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A GUIDED TOUR

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ABSTRACT

This essay surveys the literature on directed/competitive search, covering theory and applications in, e.g., labor, housing and monetary economics. These models share features with traditional search theory, yet differ in important ways. They share features with general equilibrium theory, but with explicit frictions. Equilibria are typically efficient, in part because markets price goods plus the time required to get them. The approach is tractable and arguably realistic. Results are presented for finite and large economies. Private information and sorting with heterogeneity are analyzed. Some evidence is discussed. While emphasizing issues and applications, we also provide several hard-to-find technical results.

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1 Introduction

Search theory contributes significantly to fundamental and applied research in economics, and is relevant for understanding many phenomena that are troublesome for classical theory. Examples include the coexistence of unemployment and vacancies; price or wage dispersion and stickiness; bid-ask spreads; the difficulties of bilateral trade that generate a role for money and related institutions; partnership formation; and long and variable durations in the time to execute trades in labor, housing and other markets. This essay surveys, consolidates and extends a relatively recent and, we think, a particularly interesting branch of the field called directed, or competitive, search.[1]

Consider any two-sided market with, e.g., buyers and sellers, firms and workers, borrowers and lenders, or men and women. They are trying to get together, usually in pairs, but sometimes multilaterally. Traditional search theory typically assumes the agents meet bilaterally and at random, although whether a meeting results in matching (trading or forming a relationship) can be endogenous, especially when there is heterogeneity. Directed search is different because agents have information to target their search towards particular types, or sometimes particular individuals, in the market. Moreover, traditional search models usually assume the terms of trade are determined by bargaining, or some related mechanism, after agents meet. Competitive search is again different because the terms of trade – prices, or more generally, contracts or mechanisms – are announced or posted in advance to attract agents on the other side of the market.

The label competitive search here means models with two characteristics: (a) the terms of trade are posted by agents in advance of meetings; and (b) these

[1]Here is a more or less random sample of the older literature on random search: On goods markets, see Burdett and Judd (1983), Rubinstein and Wolinsky (1987), Shi (1995) or Trejos and Wright (1995). On labor, see Mortensen and Pissarides (1994), Burdett and Mortensen (1998) or Pissarides (2000). On marriage, see Mortensen (1988), Burdett and Coles (1997) or Shimer and Smith (2000). These models can have either bargaining or price/wage posting, but posted terms do not attract counterparties as they do with directed search. The rest of this essay goes into considerable detail on how this matters.

terms direct search and hence help determine who meets whom.[2] As well as being a different philosophical approach to the study of markets, the combination of posting and directed search makes a substantial difference for substantive issues. In particular, posted prices have an allocative role, giving agents incentives to seek out particular counterparties, and this often leads to efficiency (it is sometimes said the models internalize search externalities). In addition, posting entails commitment, and this circumvents holdup problems with bargaining – which is not to say that bargaining is uninteresting, but it is good to consider alternatives. From another perspective, models with the characteristics (a) and (b) can dispense with some exogenous features from traditional search: they typically avoid having to specify a bargaining solution; and some of the models, but not all, avoid the need for a matching function. Moreover, the theory is tractable, often delivers cleaner results than alternatives, and bridges gaps between traditional search, general equilibrium and game theory.

The approach is also arguably realistic. As Howitt (2005) says, "In contrast to what happens in [random] search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters." While realism is not a unique desideratum, one could say he has a point. Even more colorfully, Hahn (1987) says, "someone wishing to exchange his house goes to estate agents or advertises – he does not, like some crazed particle, wait to bump into a buyer." And Prescott (2005) says "I think the bilateral monopoly problem has

[2]While not the first models in this class, as evidenced by Peters (1984,1991), Montgomery (1991) and other work discussed below, Moen (1997) and Shimer (1996) started a take-off phase in especially applied research, and they use the language in this way. The working papers were both 1995, as was the original version of Mortensen and Wright (2002), who according to Shimer (1996) "coined the phrase competitive search equilibrium." While we like this terminology, another meaning of the competitive search label is discussed below.

been solved. There are stores that compete. I know where the drug store and the supermarket are, and I take their posted prices as given. If some supermarket offers the same quality of services and charges lower prices, I shop at that lower price supermarket." Whether or not they realize it, these commentators are all more or less describing directed/competitive search.

There is another use for the competitive search label.[3] Research in the area often makes a concerted effort to analyze strategic aspects of markets with finite numbers of agents. One can show in several settings that, as these numbers get large, strategic considerations vanish. Sometimes a competitive search model means a limiting large economy, in the same sense a competitive Walrasian model means the set of traders is big enough to reasonably posit price taking. We present large markets, finite markets and limiting results. The limiting results are nice because we often have intuition about markets becoming competitive when the set of agents is large, but formalizing this can be difficult (see Gale 2000 and references therein). Directed search cleanly demonstrates the idea despite being far from Walrasian in the following sense: frictions take center stage, even when the set of agents is large. In particular, some sellers can have few or no customers, while others have more than they can handle, leading to rationing, unsold inventories, the coexistence of vacancies and unemployment, etc.

The theory captures a simple yet powerful idea: if you post more favorable terms customers come to you with higher probability, but not necessarily probability 1, due to capacity issues. If a restaurant only has a certain number of tables, or a firm only wants to hire a certain number of workers, it may not be smart to go where everyone else goes. Agents on both sides of the market face thus a trade off between prices and probabilities. While the theory, like general equilibrium theory, is concerned with the operation and efficiency of markets, it goes further by pricing not only quantities but also the time required to trade. This often delivers unique outcomes with remarkable efficiency properties, in con-

[3] We think this usage goes back to Peters (1994*b*), in spirit, although to be precise he called it "competitive matching equilibrium."

trast to traditional search theory, which is typically rife with inefficiencies. Yet competitive search can also accommodate complications, like private information or liquidity frictions, that may lead to multiplicity or inefficiency.

Section 2 begins with one-period models to convey the basic insights. Two versions are presented, one framed in terms of labor and the other in terms of goods markets, because, even though they are basically re-labelings, the economic interpretations and applications are different. Here we also introduce tools like matching technologies, and analyze efficiency as well as comparative statics. Section 3 embeds the market in dynamic general equilibrium to discuss phenomena such as price dispersion and stickiness, as well as on-the-job search. Section 4 presents applications in monetary economics in environments with indivisible assets, which are crude yet allow one to make important points relatively easily, and in more modern versions with divisible assets. Here the framework is natural, tractable, and complements nicely random search models of money and credit.

While Sections 2-4 start with large numbers of agents, Section 5 goes deeper into microfoundations by starting with finite markets, which imparts addition insights, then considers limits when the numbers become large. Among other reasons, this is useful because it makes clear how the relevant equilibrium concept relates subgame perfection. Section 6 goes into detail on heterogeneity and sorting by asking, who matches/trades with whom? Section 7 takes up private information, where directed search has recently proved rather useful. Section 8 expands on how agents meet and the implications for mechanism design. Section 9 covers miscellaneous other topics, including empirical evidence. Section 10 concludes. Appendices provide useful technical material that is hard to find elsewhere. In terms of style, we provide details for a few benchmark models, then discuss more or less formally extensions and applications in the literature, plus unsolved problems and directions for future research.[4]

[4]While there is no previous survey on directed search, surveys on other topics – including labor, money and housing – touch on it; see King (2003), Rogerson et al. (2005), Shi (2008), Han and Strange (2015) and Lagos et al. (2016).

2 Benchmark Models

We begin with a canonical competitive search model, with only one period, applied first to goods markets then to labor markets.

2.1 Goods Markets

There are large numbers of two types of agents called buyers and sellers, with measures N_b and N_s, and $N = N_b/N_s$ denotes the population buyer-seller ratio. One can think of buyers as households, or consumers, and sellers as retailers, but other interpretations are possible (e.g., producers buying inputs from suppliers). There are two tradable objects. There is an indivisible good q, and sellers can produce exactly one unit at cost $c \geq 0$, while buyers want to consume exactly one unit for utility $u > c$; and there is a divisible good x that anyone can produce at cost $C(x) = x$ and consume for utility $U(x) = x$. This means there are gains from trade in q, while x serves as a payment instrument buyers use to compensate sellers for their output. This can be interpreted as direct barter; more typically in the literature it is called transferable utility.[5]

For now each seller posts a price p, the amount of x buyers must pay to get q; later sellers may post something more complicated. Each buyer directs his search after observing all posted prices (all we really need is that each buyer observes at least two; see Acemoglu and Autor 2016, Theorem 13.4). For now, traders meet pairwise. In particular, suppose a set of buyers with measure n_b direct their search toward a set of sellers with measure n_s. Then the probability a seller meets a buyer is $\alpha_s = \alpha(n)$, where $n = n_b/n_s$ is the buyer/seller ratio, also called the queue length or market tightness. Similarly, the probability a buyer meets a seller

[5] Sometimes x is called money or numeraire, but these are abuses of language we cannot condone. It is obviously not money in a serious sense. It is also not a numeraire, which is a good with price normalized to 1 in the Walrasian budget equation. There is no numeraire in standard search models, which do not actually have budget equations. Below we present models that explicitly incorporate numeraire goods and money. The interpretation here, where buyers produce and sellers consume a good, with linear cost and utility, respectively, describes an internally consistent environment; it does not justify calling x numeraire or money.

is $\alpha_b = \alpha(n)/n$. As is standard, assume $\alpha_s = \alpha(n)$ is increasing and concave. Some people also assume $\alpha_b = \alpha(n)/n$ is decreasing, although that is automatic when $\alpha(n)$ is increasing and concave, given the natural restriction $\alpha(0) = 0$. In static models or discrete-time dynamics α_b and α_s are probabilities, so we impose $0 \leq \alpha_j \leq 1$; in continuous-time they are arrival rates, so we only impose $\alpha_j \geq 0$. We also usually assume differentiability, and sometimes $\lim_{n\to 0} \alpha'(n) = \infty$.

To understand this formulation, consider any two-sided market with n_1 and n_2 agents on each side, where the number of bilateral meetings between types 1 and 2 is $\mu = \mu(n_1, n_2)$. Analogous to a production function mapping inputs into output, μ is increasing, concave and CRS (constant returns to scale). Then $\alpha_1 = \mu(n_1, n_2)/n_1 = \mu(n, 1)/n$, where $n = n_1/n_2$, and $\alpha_2 = \mu(n_1, n_2)/n_2 = \mu(n, 1)$. This generalizes models of one-sided markets (e.g., Diamond 1982), and is more interesting because the α's depend on tightness even with CRS. In addition, with a two-sided specification it is natural to endogenize tightness by allowing entry on one side, a vital component in many applications. In any case, for now, a buyer seeks a seller with a particular p, but whether he finds one is random.[6]

A set of sellers posting the same p and buyers searching for them constitutes a *submarket* with tightness $n = n_b/n_s$. Thus, a submarket is characterized by (p, n). Buyers and sellers payoffs are denoted V_b and V_s. Sellers maximize V_s by posting (p, n), although it is not crucial that they post n – sellers can equivalently post only p and let buyers work out the equilibrium n for themselves. In any case, for a seller to be in business, (p, n) must deliver to buyers at least their market payoff V_b, and clearly he does not deliver more. While V_b is an equilibrium object, is taken as given by individuals. This is called the *market utility approach*, used by Montgomery (1991), McAfee (1993), Shimer (1996), Moen (1997) and many others; Peters (2000), based on Peters (1991), derives it from microfoundations, as do Julien et al. (2000) and Burdett et al. (2001), as discussed in Section 5.

[6] In Section 5 an agent finds a counterparty for sure, but may or may not trade, due to capacity constraints; those models do not need an exogenous meeting function.

The market utility approach allows us to write the sellers problem as

$$V_s = \max_{p,n} \alpha(n)(p-c) \text{ st } \frac{\alpha(n)}{n}(u-p) = V_b. \tag{1}$$

Sellers' payoff in a submarket is their trading probability times their surplus $S_s = p - c$, while buyers' payoff is their trading probability times their surplus $S_b = u - p$. In problem (1), V_b is taken as given, but it is determined below by equilibrium. We defer a more rigorous definition of equilibrium to a more general model below but the idea is basically optimization and market clearing: sellers maximize V_s subject to buyers getting V_b; and the n emerging from (1) is consistent with the set of buyers and sellers in the market. Notice sellers can get the same V_s from lower p if n is higher, and buyers can get the same V_b from higher p if n is lower. These trade-offs are a quintessential element of the theory.

One way to solve (1) is to rearrange the constraint as $p = u - nV_b/\alpha(n)$, and substitute this into the objective function to get

$$V_s = \max_{n} \{\alpha(n)(u-c) - nV_b\}. \tag{2}$$

This problem has a unique solution.[7] If the solution is interior it satisfies the FOC $\alpha'(n)(u-c) = V_b$. Then, given n, the constraint yields p uniquely, so any active submarkets must have the same (p, n). Given CRS, therefore, without loss of generality we can collapse all submarkets into one.

There are two standard ways to proceed. The first is to assume N_b and N_s are fixed. Then the equilibrium buyer-seller ratio must be the same as the population ratio, $n = N$ (market clearing). The FOC then implies $V_b = \alpha'(N)(u-c)$, and the constraint implies $p = u - NV_b/\alpha(N)$, or

$$p = \varepsilon c + (1-\varepsilon)u, \tag{3}$$

[7] Appendix A considers a generalization without perfectly transferable utility: if a buyer makes a payment p to a seller, the latter gets $\nu(p)$ while former gets $-\gamma(p)$; of course, (1) is the special case $\nu(p) = \gamma(p) = p$. Using Lagrangians, we show the SOC's hold at any solution to the FOC's, so if there is an interior solution it must be unique. However, in general, one has to check for a corner solution, where $\alpha(n)$ or $\alpha(n)/n$ hit 0 or 1, or at least assume the meeting technology is such that $\alpha(n), \alpha(n)/n \in (0,1)$ $\forall n$ to avoid corners.

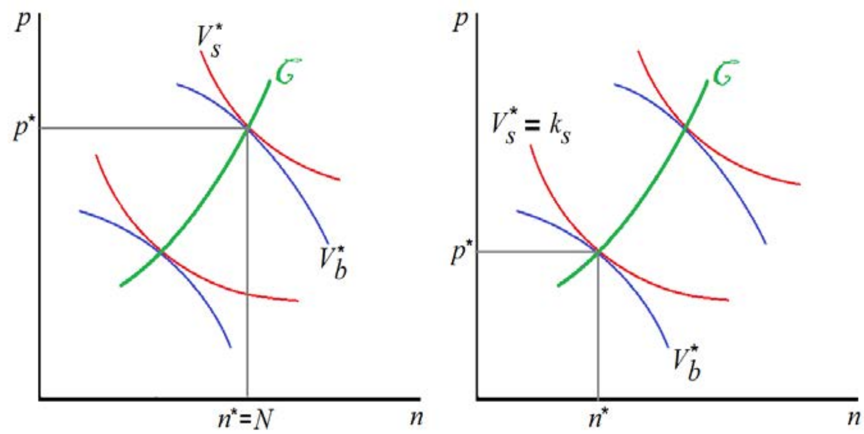

Figure 1: Equilibrium with (right) and without (left) entry by sellers

where $\varepsilon = \varepsilon(n) \equiv n\alpha'(n)/\alpha(n)$ is the elasticity of $\alpha(n)$ wrt tightness. Hence, price is a weighted average of cost and utility that splits the ex post (after meeting) surplus $S = u - c$ according to $S_b = \varepsilon(u - c)$ and $S_s = (1 - \varepsilon)(u - c)$. The ex ante (before meeting) payoffs can now be written $V_b = \alpha_b \varepsilon (u - c)$ and $V_s = \alpha_s (1 - \varepsilon)(u - c)$. This uniquely pins down the equilibrium $\langle p, n, V_b, V_s \rangle$.

The second way to proceed is to assume one side has a cost to participate, and therefore, in general, only some of them enter the market. Suppose it is sellers that have a participation cost k_s. Then in equilibrium, as long as k_s is neither too big nor too small relative to N_s, some but not all sellers enter, and we have the free entry condition $V_s = k_s$. As above, the FOC implies $V_b = \alpha'(n)(u - c)$ and the constraint implies $p = \varepsilon c + (1 - \varepsilon) u$. Now $k_s = V_s = \alpha(n)(p - c)$, from which we get n. Once again these conditions uniquely pin down $\langle p, n, V_b, V_s \rangle$.

Fig. 1, a version of which appears in Peters (1991), shows the "Edgeworth box" in (p, n) space. Indifference curves for buyers slope down, because they are willing to pay higher p if n is lower, so they can trade faster. Similarly, sellers are willing to accept lower p if n is higher. As in elementary microeconomics, efficient outcomes are points of tangency, tracing out the contract curve $\mathcal{C}$. The left panel depicts the case without entry, where $\mathcal{C}$ crosses $n = N$; the right depicts the case with entry by sellers, where $\mathcal{C}$ crosses the indifference curve $V_s = k$.

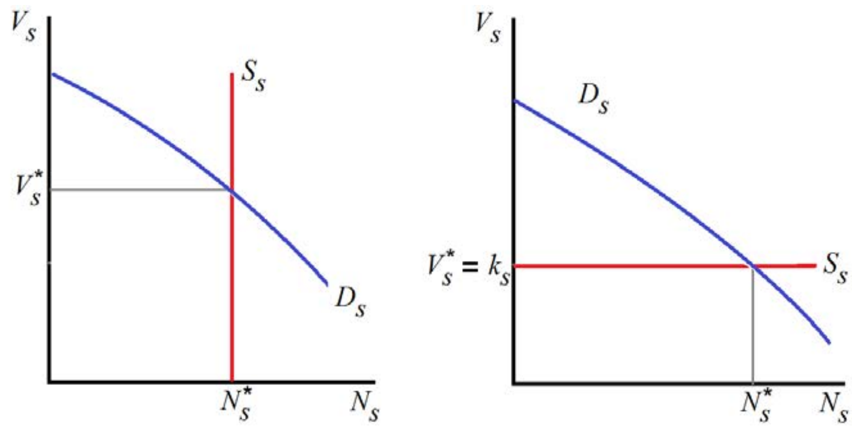

Figure 2: Equilibrium with (right) and without (left) entry by sellers

By way of comparison, consider the dual to (1),

$$V_b = \max_{p,n} \frac{\alpha(n)}{n}(u-p) \text{ st } \alpha(n)(p-c) = V_s, \tag{4}$$

which looks as if buyers post and sellers search.[8] One can check (4) leads to the same conditions for $\langle p, n, V_b, V_s \rangle$ as (1), with n fixed or with entry. Hence, it does not matter if buyers post and sellers search or vice versa (this is not always true, but a sufficient condition satisfied here is that the number of meetings given n_b buyers and n_s sellers does not depend on who searches; see fn. 32 below). There is a third interpretation, with third parties called market makers designing submarkets by posting (p, n) to attract buyers and sellers. In Moen (1997), there is a single market maker, somewhat like an auctioneer, perhaps, while in Mortensen and Wright (2002) there are competing market makers trying to extract entrance fees from participants, but competition drives the fees to 0. The outcome is the same. Hence it does not matter here who posts, buyers, sellers or market makers (again, this is not always true, as discussed below).

Fig. 2, used to describe competitive search equilibrium in Rocheteau and Wright (2005), depicts the solution to (4) as a "demand" for sellers N_s as a

[8]The methods in Appendix A can be used to show the SOC's hold at any solution to the FOC's, so the solution to (4) is unique, even in the generalization with payoffs $\nu(p)$ and $\gamma(p)$ mentioned in fn. 7. Note that this only assumes $\alpha(n)$ is concave, not $\alpha(n)/n$.

function of the "cost" V_s (given N_b, choosing n is the same as N_s). One can check "demand" is decreasing and, as shown, hits 0 at finite V_s. Without entry, in the left panel "supply" is vertical and equilibrium determines V_s. With entry by sellers, in the right panel "supply" is horizontal at k_s and equilibrium determines N_s. Indeed, one could nest these with a general upward-sloping "supply" curve by letting k_s vary with the number of homogeneous entrants, or across heterogeneous potential entrants. The point of Fig. 2, like Fig. 1, is that the theory can be described using tools from elementary microeconomics.[9]

Now consider a planner's problem with endogenous participation by sellers,

$$\max_n \left\{ \frac{\alpha(n)}{n}(u-c) - \frac{k}{n} \right\}. \tag{5}$$

The first term is the expected surplus per buyer; the second is the total entry cost of sellers per buyer, since $n_s/n_b = 1/n$. Notice something interesting: if we eliminate p from the objective function in (4) using the constraint, we get exactly the same problem as (5). Hence, the solution is the same, and this means equilibrium equilibrium is efficient.

For yet another comparison, consider bargaining instead of posting. One interpretation is that there in no communication outside of meetings, so agents cannot post terms to attract counterparties. Another is that it agents cannot commit, so that even if they can communicate talk is cheap, although this is subtle (Menzio 2007; Doyle and Wong 2013; Dutu 2013; Kim and Kircher 2015; Stacey 2016a,b). In any case, when a buyer and seller meet they now determine p by generalized Nash bargaining,

$$\max_p (u-p)^{\theta} (p-c)^{1-\theta}, \tag{6}$$

where θ is buyer bargaining power. The solution is $p = \theta c + (1-\theta) u$, which is

[9] Rocheteau and Wright (2005) study a generalization where "demand" in Fig. 2 may not be single valued or continuous, and N_s might jump downward as V_s increases. With a horizontal "supply," as in the right panel of Fig. 2, if N_s jumps existence is not a problem, and uniqueness holds at least generically. Existence may seem to be a problem with a vertial "supply," because it might hit a gap between two values of N_s when it jumps; in that case, there is an equilibrium with two submarkets posting different (n, q) but yielding the same payoff.

the same as p under posting, and hence efficient, iff $\theta = \varepsilon$. This is the well-known Hosios (1990) condition: efficiency obtains iff agents' bargaining powers are equal to the elasticity of the meeting technology with respect to their participation.[10] Hence, sellers should get a share of $u - c$ commensurate with their contribution to matching. Since this is exactly what competitive search delivers, it is often said that it induces the Hosios condition endogenously.

If $n = N$ is fixed, one can check $\partial V_s/\partial N > 0$ and $\partial V_b/\partial N < 0$, naturally, while $\partial p/\partial N \approx -\varepsilon'$ where "$a \approx b$" means "a and b have the same sign." Now $\varepsilon' < 0$, and hence $\partial p/\partial N > 0$, for many common meeting technologies but not all. Appendix E shows $\varepsilon'(n) \gtreqless 0 \Leftrightarrow \sigma(n) \gtreqless 1$, where $\sigma(n)$ is the elasticity of substitution. Consider a CES technology, $\mu(n_1, n_2) = (n_1^\gamma + n_2^\gamma)^{1/\gamma}$, $\gamma \in (-\infty, 1)$, where $\sigma = 1/(1-\gamma)$. Then $\gamma > 0 \Rightarrow \varepsilon' > 0$, $\gamma < 0 \Rightarrow \varepsilon' < 0$, and, in the Cobb-Douglas case, $\gamma = 0 \Rightarrow \varepsilon' = 0$. Does $\partial p/\partial N < 0$ make sense? Yes. To see why, first note that higher N unambiguously increases V_s and decreases V_b. These payoffs can change due to either changes in p or in the trading probabilities. By construction, $\alpha(N)$ goes up and $\alpha(N)/N$ down with N, but if they move a lot, p must go down so the changes in V_s and V_b are not too big. Hence an increase in demand along the extensive margin (higher n) can lower price, even if one can show an increase in demand along the intensive margin (higher u) implies $\partial p/\partial u > 0$ unambiguously. Similarly, with seller entry, higher k_s reduces N_s and raises n, also implying $\partial p/\partial k_s \approx -\varepsilon'$ and $\partial p/\partial u > 0$.

The finding that price might fall when the buyer-seller ratio goes up, either exogenously, or in response to changes in other factors, reflects the big idea that resource allocation is guided by both prices and probabilities. This encompasses standard general equilibrium theory, which relies exclusively on prices, and standard search theory, which relies mainly if not exclusively on probabilities, will be recurring theme in what follows.

[10] Earlier versions of this idea are in Mortensen (1982a, b) and Pissarides (1986). Hosios (1990) also shows a simple directed search model yields efficiency endogenously. Mangin and Julien (2016) provide a recent generalization and more references.

2.2 Labor Markets

Now let households be sellers, of their *time*, and firms buyers. Each firm wants to hire exactly one worker, while each household wants to land one job. Thus, n is the vacancy-unemployment ratio. Again, it does not matter here who posts and who searches. Consider a version of (1) that maximizes workers' payoffs,

$$V_s = \max_{w,n} \alpha(n)(w-b) \text{ st } \frac{\alpha(n)}{n}(y-w) = V_b, \tag{7}$$

where y is output per worker and b is the value of unemployment benefits, leisure and home production sacrificed by taking a job. Here y, b and w play the roles of u, c and p in the goods market.

Emulating Section 2.1, with $n = N$ fixed, we get $w = \varepsilon b + (1-\varepsilon)y$, $V_s = \alpha_s(1-\varepsilon)(y-b)$ and $V_b = \alpha_b\varepsilon(y-b)$. And with entry by buyers (the firms in this application), we get a similar outcome except n is endogenous and $V_b = k_b$. With $n = N$ fixed we have $\partial w/\partial N \approx -\varepsilon'$, and with entry we have $\partial n/\partial k_b < 0$ and $\partial w/\partial k_b \approx \varepsilon'$. If $\varepsilon' < 0$ then w goes up when with tightness, as one might expect, but that is not true in general, as explained above for goods markets. As other features of goods markets also carry over, we proceed to applications.

Albrecht et al. (2006), Galenianos and Kircher (2009) and Kircher (2009) let workers apply for more than one job.[11] If workers can apply to $v \in \{1, 2, ...\}$ vacancies, then it turns out there will be v distinct wages posted, and the optimal search strategy is to apply to one of each – i.e., to look for work simultaneously in v distinct submarkets. Hence, the model exhibits wage dispersion with homogeneous agents, as is relevant because a large part of empirical wage variation cannot be explained by observables (Abowd et al. 1999; Mortensen 2003). Also, consistent with the evidence, the density of posted wages can be shown to be decreasing, while by way of contrast, in models based on Burdett and Mortensen

[11] A difference in approach is this: in Albrecht et al. (2006), if two or more firms make offers to the same worker they compete à la Bertrand (see also Albrecht et al. 2003,2004); in Galenianos and Kircher (2009) or Kircher (2009), and in our presentation, firms commit to posted wages. This does not affect the efficiency results discussed below.

(1998), with homogeneous agents the density is increasing. Also, again consistent with conventional wisdom, firms offering higher wages receive more applications.

Allowing multiple applications introduces an element of portfolio choice for workers, with low-wage applications serving to reduce the downside risk. This embeds in an equilibrium setting a version of Chade and Smith's (2006) marginal improvement algorithm. For a simplified exposition, consider $v = 2$, so there are two wages posted, w_1 and $w_2 \geq w_1$, with workers sending applications to two distinct submarkets. If both pan out, they accept the highest wage; if only one pans out, they take it. Their expected payoff is therefore

$$V_s = \max_{w_1, w_2} \left\{ \alpha(n_2)(w_2 - b) + [1 - \alpha(n_2)] \, \alpha(n_1)(w_1 - b) \right\}, \tag{8}$$

where n_j is the tightness in a submarket posting w_j.

Generalizing the above methods, in the low-wage submarket, we solve

$$V_{s1} = \max_{n_1, w_1} \alpha(n_1)(w_1 - b) \text{ st } \frac{[1 - \pi(w_1)] \, \alpha(n_1)}{n_1}(y - w_1) = V_b \tag{9}$$

where $\pi(w)$ is the probability a worker rejects w_1 if offered. This looks like the problem with $v = 1$, except for $\pi(w_1)$, since $v = 1$ implies workers always accept w_1 while here they might get a better offer. Given the solution to (9), we substitute V_{s1} into (8) to obtain the problem for the high-wage submarket

$$V_s = \max_{n_2, w_2} \left\{ \alpha(n_2)(w_2 - b - V_{s1}) + V_{s1} \right\} \text{ st } \frac{[1 - \pi(w_2)] \, \alpha(n_2)}{n_2}(y - w_2) = V_b. \tag{10}$$

This again looks like a problem with $v = 1$, but now the outside option is $b + V_{s1}$, not just b. Since a higher outside option raises the posted wage, this is indeed consistent with $w_2 > w_1$. Thus we support 2 posted wages.

As regards efficiency, in Galenianos and Kircher (2009), a worker who gets a job at a high wage still enters the queue at lower wages. In the $v = 2$ example, if a fraction ρ of firms post w_1 then $n_1 = \rho n_b / n_s$, $n_2 = (1 - \rho) n_b / n_s$, $\pi(w_1) = \alpha(n_2)$ and $\pi(w_2) = 0$. To characterize equilibrium, one solves (9) and (10) with ρ set so that V_b is the same in the two submarkets. The outcome is *not* efficient. Heuristically, since workers obtaining jobs at w_2 still enter the queue at w_1, they might prevent other workers from getting low-wage jobs (a congestion effect). Neither the firms posting high wages nor the workers who obtain them take this into account, implying an unpriced externality.

However, in Kircher (2009), workers who obtain a job at a high wage no longer enter the queue for low wages, to capture the idea that firms offering jobs to workers who reject should be able to continue making offers. This implies $\pi(w_1) = 0$ because any worker in the low-wage queue by construction does not have a high-wage offer. Again the equilibrium is found by solving the two submarket problems and adjusting ρ to ensure firms get the same payoff in each. It is no surprise that this arrangement is better than the one in Galenianos and Kircher (2009); it may be more surprising that it achieves full efficiency, but it does, because the unpriced externality disappears.

Multiple-application models have a structure similar to our benchmark, and the efficiency properties carry over if there are no unpriced externalities. In related work, Wolthoff (2014) constructs a model encompassing Kircher (2009) and Galenianos and Kircher (2009), and endogenizes firms' recruitment effort. Assessing the models' empirical performance, he concludes multiple submarkets are crucial for matching the data. Gautier and Holzner (2016) introduce a more sophisticated process to bid for workers after matching, so no vacancies remains idle because workers reject them to join firms with more applicants than they need. This leads to efficiency. It is also intuitively appealing that firms can break ties among identical workers.

2.3 Summary of the Baseline Models

Table 1 provides comparative statics for goods and labor markets in the benchmark model where agents can only search in one submarket. Reported are the effects on tightness, prices or wages, ex ante payoffs and ex post surpluses, for three cases: (a) fixed populations; (b) entry by sellers; and (c) entry by buyers. Most of these are unambiguous, but as explained above some of the effects on p

or w can go either way. A few cases report $+^*$ or $-^*$ to indicate that the signs are ambiguous, in general, but $+$ or $-$ in the somewhat common case $\varepsilon' \leq 0$. While the results are the same for goods and labor markets after re-labeling, both are provided to facilitate interpretation and avoid having to translate between, e.g., $\partial p/\partial c$ and $\partial w/\partial b$.

An analogous table for bargaining would be similar, except effects reported as ε' or $-\varepsilon'$ would be 0, and those reported as $+^*$ and $-^*$ would be $+$ and $-$. We can get that with a Cobb-Douglas meeting technology, which implies $\varepsilon' = 0$. In general, however, parameter changes affect ε and the trading probabilities by enough to move prices in ways that might be counterintuitive without understanding the theory. Under bargaining the terms of trade do not change with n, because while arrival rates affect expected payoffs, they do not affect the surpluses after traders meet, and hence are irrelevant in the negotiations. There is a caveat: in some dynamic models, as discussed below, n affects continuation values and hence the bargaining outcome. That is different from competitive search, where n affects the terms of trade even in a static environment.

Table 1.1: Goods Market

(a) $n = N$ fixed

	n	p	V_b	V_s	S_b	S_s
N	$+$	$-\varepsilon'$	$-$	$+$	ε'	$-\varepsilon'$
u	0	$+$	$+$	$+$	$+$	$+$
c	0	$+$	$-$	$-$	$-$	$-$

(b) entry by sellers (firms)

	n	p	V_b	V_s	S_b	S_s
k_s	$+$	$-\varepsilon'$	$-$	$+$	ε'	$-\varepsilon'$
u	$-$	$+$	$+$	0	$+^*$	$+$
c	$+$	$+^*$	$-$	0	$-^*$	$-$

(c) entry by buyers (households)

	n	p	V_b	V_s	S_b	S_s
k_b	$-$	ε'	$+$	$-$	$-\varepsilon'$	ε'
u	$+$	$+^*$	0	$+$	$+$	$+^*$
c	$-$	$+$	0	$-$	$-\varepsilon'$	$-^*$

Table 1.2: Labor Market

(a) $n = N$ fixed

	n	w	V_b	V_s	S_b	S_s
N	$+$	$-\varepsilon'$	$-$	$+$	ε'	$-\varepsilon'$
y	0	$+$	$+$	$+$	$+$	$+$
b	0	$+$	$-$	$-$	$-$	$-$

(b) entry by sellers (households)

	n	w	V_b	V_s	S_b	S_s
k_s	$+$	$-\varepsilon'$	$-$	$+$	ε'	$-\varepsilon'$
y	$-$	$+$	$+$	0	$+^*$	$+$
b	$+$	$+^*$	$-$	0	$-^*$	$-$

(c) entry by buyers (firms)

	n	w	V_b	V_s	S_b	S_s
k_b	$-$	ε'	$+$	$-$	$-\varepsilon'$	ε'
y	$+$	$+^*$	0	$+$	$+$	$+^*$
b	$-$	$+$	0	$-$	$-\varepsilon'$	$-^*$

This concludes the presentation of the basic static models. At the risk of appearing pedantic, to highlight the main economic results, let us formalize them as follows:

Proposition 1 *In the benchmark model, when agents can search in at most one submarket, with or without entry, there is a unique equilibrium and it has a single price or wage. This is efficient. When agents can simultaneously search in* $v > 1$ *submarkets, there is a unique equilibrium and it has* v *prices or wages. This is efficient if there are no congestion externalities.*

3 Extensions and Applications

We want to move beyond static theory for many reasons, but an important one is that equilibrium meeting probabilities translate into random durations between trades, central to the study of employment/unemployment spells in labor economics, and also of interest in markets for housing, assets, etc. For goods markets, we can simply repeat the static version, assuming households trade with different sellers all the time. As this is easy, we introduce additional features, including heterogeneity, and embed the market in general equilibrium. Simply repeating the static model is less compelling for labor, which typically involves long-term relationships, so the dynamic labor extension is more intricate.[12]

3.1 Goods Markets

We now embed the Section 2.1 model in dynamic general equilibrium. To do this easily, we follow Lagos and Wright (2005) and many subsequent papers by adopting the following structure: Each period in discrete time, infinitely-lived agents interact in two ways: first there is a decentralized market, or DM, just like the one analyzed above; then there is a frictionless centralized market, or CM, as

[12] As it is impossible to cover everything, we neglect the more complicated analysis of nonstationarity markets, where buyer/seller ratios change over time as agents drop out after trading; see Peters (1991,1994*b*) and McAfee (1993).

in general equilibrium theory. One reason for this is that, rather than finalizing trades for q by paying p in spot transactions, now buyers (sellers) incur (accept) obligations for payment in the next CM. This deferred settlement arrangement – i.e., credit – is more interesting than barter or transferable utility in many applications. Indeed, an alternating-market structure is used heavily in monetary economics, as discussed in Section 4, so it makes sense to introduce it now.

To this end, continue to let V_b and V_s be the DM value functions, and now let W_b and W_s be the CM value functions. In the CM, buyers solve

$$W_b(d) = \max_{x,\ell} \{U(x) - \ell + \beta V_b\} \text{ st } x = w\ell - d, \tag{11}$$

where β is the discount factor, x is the CM numeraire, ℓ is labor, w is the wage and d is debt brought over from the previous DM.[13] Also let us assume x is produced one-for-one with ℓ, so that in equilibrium $w = 1$. Then the solution to (11) has $x = x^*$, where $U'(x^*) = 1$, and $\ell = x^* - d$. The envelope condition is $W_b'(d) = -1$. The CM problem for sellers is omitted, but similar, and also implies $W_s'(d) = -1$ (although one should expect $d < 0$ for them).

For buyers, the DM payoff is

$$V_b = \alpha_b [u + W_b(p)] + (1 - \alpha_b) W_b(0) = \alpha_b (u - p) + W_b(0) \tag{12}$$

because $W_b(p) - W_b(0) = -p$, by the envelope condition. Similarly, for sellers

$$V_s = \alpha_s (p - c) + W_s(0). \tag{13}$$

Except for the constants $W_b(0)$ and $W_s(0)$, V_b and V_s are identical to the static model. Hence, extending the benchmark to dynamic general equilibrium is easy, but is still nice, because, e.g., higher $\alpha(n)$ now means sellers trade faster, or more

[13] Having one-period debt means we can avoid tracking the distribution of d across agents as a state variable, but this is without loss of generality given quasi-linear CM utility and interior solutions for ℓ. In fact, as in Wong (2016), we could use any CM utility function satisfying $U_{11}U_{22} = U_{12}^2$, including $U = x^a (1-\ell)^a$ or $U = [x^a + (1-\ell)^a]^{1/a}$; we use quasi-linearity only to ease notation. Also, it can be any CM good that enters linearly; we use ℓ because it is common in applications, and realistic since most people do pay off debt by working. Also, we can replace x by $\mathbf{x}$ in utility and by $\mathbf{px}$ in the budget equation, where $\mathbf{p}, \mathbf{x} \in \mathbb{R}_+^n$.

often, and not just with higher probability. This fits the theory neatly into the realm of duration analysis, which has proved useful in much empirical work (e.g., Devine and Kiefer 1991). In particular, the expected times for sellers and buyers to transact are $1/\alpha(n)$ and $n/\alpha(n)$, respectively.

The next step is to consider two types of buyers, with utilities u_1 and $u_2 > u_1$, and free entry of homogeneous sellers, in which case the market segments into two distinct submarkets, $j = 1, 2$.[14] Then (p_j, n_j) in submarket j is determined as above by

$$k_s = [\alpha(n_j) - n_j\alpha'(n_j)](u_j - c) \text{ and } p_j = c + k_s/\alpha(n_j). \tag{14}$$

Note that (14) holds for $j = 1, 2$ independently, a feature called *block recursivity.* It lets us first solve for n_j in each submarket j (block 1) regardless of what is happening in other submarkets; then the number of agents in each submarket is determined (block 2) so the total number of buyers sums to the number in the economy, and free entry of sellers ensures market tightness is correct. This comes up again, and is especially convenient in environments with shocks.

Given $u_2 > u_1$, one can check $n_2 < n_1$ and $p_2 > p_1$. Thus, high-valuation buyers go to submarket 2, where they pay more but trade faster. Sellers trade slower in submarket 2 and, in equilibrium, they are indifferent between it and submarket 1. This is shown in the left panel of Fig. 3, with buyers in submarkets 1 and 2 on indifference curves denoted V_1^* and V_2^*, both of which are tangent to the sellers' common indifference curve $V_s^* = k_s$.

Now consider homogeneous buyers and two seller types, now in fixed numbers N_1 and N_2, with c_1 and $c_2 > c_1$ but the same k.[15] Suppose c_j is not too big, so all sellers participate. As shown in the right panel of Fig. 3, the market segments

[14] This is discussed further in Section 6, where we go into heterogeneity and sorting in detail; here we simply provide illustrative examples. A general result is that there will not be two different types of buyers, or two different types of sellers, in the same submarket, but one type of buyer, or one type of seller, can go to multiple submarkets.

[15] Allowing sellers' k to differ (e.g., see Julien et al. 2006*a*), but otherwise keeping them homogeneous, has little effect: there will still be only one submarket open, as in the baseline model, but now we can say *which* sellers enter – those with k below a threshold k^*.

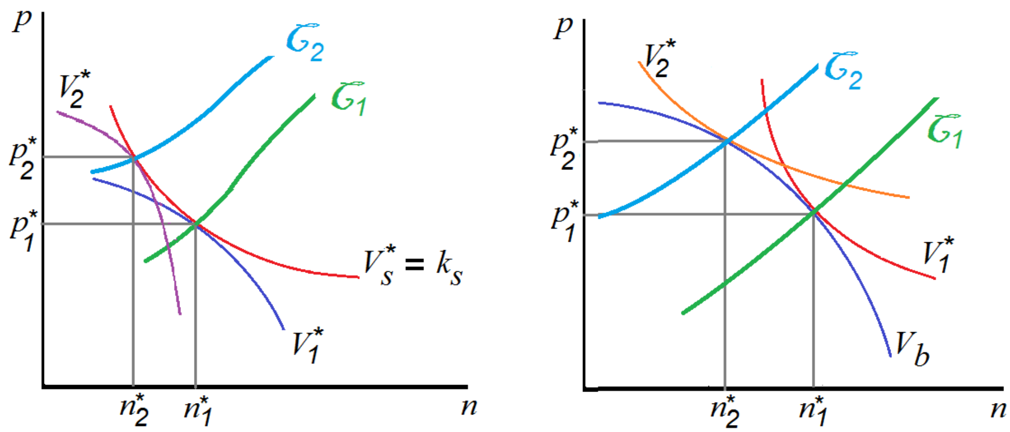

Figure 3: Heterogeneous buyers (left) or sellers (right)

into two submarkets where now buyers are indifferent between them. As usual, (p_j, n_j) is determined by

$$\alpha'(n_j)(u - c_j) = V_b \text{ and } p_j = \varepsilon(n_j) c_j + [1 - \varepsilon(n_j)] u.$$

Let us normalize $N_b = 1$ and let ξ be the fraction of buyers in submarket 1. Then the following buyer-indifference condition uniquely determines ξ:

$$\alpha'\left(\frac{\xi}{N_1}\right)(u - c_1) = \alpha'\left(\frac{1-\xi}{N_2}\right)(u - c_2),$$

where we recall that N_j is fixed here. One can check $n_2 < n_1$ and $p_2 > p_1$, so sellers in submarket 2 trade slower, while buyers trade faster but pay higher prices.

As these examples show, the theory easily accommodates deviations from the "law" of one price. With heterogeneous buyers, sellers in submarket 1 settle for p_1 even though other sellers are getting $p_2 > p_1$ in submarket 2. Why? Obviously because it takes longer to sell in submarket 2. With heterogeneous sellers, buyers in submarket 2 pay p_2 even though others are getting the good at $p_1 < p_2$ in submarket 1, for a similar reason. This is related to, yet different from, other theories of price dispersion. In Burdett and Judd (1983), e.g., buyers see a random number of prices simultaneously – they call this noisy search – and when they see more than one they pick the lowest. In equilibrium, ex ante identical sellers post different prices yet earn equal profits: those that post lower p earn less per unit, but make it up on the volume. That much is like our sellers, but Burdett-Judd buyers do not make a directed choice between paying less or trading faster, as they do here, so the economics is somewhat different.

Returning to heterogeneous buyers and homogeneous sellers, here is an application to housing based on Wright and Zhu (2017). There are a fixed number of homogeneous houses in the market, but buyers are heterogeneous, with the value to becoming a home owner distributed continuously across buyers with CDF $G(u)$ and support $[u_1, u_2]$. Now equilibrium involves a continuum of submarkets, one for each buyer type, indexed by (p_u, n_u) (this is treated more formally below). In the left panel of Fig. 3, there is now a submarket for *every point* on sellers' common indifference curve between (n_1^*, p_1^*) and (n_2^*, p_2^*), with higher u associated with higher p_u and lower n_u. Higher-valuation buyers search where their trading probabilities and prices are higher, while sellers are indifferent because of the common-sense notion that listing a house at a higher price means a longer average time on the market.[16]

Consider an expansion in demand in terms of a shift in $G(u)$ in the sense of first-order stochastic dominance. One can show this shifts the support of the price distribution to the right at least if $\varepsilon'(n) \leq 0$. It also increases n_1^* and decreases n_2^*, tightness in the highest- and lowest-price submarkets. Next consider an increase in the cost c of selling (e.g., taxes, realtor fees or apartment rents), interpretable as a contraction in supply. Again the price distribution shifts right, but now n_1^* decreases and n_2^* increases. With the total housing stock fixed, if N_s contracts the price distribution also shifts right, but n_1^* and n_2^* both rise. With entry by sellers, if k_s goes up the stock on the market contracts endogenously, with a similar impact.

[16] While it is no surprise that a big home in a nice neighborhood costs more than a small one in a bad neighborhood, the interest here is in residual price dispersion, the same way labor economists are interested in residual wage dispersion. Wright and Zhu (2016) cite empirical work finding non-negligible dispersion in listed prices for similar homes.

Disperse prices easily generate sticky prices. If market conditions change, as in the previous paragraph, the distribution reacts, but if the change is not too big the old and new supports overlap, and sellers with p in the overlapping range have *no incentive* to reprice. If demand falls, e.g., the distribution shifts left but many sellers can keep the same p – lowering it attenuates the fall in α_s but reduces profit in the long run, and equilibrium makes that a matter of indifference. This is relevant because people claim house prices look sticky in the data and find this puzzling. As Merlo et al. (2015) say, "conventional wisdom is that traditional, rational, forward-looking economic theories are unable to explain extreme price stickiness of this sort, unless there are large menu costs associated with price revisions." Others champion behavioral explanations. Here stickiness emerges without resorting to menu costs or behavioral economics. While Head et al. (2011) show similar stickiness emerges with noisy (Burdett-Judd) search, as argued above the economics of directed search is somewhat different.

Whether or not directed search explains every nuance of housing markets, it is useful for understanding various aspects. This is illustrated by Albrecht et al. (2016) in a setting where first sellers list prices, then a random number of buyers show up, as we discuss in Section 5. Each buyer can accept the listed price or make a counteroffer. If no buyers accept, the seller can accept or reject the best counteroffer. If exactly 1 buyer accepts, he gets the house at the listed price. If 2 or more accept, the seller runs an auction among them. This is consistent with empirical work (see Albrecht et al. 2016 for citations) showing that houses can sell at, above or below listed prices. More generally, it shows how directed search captures interesting features of housing markets.

In quantitative work, Diaz and Jerez (2013) build a directed search model that is consistent with cyclical properties of housing data – e.g., positive (negative) comovement of prices and sales (time on the market). They also show that search frictions amplify price volatility and propagate aggregate shocks. Head et al. (2015) use a directed search model with heterogenous sellers to think about mortgages. They show that highly-indebted home owners tend to list high prices and take longer to sell. Hedlund (2015a) develops a directed search model with heterogenous buyers and sellers, where computation is facilitated by block recursivity, and shows it can quantitatively account for cyclical dynamics in key macroeconomic housing and mortgage market variables (see also Hedlund 2016b and Garriga and Hedlund 2016). All this shows how the approach provides a fruitful avenue for future work on housing.[17]

Moving from houses back to generic goods, let us now make them divisible: DM buyers get a quantity or quality q in exchange for payment p in the next CM. Buyers' utility and sellers' cost, $u(q)$ and $c(q)$, satisfy the usual properties, plus $u(0) = c(0) = 0$ and $u(\bar{q}) = c(\bar{q})$ for some $\bar{q} > 0$. The efficient q solves $u'(q^*) = c'(q^*)$. One can call $\hat{p} = p/q$ the unit price, unless q is unobserved quality, in which case one might still call p the price. We assume both p and q are posted, although there are alternatives – e.g., perhaps due to limited commitment, there may be a posted unit price $\hat{p}$, and then in a meeting q is chosen unilaterally by the buyer (Peters 1984) or the seller (Gomis-Porqueras et al. 2016).

We also introduce a limit on how much one can promise to pay, $p \leq L$. This debt (or liquidity) constraint is exogenous here, but endogenized in Section 4. Suppose it is slack. Then, ignoring the constants $W_b(0)$ and $W_s(0)$, we have

$$V_b = \max_{p,q,n} \frac{\alpha(n)}{n} [u(q) - p] \text{ st } \alpha(n) [p - c(q)] = V_s. \tag{15}$$

Indeed, when $p \leq L$ is slack, the solution has $q = q^*$, so the problem is basically the same as the one with a fixed q, and the usual procedure yields (n^*, p^*). In particular, the generalization of (3) is $p^* = \varepsilon(n^*) c(q^*) + [1 - \varepsilon(n^*)] u(q^*)$, and the constraint is indeed slack iff $L \geq p^*$. Clearly, $\partial q / \partial N = 0$ when $p \leq L$ is slack. Without entry, we have $\partial p^* / \partial N \approx -\varepsilon$, similar to in Table 1.1(a). With entry, the results are similar to Table 1.1(b) and (c).

[17] An interesting idea is to interpret the market makers discussed above as realtors, as in Stacey (2015a). Also, Moen et al. (2016) study the decision to buy-then-sell or sell-then-buy. A housing model with directed search and two-sided hetreogeneity is Head at al. (2017).

When $L < p^*$, so the constraint binds, the results are quite different. In Appendix A we solve (15) and show the SOC's hold at any solution to the FOC's. So there is a unique equilibrium and it implies $L = g(q, n)$, where

$$g(q,n) \equiv \frac{\varepsilon(n) u'(q) c(q) + [1 - \varepsilon(n)] c'(q) u(q)}{\varepsilon(n) u'(q) + [1 - \varepsilon(n)] c'(q)}. \tag{16}$$

This condition appears in many models with liquidity considerations and Nash bargaining (see Section 4), except $\varepsilon(n)$ replaces buyers' share θ. One can check that, without entry, $\partial q/\partial L = g_q > 0$ and $\partial q/\partial N \approx \varepsilon'$, the latter taking over for $\partial p/\partial N \approx -\varepsilon'$ from the case where the constraint is slack, since now p cannot change while q can and does.

With entry by sellers, the equilibrium conditions $L = g(q, n)$ and $k_s = \alpha(n)[L - c(q)]$ imply

$$\frac{\partial n}{\partial L} > 0, \ \frac{\partial q}{\partial L} > 0, \ \frac{\partial n}{\partial k_s} > 0 \text{ and } \frac{\partial q}{\partial k_s} \approx \varepsilon'. \tag{17}$$

Again $\partial q/\partial k_s$ is ambiguous, similar to $\partial p/\partial k_s$ in Table 1.1(b). More complicated versions of this setup are studied by Rocheteau and Wright (2005), Menzio et al. (2013) and Choi (2015), sometimes using the methods of monotone comparative statics. Again, to highlight the main results, we formalize them as follows:

Proposition 2 *The dynamic equilibrium model with credit yields results similar to the static model, whether q is endogenous and p is fixed, or vice versa. Heterogeneity implies submarkets segmented by probabilities, with prices that are disperse and can look sticky.*

3.2 Labor Markets

While enduring relationships may also be relevant in goods markets – e.g., people have favorite shops or bars – in labor markets they are ubiquitous. We now work through Moen (1997), a directed search version of the standard bargaining model (e.g., Pissarides 2000). Market tightness is now $n = v/(1 - n)$, the measure of vacancies over unemployment, where e is the employment rate with a population of households normalized to 1. Also, here we use continuous time.[18]

Let V_{b1} and V_{b0} be firms' payoffs to having a worker and an open vacancy. In steady state these satisfy

$$rV_{b0} = -k + \frac{\alpha(n)}{n}(V_{b1} - V_{b0}) \tag{18}$$

$$rV_{b1} = y - w + \delta(V_{b0} - V_{b1}), \tag{19}$$

where k is the cost of a vacancy, r the discount rate, and δ the job destruction rate.[19] In words, (18) says the flow payoff to a vacancy is $-k$ plus the arrival rate of workers, $\alpha(n)/n$, times the gain to filling the position, $V_{b1} - V_{b0}$. Similarly, (19) says the flow payoff to having a worker is $y - w$ plus δ times the capital loss, $V_{b0} - V_{b1}$. Similarly, for households

$$rV_{s0} = b + \alpha(n)(V_{s1} - V_{s0}), \tag{20}$$

$$rV_{s1} = w + \delta(V_{s0} - V_{s1}). \tag{21}$$

Again it does not matter for results if firms or workers post, but the latter is easier, since $V_{b0} = 0$ (free entry) combined with (18)-(19) yield what is sometimes called the job creation curve,

$$w = y - k(r+\delta)n/\alpha(n). \tag{22}$$

Solving (18)-(19) for V_{b0} and inserting w from (22), the relevant problem is

$$rV_{s0} = b + \max_n \frac{\alpha(n)(y-b) - n(r+\delta)k}{r + \delta + \alpha(n)}.$$

The FOC implies $T(n) = 0$, where

$$T(n) \equiv \alpha'(n)(y-b) - [r + \delta + \alpha(n) - n\alpha'(n)]k, \tag{23}$$

[18] There is no CM in this environment, but it is interesting to add one (Berentsen et al. 2011; Gomis-Porqueras et al. 2013; Zhang and Huangfu 2016; Dong and Xiao 2016). Among other things, this allows firms to pay wages in CM numeraire rather than transferable utility or a fraction of output, and to finance entry by issuing CM debt or equity.

[19] In static models, free entry means $V_b = k_b$, while here it means $V_{b0} = 0$ since k is already in condition (18); not much hinges on this. We do of course need k not too big, so the market does not shut down. Also, δ is exogenous, but can be endogenized using productivity shocks (see Mortensen and Pissarides 1994), or on-the-job search (see below).

and one can check $T(0) > 0 > T(\infty)$ and $T'(n) < 0$. So there is a unique solution to $T(n) = 0$, and hence a unique equilibrium n.

It is straightforward to derive

$$\frac{\partial n}{\partial y} > 0,\ \frac{\partial n}{\partial b} < 0,\ \frac{\partial n}{\partial k} < 0,\ \frac{\partial n}{\partial \delta} < 0 \text{ and } \frac{\partial n}{\partial r} < 0. \tag{24}$$

The effects of y, b and k are consistent with Table 1.2(c), plus there are new effects of δ and r, and all accord well with intuition. One can easily show $\partial w/\partial b > 0$, $\partial w/\partial \delta < 0$ and $\partial w/\partial r < 0$. Appendix F shows $\partial w/\partial y > 0$ and $\partial w/\partial k < 0$ if $\varepsilon' \leq 0$, but this is not true in general, something one would miss by focusing exclusively on bargaining models. This is again similar to Table 1.2(c), and the economics intuition is basically the same, although in that discussion $\varepsilon' < 0$ iff $\partial w/\partial k < 0$, while now it is sufficient but not necessary, indicating there is more going on in the dynamic than the static models.

Appendix B shows the equilibrium outcome is the same as the solution to a planner's problem posed without restricting attention to steady state – i.e., the efficient n solves $T(n) = 0$ *at every date*, as in the textbook Pissarides model. This is again block recursivity: at any point in time the measure of vacancies v depends on e, but tightness $n = v/(1-e)$ does not. To continue the comparison, rearrange $T(n) = 0$ as

$$k = \frac{\alpha(n)}{n} \frac{\varepsilon(n)(y-b)}{r + \delta + \alpha(n)[1-\varepsilon(n)]}, \tag{25}$$

which equates firms' vacancy cost to their arrival rate times their share, $\varepsilon(n)$, of the appropriately-discounted surplus of a match, $y - b$. This is the same as the condition in the Pissarides model, except the elasticity ε replaces firms' share θ. It matters: if we change labor-market policy, as long as $\varepsilon'(n) \neq 0$ the effects here are different than predicted by bargaining models where θ is fixed.

An important extension involves on-the-job search, e.g., Delacroix and Shi (2006), Garibaldi and Moen (2010), Schaal (2015), Tsuyuhara (2016) and Garibaldi et al. (2016). This makes workers react differently to wage offers, and generates direct job-to-job transitions, consistent with the data (Fallick and Fleischman

2001; Christiansen et al. 2005). Following Delacroix and Shi (2006), let $\kappa > 0$ be workers' cost of search while employed. If κ is large we are back in the baseline model; for smaller κ at least some workers search while employed. As should be expected, e.g., based on models like Burdett and Mortensen (1998), on-the-job search generates wage dispersion.

Let $n(w)$ be the ratio of vacancies to job seekers in a submarket with wage w. The problem of a worker employed at w is

$$rV_{s1}(w) = w + \delta \left[V_{s0} - V_{s1}(w)\right] + \max_{w',\Sigma} \Sigma\{\alpha \left[n(w')\right] \left[V_{s1}(w') - V_{s1}(w)\right] - \kappa\}, \quad (26)$$

where $\Sigma = 1$ ($\Sigma = 0$) indicates he engages in (abstains from) search, and if $\Sigma = 1$, w' is the next wage to which he directs his search. An unemployed worker's value function is similar to a worker employed at $w = b$, except it is assumed that the former has no search cost, so $V_{s0} = V_{s1}(b) + \kappa$. Also, a standard result in this context (see Appendix C) is that workers are more selective in terms of the next targeted wage w' when their current wage w is higher.[20]

Solving for equilibrium requires finding $n(w)$. To begin, write

$$\begin{aligned} rV_{b0} &= -k + \frac{\alpha\left[n(w)\right]}{n(w)}\left[V_{b1}(w) - V_{b0}\right] \\ rV_{b1}(w) &= y - w + \left[\delta + \pi(w)\right]\left[V_{b0} - V_{b1}(w)\right], \end{aligned}$$

where the only change from the baseline model is that jobs now end with an exogenous probability δ plus the endogenous probability $\pi(w)$ that a worker gets a better offer. Now free entry implies $V_{b0} = 0$, or

$$k = \frac{\alpha\left[n(w)\right]}{n(w)} \frac{y - w}{r + \delta + \pi(w)}. \quad (27)$$

Then $\kappa > 0$ implies there is a $\underline{w}$ such that workers employed at $w \geq \underline{w}$ naturally stop searching. For firms paying $w \geq \underline{w}$, $\pi(w) = 0$, and (27) identifies the $n(w)$ that coincides with what one gets without on-the-job search.

[20] The logic is similar to the model with multiple applications in Section 2.2, where a worker takes more risk applying for a high wage because if he fails he may still get a low wage.

Next, note that under the hypothetical situation that $n(w)$ is computed this way everywhere, we can find the lowest wage at which the solution to (26) involves no search, and that identifies $\underline{w}$. Then, by way of induction, notice there is a minimum wage increment $\triangle$ that workers require to justify search (again see Appendix C). Hence, those employed at $w \in [\underline{w} - \triangle, \underline{w})$ only seek jobs with $w' \geq \underline{w}$, for which we have already determined $n(w')$. Given w, w' is the unique solution to (26), denoted $w' = \omega(w)$. Then $\pi(w) = \alpha \circ n \circ \omega(w)$, where for any functions f and g, $f \circ g(x)$ denotes the composite $f[g(x)]$. Knowing $\pi(w)$ $\forall w \in [\underline{w} - \triangle, \underline{w})$, entry condition (27) yields $n(w)$ at these wages. Repeating the procedure for $w \in [\underline{w} - 2\triangle, \underline{w} - \triangle)$ yields $\pi(w)$ and $n(w)$ at those wages, and so on, until $n(w)$ and $\pi(w)$ are determined for all w.

This establishes $n(w)$ and $\pi(w)$ $\forall w$ without reference to the distribution of employment across w, in and out of steady-state, again due to block recursivity. Starting with higher unemployment, e.g., lots of job seekers search for $w = \omega(b)$, but also lots of firms post $w = \omega(b)$, keeping $n(w)$ as determined above. Thus, we can first solve for the value functions and decision rules (block 1), then study the evolution of e from any initial condition (block 2), and only in the second step does the distribution of employment come into play. Extensions of this insight allow tractable analysis of business cycle models where aggregate productivity y is stochastic. In these models, current y is enough to compute tightness in each submarket, say $n(w, y)$, which is much easier than it would be if n depended on the distribution of w across workers. While $n(w, y)$ is different from the above construction, because y can change, the basic methods apply.[21]

To recap, there are n_w wages, $w_1 < w_2 < ...w_{n_w}$. The unemployed apply to $w_1 = \omega(b)$; workers employed at w_1 apply to $w_2 = \omega(w_1)$; and so on, until they stop searching at $\underline{w}$. It can be shown that n_w decreases with search and entry costs. It can also be shown that simple wage contracts do not induce efficiency. The situation is similar to the model in Section 2.2 with multiple applications.

[21] See Shi (2009), Menzio and Shi (2010,2011) and Schaal (2015) for more details. See Li and Weng (2017) for an extension to incorporate learning.

With on-the-job search, firms care about both recruitment and retention, and a single wage is not sufficient to balance the two. This is especially clear when all matches produce the same y, which means on-the-job search is rent seeking that has a social cost but does not increase output. However, more complicated contracts that directly specify search activity, or specify that workers compensate firms when they quit, restore to efficiency. See Menzio and Shi (2011).

Research on labor markets is a vibrant area with many possible extensions. Parallel to divisible goods in Section 3.1, one idea is to have divisible hours h, with $y(h)$ output and $c(h)$ the disutility of work, to study regulations like $h \leq \bar{h}$ or $w \geq \bar{w}$, similar to the restriction $p \leq L$. The effects of unemployment insurance are also interesting (see below). Another important extension concerns directed search over the life cycle, where trade-offs between wages and job-finding rates produce transitions that depend on age (Menzio et al. 2016). But since it is impossible to cover everything, we end with a summary of the labor models.[22]

Proposition 3 *The dynamic labor model without on-the-job search has a unique equilibrium and it is efficient. At each point in time, n solves $T(n) = 0$ and w solves (22). The outcome with on-the-job search is similar, except there is wage dispersion, and efficiency requires more complicated contracts.*

4 Monetary Economics

Monetary theory has used random matching at least since Kiyotaki and Wright (1989), and that model has been recast using directed search by Corbae et al. (2003). However, since those environments are complicated, we instead start with Julien et al. (2008), which is simpler. These models have indivisible assets. We then introduce divisible assets, which involves dealing with an endogenous

[22] While we earlier downplayed enduring relationships in goods markets, the labor models show how to handle such phenomena. This is arguably relevant for several reasons discussed in Gourio and Rudanko (2014), although they do not have directed search. Shi (2016) presents a directed search model where such relationships arise endogenously when buyers make repeat purchases, and this improves welfare. It also induces micro price dynamics, including sales, even when conditions are constant. Pursuing this is another fruitful area for future research.

distribution. While there are different ways to handle this (see surveys by Nosal and Rocheteau 2011 and Lagos et al. 2016), we focus on the approach in Lagos and Wright (2005), but also discuss Menzio et al. (2013), which interesting in this context because it relies heavily on directed search and block recursivity.

4.1 Indivisible Assets

There is a $[0, \bar{N}]$ continuum of ex ante identical agents that live forever in discrete time (it would be interesting to add entry, but we do not do so here). This environment has no centralized markets, and agents can only meet/trade bilaterally, and that is hindered by specialization: there are many different types of goods, and it is never the case in a pairwise meeting that agent i consumes what j produces and vice versa, to rule out direct barter. Assumptions on limited commitment and private information then rule out credit, so that assets have an essential role as media of exchange (Kocherlakota 2008; Wallace 2010).

Equal measures of agents consume and produce each good, and everyone has the same utility $u(q)$ for goods they consume and cost $c(q)$ for goods they produce. Goods are nonstorable. There is a storable asset that generates utility ρ each period for anyone holding it. If $\rho > 0$ this can be interpreted as a dividend, as in standard asset-pricing theory going back to Lucas (1978); if $\rho < 0$ it can be a storage cost, as in Kiyotaki and Wright (1989); and if $\rho = 0$ the asset is fiat money as defined by Wallace (1980). Individual asset holdings are restricted to $m \in \{0, 1\}$, so given a fixed supply $M \in (0, \bar{N})$, M agents have $m = 1$ and act as buyers while $\bar{N} - M$ have $m = 0$ and act as sellers.[23]

A novelty compared to the above models is that after trade the buyer becomes a seller and vice versa. Letting $\Delta = V_b - V_s$ be the value to getting an asset and

[23] This environment is from of Shi (1995) and Trejos and Wright (1995), but those papers use random search and symmetric bargaining. This is extended to generalized Nash and Kalai bargaining by Rupert et al. (2001) and Trejos and Wright (2016). There are also versions with posting and random search by Curtis and Wright (2004), or posting and noisy search by Burdett et al. (2016). Wallace (2010) and many reference therein use abstract mechanism design. The first paper to use posting and directed search is Julien et al. (2008), with recent extensions by Julien et al. (2016) and He and Wright (2016).

switching from seller to buyer, in steady state we have

$$V_b = \rho + \alpha_b [u(q) - \beta\Delta] + \beta V_b \quad (28)$$

$$V_s = \alpha_s [\beta\Delta - c(q)] + \beta V_s. \quad (29)$$

As usual, $\alpha_s = \alpha(n)$ and $\alpha_b = \alpha(n)/n$ with $n = M/(\bar{N} - M)$ and $\alpha(n)$ comes from a general meeting technology, although following Kiyotaki and Wright (1991,1993) many papers in this literature use $\mu = M\left(\bar{N} - M\right)/\bar{N}$. In any case, these are similar to Section 3.1, except the benefit of producing is having the asset next period, rather than selling for credit due in the CM.

Directed search plays two roles. First, the economy segments into markets trading different goods – as opposed to random search, where someone needing a haircut has the same chance of meeting a plumber or a barber. Second, each market segments into submarkets based on posted terms of trade. In Appendix A we take FOC's for the submarket problem and rearrange to get

$$\beta\Delta = \frac{\varepsilon(n) u'(q) c(q) + [1 - \varepsilon(n)] c'(q) u(q)}{\varepsilon(n) u'(q) + [1 - \varepsilon(n)] c'(q)}. \quad (30)$$

The RHS, denoted $g(q,n)$ in (16) is again the same as Nash bargaining except $\varepsilon(n)$ replaces θ; different from Section 3.1, instead of an exogenous limit L, the value of assets and hence the ability to pay are now endogenous.

To proceed, subtract (28)-(29) and solve for

$$\beta\Delta = \frac{\rho + \alpha_b u(q) + \alpha_s c(q)}{r + \alpha_b + \alpha_s}. \quad (31)$$

In equilibrium $n = M/(\bar{N} - M)$ determines ε, α_b and α_s. A stationary monetary equilibrium, or SME, is then a q equating the RHS's of (30) and (31), with $q \in (0, \bar{q})$, where $u(\bar{q}) = c(\bar{q})$, as required for voluntary trade. Julien et al. (2008), Julien et al. (2016) and He and Wright (2016) prove versions of these results:

Proposition 4 *For $\rho = 0$ there is a unique SME; for $\rho > 0$ there is a unique SME if ρ is small, possibly multiple SME if ρ is intermediate, and no SME if ρ is big; for $\rho < 0$ there are multiple SME if $|\rho|$ is small and no SME if $|\rho|$ is big.*

In addition to monetary equilibria, for $\rho \leq 0$ there is a nonmonetary equilibrium where assets stop circulating and agents throw them away, because $\rho \leq 0$ makes them poor stores of value, and if no one is posting $q > 0$ assets are useless as a medium of exchange. They can also stop circulating when $\rho > 0$ is very big, but now no one throws assets away, because they are always worth at least their fundamental value, ρ/r. Instead, buyers hoard them, although that is not especially robust (as in Berentsen et al. 2002, assets can circulate for any $\rho > 0$, with buyers getting q in exchange for a probability of handing over assets).

Whenever assets circulate they are worth more than their fundamental value, as is especially clear for $q > 0$ and $\rho \leq 0$, where a poor savings vehicle is valued for its liquidity – i.e., for its facilitation of transactions. Importantly, monetary and nonmonetary equilibria can coexist and there can be multiple monetary equilibria: if other sellers post higher q, the asset is more valuable, so in the spirit of directed search a given seller posts higher q to try to trade faster. Fundamentals still play a role – e.g., $\partial q/\partial \rho > 0$ if SME is unique – but this shows succinctly how liquidity is at least partly a self-fulfilling prophecy. Moreover, there are equilibria where q varies over time. Using standard methods He and Wright (2016) show the model has a continuum of perfect-foresight equilibria starting at different $q_0 > 0$, where $\lim_t q_t = 0$ or $\lim_t q_t > 0$ depending on parameters, as well as sunspot equilibria where q_t fluctuates stochastically.

To emphasize the interplay between directed search and monetary economics, consider the unique SME with $\rho = 0$, and the meeting technology commonly used in this literature, with $\bar{N} = 1$ and $\mu = M(1 - M)$. Then $M = 1/2$ is good for trade on the extensive margin, since it maximizes the number of buyer-seller meetings, but it does less well on the intensive margin, since one can check $M = 1/2$ implies $q < q^*$. Trejos and Wright (1995) show $q < q^*$ at $M = 1/2$, and argue that this is a salient feature of monetary exchange, using Nash bargaining under the assumption $\theta = 1/2$. In competitive search equilibrium, the result $q < q^*$ at $M = 1/2$ follows with no need for any such assumption, because for this specification $\varepsilon(n) = 1/2$ holds automatically at $M = 1/2$. Thus, we can derive similar results with fewer restrictions.

As another example of the connection between directed search and money economics, one can check $\partial q/\partial M < 0$ in this model if $\varepsilon' < 0$ but not in general. The intuition is similar to the discussion of $\partial p/\partial N$ in Section 2.1 or $\partial q/\partial N$ in Section 3.1, on the way resources are allocated by the probabilities plus the terms of trade, which is again missing in models with bargaining. As a final example, concerning dynamics, bargaining models also have equilibria where q varies over time, but one can argue that their microfoundations are shaky compared to versions with posting.[24] Hence, for several reasons, it is useful to study monetary economics using competitive search. For more discussion and results, see Julien et al. (2008,2016) and He and Wright (2016).

This is not to suggest that the microfoundations of monetary economics requires competitive search. Indeed, search itself is not critical, even if it may be natural for capturing the relevant frictions and generating new implications (Wallace 2016). Search is not critical in the sense that many results hold with the meeting technology $\mu = \min\left\{M, \bar{N} - M\right\}$, which lets agents on the short side of the market always trade, which with $M = \bar{N}/2$ implies all agents trade. Still, monetary models with competitive search are important in the development of the literature, as early work in the area was criticized sharply by those who find random matching and bargaining unpalatable (recall the comments by Howitt and Prescott in the Introduction). That critique is misguided, given most insights carry over, and many become sharper, with directed search and posting.[25]

[24] In brief, there is a problem in nonstationary equilibrium with the usual demonstration (Binmore et al. 1986; Binmore 1987) that Nash bargaining is the limit of strategic bargaining. Indeed, Coles and Wright (1998) argue that using Nash out of steady state with nonlinear utility is tantamount to having agents bargain myopically. Posting avoids this critique entirely.

[25] A few results change in interesting ways. With random matching, e.g., in Kiyotaki and Wright (1989) there can be equilibria where inferior assets are used as money, which they call "speculative" behavior. Corbae et al. (2003) replace random with directed search and show the unique equilibrium in a certain class uses only the best assets as money. That is interesting because it shows "speculation" requires some randomness in the meeting process, but that hardly invalidates the basic insights and methods.

4.2 Divisible Assets

We now let agents hold any $m \in \mathbb{R}_+$ and bring back the frictionless CM that convenes after each round of DM trade. One reason to have the CM is that it harnesses the distribution of m, which is otherwise complicated. Another is that the CM allows one to incorporate many elements of mainstream macro in search-based models, including standard capital and labor markets. Yet another is that we do not have to say whether agents are buyers or sellers depending on their current m, as in Section 4.1; instead we can have some called buyers that always want to consume but cannot produce in the DM, while others called sellers produce but do not consume. This is not a nonstarter with only DM trade – why would sellers work to get money if they never get to spend it? Here they work for money in the DM to spend in the CM, exactly as in Section 3.1. See Nosal and Rocheteau (2011), Lagos et al. (2016) and references therein for more on this and additional motivation for adopting the alternating CM-DM structure.

Focusing on $\rho = 0$ (fiat currency), we write buyers' CM problem as

$$W_b(m) = \max_{x,\ell,\hat{m}} \{U(x) - \ell + \beta V_{b,+1}(\hat{m})\} \text{ st } x = w\ell + \phi(m - \hat{m}) - T, \qquad (32)$$

where m is cash brought in, $\hat{m}$ is cash taken out, ϕ is its price in terms of numeraire x, and T is a lump sum tax. Other than keeping track of time with the subscript on $V_{b,+1}(\hat{m})$, (32) is like (11) with one exception: there buyers get DM goods on credit due in the next CM; here this is infeasible because of standard issues with commitment and information, so buyers must use assets as payment instruments. Still, as in Section 3.1, $W_b'(m) = \phi$ and similarly for sellers, making CM payoffs linear. Sellers do not bring cash to the DM, but buyers might, and their FOC for $\hat{m} > 0$ is $\phi = \beta V_{b,+1}'(\hat{m})$. Since m does not appear in this FOC, the $\hat{m}$ they take out does not depend on what they bring in to the CM.[26]

[26] This history independence, which makes the DM distribution of $\hat{m}$ across buyers degenerate, follows from quasilinear utility and the interiority of ℓ, but both can be relaxed as discussed in fn. 13. The distribution is not degenerate in the closely related models of Galenianos and Kircher (2008) and Dutu et al. (2012), but is still tractable due to history independence.

In the current CM, (p_{+1}, q_{+1}, n_{+1}) is posted for the next DM, where p_{+1} is the real value of the monetary payment. Since cash is a poor savings vehicle, buyers hold just enough so that $\phi_{+1}\hat{m} = p_{+1}$. Once again, it may seem natural to have sellers post and buyers search, but it is equivalent to assume the opposite. Ignoring the constants $W_b(0)$ and $W_s(0)$, as we did in (15), as well as the time subscripts, because now they are all the same, after some algebra we have

$$V_b = \max_{p,q,n} \left\{ \frac{\alpha(n)}{n} [u(q) - p] - ip \right\} \text{ st } \alpha(n) [p - c(q)] = V_s \tag{33}$$

where i is a nominal interest rate. This nominal rate is defined by the Fisher equation, $1 + i = \phi/\beta\phi_{+1}$, describing the return you would need in the next CM to give up a dollar in this CM. As is standard, in stationary equilibrium ϕM is constant, so the growth rate of the aggregate money supply pins down inflation, $\phi/\phi_{+1} = M_{+1}/M$. This plus the Fisher equation imply it is equivalent for monetary policy to peg the money growth, inflation or nominal interest rate.

Problem (33) is the same as (15) but for this important detail: buyers now must make an ex ante investment in liquidity, at cost ip_{+1}, before going to the DM. Taking the FOC for q, we get

$$in/\alpha(n) = \lambda(q) \tag{34}$$

where $\lambda(q) \equiv [u'(q) - c'(q)]/c'(q)$. Often $\lambda(q)$ is called the *liquidity premium*; it is also the Lagrange multiplier on the constraint $p \leq \phi\hat{m}$. Whatever we call it, (34) equates $\lambda(q)$ to the marginal cost of liquidity i, multiplied by the expected time until it is used, the inverse of the arrival rate $\alpha_b = \alpha(n)/n$. The FOC for n yields

$$\alpha(n)[1 - \varepsilon(n)] = \frac{V_s}{n} + \frac{iV_s}{\alpha(n)} + ic(q), \tag{35}$$

and the constraint yields $p = g(q, n)$, where g is as before. With no entry, $n = N$, (34) determines q and (35) determines V_s. With entry by sellers, $V_s = k_s$, and (34)-(35) determine (q, n) jointly.

By way of comparison, consider the planner's problem with entry by sellers,

$$\max_{q,n} \left\{ \frac{\alpha(n)}{n} [u(q) - c(q)] - \frac{k_s}{n} \right\}.$$

The FOC's are

$$u'(q) = c'(q) \tag{36}$$

$$k_s = \alpha(n)[1-\varepsilon(n)]. \tag{37}$$

Clearly, (36) implies $q = q^*$. From (34) this is the same as equilibrium iff $i = 0$, the Friedman rule for eliminating the cost of liquidity and making money equivalent to perfect credit. Then (37) determines $n = n^*$, and from (35) this is the same as equilibrium at $i = 0$. Competitive search delivers the first-best at $i = 0$.

Next, consider generalized Nash bargaining. This implies $p = v(q, n)$ with θ instead of ε, plus

$$\frac{ni}{\alpha(n)} = \frac{u'(q) - g_q(q,n)}{g_q(q,n)} \tag{38}$$

$$k_s = \frac{\alpha(n)(1-\theta)[u(q) - c(q)]}{\theta u'(q) + (1-\theta)c'(q)}. \tag{39}$$

Now (38) is the same as (34) when $\theta = 1$, and then it the same as (36) iff $i = 0$. Intuitively, for buyers to make the efficient ex ante investment in liquidity, they need all the bargaining power in the DM; otherwise, $q < q^*$ with Nash bargaining even at $i = 0$. But for sellers to make the efficient ex ante entry decision they need $\theta = \varepsilon(n)$, since that makes (39) the same as (37). This situation, described as "Friedman Meets Hosios" in Berentsen et al. (2007), presents a dilemma: it is not generally possible to have $\theta = \varepsilon(n)$ and $\theta = 1$. So even at $i = 0$ we cannot get (q^*, n^*) with Nash bargaining due to this double holdup problem.[27]

In contrast, in competitive search equilibrium $i = 0$ does achieve $q = q^*$ and $n = n^*$, so we get efficiency on both the intensive and extensive margin. As in Section 4.1, this suggests that money is intimately related to directed search, since that allows us to achieve efficiency with the two-sided investments that

[27] Other solution concepts can do better than Nash bargaining. Aruoba et al. (2007) show $q = q^*$ at $i = 0\ \forall\theta > 0$ under Kalai bargaining. Hu et al. (2009) and Gu and Wright (2016) can get $q = q^*$ when β is not too small even at $i > 0$ under more complicated mechanisms, but here we are taking bargaining as a primitive and not doing mechanism design. Also, since $q < q^*$ at $i = 0$ with Nash bargaining and $\theta < 1$, it might seem $i < 0$ is desirable – but unfortunately monetary equilibrium does not exist for $i < 0$. This is a New Monetarist version of the "zero lower bound problem" currently in vogue among New Keynesians.

are natural in this application. With or without entry, the model makes a host of interesting predictions. If $n = N$ is fixed, one can show $\partial q/\partial i < 0$ and $\partial q/\partial N > 0$, while the effects on p are complicated but at least with ε' not too big (e.g., with Cobb-Douglas) one can show $\partial p/\partial i < 0$ and $\partial p/\partial N > 0$. Things are more complicated when sellers enter endogenously, but let us suppose that the SOC's hold, which is necessarily true when i is not too big. Then one can show $\partial q/\partial i < 0$, $\partial q/\partial k_s < 0$. And at least with ε' not too big, $\partial n/\partial i > 0$, $\partial n/\partial k_s > 0$, $\partial p/\partial i < 0$ but $\partial p/\partial k_s$ is ambiguous.[28]

This is the canonical model of a monetary economy with competitive search. An early application is Lagos and Rocheteau (2005). They fix $n = N$, but introduce endogenous search effort by buyers to try to capture the "hot potato" effect of inflation, which says buyers spend money faster when i is higher. As they show, this effect does not emerge with bargaining. The reason is that i is effectively a tax on DM activity, making buyers bring less money in real terms, which lowers the gains from trade and leads to less search effort; hence they end up spending their money slower rather than faster. With competitive search, however, even though higher i lowers the total DM surplus, it can shift the terms of trade in favor of buyers at least for low i, which leads to more search effort and thus recovers the "hot potato" effect. See Liu et al. (2011) and references therein for more discussion and alternative ways to generate this effect.

In another early application, Rocheteau and Wright (2005) compare the effects of inflation in three models: competitive search, Nash bargaining, and Walrasian pricing. We saw above how competitive search delivers (q^*, n^*) at $i = 0$, while generalized Nash does not, unless $\theta = 1$; one can also show Walrasian pricing may not deliver efficiency in economies with search. Rocheteau and Wright (2009) calibrate all three models to measure the welfare cost of inflation. With random search and bargaining, this cost can be large compared to findings in an earlier literature using money-in-utility or cash-in-advance models: in Lagos and Wright

[28]See Rocheteau et al. (2016, Appendix C) for formulae for these effects, and for a more detailed analyze of the case where $\mu = \min\{n_b, n_s\}$, which of course delivers cleaner results.

(2005), going from 10% inflation to the optimal policy is worth around 5.0% of consumption, compared to around 0.5% in Lucas (2000). With competitive search, the number is around 1%, as can be understood from the result that it delivers (q^*, n^*) at $i = 0$, so a small inflation has only a second-order impact, by the envelope theorem. Nash bargaining with $\theta < 1$ calibrated to match markup data does not deliver (q^*, n^*) at $i = 0$, which is a key to the 5.0% result.

Bethune et al. (2016) pursue the issue using semi-directed search as developed by Lester (2011). By this we mean posting and directed search apply to informed buyers, called locals, while random search and bargaining apply to uninformed buyers, called tourists. In equilibrium some sellers, called local shops, cater to the informed by posting favorable terms, while others, called tourist shops, serve only the uninformed at less favorable terms. With the fraction of informed buyers, and other parameters, disciplined by data, including markup and price dispersion data, the authors find a low cost of inflation, close to pure competitive search, despite having only about 1/4 of buyers informed. However, they also find that the information distortion is inoperative when this fraction is just over 1/2, since then all tourist shops are driven out of business, making 1/4 not so low. Moreover, they show that a little inflation can be desirable, as it relatively heavily taxes the more expensive and less efficient tourist shops.

In other work, Dong (2010) allows inflation to affect the variety of goods sellers have for sale. Buyers direct their search to sellers posting attractive terms, but do not know if a seller will have a variety that suits their tastes until they show up. Inflation reduces variety in equilibrum. Quantitatively, with this effect, the cost of inflation close to 1% using competitive search, and 5% to 8% using bargaining. This research, and several related papers, underscore the importance for quantitative work of carefully modeling the microfoundations of information, specialization and price formation.[29]

[29] In a directed search model with private information about buyers' types, Ennis (2008) finds large effects of inflation on welfare (6% to 7%). Faig and Jerez (2006), in a version of Faig and Jerez (2007) with private information, show competitive search is efficient at $i = 0$, but at $i > 0$ buyers buy too little of a good they like a lot and too much of a good they like less. See also

Faig and Huangfu (2007) analyze an environment where it matters who posts the terms of trade, and not for the reasons discussed below in Section 5. Suppose market makers post terms for their submarkets, to attract buyers and sellers, as in Section 3.1. Recognizing that carrying currency is costly, given $i > 0$, a shrewd market maker proposes the following scheme: All buyers pay ϕ_b when they enter his submarket; if a buyer meets a seller he gets the goods for free; then all sellers collect ϕ_s when they exit at the close of the DM. This allows agents to share in the cost of liquidity by eliminating cash in the hands of buyers who do not meet sellers, with market makers acting somewhat like the bankers in Berentsen et al. (2007). As usual, one can concoct assumptions to rule out such arrangements – perhaps market makers cannot tell who is a buyer and who is a seller – but this is a nice example of how microstructure matters.

Dong (2011) revisits Rocheteau et al. (2008), which is like the above model except quasilinearity is replaced by indivisible labor, $\ell \in \{0, 1\}$. Following Rogerson (1998), agents in the CM trade ℓ using lotteries, getting wages in exchange for probabilities of working. As should be well known, with lotteries, given any utility function agents act *as if* they are quasilinear, so $\hat{m}$ is still degenerate. Lotteries also entail unemployment, and Rocheteau et al. (2007) derive a long-run Phillips curve fully exploitable by policy. Intuitively, inflation lowers q in the DM, that raises (lowers) x in the CM if the goods are substitutes (complements), and employment comoves with x. But they can only prove it for $\theta = 1$, which preclude any ex ante investment by sellers. Dong (2011) replaces bargaining with competitive search, and proves the result with no such restriction. As in Section 4.1, competitive search allows one to get more with less.

In terms of substance, however, these models predicts $i = 0$ is optimal even when $i > 0$ leads to lower unemployment. In related work on Phillips curve correlations, Huangfu (2009) has private information about monetary shocks.

Dong and Jiang (2014). In a model that is similar, but with indivisible goods, Carbonari et al. (2017) get the optimal $i > 0$. We do not go into more detail here because private information entails complications (discussed in Section 7).

When there is a positive shock observed only by buyers, they have an incentive to misrepresent the information to get better terms. To induce truth telling, sellers offer more output when money growth is high, and so output increases with monetary shocks. This contrasts with a Walrasian version of her model, where nominal shocks have no real effects. However, again, just because output goes up with money growth, that does not mean it is a desirable.

Also pertaining to policy, this time open market operations and quantitative easing, Rocheteau et al. (2016) use directed search to study economies with currency and liquid bonds. Equilibrium features some submarkets where sellers accept only cash, and others where they accept both assets. Acceptability is endogenized using information theory as in Lester et al. (2012): when bonds are counterfeitable and costly to authenticate, only some sellers make the requisite investment. In contrast with much of the literature on segmented asset markets based on cash-in-advance and related constraints (see Chiu 2014 for a recent example and primary references), buyers voluntarily visit venders that only take cash, even if using cash is expensive relative to bonds. The reason is that market tightness endogenously lets them trade faster with cash. This leads to heterogeneous portfolios as choices, not restrictions, and generates novel insights into the effects of monetary policy. Competitive search is ideal for this application.

Han et al. (2016) consider divisible assets and indivisible DM goods, as opposed to Section 4.1, with indivisible assets and divisible goods. This is worth studying with posting because bargaining in this environment entails extreme results: buyers bring just enough cash to the DM to compensate sellers for their cost, and hence capture the entire surplus, irrespective of bargaining power– effectively, buyers commit to not paying more than sellers' reservation price by only bringing that much. Again this precludes any ex ante investment by sellers, including entry. Directed search gets around the problem, because posting by sellers gives buyers the incentive to bring the right amount of money, with competition across sellers leading to an efficient split of the surplus.

Menzio et al. (2013) provide an alternative approach that does not have a CM and hence has a nondegenerate distribution, yet is still tractable due to block recursivity. Buyers select into submarkets as follows: those with more $\hat{m}$ prefer to higher p and q so they can trade sooner rather than later; those with less $\hat{m}$ prefer lower p and q even if it takes a little longer. Submarkets cater to their desires by having different tightness, with sellers indifferent between them. Since equilibrium separates buyers with different $\hat{m}$, their choices are independent of the distribution across individuals, as is tightness in the various submarkets, due to free entry by sellers. They prove that a unique monetary steady state exists, and characterize the $\hat{m}$ distribution. While this has not yet achieved the popularity of the alternating CM-DM framework, it has some advantages, and Sun and Zhou (2016) integrate elements of both approaches. In any case, this further speaks to the usefulness of competitive search in monetary economics.

5 Finite Markets

Directed search theory with finite numbers of agents playing a well-defined game is developed in a series of papers by Peters (1984,1991,1997,2000). The presentation here follows Julien et al. (2000) and Burdett et al. (2001). We begin with a market with $n_b = 2$ buyers and $n_s = 2$ sellers, then generalize this and describe what happens in the limit as n_b and n_s grow.

5.1 The 2×2 Market

Consider a market with 2 buyers and 2 sellers. Each seller can produce and each buyer wants to consume 1 unit of indivisible good q, with the latter paying the former p using a divisible good x that enters payoffs linearly (again this amounts transferable utility, which, we hope, no one will confuse with money after having read Section 4). The game proceeds as follows: first sellers post prices; then given $\mathbf{p} = (p_1, p_2)$, buyers decide where to go. If both buyers visit the same seller, one is chosen at random to get the good. Payoffs for buyers and sellers that trade are

$u-p$ and $p-c$, and we impose $p_j \in [c,u]$ without loss of generality. The strategy of buyer 1 is γ_{1j}, the probability he goes to seller j, and similarly for buyer 2.

Given buyer 2's strategy, payoffs for buyer 1 from visiting sellers 1 and 2 are

$$V_{b11} = \left(1-\gamma_{21}+\frac{\gamma_{21}}{2}\right)(u-p_1) \tag{40}$$

$$V_{b12} = \left(\gamma_{21}+\frac{1-\gamma_{21}}{2}\right)(u-p_2). \tag{41}$$

In words, (40) says that at seller 1 buyer 1 gets served for sure if buyer 2 goes to seller 2, which happens with probability $1-\gamma_{21}$, and gets served with probability 1/2 if buyer 2 goes to seller 1, which happens with probability γ_{21}. Given $\mathbf{p}$, it is easy to check the best response of buyer 1 is

$$\gamma_{11} = \begin{cases} 0 & \text{if } \gamma_{21} > \Gamma(\mathbf{p}) \\ [0,1] & \text{if } \gamma_{21} = \Gamma(\mathbf{p}) \\ 1 & \text{if } \gamma_{21} < \Gamma(\mathbf{p}) \end{cases} \tag{42}$$

where $\Gamma(\mathbf{p}) \equiv (u+p_2-2p_1)\,/\,(2u-p_1-p_2)$.

For any $\mathbf{p}$, equilibrium in the stage 2 game between buyers is:

1. If $p_1 \geq (u+p_2)/2$ then $(\gamma_{11},\gamma_{21}) = (0,0)$ (both buyers go to seller 2).

2. If $p_1 \leq 2p_2 - u$ then $(\gamma_{11},\gamma_{21}) = (1,1)$ (both buyers go to seller 1).

3. If $(u+p_2)/2 > p > 2p_2 - u$ there are three possible equilibria:

   - $(\gamma_{11},\gamma_{21}) = (0,1)$ (buyer 1 goes to seller 2, buyer 2 goes to seller 1);
   - $(\gamma_{11},\gamma_{21}) = (1,0)$ (buyer 1 goes to seller 1, buyer 2 goes to seller 2);
   - $\gamma_{11} = \gamma_{21} = \Gamma(p_1,p_2)$ (buyers play symmetric mixed strategies).

Using this, and taking p_2 as given, we can write the payoff for seller 1 at stage 1 as a function of p_1 as follows:

1. If $p_1 \geq (u+p_2)/2$ then $V_{s1} = 0$ (seller 1 gets no buyers).

2. If $p_1 \leq 2p_2 - u$ then $V_{s1} = p_1 - c$ (seller 1 gets both buyers).

3. If $(u + p_2)/2 > p > 2p_2 - u$ then two things can happen:

   - a pure-strategy equilibria with $V_{s1} = p_1 - c$;
   - a mixed-strategy equilibrium with $V_{s1} = [1 - (1 - \gamma)^2] (p_1 - c)$, where $\gamma = \Gamma(p_1, p_2)$, or, after simplification,

$$V_{s1} = \frac{3(u - p_2)(u - 2p_1 + p_2)(p_1 - c)}{(2u - p_1 - p_2)^2}. \tag{43}$$

Now the set of equilibria in pricing can be described as follows: One possibility is $p_1 = p_2 = u$, $\gamma_{11} = 1$ and $\gamma_{21} = 0$ (for sure buyer 1 goes to seller 1 and buyer 2 goes to seller 2), which is an equilibrium since buyers can do no better at this $\mathbf{p}$, and sellers can never do better than this. Symmetrically, $p_1 = p_2 = u$, $\gamma_{11} = 0$ and $\gamma_{21} = 1$ is an equilibrium. Burdett et al. (2001) show there are also many asymmetric equilibria supported by triggers.[30] Rather than dwelling on these, we focus on symmetric mixed-strategy equilibria, as in much of the literature, . This can be motivated by arguing pure strategies rely on a lot of *coordination*, which may be reasonable in a 2×2 market, but is less so in large markets (see Bland and Loertscher 2012 or Norman 2016 for more discussion).

Therefore, consider *non-coordinated* equilibria where buyers mix at stage 2. From the above discussion, this requires $(u + p_2)/2 > p_1 > 2p_2 - u$, shown as the unshaded region in Fig. 4. In this region, choosing p_1 to maximize V_{s1} leads to

$$p_1 = \frac{2(u^2 - p_2 c) + (u - p_2)(p_2 + 2c)}{5(u - p_2) + 2(u - c)}, \tag{44}$$

[30] Here is the idea: Pick any $\mathbf{p}$ such that it is an equilibirum for buyer 1 to go to seller 1 and buyer 2 to go to seller 2 for sure. If any seller deviates, buyers play the following equilibrium at stage 2: if $p_1 \geq (u + p_2)/2$ both go to seller 2; if $p_1 \leq 2p_2 - u$ both go to seller 1; and if $(u + p_2)/2 > p > 2p_2 - u$ they play the mixed equilibrium. Burdett et al. (2001) characterize the set $\mathcal{P}$ such that $\mathbf{p} \in \mathcal{P}$ allows no profitable deviation. Intuitively, there is no profitable deviation that makes both buyers go to the same seller at stage 2, and it is not easy to find a profitable deviation that leads to a mixed equilibrium at stage 2, because sellers do poorly in mixed equilibria, due to the chance that no one shows up. For any $\mathbf{p}$ such that profits are not too low, no seller wants to deviate and trigger mixing at stage 2. Relatedly, Camera and Kim (2016) consider an infinitely-repeated version of the model, and get multiplicity using trigger strategies as in standard repeated game theory.

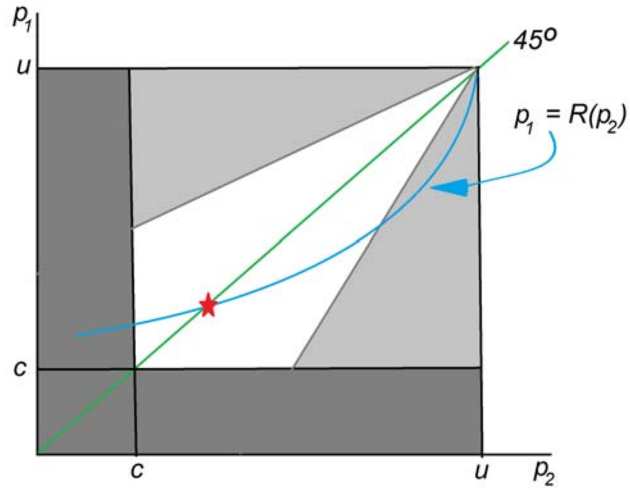

Figure 4: Best-response in price posting in the 2×2 Market

(this looks different from Burdett et al. 2001 because we do not normalize $u = 1$ and $c = 0$). Now (44) defines seller 1's best response $p_1 = R(p_2)$, and symmetrically $p_2 = R(p_1)$ for seller 2. As Fig. 4 shows, there is a unique stage 1 equilibrium in the class under consideration, $p_1 = p_2 = (u + c)/2$. Then the unique stage 2 equilibrium has $\gamma_{11} = \gamma_{12} = 1/2$, which means buyers pick sellers at random.[31]

In this equilibrium, half the time 2 buyers visit the same seller, meaning 1 buyer and 1 seller do not trade. This is friction as defined by Lagos (2000): simultaneously some buyers do not get served and some sellers have no customers. In fact, $\mu = 3/2$ is the expected number of trades, which is inefficient in the sense that $\mu = 2$ is physically feasible. Yet in another sense it is fairly good. Suppose both buyers go to seller 1 with arbitrary probability γ. Then the chance they both go to the same seller is $\gamma^2 + (1 - \gamma)^2$, which is minimized at $\gamma = 1/2$. Also, while the total expected surplus is not maximized, buyers like this equilibrium because $V_b = V_s = 3(u + c)/8$, while they get $V_b = 0$ in pure-strategy equilibrium with $p_1 = p_2 = u$. We summarize these results as follows:

[31] Although buyers pick sellers randomly in equilibrium, the fact that search can be directed off the equilibrium path of course disciplines prices on the equilibrium path. This is similar to, e.g., the threat of rejection disciplining offers, or the threat of default disciplining lending, on the equilibrium path, in bargaining and in credit arrangements, respectively.

Proposition 5 *In the* 2×2 *market there exists a unique non-coordinated equilibrium, which is* $p_j = (u + c)/2$ *and* $\gamma_{ij} = 1/2$. *The expected number of trades is* $\mu = 3/2$ *and the individual trading probabilities are* $\alpha_b = \alpha_s = 3/4$.

In addition to posting, Julien et al. (2000) consider auctions: if 1 buyer shows up he pays the posted price; and if 2 show up they bid, resulting in the Bertrand price $\bar{p} = u$. One can think of sellers posting a reserve price, denoted $\underline{p}$ below. Given this, the analogs of (40) and (41) are

$$
\begin{aligned}
V_{b11} &= (1 - \gamma_{21})(u - p_1) \\
V_{b12} &= \gamma_{21}(u - p_2),
\end{aligned}
$$

because a buyer gets 0 surplus unless he is the only one visiting a seller. A mixed-strategy equilibrium at stage 2 entails $\gamma_{11} = \gamma_{21} = (u - p_1)/(2u - p_1 - p_2)$. Then one can check sellers have a dominant strategy, $\underline{p} = (u + c)/2$. So in equilibrium, the reserve price is same as the posted price in the benchmark model. However, profit is higher: $V_s = (u + c)/2$ with auctions and $V_s = 3(u + c)/8$ with posting.

Coles and Eeckhout (2003) integrate the approaches by letting sellers post p contingent on the number of buyers that show up, say p^Q (Q for queue length). This nests pure posting with $p^1 = p^2$, and auctions with $p^2 = \bar{p} = u$ and $p^1 = \underline{p}$. One can make assumptions to preclude this – e.g., buyers do not observe Q – but suppose we allow it. Then there is an equilibrium with $p^1 = p^2 = (u + c)/2$, as in the baseline model, but there are many others, all with $p^1 = (u + c)/2$ and any $p^2 \in [c, u]$. They are not payoff equivalent, and profit is highest with auctions. To see why, note first that a seller is indifferent to posting any pair (p^1, p^2) delivering V_b to buyers, but *not* indifferent to his rival's posting. Suppose, e.g., he lowers his p to increase the probability customers come to him. That decreases the probability they go to his rival, so if they go to his rival they are more likely to get p^1 and less likely to get p^2. Ergo, stealing business is harder and as a result profit is higher when sellers post $p^1 < p^2$ rather than $p^1 = p^2$.

An exceptional case is the Coles-Eeckhout equilibrium where $p^2 = c$ is at its minimum value. In this equilibrium a buyer gets the same expected payoff, $(u-c)/2$, whether or not the other one shows up, with the expectation taken before it is determined who gets the good if both show up. Hence, a seller's deviation does not affect buyers' expected payoffs when they visit his rival, and the strategic effect in the previous paragraph is inoperative – so one might say the market utility approach is valid. We say more on this in Section 5.3, after generalizing the environment to allow any numbers of buyers and sellers.

But first we mention a way around the Coles-Eeckhout indeterminacy, due to Dutu et al. (2011), in monetary economies. They show there is an equilibrium with $p^1 = (u+c)/2$ and any $p^2 \in [c,u]$ at $i=0$. By continuity, for $i>0$ there is an equilibrium with p^1 close to $(u+c)/2$, but more care is needed to determine p^2, as buyers must decide in the CM how much $\hat{m}$ to bring to the DM. Consider equilibrium where some bring $\phi\hat{m} = p^1$ and others $\phi\hat{m} = p^2$, and they are indifferent, taking into account the cost $i\phi\hat{m}$. This provides an additional equilibrium condition to pin down p^2. Hence, the indeterminacy in Coles and Eeckhout vanishes in a monetary economy – although, as usual, the introduction of monetary considerations can engender other types of multiplicity, for different reasons. Again, directed search and monetary economics are intimately related.

5.2 The $n_b \times n_s$ Market

Consider any integer numbers of buyers and sellers, n_b and n_s. A pure price posting strategy for seller j is $p_j \in [c,u]$, and we let $\mathbf{p} = (p_1, ...p_{n_s})$. A search strategy for buyer i is $\boldsymbol{\gamma}_i = \left(\gamma_{i1}, ...\gamma_{in_s}\right)$, with γ_{ij} the probability he visits seller j, and we let $\boldsymbol{\gamma} = (\boldsymbol{\gamma}_1, ...\boldsymbol{\gamma}_{n_b})$. An equilibrium is a $(\mathbf{p}, \boldsymbol{\gamma})$ such that no one wants to deviate, and a symmetric equilibrium is one with $p_j = p$ and $\gamma_{ij} = 1/n_s$. To check for equilibrium we need to know what happens after a seller deviates. Starting at a symmetric $\mathbf{p} = (p, p...p)$, suppose some seller, say $j=1$, deviates to p_1 so that $\mathbf{p} = (p_1, \mathbf{p}_{-1})$. We now check if this is profitable, given buyers play a symmetric equilibrium after the deviation.

Let the probability any buyer visits seller 1 after his deviation be $\gamma_1 = \gamma_1(p_1, \mathbf{p}_{-1})$. A symmetric subgame-perfect equilibrium is described by p and $\gamma_1(p_1, \mathbf{p}_{-1})$ satisfying the following conditions: (a) $p_1 = p$ maximizes $V_{s1}(p_1, \mathbf{p}_{-1})$; (b) $\gamma_1^*(p_1, \mathbf{p}_{-1})$ constitutes an equilibrium in the subgame for any p_1 and $\mathbf{p}_{-1} = (p, ...p)$; and (c) on the equilibrium path $\gamma_{ij} = 1/n_s$, while after a deviation buyers go to seller 1 with probability $\gamma_1 = \gamma_1(p_1, \mathbf{p}_{-1})$ and all other sellers with probability $\bar{\gamma}_1 = (1-\gamma_1)/(n_s - 1)$.

Proposition 6 *In an $n_b \times n_s$ market, let $n = n_b/n_s$ and $\eta = 1 - 1/n_s$. Then there exists a unique non-coordinated equilibrium, and in this equilibrium every seller sets*

$$p = \frac{(1 - \eta^{n_b} - n\eta^{n_b - 1})\, u + n\eta^{n_b} c}{1 - \eta^{n_b} - n\eta^{n_b-1} + n\eta^{n_b}}, \tag{45}$$

while every buyer visits each seller with probability $1/n_s$. The expected number of trades is

$$\mu = \mu(n_b, n_s) = n_s (1 - \eta^{n_b}). \tag{46}$$

Proof: Start at $\mathbf{p} = (p, p, ...p)$, let seller 1 deviate, and consider symmetric equilibria where $\gamma_1 = \gamma_1(p_1, \mathbf{p}_{-1})$. Let the probability that at least one buyer visits seller 1 be $\alpha_{s1} = \alpha_{s1}(p_1, \mathbf{p}_{-1})$. Since he gets no customers with probability $(1-\gamma_1)^{n_b}$, clearly,

$$\alpha_{s1} = 1 - (1-\gamma_1)^{n_b}. \tag{47}$$

Let the probability a buyer trades if he visits seller 1 be $\alpha_{b1} = \alpha_{b1}(p_1, \mathbf{p}_{-1})$. Notice $n_b \gamma_1 \alpha_{b1} = \alpha_{s1}$, as the LHS is the expected number of buyers who get served by seller 1 and the RHS is his expected number of sales. This and (47) imply

$$\alpha_{b1} = \frac{1 - (1-\gamma_1)^{n_b}}{n_b \gamma_1}. \tag{48}$$

Now the profit of the deviant seller is

$$V_{s1}(p_1, \mathbf{p}_{-1}) = \alpha_{s1}(p_1 - c) = [1 - (1-\gamma_1)^{n_b}]\,(p_1 - c), \tag{49}$$

and the payoff to a buyer visiting him is

$$V_{b1}(p_1, \mathbf{p}_{-1}) = \alpha_{b1}(u - p_1) = \frac{1 - (1 - \gamma_1)^{n_b}}{n_b \gamma_1}(u - p_1). \tag{50}$$

Also, the payoff to a buyer visiting seller $j \neq 1$ is

$$V_{bj}(p_1, \mathbf{p}_{-1}) = \frac{1 - (1 - \bar{\gamma}_1)^{n_b}}{n_b \bar{\gamma}_1}(u - p). \tag{51}$$

Given this, the FOC for maximizing π_1 is

$$0 = \frac{\partial V_{s1}}{\partial p_1} = \alpha_{s1} + (p_1 - c)\frac{\partial \alpha_{s1}}{\partial p_1} = \alpha_{s1} + n_b(1 - \gamma_1)^{n_b - 1}(p_1 - c)\partial \gamma_1 / \partial p_1. \tag{52}$$

In a symmetric mixed equilibrium in the subgame, buyers are indifferent between visiting any seller, which means γ_1 satisfies

$$\frac{1 - (1 - \gamma_1)^{n_b}}{n_b \gamma_1}(u - p_1) = \frac{1 - (1 - \bar{\gamma}_1)^{n_b}}{n_b \bar{\gamma}_1}(u - p). \tag{53}$$

Over the range $\gamma_1 \in (0, 1)$, we can derive $\partial \gamma_1 / \partial p_1$ and insert it into (52), then simplify using $p_1 = p$ and $\gamma_1 = 1/n_s$ to verify that a deviation is not profitable iff p solves (45). Hence there is a unique symmetric equilibrium where buyers mix. Galenianos and Kircher (2012) prove there are no asymmetric equilibrium where buyers mix and sellers use pure strategies. All that remains is to show μ satisfies (46), but that follows directly from (47) and (48) with $\gamma_1 = 1/n_s$. ■

Several remarks are in order. First, (45) endogenously gives p as a weighted average of c and u, which might not be apparent from Burdett et al. (2001) because they normalize $u = 1$ and $c = 0$. In fact, the weight on u is the probability a seller gets at least 2 buyers, and the weight on c is probability he gets just 1. Second, as in the 2×2 game, in equilibrium buyers visit sellers at random, but the fact that search can be directed still disciplines prices. Third, notice that p is a smooth function of n_b and n_s, which we think is nice. To say why, let n_s be large, and note that as n_b goes from $n_s - 1$ to $n_s + 1$ frictionless equilibrium theory predicts p jumps from c to u. As shown in Fig. 5, with competitive search the discrete jump gets smoothed out by the frictions, which is one reason Peters (1984,1991) and others advocate the approach in the first place.

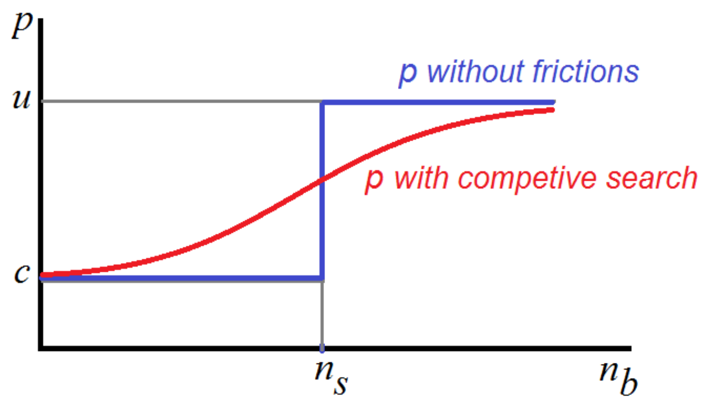

Figure 5: Price and market tightness

Fourth, (46) endogenously gives $\mu(n_b, n_s)$ as an urn-ball meeting technology, used in economics at least since Butters (1977) and Hall (1979). The name reflects the fact that the number of buyers up at a seller is binomially distributed, like putting n_b balls in n_s urns in elementary probability theory, which converges to a Poisson distribution as $n_b, n_s \to \infty$ for fixed $n = n_b/n_s$, with e^{-n} the probability a seller gets no buyers.[32] Also note that $\mu(n_b, n_s)$ displays DRS for finite n_b and n_s, but quickly converges to CRS as n_b and n_s grow. Finally, while we appeal to Galenianos and Kircher (2012) to claim uniqueness with homogeneous sellers, Kim and Camera (2014) extend this to heterogeneity. These papers also consider more general meeting technologies, risk aversion and private information.

Versions of the above results can be found in Peters (2000), Julien et al. (2000) or Burdett et al. (2001). Using standard formulae, they also imply:

[32] From this it should be evident that it can matter who posts and who searches in finite markets, since throwing n_b balls into n_s urns is not the same as throwing n_s balls into n_b urns (Kultti 2000; Halko et al. 2008). Indeed, it matters even when $n_b, n_s \to \infty$, where (based on Proposition 7 below) the number of matches is $\mu^s = n_s(1 - e^n)$ if sellers post and $\mu^b = n_b\left(1 - e^{1/n}\right)$ if buyers post. As Delacroix and Shi (2016) point out, in this case $\mu^s > \mu^b$ iff $n_s < n_b$, and hence we generate more meetings when the short side posts and the long side searches. They also show it does not matter when the matching function is symmetric in the sense that the number of meetings $\mu(n_b, n_s)$ does not depend on who posts and who searches, which is a maintained assumption in the baseline models. Delacroix and Shi (2016) also consider asymmetric technologies and other generalizations.

Proposition 7 *Let* $n_b, n_s \to \infty$ *holding* $n = n_b/n_s$ *fixed. Then*

$$p(n_b, n_s) \to [1 - \varepsilon(n)]\, u + \varepsilon(n)c, \tag{54}$$

where $\varepsilon(n) = ne^{-n}/\left(1 - e^{-n}\right)$ *is the usual elasticity, and*

$$\alpha_b(n_b, n_s) \to \left(1 - e^{-n}\right)/n \text{ and } \alpha_s(n_b, n_s) \to 1 - e^{-n}. \tag{55}$$

As they did for the 2×2 market, Julien et al. (2000) also consider auctions in $n_b \times n_s$ markets. They show that as $n_b, n_s \to \infty$ the reserve price goes to $\underline{p} = c$, and payoffs are the same as under posting (see Appendix D for details). They also consider a dynamic labor market version, which generates steady state unemployment and wage dispersion. Also, we mention that Julien et al. (2000) and Burdett et al. (2001), with entry, equilibrium is not efficient in the $n_b \times n_s$ case, because the reserve price is positive, and because the matching functioni exhibits decreasing returns to scale for finite numbers. See Julien et al. (2005,2006*b*,2011) for a discussion how this relates to efficiency in Mortensen (1982*b*).

5.3 Issues, Applications and Extensions

The above methods allows us to determine p without an exogenous bargaining solution, and to determine α_s and α_b without an exogenous meeting technology. While this is attractive, we keep an open mind. For the terms of trade, bargaining better captures situations without commitment and hence with holdup problems. For the trading probabilities, (55) is a special case of models where α_s and α_b come from a general meeting technology, which is relevant empirically to the extent that urn-ball functions can perform poorly when confronted with data (Petrongolo and Pissarides 2001). There are trade-offs, and different approaches may be appropriate in different applications.

Having said that, one clear advantage to finite numbers concerns off-equilibrium beliefs: all we need is subgame perfection. With a continuum, however, things gets tricky. Given a candidate equilibrium $\mathbf{p}$, if one seller from a continuum deviates, what is the best response of buyers? If the deviator is measure 0 there is no response. The market utility approach skirts this issue in a way that is not entirely satisfactory. As an alternative, Galenianos and Kircher (2009) posit a set of artificial sellers with measure δ that exogenously post every p in the relevant range. They then properly evaluate deviation payoffs for $\delta > 0$, and focus on equilibria that obtain when $\delta \to 0$. This works, but is slightly cumbersome.

An alternative going back to Montgomery (1991) is to use the market utility approach in $n_b \times n_s$ markets by solving

$$V_s = \max_{p,\gamma} \alpha_s(p - c) \text{ st } \alpha_b(u - p) = V_b. \tag{56}$$

In (56), α_s is the probability a seller gets at least 1 buyer, α_b is the probability a buyer visiting the seller gets served, and both depend on the probability buyers visit the seller γ, as derived in Section 5.2. Eliminating p using the constraint and taking the FOC with respect to γ, we get $(1-\gamma)^{n_b-1}u = V_b$. In symmetric equilibrium, all sellers post the same p and $\gamma = 1/n_s$. Hence $V_b = \eta^{n_b-1}u$, where as above $\eta = 1 - 1/n_s$, and the constraint yields

$$p = \frac{\left(1 - \eta^{n_b} - n\eta^{n_b-1}\right)u + n\eta^{n_b-1}c}{1 - \eta^{n_b}}. \tag{57}$$

This is nice, but not quite right – in a 2×2 market, e.g., (45) gives $p = (u+c)/2$ while (57) gives $p = (u + 2c)/3$. To be fair, Montgomery solves the 2×2 model correctly, and acknowledges it is a "short cut" in the general case to take V_b as given after a seller deviates. The difference between (45) and (57) is the presence of $-n\eta^{n_b-1} + n\eta^{n_b}$ in the denominator of the former, so the Burdett et al. price is higher than the Montgomery price, because it is less attractive for a seller to lower his p when he recognizes this increases buyers' utility. However, as $n_b, n_s \to \infty$ holding n fixed, this consideration vanishes, so Montgomery's approach gives the right answer in large markets. In small markets, his approach can be misleading – e.g., it yields efficiency in versions with entry or where sellers are heterogeneous, but only because it neglects relevant strategic effects.[33]

[33] Galenianos et al. (2011) show the correct approach with heterogeneous sellers implies too

Galenianos et al. (2011) and Galenianos and Kircher (2012) propose a hybrid approach, solving (56) with $V_b = V_b(\gamma)$. This means sellers must offer buyers their market utility, but recognizes that it should be computed *after* a deviation. Given a symmetric mixed equilibrium after the deviation, and using (51) and $\bar{\gamma} = (1-\gamma)/(n_s - 1)$, we have

$$V_b(\gamma) = \frac{1 - \left(1 - \frac{1-\gamma}{n_s - 1}\right)^{n_b}}{n_b \left(\frac{1-\gamma}{n_s-1}\right)} (u - p^*) . \tag{58}$$

Solving (56) with this V_b, then imposing equilibrium, we get the p in (45), which is correct for any (n_b, n_s). Hence we get the right result for small markets, but the method and notation are similar to the earlier analysis of large markets.

Norman (2016) offers another approach, based on population uncertainty (Myerson 1998,2000). Suppose n_b and n_s are independently drawn from Poisson distributions, where sellers do not see the realization, while buyers see n_s and prices when they choose search strategies. This justifies the usual focus on symmetric equilibrium, where buyers use mixed strategies, since he shows any equilibrium is payoff equivalent to that (thus eliminating the other equilibria mentioned in fn. 30). The Poisson assumption makes the model tractable. Also, this model belongs to a general class in which prices are strategic complements, which implies there is a unique equilibrium if there is a unique symmetric equilibrium. As well, as usual, as $n_b, n_s \to \infty$ his outcome approaches our benchmark results. More generally, Norman's ostensibly minor change in the environment generates many results in a tractable way, as should be useful in future applications.[34]

On a less technical note, Burdett et al. (2001) consider two types of sellers with different capacity: n_1 of them have 1 unit for sale; n_2 have 2 units; and

much trade at high-cost sellers because the strategic effects increase prices more at low-cost sellers. Price ceilings (or minimum wages in a labor market) can restore efficiency.

[34] He also generalizes Galenianos and Kircher (2012) and Kim and Camera (2014). First, he shows sellers' profit is strictly concave in the relevant range. Then he shows his reduced-form game is supermodular under more general conditions than previous authors. This is useful because supermodular games have a smallest and largest equilibrium, and the existence of pure strategy equilibrium is simple. Because profit is strictly concave, mixed strategies by sellers are ruled out. He also provides a simple test for uniqueness with symmetric sellers.

$n_s = n_1 + 2n_2$ is the total quantity on the market. This implies a matching function $\mu(n_b, n_1, n_2)$, which in general does not reduce to a function of only (n_b, n_s). Intuitively, the coordination friction is worse when there are more sellers with 1 unit and fewer with 2, holding n_s constant. In a labor application, this helps account for a changing Beveridge curve in the data, as the relationship between unemployment and total vacancies depends shifts with the mix of big and small employers. .Lester (2010) lets firms choose how many positions to create, making the distribution of vacancies endogenous. He derives the matching function and studies the implications of various shocks, depending on whether job creation occurs via entry by new firms or expansion by existing firms.[35]

In another application, Lester (2011) introduces *semi-directed* search: some buyers, called locals, are informed and direct their search to sellers posting favorable terms; others, called tourists, are uninformed and search randomly. He has free entry of sellers. In a large market, equilibrium features two types of sellers: local shops post low prices to attract the informed; tourist shops exploit the uninformed. In fact, there are *at most* two types of sellers, since if the fraction of informed buyers is above a threshold, which is strictly below 1, no tourist shops opens. Lester also analyzes markets with small numbers, but only under a parameter restriction that guarantees no tourist shops open, so there is only one price posted. Recently, Huang (2016) makes progress on relaxing this restriction, at least for $n_s = 2$. He shows there is a unique equilibrium, and it involves sellers randomizing over p in discrete infinite set $\{p_1, p_2, ...\} \subset [c, u]$.

The setup is useful for studying changes in information. Lester (2011) shows that increasing the fraction of informed buyers can increase or decrease prices, depending on parameters. This is contrary to conventional wisdom, and to several papers where prices fall when consumers are better informed (Salop and Stiglitz; Varian 1980; Burdett and Judd 1983; Stahl 1989). However, Lester's result requires finite numbers: the fraction of informed buyers actually does not

[35] See .Tan (2012), Li and Tain (2013), Kultti and Mauring (2014) and Godenhielm and Kultti (2015) for more on this topic.

affect prices with a continuum. Gomis-Porqueras et al. (2015) pursue this when the number of informed buyers is endogenized by costly advertising. They show more information can raise prices even in large markets. Bethune et al. (2016) also show more information can raise or lower prices in large markets in monetary economies (see also Dong et al. 2016). Intuitively, if buyers' bargaining power is low at tourist shops, they get more from a marginal dollar at local shops. With better information, the fraction of local shops rises, and in response buyers bring more money, which allows sellers to raise prices.

This is relevant because, as Ellison and Ellison (2005) say, "evidence from the Internet... challenged the existing search models, because we did not see the tremendous decrease in prices and price dispersion that many had predicted." Similarly, according to Baye et al. (2006), "Reductions in information costs over the past century have neither reduced nor eliminated the levels of price dispersion observed." As the previous paragraph indicates, not all search models predict prices fall with information. And it is even more obvious that price dispersion need not fall with information: when the fraction of informed buyers is 0 or 1, we are in a pure random or a pure directed search world, and both have no dispersion; when the fraction is between 0 and 1 there is price dispersion; and so it is obviously nonmonotone in information.

Let us now return to Coles and Eeckhout (2003), by allowing sellers to post p^Q where Q is the number of buyers that show up. As in the 2×2 market, indeterminacy obtains.[36] However, Selcuk (2012) perturbs the environment by having risk-averse buyers, so $V_b = \mathbb{E}U\left(S_b^Q\right)$, where $S_b^Q = \left(u - p^Q\right)/Q$ is buyers' surplus when Q buyers show up. He shows that $U'' < 0$ eliminates the indeterminacy: there is a unique equilibrium, and it features $p^Q = p \ \forall Q$. In particular,

[36] Related to the discussion at the end of Section 5, there is one exceptional equilibrium where buyers' expected payoff is independent of Q. Thus, a seller's deviation does not affect buyers' expected payoff at his rivals, so one might say the market utility approach is valid. Moreover, if we add entry, e.g., it will be efficient, even with finite numbers, because the strategic impact of pricing is inoperative in this equilibrium.

if $U(S) = S^a$ with $a \in (0,1)$, in a market with $n = n_b/n_s$ all sellers post

$$p = \frac{(1 - e^{-n} - ne^{-n})\, u + ane^{-n} c}{1 - e^{-n} - ne^{-n} + ane^{-n}}.$$

Notice $\partial p/\partial a < 0$, and $p \to u$ as $a \to 0$. Also notice that $a = 1$ implies p is the same as (54). Risk aversion seems an important extension to many applications of directed search, some of which are discussed further below.

In fact, posting a contingent p^Q is still restrictive – why can't buyers make or receive transfers even if they do not get served? Again, one might make assumptions to preclude this, but suppose we allow it. As in Jacquet and Tan (2012), for any mechanism in a general class, the outcome can be implemented by having a buyer who gets the good pay p and having others that show up pay ϕ, which can be positive or negative. If $\phi = p - u$ then buyers are fully insured: their payoff is the same whether or not they get served. Jacquet and Tan (2012) show that when buyers are risk averse there is a unique equilibrium and it features full insurance, $\phi = p - u$. Thus one might say the market utility assumption holds, and that this provides strategic foundation for the results in Montgomery (1991), including his efficiency results (recall fn. 33).

Returning to risk neutrality, Geromichalos (2012) explores several additional extensions in $n_b \times n_s$ markets. First, he gives sellers the capacity to each serve up to κ customers, and post mechanisms announcing the number they will serve, which cannot exceed κ but could be less. He also allows p^Q to be contingent on the queue of buyers that show up, and allows payments from those who get served and those who do not. As in the previous paragraph, it suffices to consider mechanisms where buyers that get served pay p while buyers that show up but do not get served pay ϕ. He then shows that in a particular sense this does not matter in large markets – it is payoff equivalent to have sellers simply post one price, as in the baseline model.

To illustrate his methods, let us set capacity to $\kappa = 1$. Then, as shown in

Appendix F, in the limit as $n_b, n_s \to \infty$ with $n = n_b/n_s$ fixed, equilibrium satisfies

$$(p-c)\left(1-e^{-n}\right)+n\phi=(u-c)\left(1-e^{-n}-ne^{-n}\right). \tag{59}$$

This pins down $V_s = (u-c)(1-e^{-n}-ne^{-n})$, which is the same as the outcome described in Proposition 6. Intuitively, any combination (p, ϕ) satisfying (59) is payoff equivalent to the outcome when each seller simply posts p. However, this needs to be reconciled with Virag (2011), who shows that for any market size, and the same type of mechanism (p, ϕ), there is always an equilibrium where sellers extract the entire surplus and market utility is 0. Hence, it is not true that all equilibria converge to the limit in (59). The difference stems from the fact that Geromichalos (2012) excludes the possibility of infinite ϕ: as Virag (2011) makes clear, the convergence does apply if there is a bound on ϕ, in which case the limit in (59) is valid.

Geromichalos (2014) is a sequel on the Bertrand paradox – the idea that duopoly models often have an equilibrium where price equals marginal cost (this is a paradox because one might expect that to require many sellers). A potential resolution discussed in the literature involves capacity constraints: if a firm cannot meet market demand, a rival charging more can still get customers. He argues this is a special case of the idea that buyers' payoffs might fall with realized demand at a given location, and considers three examples: capacity constraints; congestion effects; and prices that depend on Q as in Coles and Eeckhout (2003). These all resolve the paradox by making buyers' payoffs at a seller's location fall with the number that show up, so buyers may not all go to a seller just because his p is lower. As Geromichalos says, while the related literature on industrial organization typically specifies demand exogenously, directed search theory generates it endogenously as a function of strategic behavior, and is therefore ideal for studying these issues.

6 Heterogeneity and Sorting

We now discuss in more depth heterogeneity on one or both sides of the market. The presentation starts with an abstract formulation and definition of equilibrium, then covers specific applications from the literature.[37]

6.1 General Framework

Consider any two-sided market with heterogeneous agents: there are types t_1 and t_2 on each side of the market, with distributions $N_1(t_1)$ and $N_2(t_2)$ on supports $\mathcal{T}_1$ and $\mathcal{T}_2$. This encompasses buyers and sellers with heterogeneous valuations or costs, workers and firms with heterogeneous productivity, and a general theory of partnership formation when agents have heterogeneous attributes. The type distributions are fixed for now, but can be determined by entry. Also, we focus mainly on the case where types are drawn ex ante, before decisions are made, but mention how to have types drawn after meeting counterparties.

Each agent from side 1 posts a mechanism $s \in \mathcal{S}$, which includes all information side 2 sees. Thus, e.g., with pure price posting $s = p$ and $\mathcal{S} = \mathbb{R}_+$, while if side 1 also states his type then $s = (p, t_1)$ and $\mathcal{S} = \mathbb{R}_+ \times \mathcal{T}_1$, where for notational convenience he can state any $t_1 \in \mathcal{T}_1$, but when types are observable we assign payoff $-\infty$ to lying. Richer mechanisms include auctions, where agents in meetings have actions like bidding. In such cases, an action for side i is denoted $a_i \in \mathcal{A}_i$. We assume mechanisms can be ordered, and for side 1 let $n_1(s, a_1, t_1)$ be the mass of types weakly below t_1 posting s or lower, and who play action a_1 or lower if matched. With a slight abuse of notation, let $n_1(s)$ be the marginal, i.e., the mass posting s or lower. Similarly, distributions for side 2 indicate where they direct their search and what they do if matched. Then the equilibrium concept is based on the theory of large games (e.g., Mas-Collel 1984), where an individual's

[37] While some of this material is quite technical, at least relative to Sections 2-4, we think it is important to be precise at this point. Also, we think it is best to see this framework after the finite models in Section 5, which can be somewhat messy, but at lest they avoid some measure-theoretic complications dealt with here.

payoff is determined by his behavior, and the distribution of others' behavior described here by n_1 and n_2, but not on the behavior of any other particular individual.

As applied to competitive search by Eeckhout and Kircher (2010) and Peters (2010), a key variable is tightness in the submarket with mechanism s, determined on the equilibrium path by the ratio of side 2 to side 1 agents in this submarket. If a submarket attracts a mass of agents on both sides, this ratio is determined in the obvious way. If one lets these masses shrink to have 0 measure, as is often natural in applications, the limit is the Radon-Nikodym derivative, $n(s) = dn_2(s)/dn_1(s)$.[38] Other keys variables are the set of types on side 1 posting any s, and the set on side 2 searching for s. Let $\phi_i(t_i, a_i; s)$ be the distribution of types from side i playing action a_i in submarket s. For the mechanisms that are posted in equilibrium, this has to be consistent with the conditional of $n_i(s, a_i, t_i)$ on s, and is almost everywhere uniquely determined by n_1 and n_2, as is n.[39] However, this does not determine n and ϕ_i off the equilibrium path, as discussed below.

In sum, in addition to his own type, an agent cares about these objects in a submarket: the probability of trade; the distribution of types on the other side; and payoffs within matches. These depend on: his choice of s and a_i; the expected number of other traders determined by n; and the distribution of types and actions of other traders determined by ϕ. Consider type t_i from side i in a submarket with mechanism s, expecting tightness n and distribution ϕ. Denote his probability of trade by $\tilde{\alpha}_i(s, a_i, t_i; n, \phi)$, his payoff in a pair $t = (t_1, t_2)$ given $a = (a_1, a_2)$ by $v_i(s, a, t)$, and his payoff from not trading by $\underline{v}_i(s, a, t)$. His

[38] Technically the Radon-Nikodym derivative ensures that for any subset $\mathcal{S}' \subseteq \mathcal{S}$ the integral $\int_{s \in \mathcal{S}'} n(s) dn_1(s)$ is the same as the associated number of side 2 agents searching for $s \in \mathcal{S}'$, $\int_{s \in \mathcal{S}'} dn_2(s)$. Whenever we have masses of agents on both sides, the Radon-Nikodym derivative represents the ratio correctly. As it is never optimal for side 1 to post an s that attracts no one from side 2, we can assume $\sup n_1(s)$ is a subset of $\sup n_2(s)$, where $\sup n_i(s)$ is the support of n_i, which is necessary for the Radon-Nikodym derivative to be well-defined.

[39] Note that $n_i(s, a_i, t_i)$ can have a mass larger than 1 if there are more than a unit measure of agents on side i. Let $\bar{n}_i$ be the total mass of agents attempting to trade on side i. Then $\phi_i(t_i, a_i; s)$ has to be a distribution of $n_i(s, a_i, t_i)/\bar{n}_i$ conditional on s. While ϕ_2 might not be almost everywhere determined if there is a positive measure of mechanisms that are not visited by anyone, in this case the matching probability is zero and beliefs do not matter.

expected payoff is then

$$V_i(s, a_i, t_i; n, \phi) = \tilde{\alpha}_i(s, a_i, t_i; n, \phi)\mathbb{E}v_i(S, a, t) + [1 - \tilde{\alpha}_i(s, a_i, t_i; n, \phi)]\,\mathbb{E}\underline{v}_i(S, a, t) \tag{60}$$

where expectations are with respect to ϕ on the other side of the market.

As elsewhere in this survey, off-equilibrium is tied to the notion of subgame perfection. Let the market utility $U(t_2)$ be the supremum of $V_2(s, a_2, t_2; n, \phi)$ over s posted in equilibrium and actions $a_2 \in A_i$. We capture this notion by:

> Condition P: If $n(s) > 0$ then the support of $\phi_2(t_2, a_2; s)$ is non-empty and includes only $(t_2, a_2) \in \mathcal{T}_2 \times \mathcal{A}_2$ st $V_2(s, a_2, t_2; n, \phi) = U(t_2)$.

Thus, a positive number of side 2 agents can only be expected if those agents get their market utility, and beliefs include only such agents. It is also convenient to impose that beliefs exclude strictly-dominated combinations:[40]

> Condition B: The support of beliefs ϕ_1 does not allow combinations $(t_1, a_1; s)$ such that (a_1, s) is strictly dominated for t_1 whenever there is $(a_1'; s)$ that is not dominated for some t_1'.

Equilibrium is defined as a distribution of strategies – what to post and where to search – plus tightness and type distributions in every submarket satisfying the natural conditions. To make this precise, let $\sup n_i$ be the support of n_i, let s_0 be a fictitious mechanism that yields zero utility, the outside option, and let $\mathcal{S}_0 = \mathcal{S} \cup \{s_0\}$ be the set of mechanisms plus this option. Then we have:

Definition 1 *Equilibrium is a list of functions $\langle n_1, n_2, n, \phi_1, \phi_2, U \rangle$ such that:*

1. *(maximization)* $V_i(s, a_i, t_i; n, \phi) \geq V_i(s', a_i', t_i; n, \phi)$ $\forall (s, a_i, t_i) \in \sup n_i$ *and* $(s', a_i') \in \mathcal{S}_0 \times \mathcal{A}_i$*;*

[40] Even when there are large costs to lying, some restriction is necessary to ensure that agents believe announcements off the equilibrium path, and excluding strictly dominated combinations does the trick. Other standard refinements from signalling theory can also be used, although we note in passing that signalling has not received that much attention in the directed search literature.

2. *(consistency)* ϕ *is consistent with the conditional distributions of* n_1 *and* n_2*, while* n *is consistent with their Radon-Nikodym derivative;*

3. *(perfection) Conditions P and B hold;*

4. *(feasibility)* $\forall \mathcal{T}_i' \subseteq \mathcal{T}_i$

$$\int_{\mathcal{S}\times\mathcal{A}_i\times\mathcal{T}_i'} dn_i(s, a_i, t_i) \leq \int_{\mathcal{T}_i'} dN_i(t_i),$$

with $=$ *if* $\max_{(s,a_i)\in\mathcal{S}\times\mathcal{A}_i} V_i(s, a_i, t_i; n, \phi) > 0$ *for almost all* $t_i \in \mathcal{T}_i'$.

To be clear, the first condition says that agents participate in submarkets that maximize their payoffs given their beliefs. The second says beliefs are consistent with these strategies. The third captures off-equilibrium behavior. The inequality in the fourth says that the measure of agents in submarkets can never exceed their measure in the population, but could be less if some agents abstain from trade and take their outside options; if there are strictly positive returns, they do not abstain, and this holds with equality.

While the above definition does not incorporate entry, that can easily be amended – e.g., with entry by side 1 at cost $k(t_1)$, we replace their maximization condition by $V_1(s, a_1, t_1; n, \theta) \leq k(t_1)$, with $=$ if $(s, a_1, t_1) \in \sup n_1$. Also, this setup assumes types are drawn ex ante, but we can alternatively have side 2 draw their types after making search decisions. Consider $n_2(s, a_2, t_2)$. If type is realized after entering a submarket, its conditional $n_2(a_2, t_2|s)$ must have a marginal equal to the distribution of types $N_2(t_2)/N_2$, where N_2 is the total mass of type 2. Among possible conditional distributions, consider the one that maximizes the expectation of $V_2(s, a_2, t_2; n, \phi)$ over a_2 and t_2, and let U be the highest such expectation across s. Then the analog to Condition P is this: for s off the equilibrium path, ϕ_2 has to coincide with one of these maximizers, and the maximal value equals market utility U if tightness is strictly positive. Also, the maximization condition for side 2 is adjusted as follows: if s is in the support of $n_2(s)$ then $\mathbb{E}_{n_2(a_2,t_2|s)}V_2(s, a_2, t_2; n, \phi) = U$, which says that on the equilibrium path market utility is achievable.

6.2 Sorting

Consider the case of a bilateral meeting technology and price posting, as in Sections 2-4, where types are known ex ante and observable, and a mechanism lists a price, the type of agent 1 and the type of agent 2 he aims to attract.[41] In our general notation, $s = (p, t_1, t_2)$ from set $\mathcal{S} = \mathbb{R}_+ \times \mathcal{T}_1 \times \mathcal{T}_2$. There are no actions because in a meeting agents simply trade at the posted price. This is relatively tractable since trading probabilities only depend on n and types are observed, implying $\tilde{\alpha}_1 \left[(p,t), t_1, t_2; n, \phi\right] = \alpha_1(n)$ and $\tilde{\alpha}_2 \left[(p,t), t_1, t_2; n, \phi\right] = \alpha_2(n)$, where $\alpha_1(n) = \alpha(n)$ and $\alpha_2(n) = \alpha(n)/n$, as in the baseline model.

Here trade yields a surplus $S(t_1, t_2)$ that depends on the types, while no trade yields 0 utility. Let $S_i(t_1, t_2)$ be the surplus of side i, which allows private valuations, $S_i(t_1, t_2) = t_i$. In this case, and, more generally, whenever one side's payoffs do not directly depend on the other side's type, it is not actually necessary for the mechanism to specify this type. Another interesting case is pure public goods, $S_1(t_1, t_2) = S_2(t_1, t_2) = (t_1 + t_2)/2$, where the absence of contractual type specification can lead to adverse selection, as discussed below. Unless otherwise noted, payoffs are increasing in type. Since side 2 ends up paying p to side 1, and assuming transferable utility, the actual payoff from trade for each side is $v_1 \left[(p,t), a, t\right] = S_1(t_1, t_2) + p$ and $v_2 \left[(p,t), a, t\right] = S_2(t_1, t_2) - p$.

Again, assume for now that agents get utility $-\infty$ from lying, to capture observable types with our formalization. Then side 1 agents reveal their types and are approached only by the desired types from side 2. Hence, we can treat beliefs ϕ_1 and ϕ_2 as degenerate, and the expected transfer is simply the trading

[41] Shi (2001) studies this for a specific matching function; our exposition follows the generalization in Eeckhout and Kircher (2010*b*). Davoodalhosseini (2015) modifies Eckhout and Kircher by introducing free entry and private information about side 2's type, which affects side 1's payoff. See also Jerez (2014) and Mortensen and Wright (2003), where the latter actually studies a dynamic model, with endogenous flows into the market, but only considers additively separable payoffs. The treatment mainly focuses on bilateral meetings and heterogeneity with observable types. Less is known about the case of multilateral meetings, with a few exceptions, like Shimer (2005), Julien et al. (2005) or Albrecht et al. (2014).

probability times price:

$$V_1\left[(p,t_1,t_2),a_1,t_1;n,\phi\right] = \alpha(n)\left[S_1(t_1,t_2)+p\right] \tag{61}$$

$$V_2\left[(p,t_1,t_2),a_2,t_2;n,\phi\right] = \frac{\alpha(n)}{n}\left[S_2(t_1,t_2)-p\right]. \tag{62}$$

An announcement by type t_1 maximizes (61) given that type t_2 gets $U(t_2)$, his market utility, where the endogenous variables are the type t_2 of the counterparty, the price p, and tightness n:

$$\max_{p,n,t_2} \alpha(n)\left[S_1(t_1,t_2)+p\right] \text{ st } \frac{\alpha(n)}{n}\left[S_2(t_1,t_2)-p\right] = U(t_2).$$

Introducing a change of variable, $z = S_2(t_1,t_2) - p$, we rewrite this as

$$\max_{z,n,t_2} \alpha(n)\left[S(t_1,t_2)-z\right] \text{ st } \frac{\alpha(n)}{n} z = U(t_2). \tag{63}$$

As in Shi (2001), side 1 gets the total surplus and compensates side 2 through a transfer z, further reducing the problem to

$$\max_{n,t_2} \alpha(n)S(t_1,t_2) - nU(t_2). \tag{64}$$

A difference from earlier applications is that t_2 is now an argument of $U(t_2)$. Nevertheless, one can show $U(t_2)$ is increasing, and hence differentiable almost everywhere, so the solution is still described by the FOC's.

As a special case, consider first one-sided heterogeneity, where only agents on side 2 differ. Then $S_1(t) = S(t) = t_2$ and $S_2(t) = 0$, so that side 1 reaps all the gains from trade, and compensation z in the transformed problem coincides with the true transfer price.[42] The FOC's are then

$$\alpha'(n)u - U(t_2) = 0 \tag{65}$$

$$\alpha(n) - nU'(t_2) = 0. \tag{66}$$

Each type t_2 is associated with a $n(t_2)$ and $U(t_2)$ solving these conditions, similar to Section 3.1.

[42] Since only the joint surplus $S(t)$ matters in (64) the analysis is unchanged under private valuation, $S_1(t_1,t_2) = 0$ and $S_2(t_1,t_2) = t_2$, except z in this case does not correspond to the actual price.

To proceed, notice that (65) and (66) hold $\forall t_2$, so we can differentiate the former and use the result to eliminate $U'(t_2)$ from the latter, resulting in

$$n'(t_2) = -\frac{\sigma[n(t_2)]n(t_2)}{\varepsilon(n)t_2}, \tag{67}$$

with $\varepsilon(n)$ and $\sigma(n)$ are the elasticity and the elasticity of substitution of the matching function, respectively, as in the baseline model. This differential equation determines $n(t_2)$ up to a constant. Clearly $n'(t_2) < 0$, so higher t_2 select into submarkets with lower tightness, and hence trade faster, as in Section 3.1, but still the compensation z is higher under mild conditions on the matching function, because side 1 collects all the surplus in the transformed problem but rewards better partners with higher compensation and matching prospects.

For a example with $\alpha(n) = n^\gamma$, we can explicitly solve $n(t_2) = \xi t_2^{-\gamma}$ up to the constant $\xi > 0$. Adding entry, $V_1 = k$, we pin down $\xi = k^{1/\gamma}$, $n(t_2)$, and the rest of the variables in each submarket. Without entry, ξ comes from market clearing,

$$\int n(t_2)^{-1} dN_2(t_2) = N_1, \tag{68}$$

where the RHS is the number of agents on side 1, and the LHS integrates tightness over side 2. Along with (67), this fully characterizes equilibrium. Given $z = t_2 - p$, in this example, substitution of (65) into the constraint in (63) yields $z = \gamma t_2$. Thus, each submarket satisfies the relevant version of the Hosios condition, and this entails efficiency.

With heterogeneity only on side 1 rather than side 2, as mentioned in Section 3.1, identical agents from side 2 have to be compensated for matching with different agents on side 1. Since again better types (now on side 1) are matched faster, the agents they match with trade slower. So for side 2 higher compensation is negatively related to the trading probability, the opposite of the previous case. Intuitively, prices have two roles, managing heterogeneity and trading probabilities. Hence, the relationship between the terms of trade and probability of trade generally depends on the type of heterogeneity.

Two-sided heterogeneity makes this even more clear. When there are non-zero continuous densities on both sides, the analysis remains tractable when it can be compressed into a differential equation, as above, but the issue of who trades with whom is more complicated. One concern in the literature is assortative matching (see Chade et al. 2016 for a survey). In particular, when do we get positive (negative) assortative matching, which means higher types on one side match with higher (lower) types on the other?

With two-sided heterogeneity, the FOC's from problem (64) for type t_1 are

$$\alpha'(n)S(t_1,t_2) - U(t_2) = 0 \tag{69}$$

$$\alpha(n)S_{t_2}(t_1,t_2) - nU'(t_2) = 0 \tag{70}$$

where S_{t_2} is the partial derivative. Let $t_1^*(t_2)$ indicate the type on side 1 that matches with type t_2 on side 2, and $n^*(t_2)$ the associated tightness, solving (69)-(70). Positive sorting means $t_1^{*\prime}(t_2) > 0$. The SOC's reduce to

$$t_1^{*\prime}(t_2)\left\{\frac{SS_{t_1t_2}}{S_{t_1}S_{t_2}} - \sigma\left[n^*(t_2)\right]\right\} \geq 0. \tag{71}$$

Hence positive sorting obtains if the expression in braces is positive; one can also show the converse is true (Eeckhout and Kircher 2010*b*). If the total surplus displays CRS, (71) is especially tractable, and nicely illustrates a trade-off between the elasticity of substitution in the surplus and the elasticity of substitution in the matching function.

Many standard market models (again Chade et al. 2016) imply positive sorting under a condition known as supermodularity, $S_{t_1t_2} \geq 0$. In the present formulation, (71) is stronger, because standard assumptions implies $\sigma > 0$, as well as $S, S_{t_1}, S_{t_2} > 0$. Indeed, if $S(t_1,t_2) = t_1 + t_2$, which is weakly supermodular, we get negative sorting. Intuitively, this is because it does not matter for the surplus who matches with whom, but higher types should match with a higher probability, which requires a lower probability for the other side. The losses from reduced matching prospects are minimized if the agents on the other side are low types.[43]

[43] How much supermodularity is needed to overturn this effect depends on the matching

This holds if more potential partners increases one's matching prospects, but ceases to apply if the matching function approaches the inelastic limit, $\sigma \to 0$, in which case the sorting condition (71) becomes standard.

As Eeckhout and Kircher (2010b) discuss, this characterization also holds for finite numbers L_1 and L_2 of types on each side. Since each combination of types trades in a different submarket, in principle there could be up to $L_1 \times L_2$ submarkets. Sorting reduces this complexity substantially, however, since only adjacent types trade – i.e., if there is sorting, and t_1 trades with both t_2 and t_2', then all types t_2'' in between also trade only with t_1. This implies that there are no more than $L_1 + L_2$ submarkets.

With a continuum of types the system again reduces to a differential equation: totally differentiating (69) and using (70), we get

$$n^{*\prime} = -\left[\frac{\sigma\left(n^*\right)}{\varepsilon\left(n^*\right)}S_{t_2}(t_1^*, t_2) + \frac{\alpha'(n^*)}{\alpha''(n^*)}S_{t_1}(t_1^*, t_2)t_1^{*\prime}\right]\frac{n^*}{S(t_1^*, t_2)}, \tag{72}$$

where n^* and t_1^* are functions of t_2, generalizing (67) from the case of one-sided heterogeneity. As in Shi (2001), feasibility implies

$$N_2'(t_2) = N_1'\left[t_1^*\left(t_2\right)\right]t_1^{*\prime}(t_2)n^*\left(t_2\right), \tag{73}$$

where N_1' and N_2' are the type densities. The system (72)-(73) with positive sorting has the end-point requirements that the highest agents are matched, and either all the agents are matched or the last unmatched type obtains 0 payoff. Shi (2001) proves the planner's problem yields the same results.

The above sorting conditions can be discussed independent of prices. To see how the transfer z varies by type, notice from (69) that the increase in side 2 payoff is the marginal contribution of another such agent to those on side 1, which is the increase in matching probability times the payoff. Jerez (2014) exploits this to extend existence results from standard sorting theory. Also notice that the constraint in (63) yields $z^*(t_2)$. Thus, $z^*(t_2) = \varepsilon\left[n^*(t_2)\right]S\left[t_1^*(t_2), t_2\right]$, similar to the one-sided case, but depending on both types. Since $z^*(t_2) = S_2\left[t_1^*(t_2), t_2\right] - p^*(t_2)$,

function – e.g., it requires root-supermodularity with urnball, and more with Cobb-Douglas.

this yields the price paid by t_2. It is elegant (or at least comforting) to see that this reduces to the homogeneous-agent baseline with $S_1 = -c$ and $S_2 = u$, in which case $p^*(t_2) = p^* = [1 - \varepsilon(n^*)]\, u + \varepsilon(n^*)c$.

If the type distributions and matching function are symmetric and sorting is positive, types pair up perfectly, and every submarket has $n^*(t_2) = 1$. In this case all agents trade with equal probability despite different prices. While it may seem desirable to match higher side 1 types at a higher rate, they are being paired with higher side 2 types, and we do not want to lower their matching probabilities. In general, the relationship between prices and probabilities can be complex. Starting from symmetric type distributions, where all agents are matched at the same rate, if we spread out types on one side it becomes more important to match them with higher probability, and hence to match their potential partners with lower probability. A interesting research agenda concerns links between outcomes (prices, payoffs, etc.) and dispersion, or other properties of the distributions, on both sides. See Chang (2014) and Mangin (2017) for applications, to asset and labor markets, respectively.

7 Private Information

The next step is to consider private information about types, including workers with better knowledge than their employers about their skills (or vice versa), and sellers with better knowledge than buyers about their wares (or vice versa). Directed search is a natural way to study these markets. We present this in two parts, corresponding to match-specific and individual-specific (i.e., ex post and ex ante) heterogeneity. Also, for now the focus is on bilateral meetings; multilateral meetings are deferred to Section 8.

7.1 Match-Specific Models

There are several competitive search models in this first category: Faig and Jerez (2005) study static goods markets where buyers have private information about their valuation of match-specific goods; Moen and Rosen (2011) study steady state in labor markets where workers have private information about match quality and effort; and Guerrieri (2008) analyzes dynamics in labor markets where workers have private information about the match-specific disutility from work. Among other issues, these studies ask if equilibrium is efficient. In the baseline models analyzed above, competitive search equilibria are typically constrained efficient, but it is less clear what happens under private information. Faig and Jerez (2005) show in some settings that equilibrium is inefficient. Guerrieri (2008) shows it is efficient in steady state but (generically) not in equilibrium when the economy is not in steady state.[44]

Consider match-specific private information in the simple goods market of Section 2.1, where side 1 agents are sellers and side 2 buyers. Sellers are homogenous, and can produce one unit at zero cost; buyers get utility t_2 from the good, randomly drawn and privately observed after matching. There is a continuous distribution of buyer types N_2 with density $f(t_2)$, CDF $F(t_2)$ and support $\mathcal{T}_2$, and total mass normalized to 1. The mass N_1 of sellers is endogenized by entry at cost k. Since all heterogeneity is on the seller side, we drop the subscript for side 2 when there is no risk of confusion.

Each seller decides on whether to enter the market and, if so, what contract to post. Thanks to the revelation principle, without loss of generality, a contract can be restricted to the set of incentive compatible, individually rational, direct revelation mechanisms $\mathcal{S}$. The action a required from the buyer is to report his type. A contract $s : \mathcal{T} \mapsto [0,1] \times \mathbb{R}_+$ specifies a menu of trading probabilities $e(a)$ and transfers $p(a)$ for each matched buyer who reports utility type $a \in \mathcal{T}$. The probabilities $e(a)$ must be specified since in general lotteries may be used – even after matching, trade may occur with probability $e(n) < 1$. Buyers observe posted contracts $s(a) = [e(a), p(a)]_{a\in\mathcal{T}} \in \mathcal{S}$ and decide where to direct their

[44] In related work, Guerrieri (2007) shows that introducing match-specific heterogeneity in the labor market, with or without private information, does not help to amplify volatility relative to a representative-agent benchmark.

search; the set of sellers posting the same s and buyers searching for it constitutes a submarket with tightness $n(s)$. Given heterogeneity is match-specific, the matching probabilities in each submarket are simply $\alpha(n)$ and $\alpha(n)/n$.

In a match, the buyer's utility realization leads to a decision to trade or not. Entry implies that equilibrium seller profits equal k, while U is a buyers' ex ante market utility and $v(s,t,a)$ his ad interim utility if he matches at s, draws t and reports a,

$$v(s,t,a) = e(a)[t - p(a)].$$

A mechanism $s \in \mathcal{S}$ is incentive compatible and individually rational if[45]

$$v(s,t,t) \geq v(s,t,a) \; \forall t, a \in \mathcal{T} \tag{74}$$

$$v(s,t,t) \geq 0 \; \forall t \in \mathcal{T} \tag{75}$$

In this environment the equilibrium can be characterized by a value for buyers' market utility U, a set of posted mechanisms $\mathcal{S}^P = \mathcal{S}$, and a tightness function $n(s)$ defined on $\mathcal{S}$, such that the following conditions are satisfied: $\forall s(t) = [e(t), p(t)]_{t \in \mathcal{T}} \in \mathcal{S}$ we have:

a) seller maximization and entry,

$$\alpha\left[n(s)\right] \int_{t \in \mathcal{T}} e(t)p(t)dF(t) - k \leq 0, \tag{76}$$

with equality if $s \in \mathcal{S}^P$;

b) buyer maximization,

$$\frac{\alpha\left[n(s)\right]}{n(s)} \int_{t \in \mathcal{T}} e(t)[t - p(t)]dF(t) \leq U, \tag{77}$$

and $n(s) \geq 0$ with complementarity slackness, where

$$U = \max_s \frac{\alpha\left[n(s)\right]}{n(s)} \int_{t \in \mathcal{T}} e(t)[t - p(t)]dF(t).$$

[45] To link this to the definition of equilibrium in Section 6.1, observe that incentive compatibility ensures that sellers' beliefs about the action of buyers place all mass on truthtelling. With slight abuse of notation we therefore drop beliefs about actions, and the only relevant beliefs concern type. But since types are drawn ex post, $\phi(t) = F(t)$ independent of the posted s.

These are immediate implications of the maximization condition in the general definition. Note also that feasibility is not stated explicitly, since free entry ensures that sellers enter to satisfy the desired market tightness. Perfection would additionally fix (77) at equality even off equilibrium (whenever possible), but here this does not provide additional bite. And consistency is ensured by $\phi(t) = F(t)$.

Generalizing the results in the baseline model, one can show that the equilibrium can be characterized by maximizing buyer's payoff subject to (74), (75) and the entry condition. Appendix 10 reduces the dimensionality of the constraints following methods in mechanism design (e.g., Myerson 1981). Then one can show there is a unique equilibrium, and only one type of contract $s(t) = [e(t), p(t)]$ is posted, where the nondecreasing trading probability $e(t)$ and associated market tightness solve

$$U = \max_{e(.),n} \frac{\alpha(n)}{n} \int_{t\in\mathcal{T}} e(t)tdF(t)$$
$$\text{st } \alpha(n) \int_{t\in\mathcal{T}} e(t) \left[t - \frac{1-F(t)}{f(t)} \right] dF(t) \geq k.$$

Entry by sellers implies buyers get the entire surplus. Equilibrium maximizes the surplus, given we cover sellers' entry cost, subject to buyers getting the information rents required for them to truthfully reveal their types. In equilibrium only buyers that draw $t \leq \hat{t}$ trade, with $\hat{t}$ endogenous. In this static environment, equilibrium is constrained efficient; in a dynamic extension of the model, when the economy is not in steady state, equilibrium is generically not constrained efficient (Guerrieri 2008).

7.2 Individual-Specific Models

The literature on competitive search with private information about ex ante characteristics focuses on adverse selection, and either signalling or screening, depending on whether the informed or uninformed side posts. Guerrieri et al. (2010), Shao (2014), Chang (2014), Guerrieri and Shimer (2014,2015), Chen et al. (2016), Williams (2016) and Davoodalhosseini (2017) all consider screening problems à la Akerlof (1970). Consider Guerrieri et al. (2010), which involves a static environment with, in a language they adopt, homogenous *principals* on side 1 and heterogeneous *agents* on side 2. Type is private information and has common value for principals and agents. Principals post contracts, and agents decide where to search, given beliefs about submarkets' tightness and composition of types. Guerrieri et al. (2010) show that there exists a fully separating equilibrium, and it is unique.[46]

Consider a simple extension of the basic model in Section 2.1, where side 1 agents are buyers and side 2 sellers. Buyers are ex ante homogenous and can enter at cost k. A mass 1 of heterogenous sellers can provide an indivisible good at 0 cost, where a fraction π_h produce high quality and $\pi_l = 1 - \pi_h$ produce low quality, with respective values $\tilde{v}_1^h$ and $\tilde{v}_1^l < \tilde{v}_1^h$ to buyers. Quality is private information to sellers before trade. If a seller does not trade he gets $\tilde{v}_2^h$ or $\tilde{v}_2^l$, depending on quality, where $\tilde{v}_2^t < \tilde{v}_1^t$, $t \in \{l, h\}$, so there are always gains from trade. Each buyer in the market posts $s \in \mathcal{S} = [0,1] \times R_+$, and sellers choose where to search. A contract $s = (e, p)$ specifies a trading probability e and a transfer p without loss of generality – the same outcome obtains if buyers post menus of contracts, $\left[(e^l, p^l), (e^h, p^h)\right]$.

Any s is associated with a submarket with tightness $n(s)$ and a fraction $\phi(t; s)$ of type t sellers, with $\phi(l; s) + \phi(h; s) = 1$, so the probability a seller matches is $\alpha[n(s)]$, and the probability a buyer matches with a seller of type t is $\alpha[n(s)]\,\phi(t; s)/n(s)$. For $s = (e, p)$ define

$$v_1^t(s) = e(\tilde{v}_1^t - p) \text{ and } v_2^t(s) = ep + (1 - e)\,\tilde{v}_2^t,$$

where $v_2^t(s)$ and $v_1^t(s)$ are the payoffs of type t sellers in submarket s conditional on meeting a buyer, and the payoffs of buyers in submarket s conditional meeting a type t seller.

[46] This constitutes an alternative solution to the non-existence problem in Rothschild and Stiglitz (1976), based on the combination of a capacity constraint and an endogenous rationing rule emerging from the competitive search setting. Papers exploring related competitive equilibrium notions are Gale (1992), Dubey and Geanakoplos (2002), Inderst and Wambach (2002), Inderst (2005).

In this relatively simple setting, equilibrium can be defined by a pair of sellers' market utilities $[U(l), U(h)]$, a market tightness function $n(s)$ and a market composition function $\phi(t; s)$, both defined over $\mathcal{S}$, a CDF $n_1(s)$, and a set of posted contracts $\mathcal{S}^P \in \mathcal{S}$, satisfying the following conditions:[47]

a) buyer maximization and entry,

$$\frac{\alpha\left[n(s)\right]}{n(s)} \sum_{j=\{l,h\}} \phi(j; s) v_1^t\left(s\right) - k \leq 0$$

$\forall s \in \mathcal{S}$, with equality if $s \in \mathcal{S}^P$;

b) seller maximization,

$$\alpha\left[n(s)\right] v_2^t\left(s\right) + \left\{1 - \alpha\left[n(s)\right]\right\} \tilde{v}_2^t \leq U(t)$$

$\forall s \in \mathcal{S}$ and $t \in \{l, h\}$, with equality if $n(s) < \infty$ and $\phi(j; s) > 0$, where

$$U(t) = \max_{s \in \mathcal{S}} \alpha\left[n(s)\right] v_2^t\left(s\right) + \left\{1 - \alpha\left[n(s)\right]\right\} \tilde{v}_2^t;$$

c) and feasibility,

$$\int_{\mathcal{S}^P} \phi(j; s) n\left(s\right) dn_1\left(s\right) \leq \pi_j$$

$\forall j$, with equality if $U(t) > 0$.

Guerrieri et al. (2010) prove that equilibrium is fully separating and characterized by two simple problems: Any seller $t \in \{l, h\}$ chooses a contract s and faces tightness n, where (s, n) solves

$$U(t) = \max_{s \in \mathcal{S}, n} \alpha\left(n\right) v_2^t\left(s\right) + \left[1 - \alpha\left(n\right)\right] \tilde{v}_2^t \tag{78}$$

$$\text{st } \frac{\alpha\left(n\right)}{n} v_1^t\left(s\right) \geq k \text{ and } \alpha\left(n\right) v_2^{t'}\left(s\right) + \left[1 - \alpha\left(n\right)\right] \tilde{v}_2^{t'} \leq U(t') \text{ for } t' < t.$$

[47] The first two conditions are direct consequences of maximization in the general definition of equilibrium in Section 6.1 coupled with perfection. Notice the second one requires maximization on and off equilibrium – if a buyer expects a seller of a given type to search for a given contract, even off equilibrium, it must be weakly optimal for that seller to do so. This delivers uniqueness. In contrast to Section 7.1, we must specify feasibility here because free entry of buyers alone is not sufficient to ensure the right market tightness per type. It also ensures consistency.

As might be expected with adverse selection (going back to Mirelees 1971), individually rational is binding for low quality sellers, incentive compatibility is binding for high quality sellers, and equilibrium features less trade in the submarket with high quality. Equilibrium is not generally efficient, although taxes can be used to correct the inefficiencies along both the extensive and intensive margins (Davoodalhosseini 2017).

Guerrieri and Shimer (2014) build on this general tucture to study dynamic markets with heterogenous assets. Intuitively, by selling in a "less liquid" market, sellers convey that their assets have higher value. They then analyze financial crises resulting from increases in information frictions, and show how this can generate a fire sale and flight to quality. Also, asset prices in these models depend on the dynamics, since investors expect to resell assets in the future and hence value liquidity, and buyers can shift demand to classes of assets less susceptible to informational problems. Guerrieri and Shimer (2015) introduce another dimension of private information, albeit in a static setting. Namely, investors may not know the quality of assets nor the impatience of sellers. This more complex problem generates multiple equilibria with interesting efficiency implications.

Chang (2014) analyzes similar issues in dyθnamic financial markets, where again private information reduces liquidity and can generate fire sales. She has a second type of private information, as in Guerrieri and Shimer (2015), but she collapses the two dimensions of private information into one, which avoids multiple equilibria. In general, we think this is a very promising approach to information issues in labor, asset and other markets, and encourage more research along these lines. Especially for asset markets, future work could profit from connecting these kinds of models more closely to the theories of liquidity summarized in Section 4 above.[48]

[48] Davoodalhosseini (2012) studies directed search when sellers have private information about the quality of their wares, and some buyers are uninformed while others are informed, similar to the random search model of Williamson and Wright (1994). See also Kurlat (2015). Other related work with random search includes Inderst (2005), Chiu and Koeppl (2011) and Camargo and Lester (2012).

These papers focus on screening: the uninformed party posts the contracts. Delacroix and Shi (2014) study a signalling model where price-posting sellers have private information about their quality t, chosen prior to meetings. In this environment, the price a seller posts can signal quality and direct buyers' search. They find a unique equilibrium and it is separating. When the quality differential is large, it is also efficient; otherwise, the quality of goods or the amount of trade can be inefficient, interestingly, due to a conflict between the roles of prices in directing search and signaling quality. Clearly, these are interesting issues, and even with some impressive progress, there is more work to be done on signalling and directed search.

8 Meetings and Mechanisms

Here we consider private information with multilateral meetings, first with a focus on competing auctions, and then on more general mechanisms.

8.1 Private Information and Multilateral Meetings

For this discussion we merge individual- and match-specific private information, and focus mostly on one-sided heterogeneity. Consider a distribution $N_2(t_2)$ with support $[0, \bar{t}_2]$ on side 2, and a mass N_1 of homogeneous agents on side 1, again with side 1 called sellers and side 2 buyers. We also stick with private valuations, $S_2(t) = t > 0$ and $S_1(t) = 0$, assume a buyer's type t is private information, and focus on sellers using mechanisms that do not specify types, like auctions. In this relatively simple environment the literature is mainly concerned with the case where payoffs can be replicated using direct revelation mechanisms: in a meeting, the buyer reports t, and payoffs are delivered as a function of the report.[49]

Here is the issue: with bilateral meetings, it is standard that sellers do not need to post anything more complicated than a price (e.g., Riley and Zeckhauser

[49] It is possible to have buyers with more information than their own type, e.g., which mechanisms are posted, and in principle a seller could try to elicit this, but that requires a broader notion of type; see McAfee (1993), Epstein and Peters (1999) and Peters (1999).

1983); but with multilateral meetings, under mild assumptions a seller should run an auction with reserve price $\underline{p}$ that generally can depend on the type distribution. Now, a priori one might think it is inefficient if sellers extract rents via reserve prices, since it is possible that all the buyers that show up have $t > 0$ but $t < \underline{p}$. Are their gains from trade left on the table? Extracting rents via standard price posting, as in previous sections, does not resolve this, because that would not generally deliver the good to the buyer with highest valuation. In fact, the results depend on the way that buyers and sellers meet.[50]

In the approach used by McAfee (1993), Peters (1997*a*) and Peters and Severinov (1997), each buyer directs his search toward mechanisms they most prefer, with multilateral meetings and coordination frictions as in Section 5. Given n, with probability $P_0(n) = e^{-n}$ a seller gets no buyers, with probability $P_1(n) = ne^{-n}$ he gets one, and with probability $P_l(n) = n^l e^{-n}/l!$ he gets l buyers. These authors show that restricting attention to standard auctions with reserve prices is without loss of generality for an individual seller, independent of the mechanisms posted by other sellers. Hence we proceed by having sellers use auctions with reserve prices, and refer to the mechanism simply by $\underline{p}$, with no further actions for sellers.

For each buyer, let a_2 be his bid. In a first price auction, his payoff conditional on trading is $S_2(s, a_2, t) = t - a_2$, and the seller's payoff is $S_1(s, a_2, t) = a_2$ if $a_2 \geq \underline{p}$ as specified in s, and the payoff from no trade is 0. Now $\phi_2(t, a_2; \underline{p})$ is the probability a buyer who directs his search to $\underline{p}$ has a type weakly below t and bids weakly less than a_2. Let $\phi_2(a_2; \underline{p}, \mathcal{T}_2)$ be the corresponding marginal probability that any type who approaching mechanism $\underline{p}$ bids weakly below a_2. Equivalently, let the probability any buyer bids weakly above a_2 be $\Phi_2(a_2; \underline{p}, \mathcal{T}_2)$. The queue of buyers who pay at least $\underline{p}$ is $n\Phi_2(\underline{p}; \underline{p}, \mathcal{T}_1)$, and since a seller trades as long as he

[50] With random matching $\underline{p} > 0$ would be the seller's revenue-maximizing choice. The analysis reduces to the well-studied problem of a monopolist seller.

meets at least one such buyer,

$$\tilde{\alpha}_1(\underline{p}, t; n, \Phi_2) = 1 - P_0\left[n\Phi_2(\underline{p}; \underline{p}, \mathcal{T}_1)\right] = 1 - e^{-n\Phi_2(\underline{p};\underline{p},\mathcal{T}_1)}. \tag{79}$$

For buyers the probability of trade is 0 if $a_2 < \underline{p}$; otherwise it is $\tilde{\alpha}_2(\underline{p}, a_2, t; n, \phi) = P_0\left[n\Phi_2(\underline{p}; \underline{p}, \mathcal{T}_1)\right] = e^{-n\Phi_2(a_2;\underline{p},\mathcal{T}_1)}$, assuming here that ϕ does not have a mass point at a_2 (see Kim and Kircher 2015 for the general case).

Peters and Severinov (1997) start with finite numbers, then take the limit, as in Section 5. As discussed above, there are two cases: (i) buyers draw value t before deciding where to search; (ii) they draw t after meeting a seller. In equilibrium, in the limit as the market gets large, case (i) implies sellers post a reserve price equal to their outside option, $\underline{p} = 0$. This validates the finding in McAfee (1993), who considers a finite economy but ignores market power, as we discussed in Section 5. Peters and Severinov (1997) conjecture that in case (ii) sellers post $\underline{p} > 0$, but Albrecht et al. (2012) prove $\underline{p} = 0$ in that case as well.[51] They also show this with second price auctions, for both cases (i) and (ii), in large markets, and they show that entry by sellers is efficient. Second price auctions where bids trivially coincide with types can be captured here if the payoff S_i is allowed to depend on n and ϕ. The actions of the other buyers not only affect the meeting probability but also the transfer price conditional on meeting. In this context Mangin and Julien (2016) generalize the Hosios condition to explain efficiency results in Albrecht et al. (2014).

The result $\underline{p} = 0$ is striking, as it implies auctions are in fact efficient: it is never the case that all buyers at a given seller have $t > 0$ but $t < \underline{p}$. Even though buyers are locked in at the time of bidding, the fact that they choose where to search based on the posted $\underline{p}$ dissipates sellers' monopoly power – they still earn rents due to the frictions, but the usual monopoly considerations vanish.

[51] The efficiency of the reserve price is shown by Julien et al. (2000) in large markets with homogeneous agents. With heterogeneous sellers Julien et al. (2005) show that a market where all sellers use efficient reserve price induces buyers to make efficient choices across sellers, even with finite numbers. Taking all strategic considerations into account, this carries over to 2×2 markets (Julien et al. 2002), but not $n_b \times n_s$ markets (Kim and Kircher 2015).

The main driving force behind Peters and Severinov's argument for case (i) can be illustrated as follows: Suppose all sellers offer the same $\underline{p} > 0$. Then any seller trades with probability less than 1 due to the frictions. Now a deviating seller posting $\underline{p} - \varepsilon$ attracts additional buyers, those with type above $\underline{p} - \varepsilon$ for whom the original $\underline{p}$ is too high. In a large market the deviant seller's trading probability jumps, since there are many of such buyers, making the deviation profitable. Hence, the only possibility in a large market is $\underline{p} = 0$. This coincides with the planner's outcome, since the most trade occurs if buyers visit sellers at random, if the goes good to the highest type, without precluding anyone outright from the market. Interestingly, even though in case (ii) there is ex ante information that can be used when selecting where to search, the efficient equilibrium outcome (random search and where the good goes to the highest bidder) does not use that information. Thus, even in the less informative case (ii), the same outcome occurs. Competitive search is efficient, independent of when types are drawn, which is especially nice here because the efficient mechanism is so simple: $\underline{p}$ does not depend on the distribution of t.[52]

In case (i) where types are known ex ante, it is useful to expand on the benefit of random visits, which contrasts with the finding that it is beneficial to separate types into different submarkets with bilateral meetings. With multilateral meetings, it is beneficial to have multiple types in the same submarket. With urn-ball meetings, e.g., a seller can first consider all the buyers, then trade with the one having the highest valuation. Separation would place many high t buyers in the same submarket, leading to congestion. As Eeckhout and Kircher (2010a) emphasize, with two types, high valuation buyers trade even if low valuation buyers are present because they outbid them. If there are $\bar{N}_2$ high types and they trade with a subset $\bar{N}_1 \leq N_1$ of sellers, their matching probability is $\left(1 - e^{-\bar{N}_2/\bar{N}_1}\right) / \left(\bar{N}_2/\bar{N}_1\right)$, which is maximized at $\bar{N}_1 = N_1$, when all sellers trade

[52] While many insights from the homogeneous seller model carry over to heterogeneous sellers, the equivalence between ex ante and ex post type revelation does not. In this case, buyers sort if they know their type ex ante, as discussed below.

with high types. Also, the total number of trades is maximized when buyers are allocated randomly due to the concavity of the matching function. Hence, if buyers search randomly, the number of trades by high types and the total number of trades are both maximized.[53]

Random assignment is only sensible when sellers are identical, however. If they differ in cost t_1 with support $[\underline{t}_1, 0]$ and payoff $S_1(s, a, t) = a_2 + t_1$, trades between types with negative total surplus $S(t_1, t_2) = t_2 + t_1 < 0$ are not beneficial. Yet the main insights discussed above still apply, and in particular, in equilibrium sellers use a reserve price equal to their cost. One can prove a notion of sorting: each buyer type t_2 has a cutoff seller type $t_1^*(t_2)$ and trades randomly with all types $t_1 \geq t_1^*(t_2)$. Higher t_2 buyers are willing to pay more, so $t_1^{*\prime}(t_2) < 0$ (Peters 1997*a*). This is similar to Section 6.2, but now sorting is imperfect because a buyer not only trades with seller $t_1^*(t_2)$, but with all higher types. This corresponds to the planner's outcome where the chance of trading is increased by spreading these buyers across sellers.

The result obtains whether buyers only observe the reserve price, or also the type of seller, since given $\underline{p}$ they do not care about the latter. It also obtains if they can only see s, given a penalty for lying, and not also $\underline{p}$. Clearly sellers accept the highest bid, unless they are all below cost, and buyers do not submit bids with $a_2 + t_1 < 0$, which would be rejected. Effectively this is like a first price auction with $\underline{p}^*(t_1) = t_1$ (Julien et al. 2005). Finally, even if there is no penalty for lying, and messages are cheap talk, there is an still equilibrium where sellers truthfully reveal type.[54] This highlights the efficiency role of cheap talk without commitment, as in housing and labor markets where advertised prices and wages might send

[53] Note that pooling multiple types into one submarket in this way is efficient even though separation satisfies a Hosios condition, because that condition does not allow for pooling. Mangin and Julien (2016) show that pooling types in one market, using a selection process among buyers like a second price auction, satisfies their generalized Hosios condition.

[54] It is important that buyers know the correct type t_1 to know where to attempt to trade and what to bid. In a cheap talk environment sellers post only a message $s \in \mathcal{T}_1$ about their type, but there is no penalty from lying. The question is whether there is an equilibrium in which sellers send truthful messages in the first stage. Indeed this exists and it attains the efficient sorting of the fully competitive model (Kim and Kircher 2015).

messages but are typically not binding (Albrecht et al. 2016). Attaining full using efficiency only cheap talk depends on details, but some room for directing search seems to be a common feature (Menzio 2005).

These results concern private information about buyer types, but the trading patterns are unchanged if buyer types are observable and sellers post menus of prices, one for each buyer type, plus a priority rule determining who gets served when many show up. This is studied by Shi (2002) and Shimer (2005), who find that mixing types in a submarket is a robust outcome. With strict supermodularity, $S_{t_1 t_2} > 0$, Shimer (2005) shows the following: if sellers of type t_1 attract a strictly positive ratio of buyers of type t_2, then sellers of type $t_1' < t_1$ attract a lower ratio of such buyers. In the limiting case $S(t_1, t_2) = t_1 + t_2$, however, the ratios stay constant, as in the private valuation case discussed above. In either case, output is maximized, because auctions ensure the highest valuation buyers get the goods. General conditions for higher type sellers being more likely to match with higher type buyers are not yet established for a general $S(t_1, t_2)$ and multilateral meetings.

The relation between observable types and private information requires private values. Auster and Gottardi (2017) consider adverse selection with urn-ball meetings. Given low- and high-quality traders on side 2, they show all agents from side 1 post the trading mechanism. The good is always given to those on side 2 who report low quality, unless no such agents show up. Even in that case, high-quality agents may not trade with probability 1, for incentive reasons. Most interesting is the question whether one should allow for arbitrary mechanisms or restrict attention to price posting, which then is the same as the bilateral setting studied earlier. They show that adverse selection can make the bilateral setup – i.e., requiring price posting and the random selection of the agent that trades – better than general mechanisms that distinguish between traders based on their announcements, even if the underlying matching is multilateral.

Julien and Roger (2016) combine adverse selection and moral hazard, assuming a stochastic relationship between output and unobserved effort. Homogeneous principals post menus of contracts to attract heterogeneous agents, with bilateral or multilateral meetings. This can be captured in our framework with actions for side 2, with $S_1(t_1, t_2, a) = \mathbb{E}(y|a) - \mathbb{E}p(y)$ and $S_2(t_1, t_2, a) = \mathbb{E}p(y) - c(a; t_2)$. Contracts depend on realized output, and on the number of agents if meetings are multilateral. These contracts also select one of the agents at random when multiple agents show up, but, unlike Moen and Rosen (2011), here agents are risk averse and do not have limited liability. The unique equilibrium contract actually does not depend on the number of agents who show up, and all principals offer the same menu. Equilibrium is constrained efficient on the intensive (effort) and extensive (entry) margins, assuming we can make transfers to agents who show up but are not selected (similar to Jacquet and Tan 2012).

Relatedly, Tsuyuhara (2016) considers a dynamic version with moral hazard and on-the-job search. He shows the existence of block recursive equilibrium, where wages and worker effort increase with tenure, while (voluntary and involuntary) job separation decrease with tenure. Other interesting work includes Lester et al. (2016), where sellers compete on posted asking prices where agents can trade immediately or wait for an auction (as is an option on Ebay). This combines elements of optimal stopping models and competitive search. By expanding the set of mechanisms that sellers can use, they generate multiple equilibria, but they are payoff equivalent and efficient. Kennes and Schiff (2008) assume sellers have private information about the quality of their goods, and multilateral meetings. A intermediary verifying quality sells information to buyers and sells accreditation to sellers. This can be welfare improving or reducing. Again, this is interesting work but there is more to be done in this important area.

8.2 Mechanism Design

There is a large literature on mechanism design and auctions, where competition for buyers is a nontrivial element, but less work on how the meeting process matters. In the private valuation case, with multilateral meetings, we saw competition for buyers results in a standard auction with reserve price equal to cost, $\underline{p} = 0$. This is nice and simple, but raises concerns: why don't sellers use two margins, one to attract buyers and one to guarantee buyers who show up reveal their types? Why does competition stop at $\underline{p} = 0$ price, rather than $\underline{p} < 0$, which attracts even more buyers? Should there be an entry payment to attract buyers and a reserve price to select the best type? When does it suffice to have only an entry payment or simply post a price?

The general issue is to know how mechanism design is affected by the way agents meet. While this has not attracted much attention, we can review some advances in settings with ex ante private values for buyers, $S_2(t) = t_2$, and homogeneous sellers, $S_1(t) = 0$, again dropping the type subscript. Following Eeckhout and Kircher (2010a), Lester et al. (2015) and Cai et al. (2015,2016), consider a general meeting function $P_l(n)$, instead of urn-ball. This is natural e.g., if each buyer approaches one seller but the latter seller only has time to deal with a random subset of L buyers. If $L = 1$ this is a bilateral meeting technology; if $L = \infty$ it is urn-ball; and other intermediate case may be equally plausible.

To see how this affects the optimal mechanism choice, two observations are key: First, posting an optimal entry payment and a standard auction with a reserve price is as least as profitable as any direct revelation mechanism, so we can focus on a second price auction, where buyers pay an entry fee and then bid. Second, equilibrium is constrained efficient, which means the reserve price equals cost, and the surplus added across sellers is maximized. Consider seller j facing tightness n and a distribution of buyers $\phi_2(t; j)$, where we sometimes suppress j. His expected surplus is[55]

$$\sum_{l=1}^{\infty} P_l(n) \int_{t \in \mathcal{T}_2} t d\phi_2^l(t) = \int_{t \in \mathcal{T}_2} \left[1 - \sum_{l=1}^{\infty} P_l(n) \phi_2^l(t) \right] dt.$$

[55]The first expression says that if there are l bidders the one with the highest valuation gets the good, which has distribution $\phi_2^l(t)$, and this is added over all possible number l of bidders. The second interchanges integration and summation.

Letting $\zeta(\hat{n}, n) = 1 - \Sigma_l P_l(n)(1 - \hat{n}/n)^l$, we have

$$V_1 = \int_{t \in \mathcal{T}_2} \zeta(n\left[1 - \phi_2(t)\right], n)dx. \tag{80}$$

In a submarket with tightness n, and $\hat{n}$ high types, $\zeta(\hat{n}, n)$ is the chance a seller meets at least one high buyer type. Simple restrictions on the meeting function imply ζ is concave in each variable, if not jointly. Knowing $P_l(n)\ \forall(n, l)$ implies knowing ζ, and vice versa. Given this, consider two buyers, a lower type $\underline{t}$ and a higher type $\bar{t}$. Suppose there is a measure 1/2 of sellers in a submarket with $\hat{n}_1$ high types and n_1 buyers in total, and another measure 1/2 of sellers in a submarket with $\hat{n}_2$ high type and n_2. By (80), the surplus per seller is $\underline{t}\zeta(n_j, n_j) + \left(\bar{t} - \underline{t}\right)\zeta(\hat{n}_j, n_j)$, where the first term says that as long as the seller meets someone he creates at least value $\underline{t}$, while the second says that if he meets a high type he creates additional $\bar{t} - \underline{t}$. The expected surplus in this market is

$$1/2 \sum_{j=1}^{2} \left[\underline{t}\zeta(n_j, n_j) + \left(\bar{t} - \underline{t}\right)\zeta(\hat{n}_j, n_j)\right].$$

If instead we sent buyers randomly to sellers the expected surplus would be

$$\underline{t}\zeta(\frac{n_1 + n_2}{2}, \frac{n_1 + n_2}{2}) + \left(\bar{t} - \underline{t}\right)\zeta(\frac{\hat{n}_1 + \hat{n}_2}{2}, \frac{n_1 + n_2}{2}).$$

If $\zeta(\hat{n}, n)$ is concave then pooling everyone in one submarket is better. This is a good insight: concavity of ζ implies buyers should be assigned randomly. Moreover, the result holds for arbitrary distributions of types. If ζ is not globally concave, there are always type distributions for which segregation rather than random matching is desirable under mild conditions.

This can be illustrated in the example where sellers can deal with up to L buyers, where the resulting ζ is never globally concave except when $L = \infty$. Here the lack of concavity is more severe with small L. Markets in which sellers are more constrained in their meetings are naturally more inclined to separation, in which case they can opt for simple price posting. Eeckhout and Kircher (2010a) highlight a natural role for simple price posting: if sellers are more constrained in their meetings, they want to segregate buyer types into different submarkets, and since they can then infer buyer type they have no need for complicated mechanisms at the time of meeting (see also Auster and Gottardi 2017). However, to ensure price posting is optimal for any distribution of buyer types, we need bilateral meetings (Cai et al. 2015).

Another question concerns the importance of entry fees. Cai et al. (2016) show it is optimal for a seller that attracts n to post auctions with no reserve price but an entry payment τ. One can show $\tau = 0$ if $\zeta_2 = 0$ everywhere, a feature Lester et al. (2015) call *invariance*. When this applies, it is optimal to pool types, intuitively, because it means meetings between high type buyers and sellers are not reduced by having more low type buyers in the submarket. Pooling is then optimal because competition between high and low types is dealt with by the mechanism rather than the meeting process – meeting externalities are absent. In sum, the meeting process has substantial implications for mechanism choice, and we need more work on these issues.

9 Other Topics

We now touch on a few important topics not covered above.

9.1 The Nash Program

The quest for strategic foundations for axiomatic bargaining is dubbed "the Nash program" by Binmore (1987). As Serrano (2005) nicely puts it, "Similar to the microfoundations of macroeconomics, which aim to bring closer the two branches of economic theory, the Nash program is an attempt to bridge the gap between the two counterparts of game theory (cooperative and non-cooperative). This is accomplished by investigating non-cooperative procedures that yield cooperative solutions as their equilibrium outcomes." See also Osborne and Rubinstein (1990). Directed search and posting can be viewed from this perspective: it provides an explicit description of a market, with finite numbers of participants, or in limiting cases with large numbers, where traders end up sharing the gains from trade in a way that can be interpreted in terms of generalized Nash, with bargaining powers and threat points determined by market conditions. In an $n_b \times n_s$ market, (45) gives p as a weighted average of c and u, where the weight on u is the probability a seller gets at least 2 buyers; in particular, in a 2×2 market, $p = (u + c)/2$, consistent with the original Nash solution.

This provides an alternative to demonstrating that generalized Nash is the limit of a non-cooperative game, with bargaining power and threat points determined by details like the probability a player gets to make the next offer or how much he loses from delay (Binmore et al. 1986). Competitive search is therefore a contribution to the Nash program. Note that with bargaining the share parameter θ is typically assumed to be structural. Competitive search demonstrates that the division of the surplus is not generally constant, but changes with economic conditions, including changes in policy. Ignoring that can be problematic, although it can also be justified in some cases – e.g., with a Cobb-Douglas matching technology the shares of the surplus are indeed invariant. Still, one ought to acknowledge that this is a special case.

Similarly, in competitive search equilibrium the shares of the surplus will vary in the cross section, not only with respect to time or policy, with a different split in different submarkets into which traders select. This is interesting and important. Typically, in markets with two-sided ex ante investments, there is no single value of θ that delivers efficiency independent of other parameters – i.e., there is not way to satisfy more than one Hosios condition with a single choice of θ. Competitive search delivers efficiency endogenously in many of these situations. Hence it not only provides a microfoundation for sharing the surplus, as assumed in bargaining models, it dominates bargaining as a mechanism. Of course, competitive search (usually) assumes commitment, but at least it is good to know what we are buying into.

9.2 Large Firms

Most of the literature on labor search, directed or otherwise, concentrates on jobs rather than firms: they treat each job as an entity unto itself without specifying how collections of jobs aggregate into firms. This is fine for many purposes but not, e.g., to discuss observations on firm size or growth. Davis et al. (2013) show that matching efficiency varies substantially, and linearly, with a firm's growth rate. To speak to such facts, a promising avenue is to postulate that if a firm has L workers it produces $f(L)$, and can post v vacancies at cost $k(v)$, generalizing the baseline formulation where $k = k(1)$ is the cost of posting a single vacancy. At the level of an individual vacancy the model is unchanged, but aggregation differs, and if f is strictly concave or k strictly convex then firms grow to a finite endogenous size. Hawkins (2013) presents such a model with finite numbers, which complicates the analysis. Most subsequent work assumes the law of large number applies, so that a firm posting any number of vacancies gets a deterministic number of new workers.

In Kaas and Kircher (2015), a strictly concave $k(v)$ leads to a slow growth path for firms, and those that want to grow faster post both higher wages and more vacancies. The former induces more hires per vacancy, consistent with the above-mentioned facts in Davis et al. (2013). Also, they decentralize the appropriate planner's problem, in contrast to models with multi-worker bargaining (Stole and Zwiebel 1996; Smith 1999; Brugemann et al. 2015). Menzio and Moen (2010) show how incomplete contracts that specify the wage but do not guarantee employment imply wage floors that insure workers with respect to modest shocks but not large shocks. As in Kaas and Kircher (2015), they use an extended version of block recursivity to achieve tractability. Garibaldi and Moen (2010), Schaal (2015) and Garibaldi et al. (2016) incorporate on-the-job search, but only if f or k are linear; the general case is still outstanding.

In other work along these lines, Eeckhout and Kircher (2016) introduce two-sided heterogeneity into a large-firm model with linear k but convex f to study the sorting and wage implications. Julien et al. (2016) study large firms as production teams in a competitive search setting, related to the "island matching" model in Mortensen (2009). In each period, a firm can post one or more vacancies and may loose one or more workers, and may temporarily shut down if a minimum number of workers is necessary to operate profitably. The optimal size of a firm (team) depends on the extent of frictions, and the complementarity between workers within a team leads to wage dispersion, suggesting that the proliferation of human resource management emphasizing team production can contribute to increasing wage inequality. While there is not that much work with large firms in this literature, what there is suggests it is an interesting area for future research.

9.3 Evidence

There are several ways in which directed and random search can differ empirically. Obviously, both are stark, and neither captures every detail of the underlying market, but there are differences that can serve as hypotheses for testing. One commonly noted difference is that the number of applications firms receive should not vary with their wages under random search, and should increase with their wages under directed search, holding other things equal (it is important to consider applications rather than hires, since higher wage firms might be more successful in hiring even under random search). The main challenge in testing this is to find a situations where two otherwise similar vacancies offer different wages, since if the vacancies are not otherwise similar, they may require different qualifications, hours, etc.

We first mention that many workers do not seem in a position to bargain over wages, but take offers as given. This is the case reported by more than two thirds of workers both in the US and Germany, according to Hall and Kruger (2012) and Brenzel et al. (2014). Hall and Kruger also highlight that nearly half of the workers in their sample report that even before their applications they were highly certain about wages. This is particularly pronounced for lower-qualification jobs, and bodes well for directed search. Obviously, whether or not a firm posts a wage is an observable component and will differentially attract applicants. Michelacci and Suarez (2006) study a setting where firms can post either wages or commit to bargain, and predict that posting is used for jobs where qualifications matter less while bargaining is used when qualifications are matter more, as then firms need wage flexibility to deal with quality differentials in the applicant pool.

Regarding the relation between wages and the number of applications, non-experimental evidence is mixed and depends importantly on the ability to control for variations in wages that are unrelated to job requirements. Using survey evidence on a limited number of US firms in the 1980s, Holzner et al. (1991) and Faberman and Menzio (2015) do not find support for the number of applications increasing with the wage, controlling for occupational code and other observables. Faberman and Menzio even find higher wages are associated with fewer applications, but the effect shrinks with the inclusion of more observables. This raises the possibility that different jobs are targeted to different groups of applicants and higher wages mainly compensate for higher skills.[56]

This raises the question whether one can obtain better controls to make job requirements more comparable. Marinescu and Wolthoff (2015) use a much larger data set of online job postings at the US website *careerbuilder.com* and control for job titles, which are orders of magnitude finer than the two-digit occupational codes used in the above-mentioned work. They find that applications increases with wages. They also highlight that other features of the job description, especially qualifiers such as "senior" or "experienced" seem to convey substantial information about the attractiveness of the job and explain a large part of wage variation. This is related to work on cheap talk, where messages like "senior" or "experienced" can signal the eventual wage that can be obtained (Menzio 2005; Kim and Kircher 2015). In any case, search among similar jobs seems to be positively geared to higher posted wages, and to other identifiers that are correlated

[56] In a directed search model with homogeneous firms and workers differing in productivity, higher wages can attract fewer but better applicants.

with the attractiveness of the job.

Banfi and Villena-Roldan (2016) have information on posted wages on a Chilean job-search website, plus intended wages, even if these are not shown to job seekers. They show that higher intended wages substantially increase the number of applicants after controlling for job title. This highlights the idea that employers are able to communicate effectively throughout job descriptions, even if they do not include it outright. Effects are larger for publicly posted wages, though, suggesting a significant direct channel.

To make sure wages are not capturing other elements of the job, e.g., skill requirements, one approach is to randomize wages. This ensures orthogonality at least to the observed dimensions of the job advertisement. Dal Bo et al. (2013) exploit truly randomized wage offers for civil servant positions in Mexico, and find higher wages attract more and better applicants. Belot et al. (2016) use randomized wages in a small online job market and also find higher wages attract substantially more interest from job seekers. These observations can be taken as further support for directed search. However, while this evidence is hard to reconcile with the simplest random search models, minor extensions of that model could help.[57]

The reservation wage property is pervasive in random search, but not directed search, where workers typically cannot apply to all the jobs they like. Because they cannot, they are selective, and may apply to low wages even if they are aware of higher wage options, because they understand that it is harder to get latter. A test between theories can therefore directly focus on the reservation wage property. Belot et al. (2016) conduct a field experiment where otherwise identical jobs offer different wages. They find more workers apply to high wage

[57] Suppose workers encounter vacancies at random and observe the wage, as in random search with posting (e.g., Burdett and Mortensen 1998), with a slight twist: workers only bother to apply for jobs if they would actually accept them. This has no effect on actual hires, but now applications are only recorded by the firm if the job exceeded their reservation wage, which can explain the findings mentioned in the text because firms with higher wage offers are more likely to meet workers' reservation wages.

jobs, but a significant share apply only to the low wage job, even though they see the high wages on their search engine. This lends support to the idea that workers are selective in where they apply.

Directed search has also been studied in laboratory experiments. Cason and Noussair (2007) ask if the strategic considerations in the finite market models discussed in Section 5 are important. The answer is yes – their experiments with small numbers of traders are broadly consistent with on Burdett et al. (2001), although there are some discrepancies, e.g., prices can exceed the equilibrium prediction when there are two sellers. In Anbarci and Feltovich (2013), in one treatment sellers post as in Burdett et al., and in another they post as in Coles and Eeckhout (2003), with prices contingent on the number of buyers that show up. They consider 2×2, 2×3 and 3×2 market versions, and find that allowing contingent pricing does not affect seller payoffs. See also Anbarci and Feltovich (2017). Moen et al. (2015) vary both the capacity constraints and information, and find that observed play corresponds closely to equilibrium. Kloosterman (2016) finds that firms offer wages close to, if a little lower than, the theory's predictions, but are highly variable. Also, workers apply more often to high wages although not quite to the extent predicted.

Godoy and Moen (2011) study another difference between directed and random search in the context of on-the-job search. In random search, a worker who is currently employed at w_1 will take any $w > w_1$ that he encounters. A similar worker employed at w_2 will do the same with and $w > w_2$. If we see both move to a new job with wages above w_1 and w_2, the wage distribution upon moving for both should be identical. As discussed above, that is not the case with directed on-the-job search, since workers who are employed at a higher w tend to target new jobs at still higher wages. Their evidence supports this prediction. In a similar vein, Braun et al. (2016) study whether unemployment insurance affects subsequent wages beyond the reservation wage, and also find evidence of directed search.

Li et al. (2015*a*) show how a directed search model can be used to understand worker transitions. The theory is a dynamic extension of Peters (2010) in which workers have privately known types that are observable to firms once they apply. It shows how the wage offer distribution can be derived from the accepted wage distribution and the employment distribution by solving a differential equation. This relationship is used to derive an outcome distribution that can be used to study transitions and test the theory. Li et al. (2015*b*) use a similar approach to show that for any smooth wage distribution there is an equilibrium where unemployed workers choose reservation wages as a strictly increasing function of their type, then apply with equal probability to all positions that offer more than that wage. They show two results. First, workers' wages throughout their lives are correlated, but very imperfectly because equilibrium involves a lot of mismatch. Second, the variance of future income is a decreasing function of the current wage, i.e., high wage workers have more stable lifetime income.

Herkenhoff (2013) develops a quantitative model with directed search and risk aversion to study how raising credit access among the unemployed affects business cycles. He has directed search in labor and credit markets, and models easier access to credit as an increase in matching efficiency. As credit expanded over the last 40 years, better access to credit this slows business cycle recoveries, but is still welfare improving. Since the model is block recursive, he can estimate it and solve for transitions. Relatedly, Herkenhoff et al. (2016) develop a sorting model of risk aversion and credit, which is again tractable due to directed search. In particular, workers with low assets search for easy-to-find jobs, but these may be poor matches. Estimating the model, they find that credit constraints tightening as they did in 2007-2009 generates enough mismatch to depress productivity by 0.25%, which persists over time, and is equivalent to a 425 million dollar reduction in output per annum.[58]

[58] As regards related work, we already mentioned Menzio et al. (2016), who combine life cycle considerations with directed search in the labor market. They study how the trade-offs between wages and job-finding rates, and hence transitions, depend on age. See also Carillo-Tudela and Visschers (2014).

All of this suggests directed search can help rationalize several features of the data. Whether it can fully account for the observations, or whether perhaps a hybrid solution with both random and directed search is more suitable, awaits empirical scrutiny. The estimation in Engelhard and Rupert (2016) may suggest the need for a hybrid approach, since they largely reject the implications of a simple competitive search model, although they fail to reject directed search with heterogeneous workers. terms of submarket search and efficient surplus sharing. Alternatively, as they acknowledge, this might simply reflect the need to incorporate elements like on-the-job search and differences in productivity, which are crucial in rationalizing wage dispersion in other models (e.g., Hornstein et al. 2011). There is ample unexplored territory for empirical work on directed search, in general, and on comparing it to random search, in particular.

9.4 Miscellany

Lester et al. (2014) study over-the-counter financial markets in a generalized version of Duffie et al. (2005) using directed search. Watanabe (2010,2015) considers directed search in a model of intermediaries (middlemen). His middlemen have large inventories, so they are less likely to stock out, similar to the situation when some sellers has multiple units for sale in Burdett et al. (2001). In particular, if two firms could merge, then if one gets 2 customers and the other 0, and if they can share them, it would a profitable venture. Another related paper is Gautier et al. (2016), who use directed search to analyze two types of middleman, those who hold inventories they get from sellers to retrade to buyers, and those who offer platforms for buyers and sellers to trade with each other. This is very interesting, but it seems there is much more to do on directed search and middlemen.

Gonzalez and Shi (2009) study differences in worker arrival rates that are not known – they have to learn over time. This pioneers new ways to analyze dynamic markets, and rationalizing a discouraged worker effect. Chade et al. (2014) and Nagypal (2004) develop equilibrium models of directed college choice where applicants can simultaneously send out many applications. All these interesting applications can serve as guides for future research.

Some papers study markets with two-sided investment, including Acemoglu and Shimer (1999*a*), Masters (2011) and Jerez (2016). In the latter, competitive search entails both commitment to and advertising of payoff relevant characteristics. An example from labor is when workers and firms invest in human and physical capital prior to entering the market. When the characteristics of jobs and workers are posted, efficiency obtains. However, commitment without advertising leads to unravelling and inefficiency.

Some papers study in detail the impact of unemployment insurance using direted search, including Acemoglu and Shimer (1999*b*), Geromichalos (2015) and Golosov et al. (2013). In Geromichalos (2015) workers that do not match receive b from the government, financed by a lump sum tax on firms. This makes it cheap for firms to be aggressive in posting, because they know their competitors all contribute to the unemployment caused by attracting more workers than they want to hire In equilibrium wages are too high and entry is too low. This can be corrected by experience rating (making taxes depend the unemployment a firm causes). It is also corrected without taxes if firms post contracts as in Jacquet and Tan (2012), with one payment to those hired and another to those not hired. Golosov et al. (2013) study optimal unemployment insurance (Acemoglu and Shimer 1999*b* only maximize output). They find it is optimal to insure workers against the risk of not getting hired, but not to redistribute across workers applying to different types of jobs. Clearly directed search raises new issues in this important application, and hence merits even more research.

10 Conclusion

This completes our directed tour through the theories and applications of directed search. Like search theory in general, the models contribute substantially to our understanding of phenomena like the coexistence of unemployment and vacancies, the fact that some shops have unsold inventories while others stock out, price dispersion and price stickiness, and random durations in the time to execute trades in many markets. Different from traditional search, in competitive search equilibrium the terms of trade are posted and agents use this to target counterparties. In particular, posted prices have an allocative role, and this typically leads to efficiency, at least without additional complications like private information or liquidity considerations.

To quote from the survey on labor markets by Rogerson et al. (2005): "Although there are several important modeling decisions in equilibrium search theory... two questions are paramount. First, how do agents meet? In particular, is search random, so that unemployed workers are equally likely to locate any job opening, or directed, so that for example firms can attract more applicants by offering higher wages? Second, exactly how are wages determined? Do matched workers and firms bargain, or are wages posted unilaterally before they meet?" As that survey says, competitive search offers particular answers to both questions that can avoid the "black box" of bargaining and (sometimes) of matching functions. And, obviously, this applies not just to labor, but too many if not most markets.

Competitive search can be more tractable and can yield cleaner results than alternatives. It is also arguably more realistic, depending on the application. It also lets us study both finite markets and limiting large markets, where we reiterate that the frictions do not go away in the latter case. Again, the theory captures the idea that if you post more favorable terms then potential customers come to you with a higher probability, but not generally with probability 1. Agents on both sides of the market trade off prices and trading probabilities, which nicely generalizes general equilibrium theory. In particular, with homogeneous buyers and sellers the price and probability of trade are the same everywhere, but the out-of-equilibrium options to post different terms and to search differently disciplines in-equilibrium behavior. For these reasons and more, it seems easy to predict a bright future for the directed search approach. We hope this essay inspires readers to learn more about the field, and to contribute to its ongoing development.

Appendix A

We consider two scenarios. The first has indivisible q, like our baseline model, but does not necessarily have perfectly transferable utility: if a buyer makes a payment p to a seller, the latter gets $\nu(p)$ while former gets $-\gamma(p)$, where $\nu(0) = \gamma(0) = 0$, $\nu'(q) > 0$, $\gamma'(q) > 0$, $\nu''(q) \leq 0$ and $\gamma''(q) \geq 0$ $\forall q > 0$. Transferable utility, as in Section 2.1 is the special case, $\nu(p) = \gamma(p) = p$. The generalization of problem (1) is

$$V_s = \max_{p,n} \alpha(n)\left[\nu(p) - c\right] \text{ st } \frac{\alpha(n)}{n}\left[u - \gamma(p)\right] = V_b.$$

Form the Lagrangian

$$\mathcal{L} = \alpha(n)\left[\nu(p) - c\right] + \Lambda\left\{\frac{\alpha(n)}{n}\left[u - \gamma(p)\right] - V_b\right\}.$$

The FOC's are:

$$\begin{aligned}
\mathcal{L}_n &= \alpha'(\nu - c) + \frac{\Lambda(u-\gamma)(n\alpha' - \alpha)}{n^2} = 0 \\
\mathcal{L}_p &= \alpha\nu' - \frac{\Lambda\alpha}{n}\gamma' = 0 \\
\mathcal{L}_\Lambda &= \frac{\alpha(n)}{n}(u-\gamma) - V_b = 0
\end{aligned}$$

Notice $\mathcal{L}_p = 0$ implies $\Lambda = n\nu'/\gamma'$. Then $\mathcal{L}_n = 0$ implies $\varepsilon(\nu - c)\gamma' = (1-\varepsilon)(u-\gamma)\nu'$, like generalized Nash bargaining except $\varepsilon = n\alpha'(n)/\alpha(n)$ replaces θ. At any solution to the FOC's, the bordered Hessian is

$$H = \begin{bmatrix}
\frac{\alpha''(u-\gamma)\nu'}{\varepsilon\gamma'} + \frac{2\alpha(1-\varepsilon)(u-\gamma)\nu'}{n^2\gamma'} & \frac{\alpha\nu'}{n} & -\frac{\alpha(1-\varepsilon)(u-\gamma)}{n^2} \\
\frac{\alpha\nu'}{n} & \frac{\alpha(\gamma'\nu''-\nu'\gamma'')}{\gamma'} & -\frac{\alpha\gamma'}{n} \\
-\frac{\alpha(1-\varepsilon)(u-\gamma)}{n^2} & -\frac{\alpha\gamma'}{n} & 0
\end{bmatrix}$$

and its determinant, after simplification, is

$$|H| = -\left(\frac{\alpha}{n}\right)^2(u-\gamma)\left[\frac{\alpha''\nu'\gamma'}{\varepsilon} + \frac{\alpha(1-\varepsilon)^2(u-\gamma)(\gamma'\nu''-\nu'\gamma'')}{n^2\gamma'}\right].$$

Standard assumptions on α, ν and γ imply $|H| > 0$, so the SOC's hold. As a special case this applies to $\nu(p) = \gamma(p) = p$. Also, the same method applies to the dual problem (4).

Now consider divisible goods with p fixed, as in Section 3.1's credit model with a binding constraint, where $p = L$, or Section 4.1's monetary model with indivisible assets, where $p = \beta\Delta$. Form the Lagrangian

$$\mathcal{L} = \alpha(n)[p - c(q)] + \Lambda \left\{ \frac{\alpha(n)}{n}[u(q) - p] - V_b \right\}.$$

With p is fixed, the FOC's are:

$$\begin{aligned}
\mathcal{L}_n &= \alpha'(p-c) + \frac{\Lambda(u-p)(n\alpha' - \alpha)}{n^2} = 0 \\
\mathcal{L}_q &= -\alpha c' + \frac{\Lambda\alpha}{n} u' = 0 \\
\mathcal{L}_\Lambda &= \frac{\alpha(n)}{n}(u-p) - V_b = 0
\end{aligned}$$

Now $\mathcal{L}_q = 0$ implies $\Lambda = nc'/u'$. Then $\mathcal{L}_n = 0$ implies $\varepsilon(p-c)u' = (1-\varepsilon)(u-p)c'$, or $p = v(q,n)$ with $v(q,n)$ given by (16), again like generalized Nash except ε replaces θ. At any solution to the FOC's, the bordered Hessian is

$$H = \begin{bmatrix} \frac{\alpha''(u-p)}{\varepsilon}\frac{c'}{u'} + \frac{2\alpha(1-\varepsilon)(u-p)}{n^2}\frac{c'}{u'} & -\frac{\alpha c'}{n} & -\frac{\alpha(1-\varepsilon)(u-p)}{n^2} \\ -\frac{\alpha c'}{n} & \frac{\alpha(c'u''-u'c'')}{u'} & \frac{\alpha u'}{n} \\ -\frac{\alpha(1-\varepsilon)(u-p)}{n^2} & \frac{\alpha u'}{n} & 0 \end{bmatrix}$$

and its determinant is

$$|H| = -\left(\frac{\alpha}{n}\right)^2 (u-p)\left[\frac{\alpha'' c' u'}{\varepsilon} + \frac{\alpha(1-\varepsilon)^2(u-p)(c'u'' - u'c'')}{n^2 u'}\right].$$

Standard assumptions imply $|H| > 0$, so again the SOC's hold. The same method applies to the dual problems. ■

Appendix B

Recall the continuous-time planner problem in Section 3.2. Normalizing the measure of households to $N_s = 1$, we let the state variable be employment e, and let the control be the measure of vacancies posted v. The law of motion is $\dot{e} = (1-e)\alpha(n) - \delta e$, where $n = v/(1-e)$. Denote the value function by $J(e)$ and write the problem as

$$rJ(e) = \max_v \left\{ ey + (1-e)b - vk + J'(e)\left[(1-e)\alpha\left(\frac{v}{1-e}\right) - \delta e\right]\right\}. \quad (81)$$

In case it is not obvious, we derive this, following Shimer (2004). Consider the integral form of the problem

$$J\left[e\left(t\right)\right] = \int_t^\infty e^{-r(s-t)} \left\{e\left(s\right)y - \left[1-e\left(s\right)\right]b - vk\right\} ds.$$

The objective function is household utility net of vacancy posting costs. Differentiating with respect to time, we get

$$J'\left[e\left(t\right)\right]\dot{e}\left(t\right) = -e\left(t\right)y - \left[1-e\left(t\right)\right]b + rJ\left[e\left(t\right)\right].$$

Using this to replace $\dot{e}\left(t\right)$ in the objective function, we arrive at (81).

One can show the value function is linear, $J\left(e\right) = A_0 + A_1 e$. This is easiest in discrete time, as in Rogerson et al. (2005), where it is easy to check the mapping analogous to (81) is a contraction, with $J\left(e\right)$ its unique fixed point. It is also easy to check that this mapping takes linear functions into linear functions. Since the set of linear functions is closed, the fixed point $J\left(e\right)$ is linear. To get the result in continuous time one can taking the limit of the discrete time model as the period length shrinks to 0, a standard technique in search theory (e.g., Mortensen 1986). See Wright (2001) for more details.

Therefore the FOC is

$$k = A_1 \alpha'\left(\frac{v}{1-e}\right), \tag{82}$$

which implies $n = v/\left(1-e\right)$ is independent of e. Differentiating (81), we get

$$rA_1 = y - b - A_1\left[\alpha\left(\frac{v}{1-e}\right) - \alpha'\left(\frac{v}{1-e}\right)\frac{v}{1-e} + \delta\right]. \tag{83}$$

Using (83) to eliminate A_1 from (82), we arrive at $T\left(n\right) = 0$, the steady state equilibrium condition in the text. Hence, at every point in time, the planner's n is the same as the steady state equilibrium n. ■

Appendix C

Recall on-the-job search from Section 3.2. We claim there is a $\underline{w}$ such that workers employed at $w \geq \underline{w}$ stop searching. To begin, for a worker employed at w_1 searching for w_1' and another employed at $w_2 > w_1$ searching for w_2', the fact that both are behaving optimally implies

$$\begin{aligned}
\alpha\left[n(w_2')\right]\left[V_{s1}(w_2') - V_{s1}(w_2)\right] &\geq \alpha\left[n(w_1')\right]\left[V_{s1}(w_1') - V_{s1}(w_2)\right] \\
\alpha\left[n(w_2')\right]\left[V_{s1}(w_2') - V_{s1}(w_1)\right] &\leq \alpha\left[n(w_1')\right]\left[V_{s1}(w_1') - V_{s1}(w_1)\right].
\end{aligned}$$

Now subtraction implies

$$\alpha\left[n(w_2')\right]\left[V_{s1}(w_1)-V_{s1}(w_2)\right]\geq\alpha\left[n(w_1')\right]\left[V_{s1}(w_1)-V_{s1}(w_2)\right].$$

Since $V_{s1}(w)$ is strictly increasing, $V_{s1}(w_1) < V_{s1}(w_2)$ and hence $\alpha\left[n(w_2')\right] < \alpha\left[n(w_1')\right]$. So the worked employed at w_2 searches in a submarket with a higher wage and lower success rate than the worker employed at w_1.

We now show there is a minimum wage increment $\triangle$ workers require to justify search cost κ. The gain $V_{s1}(w_0)-V_{s1}(w)$ is bounded by $(w'-w)/(\rho+\delta)$, the difference in wages over the maximum time on a job until exogenously destroyed. A employed worker that searches needs a gain that at least makes up for the cost, $V_{s1}(w_0)-V_{s1}(w)\geq\kappa$. Hence, $\kappa\leq(w'-w)/(\rho+\delta)$, or $\Delta\geq\kappa(w'-w)$. In particular, if w is high enough, a worker stops searching. ■

Appendix D

Consider auctions instead of posting in the $n_b\times n_s$ market. For seller i the payoff is $u-c$ unless only 1 buyer shows up, in which case it is p_i-c, or no buyers show up, in which case it is 0. Therefore,

$$V_{si}=n_b\gamma_i\left(1-\gamma_i\right)^{n_b-1}(p_i-c)+\left[1-n_b\gamma_i\left(1-\gamma_i\right)^{n_b-1}+\left(1-\gamma_i\right)^{n_b}\right](u-c). \quad (84)$$

For a buyer the payoff from visiting seller i is $u-p_i$ if he is alone, and 0 otherwise. In equilibrium where buyers mix, therefore,

$$\left(1-\gamma_i\right)^{n_b-1}(u-p_i)=\left(1-\gamma_j\right)^{n_b-1}(u-p_j). \quad (85)$$

Suppose seller i deviates to p_i. Given buyers mix symmetrically, $\gamma_1+(n_s-1)\gamma_j=1$, and

$$\left(1-\gamma_i\right)^{n_b-1}(u-p_i)=\left(1-\frac{1-\gamma_i}{n_s-1}\right)^{n_b-1}(u-p_j). \quad (86)$$

Implicit differentiation and simplification implies

$$\frac{\partial\gamma_i}{\partial p_i}=-\frac{(1-\gamma)(n_s-1)}{(n_b-1)(u-p)n_s}$$

around the equilibrium values of $p_i=p$ and $\gamma=1/n_s$. Taking the FOC from maximizing V_{si} with respect to p_i and simplifying, we get

$$p=\frac{(n-\frac{1}{n_s})u+(n_s-2+\frac{1}{n_s})c}{n+n_s-2}.$$

As $n_b, n_b \to \infty$ holding n fixed, $p \to c$ and $\pi \to (1 - e^{-n} - ne^{-n})(u - c)$. This is the same as the payoff under posting. ■

Appendix E

Consider any CRS technology $\mu(n_1, n_2)$ and let $n = n_1/n_2$. We claim $\sigma(n) \gtrless 1 \Leftrightarrow \varepsilon'(n) \gtrless 0$, where $\varepsilon(n) = n\alpha'(n)/\alpha(n)$, and $\sigma(n)$ is the elasticity of substitution,

$$\sigma(n) = \frac{dn}{d(\mu_2/\mu_1)} \frac{\mu_2/\mu_1}{n}. \tag{87}$$

As in standard production theory, σ measures the degree of complementarity between inputs. Clearly, $\mu(n_1, n_2) = n_2\alpha(n) = n_1\alpha(n)/n$, which implies $\mu_1 = \alpha'(n)$ and $\mu_2 = \alpha(n) - \alpha'(n)n$. Therefore, $\mu_2/\mu_1 = \alpha(n)/\alpha'(n) - n$ and $d(\mu_2/\mu_1)/dn = -\alpha(n)\alpha''(n)/\alpha'(n)^2$. Given this, (87) implies

$$\sigma(n) = -\frac{\alpha'(n)^2}{\alpha(n)\alpha''(n)} \frac{\alpha(n) - \alpha'(n)n}{n\alpha'(n)} = -\frac{\alpha'(n)[1 - \varepsilon(n)]}{\alpha''(n)n}.$$

Now algebra implies

$$\begin{aligned}
\varepsilon'(n) &= \frac{[\alpha''(n)n + \alpha'(n)]\alpha(n) - n\alpha'(n)\alpha'(n)}{\alpha(n)^2} \\
&= \frac{\alpha'(n)}{\alpha(n)}[1 - \varepsilon(n)][1 - 1/\sigma(n)].
\end{aligned}$$

This proves $\varepsilon'(n) \gtrless 0 \Leftrightarrow \sigma(n) \gtrless 1$.

For the specification $\mu(n_1, n_2) = n_2(1 - e^{-n_1/n_2})$ discussed in Section 5, $\alpha(n) = 1 - e^{-n}$ and $\varepsilon(n) = ne^{-n}/(1 - e^{-n})$. Moreover,

$$\begin{aligned}
\sigma(n) &= \frac{1 - e^{-n} - ne^{-n}}{n(1 - e^{-n})} < 1 \\
\varepsilon'(n) &= \frac{e^{-n}(1 - e^{-n} - n)}{(1 - e^{-n})^2} < 0,
\end{aligned}$$

because $1 - e^{-n} < n$. As discussed in Section 4, another common specification is the Kiyotaki-Wright matching function, $\mu(n_1, n_2) = n_1 n_2/(n_1 + n_2)$. This implies $\alpha(n) = n/(1+n)$, $\sigma(n) = 1/2$, $\varepsilon(n) = 1/(1+n)$ and $\varepsilon'(n) = -1/(1+n)^2 < 0$. The CES function is $\mu(n_1, n_2) = (n_1^a + n_1^a)^{1/a}$, where $a \in (-\infty, 1)$. This implies $\alpha(n) = (1 + n^a)^{1/a}$, $\sigma(n) = 1/(1-a)$ and $\varepsilon(n) = n^a/(1 + n^a)$. Clearly, $\sigma \gtreqless 1 \Leftrightarrow a \gtreqless 0 \Leftrightarrow \varepsilon'(n) = an^{\gamma-1}/(1+n^a)^2 \gtreqless 0$, providing a simple example with $\varepsilon' > 0$. A special case is the Cobb-Douglas function, $\mu(n_1, n_2) = n_1^a n_2^{1-a}$, which implies $\alpha(n) = n^a$, $\sigma(n) = 1$ and $\varepsilon'(n) = 0$. ■

Appendix F

Consider the model in Section 3.2. From (22),

$$\begin{aligned}\frac{\partial w}{\partial y} &= 1 - \frac{(r+\delta)\,k\,(1-\varepsilon)}{\alpha}\frac{\partial n}{\partial y} \\ &= \frac{\alpha\alpha''(y-b+nk)+\alpha'\,(r+\delta)\,k\,(1-\varepsilon)}{\alpha\alpha''(y-b+nk)}.\end{aligned}$$

Ignoring terms that do not affect the sign and using $T(n) = 0$ to eliminate k, we get

$$\begin{aligned}\frac{\partial w}{\partial y} &\approx -\alpha\alpha''(y-b+nk) - \alpha'\,(r+\delta)\,k\,(1-\varepsilon) \\ &\approx -\alpha\alpha''\,(r+\delta+\alpha) - \alpha'^2\,(r+\delta)\,(1-\varepsilon) \\ &\approx -\alpha\,(r+\delta+\alpha)\,\varepsilon' + \alpha\alpha'\,(1-\varepsilon) + (r+\delta)\,\alpha'\,(1-\varepsilon)^2,\end{aligned}$$

where the last line follows from eliminating α'' using $\varepsilon' = (\alpha\alpha' + n\alpha\alpha'' - n\alpha'^2)/\alpha^2$ and simplifying. Hence, $\varepsilon' \leq 0 \Rightarrow \partial w/\partial y > 0$. The proof for $\partial w/\partial k$ is similar. ■

Appendix G

Standard results imply the incentive compatibility and individual rationality constraints can be rewritten

$$v(t_2, t_2) = v(\underline{t}_2, \underline{t}_2) + \int_{\underline{t}_2}^{t_2} e(s)ds, \tag{88}$$

plus $v(\underline{t}_2, \underline{t}_2) \geq 0$ and $e(.)$ nondecreasing. Using (88), we obtain

$$\int_{t\in\mathcal{T}} v(t_2, t_2)dN_2(t_2) = v(\underline{t}_2, \underline{t}_2) + \int_{t\in\mathcal{T}}\int_{\underline{t}_2}^{t_2} e(s)ds n_2(t_2)dt_2.$$

After integrating the last term by parts, we rewrite this as

$$\int_{t\in\mathcal{T}} v(t_2, t_2)dN_2(t_2) = v(\underline{t}_2, \underline{t}_2) + \int_{t\in\mathcal{T}} \frac{1-N_2(t_2)}{n_2(t_2)}e(t_2)dN_2(t_2).$$

Using this and the definition of $v(t_2, t_2)$, we rewrite $v(\underline{t}_2, \underline{t}_2) \geq 0$ as

$$\int_{t\in\mathcal{T}} e(t)\left[t - p(t) - \frac{1-N_2(t_2)}{n_2(t_2)}\right]dN_2(t_2) \geq 0.$$

Hence, the relevant problem is:

$$U = \max_{e(.),p(.),n} \frac{\alpha(n)}{n} \int_{t_2 \in \mathcal{T}} e(t_2) \left[t_2 - p(t_2) \right] dN_2(t_2)$$

$$\text{st} \int_{t_2 \in \mathcal{T}} e(t_2) \left[t_2 - p(t_2) - \frac{1 - N_2(t_2)}{n_2(t_2)} \right] dN_2(t_2) \geq 0$$

$$\alpha(n) \int_{t_2 \in \mathcal{T}} e(t_2) p(t_2) dN_2(t_2) = k$$

Using the second constraint to eliminate $p(t_2)$, we reduce this to the problem discussed in the text. ■

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