynamic competitive search. Firms who post contracts at time $t+1$ do not take into account the informational cost they impose on contracts designed by other firms at time t, by affecting the

[3] This corresponds to the typical *at will* employment contracts enforced in the United States.

workers' outside option. This externality can be the source of constrained inefficiency. The social planner takes into account the impact that the continuation utility of unemployed workers has on current contracts, and can improve upon the equilibrium allocation.

The main result of the paper is that, under asymmetric information and the workers' participation constraint, the competitive search equilibrium is constrained inefficient whenever the economy is away from the steady state. Imagine that the social planner decreases the continuation utility for unemployed workers. An intertemporal trade-off emerges, that is not taken into account by the competitive equilibrium. On one hand, the outside option for a worker who meets a firm today decreases, making it easier for the firm to extract information and increasing job creation today. On the other hand, the social planner has promised to give less utility to the unemployed workers from tomorrow onward. This means that workers will receive smaller informational rents in the future, reducing job creation tomorrow.

When the economy is at the steady state level, the flow of workers out of unemployment, who enjoy the informational gain from a reduction of the outside option, is perfectly offset by the flow of workers into unemployment, who are damaged by a future lower expected utility. When the economy is away from the steady state, the inefficiency depends on the equilibrium dynamics of the unemployment rate. I characterize the direction of the inefficiency and show that it depends on whether the initial unemployment rate is above or below the steady state level. Consider a competitive equilibrium. If the initial unemployment rate is above the steady state level, this means that the mass of potential matches is higher today relative to tomorrow. Hence, the shadow cost of informational extraction is relatively high today and the social planner would like to reduce the continuation utility of unemployed workers in order to achieve higher job creation today. On the contrary, when the unemployment rate is rising, the planner would like to increase the continuation utility of unemployed workers in order to increase job creation tomorrow. The planner can indeed manipulate the continuation utilities by changing the future choices of vacancy creation and hiring margins.

Finally, I explore an alternative environment in which unemployed workers own a transferable endowment which can be seized by the social planner. I show that if these resources are high enough, the social planner can use them to finance the informational rents of employed workers,[4] restoring the full information allocation.[5] Moreover, when these resources are even higher, the full information allocation can be decentralized also by bond posting in private contracts.

Related Literature. My work is related to a vast literature on search theoretic models of the

[4] An example of feasible policy that I explore in the static analysis is one of subsidizing job creation by taxing lump-sum all the workers.

[5] What is needed to restore the full information allocation is that the economy is able to appropriate enough resources to cover the informational costs without distorting the allocation. In this alternative environment those resources come from home production of unemployed workers. An economy with access to enough external resources could achieve the same outcome.

labor market, surveyed by Rogerson, Shimer and Wright (2005). The conventional model builds on Diamond (1982), Mortensen (1982a, 1982b), Pissarides (1984, 1985) and Mortensen and Pissarides (1994).[6] By combining random matching and Nash bargaining, it does not generically achieve efficiency, as shown in Hosios (1990). Departing from this benchmark, more recently, Shimer (1996), Moen (1997), Acemoglu and Shimer (1999a), introduce the equilibrium notion of competitive equilibrium, which combines directed search and wage posting, internalizing the search externality. A series of papers highlights the robustness of the efficiency properties of competitive search.[7] In my model, I use competitive search as equilibrium concept, with the explicit purpose of eliminating the inefficiency coming from the standard search externality and highlighting the novel externality generated by informational frictions.

My work is also related to a growing literature on asymmetric information in search environments. In particular, Shimer and Wright (2004) and Moen and Rosen (2005) analyze, as in my model, labor markets where trading frictions interact with asymmetric information, using competitive search. However, they both explore a static environment, where, as I will show, even in the presence of informational frictions, competitive search keeps its efficiency property. Shimer and Wright (2004) analyze an economy where the employer has some private information about the match and the worker a private effort choice. They show that under mild regularity assumptions, in their environment, contracts take a simple form with at most two wages. In the same spirit, Moen and Rosen (2005) study a competitive search equilibrium with private information on the workers' side. They focus mainly on the impact of asymmetric information on the responsiveness of the unemployment rate to productivity shocks. Moreover, they show that cross subsidization between workers and firms can restore the full information allocation. This result is similar to the one I derive for the case of trasferable endowment.[8]

Another related paper is Faig and Jerez (2004) who propose a theory of commerce, where buyers have private information about their willingness to pay for a product. They also show that the static model is constrained efficient, if the social planner cannot transfer utility across agents. However, they point out that another source of inefficiency can be the non-linearity of the production function. They calibrate a dynamic version of their model embodied in a neoclassical framework where the existence of capital induces a non-linear production function. They show that the welfare losses of competitive search are negligible. In a similar spirit, Wolinsky (2005) analyzes the efficiency properties of a sequential procurement model where a small buyer cannot commit to a mechanism and finds inefficient equilibria. However, in his model the inefficiency arises because of contracting

[6] See Pissarides (2000) for a general treatment.

[7] For example, Acemoglu and Shimer (1999b) show that competitive search is efficient even with ex-ante investments, Mortensen and Wright (2002) generalize results on price determination and show how competitive search achieves efficiency by exploiting all gains from trade. Hawkins (2005) shows that even when a firm can hire more workers, competitive search is efficient when firms post contracts that are general enough.

[8] See subsection 3.2.

restrictions. The fact that the seller's effort is not contractible distorts the buyer's search intensity. In my paper, private contracts are unrestricted and the equilibrium inefficiency comes from a general equilibrium effect.

My work is also indirectly related to a growing literature more focused on the impact of asymmetric information on the business cycle. Reacting to Shimer (2005) and Hall (2005), they have proceeded in the direction of some form of wage rigidity in order to match the business cycle unemployment volatility.[9]

Finally, from a methodological standpoint my paper is related to the literature on mechanism design with asymmetric information, e.g. Mirrlees (1971), Myerson (1981), Myerson and Satterthwaite (1981), Laffont and Maskin (1980).

The paper is organized as follows. Section 2 introduces the static environment of the economy, defines and characterizes the competitive search equilibrium. Section 3 analyzes the efficiency properties of the static economy both for the benchmark environment and for alternative settings. Section 4 describes the dynamic environment, defines and characterizes the dynamic competitive search equilibrium. Section 5 describes the dynamic welfare properties of the model and derives the main result that competitive search, away from the steady state, is constrained inefficient. Finally, Section 6 concludes.

2 Static Economy

The crucial ingredient of my model is the interaction of informational asymmetry and trading frictions, when there is a participation constraint on the worker's side. This section introduces the static version of a decentralized economy highlighting this interaction. I define and characterize the competitive search equilibrium for this economy.

Environment. The economy is populated by a continuum of measure 1 of workers and a large continuum of potential employers. Both workers and employers are risk-neutral and *ex-ante* homogeneous. Workers can search freely, while employers need to pay an entry cost k to post a vacancy. Each worker wants to match an employer and each employer with an open vacancy wants to match only one worker. When a match is formed, the disutility of labor θ is drawn randomly from the cumulative distribution function $F(.)$, with support $\Theta \equiv [\underline{\theta}, \overline{\theta}]$, and is observed privately by the worker.[10] I assume that the cumulative distribution function $F(.)$ is differentiable, with $f(.)$ being

[9] Among others, Kennan (2004) constructs a model with hidden information and bargaining, Menzio (2004) assume employers have private information about productivity and contracts are non-binding, Nagypal (2004) combines workers' heterogeneity, asymmetric information and on-the-job search and Hall and Milgrom (2005) construct an equilibrium characterized by Nash-type bargaining where the threat points are the payoffs of endless delay. None of these papers explore the welfare properties of the models.

[10] The value θ can also be interpreted as the cost of effort that the worker has to exert to make the match productive, which depends on the specificity of the match.

the associated density function, and that it satisfies a monotone hazard rate condition, that is, $d\left[F(\theta)/f(\theta)\right]/d\theta > 0$. The net surplus of the match is given by $y-\theta$, where y represents the amount of output generated by a productive match. The value of y is common to all the matches and is exogenously given in the benchmark model.[11]

At the beginning of the period employers can open a vacancy at a cost k which entitles them to post an employment contract $\mathcal{C}\in\mathbb{C}$, where $\mathbb{C}$ is the set of *ad interim* incentive compatible and individually rational mechanisms. As I describe below, a contract $\mathcal{C}:\Theta\mapsto[0,1]\times\mathbb{R}_{+}$ specifies the hiring probability and the wage for each matched worker who reports type θ. Therefore the strategy of a firm is a pair $(\sigma,\mathcal{C})\in\{0,1\}\times\mathbb{C}$ where σ denotes the decision of posting a vacancy and $\mathcal{C}$ is the posted contract. Next, each worker observes all the contracts posted and decides where to apply. He chooses a contract $\mathcal{C}\in\mathbb{C}^{P}\subset\mathbb{C}$, where $\mathbb{C}^{P}$ denotes the set of contracts posted by active firms. After workers start to search for a specific contract, matching takes place and for each match the draw θ is realized and is private information of the worker. The worker's behavior is described by a map $(a,s):\Theta\mapsto\Theta\times\{0,1\}$ that for each type θ specifies a report $\hat{\theta}=s(\theta)$ and a participation decision $a(\theta)$. The worker can either implement the contract, that is choose $a(\theta)=1$, or walk away, that is choose $a(\theta)=0$. If he walks away he gets b, a flow of non-transferable utility from leisure. In section 3.2, I will extend the analysis to the case where b is transferable and can be interpreted as home production or unemployment benefit.

Trading frictions in the labor market are modeled through random matching and can be thought of as coordination frictions, as in Burdett, Shi and Wright (2001). Employers and workers know that their matching probabilities will depend on the contract that they respectively post and seek for. Each type of contract $\mathcal{C}$ is associated with a labor submarket, where a mass $v(\mathcal{C})$ of employers posts contracts of type $\mathcal{C}$ and a mass $u(\mathcal{C})$ of unemployed workers applies for jobs at firms offering that type of contract. I assume that each submarket is characterized by a constant returns to scale matching function $m(v(\mathcal{C}),u(\mathcal{C}))$ and by an associated "tightness" $\gamma(\mathcal{C})=v(\mathcal{C})/u(\mathcal{C})$.[12] Hence, for each contract $\mathcal{C}$, I can define the function $\mu(\gamma)\equiv m(\gamma,1)$, which represents the probability of a worker applying for $\mathcal{C}$ meeting an employer posting it. On the other hand, the probability of a firm posting $\mathcal{C}$ meeting a worker applying for it is represented by the non-increasing function $\mu(\gamma)/\gamma$.

Assumption A1. The function $\mu(\gamma):[0,\infty)\longmapsto[0,1]$ satisfies the following conditions:

(i) $\mu(\gamma)\leq\min\{\gamma,1\}$;[13]

(ii) for any γ such that $\mu(\gamma)<\min\{\gamma,1\}$, $\mu(\gamma)$ is twice differentiable with $\mu'(\gamma)>0$ and

[11] When the asymmetric information is on the side of the employer, the analysis is similar. The equilibrium allocation still exhibits less trade than in the full information case. However, now firms appropriate ex-post the informational rents required by incentive compatibility.

[12] In order to simplify the notation, from now on I am going to drop the dependence of u, v and γ on the contract $\mathcal{C}$, whenever it does not cause any confusion.

[13] With discrete time, this condition ensures that both $\mu(\gamma)$ and $\mu(\gamma)/\gamma$ are proper probabilities.

$\mu''(\gamma) < 0.$

This assumption allows me to consider matching functions that either are everywhere differentiable or have one or two kinks. The standard matching functions considered in the literature are covered by one of these two classes. The first category includes the exponential case, while the properly *modified*[14] linear and Cobb Douglas case falls into the second one.

In a decentralized economy the consumption of employed workers is given by the wage. Moreover, the consumption of unemployed workers must be equal to the value of leisure b, where unemployed workers are both unmatched workers and workers who have been matched but have not been hired. Assume that $y > b + \underline{\theta}$ in order to make the problem interesting.[15]

Employment Contracts. Without loss of generality, by invoking the Revelation Principle, I can restrict attention to direct revelation mechanisms, corresponding to a mapping $\mathcal{C} : \Theta \mapsto [0,1] \times \mathbb{R}_+$, specifying for each matched worker who reports type θ, the hiring probability $e(\theta) \in [0,1]$ and the wage $\omega(\theta) \in \mathbb{R}_+$. The contract must be incentive compatible and individually rational, that is, it has to ensure that the worker reveals truthfully his type and chooses to participate in the employment relationship after the draw has been realized. Individual rationality can be interpreted as a "no-commitment" assumption on the side of the worker. The no-commitment constraint on the worker's side can represent the typical *at will* employment contracts widespread in the United States. Instead, firms can fully commit to the posted contract.

Let $v(\theta, \hat{\theta})$ denote the *ad interim utility* for worker of type θ revealing $\hat{\theta}$, associated with a contract $\mathcal{C}$,[16] with

$$v(\theta, \hat{\theta}) \equiv \omega(\hat{\theta}) - e(\hat{\theta})\theta + [1 - e(\hat{\theta})]b. \tag{1}$$

An employment contract is *incentive-compatible* whenever it satisfies

$$v(\theta, \theta) \geq v(\theta, \hat{\theta}) \text{ for all } \theta, \hat{\theta} \in \Theta \tag{IC}$$

and *individually rational* whenever

$$v(\theta, \theta) \geq b \text{ for all } \theta \in \Theta. \tag{IR}$$

I define $\mathbb{C}$ the set of incentive compatible and individually rational direct mechanisms.

Following a standard result in the mechanism design literature,[17] I can reduce the dimensionality of the constraints. In particular, I can state the following lemma.

[14] From now on I define the *modified* version of a function $\hat{\mu}(\gamma)$, the function $\mu(\gamma) = \min\{\hat{\mu}(\gamma), \gamma, 1\}$.

[15] Notice that if $y < b + \underline{\theta}$ then even with full information the equilibrium would be characterized by zero trade.

[16] In order to simplify the notation, I am going to drop the dependence of $v(.,.)$ on the contract $\mathcal{C}$, since it does not cause any confusion.

[17] Among others, Mirlees (1971), Myerson (1981), Myerson and Satterthwaite (1981), Laffont and Maskin (1980).

Lemma 1 *Conditions IC and IR are equivalent to* $e(.)$ *non-increasing and*

$$v(\theta,\theta) = v\left(\overline{\theta},\overline{\theta}\right) + \int_{\theta}^{\overline{\theta}} e(y)\, dy \text{ for all } \theta \in \Theta, \tag{IC'}$$

$$v\left(\overline{\theta},\overline{\theta}\right) \geq b. \tag{IR'}$$

Proof. The proof that IC is equivalent to IC' and $e(\theta)$ non-increasing is standard and hence omitted.[18] Then, using the monotonicity of $e(\theta)$, the individual rationality constraints IR can be reduced to the one for the worst type $\overline{\theta}$, completing the proof. ■

This Lemma allows me to separate the problem of finding an optimal allocation from the problem of finding a wage schedule that implements it.

Define $v(\theta,\theta) - v\left(\overline{\theta},\overline{\theta}\right)$ as the *informational rent* of a worker of type $\theta \leq \overline{\theta}$, that is, the additional utility that such a worker must receive in order to reveal his own type. Condition IC' ensures that no worker would gain by pretending to have a higher disutility from working than the realized one. Moreover, condition IR' ensures that the worse type does not expect an utility level lower than the one he could get by staying in autarky.

Finally, the large number of potential firms ensures free entry, imposing that the value of an open vacancy must be zero in equilibrium, that is,

$$\frac{\mu(\gamma)}{\gamma} \int_{\underline{\theta}}^{\overline{\theta}} [e(\theta)\, y - \omega(\theta)]\, dF(\theta) = k. \tag{2}$$

2.1 Static Competitive Search Equilibrium

I now define the concept of competitive search equilibrium in this economy, I prove that it always exists, is unique and I show how to characterize it.

Definition 1 *A static symmetric Competitive Search Equilibrium (CSE) is a set of incentive-compatible and individually rational contracts* $\mathbb{C}^*$ *together with a function* $\Gamma^* : \mathbb{C} \longmapsto \mathbb{R}_+ \cup \infty$ *and a utility level* $U^* \in \mathbb{R}_+$ *satisfying*

(i) *employers'* ***profit maximization*** *and* ***free-entry:*** $\forall \mathcal{C} \equiv [e(\theta), \omega(\theta)]_{\theta\in\Theta}$,

$$\frac{\mu\left(\Gamma^*(\mathcal{C})\right)}{\Gamma^*(\mathcal{C})} \int_{\underline{\theta}}^{\overline{\theta}} [e(\theta)\, y - \omega(\theta)]\, dF(\theta) - k \leq 0$$

subject to incentive compatibility IC and individual rationality IR, with equality if $\mathcal{C} \in \mathbb{C}^*$*;*

[18] See Mas-Colell, Winston and Green (1995), Proposition 23.D.2, p. 888.

(ii) *workers' **optimal job application**:* $\forall \mathcal{C} \equiv [e(\theta), \omega(\theta)]_{\theta \in \Theta}$,

$$U^* \geq \mu(\Gamma^*(\mathcal{C})) \int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)] \, dF(\theta) + b$$

and $\Gamma^*(\mathcal{C}) \geq 0$ *with complementarity slackness, where* U^* *is given by*

$$U^* = \max_{\mathcal{C}'} \mu(\Gamma^*(\mathcal{C}')) \int_{\underline{\theta}}^{\overline{\theta}} [\omega'(\theta) - e'(\theta)(\theta + b)] \, dF(\theta) + b$$

or $U^* = b$ *if* $\mathbb{C}^*$ *is empty.*

In equilibrium, both firms and workers know which market tightness is associated with each contract, that is, they know the function $\Gamma^*(\mathcal{C})$. Given that, profit maximization ensures that firms post the incentive compatible and individually rational contract that maximizes their profits, anticipating the tightness associated even to contracts not offered in equilibrium. This ensures that there are no profitable deviations for the firm and free entry drives profits to zero. Moreover, optimal job application ensures that workers choose which type of contracts to look for, so as to maximize their *ex-ante* utility. In particular, notice that the tightness associated with contracts that are not optimal is zero, since firms will never post those contracts anticipating that they will not be able to attract workers.

It follows that the equilibrium unemployment rate of workers applying to firms posting a contract of type $\mathcal{C}$ is given by

$$u(\mathcal{C}) = 1 - \mu(\Gamma^*(\mathcal{C})) \int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \, dF(\theta)$$

and is affected by both matching and informational frictions. In fact job creation depends not only on the equilibrium matching probability, through $\mu(\Gamma^*(\mathcal{C}))$, but also on the equilibrium hiring decision, once the match is realized, through $\int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \, dF(\theta)$.

Generalizing the standard result in the search literature,[19] I can show that the symmetric competitive search equilibrium is such that the utility of an unemployed worker is maximized subject to the zero profit condition for the employer, the incentive and the participation constraint for the worker.

Proposition 1 *If* $\{\mathbb{C}^*, \Gamma^*, U^*\}$ *is an equilibrium, then any* $\mathcal{C}^* \in \mathbb{C}^*$ *and* $\gamma^* = \Gamma^*(\mathcal{C}^*)$ *solves*

$$U = \max_{e(\theta), \omega(\theta), \gamma} \mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)] \, dF(\theta) + b \tag{P1}$$

subject to $e(\theta) \in [0, 1]$*, non-negative consumption, the incentive constraint IC', together with the*

[19] Moen (1997), Acemoglu and Shimer(1999a) analyze a competitive search equilibirum when information is complete. Shimer and Wright (2004) define a competitive search equilibrium with bilateral asymmetric information.

monotonicity of $e(.)$*, the participation constraint IR' and the free-entry condition (2). Conversely, if a pair* $\{\mathcal{C}^*, \gamma^*\}$ *solves the program P1, then there exists an equilibrium* $\{\mathbb{C}^*, \Gamma^*, U^*\}$ *such that* $\mathcal{C}^* \in \mathbb{C}^*$ *and* $\gamma^* = \Gamma^*(\mathcal{C}^*)$*.*

Proof. See Appendix. ■

Proposition 1 shows how a CSE must solve Problem P1. The next Proposition shows how it can be equivalently described by a tightness γ and a hiring function $e(\theta)$ solving a simplified program P2 and by an associated wage function $\omega(\theta)$ which can be constructed such that the incentive and the participation constraints are satisfied.

Proposition 2 *Any function* $[e(\theta)]_{\theta\in\Theta}$ *and* γ *which solve Problem P1 solves also*

$$U = \max_{e(.),\gamma} \mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} e(\theta)[y-\theta-b]\, dF(\theta) + b - \gamma k \tag{P2}$$

s.t.

$$\mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \geq \gamma k \tag{3}$$

and $e(.)$ *non-increasing.*

Furthermore, for any function $[e(\theta)]_{\theta\in\Theta}$ *and* γ *solving problem P2, there exists a function* $[\omega(\theta)]_{\theta\in\Theta}$ *such that the contract* $\mathcal{C} = [e(\theta), \omega(\theta)]_{\theta\in\Theta}$ *and* γ *solve problem P1.*

Proof. See Appendix. ■

Free-entry implies that the entire surplus of the economy accrues to workers. Hence, the competitive search equilibrium maximizes the net surplus of the economy, subject to the constraint that the net output must cover both the *ex-ante* cost of vacancy creation, that is, γk, and the informational rents of all the workers that are hired, that is, $\int_{\underline{\theta}}^{\overline{\theta}} e(\theta) F(\theta)\, d\theta$.[20] Under full information, the maximization problem is unconstrained and the equilibrium coincides immediately with the social optimum.

Finally, the following Proposition establishes the existence and uniqueness of a CSE. The argument relies on the result of Proposition 2, that the existence of a solution to problem P2 is sufficient to prove the existence of a solution for problem P1.

Proposition 3 *A Competitive Search Equilibrium exists and is unique.*

[20] From condition IC', using integration by parts, it follows that the average information rents are:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[v(\theta,\theta) - v\left(\overline{\theta},\overline{\theta}\right) \right] dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} e(\theta) F(\theta)\, d\theta.$$

Proof. See Appendix. ■

The proof proceeds by showing the existence of a solution for the relaxed version of problem P2 without assuming that $e(\theta)$ is monotone and, then, by checking that the optimal $e(\theta)$ is effectively monotone, implying that it is also the solution to the original problem.

Equilibrium Characterization. Proposition 2 allows me to characterize the competitive search equilibrium of the static economy in a simple way. Proposition 3 proves that Problem P2 has a unique solution and that the first order conditions are necessary and sufficient to characterize it. The analysis proceeds by focusing on the relaxed problem without the monotonicity assumption on $e(\theta)$. Then, using pointwise maximization for $e(\theta)$, I show that the trading area can be fully described by a cut-off value $\hat{\theta}$ such that

$$e(\theta) = \begin{cases} 1 \text{ if } \theta \leq \hat{\theta} \\ 0 \text{ if } \theta > \hat{\theta} \end{cases}$$

implying that the optimal $e(\theta)$ is in fact monotone. When the constraint is binding[21] and $\mu(\gamma)$ is everywhere differentiable, the equilibrium can be characterized by an array $\hat{\theta}$, γ and λ satisfying the conditions

$$\hat{\theta} = y - b - \lambda \frac{F(\hat{\theta})}{f(\hat{\theta})}, \tag{4}$$

$$\mu'(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \lambda \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k \tag{5}$$

and the binding constraint

$$\frac{\mu(\gamma)}{\gamma} \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k. \tag{6}$$

The variable λ represents a normalized version of the shadow value of the *informational rents*. I define $\lambda \equiv \hat{\lambda}/(1+\hat{\lambda})$, where $\hat{\lambda}$ is the Lagrangian multiplier attached to the constraint of problem P2. From equation (4) it follows that the trading cut-off $\hat{\theta}$ is decreasing in λ, that is, as the constraint is tighter, the shadow value that workers have to receive in order to reveal their information increases and the equilibrium is characterized by less trade. Notice that, when $\mu(\gamma)$ is not differentiable at some point, as I describe in the Appendix, equation (5) will be replaced by inequalities involving the left and right derivatives of $\mu(\gamma)$ when the solution will be at the points of non differentiability.

Notice that when $\lambda = 0$, the constraint (3) is slack and γ is simply determined by (5). Then, the full information allocation is achieved. This is possible only when the *ex-ante* cost k is zero. As shown in the next Lemma, incentive compatibility would drive employers to zero profits *ex-post*, if

[21] When the constraint is not binding, the informational problem is irrelevant and the competitive search equilibrium is standard.

the full information allocation would be implemented, contradicting the possibility of an equilibrium where they have to pay a positive cost *ex-ante*.

Lemma 2 *If $k > 0$, then the solution to problem P2 requires $\lambda > 0$, where $\lambda = \hat{\lambda}/(1-\hat{\lambda})$ and $\hat{\lambda}$ is the Lagrangian multiplier attached to the constraint.*

Proof. The proof proceeds by contradiction. Let assume that the solution to problem P2 is an array $\hat{\theta}$, γ and λ with $\lambda = 0$. Then, Proposition 3 implies that $\hat{\theta}$ and γ have to satisfy equations (4), (6) and (5) with $\lambda = 0$. Using (4), equation (6) can be rewritten as

$$\frac{\mu(\gamma)}{\gamma}\int_{\underline{\theta}}^{\hat{\theta}}\left[\hat{\theta}-\theta-\frac{F(\theta)}{f(\theta)}\right]dF(\theta)=k.$$

Integration by parts implies that

$$\int_{\underline{\theta}}^{\hat{\theta}}\left[\hat{\theta}-\theta\right]dF(\theta)=\int_{\underline{\theta}}^{\hat{\theta}}F(\theta)\,d\theta.$$

It follows that it must be $k = 0$ yielding a contradiction and completing the proof. ■

This argument highlights the main channel driving the misalignment between *ex-ante* and *ex-post* efficiency, which keeps the economy away from the full information allocation. *Ex-post* allocative distortions are necessary to induce employers to open vacancies *ex-ante* and make the economy productive. From now on, I focus on $k > 0$ such that the constraint is binding and the informational problem interesting.

From incentive compatibility it follows that the optimal wage schedule must take the following form:

$$\omega(\theta)=\begin{cases}\omega(\bar{\theta})+\hat{\theta}+b & \text{if } \theta\leq\hat{\theta}\\ \omega(\bar{\theta}) & \text{if } \theta>\hat{\theta}\end{cases}.$$

Notice that when $k > 0$ and $\lambda > 0$, then $\omega(\bar{\theta}) = 0$. Then, a constant wage is paid only to workers who are effectively hired and is equal to the disutility of the marginal hired worker plus the outside option. In fact, if two hired workers with different types receive different wages, then the worse type would always pretend to be the best in order to get a higher compensation. Moreover, the marginal hired worker would have no incentive to lie if he is indifferent about being unemployed, that is, if he is compensated exactly for his disutility and for the working opportunity cost b. It follows that the wage is increasing in the trading cut-off $\hat{\theta}$. The more trade is generated, the higher the wage must be in order to induce the marginal hired worker to reveal his type.

Remark 1 *The static economy is equivalent to a reduced form economy where firms post a constant wage and workers apply for jobs.*

3 Static Efficiency

It is an established result that competitive search correctly internalizes the externality generated by matching frictions. This section investigates the efficiency properties of a static equilibrium when there are both matching frictions and informational imperfections. Does competitive search equilibrium still achieve efficiency?

In the benchmark environment, the workers' outside option b is assumed to be leisure, which is non-transferable and cannot be wastefully destroyed. Consumption must be non negative. The worker's disutility from a match θ is his own private information. Hence, the relevant Social Planning problem must be constrained by incentive compatibility and individual rationality on the workers' side and ends up being very similar to problem P2. The only difference is that the planner could potentially transfer resources to the unemployed workers. However, I show that such a transfer is not desirable and the competitive search equilibrium reaches constrained efficiency in the static setting.

Before turning to the dynamic set-up I also study two variations on the original environment by relaxing the assumption on b: in the first one the social planner can destroy wastefully b, but cannot transfer it, and in the second one b can be freely transferred.

Social Planner. In the static economy, the social planner controls the matching process by deciding how many vacancies to open at the beginning of the period. He does not observe the types of the matched workers and has to induce them to truthfully reveal their match-specific disutility. Moreover, the planning problem is also subject to a participation constraint on the side of the workers, who can decide not to produce and enjoy b. Remember that b is leisure and cannot be destroyed or transferred and that negative consumption is not allowed. Given these environmental constraints, together with the resource constraint of the economy, the social planner decides how to allocate consumption among employed and unemployed workers.

An *allocation* is a pair of functions $[c(\theta), e(\theta)]_{\theta \in \Theta_w}$ representing the consumption and the hiring probability for a matched worker who reports type θ, a value for consumption of unmatched workers C_u, and a value γ denoting the tightness of the market.

As in the case of private contracts, also here, the Revelation Principle allows me to restrict attention to direct revelation mechanisms, without loss of generality. Following the analysis of the previous section, an allocation is *incentive-compatible* when, for all $\theta \in \Theta$, $e(.)$ is non-increasing and

$$v^{SP}(\theta, \theta) = v^{SP}\left(\overline{\theta}, \overline{\theta}\right) + \int_{\theta}^{\overline{\theta}} e(y)\, dy. \tag{7}$$

where the *ad-interim* utility for a worker of type θ reporting type $\hat{\theta}$, in the centralized economy, is given by

$$v^{SP}(\theta, \hat{\theta}) = c(\hat{\theta}) - e(\hat{\theta})\theta.$$

Moreover there is a *participation constraint* coming from a lack of commitment on the worker's side together with the assumption that b cannot be destroyed or transferred and that negative consumption is not allowed. It requires that all workers who participate in the society, both if matched and unmatched, consume more than the private utility b that they can appropriate by not participating, that is,

$$C_u \geq b \text{ and } c(\theta) \geq b \text{ for all } \theta \in \Theta. \tag{8}$$

Finally, the *resource constraint* for the static economy ensures that aggregate consumption is covered by aggregate net production, that is,

$$\mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} c(\theta) dF(\theta) + (1 - \mu(\gamma)) C_u \leq \mu(\gamma)(y - b) \int_{\underline{\theta}}^{\overline{\theta}} e(\theta) dF(\theta) + b - \gamma k. \tag{9}$$

I can now define a feasible and a constrained efficient allocation.

Definition 2 *An allocation is feasible iff it (i) is incentive-compatible, that is, satisfies (7) together with the monotonicity of* $e(.)$, *(ii) satisfies the participation constraint (8) and (iii) satisfies the resource constraint (9).*

Definition 3 *A constrained efficient allocation maximizes workers' ex-ante utility*

$$\mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} [c(\theta) - e(\theta)\theta] dF(\theta) + (1 - \mu(\gamma)) C_u$$

subject to feasibility.

The maximization problem defining a constrained efficient allocation can be expressed, after some algebra, as

$$U(b) = \max_{e(\theta), C_u, \gamma} \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} [y - \theta - b] dF(\theta) + b - \gamma k \tag{P3}$$

s.t.

$$\int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \geq \left(\frac{1 - \mu(\gamma)}{\mu(\gamma)} \right) (C_u - b) + \frac{\gamma}{\mu(\gamma)} k$$

$$C_u \geq b.$$

The next proposition shows that the competitive search equilibrium is constrained efficient in the static setting

Proposition 4 *Assume that* b *cannot be destroyed or transferred and that negative consumption is not allowed, then a static Competitive Search Equilibrium is constrained efficient.*

Proof. First, notice that C_u does not appear in the objective function so that the social planner cannot do worse by choosing $C_u = b$. Then problem P3 becomes equivalent to problem P2 and competitive search is constrained efficient. ■

It is interesting to notice that, as in the decentralized equilibrium, the social optimum does not reach the full information allocation, that is, $\lambda > 0$. The result is driven by the binding participation constraint for the workers. The proof shows clearly that the Planner could do better by reducing the consumption of unemployed workers, which does not affect the objective function, below the level of leisure b. Hence, in the next two subsections, I explore two variations of the main environment: in the first one the social planner can destroy wastefully b, but cannot transfer it, and in the second one b can be freely transferred.

3.1 Money Burning can be Desirable.

I now show how reducing b wastefully, what I refer to as *money burning*, can generate a Pareto improvement. For simplicity, assume that all the workers are unemployed *ex-ante*. Employers get zero profits in expectation due to the free entry assumption. Then, the social welfare coincides with the *ex-ante* value of being unemployed $U(b)$, as defined in problem P3. A Pareto improvement is feasible when $U'(b) < 0$. Considering this alternative environment is useful to highlight, in a simple way, the crucial mechanism of the paper. The source of dynamic constrained inefficiency will come from the fact that it can be socially optimal to reduce *ex-ante* the workers' outside option, which here is exogenously given by b. In the dynamic economy, the workers' outside option is endogenous and corresponds to the continuation value of being unemployed, generating an externality that is not internalized by the competitive search equilibrium. As I will show in section 5, the externality comes from the fact that firms who post contracts at time $t+1$ do not internalize the informational cost that they impose on contracts designed by other firms at time t.

I show that for an interesting class of functions $\mu(\gamma)$, there always exists an open set of parameters such that money burning is Pareto improving. Suppose $U(b)$ is differentiable[22] and define

$$g \equiv 1 - \mu(\gamma) F(\hat{\theta}) - \lambda.$$

Notice that g has the same sign of $U'(b)$, which represents the effect of the workers' outside option on welfare. There is a direct positive effect coming from the fact that, as the outside option is higher, workers who end up being unemployed will be better off. This force is summarized by $1-\mu(\gamma) F(\hat{\theta})$, which represents the *ex-ante* probability of being unemployed at the end of the period. However, there is a negative indirect effect coming from the tightness of the informational constraint, represented by λ. As the outside option increases, the shadow cost of revealing information is higher,

[22] It is easy to show that when γ, $\hat{\theta}$ and $\hat{\lambda}$ are uniquely defined, then $U(b)$ is differentiable. In fact this is always the case in the rest of the analysis.

since workers have a higher opportunity cost of remaining in the employment relationship. When the indirect effect dominates making g negative, a Pareto improvement can be implemented by reducing *ex-ante* the workers' outside option.

Proposition 5 *Whenever $g < 0$, the competitive search equilibrium allocation can be Pareto improved by reducing b.*

Proof. The proof is straightforward. Notice that the last constraint of problem P3 will be always binding and then can be eliminated by substituting for $C_u = b$. Then, from the Envelope condition $dU/db = 1 - (1-\lambda)^{-1}\mu(\gamma)\,F(\hat{\theta})$ where $\lambda = \hat{\lambda}/(1+\hat{\lambda})$ and $\hat{\lambda}$ is the multiplier attached to the constraint, which implies that $dU/db < 0$ iff $g < 0$. ■

There are two extreme cases that can lead to g being negative: first, if the constraint is extremely tight, that is, $\lambda \to 1$ and second, if the probability of staying unemployed is extremely small, that is, $\mu(\gamma)\,F(\hat{\theta}(\lambda)) \to 1$. The next two Propositions show that for an interesting class of functions $\mu(\gamma)$, there exists a set of the parameter space (y, k) such that g is negative. My intent is not to fully characterize the set of parameters generating optimal money burning, but to deliver sufficient conditions for this set to exist. Then, I will deliver some insights on its characterization, using the Cobb-Douglas case.

First, I consider two general families of functions $\mu(\gamma)$, which include the specifications commonly used in the search literature: the family of functions everywhere differentiable satisfying a particular restriction on the elasticity stated in assumption A2, and the family of functions reaching 1 with a kink and possibly exhibiting an additional kink when $\mu(\gamma) = \gamma$ at $\gamma > 0$, as specified in assumption A3. Proposition 6 shows that g can be negative for each of these families of functions $\mu(\gamma)$.

The first class of functions $\mu(\gamma)$ includes the exponential case. Functions belonging to this class have to satisfy assumption A1 and A2, where the latter is defined as follows.

Assumption A2. Assume $\mu(\gamma)$ is strictly concave and everywhere differentiable. Furthermore, $\lim_{\gamma\to 0}\mu'(\gamma) = \mu'(0) > 0$ and $\lim_{\gamma\to 0}\eta'(\gamma) > -\mu'(0)\,f(\bar{\theta})\int_{\underline{\theta}}^{\bar{\theta}} F(\theta)\,d\theta$ where $\eta(\gamma) = \gamma\mu'(\gamma)/\mu(\gamma)$ denotes the elasticity of $\mu(\gamma)$ and $\eta'(\gamma) \leq 0$.

The second class of functions $\mu(\gamma)$ that I consider includes for example the Leontief specification, $\mu(\gamma) = \min\{\gamma, 1\}$, and the properly modified Cobb-Douglas case, $\mu(\gamma) = \min\{A\gamma^{\alpha}, \gamma, 1\}$ with $\alpha \in (0, 1]$ and $A > 0$. Functions that belong to this class have to satisfy assumptions A1 and A3, where the latter is defined below.

Assumption A3. Assume there exist two cutoffs $\underline{\gamma} \leq \bar{\gamma} < \infty$ such that

$$\begin{aligned} \mu(\gamma) &= \gamma \text{ for } \gamma \leq \bar{\gamma} \\ \mu(\gamma) &= 1 \text{ for } \gamma \geq \underline{\gamma} \end{aligned}$$

with $\lim_{\gamma \searrow \underline{\gamma}} \mu'(\gamma) < 1$ and $\lim_{\gamma \nearrow \bar{\gamma}} \mu'(\gamma) > 0$. Furthermore, assume $\mu(\gamma)$ is strictly concave and exhibits non increasing elasticity for any $\gamma \in (\underline{\gamma}, \bar{\gamma})$.

The following Proposition shows that money burning can Pareto improve the competitive search equilibrium for functions $\mu(\gamma)$ that belong to one of these families of functions, that is, that satisfy A1 and either A2 or A3.

Proposition 6 *Consider any $\mu(.)$ satisfying Assumption A1 and either A2 or A3. For given $F(.)$ and b, there exists an open set of the parameter space (k, y) such that $g < 0$ at the competitive search equilibrium.*

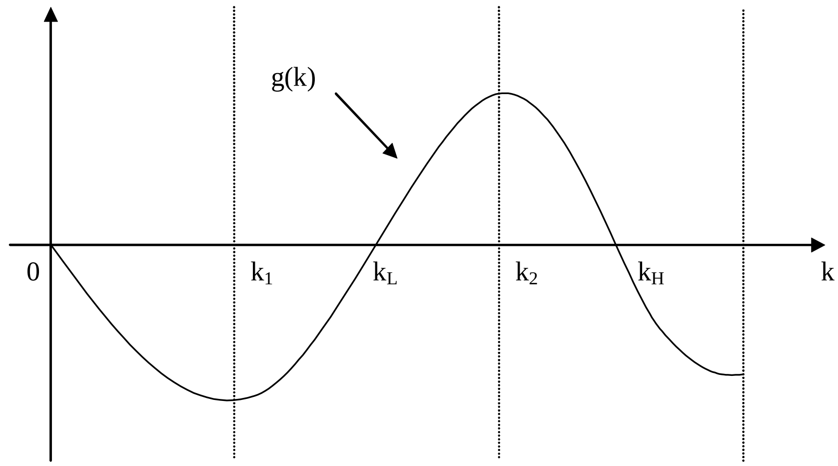
Proof. See the Appendix. ∎

The proof highlights the two forces potentially driving money burning. On one hand, inefficiency can arise when the shadow cost of revealing information is high enough, that is, for $\lambda \to 1$, as I show as being a possibility for the first class of functions and for a subfamily of the second class.[23] On the other hand, inefficiency can arise when $\mu(\gamma) F(\hat{\theta}(\lambda)) \to 1$ and the mass of unemployed workers enjoying b at the end of the period is infinitesimal. This can happen if y is big enough, for the subfamily of functions of the second class for which $\underline{\gamma} < \bar{\gamma}$ and $\mu'(\gamma) < \mu(\gamma)/\gamma$ for any $\gamma \in (\underline{\gamma}, \bar{\gamma})$.

In order to give a flavor of the impact of k on money burning, in the next Proposition, I analyze the *modified* Cobb Douglas case[24] and characterize how g depends on k. Intuition suggests that k has two first-order opposite effects on inefficiency: it reduces the number of vacancies posted and it increases the shadow cost of information revelation, that is, λ. On the top of these, there is an indirect effect of k on λ. This effect is shut down in the Cobb Douglas case, making it analytically tractable. I show that g behaves as in the picture below, highlighting the opposite effects that k can have on money burning. For y big enough, g can be negative both when k is big enough to make the cost of revealing information extremely high, that is, $\lambda \to 1$, and when k is small enough to make the market very tight so that the probability of staying unemployed is extremely small, that is, $\mu(\gamma) F(\hat{\theta}(\lambda)) \to 1$.

[23] This subfamily of the class of functions satisfying assumptions A1 and A3 reduce to the Leontief case $\mu(\gamma) = \min\{\gamma, 1\}$.

[24] This is a special case of the functions satisfying Assumption A2, so that the previous Proposition has already shown that there exists a parameter subspace for which $g < 0$.



Proposition 7 *Assume* $\mu(\gamma) = \min\{A\gamma^{\alpha}, \gamma, 1\}$ *with* $A \in (0,1]$ *and* $\alpha \in [0,1]$.[25] *Furthermore, assume* $y > b+\bar{\theta}+\tilde{\lambda}F(\bar{\theta})/f(\bar{\theta})$, *where* $\tilde{\lambda} \equiv \alpha D/[1-\alpha(1-D)]$ *with* $D \equiv f(\bar{\theta})\int_{\underline{\theta}}^{\bar{\theta}} F(\theta)\, d\theta$. *Then, either* g *is always negative, or there exist* k_L *and* k_H *such that* $0 < k_L \leq k_H < \bar{k}$ *and at the competitive search equilibrium* $g < 0$ *for* $k \in [0, k_L]$ *and* $k \in [k_H, \bar{k}]$, *where* $\bar{k} \equiv \int_{\underline{\theta}}^{\hat{\theta}(1)} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)}\right] dF(\theta)$ *with* $\hat{\theta}(1) = b + \hat{\theta}(1) + F(\hat{\theta}(1))/f(\hat{\theta}(1))$.

Proof. See the Appendix. ■

3.2 Transferability can Restore Full Information

Now I explore a second alternative environment in which the worker's outside option b is freely transferable. In this case, I interpret b as home production. If b is high enough, the social planner can achieve the full information allocation, by transferring utility between unemployed and employed workers. In particular, given risk neutrality, the social planner could tax, at no social cost, the unemployed workers in order to finance the informational rents of matched workers. Similarly, the full information allocation could be achieved if the planner had access to enough external resources.

Allowing for the possibility of transferring resources is basically equivalent to reducing the participation constraint to the constraint of non-negative consumption. From problem P3, it appears that, if b is big enough, the Social planner can make a negative net transfer to the unemployed, $C_u - b$, so as to achieve the full information efficient allocation, which is characterized by the cut-off value $\hat{\theta}^{FI}$ such that

$$\hat{\theta}^{FI} = y - b \qquad (10)$$

[25] Note that the Leontief case is a particular case of this modified Cobb Douglas with $\alpha = 1$.

and γ^{FI} implicitly defined by

$$\mu'\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\hat{\theta}^{FI}}[y-\theta-b]\,dF(\theta)=k. \tag{11}$$

What allows the planner to achieve the first best is the reduction of the relative value of being unemployed with respect to the one of being employed, that is, the sum of the outside option and the opportunity cost of being employed for the marginal hired worker. In particular, next Proposition shows that subsidizing job creation can restore the full information outcome.

Proposition 8 *Suppose that b is transferable and the following inequality holds*

$$b\geq k\gamma^{FI}.$$

Then the full information allocation can be decentralized by subsidizing job creation with a lump-sum tax on workers, both employed and unemployed.

Proof. Assume that the social planner gives a subsidy τ to the firms, by taxing lump-sum employed and unemployed workers. Budget balance imposes that the subsidy τ is covered by a tax equal to $\tau\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}}e(\theta)\,dF(\theta)$. Then, the social planning problem maximizes

$$\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}}e(\theta)(y-\theta-b)\,dF(\theta)+b-\gamma k$$

subject to

$$\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}}e(\theta)\left(y-\theta-b+\tau-\frac{F(\theta)}{f(\theta)}\right)dF(\theta)\geq\gamma k.$$

I guess that $\hat{\theta}^{FI}$, γ^{FI}, τ and $\lambda=0$ are a solution and then I verify it. The social planner can choose the minimal transfer τ which makes the constraint satisfied with equality at the full information allocation, that is, the τ such that

$$\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\hat{\theta}^{FI}}\left(y-\theta-b+\tau-\frac{F(\theta)}{f(\theta)}\right)dF(\theta)=\gamma^{FI}k$$

yielding $\lambda=0$. Once $\lambda=0$, the first order conditions with respect to $e(.)$ and γ are equivalent to equations (10) and (11), yielding $\hat{\theta}=\hat{\theta}^{FI}$ and $\gamma=\gamma^{FI}$. Finally, notice that such a τ is feasible only when $b\geq\tau\mu(\gamma)F(\hat{\theta})$, that is, when b is high enough to cover the informational rents. In fact the minimal b sufficient to restore the full information allocation is such that

$$\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\hat{\theta}^{FI}}\left(y-\theta-\frac{F(\theta)}{f(\theta)}\right)dF(\theta)=\gamma^{FI}k$$

which, using the first order condition with respect to $e(\theta)$ and integration by parts, yields

$$b \geq k\gamma^{FI}$$

completing the proof. ■

The proof of the previous Proposition suggests that the full information allocation can be restored also if there are enough resources to finance both the optimal vacancy creation and the informational rents necessary to sustain the optimal job creation. The next remark follows naturally.

Remark 2 *If the social planner has access to enough external resources, then he achieves the full information allocation.*

These results beg the question wether the competitive search equilibrium can achieve the full information allocation when b is transferable. Indeed, in the next proposition I show that this is the case when b is high enough.

Proposition 9 *Suppose that b is transferable and the following inequality holds*

$$b \geq \frac{k\gamma^{FI}}{\mu(\gamma^{FI})},$$

then the competitive search equilibrium achieves the full information allocation.

Proof. The competitive search equilibrium can be now defined exactly as in section 2, except for the participation constraints IR which now become

$$v(\theta, \theta) \geq 0 \text{ for all } \theta \in \Theta.$$

Hence, the constraint in problem P2 can be replaced by

$$\mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) + \mu(\gamma) b \geq \gamma k.$$

Substituting the full information values for γ and $e(\theta)$ and integrating by parts as in the proof of Lemma 2, it follows that

$$\mu(\gamma^{FI}) b \geq k\gamma^{FI},$$

completing the proof. ■

Notice that

$$\frac{k\gamma^{FI}}{\mu(\gamma^{FI})} \geq k\gamma^{FI}.$$

This implies that the social planner can restore the full information allocation for a larger set of parameters than competitive search can. The difference comes from the fact that firms cannot extract resources from workers who are unmatched, while the social planner can impose an *ex-ante* entry cost for the search market.[26]

When b is transferable, as in the benchmark case, the optimal wage schedule takes the form

$$\omega(\theta) = \begin{cases} \omega(\bar{\theta}) + \hat{\theta} + b & \text{if } \theta \leq \hat{\theta} \\ \omega(\bar{\theta}) & \text{if } \theta > \hat{\theta} \end{cases}.$$

However, now firms can set a negative value for $\omega(\bar{\theta})$ as long as

$$\omega(\bar{\theta}) \geq -b. \tag{12}$$

This contract has a natural interpretation as bond posting. The firms ask workers to sign a contingent promise, after the match and before they observe the realization of the shock. Matched workers sign a promise that they will pay an application fee of value $-\omega(\bar{\theta})$. On top of that, if they are hired, they will receive a wage of value $\hat{\theta} + b$. The constraint (12) means that workers can credibly promise to pay *ex-post* a value not greater than the value of the home production they obtain when unemployed.

4 Dynamic economy.

This section introduces the dynamic environment of this economy leading to the main result of the paper: the dynamic competitive search equilibrium can be constrained inefficient. The crucial difference between the static and the dynamic environment is that the worker's outside option is exogenously given in the former, while in the latter, it is endogenously determined as the equilibrium continuation utility of unemployed workers. Inefficiency arises because the worker's outside option affects the *ex-ante* cost of information revelation, generating a novel externality. The social planner can improve upon the decentralized economy by internalizing this *informational externality.*

Environment. Consider an economy with infinite horizon and discrete time. Both workers and employers have linear preferences and discount factor β. The search and production technologies are natural generalizations of the static setting. At the beginning of each period t employers can be either productive or not. Workers can be either employed or unemployed. Non-productive employers can open a vacancy at a cost k which entitles them to post an employment contract

[26] Though, allowing for a broader interpretation of competitive search, I could think of market makers who impose an application fee to all the workers who search for a match. This delivers a problem that is isomorphic to the one of the social planner I have described above. In this case, the competitive search equilibrium will be able to restore the full information allocation exactly for the same set of parameters, that is for $b \geq k\gamma^{FI}$.

$\mathcal{C}_t \in \mathbb{C}$ where $\mathbb{C}$ is the set of *ad interim* incentive compatible and individually rational mechanisms. As I describe below, a contract $\mathcal{C}_t : \Theta \mapsto [0,1] \times \mathbb{R}_+$ specifies the hiring probability and the wage for each matched worker at time t, who reports type θ. Therefore at each time t, a non-productive firm chooses a pair $(\sigma_t, \mathcal{C}_t) \in \{0,1\} \times \mathbb{C}$ where σ_t denotes the decision of posting a vacancy. Next, each unemployed worker observes all the contracts posted and decides where to apply. He chooses a contract $\mathcal{C}_t \in \mathbb{C}_t^P \subset \mathbb{C}$, where $\mathbb{C}_t^P$ denotes the set of contracts posted by active firms at time t. As in the static environment, each contract $\mathcal{C}_t$, is associated to a specific γ_t so that employers and workers know that their matching probabilities will depend on the contract that they respectively post and seek for. After workers start to search for a specific contract, matching takes place and, for each match, the draw θ is realized and is private information of the worker. The behavior of a worker who is matched at time t is described by a map $(a_t, s_t) : \Theta \mapsto \Theta \times \{0,1\}$ that for each type θ specifies a report $\hat{\theta}_t = s(\theta)$ and a participation decision $a_t(\theta)$. After he sees his type, the worker can either implement the contract, that is choose $a_t(\theta) = 1$, or walk away, that is choose $a_t(\theta) = 0$. If he walks away, he stays in autarky for one period, gets a non-transferable utility from leisure b, enters an anonymous pool of unemployed workers and look for another match next period. If the worker is effectively hired, the parties are productive until separation, which happens according to a Poisson process with parameter s. The worker's disutility θ is constant for the duration of the match.

In a decentralized economy, as in the static setting, the consumption of employed workers is given by the contracted wage, which is fixed at the time of the match. Moreover, the consumption of unemployed workers, that is, both unmatched workers and workers who matched but have not been hired, is equal to the value of leisure b.

Employment Contracts and Bellman Values. Invoking the Revelation Principle, without loss of generality, I can again restrict attention to incentive-compatible and individually rational direct revelation mechanisms, corresponding to a mapping $\mathcal{C}_t : \Theta \mapsto [0,1] \times \mathbb{R}_+$, specifying for each matched worker at time t who reports type θ, the hiring probability $e_t(\theta) \in [0,1]$ and the wage $\omega_t(\theta) \in \mathbb{R}_+$ which is paid at the beginning of the productive relationship. Notice that I can restrict attention to the set $\mathbb{C}$ of *ad interim* incentive compatible and individually rational mechanisms described above, due to the unemployed anonymity assumption. All the unemployed workers searching for a job cannot be distinguished, so that contracts cannot be conditioned on the past employment history.

Linear preferences, together with the fact that types are fixed over time within a match, imply that the wage profile over the life of the relationship is irrelevant for the analysis. Therefore, I can assume, without loss of generality, that the wage is fully paid at the moment of the match. Analogously I assume that the whole disutility generated over the life of the match, $\alpha\theta$, takes place at the beginning of the relationship. It follows that the continuation value of being employed *net of wages and disutility* at time t, V_t, which from now on I will refer to simply as the continuation utility of employed workers, represents just the discounted expected value of being separated and becoming unemployed, that is,

$$V_t = \beta s U_{t+1} + \beta (1 - s) V_{t+1}. \tag{13}$$

Moreover the continuation value of an unemployed worker at time t is given by

$$U_t = b + \beta\mu\left(\gamma_{t+1}\right)\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega_{t+1}(\theta) - e_{t+1}(\theta)\left(\alpha\theta - V_{t+1} + U_{t+1}\right)\right] dF(\theta) + \beta U_{t+1} \tag{14}$$

where $\alpha \equiv [1 - \beta (1 - s)]^{-1}$.

The *ad interim* utility of a worker of type θ, who reports to be of type $\hat{\theta}$ at time t, is given by

$$v_t(\theta, \hat{\theta}) = [\omega_t(\hat{\theta}) - e_t(\hat{\theta})(\alpha\theta - V_t)] + [1 - e_t(\hat{\theta})]U_t \tag{15}$$

and the expected revenues of the firm, after a match, is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[e_t(\theta)\alpha y - \omega_t(\theta)\right] dF(\theta).$$

The large number of potential firms ensures free entry and implies that the value of an open vacancy will be zero at each time, that is,

$$\beta\frac{\mu(\gamma_t)}{\gamma_t}\int_{\underline{\theta}}^{\overline{\theta}} \left[e_t(\theta)\alpha y - \omega_t(\theta)\right] dF(\theta) = k. \tag{16}$$

A natural generalization of the static analysis, gives that a contract $\mathcal{C}_t$ is *incentive-compatible* and *individually rational* whenever $e_t(.)$ is non-increasing and the following conditions hold:

$$v_t(\theta, \theta) = v_t\left(\overline{\theta}, \overline{\theta}\right) + \alpha\int_{\theta}^{\overline{\theta}} e_t(y)\, dy \text{ for all } \theta \in \Theta \tag{IC'}$$

and

$$v_t\left(\overline{\theta}, \overline{\theta}\right) \geq U_{t-1}. \tag{IR'}$$

Following the static analysis, the informational rents for a worker of type θ who meets a firm at time t are $v_t(\theta, \theta) - v_t\left(\overline{\theta}, \overline{\theta}\right)$, as define by equation IC'.

4.1 Dynamic Competitive Search Equilibrium

This section defines the dynamic version of the Competitive Search Equilibrium. Generalizing the static definition, a dynamic Competitive Search Equilibrium, in sequential terms, is a sequence of sets of incentive-compatible and individually rational contracts $\{\mathbb{C}_t^*\}$ and a sequence of tightness functions $\{\Gamma_t^*\}$, where $\Gamma_t^* : \mathbb{C}_t^* \longmapsto \mathbb{R}_+ \cup \infty$, such that, at any t employers maximize profits and workers apply optimally for jobs, taking as given the future sequence of sets of contracts, $\{\mathbb{C}_{t+1}\}, \{\mathbb{C}_{t+2}\}, ...,$ and tightness functions, $\{\Gamma_t^*\}, \{\Gamma_{t+1}^*\}, ...$

In order to simplify the analytical treatment, we can introduce an equivalent definition of the dynamic competitive search equilibrium in recursive terms.

The first thing to notice is that the pair of continuation utilities for unemployed and employed workers, U and V, are a sufficient statistic for future sets of $\mathbb{C}'s$ and $\Gamma's$. This allows me to describe the dynamic competitive search equilibrium in a recursive way, as shown by the following Definition.

Definition 4 *A dynamic Competitive Search Equilibrium (CSE) is a sequence of sets of incentive-compatible and individually rational contracts* $\{\mathbb{C}_t^*\}$*, a sequence of functions* $\{\Gamma_t^*\}$*, where* $\Gamma_t^* : \mathbb{C}_t^* \longmapsto \mathbb{R}_+ \cup \infty$*, and a sequence of pairs of continuation utility levels* $\{U_t^*, V_t^*\}$*, where* $(U_t^*, V_t^*) \in \mathbb{R}_+^2$ *for any* t*, satisfying*

(i) *employers'* ***profit maximization*** *and* ***free-entry*** *at each time* t*:* $\forall \mathcal{C}_t \equiv [e_t(\theta), \omega_t(\theta)]_{\theta \in \Theta}$*,*

$$\frac{\mu(\Gamma_t^*(\mathcal{C}_t))}{\Gamma_t^*(\mathcal{C}_t)} \beta \int_{\underline{\theta}}^{\overline{\theta}} [e_t(\theta)\alpha y - \omega_t(\theta)] \, dF(\theta) - k \leq 0$$

subject to incentive compatibility IC and individual rationality IR, with equality if $\mathcal{C}_t \in \{\mathbb{C}_t^*\}$*;*

(ii) *workers'* ***optimal job application*** *at each time* t*:* $\forall \mathcal{C}_t \equiv [e_t(\theta), \omega_t(\theta)]_{\theta \in \Theta}$*, for given* V_t *and* U_t

$$U_{t-1}^* \geq b + \beta\mu(\Gamma_t^*(\mathcal{C}_t)) \int_{\underline{\theta}}^{\overline{\theta}} [\omega_t(\theta) - e_t(\theta)(\alpha\theta - V_t + U_t)] \, dF(\theta) + \beta U_t$$

and $\Gamma_t^*(\mathcal{C}_t) \geq 0$*, with complementarity slackness, where*

$$U_{t-1}^* = \max_{\mathcal{C}_t'} b + \beta\mu(\Gamma_t^*(\mathcal{C}_t')) \int_{\underline{\theta}}^{\overline{\theta}} [\omega_t'(\theta) - e_t'(\theta)(\alpha\theta - V_t + U_t)] \, dF(\theta) + \beta U_t$$

or $U_{t-1}^* = b + \beta U_t$ *if* $\{\mathbb{C}_t^*\}$ *is empty, and*

$$V_{t-1}^* = \beta s U_t + \beta(1 - s) V_t.$$

The definition of the dynamic equilibrium is a natural generalization of the static version. At each point in time employers maximize profits and workers apply optimally for jobs, both taking as given the future values of being employed and unemployed and aware that a market tightness is associated with each contract, even if not offered in equilibrium, according to the function $\Gamma_t^*(\mathcal{C}_t)$. Moreover, profits are driven to zero at each point in time by free entry.

It follows that the unemployment rate of workers applying to firms posting a contract of type $\mathcal{C}_t$ at time t is given by

$$u_{t+1}(\mathcal{C}_{t+1}) = u_t(\mathcal{C}_t)\left[1-\mu\left(\Gamma_t^*(\mathcal{C}_t)\right)\int_{\underline{\theta}}^{\overline{\theta}} e_t(\theta)\, dF(\theta)\right] + (1-u_t(\mathcal{C}_t))\, s. \tag{17}$$

Generalizing the static result, the next Proposition states a dynamic characterization of a symmetric competitive search equilibrium in recursive terms.

Proposition 10 *If $\{\mathbb{C}_t, \Gamma_t, U_t, V_t\}_{t=0}^{\infty}$ is a Competitive Search Equilibrium, then any pair $(\mathcal{C}_t^*, \gamma_t^*)$ with $\mathcal{C}_t^* \in \mathbb{C}_t$ and $\gamma_t^* = \Gamma_t^*(\mathcal{C}_t^*)$ satisfy the following*

(i) *for given pair U_t and V_t, for any time t, $\mathcal{C}_t = [e_t(\theta), \omega_t(\theta)]_{\theta\in\Theta}$ and γ_t solve*

$$\max_{e_t(.),\omega(.),\gamma_t} b + \beta\mu(\gamma_t)\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega_t(\theta) - e_t(\theta)(\alpha\theta - V_t + U_t)\right] dF(\theta) + \beta U_t \tag{P4}$$

subject to $e_t(\theta) \in [0,1]$, non-negative consumption, the incentive compatibility constraint IC' together with the monotonicity assumption on $e_t(\theta)$, the individual rationality constraint IR' and the free-entry condition (16);

(ii) *for given $\{\mathcal{C}_t, \gamma_t\}_{t=0}^{\infty}$, then $\{U_t, V_t\}_{t=0}^{\infty}$ evolve according to (13) and (14).*

Conversely, if a sequence $\{\mathcal{C}_t^*, \gamma_t^*\}_{t=0}^{\infty}$ solves the program P4, then there exists an equilibrium $\{\mathbb{C}_t^*, \Gamma_t^*, U_t^*, V_t^*\}_{t=0}^{\infty}$ such that $\mathcal{C}_t^* \in \mathbb{C}_t^*$ and $\gamma_t^* = \Gamma_t^*(\mathcal{C}_t^*)$.

In the rest of the analysis I adopt a recursive notation, dropping the t whenever this causes no confusion, and denoting a variable at time $t-1$ with a $-$ sign.

Proposition 10 shows that for given U and V, a (symmetric) equilibrium incentive-compatible and individually-rational contract $\mathcal{C}$ and tightness γ must solve Problem P4. The next Proposition shows that the equilibrium can be equivalently described by a hiring function $e(\theta)$ and a tightness γ that solve a simplified program P5. Given $e(\theta)$ and γ, an associated wage function $\omega(\theta)$ can be constructed so that the constraints IC' and IR' are satisfied.

Proposition 11 *For given U and V, any function $[e(\theta)]_{\theta\in\Theta}$ and γ which solve Problem P4, solve also*

$$\begin{aligned} U^-(U,V) &= \max_{e(.),\gamma} \beta\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}} e(\theta)\left[\alpha(y-\theta) + V - U\right] dF(\theta) \\ &\quad + b - \gamma k + \beta U \end{aligned} \tag{P5}$$

s.t.

$$\beta\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}} e(\theta)\left[\alpha\left(y - \theta - \frac{F(\theta)}{f(\theta)}\right) + V - U\right] dF(\theta) \geq \gamma k. \tag{18}$$

Furthermore, for any function $[e(\theta)]_{\theta\in\Theta}$ *and* γ *which solve problem P5, there exists a function* $[\omega(\theta)]_{\theta\in\Theta}$ *such that the contract* $\mathcal{C} = [e(\theta), \omega(\theta)]_{\theta\in\Theta}$ *and* γ *solve problem P4.*

Proof. The proof is similar to the one of Proposition 2 and is therefore omitted. ■

Equilibrium Characterization. The characterization of the equilibrium allocation for a given continuation utility gap between unemployed and employed workers, $U - V$, is very similar to the static one. In fact, in the dynamic setting, $U - V$ represents the effective outside option of the workers. A generalization of Proposition 3 proves that Problem P5 has a unique solution and that the first order conditions are necessary and sufficient to characterize it. I proceed by studying the relaxed problem without the monotonicity assumption on $e(\theta)$. Then, using pointwise maximization with respect to $e(\theta)$ I show that the trading area can be fully described by a cut-off value $\hat{\theta}$ such that

$$e(\theta) = \begin{cases} 1 \text{ if } \theta \leq \hat{\theta} \\ 0 \text{ if } \theta > \hat{\theta} \end{cases},$$

implying that the optimal $e(\theta)$ is effectively non-increasing. When the constraint (18) is binding[27] and $\mu(\gamma)$ is everywhere differentiable the equilibrium can be characterized, for given $U - V$, by an array $\hat{\theta}$, γ and λ satisfying the first order conditions, respectively, for $\hat{\theta}$ and γ

$$\alpha\left(y - \hat{\theta} - \lambda\frac{F(\hat{\theta})}{f(\hat{\theta})}\right) - (U - V) = 0, \tag{19}$$

$$\beta\mu'(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}\left[\alpha\left(y - \theta - \lambda\frac{F(\theta)}{f(\theta)}\right) - (U - V)\right]dF(\theta) = k \tag{20}$$

and the condition

$$\beta\mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}\left[\alpha\left(y - \theta - \frac{F(\theta)}{f(\theta)}\right) - (U - V)\right]dF(\theta) = \gamma k. \tag{21}$$

The variable λ represents a normalized version of the shadow value of the *informational rents.*[28] Notice that when $\lambda = 0$, the constraint (18) is slack and γ is simply determined by (20). Then, the full information allocation is achieved. Clearly, asymmetric information reduces job creation, as the surplus of the economy must cover not only the cost of vacancy creation but also the rents needed to extract information from the workers. As intuition suggests, $\hat{\theta}$ decreases with λ. When $\mu(\gamma)$ is not differentiable at some points, equation (20) will be replaced by inequalities involving the left and right derivatives of $\mu(\gamma)$ at the points of non differentiability.

[27] When the constraint is not binding, the informational problem is irrelevant and the competitive search equilibrium is constrained efficient as in the standard result.

[28] Similarly to the static setting, $\lambda \equiv \hat{\lambda}/(1+\hat{\lambda})$, where $\hat{\lambda}$ is the Lagrangian multiplier attached to the constraint (18).

Similarly to the static setting, whenever the cost of posting a vacancy k is positive, then $\lambda > 0$ and the equilibrium is away from the full information allocation.

Lemma 3 *If $k > 0$, then the solution to problem P5 requires $\lambda > 0$, where $\lambda = \hat{\lambda}/(1-\hat{\lambda})$ and $\hat{\lambda}$ is the Lagrangian multiplier attached to constraint (18).*

Proof. The proof is analogous to the proof of Lemma 2 and therefore omitted. ■

Proposition 11 shows that at each point in time the competitive search equilibrium γ and $e(\theta)$ are functions only of the expected values of V and U, which evolve according to the law of motions (13) and (14). Hence, the equilibrium U^{CE} and V^{CE} corresponds to a fixed point, given by

$$U^{CE} = \frac{\alpha\beta\mu\left(\gamma^{CE}\right)\int_{\underline{\theta}}^{\hat{\theta}^{CE}}\left[y-\theta\right]dF\left(\theta\right)+b-\gamma^{CE}k}{\left(1-\beta\right)\left[1+\alpha\beta\mu\left(\gamma^{CE}\right)F(\hat{\theta}^{CE})\right]} \tag{22}$$

$$V^{CE} = \frac{\beta s}{1-\beta\left(1-s\right)}U^{CE} \tag{23}$$

where, the equilibrium $\hat{\theta}^{CE}$ and γ^{CE} solve Problem P5, for $U = U^{CE}$ and $V = V^{CE}$, that is, satisfy equations (19), (20) and (21). The unemployment rate u, which is the only state variable of the economy, does not affect this problem. This implies that γ and $e(\theta)$ (and $\omega(\theta)$ as well) together with V and U, achieve the steady state values directly in the first period and stay constant over time.[29] The transitional dynamics of the competitive search equilibrium will then be characterized uniquely by the transition of the unemployment rate.

Steady State. Denote the steady state competitive search equilibrium by

$$SS = \left\{\left[e^{CE}\left(\theta\right)\right]_{\theta\in\Theta}, \gamma^{CE}, U^{CE}, V^{CE}, u^{SS}\right\}.$$

In steady state not only $e(\theta)$, γ, V and U are constant, but also the unemployment rate u is. The steady state equilibrium is given by equations (19), (21) and (20) with $U = U^{CE}$ and $V = V^{CE}$, (22), (23) and

$$u^{SS} = s\left[s+\mu\left(\gamma^{CE}\right)F(\hat{\theta}^{CE})\right]^{-1}. \tag{24}$$

In the analysis of the static economy, I have shown how decreasing the worker's outside option b can generate a Pareto improvement. In the dynamic environment, the effective worker's outside option corresponds to $U^{CE} - V^{CE}$ and is endogenously determined in equilibrium. In a decentralized economy agents' decisions determine the continuation utility of unemployed workers, without internalizing the cost in terms of efficient creation imposed by the informational rents.

[29] Note that in equilibrium the continuation utility of the employed turns out to be smaller than the continuation utility of the unemployed. This is natural if one thinks that I define the continuation value of the employed net from wage and disutility.

5 Dynamic Efficiency

In this section I explore the efficiency properties of the dynamic competitive search equilibrium when matching frictions interact with informational asymmetry.

The static analysis shows that in the benchmark environment, where b cannot be transferred or destroyed, the competitive search equilibrium is constrained efficient. On the contrary, in the analogous dynamic environment, the social planner can improve the decentralized equilibrium allocation, because of an extra instrument to provide incentives. The planner can reward or punish workers reporting a low type, not only through the instantaneous consumption level, but also through continuation utilities. This allows the planner to internalize the externality coming from informational imperfections.

First, I analyze a simple example of inefficiency, driven by the same mechanism described in section 3 as *money burning*: changing the worker's outside option can lead to a Pareto Improvement. In the benchmark static environment b is exogenously given and cannot be reduced. However, in a dynamic setting, the social planner can improve the *ex-ante* welfare by affecting the workers' expected continuation utility, through future allocations.

Then, I characterize the social planning problem and show that the competitive equilibrium is constrained inefficient whenever the unemployment rate is away from the steady state level. Moreover, I show that the steady state competitive search equilibrium, although it satisfies the necessary conditions for constrained efficiency, is not socially optimal according to the utilitarian welfare criterion. In particular, the informational asymmetry makes employed people, who enjoy the informational rents, better off than an utilitarian planner would prescribe.

Finally, I show that in the alternative environment where b is transferable and high enough the social planner can achieve the full information allocation, as it was the case in the static economy.

Social Planner. Analogously to the static setting, the social planner controls the matching process by deciding how many vacancies to open at the beginning of each period. He does not observe the types of the matched workers and has to induce them to truthfully reveal them. Moreover, there is lack of commitment from the side of the workers, who can always decide not to produce and to consume their private non-transferable utility b. I impose on the planner the same anonymity restriction that I impose on the decentralized economy: the pool of unemployed workers is anonymous. Once workers decide to consume b at time t, they can always join back the pool of unemployed at time $t+1$. Given these constraints, together with the resource constraint of the economy, the social planner decides how to allocate intertemporally non-negative consumption among employed and unemployed workers.

An allocation is a sequence of functions $[e_t(\theta)]_{\theta\in\Theta}$ representing the hiring probability for a worker who meets an employer at time t and reports type θ and a sequence of functions $[c_t(\theta, s)]_{\theta\in\Theta s\geq t}$ denoting the consumption at time s of the worker hired at time t reporting type θ, a sequence of consumption values for unmatched workers C_t^U, a sequence of consumption values C_t^V for employed workers who matched at time $\tau \leq t$, and a sequence of tightness values γ_t.[30]

Given that agents have linear utility, the path of consumption does not matter and I can restrict attention to the case in which $c_t(\theta, s) = c_t(\theta)$. Moreover, given that types are fixed over time within a match and there is no commitment problem after the match is implemented, the consumption profile over time is irrelevant for the analysis as in the competitive equilibrium. Thus, without loss of generality, I can characterize the efficient allocation as if the whole consumption $\alpha c_t(\theta)$ and disutility $\alpha\theta$ took place at the beginning of the relationship. This implies that the *net continuation value* of being employed V, which I will refer to as the employed continuation utility, represents just the value of a current transfer plus the discounted expected value of being separated and becoming unemployed in the future, that is,

$$V_t = C_t^V + \beta s U_{t+1} + \beta(1-s) V_{t+1}.$$

The value of being unemployed at time t is instead

$$U_t = C_t^u + \beta\mu(\gamma_{t+1}) \int_{\underline{\theta}}^{\overline{\theta}} e_{t+1}(\theta)\left[\alpha\left[c_{t+1}(\theta) - \theta\right] + V_{t+1} - U_{t+1}\right] dF(\theta) + \beta U_{t+1}.$$

The *ad interim* utility of a matched worker of type θ, who reports to be of type $\tilde{\theta}$ at time t is given by

$$v_t^{SP}(\theta, \tilde{\theta}) = e_t(\tilde{\theta})\left[\alpha[c_t(\tilde{\theta}) - \theta] + V_t - U_t\right] + U_t \quad \text{for all } \tilde{\theta}, \theta \in \Theta.$$

A straightforward generalization of the static analysis gives that an allocation is *incentive-compatible* when $e(.)$ is non-increasing and

$$v_t^{SP}(\theta, \theta) = v_t^{SP}(\overline{\theta}, \overline{\theta}) + \alpha \int_{\theta}^{\overline{\theta}} e_t(y)\, dy \text{ for all } \theta \in \Theta. \tag{IC'}$$

Unemployed workers can choose at any point in time to stay in autarky, enjoy an instantaneous utility from leisure of value b and go back to the anonymous pool of unemployed at the beginning of the following period. Hence, an allocation satisfies the *participation* constraint, whenever

(i) $C_t^U \geq b$

(ii) $v_t^{SP}(\theta, \theta) \geq b - C_t^U + U_t$ for any θ.

Notice that when an allocation is incentive compatible condition *(ii)* reduces to

$$v^{SP}(\overline{\theta}, \overline{\theta}) \geq b - C_t^U + U_t.$$

[30] Without loss of generality, because of linear preferences, I assume that matched workers who are not hired at time t get the same consumption , C_t^U, of the unmatched ones.

Hence, next lemma follows.

Lemma 4 *An incentive compatible allocation that satisfies the participation constraint must satisfy* $e(.)$ *non-increasing,* $C_t^U \geq b$ *and*

$$\int_{\underline{\theta}}^{\overline{\theta}} e_t(\theta) \left[\alpha \left(c_t(\theta) - \theta - \frac{F(\theta)}{f(\theta)} \right) + V_t - U_t \right] dF(\theta) + C_t^U - b \geq 0. \tag{25}$$

The *intertemporal resource constraint* ensures that aggregate consumption is covered by aggregate output, that is,

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ u_t \left[\beta\mu(\gamma_{t+1}) \int_{\underline{\theta}}^{\overline{\theta}} e_{t+1}(\theta)\, c_{t+1}(\theta)\, dF(\theta) + C_t^U \right] + (1-u_t)\, C_t^V \right\} \\ \leq & \sum_{t=0}^{\infty} \beta^t u_t \left[\mu(\gamma_{t+1}) \int_{\underline{\theta}}^{\overline{\theta}} e_{t+1}(\theta)\, \alpha y dF(\theta) + b - \gamma_{t+1} k \right], \end{aligned}$$

where u_t follows the law of motion

$$u_{t+1} = u_t \left[1 - \mu(\gamma_t) \int_{\underline{\theta}}^{\overline{\theta}} e_t(\theta)\, dF(\theta) \right] + (1-u_t)\, s \tag{26}$$

Assume that the social planner can transfer resources intertemporally at the fixed interest rate $r = \beta^{-1} - 1$, by borrowing at the beginning of time t at price β and by paying back at the beginning of time $t+1$. I assume that the economy does not have external resources so that the intertemporal resource constraint has to hold. Define P_t as the net resources of the planner at time t, that is,

$$P_t = \sum_{j=t}^{\infty} \beta^t \left\{ u_t \left[\beta\mu(\gamma_{t+1}) \int_{\underline{\theta}}^{\overline{\theta}} e_{t+1}(\theta) \left[\alpha y - c_{t+1}(\theta) \right] dF(\theta) - C_t^U \right] - (1-u_t)\, C_t^V \right\}.$$

Then, the resource constraint can be written in recursive terms by using the state variable P_t as

$$\begin{aligned} P_t \leq\; & u_t \left[\beta\mu(\gamma_{t+1}) \int_{\underline{\theta}}^{\overline{\theta}} e_{t+1}(\theta)\, \alpha(y - c_{t+1}(\theta)) dF(\theta) + b - \gamma_{t+1} k - C_t^u \right] \\ & - (1-u_t)\, C_t^V + \beta P_{t+1} \end{aligned} \tag{RC}$$

for any t.

Definition 5 *An allocation is feasible iff it (i) is incentive-compatible, (ii) satisfies the participation constraint and (iii) satisfies the resource constraint for any* t*.*

5.1 An Example of a Feasible Pareto Improvement.

I now show, with a simple example, that in a dynamic economy the competitive search equilibrium can be constrained inefficient. In this example, the mechanism driving inefficiency is the natural generalization of the static *money burning* result described in section 3. When all the workers are *ex-ante* unemployed, the economy can achieve a Pareto Improvement by reducing the worker's effective outside option.

Let assume that the economy is at the competitive search equilibrium. From the previous section it means that $e_t(\theta)$, γ_t, U_t and V_t are equal to the steady state competitive equilibrium level, CE, for any $t = 0, 1, \ldots$.

Define the analogous of g for the dynamic economy:

$$g \equiv 1 - \mu(\gamma^{CE})F(\hat{\theta}^{CE}) - \lambda^{CE}. \tag{27}$$

The value g represents a monotonic transformation of the effect of the workers' outside option on the *ex-ante* value of being unemployed at the competitive search equilibrium. There is a direct positive effect coming from the probability of being unemployed at the end of the period, $1-\mu(\gamma^{CE})F(\hat{\theta}^{CE})$, and a negative effect coming from the tightness of the informational constraint, represented by λ^{CE}, since, as the outside option increases, it is more costly to reveal information.

Assume that all the workers start out as unemployed, so that the *ex-ante* welfare coincides with the *ex-ante* value of being unemployed, that is, $W = U_0$. Thus, when g is negative it would be Pareto improving to reduce the workers' outside option.

Proposition 12 *Whenever $g < 0$, the competitive search equilibrium allocation is Pareto inefficient.*

Proof. The proof proceeds by construction. Consider the feasible allocation where $\hat{V}_t^* = \hat{V}^{CE}$ for any $t = 1, 2, \ldots$, $\hat{\theta}_t^* = \hat{\theta}^{CE}$ and $U_t^* = U^{CE}$ for any $t = 2, 3, \ldots$; $\gamma_t^* = \gamma^{CE}$ for any $t = 3, 4, \ldots$ and $\gamma_2^* = \gamma^{CE} + \varepsilon$, U_1^* is given by

$$U_1^* = \beta\mu\left(\gamma_2^*\right)\int_{\underline{\theta}}^{\hat{\theta}^{CE}}\left[\alpha\left(y-\theta\right)+\hat{V}^{CE}-U^{CE}\right]dF\left(\theta\right)+b-\gamma_2^*k+\beta U^{CE},$$

$\hat{\theta}_1^*$ and γ_1^* solve problem P2 at time 0 and W^* is given by

$$W^* = \beta\mu\left(\gamma_1^*\right)\int_{\underline{\theta}}^{\hat{\theta}_1^*}\left[\alpha\left(y-\theta\right)+\hat{V}^{CE}-U_1^*\right]dF\left(\theta\right)+b-\gamma_1^*k+\beta U_1^*.$$

First, the Envelope Condition shows that welfare *ex-ante* can be improved by reducing U_1 whenever $g < 0$. It follows that if in period 2, I perturb the competitive equilibrium level of γ, choosing

$\gamma_2^* = \gamma + \varepsilon$, leaving everything else at the level of competitive equilibrium, then, by definition, U_1^* will be marginally lower than the competitive equilibrium level, thus increasing *ex-ante* welfare W^*, so that for any $\hat{\theta}_1$ and γ_1 the allocation proposed in the proposition is Pareto improving. Finally, $\hat{\theta}_1^*$ and γ_1^* solve problem P2 for given $\hat{V}^{CE}$ and U_1^*, so that the allocation is also feasible, completing the proof. ■

An equilibrium is constrained inefficient if there exists a feasible allocation that Pareto dominates it. It follows that a dynamic competitive search equilibrium is constrained inefficient whenever g is negative at the equilibrium. I naturally extend the static analysis and show that the dynamic competitive search equilibrium can be constrained inefficient for an interesting class of functions $\mu(.)$. In Section 2, I have shown that for $\mu(.)$ satisfying assumptions A1 and A2 or A3, for a given b there exists a set of the parameter space (k, y) such that the static version of g is negative. Now, I show that varying b for any pair (k, y), I can adjust the equilibrium value of unemployed workers, U^{CE}, so that there exists a set of the parameter space (b, k, y) such that the dynamic version of g is negative. The next proposition is the counterpart of Proposition 6 in the static analysis.

Proposition 13 *For any $\mu(.)$ satisfying Assumption A1 and either A2 or A3 and given $F(.)$, there exists an open set of the parameter space (b, k, y) such that the dynamic competitive search equilibrium is constrained inefficient.*

Proof. See Appendix. ■

5.2 The Social Planning Problem

I now characterize the constrained efficient allocation. The social planner, for a given initial rate of unemployment u_0, chooses a Pareto optimal pair U_0 and V_0. The problem can be written in a recursive form, by maximizing at a given t

$$C_t^u + \beta\mu\left(\gamma_{t+1}\right)\int_{\underline{\theta}}^{\hat{\theta}_{t+1}} [\alpha(c_{t+1}(\theta) - \theta) + V_{t+1} - U_{t+1}]dF(\theta) + \beta U_{t+1}$$

subject to feasibility as described in Definition 5, the law of motion of u_t given by (26) and the promise-keeping constraint

$$V_t = C_t^V + \beta s U_{t+1} + \beta(1-s)V_{t+1}.$$

In order to analyze the constrained efficient allocation, it is convenient to approach the social planner problem from a dual perspective. The social planner Bellman equation is a function of three state variables: the promised utility to employed workers, V, the promised utility to unemployed workers, U, and the unemployment rate, u. The planner maximizes the net resources of the economy, subject to two promise-keeping constraints for V and U, the law of motion of u and incentive compatibility and individual rationality, as summarized by Lemma 4. I study a relaxed version of the problem, where I do not impose the monotonicity of $e(.)$ and then I verify that the result of the optimization gives a monotone $e(.)$. Indeed, pointwise maximization, as in the competitive equilibrium analysis, implies that there exists a threshold $\hat{\theta}$ such that $e(\theta) = 1$ iff $\theta \leq \hat{\theta}$. Moreover, when $k > 0$ the constraint (25) is binding for any U and V,[31] so that I can solve for the optimal allocation substituting for $c(\theta)$ and recovering it from the binding constraint at the optimum.

The Bellman equation can be written as:

$$
\begin{aligned}
P\left(V^{-},U^{-},u^{-}\right) \; = \; & \max_{\substack{C^u,C^V \\ \hat{\theta},\gamma,u',V',U'}} \; u^{-}\beta\mu(\gamma)\int_{\underline{\theta}}^{\hat{\theta}}\left[\alpha\left(y-\theta-\frac{F(\theta)}{f(\theta)}\right)+V-U\right]dF(\theta) \\
& +u^{-}\left[(1-\beta\mu(\gamma))\left(b-C^U\right)-\gamma k\right]-\left(1-u^{-}\right)C^V+\beta P(V,U,u)
\end{aligned}
\tag{P6}
$$

s.t.

$$[\nu] \qquad V^{-} = C^V + \beta s U + \beta(1-s)V$$

$$[\eta] \qquad U^{-} = C^U + \beta\mu(\gamma)\alpha\int_{\underline{\theta}}^{\hat{\theta}}\frac{F(\theta)}{f(\theta)}dF(\theta) + \beta\mu(\gamma)\left(b-C^U\right)+\beta U$$

$$[\pi] \qquad u = u^{-}\left[1-\mu(\gamma)F(\hat{\theta})\right]+\left(1-u^{-}\right)s$$

$$[\chi] \qquad C^u \geq b$$

I assume that the social planner, as the market economy, does not have access to any external resources. Therefore, for a given u_0, U_0 and V_0 are on the Pareto frontier iff $P(V_0,U_0,u_0) = 0$.

Next, I show the main result of the paper: away from the steady state, a competitive equilibrium is always constrained inefficient. Recall that the competitive equilibrium allocation, $\hat{\theta}^{CE}$, γ^{CE}, V^{CE} and U^{CE}, achieves immediately the steady state level, while the unemployment rate evolves slowly to the steady state, starting at a given u_0.

Proposition 14 *If $u_0 \neq u^{SS}$, then the competitive search equilibrium allocation is constrained inefficient.*

Proof. See Appendix. ■

The proof proceeds by contradiction. I assume that the competitive allocation solves the social planning problem and then I show that the necessary first order conditions are violated. In particular, the optimality condition that is violated is the one determining the level of the promised utility for the unemployed workers. The proof highlights the mechanism driving the inefficiency. The direction of the inefficiency depends on the dynamics of the unemployment rate. Constrained efficiency requires to set the optimal promised utility to unemployed workers, U_{t+1}, so that the

[31] The proof of this statement is very similar to the proof of Lemma 2.

marginal benefit of increasing it in terms of social surplus at time t equates the marginal cost at time $t+1$. In particular, the planner needs to satisfy the following Euler equation

$$-P_{U_{t+1}} = \left\{\nu_t \left(1-u_t\right) s + \eta_t[1-\mu\left(\gamma_t\right) F(\hat{\theta}_t)]u_t\right\} - \left(1-\eta_t\right) u_t \mu\left(\gamma_t\right) F(\hat{\theta}_t) \qquad (28)$$

where η_t represents the shadow cost of increasing U_t, ν_t the shadow cost of increasing V_t and from the Envelope condition

$$P_{U_{t+1}} = -\eta_{t+1} u_t.$$

The right-hand side of equation (28) represents the net benefit at time t of a one unit increase of U_{t+1} for the social planner. The direct effect is to increase by η_t the utility of unemployed workers who will not be hired at the end of the period, $[1-\mu\left(\gamma_t\right) F(\hat{\theta}_t)]u_t$, and by ν_t the utility of employed workers who will be separated, $\left(1-u_t\right) s$. Moreover there is an indirect effect coming from the fact that an increase of U_{t+1} makes tighter the incentive constraint of the workers hired at time t, that is, $u_t \mu\left(\gamma_t\right) F(\hat{\theta}_t)$. The latter effect imposes a cost of $1-\eta_t$ per worker hired.

The left-hand side of the equation represents, instead, the cost of a one unit increase of U_{t+1} in terms of resources at time $t+1$. The cost of giving one additional unit of promised utility to each unemployed worker at time $t+1$ is smaller than one because increasing that utility allows the planner to increase the informational rents of workers who are hired at time $t+1$ and, thus, increase the social surplus in the future.

The Euler condition (28), by equating the right and the left-hand sides, implies that the evolution of u_t affects the dynamics of the shadow cost of informational extraction. In particular, when the unemployment rate is decreasing, the social planner wants to reduce the worker's effective outside option because he wants to extract information from the unemployed workers who match today who are relatively more abundant. On the contrary, when the unemployment rate is rising, then there is a gain from increasing the worker's effective outside option because it makes it less costly to extract information from unemployed workers who match tomorrow. The planner can indeed manipulate the continuation utilities by changing the future choices of vacancy creation and hiring margins. In steady state the flows in and out of unemployment are equal and the cost of extracting information is invariant over time. Recall that in competitive equilibrium the allocation $\hat{\theta}$ and γ and the continuation utilities U and V are constant over time. Hence, the equilibrium dynamics are characterized only by the evolution of the unemployment rate. It follows that the competitive equilibrium cannot meet this optimality condition away from the steady state. Denote with $\mathcal{L}$ the Lagrangian associated to P6. Then, we can write:

$$\frac{\partial \mathcal{L}}{\partial U_{t+1}} \gtreqless 0 \text{ if } \left(u_{t+1}^{CE} - u_t^{CE}\right) \lambda^{CE} \gtreqless 0.$$

The steady state competitive search equilibrium satisfies the necessary conditions of the social planning problem. In fact, when the mass of unemployed workers is constant over time the externality is muted. When the unemployment rate is at the steady state level, the flow of workers out of unemployment, who enjoy the informational gain from a reduction of the outside option, are offset by the flow of workers into unemployment, who are damaged by a future lower expected value.[32]

Proposition 14 generalizes the "money burning" example of Pareto improvement discussed in subsection 5.1. That example relies on the extreme case of all the workers being unemployed *ex ante*, which, by construction, implies that the unemployment rate is decreasing to the steady state level.[33] Hence, it focuses only on one direction of the inefficiency. If we assume by contradiction that the competitive equilibrium is constrained efficient, equation (28) could be rewritten as

$$u_{t+1}\left(1-\lambda^{CE}\right)=u_t g+\left(1-u_t\right)s$$

where g is expression (27) in subsection 5.1. Recall that g represents the impact of an increase of the promised utility to unemployed workers tomorrow. There are two reasons why the general dynamic analysis differs from the simple "money burning" in the example, where $u_t = 1$ and $g < 0$. First, even though the impact of an increase of the promised utility tomorrow on workers today is overall positive, that is, $g > 0$, the relatively high mass of potential matches today would make the social planner decrease the promised utility U_{t+1}. Second, if there is a mass of employed workers today, that is, $u_t < 1$, then it is possible that the direction of the inefficiency is reversed and in particular it will be the case exactly when $u_t < u_{t+1}$.

5.3 Utilitarian welfare

Now I want to consider a particular welfare criterion commonly used in the literature: the utilitarian criterion. It is interesting to notice that the steady state competitive search equilibrium does not maximize the utilitarian welfare function. When utility is perfectly transferable the Pareto frontier is linear and this criterion can be used without loss of generality in order to determine the efficient allocation. However, in the context of this environment where b is not transferable, the Pareto frontier is typically not linear. Nevertheless, the utilitarian welfare represents the long run expected welfare of a worker *ex-ante*. For this reason, it is interesting to note that the steady state competitive search equilibrium does not maximize welfare according to this criterion due to the informational asymmetry which makes unemployed people worse off than what the utilitarian planner would prescribe.

Corollary 1 *If P is differentiable, then the competitive search steady state equilibrium does not*

[32] Notice that unfortunately problem P6 is not concave so that I cannot state that the first order conditions are also sufficient to characterize the constrained efficient allocation.

[33] The Pareto improvement built on the money burning mechanism works more generally for a high enough ex-ante unemployment rate.

maximize the utilitarian welfare function.

Proof. The proof of the previous Proposition shows that the competitive search steady state equilibrium satisfies the first order condition of the social planner problem. Then, either it is not an optimum, since the problem is not concave, and then it cannot maximize the utilitarian welfare function, or it is a point on the Pareto frontier. If this is the case, from the Envelope conditions, given that P is assumed differentiable, then

$$P_U = -u^{CE}(1-\lambda^{CE})$$

$$P_V = -\left(1-u^{CE}\right).$$

It follows directly that the allocation does not maximize the total output of the economy, equal to $uU + (1-u)V$, as long as $\lambda^{CE} > 0$, given that

$$\frac{dU}{dV} = -\frac{1-u^{CE}}{u^{CE}(1-\lambda^{CE})} < -\frac{1-u^{CE}}{u^{CE}}$$

completing the proof. ■

5.4 Transferability restores Full Information.

Finally, I consider the alternative environment in which b is freely transferable. As in the static setting, under this relaxed assumption, a net lump-sum transfer from unemployed workers to employed workers, τ_t at time t, makes the competitive equilibrium reach the full information allocation, γ^{FI} and $e^{FI}(.)$, if b is high enough. Budget balance requires that employed workers receive a transfer of $(1-u_t)^{-1} u_t \tau_t$ at time t. Notice that this policy can be interpreted, as in the static section, as a subsidy to job creation financed by a lump-sum tax on workers both employed and unemployed.

Proposition 15 *When b is transferable and high enough, lump-sum transfers from unemployed to employed workers can make the competitive search equilibrium achieve the full information allocation.*

Proof. From the informational constraint, there must exist an H such that

$$\beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\left[\alpha(y-\theta)+H-\frac{F(\theta)}{f(\theta)}\right]dF(\theta) = \gamma^{FI}k$$

so that a competitive equilibrium allocation reaching the full information outcome requires $V_t - U_t \geq H\ \forall t$. The equilibrium has to satisfy

$$U_t = \beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\left[\alpha(y-\theta)+V_{t+1}-U_{t+1}\right]dF(\theta) + b - \tau_t - \gamma^{FI}k + \beta U_{t+1}$$

$$V_t = \left(\frac{u_t}{1-u_t}\right)\tau_t + \beta s U_{t+1} + \beta(1-s)V_{t+1}$$

$$u_{t+1} = u_t\left[1-\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\,dF(\theta)\right] + (1-u_t)\,s$$

Then, imposing that $V_t - U_t = H$ for any t and combining the equations above it follows that

$$\begin{aligned} H &= \hat{V}_t - U_t = \left(\frac{1}{1-u_t}\right)\tau_t - \alpha(y-\theta)\,\beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\,dF(\theta) - b + \gamma^{FI}k \\ &\quad -\beta s H - \beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\,dF(\theta)\,H + \beta H \end{aligned}$$

so that the full information allocation is achievable in competitive equilibrium by imposing the following tax

$$\begin{aligned} \tau_t &= (1-u_t)\left(\alpha(y-\theta)\,\beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\,dF(\theta) + b - \gamma^{FI}k\right) H \\ &\quad + (1-u_t)\left(1-\beta+\beta s+\beta\mu\left(\gamma^{FI}\right)\int_{\underline{\theta}}^{\overline{\theta}} e^{FI}(\theta)\,dF(\theta)\right) H \end{aligned}$$

When $b \geq \tau_t$ the policy is feasible completing the proof. ■

6 Conclusions

In this paper, I have modeled the interaction between informational imperfections and matching frictions in labor markets. My focus has been to analyze the ability of labor markets with these features to decentralize the efficient allocation of resources.

The two crucial assumptions of my setup are: the presence of asymmetric information and the fact that workers' always have the option to quit and go back to the unemployed pool. In this setup a new type of externality arises, which can lead to inefficient job creation. In order to highlight the role of this externality, I have used the equilibrium notion of competitive search, which correctly internalizes the search externality generated by matching frictions. I have shown that, under asymmetric information, the efficiency property of competitive search may fail. All along, I have framed the efficiency analysis in terms of constrained efficiency, by defining a social planning problem subject to the same constraints faced by the decentralized economy.

My model shows that the competitive search equilibrium is constrained inefficient whenever the unemployment rate is away from the steady state level. A natural business cycle interpretation would suggest that decentralized economies may react inefficiently to booms and to recessions. In particular, there is insufficient creation in recessions and excessive creation in booms. An interesting area for future research is to add aggregate shocks and to study explicitly the business cycle implications of the model. In particular, it would be interesting to design optimal policies to restore efficiency, for example in terms of optimal subsidies to job creation. The model suggests that countercyclical subsidies may be an optimal response to business cycle shocks.

Appendix

Proof of Proposition 1. The proof follows closely Acemoglu and Shimer (1999a) and proceeds in two steps: step 1 shows that any equilibrium solves problem P1 and step 2 shows that any solution to P1 is part of an equilibrium.

Step 1. Let $\{\mathbb{C}, \Gamma, U\}$ be an equilibrium with $\mathcal{C}^* \in \mathbb{C}$ and $\gamma^* = \Gamma(\mathcal{C}^*)$. I show that $\{\mathcal{C}^*, \gamma^*\}$, where $\mathcal{C}^* = [e^*(\theta), \omega^*(\theta)]_{\theta\in\Theta}$, solves P1. First, profit maximization ensures that $\{\mathcal{C}^*, \gamma^*\}$ solves constraint (2) together with IC and IR.

Suppose now that another pair $\{\mathcal{C}, \gamma\}$ satisfies IC, IR and achieves an higher value of the objective, that is,

$$\mu(\gamma)\int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)]\, dF(\theta) + b > U,$$

I show that it must violates constraint (2). Since $\{\mathbb{C}, \Gamma, U\}$ is an equilibrium, optimal job application implies

$$\mu(\Gamma(\mathcal{C}))\int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)]\, dF(\theta) + b \leq U$$

and given IR this implies that $\mu(\Gamma(\mathcal{C})) < \mu(\gamma)$ and so $\Gamma(\mathcal{C}) > \gamma$. Then, combining this with profit maximization, it follows that

$$\frac{\mu(\Gamma(\mathcal{C}))}{\Gamma(\mathcal{C})}\int_{\underline{\theta}}^{\overline{\theta}} [e(\theta)y - \omega(\theta)]\, dF(\theta) - k < \frac{\mu(\gamma)}{\gamma}\int_{\underline{\theta}}^{\overline{\theta}} [e(\theta)y - \omega(\theta)]\, dF(\theta) - k \leq 0.$$

This implies that $\{\mathcal{C}, \gamma\}$ violates (2), completing the proof of the first step.

Step 2. This step shows that for any solution $\{\mathcal{C}^*, \gamma^*\}$ to problem P1, there is an equilibrium $\{\mathbb{C}, \Gamma, U\}$ with $\mathbb{C} = \{\mathcal{C}^*\}$ and $\Gamma(\mathcal{C}^*) = \gamma^*$. Set

$$U = \mu(\gamma^*)\int_{\underline{\theta}}^{\overline{\theta}} [\omega^*(\theta) - e^*(\theta)(\theta + b)]\, dF(\theta) + b$$

and let $\Gamma(\mathcal{C})$ satisfy

$$U = \mu(\Gamma(\mathcal{C}))\int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)]\, dF(\theta) + b,$$

or $\Gamma(\mathcal{C}) = 0$ if either IC or IR are not satisfied. It follows that $\{\mathbb{C}, \Gamma, U\}$ satisfies the optimal application for jobs.

To complete the proof I now show that it also satisfies firms' profit maximization. Suppose by contradiction that it is violated by a pair $\{\mathcal{C}', \gamma'\}$ which satisfies IC and IR, but such that

$$\frac{\mu\left(\Gamma\left(\mathcal{C}'\right)\right)}{\Gamma(\mathcal{C}')} \int_{\underline{\theta}}^{\overline{\theta}} \left[e'\left(\theta\right) y - \omega'\left(\theta\right)\right] dF\left(\theta\right) - k > 0.$$

Then, I can choose $\gamma' > \Gamma\left(\mathcal{C}'\right)$ such that

$$\frac{\mu\left(\gamma'\right)}{\gamma'} \int_{\underline{\theta}}^{\overline{\theta}} \left[e'\left(\theta\right) y - \omega'\left(\theta\right)\right] dF\left(\theta\right) - k = 0.$$

Then, by the construction of Γ, $\gamma' > \Gamma\left(\mathcal{C}'\right)$ and IR imply

$$U < \mu\left(\gamma'\right) \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega'\left(\theta\right) - e'\left(\theta\right)\left(\theta + b\right)\right] dF\left(\theta\right) + b,$$

so that the pair $\{\mathcal{C}', \gamma'\}$ satisfies all the constraints, but generates an higher value for the objective function, yielding to a contradiction.

Proof of Proposition 2. The proof proceeds in two steps. First I show that any pair $\{e\left(\theta\right), \gamma\}$ solving problem P1 is also a solution to problem P2 and then I show that for any such a pair, I can construct a wage function $\omega\left(\theta\right)$ which solves problem P1.

Step 1. First, from constraints IC' and IR'

$$\int_{\underline{\theta}}^{\overline{\theta}} v\left(\theta, \theta\right) dF\left(\theta\right) = v\left(\overline{\theta}, \overline{\theta}\right) + \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\theta}^{\overline{\theta}} e\left(y\right) dy\right] dF\left(\theta\right), \tag{29}$$

$$v\left(\overline{\theta}, \overline{\theta}\right) \geq b. \tag{30}$$

From integration by parts it follows

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\theta}^{\overline{\theta}} e\left(y\right) dy\right] dF\left(\theta\right) = \int_{\underline{\theta}}^{\overline{\theta}} e\left(\theta\right) F\left(\theta\right) d\theta,$$

which combined with (29) gives

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega\left(\theta\right) - e\left(\theta\right)\left(\theta + b\right)\right] dF\left(\theta\right) + b = \int_{\underline{\theta}}^{\overline{\theta}} e\left(\theta\right) \frac{F\left(\theta\right)}{f\left(\theta\right)} dF\left(\theta\right) + v\left(\overline{\theta}, \overline{\theta}\right).$$

Using (30) I get

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega\left(\theta\right) - e\left(\theta\right)\left(\theta + b + \frac{F\left(\theta\right)}{f\left(\theta\right)}\right)\right] dF\left(\theta\right) \geq 0.$$

Then, a relaxed version of problem P1, where I leave to check the monotonicity of $e\left(\theta\right)$ for the end, can be rewritten as

$$U = \max_{e(\theta),\omega(\theta),\gamma} \mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} [\omega(\theta) - e(\theta)(\theta + b)]\, dF(\theta) + b$$

s.t.

$$\mu(\gamma) \int_{\underline{\theta}}^{\overline{\theta}} [e(\theta)\, y - \omega(\theta)]\, dF(\theta) = \gamma k$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) - e(\theta)\left(\theta + b + \frac{F(\theta)}{f(\theta)}\right)\right] dF(\theta) \geq 0$$

where I can eliminate the wage from the program by using the free-entry condition, ending up with problem P2 exactly.

Step 2. Now I show that for any $\{e(\theta), \gamma\}$ solving problem P2, I can construct a wage which, together with the same $\{e(\theta), \gamma\}$, satisfies problem P1. In particular pointwise maximization gives that at the optimum

$$e(\theta) = \begin{cases} 1 & \text{if } \theta \leq \hat{\theta} \\ 0 & \text{if } \theta > \hat{\theta} \end{cases}$$

where $\hat{\theta}$ is implicitly defined by

$$\hat{\theta} = y - b - \lambda \frac{F(\hat{\theta})}{f(\hat{\theta})}.$$

I can construct the wage schedule

$$\omega(\theta) = \begin{cases} \hat{\theta} + b & \text{if } \theta \leq \hat{\theta} \\ 0 & \text{if } \theta > \hat{\theta} \end{cases},$$

which satisfies the IC constraints, completing the proof.

Proof of Proposition 3.

Step 1. Existence.

First, notice that Proposition 2 shows that for any solution $e^*(\theta)$ and γ^* of problem P2 there exists a function $\omega^*(\theta)$ such that $\omega^*(\theta)$, $e^*(\theta)$ and γ^* are a solution of problem P1. Then the existence of a solution of problem P2 is sufficient for the existence of a solution to problem P1.

Next, to show existence of a solution to problem P2, I first show that there exists a solution to the relaxed version of P2, without assuming the monotonicity condition on $e(\theta)$, and then I show that $e(\theta)$ is in fact monotone, implying that there exists a solution to the original problem P2.

Pointwise maximization, together with the monotone hazard rate assumption, implies that there exists a threshold $\hat{\theta}$ such that $e(\theta) = 1$ if $\theta < \hat{\theta}$ and $e(\theta) = 0$ otherwise. This shows directly that $e(\theta)$ is in fact non-increasing and that a solution to the relaxed version of problem P2 is also a solution of the original problem. This allows me to reduce the control variables to $\hat{\theta}$ and γ. The relaxed problem can be written as

$$U = \max_{\hat{\theta},\gamma} \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} [y - \theta - b] \, dF(\theta) + b - \gamma k$$

s.t.

$$F(\hat{\theta},\gamma) \equiv \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) - \gamma k \geq 0.$$

It is straightforward to see that the objective function is continuous in $\hat{\theta}$ and γ and that the constraint set is compact, since $F(\hat{\theta},\gamma)$ is continuous in both its arguments and is not empty, since, for example, $\gamma = 0$ and any $\hat{\theta}$ satisfies it. Existence follows directly.

Step 1. Uniqueness.

Proposition 2 shows that an equilibrium can be characterized by and array $\hat{\theta}$, γ and λ that must satisfy equations (4), (5) and (6). Notice that equation (4) defines implicitly $\hat{\theta}$ as a function of λ with $\partial\hat{\theta}/\partial\lambda < 0$ which can be substituted for into equations (5) and (6). Now there are two equations in two unknown, $\hat{\theta}$ and λ:

$$f_1(\gamma,\lambda) = \mu'(\gamma) \int_{\underline{\theta}}^{\hat{\theta}(\lambda)} \left[y - \theta - b - \lambda \frac{F(\theta)}{f(\theta)} \right] dF(\theta) - k$$

$$f_2(\gamma,\lambda) = \frac{\mu(\gamma)}{\gamma} \int_{\underline{\theta}}^{\hat{\theta}(\lambda)} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) - k$$

Notice that $f_1(\gamma,\lambda)$ and $f_2(\gamma,\lambda)$ define implicitly two functions, which I name $\gamma_1(\lambda)$ and $\gamma_2(\lambda)$. Then, the implicit function theorem implies that

$$\frac{d\gamma_1(\lambda)}{d\lambda} = -\frac{\frac{\partial f_1(\gamma,\lambda)}{\partial\lambda}}{\frac{\partial f_1(\gamma,\lambda)}{\partial\gamma}} > 0 \text{ and } \frac{d\gamma_2(\lambda)}{d\lambda} = -\frac{\frac{\partial f_2(\gamma,\lambda)}{\partial\lambda}}{\frac{\partial f_2(\gamma,\lambda)}{\partial\gamma}} > 0,$$

since

$$y - \hat{\theta} - b - \frac{F(\hat{\theta})}{f(\hat{\theta})} < y - \hat{\theta} - b - \lambda\frac{F(\hat{\theta})}{f(\hat{\theta})} = 0.$$

It follows that the two curves must intersect at most once. Moreover, given that I have proved existence in the previous step, they must intersect exactly at one point, completing the proof.

Proof of Proposition 6. The proof separately analyzes the case of $\mu(\gamma)$ satisfying assumptions A1 and A2 and $\mu(\gamma)$ satisfying A1 and A3.

Case 1. $\mu(\gamma)$ satisfies A1 and A2.

Recall that for any given y and k, the equilibrium $\hat{\theta}$, γ and λ must satisfy equations (4), (6) and

(5). Notice that using equation (4) and integration by parts yields

$$\int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \lambda \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = (1-\lambda) \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)\, d\theta + \lambda \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})}$$

Moreover,

$$\int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = \lambda \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})}.$$

Then, I can rewrite equations (6) and (5) as

$$\lambda \frac{\mu(\gamma)}{\gamma} \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})} = k, \tag{31}$$

$$\mu'(\gamma) \left[(1-\lambda) \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)\, d\theta + \lambda \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})} \right] = k \tag{32}$$

and, when γ is an interior solution, by combining them it follows

$$\frac{\lambda}{1-\lambda} = \left[\frac{\eta(\gamma)}{1-\eta(\gamma)} \right] \frac{f(\hat{\theta})}{[F(\hat{\theta})]^2} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)\, d\theta, \tag{33}$$

where $\eta(\gamma)$ denotes the elasticity of $\mu(\gamma)$, that is,

$$\eta(\gamma) = \frac{\gamma \mu'(\gamma)}{\mu(\gamma)}.$$

Equations (4), (31) and (33) define an equilibrium $\hat{\theta}$, γ and λ.

Consider a family of economies parametrized by a pair (ε, δ) for (ε, δ) belonging to a (one-sided) neighborhood of $(0,0)$ such as $\mathcal{I} \equiv (0, \bar{\varepsilon}) \times (0, \bar{\delta})$ with $\bar{\varepsilon}$ and $\bar{\delta}$ strictly positive. For each pair (ε, δ), set the parameters k and y such that

$$k(\varepsilon, \delta) = \lambda(\varepsilon, \delta) \frac{\mu(\varepsilon)}{\varepsilon} \frac{[F(\bar{\theta} - \delta)]^2}{f(\bar{\theta} - \delta)} \tag{34}$$

and

$$y(\varepsilon, \delta) = b + \bar{\theta} - \delta + \lambda(\varepsilon, \delta) \frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)}, \tag{35}$$

where $\lambda(\varepsilon, \delta)$ solves

$$\frac{\lambda(\varepsilon, \delta)}{1 - \lambda(\varepsilon, \delta)} = \left[\frac{\eta(\varepsilon)}{1 - \eta(\varepsilon)} \right] \frac{f(\bar{\theta} - \delta)}{[F(\hat{\theta})]^2} \int_{\underline{\theta}}^{\bar{\theta} - \delta} F(\theta)\, d\theta$$

and $\eta(\varepsilon) = \varepsilon \mu'(\varepsilon) / \mu(\varepsilon)$. Notice that (34) and (35) make $\gamma = \varepsilon$ and $\hat{\theta} = \bar{\theta} - \delta$ an equilibrium solution, given that Proposition 3 shows that the first order conditions have a unique solution. Notice that, as we assumed, given that $\varepsilon > 0$, γ is an interior solution.

Next I show that equations (34) and (35) define a continuous and invertible mapping between the space (ε, δ) and the space (k, y). The determinant of the Jacobian of the bidimensional function $f(\varepsilon, \delta)$, where $f_1 \equiv k(\varepsilon, \delta)$ and $f_2 \equiv y(\varepsilon, \delta)$, is

$$
\begin{aligned}
\det J(\varepsilon, \delta) \quad = \quad & -\lambda(\varepsilon, \delta) \frac{1}{\varepsilon} \left[\mu'(\varepsilon) - \frac{\mu(\varepsilon)}{\varepsilon} \right] \frac{[F(\bar{\theta} - \delta)]^2}{f(\bar{\theta} - \delta)} \\
& \left[1 + \lambda(\varepsilon, \delta) \partial \frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)} / \partial \left(\bar{\theta} - \delta \right) - \frac{\partial \lambda(\varepsilon, \delta)}{\partial \delta} \frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)} \right] \\
& - \frac{\mu(\varepsilon)}{\varepsilon} \frac{\partial \lambda(\varepsilon, \delta)}{\partial \varepsilon} \frac{[F(\bar{\theta} - \delta)]^2}{f(\bar{\theta} - \delta)} \left[1 - \lambda(\varepsilon, \delta) \right],
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial \lambda(\varepsilon, \delta)}{\partial \delta} \quad = \quad & \frac{\eta(\gamma)(1 - \eta(\gamma))}{[1 - \eta(\gamma)(1 - D(\delta))]} \left(\frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)} \right)^{-1} \\
& \left[D(\delta) \left(1 - \frac{dF(\bar{\theta} - \delta)/f(\bar{\theta} - \delta)}{d\left(\bar{\theta} - \delta\right)} \right) + 1 \right]
\end{aligned}
$$

and

$$
\frac{\partial \lambda(\varepsilon, \delta)}{\partial \varepsilon} = \frac{\eta'(\varepsilon) D(\delta)}{[1 - \eta(\varepsilon)(1 - D(\delta))]}
$$

since $\eta'(\varepsilon) < 0$, with

$$
D(\delta) \equiv \left(\frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)} \right)^{-1} F(\bar{\theta} - \delta)^{-1} \int_{\underline{\theta}}^{\bar{\theta} - \delta} F(\theta) \, d\theta.
$$

Notice that $\det J(\varepsilon, \delta) > 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$, where $\mathcal{I}$ is small enough. This is easy to see. In fact $\partial \lambda(\varepsilon, \delta)/\partial \varepsilon < 0$, given that $\eta'(\varepsilon) < 0$ and $\mu'(\varepsilon) < \mu(\varepsilon)/\varepsilon$ for any $\varepsilon > 0$, given that $\mu(.)$ is strictly concave. Moreover either $\partial \lambda(\varepsilon, \delta)/\partial \delta$ is positive, or, if negative, $\lim_{(\varepsilon, \delta) \to (0,0)} \partial \lambda(\varepsilon, \delta)/\partial \delta = 0$ so that we can choose $\mathcal{I}$ small enough so that $\partial \lambda(\varepsilon, \delta)/\partial \delta$ is sufficiently close to zero for $(\varepsilon, \delta) \in \mathcal{I}$, completing the argument. Notice that this is where the assumption of strict concavity of $\mu(.)$ is required.

Then, think of g as a function of (ε, δ), that is,

$$
g(\varepsilon, \delta) \equiv 1 - \mu(\varepsilon) F(\bar{\theta} - \delta) - \lambda(\varepsilon, \delta).
$$

Thus, if for a small enough neighborhood $\mathcal{I}$, $g(\varepsilon, \delta) < 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$, then there exists an open set of the space (k, y) for which $g < 0$.

Next, I show that $\lim_{\varepsilon \to 0} g(\varepsilon, \delta) = 0$ for any $\delta < \bar{\theta} - \underline{\theta}$. Given that $\mu(0) = 0$ and $\mu(\gamma)$ is everywhere differentiable it follows that

$$
\mu'(0) = \lim_{\varepsilon \to 0} \frac{\mu(\varepsilon)}{\varepsilon} \Rightarrow \lim_{\varepsilon \to 0} \eta(\varepsilon) = 1.
$$

Then, equation (33) yields

$$\lim_{\varepsilon \to 0} \lambda(\varepsilon, \delta) = 1 \; \forall \delta < \bar{\theta} - \underline{\theta},$$

given that, by construction, equation (35) implies $\hat{\theta} = \bar{\theta} - \delta > \underline{\theta}$ for $\delta < \bar{\theta} - \underline{\theta}$, completing the argument. It follows that

$$\lim_{\varepsilon \to 0} g(\varepsilon, \delta) = 0 \; \forall \delta < \bar{\theta} - \underline{\theta}.$$

Then, to show that $g(\varepsilon, \delta) < 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$ where $\mathcal{I}$ is small enough, it is sufficient to show that $\partial g(\varepsilon, \delta) / \partial \varepsilon < 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$.

Notice that

$$\left. \frac{\partial g(\varepsilon, \delta)}{\partial \varepsilon} \right|_{(\varepsilon,\delta) \in \mathcal{I}} = -\mu'(\varepsilon) F(\bar{\theta} - \delta) - \left. \frac{\partial \lambda(\varepsilon, \delta)}{\partial \varepsilon} \right|_{(\varepsilon,\delta) \in \mathcal{I}}.$$

Equation (33) yields

$$\lim_{(\varepsilon,\delta) \to (0,0)} \frac{\partial \lambda(\varepsilon, \delta)}{\partial \varepsilon} = \lim_{\varepsilon \to 0} \frac{\eta'(\varepsilon)}{f(\bar{\theta}) \int_{\underline{\theta}}^{\bar{\theta}} F(\theta) \, d\theta}.$$

Moreover $\lim_{\delta \to 0} F(\bar{\theta} - \delta) = 1$ and, by assumption, $\lim_{\varepsilon \to 0} \mu'(\varepsilon) = \mu'(0) > 0$ and $\lim_{\varepsilon \to 0} \eta'(\varepsilon) > -\mu'(0) f(\bar{\theta}) \int_{\underline{\theta}}^{\bar{\theta}} F(\theta) \, d\theta$. It follows that I can choose $\bar{\varepsilon}$ and $\bar{\delta}$ small enough such that $\partial g(\varepsilon, \delta) / \partial \varepsilon|_{(\varepsilon,\delta) \in \mathcal{I}} < 0$, completing the proof.

Case 2. $\mu(\gamma)$ satisfies A1 and A3.

The proof of this case proceeds in two steps. First I show that there exists a $\bar{k}$ such that either $k \geq \bar{k}$ and $\gamma = 0$ or $k < \bar{k}$ and $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, which is the interesting case. Then, focusing on this case, I divide the family of functions $\mu(\gamma)$, satisfying assumptions A1 and A3 into two subfamilies and I show that in both cases, there exists a parameter set such that g is negative.

Step 1.

In order to analyze the competitive search equilibrium allocation with two points of non-differentiability $\underline{\gamma}$ and $\bar{\gamma}$, first notice that, when the constraint is binding, for given y and k, the equilibrium still has to satisfy equations (4) and (6).

Define $\tilde{\theta}$ such that

$$y - b - \tilde{\theta} - \frac{F(\tilde{\theta})}{f(\tilde{\theta})} = 0.$$

From equation (4), it follows that $\hat{\theta} \geq \tilde{\theta}$. Moreover notice that

$$\tilde{\theta} = \arg\max_{\hat{\theta}} \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta)$$

and I can define $\bar{k}$ the maximum value

$$\bar{k} \equiv \int_{\underline{\theta}}^{\tilde{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) > 0,$$

since $y > b + \underline{\theta}$. For any $k \geq \bar{k}$, given that $\mu(\gamma)/\gamma \leq 1$, the only feasible equilibrium is characterized by zero vacancy, because the total surplus is not enough to cover the cost of information revelation and the cost of creating a vacancy. So, from now on, let $k < \bar{k}$.
It is, then, straightforward to see that the optimal $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. In fact, $\gamma < \underline{\gamma}$ cannot be an equilibrium given that $\mu'(\gamma) = \mu(\gamma)/\gamma = 1$ and

$$\int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \lambda \frac{F(\theta)}{f(\theta)} \right] dF(\theta) > \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k.$$

Moreover, $\gamma > \bar{\gamma}$ cannot be an equilibrium since $\mu'(\gamma) = 0$, $\mu(\gamma)/\gamma = 1/\bar{\gamma}$ and

$$0 < \frac{1}{\bar{\gamma}} \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k.$$

It follows that the equilibrium can be characterized by a tuple $\hat{\theta}$, γ and λ solving equations (4), (6) and

$$\mu'(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \left[y - \theta - b - \lambda \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \begin{cases} < k & \text{if } \gamma = \underline{\gamma} \\ = k & \text{if } \gamma \in (\underline{\gamma}, \bar{\gamma}) \\ > k & \text{if } \gamma = \bar{\gamma} \end{cases}, \tag{36}$$

where $\mu'(\gamma) = \lim_{\gamma \searrow \underline{\gamma}} \mu'(\gamma)$ when $\gamma = \underline{\gamma}$ and $\mu'(\gamma) = \lim_{\gamma \nearrow \bar{\gamma}} \mu'(\gamma)$ when $\gamma = \bar{\gamma}$.

Step 2.

The family of functions $\mu(\gamma)$ satisfying Assumptions A1 and A2 can be divided into two subfamilies: the family of functions for which $\underline{\gamma} = \bar{\gamma}$, that is, the Leontief case $\mu(\gamma) = \min\{\gamma, 1\}$, and the family of function for which $\underline{\gamma} < \bar{\gamma}$ and $\mu'(\gamma) < \mu(\gamma)/\gamma$ for any $\gamma \in (\underline{\gamma}, \bar{\gamma})$.

Subfamily I: $\mu(\gamma) = \min\{\gamma, 1\}$.[34]

It follows from the previous analysis that, when $\underline{\gamma} = \bar{\gamma} = 1$ as in the Leontief case and $k < \bar{k}$, then in equilibrium $\gamma = \mu(\gamma) = 1$ and (5) is not binding. Then, equations (4) and (6) complete the equilibrium characterization.
Think of g as a function of λ

$$g(\lambda) \equiv 1 - \mu(\gamma) F(\hat{\theta}(\lambda)) - \lambda,$$

where, for this first case, I think of $\hat{\theta}$ as a function of λ implicitly defined by equation (4), that is,

$$\hat{\theta}(\lambda) = y - b - \lambda \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))}. \tag{37}$$

[34] Note that $\eta = 1$ represents the extreme case of frictionless labor market. The Proposition can be easily extended to the more general Leontief case where $\mu(\gamma) = \min\{\eta\gamma, 1\}$ with $\eta \in (0, 1]$ and the labor market is frictional for any $\eta < 1$.

Notice that

$$g(1) = -F(\hat{\theta}(1)) < 0,$$

since $y > b + \underline{\theta}$ implies $\hat{\theta}(1) > \underline{\theta}$. Given that $g(\lambda)$ is a continuous function, then there exists a set of $\lambda \leq 1$ such that $g(\lambda)$ is negative. For each λ in this set, I can set the parameter k such that

$$k = \int_{\underline{\theta}}^{\hat{\theta}(\lambda)} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta). \tag{38}$$

Observe that

$$\frac{dk}{d\lambda} = \left[y - \hat{\theta}(\lambda) - b - \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} \right] f(\hat{\theta}(\lambda)) \frac{d\hat{\theta}(\lambda)}{d\lambda} > 0,$$

since from equation (37) it follows that

$$y - \hat{\theta}(\lambda) - b - \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} \leq y - \hat{\theta}(\lambda) - b - \lambda \left[\frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} \right] = 0$$

and, given the monotone hazard rate assumption, that $d\hat{\theta}(\lambda)/d\lambda < 0$. Thus, for any $y > b + \underline{\theta}$, there exists a $\hat{k} < \bar{k}$ such that $g < 0$ for any $k \in (\hat{k}, \bar{k}]$, completing the proof for the Leontief case.[35]

Subfamily II: $\underline{\gamma} < \bar{\gamma}$ and $\mu'(\gamma) < \mu(\gamma)/\gamma \; \forall \gamma \in (\underline{\gamma}, \bar{\gamma})$.

Assume that the equilibrium is characterized by an interior solution for $\gamma \in (\underline{\gamma}, \bar{\gamma})$. Then, as derived in the proof of Proposition 6, the equilibrium γ, $\hat{\theta}$ and λ must satisfy

$$\hat{\theta} = y - b - \lambda \frac{F(\hat{\theta})}{f(\hat{\theta})}, \tag{39}$$

$$\lambda \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})} \frac{\mu(\gamma)}{\gamma} = k, \tag{40}$$

$$\frac{\lambda}{1-\lambda} = \left[\frac{\eta(\gamma)}{1 - \eta(\gamma)} \right] \frac{f(\hat{\theta})}{[F(\hat{\theta})]^2} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)\, d\theta, \tag{41}$$

where $\eta(\gamma)$ denotes the elasticity of $\mu(\gamma)$, that is,

$$\eta(\gamma) = \frac{\gamma \mu'(\gamma)}{\mu(\gamma)}.$$

Consider a family of economies parametrized by a pair (ε, δ) for (ε, δ) belonging to a (one-sided) neighborhood of $(0,0)$ such as $\mathcal{I} \equiv (0, \bar{\varepsilon}) \times (0, \bar{\delta})$ with $\bar{\varepsilon}$ and $\bar{\delta}$ strictly positive.
Think of λ as a function of ε and δ, $\lambda(\varepsilon, \delta)$, defined implicitly by equation (41) with $\gamma = \bar{\gamma} - \varepsilon$ and

[35] In particular, $g(0) > 0$ and $g'(.) < 0$ so that, for any $y > \tilde{y}$, there exists a $\hat{k}$ such that $g < 0$ for any $k \in (\hat{k}, \bar{k}]$ and $g > 0$ for any $k \in [0, \hat{k}]$. The Leontief case is a special case of Proposition 7 that I discuss below.

$\hat{\theta} = \bar{\theta} - \delta$. Notice that $\lambda(\varepsilon, \delta)$ is increasing in ε, given the monotone elasticity assumption. The Proposition assumes that there exists a $\bar{\gamma} < \infty$ such that $\mu(\bar{\gamma}) = 1$ and $\lim_{\gamma \nearrow \bar{\gamma}} \eta(\gamma) > 0$, which implies $\lambda(\varepsilon, \delta) > 0$ for any $\varepsilon > 0$, since $\lim_{\varepsilon \to 0} \lambda(\varepsilon, \delta) > 0$ and $\partial \lambda(\varepsilon, \delta)/\partial \varepsilon > 0$ for any $\delta < \bar{\theta} - \underline{\theta}$.
For each pair (ε, δ), set the parameters k and y such that

$$k = \lambda(\varepsilon, \delta) \frac{\mu(\bar{\gamma} - \varepsilon)}{\bar{\gamma} - \varepsilon} \frac{[F(\bar{\theta} - \delta)]^2}{f(\bar{\theta} - \delta)} \tag{42}$$

and

$$y = b + \bar{\theta} - \delta + \lambda(\varepsilon, \delta) \frac{F(\bar{\theta} - \delta)}{f(\bar{\theta} - \delta)}, \tag{43}$$

which make $\gamma = \bar{\gamma} - \varepsilon$ and $\hat{\theta} = \bar{\theta} - \delta$ an equilibrium solution. Notice that, as we have assumed, γ is an interior solution given that $\varepsilon > 0$.
Then, think of g as a function of (ε, δ), that is,

$$g(\varepsilon, \delta) \equiv 1 - \mu(\bar{\gamma} - \varepsilon) F(\bar{\theta} - \delta) - \lambda(\varepsilon, \delta).$$

Since equations (42) and (43) define a continuous and invertible[36] mapping between the space (ε, δ) and the space (k, y), if for a small enough neighborhood $\mathcal{I}$, $g(\varepsilon, \delta) < 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$, then there exists an open set of the space (k, y) for which $g < 0$.
I can choose $\bar{\varepsilon}$ and $\bar{\delta}$ small enough such that $g(\varepsilon, \delta) < 0$ for any $(\varepsilon, \delta) \in \mathcal{I}$, given that

$$\lim_{(\varepsilon,\delta) \to (0,0)} g(\varepsilon, \delta) = -\lambda(\varepsilon, \delta) < 0,$$

completing the proof.

Proof of Proposition 7. The proof proceeds in two steps. First I characterize the equilibrium solution. Then, I describe a set of the parameter space such that g is negative.

Step 1.
First of all, notice that

$$\mu(\gamma) = \begin{cases} \gamma & \text{if } \gamma \leq \underline{\gamma} \\ A\gamma^{\alpha} & \text{if } \underline{\gamma} < \gamma \leq \bar{\gamma} \\ 1 & \text{if } \gamma > \bar{\gamma} \end{cases},$$

where

$$\underline{\gamma} \equiv A^{\frac{1}{1-\alpha}} \text{ and } \bar{\gamma} \equiv A^{-\frac{1}{\alpha}}.$$

Moreover, as described in the proof of Proposition 6, given that $y > b + \underline{\theta}$, there exists a $\bar{k} > 0$

[36] The proof of invertibility is analogous to the one in the proof of Proposition 6 and thus requires strict concavity of $\mu(.)$ and non-increasing elasticity.

defined as

$$\bar{k} \equiv \int_{\underline{\theta}}^{\tilde{\theta}} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta),$$

where $\tilde{\theta}$ solves $y - b - \tilde{\theta} - F(\tilde{\theta})/f(\tilde{\theta}) = 0$. I assume $k < \bar{k}$[37], so that the equilibrium allocation must satisfy equations (4), (6) and (36) for $\gamma \in [\underline{\gamma}, \bar{\gamma}]$.

Think of γ as a function of k. Next, I show that there exist two cut-off values k_1 and k_2 such that $\gamma(k) = \bar{\gamma}$ for any $k < k_1$, $\gamma(k) = \underline{\gamma}$ for any $k > k_2$ and $\gamma(k) \in (\underline{\gamma}, \bar{\gamma})$ for any $k \in (k_1, k_2)$.

When $k \in (k_1, k_2)$, there is an interior solution with $\gamma \in (\underline{\gamma}, \bar{\gamma})$ such that, as derived in the proof of Proposition 6, the equilibrium $\hat{\theta}$, γ and λ must satisfy

$$\hat{\theta} = y - b - \lambda \frac{F(\hat{\theta})}{f(\hat{\theta})}, \tag{44}$$

$$\lambda \frac{[F(\hat{\theta})]^2}{f(\hat{\theta})} \frac{\mu(\gamma)}{\gamma} = k, \tag{45}$$

$$\frac{\lambda}{1-\lambda} = \left[\frac{\alpha}{1-\alpha} \right] \frac{f(\hat{\theta})}{[F(\hat{\theta})]^2} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)\, d\theta, \tag{46}$$

since the elasticity of $\mu(.)$, $\eta(\gamma(k)) = \gamma\mu'(\gamma)/\mu(\gamma) = \alpha$ for any $\gamma \in (\underline{\gamma}, \bar{\gamma})$.

Notice that, from equations (44)-(46) it follows that when $k \in (k_1, k_2)$ γ depends on k, while λ and $\hat{\theta}$ are constant. More precisely, using the implicit function theorem, equation (44) defines $\hat{\theta}$ as a continuous increasing function of λ and, using that, equation (46) defines implicitly $\lambda = \tilde{\lambda}$ for any $k \in (k_1, k_2)$. Then, $\tilde{\lambda}$ is implicitly defined by

$$\tilde{\lambda} = \frac{\alpha D(\hat{\theta}(\tilde{\lambda}))}{1 - \alpha(1 - D(\hat{\theta}(\tilde{\lambda})))},$$

where $D(\hat{\theta}(\tilde{\lambda})) \equiv f(\hat{\theta}(\tilde{\lambda})) \int_{\underline{\theta}}^{\hat{\theta}(\tilde{\lambda})} F(\theta)\, d\theta / [F(\hat{\theta}(\tilde{\lambda}))]^2$. Notice that the monotone hazard rate assumption implies

$$E\left[F(\theta)/f(\theta) \mid \theta < \hat{\theta} \right] < F(\hat{\theta})/f(\hat{\theta}).$$

so that $D(\hat{\theta}(\tilde{\lambda})) < 1$ and $\tilde{\lambda} < 1$ as well. Thus, using $\lambda = \tilde{\lambda}$ and $\hat{\theta}(\tilde{\lambda})$, equation (45) defines implicitly $\gamma(k)$.

Now, define

$$k_1 \equiv \frac{1}{\bar{\gamma}} \int_{\underline{\theta}}^{\hat{\theta}(\tilde{\lambda})} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta)$$

and

$$k_2 \equiv \int_{\underline{\theta}}^{\hat{\theta}(\tilde{\lambda})} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta).$$

[37] Excluding equilibria with zero vacancy, that is such that $k \geq \bar{k}$, ensures that $\gamma = 0$ is never optimal.

From equation (4), (6) and (36), it follows that an equilibrium for any $k \leq k_1$ is characterized by $\gamma = \bar{\gamma}$ and $\lambda(k)$ implicitly defined by

$$\int_{\underline{\theta}}^{\hat{\theta}(\lambda(k))} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = \bar{\gamma} k,$$

so that by definition $\lambda(k_1) = \tilde{\lambda}$. Instead, an equilibrium for any $k \geq k_2$ is characterized by $\gamma = \underline{\gamma}$ and $\lambda(k)$ implicitly defined by

$$\int_{\underline{\theta}}^{\hat{\theta}(\lambda(k))} \left[y - \theta - b - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k,$$

so that by definition $\lambda(k_2) = \tilde{\lambda}$.

Notice that for any $k \leq k_1$

$$\frac{d\lambda(k)}{dk} = \left[\frac{d(\hat{\theta}(\lambda))}{d\lambda} \left[y - \hat{\theta}(\lambda) - b - \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} \right] f(\hat{\theta}(\lambda)) \right]^{-1} > 0 \tag{47}$$

since equation (4) implies

$$y - \hat{\theta}(\lambda) - b - \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} < y - \hat{\theta}(\lambda) - b - \lambda \frac{F(\hat{\theta}(\lambda))}{f(\hat{\theta}(\lambda))} = 0$$

and using the implicit function theorem and the monotone hazard rate assumption, it follows directly that $d(\hat{\theta}(\lambda))/d\lambda < 0$. An analogous argument shows that $d\lambda(k)/dk > 0$ for any $k \geq k_2$. Notice that the assumption $y > b + \underline{\theta}$ yields $\bar{k} > k_2$.

It follows that we can think of $\lambda(k)$, as a non decreasing function of k with $\lambda(k) = 0$ for $k = 0$ and $\lambda(k) = 1$ for $k = \bar{k}$. Moreover it is strictly increasing for $k < k_1$ and $k > k_2$ and is constant at $\tilde{\lambda}$ for $k_1 \leq k \leq k_2$.

Step 2.

Think of g as a function of k, that is,

$$g(k) \equiv 1 - \mu(\gamma(k)) F(\hat{\theta}(\lambda(k))) - \lambda(k).$$

Then,

$$g(0) = 1 - F(\hat{\theta}(0)) = 0 \text{ and } g\left(\bar{k}\right) = -\underline{\gamma} F(\hat{\theta}(1)) < 0,$$

where $g(0) = 0$ since $\hat{\theta}(0) = \bar{\theta}$, given that by assumption $y > b + \bar{\theta} + \tilde{\lambda} F(\bar{\theta})/f(\bar{\theta}) > b + \bar{\theta}$.

Moreover $g'(k) < 0$ for $0 < k < k_1$, since $\gamma = \bar{\gamma}$, $\mu(\bar{\gamma}) = 1$ and

$$g'(k) = -\left[1 + f(\hat{\theta}(\lambda(k)))\frac{d\hat{\theta}(\lambda(k))}{d\lambda(k)}\right]\frac{d\lambda(k)}{dk} < 0,$$

where $d\hat{\theta}(\lambda(k))/d\lambda(k) < 0$ and $d\lambda(k)/dk > 0$ by equation (47). With a similar argument we can show that $g'(k) < 0$ for $k_2 < k < \bar{k}$. Instead, for $k_1 < k < k_2$, $d\lambda(k)/dk = 0$ but γ is interior and strict concavity of $\mu(\gamma)$ implies

$$g'(k) = -\mu'(\gamma(k))\,\mu''(\gamma(k))\,F(\hat{\theta}(\lambda(k)))\int_{\underline{\theta}}^{\hat{\theta}(\lambda(k))} F(\theta)\,d\theta > 0.$$

Then, $g(k)$ has a minimum in k_1 and a maximum in k_2. Notice that $g(k_1) = 1 - F(\hat{\theta}(\tilde{\lambda})) - \tilde{\lambda}$, so that

$$g(k_1) < 0 \Leftrightarrow F(\hat{\theta}(\tilde{\lambda})) > 1 - \tilde{\lambda}.$$

Thus, $y > b + \bar{\theta} + \tilde{\lambda}/f(\bar{\theta})$ implies $\hat{\theta}(\tilde{\lambda}) = \bar{\theta}$, so that $F(\hat{\theta}(\tilde{\lambda})) = 1 > 1 - \tilde{\lambda}$ given that $\tilde{\lambda} < 1$. Moreover $g(k_2) = 1 - \underline{\gamma}F(\hat{\theta}(\tilde{\lambda})) - \tilde{\lambda}$ yielding

$$g(k_2) < 0 \Leftrightarrow \frac{1 - \tilde{\lambda}}{\underline{\gamma}} < 1.$$

Thus, if $\underline{\gamma} > 1 - \tilde{\lambda}$ then $g(k_2) < 0$ leading $g(k) < 0$ for any $k < \bar{k}$. If, otherwise, $\underline{\gamma} < 1 - \tilde{\lambda}$, then $g(k_2) > 0$. In the latter case, it follows, by a continuity argument, that there exist k_L and k_H such that $k_1 < k_L < k_2 < k_H < \bar{k}$ so that in equilibrium $g < 0$ for $k \in [0, k_L]$ and $k \in [k_H, \bar{k}]$, completing the proof.

Proof of Proposition13. The proof analyzes separately the two cases of functions $\mu(\gamma)$ satisfying assumptions A1 and, respectively, A2 and A3.

Case 1. $\mu(\gamma)$ satisfies A1 and A2.
First, notice that at the competitive equilibrium, (23) implies that $\hat{V}^{CE} - U^{CE} = -(1-\beta)\,\alpha U^{CE}$. For now, fix $(1-\beta)\,\alpha U^{CE} = \hat{b}$. Then, equations (19), (21) and (20) define an equilibrium $\hat{\theta}^{CE}$, γ^{CE} and λ^{CE} for $\hat{b}$ given, and, for $\alpha = 1$, are equivalent to equations (4), (6) and (5) that define a static equilibrium where $b = \hat{b}$. Thus, an argument analogous to the case 1 of the proof of Proposition 6, adjusted for $\alpha < 1$, shows that for each $\hat{b}$ and a given $F(.)$ and $\mu(.)$ satisfying Assumption A1 and A2, there exists an open set of the parameter space (k, y) such that $g = 1 - \mu(\gamma^{CE})F(\hat{\theta}^{CE}) - \lambda^{CE} < 0$. In particular such a set of (k, y) is characterized by $\gamma^{CE} \to 0$ and $\hat{\theta}^{CE} \to \bar{\theta}$. Notice that, from equation (22), there exists a continuous and invertible mapping between $\hat{b}$ and b such that $(1-\beta)\,\alpha U^{CE} = \hat{b}$, so that for any open set of $\hat{b}$, there exists a correspondent open set of b. The

last thing left to check is that $b > 0$, which is the case whenever

$$\hat{b} > \alpha^2 \beta \mu \left(\gamma^{CE}\right) \int_{\underline{\theta}}^{\hat{\theta}^{CE}} [y - \theta] \, dF(\theta) - \gamma^{CE} k. \tag{48}$$

Notice that when $\gamma^{CE} \to 0$ and $\hat{\theta}^{CE} \to \bar{\theta}$ the right-hand side of the equation is zero, competing the proof.

Case 2. $\mu(\gamma)$ satisfies A1 and A3.

An analogous argument to the proof of the previous case, using the case 2 of Proposition 6, shows that for given $\hat{b}$, $F(.)$ and $\mu(.)$ satisfying Assumption A1 and A3, there exists an open set of the parameter space (k, y) such that $g < 0$. Now such a set of (k, y) is characterized by $\gamma^{CE} \to \bar{\gamma}$ and $\hat{\theta}^{CE} \to \bar{\theta}$. Moreover, given the expressions for (k, y) in the proof of Proposition 6, I can choose an open set of $\hat{b}$ such that

$$\hat{b} > \frac{\alpha^2 \beta}{1 - \alpha^2 \beta} \left[\bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta) \right] - \frac{1}{f(\bar{\theta})} \lim_{(\varepsilon, \delta) \to (0,0)} \lambda(\varepsilon, \delta),$$

where $\lambda(\varepsilon, \delta)$ does not depend on $\hat{b}$, competing the proof.

Proof of Proposition 14. The Proof proceeds by contradiction: assume that the competitive equilibrium is constrained efficient and then show that it is impossible. Note that if the competitive equilibrium allocation $\hat{\theta}^{CE}$, γ^{CE}, V^{CE}, U^{CE} is constrained efficient for a given initial value of unemployment rate u_0, then it must solve the social planner problem, that is, it must satisfy the first order conditions:

$$\alpha(y - \hat{\theta}) + V' - U' - \alpha(1 - \eta) \frac{F(\hat{\theta})}{f(\hat{\theta})} - \pi = 0, \tag{49}$$

$$\begin{aligned} & \beta \mu'(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \left[\alpha(y - \theta) + V' - U' - \alpha(1 - \eta) \frac{F(\theta)}{f(\theta)} - \pi \right] dF(\theta) \\ = \; & k + \beta \mu'(\gamma)(1 - \eta)\left(b - C^U\right), \end{aligned} \tag{50}$$

$$\chi = u(1 - \eta)(1 - \beta \mu(\gamma)), \tag{51}$$

$$\nu \leq 1 \text{ when } C^V \geq 0, \tag{52}$$

$$\beta P_V' + u \beta \mu(\gamma) F(\hat{\theta}) + \nu(1 - u) \beta (1 - s) = 0, \tag{53}$$

$$\beta P_U' - u \beta \mu(\gamma) F(\hat{\theta}) + \nu(1 - u) \beta s + \eta u \beta = 0, \tag{54}$$

$$P_u' = \pi, \tag{55}$$

where the multipliers η, π, χ, ν must be such that

$$V = C^V + \beta s U' + \beta (1-s) V', \tag{56}$$

$$U = C^U + \beta \mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \frac{F(\theta)}{f(\theta)} dF(\theta) + \beta U', \tag{57}$$

$$u' = u \left[1 - \mu(\gamma) F(\hat{\theta}) \right] + (1-u) s, \tag{58}$$

$$C^U \geq b. \tag{59}$$

Moreover, recall that when $k > 0$ the informational constraint is binding, that is, it must be true that

$$\int_{\underline{\theta}}^{\overline{\theta}} e(\theta) \left[\alpha \left(c(\theta) - \theta - \frac{F(\theta)}{f(\theta)} \right) + V' - U' \right] dF(\theta) + C^U - b = 0 \tag{60}$$

and at the optimum

$$P = u \left[\beta\mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \alpha (y - c(\theta)) dF(\theta) + b - \gamma_t k - C^U \right] - (1-u) C^V + \beta P'. \tag{61}$$

Finally, the Envelope conditions are

$$P_U = -u\eta, \tag{62}$$

$$P_V = -(1-u)\nu, \tag{63}$$

$$\begin{aligned} P_u &= \beta\mu(\gamma) \int_{\underline{\theta}}^{\hat{\theta}} \left[\alpha(y-\theta) + V' - U' - \frac{F(\theta)}{f(\theta)} \right] dF(\theta) \\ &\quad + \left[(1-\beta\mu(\gamma)) \left(b - C^U\right) - \gamma k \right] + C^V + \beta\pi \left[1 - \mu(\gamma) F(\hat{\theta}) - s \right]. \end{aligned} \tag{64}$$

Now guess that the steady state competitive search equilibrium $\hat{\theta}^{CE}$, γ^{CE}, u^{CE}, V^{CE}, U^{CE}, $C^U = b$, $C^V = 0$ and $B = 0$ is a solution to this problem.
Moreover, recall that $\hat{\theta}^{CE}$, γ^{CE}, U^{CE}, V^{CE}, u_t^{CE} and the normalized multiplier λ^{CE} must satisfy $C^U = b$, $C^V = P = 0$ and solve the system of equations

$$\alpha(y - \hat{\theta}^{CE}) + V^{CE} - U^{CE} - \alpha\lambda^{CE} \frac{F(\hat{\theta}^{CE})}{f(\hat{\theta}^{CE})} = 0, \tag{65}$$

$$\beta\mu'\left(\gamma^{CE}\right) \int_{\underline{\theta}}^{\hat{\theta}} \left[\alpha(y - \theta - (1-\beta) U^{CE}) - \alpha\lambda^{CE} \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = k, \tag{66}$$

$$\beta\mu\left(\gamma^{CE}\right) \int_{\underline{\theta}}^{\hat{\theta}} \left[\alpha(y - \theta - (1-\beta) U^{CE}) - \alpha \frac{F(\theta)}{f(\theta)} \right] dF(\theta) = \gamma^{CE} k, \tag{67}$$

$$U^{CE} = \frac{\alpha\beta\mu\left(\gamma^{CE}\right)\int_{\underline{\theta}}^{\hat{\theta}^{CE}}\left[y-\theta\right]dF\left(\theta\right)+b-\gamma^{CE}k}{\left(1-\beta\right)\left[1+\alpha\beta\mu\left(\gamma^{CE}\right)F(\hat{\theta}^{CE})\right]}, \tag{68}$$

$$V^{CE} = \frac{\beta s}{1-\beta\left(1-s\right)}U^{CE}, \tag{69}$$

and

$$u_{t+1} = u_t\left[1-\mu\left(\gamma^{CE}\right)F(\hat{\theta}^{CE})\right]+\left(1-u_t\right)s.$$

First, combining equations (49) and (50) with equations (65) and (66), it follows

$$\begin{cases} \alpha\left(1-\eta-\lambda^{CE}\right)\frac{F(\hat{\theta}^{CE})}{f(\hat{\theta}^{CE})}+\pi=0 \\ \alpha\left(1-\eta-\lambda^{CE}\right)\int_{\underline{\theta}}^{\hat{\theta}}\frac{F(\theta)}{f(\theta)}dF\left(\theta\right)+\pi\int_{\underline{\theta}}^{\hat{\theta}^{CE}}dF\left(\theta\right)=0 \end{cases}.$$

Given that

$$\frac{F(\hat{\theta}^{CE})}{f(\hat{\theta}^{CE})}\left[\frac{\int_{\underline{\theta}}^{\hat{\theta}}\frac{F(\theta)}{f(\theta)}dF\left(\theta\right)}{\int_{\underline{\theta}}^{\hat{\theta}^{CE}}dF\left(\theta\right)}\right]^{-1}>1,$$

it must be that

$$\eta = 1-\lambda^{CE} \quad \text{and} \quad \pi = 0.$$

It follows that $\chi > 0$ and $C^U = b$ as we assumed.
Notice that, using the Envelope condition (62), I can rewrite equation (53) as

$$\nu_{t+1}\left(1-u_{t+1}\right) = u_t\mu\left(\gamma\right)F(\hat{\theta})+\nu_t\left(1-u_t\right)\left(1-s\right),$$

which for $\nu_{t+1} = \nu_t = \nu^{CE}$ and $\gamma = \gamma^{CE}$, $\hat{\theta} = \hat{\theta}^{CE}$, using the law of motion for u, yields

$$\left(1-\nu^{CE}\right)\left[u_{t+1}-u_t-\left(1-u_t\right)s\right]=0,$$

implying $\nu^{CE} = 1$, as long as job creation is different from zero. Moreover, using the Envelope condition (63), I can rewrite equation (54) as

$$\left(1-\eta_{t+1}\right)u_{t+1} = \left(1-\eta_t\right)u_t+\left(1-\nu_t\right)\left(1-u_t\right)s,$$

which, when $\nu^{CE} = 1$, $\gamma = \gamma^{CE}$, $\hat{\theta} = \hat{\theta}^{CE}$, $1-\eta_{t+1} = 1-\eta_t = \lambda^{CE}$, using the law of motion of u, yields

$$\left(u_t^{CE}-u_{t+1}^{CE}\right)\lambda^{CE}=0.$$

A contradiction follows immediately as long as the unemployment rate is not at the steady state value and λ^{CE} is different from zero, which is the case whenever $k > 0$, as we assumed. This implies that, away from the steady state, the competitive search equilibrium is constrained inefficient completing the proof.

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