Intraday Liquidity and Money Market Dislocations*

Adrien d’Avernas†  Quentin Vandeweyer‡

[First Draft: June 2020]
This Draft: July 2021

Abstract

This paper relates recent disruptions in money markets to intraday liquidity regulations required under Basel III. We find that these regulations have inadvertently reduced intraday liquidity and made the supply curve of repo inelastic. Under this new regime, intraday liquidity suddenly becomes scarce and constrains the supply of repo by banks, leading to sharp increases in rates. Consistent with empirical observations, our model predicts that these spikes are more likely when Treasury debt held by shadow banks is large relative to central bank reserves and settlement volumes are high. Our model further implies that an increased risk of disruption generates a rise in Treasury spreads, as observed in March 2020.

*We would like to thank Arvind Krishnamurthy, Simon Potter, Eric Fischer, and Moritz Lenel for their valuable discussions as well as participants in seminars, workshops and conferences at the University of Chicago: Booth, NYU: Stern, Wharton macro-finance, the BI-SHoF Conference 2020, and the 2020 Macro-Finance Society Fall Meeting, and the mini-symposium on Funding Markets. We acknowledge gracious support from the Fama-Miller Center for Research in Finance and thank Livia Amato for excellent research assistance.

†Stockholm School of Economics
‡University of Chicago: Booth School of Business
1 Introduction

In recent years, short-term money market rates have been characterized by increased volatility and sporadic spikes with a high prevalence on quarter-ends. Notably, on December 31, 2018, September 16, 2019, and March 17, 2020, repo rates\(^1\) jumped up by around, 260, 380, and 60 bps, respectively (Figure 1). These sudden increases in repo rates surprised most market participants and prompted a strong reaction by the Federal Reserve (the Fed). While the Fed does not directly target repo rates, the repo market is a prime source of funding for many financial institutions, including insurance companies, asset managers, and institutional investors. For this reason, lasting disruptions in this market have the potential to amplify negative financial shocks and cause severe adverse effects to the economy, as observed during the 2008-2009 financial crisis (Brunnermeier, 2009; Gorton, 2009). Hence, the Fed intervened forcefully in September 2019 to reverse the tapering of its balance sheet and added more than $400 bn of reserves in its first open market operations since 2008.

As of today, it remains an open question as to why sudden pressures arise in these markets and whether these reflect structural issues. Recent empirical evidence points to the emergence of intermittent shortages of *intraday* liquidity. For example, Copeland, Duffie, and Yang (2021) document a strong correlation between intraday payment timing stresses and the occurrence of repo market disruptions. Correa, Du, and Liao (2020) show how global banks are mobilizing their excess reserves buffer to lend into the periodic dislocations of money markets—an operation they refer to as *reserve-draining intermediation*. The main contribution of this paper is to propose a theory connecting *intraday* liquidity to *overnight* liquidity that is able to explain these facts as a consequence of the introduction of new liquidity regulation mandated by the Basel III Accords. According to these new regulations, banks have to create a buffer of liquid assets that can be used to meet outflows in stressed scenarios. Unlike pre-Basel III requirements, these liquidity regulations mandate banks to keep this buffer of reserves at all points during the day—i.e. not only at the end of the day. We find that such intraday regulations can substantially reduce the ability of banks to lend in money markets and provide reserve-draining intermediation services to shadow banks, leading to substantial spikes in money market rates. The model further predicts that these spikes are more likely (i) on days of high settlement volumes, (ii) when the supply

\(^{1}\)Repurchase agreements (repos) are short-term collateralized loans with an estimated outstanding daily volume over $15 trillion according to ICMA.
of central bank reserves is relatively low, (iii) when the supply of Treasuries outstanding is relatively high, and (iv) that Treasury yield spreads increase when the probability of a repo spike becomes larger. We find empirical evidence in support of these four predictions.

Our macro-banking model extends from traditional theories of monetary policy implementation à la Poole (1968) to include a repo market in which leveraged shadow banks (i.e., hedge funds and securities dealers) trade liquidity with banks. The model features two main elements. First, banks are subject to an intraday stress-test requiring these institutions to hold on to a portion of reserves at any point of the day and in proportion to their expected gross outflows. Second, both the (inter-bank) fed funds and the (bank-to-shadow bank) repo markets are assumed to instantaneously settle in central bank reserves. That is, the action of lending in those markets triggers an outflow of reserves for banks. These assumptions are made to match important institutional details of US money markets such as its reliance on a Real Time Gross Settlement (RTGS) system and the existence of intraday liquidity regulations, which we introduce in Section 2.

In the model, households allocate their wealth between deposits with traditional banks and repos with shadow banks, according to their relative preferences for the two liquid assets. These preferences are subject to shocks with implications for the funding of banks and shadow banks. When households re-balance their portfolio from repo to deposit, banks find themselves with a surplus of funds, whereas shadow banks are in a deficit. As long as the repo rate is weakly above their valuation for reserves, banks benefit from providing reserves-draining intermediation to shadow banks by swapping reserves for repos on their books. In an economy not subject to the intraday stress-test constraint, banks elastically lend in repo markets and thereby prevent repo rates from rising far above the interest on reserves. Our flow-centric model highlights the crucial role of the Fed intraday liquidity overdrafts—allowing banks to run a negative balance on their reserves account within the day. By making use of this facility, banks are able to lend in repo markets even when doing so requires more reserves to settle the position than available on their balance sheets.

Our first theoretical finding is that introducing an intraday liquidity requirement in this setting creates a strict constraint on the amount that banks can lend in the repo market. Once a certain threshold is reached, the repo rate rapidly shoots up above the discount window rate as banks lacking intraday liquidity cannot take advantage of arbitrage spreads.

---

2In reality, households only invest in repo markets indirectly through money market funds. For simplicity, we omit to model these institutions and assume that households can directly invest in repos. Shadow banks should therefore be interpreted in the context of the paper as leveraged non-bank institutions borrowing in the repo market such as relative-value hedge funds, and securities broker-dealers.
between fed funds and repo rates. As intraday liquidity regulations require banks to hold a positive amount of reserves at any point during the day, banks stop lending when hitting their constraints and the supply of repo loses its elasticity. Under this regime, there is no injection of intraday reserves at the margin by the Fed through its overdraft, and trades have to settle using the fixed amount of reserves available. This feature explains the puzzling fact that daylight Fed overdrafts have ceased to be a good indicator for liquidity shortages (Kroeger, McGowan, and Sarkar, 2017) and the inelasticity of the supply of repo in the model rationalizes the strong non-linear dynamics observed in repo rates.

In an economy in which banks are subject to intraday liquidity regulation, repo rates are determined by the interaction between an inelastic demand for repo financing from shadow banks holding Treasury securities and a sometimes-constrained repo supply from banks. Various factors affect the balance between these two forces and, hence, the probability of hitting banks’ intraday limits. First, the size of the central bank balance sheet plays a crucial role in determining the total supply of reserves available to banks for settlement. As reserves allow to settle repo transactions, an economy with more reserves can sustain larger repo demand shocks before generating a surge in repo rates.

Second, an essential contribution of this paper is to point to fiscal policy as an important driver of money market imbalances. In the model, fiscal policy can affect both the demand and supply of repos through three channels. On the supply side, the issuance of new Treasury debt and payment of household tax liabilities have the side effects of draining reserves away from banks’ accounts to the Treasury account at the central bank. These operations take away from the pool of reserves that banks can use. The issuance of new Treasury debt also has a second effect on banks’ ability to lend in repo markets. Because larger outflows are anticipated on issuance days, stress-tests require banks to hold more reserves, which means that fewer reserves are available for money market lending. Moreover, on the demand side, a surge in the quantity of Treasury debt outstanding leads to an increase in repo demand from shadow banks. An expansion of public debt generates an increase in both households’ tax liabilities and assets to invest. When households invest a large portion of these assets in deposits with banks while shadow banks hold a large portion of the newly issued T-bonds, the financial system increases its reliance on bank-to-shadow bank repo lending. This increase in repo volumes eventually pushes banks’ repo supply closer to the constraints

---

3 Pozsar (2019) makes a similar point and notes: “The payments system morphed into a “token” system under Basel III... [A]’s liquidity rules require large money center banks—which, under Basel III, we call globally systemically important banks or G-SIBs—to pre-fund their 30-day outflows, intraday liquidity needs and resolution liquidity needs.”
and makes a repo spike more likely. We document this pattern in the run-up to September 2019 with banks increasing repo lending around $200 billion and shadow banks increasing repo borrowing by a similar magnitude to absorb newly issued Treasuries in 2018.

The model also features striking implications for the relationship between repo and Treasury markets. In our dynamic model, shadow banks anticipate the probability of banks hitting their constraints and the repo market spiking. To compensate for heightened liquidity risk brought about by expectations of disrupted repo markets, shadow banks require larger excess returns for holding Treasury securities. Therefore, the model predicts that when the economy moves closer to intraday constraints, spreads between T-bond yields and interest on reserves experience a surge ahead of a repo market disruption. This prediction is observed in March 2020 when the Covid-crisis hit financial markets.

When applied to the September 2019 spike, our analysis explicates the event as a combination of two slow-moving factors and two triggers. First, the supply of reserves decreased gradually from August 2014 to September 2019, thereby pushing banks closer to their intraday liquidity limits. Second, the structural demand for repo increased due to an increase in dealers’ holdings of Treasuries. Finally, the combination of new Treasury issuance and tax settlement further decreased the supply of reserves available to banks to the point of reaching a tightened intraday liquidity constraint. At this point, no further adjustment in banks’ balance sheets was possible and repo rates surged as a manifestation of the high liquidity risk faced by shadow banks such as relative-value hedge funds with large repo-leveraged positions in Treasuries.

**Related Literature** This paper contributes to a literature linking the pricing of money market assets to post-crisis regulation. On the theory side, Bech and Klee (2011) find that banks’ limits to arbitrage are responsible for fed funds rates trading below the interest on reserves while Bech and Keister (2017) focus on the impact of liquidity coverage ratio regulation. Andersen, Duffie, and Song (2019) demonstrate that the funding value adjustments of major dealers are debt overhang costs to their shareholders resulting in intermediation spreads. On the empirical side, Anbil and Senyuz (2018) and Munyan (2015) look at tri-party repos data and show that some foreign banks engage in window dressing in response to the introduction of leverage ratio requirements. Duffie and Krishnamurthy (2016) show that frictions in U.S. dollar money markets limits the pass-through of the Federal Reserve’s monetary policy. In its focus, the paper is close to the empirical work of Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020), Correa, Du, and Liao (2020) and Ava-
los, Ehlers, and Eren (2019) exploring potential explanations for the September 2019 repo rate spike. These studies point to the role of large global dealer-banks with balance sheet constraints. We complement this literature by focusing on the effect of intraday liquidity regulations. In a contemporaneous work, Copeland, Duffie, and Yang (2021) come to a similar conclusion that the supply of reserves is scarcer than previously thought due to a shortage of intraday liquidity proxied in their work by the timing of intraday payments. Yang (2020) proposes a microeconomic model in which repo spikes appear as a consequence of strategic complementarity in intraday payment timing among banks. Related to our result that expectations of larger repo spreads may give rise to a surge in Treasury spreads, He, Nagel, and Song (2021), Ma, Xiao, and Zeng (2020), Schrimpf, Shin, and Sushko (2020), and Di Maggio, Kermani, and Palmer (2019) document the role of strong selling pressure from foreign investors and exacerbated by leveraged cash-bond future arbitrage strategies. We complement this literature by pointing to the theoretical possibility that anticipations of future funding market disruption might have played a role in the unexpected rise in Treasury spreads in March 2020. Our paper also relates to a literature on the implementation of monetary policy following the seminal work from Poole (1968), adapted to dynamic OTC markets by Afonso and Lagos (2015), and to a macroeconomic framework by Bianchi and Bigio (2014) and Piazzesi and Schneider (2018). These models capture fed funds market dynamics with banks exchanging scarce reserves to mitigate their risk of borrowing at the discount window. Our paper extends this approach to include a repo market in which non-bank institutions are trading with banks. In this regard, the paper also relates to Pozsar, Adrian, Ashcraft, and Boesky (2013), Chernenko and Sunderam (2014), Sunderam (2014), and Adrian and Ashcraft (2016) which document the critical role of these institutions, collectively referred to as “shadow banks”, in creating liquidity services. Li and Krishnamurthy (2021) share with this work in a setting where households substitute the liquidity benefits across Treasuries, bank deposits, and shadow bank liabilities. Lastly, our paper is linked to the literature on the repo market and its central role in financial stability. Gorton and Metrick (2012) and Krishnamurthy, Nagel, and Orlov (2014) document a rise in margin requirements in the repo market in the 2008 crisis and argue that a run in repo markets amplified the crisis. The main innovation of this paper is to propose a macro-financial model of post-crisis money markets and highlight the role of intraday liquidity requirements as a critical piece of regulation to explain repo spikes. Our framework combines a general equilibrium setting with relevant micro-institutional frictions. This feature is key to account for how movements in aggregate variable such as the size and composition of central bank and treasury balance sheets can affect asset prices.
2 Facts

In this section, we discuss four sets of facts at the core of our analysis: the recurrence of repo spikes, the introduction of new liquidity regulations following Basel III, the tapering of the Fed’s balance sheet, and the increase in shadow banks’ Treasury position financed by repo from banks.

Recurring Spikes  In a repo transaction, an institution sells an asset to another institution at a given price and commits to repurchase the same asset from the second party at a different (typically lower) price at a future date. Although a repo is structured legally as a sale and repurchase of securities, it behaves economically like a collateralized loan. If the seller defaults before the maturity of the repo expires, the buyer retains the asset, which acts as collateral to mitigate the credit risk that the buyer (lender) has on the seller (borrower). For this reason, the return on the transaction generated by the difference in prices is referred to as a repo rate. Figure 1 displays the time series of the repo rates from January 2010 to January 2020. Repo rates have been characterized by increasing volatility, with peaks culminating at more than 275 bps in December 2018 and more than 400 bps in September 2019 above the interest paid on reserves by the Fed. One can also see that spikes of lower intensity are present throughout the rest of the series and tend to be located at fixed intervals corresponding to quarter-ends.\footnote{Munyan (2015) provides evidence of repo deleveraging behaviors from foreign banks with regulatory metrics being computed on a snapshot of their balance sheets on quarter-ends.} Notably, a spike of more than 70 bps could already be observed in September 2016. We also note that many quarter-ends do not display any sign of pressure on rates and that the largest repo spike took place in the middle of the month of September 2019 and not at a quarter-end. For instance, figure 2 show that US banks—although typically lending more when the GCF-IOR spread is higher—actually decreased their lending volumes on the week of September 16, 2019, despite the GCF spreads moving to historically high levels.

[Figure 2 about here]

New Regulation  In the aftermath of the 2008 financial crisis, policymakers worldwide introduced new regulations to address vulnerabilities in the financial system. Notably, following the Basel Committee—which introduced a set of rules collectively referred to as Basel III—tighter capital and liquidity regulations were introduced. In the debates that have fol-
lowed the September events, three types of regulation have been incriminated as possible drivers of the spike.

First, capital regulation, such as the Supplementary Leverage Ratio (SLR), compels banks to hold a given share of their liabilities in equity to protect debtors and depositors. The supplementary leverage ratio generally applies to financial institutions with more than $250 billion in total consolidated assets. It requires them to hold a minimum ratio of 3%, measured against their total leverage exposure, with more stringent requirements for the largest and most systemic financial institutions. As argued by Duffie (2018), the introduction of the SLR entails that the space on the balance sheets of major dealer banks is today more expensive than before the 2008 financial crisis. Accordingly, large dealer-banks have increased intermediation spreads, resulting in an increase in funding costs for other institutions.

Second, the 2008 financial crisis illustrated how quickly market liquidity could evaporate and prompted regulators to introduce additional liquidity regulations. First, the Liquidity Coverage Ratio (LCR) requires that banks hold adequate stock of unencumbered high-quality liquid assets (HQLAs)—i.e., cash or assets that can be converted into cash quickly through sales with no significant loss of value. HQLAs include level 1 assets, which can be held without limit or haircut, and level 2 assets, which cannot exceed 40% of the liquidity reserve capped at a maximum of 40% and receive a 15% haircut. In practice, Level 1 HQLAs include both reserves at the Fed and US Treasuries, whereas level 2 HQLAs include mortgage-backed securities guaranteed by government-sponsored enterprises.

Third, the LCR is complemented by various internal Liquidity Stress Tests (LSTs). According to LCR, all categories of Level 1 HQLAs—such as reserves and Treasuries—are treated as substitutes. However, according to the Fed vice-chair for the supervision, Randal K. Quarles, it may, in practice, be difficult to liquidate a large stock of Treasury securities to meet large day one outflows (Quarles, 2020). Within Regulation YY’s enhanced prudential standards and Resolution Liquidity Adequacy and Positioning (RLAP), large firms are required to conduct internal liquidity stress tests, and supervisors expect firms to ensure that their liquidity buffers can cover estimated day-one outflows without reliance on the Federal Reserve. For institutions with large intraday outflows such as clearing banks or banks with a broker-dealer subsidiary, these liquidity buffers can be sizable—e.g., in 2008-2009, several firms experienced outflows exceeding tens of billions of dollars in a single day. Therefore, these firms are required to hold substantial quantities of reserves at the Fed—rather than Treasuries—as a complementary liquidity buffer to their LCR.5

5Before September 2019, the supervisory guidelines for banks’ internal liquidity stress tests were kept se-
The notion that the September spike is connected to bank intraday liquidity hurdles has been discussed in policy circles. In May 2019 already, Pozsar (2019) argued that intraday liquidity had been much scarcer than commonly admitted, and the repo market was likely to face severe disruptions going forward. Following the September 2019 spike when asked on how the Fed would adjust to the event, its chairman, Jerome Powell, pointed out that: “It used to be a common thing for banks to have intraday liquidity from the Fed, what is called “daylight overdrafts.” That’s something we can look at. Also, there are just a few technical things that we can look at that would perhaps make the liquidity that we have—which we think is ample in the financial system—move more freely and be more liquid, if you will.”

Moreover when asked why JP Morgan did not lend in repo when spreads were large in September despite holding large amount of reserves, its chairman, James Dimon, answered: ”[…] we have $120 billion in our checking account at the Fed, and it goes down to $60 billion and then back to $120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there’s extreme stress during the course of the day, it doesn’t go below zero. If you go back to before the crisis, you’d go below zero all the time during the day. So the question is, how hard is that as a red line?”

[Figures 3 about here]

**Scarce and Volatile Reserves** Figure 3 displays the evolution of the Fed’s liabilities. Responding to the financial turmoil, the Fed started in 2008 to purchase large amounts of long-term securities—a policy instrument commonly referred to as Quantitative Easing (QE). As a consequence, the volume of reserves held by banks had increased by a factor of 40, leading to a doubling of the total size of their combined balance sheet. In November 2010, the Fed announced the second round of QE, and further increased its balance sheet by $600 billion by the end of the second quarter of 2011. The third round of QE was secret, and it was unclear if regulators were indeed drawing a distinction between reserves and cash-equivalents such as Treasuries. Vice-Chairman Randal K. Quarles recently clarified this point in a series of declarations. For instance, on February 6, 2020, he mentions that “supervisors expect firms to estimate day-one outflows and to ensure that their liquidity buffers can cover those outflows without reliance on the Federal Reserve. For firms with large day-one outflows, reserves can meet this need most clearly.” In the same speech, he opens the door for taking discount window liquidity as part of the stress-test so that: “[i]f firms could assume that this traditional form of liquidity provision from the Fed was available in their stress-planning scenarios, the liquidity characteristics of Treasury securities could be the same as reserves, and both assets would be available to meet same-day needs.” Quarles (2020).

In the US, only a restricted set of institutions—including depository institutions, Federal Home Loan Banks and Government-Sponsored Enterprises—have an account at the Fed and can directly hold reserves.
then announced in September 2012, leading to an additional increase in the Fed’s balance
sheet of over $1.5 trillion by 2015. Until October 2017, the Fed kept its balance sheet at a
stable size. At this juncture, it started to “normalize” its balance sheet by not reinvesting
a part of maturing securities. The intention was to reduce the balance sheet at an initial
of pace $30 billion per month, then adjusted to $15 billion per month, up to September
2019. At this point, reserves were anticipated to “likely still be somewhat above the level of
reserves necessary to efficiently and effectively implement monetary policy. In that case, the
Committee [anticipated] that it [would] likely hold the size of the SOMA portfolio roughly
constant for a time. During such a period, persistent gradual increases in currency and other
non-reserve liabilities would be accompanied by corresponding gradual declines in reserve
balances to a level consistent with efficient and effective implementation of monetary policy”
Board of Governors of the Federal Reserve System (2019). In September 2019—as repo rates
suddenly hiked to more than 600bps—the Fed reversed course and started to increase its
balance sheet again, initially by lending in the repo market and eventually through direct
purchases of Treasury bills.

This evolution in the Fed’s balance sheet size was also accompanied by a change in the
composition of its liabilities. After the 2008 financial crisis, the Fed started offering overnight
liquidity services to a growing number of non-bank institutions—ultimately leading to a
further reduction in the supply of reserves to banks (Pozsar, 2017). First, the US Treasury
no longer keeps its cash balances with private banks but rather with the Fed at its Treasury
General Account (TGA). These balances can run as high as $400 billion and are highly
volatile, in particular during tax payment and debt issuance periods, as well as around
quarter-end and year-end. For instance, in September 2019, TGA balances increased by
more than $150 billion within two weeks, thereby removing a similar amount of reserves
from the stock available to banks. Second, the Fed allowed some foreign central banks to
move their short-term balances to its balance sheet within the Foreign Reverse Repo Facility
(FRRP). In September 2019, the FRRP reached a peak of $300 billion. Third, in 2014, the
Fed let money market funds access its balance sheet through the Domestic Reverse Repo
Facility (DRRP). The DRRP became an important element in the Fed’s monetary policy
implementation strategy by creating a floor under which money market funds would not
lend to the repo market. The DRRP was heavily used by money market funds, which would
deposit on average $150 billion with the Fed per day up to 2018, when a large supply of
Treasury bills pushed repo rates above the DRRP rate.

Importantly, these different items on the liability side of the Fed’s balance sheet are not
under its direct control. The Fed decides how much it remunerates the facility, but it does not control the quantities that institutions—including the Treasury—can deposit at the facilities. When taken in combination—a gradual reduction in balance sheet size and the introduction of volatile facilities to non-bank institutions—these different factors have made the quantity of reserves available to banks gradually scarcer and more volatile. In September 2019, the quantity of reserves hits a minimum after two years of gradual reduction and a seasonal surge in TGA balances.

[Triples 4, 5, and 6 about here]

**Treasury Supply and Repo Volumes** Finally, we document the relationship between the sharp rise in outstanding stocks of Treasuries available to the public (i.e., net of Fed holdings) and an increase in repo volumes from banks to shadow banks in the run-up to September 2019. Figure 4 plots the net supply of Treasuries outstanding between January 2010 and September 2019. The shaded area represents the portion of these Treasuries held by the Fed and, therefore, not available to the public. One observes a large increase in the amount of Treasuries available to the public between 2017 and 2019 brought about by two factors. First, in Q4 2017, the Fed started to unwind its Treasury portfolio. Second, since the beginning of the Trump Administration, the US has run larger deficits than in previous years. Figure 5 shows that domestic banks and shadow banks have largely absorbed this increase in Treasury securities supply. Figure 6 shows that the rise in Treasury holdings from shadow banks is concomitant to a similar-sized increase in their repo borrowing from banks. According to Barth and Kahn (2021) this association can be explained by a sharp surge in activity from hedge funds arbitraging the cash-future basis with highly leveraged Treasury portfolios financed by general collateral repos. Lenel (2020) documents a closely related relationship between the supply safe assets and the quantity of collateralized loans in the US economy.

3 Model

3.1 Environment

The model is a general equilibrium extension of Poole (1968) featuring shadow banks and a repo market. Formally, we let \((\Omega, \mathcal{F}, \mathbb{P})\) be a probability space that satisfies the usual conditions and assume that all stochastic processes are adapted. Time is discrete and
infinite. Any period has two stages: the morning and the afternoon. The economy is populated by a continuum of households, bankers running traditional and shadow banks, as well as a treasury and a central bank. Figure 9 provides an illustration of sectoral balance sheets with interlocking positions.

**Market Structure** There are two goods in positive supply: a securitized productive capital $k$ and a final consumption good $y$ produced by capital. The Treasury issues T-bonds $b$ against future households tax liabilities $\tau$. The central bank holds some of the outstanding T-bonds by issuing reserves $m$ to the traditional banking sector and the Treasury. Households and shadow banks cannot hold reserves. Households hold their wealth in deposits $d$ issued by traditional banks and repurchase agreements (repo) $p$ issued by traditional and shadow banks. Traditional banks hold securitized capital $k$ valued at price $q$, reserves at the central bank $m$ that can be traded as fed funds $f$, and repos $p$. Shadow banks finance a portfolio of T-bonds $b$ with repo $p$ from both banks and households.

**Timing** Figure 10 summarizes the timing of the economy. Subscript $t^-$ indicates variables and prices set in the morning, while the subscript $t^+$ indicates variables and prices set in the afternoon. $X_{t^-}$ and $X_{t^+}$ represent the state space of the economy in the morning and
in the afternoon of time $t$ respectively. Uppercase variables designate aggregate variables. Markets for the consumption good, T-bonds, securitized capital, deposits, and bank equity clear in the morning. The repo and fed funds markets clear in the afternoon. The model features two types of shocks. First, an aggregate shock $\alpha^h_t$ to households’ preference for liquid assets (deposits and repo) occurs in the afternoon and results in an aggregate deposit flow. Second, as in Poole (1968), deposits are subject to idiosyncratic shocks in the late afternoon.

Preferences and Technology  Bankers and households are risk neutral. Households also value liquidity services modeled as deposit-and-repo-in-the-utility with a Leontief aggregator between deposits and repos:

$$U^h(c^h_t, d^h_t, p^h_t, \alpha^h_t) = c^h_t + \varphi \min \left\{ \frac{d^h_t}{\alpha^h_t K^r_t}, \frac{p^h_t}{(1 - \alpha^h_t)K^r_t} \right\}^{1-\gamma} / 1 - \gamma.$$  

In previous expression, the parameter $\varphi$ governs the weight of liquidity services relative to the numeraire consumption. The preference for liquidity has decreasing returns parameterized by $\gamma$. Liquidity services are scaled by the aggregate supply of capital so that the utility derived from holding liquid assets does not depend on the size of the economy. The preference shock $\alpha^h_t$ realizing in the afternoon determines the weight that each type of liquidity receives in generating aggregate liquidity benefits and is uniformly distributed between $\underline{\alpha}$ and 1.

Each period, the stock of capital produces $a$ units of consumption good. All units of capital are pooled in an economy-wide diversified vehicle in quantity $K^r_t$ with price $q^r_t$. The
expected return on this securitized capital is given by:

\[ 1 + r_t^k = E_r \left[ \frac{a + q_r + 1}{q_r} \right] . \]

Without loss of generality, since there is no capital growth or depreciation, we normalize the size of our economy by setting \( K_t = 1 \).

**Intraday Flows** We specify the timing of intraday flows. In the afternoon, the repo market opens and traditional banks have the option to lend (reverse) repo to shadow banks. We assume that repos do not roll over and require new settlement every day. This operation triggers an outflow of reserves. Hence for traditional banks, repo lending amounts to swapping a quantity \( p_r \) of reserves into repo. Shadow banks use borrowed repos to clear daylight overdrafts positions \( o_r \) with traditional banks. These overdrafts are essential to allow shadow banks to finance T-bonds in the morning ahead of the opening of the repo market.\(^7\) The intraday laws of motion for deposits \( d_r \), overdrafts \( o_r \), and reserves \( m_r \) are given by:

\[ d_r = d_r + \Delta d_r, \quad o_r = o_r - \Delta o_r, \quad m_r = m_r + \Delta d_r + \Delta o_r - p_r. \]

We do not impose a nonnegativity constraint on \( m_r \) and interpret this ability for reserves to become negative in the morning as a temporary intraday overdraft provided by the central bank.\(^8\)

**Deposit Shocks** Following Poole (1968), at the end of the afternoon, traditional banks are subject to a deposit shock \( \Delta d_r \) that results in reserve transfers. These shocks are meant to capture that demandable deposits have stochastic maturity from the point of view of traditional banks, and therefore carry liquidity risk. Traditional banks with net deposit

---

\(^7\)Hedge funds and securities broker-dealers typically rely on clearing banks such as Bank of New York Mellon and JPMorgan Chase for daylight overdraft financing of auctioned Treasury debt.

\(^8\)In the US, the Fed allows qualifying banks to overdraw on their Fed accounts in order to make payments via Fedwire. Most developed economies have similar facilities. The need for deep and cheap central bank daylight overdrafts is a product of the generalization of Real-Time Gross Settlement (RTGS) systems that started in the 80s to reduce banks’ intraday credit risk exposure. Under RTGS, banks have to pre-fund all intraday outflows with reserves, which could not be done without a dramatic increase in reserves supply. As before the 2008 financial crisis, such an increase in reserves was not conceivable, central banks chose to rely on favorable daylight overdrafts embedded within the RTGS system, thereby allowing the supply of liquidity to grow without bound for the payment system to clear during the day while still being scarce overnight (Mehrling, 2020).
outflows late in the day have to transfer reserves to other banks and may end up with less reserves than required by regulation (see below). This feature generates a motive in traditional banks for holding reserves as a buffer against the deposit shocks. We specify gross intraday deposit inflows $\Delta d_{it}^i$ and deposit outflows $\Delta d_{it}^o$ as:

$$
\Delta d_{it}^o + \epsilon_{it}^o = d_{it} - \Delta d_{it}^i - \Delta d_{it}^o,
$$

where $\mu^o_{it}$ and $\mu^i_{it}$ are the average outflow and inflows per unit of deposit while $\epsilon^o_{it}$ and $\epsilon^i_{it}$ are idiosyncratic shocks distributed according to a distribution truncated over $[0, \sigma]$ where $\sigma < 1$. Note that the average outflow $\mu^o_{it}$ may not equal to the average inflow $\mu^i_{it}$ as the aggregate demand for deposits changes in the afternoon following the household preference shock.\(^9\)

**Regulation** To explore the role of regulation in the repo market, we posit that traditional banks are subject to two constraints matching the regulatory practice in the US described in Section 2. First, as in Poole (1968), we assume that traditional banks are subject to a Reserve Requirement (RR):

$$
m_{it} \geq \chi m d_{it},
$$

where $\chi_m$ is the regulatory reserve ratio set by the regulator. If the reserve requirement is not satisfied, a traditional bank has to borrow reserves at the discount window with additional cost $r^w > 0$—i.e., the discount window rate is the sum $r^m_t + r^w$.

Second, traditional banks are subject to an intraday Liquidity Stress-Test (LST) requirement according to which they have to hold enough liquid assets to cover their intraday outflows of deposit without relying on expected inflows. To match supervisory practices, we assume that only reserves can be used to meet this requirement. A traditional bank is compliant with LST if:

$$
m_{it} - \max\{0, p_{it}\} - \max\{0, f_{it}\} \geq \chi m d_{it} + \Delta^o d_{it},
$$

That is, traditional banks need to have a buffer of reserves to cover daylight outflows of deposits in excess of what is required by the reserve requirement (RR) and used in repo and

\(^9\)The average outflow of deposits is given by: $\mu^o_{it} = \max\{0, -\Delta D^h_t\}/D_{it}$. Indeed, for markets to clear, the average outflow of deposits from traditional banks must correspond to the aggregate decrease in the demand for deposits from households.
fed funds transactions. The maximum operators enforce that the traditional bank cannot rely on money markets in the afternoon to satisfy LST in the morning. These restrictions are set in the morning using stress test models such that this requirement is breached during the day with a $\zeta$ probability corresponding to the regulator tolerance for breaching LST.

### 3.2 Agents’ Problems

**Bankers** The problem of a banker can be written in recursive form as:

$$V^b_t(n^b_t; X_t) = \max_{c^b_t \leq n^b_t, m_t \geq 0, q_t \geq 0} \mathbb{E}_t \left[ c^b_t + \beta V^b_{t+1}(n_{t+1}; X_{t+1}) \right],$$

subject to the morning balance sheet constraint:

$$n^b_t + n^b_t = n^b_t - c^b_t,$$

where returns on their portfolio are such that:

$$n^b_{t+1} = n_{t+1} + n^b_{t+1}.$$

In the morning, banker consume $c_t$ and decide how much wealth to invest in traditional $n_t$ and shadow banks $n_t$. The parameter $\beta$ is the state price density for future dividends.

**Traditional Banks** The problem of traditional banks can be written as:

$$\max_{k_t \geq 0, d_t \geq 0, q_t \geq 0} \mathbb{E}_t \left[ \max_{p_t, f_t} \mathbb{E}_t \left\{ n_{t+1} \right\} \right],$$

subject to the morning balance sheet constraint:

$$q_t k_t + m_t + o_t = n_t - c_t + d_t,$$

where returns on their portfolio are such that:

$$n_{t+1} = n_t - c_t + k_t r^k_t + m_t r^m_t + o_t r^o_t + p_t r^p_t + f_t r^f_t - d_t r^d_t - (\chi d_t - m_t) r^m_t 1 \{m_t < \chi^m d_t \}.$$
The variables $r^k_t$, $r^m_t$, $r^p_t$, $r^d_t$, and $r^f_t$ are the returns (or interest rates) on capital, reserves, overdrafts, deposits, repo and fed funds, respectively. In the morning, traditional banks issue deposits $d_t$, invest in capital $k_t$ and in reserves $m_t$ and open credit lines in the form of daylight overdrafts $o_t$ to shadow banks. In the afternoon, they decide on how much to lend in repos $p_t$ and fed funds $f_t$. If at the end of the day the stock of reserves falls short of the reserve requirement (RR), banks have to pay the discount window penalty $r^w_t$.

Whenever shadow banks fail to fully reimburse this overdraft by the end of the day ($o_t \geq 0$), they incur expensive overnight credit rates at a penalty cost $\lambda > 1/\beta - 1$ with their clearing bank.10 An interpretation of this charge is the opportunity cost for traditional banks to provide overnight short-term funds while constraints by LST.

**Shadow Banks** The problem of shadow banks can be written as:

$$\max_{\omega_t \leq \omega_t, o_t \geq 0, n_t \geq 0} \mathbb{E}_t \left[ \max_{\mathbb{E}_t} \mathbb{E}_t \{ n_{t+1} \} \right],$$

subject to the morning balance sheet constraint:

$$b_t = n_t - o_t - \omega_t,$$

where returns on their portfolio are such that:

$$n_{t+1} = n_t - o_t + b_t - \omega_t - P_t - \delta_t - r^d_t - o_t - \omega_t - \lambda \max\{0, o_t\}.$$ 

All variables have a similar interpretation as for banks with an lower bar notation to designate variables specific to shadow banks. In the morning, shadow banks consume and purchase treasuries $b_t$ with an overdraft at a traditional bank. In the afternoon, shadow banks transfer funds raised in repo markets to banks in order to net their position. The intraday laws of motion for bonds $b_t$ and repo $P_t$ are given by:

$$b_t = b_{t-1} + \Delta b_t, \quad \omega_t = \omega_{t-1} + \Delta \omega_t,$$

10 According to Pozsar (2019), daylight overdrafts at the Bank of New York Mellon Corporation cost 60 bps per annum per minute for less than $5 billion, and 120 bps per annum per minute for amounts greater than $5 billion—all collateralized. Furthermore, the pricing of and reputational risk around daylight overdrafts are a strong deterrent for dealers to use overdrafts frequently and liberally. The daylight overdrafts turn into overnight general collateral (GC) repo if they are not paid back by “sunset.”
Daylight overdrafts that are rolled over to the next morning are priced at the penalty rate $\lambda$.

**Central Bank** The central bank controls the supply of liquid assets available to the banking sector and the Treasury by swapping reserves for T-bonds (and conversely) through open market operations. The central bank decides simultaneously on the stock of reserves $M_t$ and the amount of T-bonds held by the central bank $\overline{B}_t$ (and hence, not available to other agents), subject to the balance sheet constraint:

$$\overline{B}_t = M_t.$$ 

The upper bar notation differentiates the central bank’s holdings of T-bonds $\overline{B}_t$ from the bonds issued by the Treasury $B_t$. For simplicity, we assume that the central bank operates with zero net worth and transfers all seigniorage and discount window revenues to the Treasury.

**Treasury** The Treasury issues T-bonds against future tax liabilities of households and has access to reserves through its treasury account at the central bank. At the beginning of each period, the government decides on the quantity of new bonds to issue $\Delta B_t$ and how much taxes to settle $\Delta T_t$. T-bonds issuance follows:

$$B_t = B_{t-1} + \Delta B_t.$$ 

For simplicity, we also abstract from direct government expenditures so that T-bonds are fully backed by future lump-sum tax policies. Accordingly, the net present value of future tax liabilities must equal the outstanding amount of T-bonds minus the quantity of reserves in the treasury account: $T_t = B_t - G_t$, where $T_t = T_{t-1} + \Delta T_t$. Additionally, the government budget constraint is such that the transfers to households are given by:

$$\tau_t T_t = r^b_t B_t + r^m_t M_t - r^m_t G_t - r^b_t \overline{B}_t - r^w M^w_t,$$

where $M^w_t$ is the quantity of reserves borrowed at the discount window at the end of the day and $\tau_t$ is an exogenous tax policy on households.\(^{11}\) Because the central bank provides liquidity to the economy, reserves earn a liquidity premium and receive a lower interest than T-bonds $r^m_t < r^b_t$. Thus, the discounted value of future taxes is lower than the tax liability.

\(^{11}\)We also impose that $G_t < M_t$, otherwise there would not be enough reserves for the treasury account to function.
and the value of liquidity insurance benefit of reserves $L_t$ is:

$$L_t = T_t - \mathbb{E}_t \left[ \sum_{j=t+1}^{\infty} \beta^{j-t} \tau_j T_j \right].$$

**Households**  Households are risk neutral, maximize their lifetime utility of consumption, and discount the future at rate $\beta$. They also receive utility from holding liquid assets such as deposits and repos and have to pay taxes $\tau_t$ to the Treasury, where $t^h_t$ is the net present value of future tax liabilities.

The problem of the households can be written in recursive form as:

$$V^h_t (n^h_t, X_t) = \max_{c^h_t \leq n^h_t, d^h_t \geq 0, \beta \geq 0} \mathbb{E}_t \left[ \max \mathbb{E}_t \left\{ U^h (c^h_t, d^h_t, p^h_t, \alpha^h_t) + \beta V^h_{t+1} (n^h_{t+1}, X_{t+1}) \right\} \right]$$

subject to the balance sheet constraint:

$$d^h_t + l^h_t = n^h_t - c^h_t + t^h_t,$$

where returns on their portfolio are such that:

$$n^h_{t+1} = n^h_t - c^h_t + d^h_t r^d_t + p^h_t r^p_t - t^h_t \tau_t.$$

The relative demand for repo $p^h_t$ depends on the liquidity preference shock $\alpha^h_t$ realized in the afternoon. Thus, this shock generates uncertainty for the aggregate supply of short-term fund to shadow banks and intraday flows: $d^h_t = d^h_t - p^h_t$. The value of tax rebates from the treasury $t^h_t$ is non-tradable.\(^\text{12}\)

### 3.3 Equilibrium

**Definition 1.** *Given an initial allocation of all asset variables at $t = 0$, monetary policy decisions $\{M_t : t \geq 0\}$, fiscal policy decisions $\{B_t, T_t : t \geq 0\}$, and household’s liquidity preference shocks $\{\alpha^h_t, \beta : t \geq 0\}$, a sequential equilibrium is a set of adapted stochastic processes for (i) prices $\{d^k_t, r^m_t, r^o_t, r^h_t, r^d_t, r^p_t, r^f_t, t^h_t : t \geq 0\}$; (ii) individual controls for bankers $\{c^h_t, n^h_t, \beta : t \geq 0\}$; (iii) traditional banks $\{k^m_t, m^o_t, \alpha^h_t, \beta : t \geq 0\}$, (iv)\(^\text{12}\)This assumption simplifies the analysis such that the value of tax rebates stays on the balance sheet of households and $n^h_t \geq t^h_t$ at all times. The households can consume the flows that it receives from the tax rebates but cannot exchange the future tax rebates against consumption goods."
shadow banks \{b_1, \ldots, b_T : t \geq 0\}; (v) households \{c^h_{i}, d^h_{i}, p^h_{i} : t \geq 0\}; (vi) value of liquidity services and tax liabilities \{l^h_{i}, L^h_{i}, t^h_{i}, T^h_{i} : t \geq 0\}; and (vi) agents’ net worth \{n^b_{i}, n^b_{i} : t \geq 0\} such that:

1. Agents solve their respective problems defined in equations (2), (1), (3), and (4).

2. Morning markets clear:

(a) capital: \[ \int_{0}^{1} k_t(i)di = 1, \]
(b) deposits: \[ \int_{0}^{1} d^h_{i}(h)dh = \int_{0}^{1} d^r(i)di, \]
(c) T-bonds: \[ \int_{0}^{1} b_t(j)dj + B_t = B_r, \]
(d) reserves: \[ \int_{0}^{1} m_t(i)di + G_t = M_t, \]
(e) overdrafts: \[ \int_{0}^{1} o_t(i)di = \int_{0}^{1} o_r(j)bj, \]
(f) output: \[ \int_{0}^{1} c^h_{i}(h)dh + \int_{0}^{1} c^b_{i}(b)db = a, \]

3. Afternoon markets clear:

(g) repo: \[ \int_{0}^{1} p^r_t(i)di + \int_{0}^{1} p^b_t(h)dh = \int_{0}^{1} p_r(j)dj, \]
(h) fed funds: \[ \int_{0}^{1} f^r_t(i)di = 0, \]

4. The aggregation of liquidity services and tax liabilities are consistent:

\[ \int_{0}^{1} l^h_{i}(h)dh = L_t, \]
\[ \int_{0}^{1} t^h_{i}(h)dh = T_t. \]

Resolution Method We describe our approach to solving the model informally in this section and leave technicalities to Section A. First, we note that it is without loss of generality to solve the model for a representative banker and household. As the probability of having to borrow at the discount window is increasing in deposits \(d_t\) and decreasing in both reserves \(m_t\) and overdrafts \(o_t\), all traditional banks choose the same ratio of deposits to reserves and overdrafts. Since all agents are risk-neutral, the distribution of wealth across banks does not matter for liquidity risk management decisions. Furthermore, none of the constraints can ever be binding for one particular bank without binding for all other banks. Otherwise, a
banker managing a constrained bank would have an incentive to change its wealth allocation to relax the constraint as its marginal valuation would not equal the market price.

In addition, risk neutrality further allows to abstract from forces which are not the focus of this paper, such as wealth dynamics, risk aversion, and intertemporal consumption smoothing. As a consequence of this assumption, households and bankers are indifferent between multiple equilibrium allocations as long as the expected returns at time \( t \) on all assets is equal to their time discount rate. For example, bankers are indifferent between an infinity of equilibria (e.g., with different repo rate \( p_t \), interest rates on reserves \( r^{m}_t \), and consumption allocations \( c^b_t \)), earning the same expected return on all assets. Thus, given the monetary and fiscal policy decisions, the economy admits a multiplicity of equilibria.

Notwithstanding this multiplicity, an important property of the model is that once consumption is chosen and \( \phi^p_t > 0 \), there is a unique equilibrium that solves agents’ maximization problems and satisfies the market clearing conditions.\(^{13}\) We take advantage of this property and define agents’ post-consumption wealth (and net of the value of liquidity services) to pin down an unique equilibrium in prices, defined as:

\[
\begin{align*}
\tilde{n}_t^h &\equiv n_t^h - c_t^h - \ell_t^h, \\
\tilde{n}_t^b &\equiv n_t^b - c_t^b.
\end{align*}
\]

Doing so allows to perform comparative statics analysis on money market rates irrespective of allocation within wealth-consumption pairs. In Appendix A, we characterize the existence of the unique Markov perfect equilibrium in the state space given by: \((\tilde{n}_t^h, M, B, T)\). Importantly, the relative shares of wealth invested in traditional and shadow banks are pinned down by the market for deposits. Indeed, the demand for deposits (as a function of the deposit interest rate \( r^{d}_t \)) by households is monotonically increasing and the supply of deposits by traditional bankers is monotonically decreasing. In the following section, we drop the time subscript but keep the + subscript to distinguish afternoon variables.

When performing comparative statics in the next section, we formally define the short-term impact of a change in fiscal or monetary policy on an equilibrium policy function, such as the probability of a repo spike \( \phi^p(\tilde{n}_t^h, M, B, T) \), as the change of that policy function with respect to fiscal and monetary policy variables while holding the relative shares of

\(^{13}\)When \( \phi^p_t = 0 \), LST is never binding, and the supply of repo by banks is perfectly elastic such that any demand for repo by shadow banks satisfies the maximization problems and clear the markets without impacting prices. Thus, the equilibrium is unique in prices and relevant allocations, but not in banker’s investment shares into the traditional and shadow banking sector.
wealth invested in traditional and shadow banks constant. That is, we study the impact of these policies on the balance sheet size and intraday flows without letting the bankers change their investment allocations into the traditional and shadow banking sectors. To do so, we define the function $\tilde{\varphi}(n(\tilde{n}^h, M, B, T), M, B, T) = \varphi(\tilde{n}^h, M, B, T)$, and take the partial derivative of that function with respect to the policy variables. For example, $\partial \tilde{\varphi}(n, \tilde{n}^h, M, B, T)/\partial B$.

Subsequently, when studying the long-term impact of a change in fiscal or monetary policy on an equilibrium policy function, we do not constrain the investment allocations into the traditional and shadow banking sectors. Thus, we simply take the partial derivative with respect to the equilibrium policy function with respect to the policy variables. For example, $\partial \varphi(\tilde{n}^h, M, B, T)/\partial B$.

4 Theoretical Analysis

In this section, we study the implications of the introduction of intraday LST constraint for price and quantity dynamics in fed funds, repo and T-bond markets. We derive asset pricing equations for each assets in the economy. These equations depend on the constraints banks face, which in turn depends on aggregate portfolio choices and intraday flows. As a benchmark, we first derive the model’s implications for an economy in which the only piece of liquidity regulation is the reserve requirement. We then study the dynamics of an economy in which banks are also subject to the intraday liquidity constraint.

4.1 Solving

To streamline the analysis and avoid marginal cases, we make some technical restrictions on the parameter space. Moreover, to simplify the exposition of analytical results, we assume that the idiosyncratic deposit shocks $\varepsilon^o$ and $\varepsilon^i$ are distributed according to a uniform distribution over $[0, \sigma]$. Finally, we define the size of the reserve buffer required to satisfy the LST requirement in equation (LST) as $\Phi^d \equiv \chi^m + \sigma (1 - \zeta) + \mu^o$. Proofs and derivations are relegated to Appendix A.

Assumption 1. LST does not constrain the issuance of deposits by traditional banks in the morning: $M - G > \Phi^d d$.

Assumption 2. The equilibrium quantity of overdrafts is below the quantity of reserves in the economy: $o < m$. 

22
**LST Risk**  Given the distribution of the outflow deposit shock $\varepsilon_o$ and denote the morning probability that a bank hits the constraint in the afternoon by:

$$\phi^p \equiv \mathbb{P}[p^k < q + \Phi^d d - m].$$

The constraint is binding when the supply of repo by households is lower than the sum of the minimum required to satisfy LST and net the shadow bankers’ overdrafts. If in the afternoon the LST constraint is binding, we set $\phi^p = 1$ such that $\phi^p = \mathbb{P}[\phi^p = 1]$.

**Bond and Capital Markets**  Shadow banks invest in capital and T-bonds until the return on their T-bond portfolio equals the expected cost of funds in the repo market:

$$r^b = \mathbb{E}[r_p].$$

(5)

Given Assumption 1, traditional banks have the same discount factor as shadow banks and the return on capital is equal to the return on T-bonds: $r^b = r^k$.

**Reserves**  By holding reserves, traditional banks receive interest on reserves, reduce the probability of having to borrow at the discount window, and increase their lending capacity in afternoon money markets. Hence, banks’ first order conditions yield:

$$r^k - r^m = \phi^m r^w + \phi^p (\mathbb{E}[r^p | \phi^p = 1] - r^m - \phi^m (r^p + r^w)),$$

where $\phi^m = \mathbb{P}[m + \Delta d_s - \Delta p_s < \chi d]$ is the probability of having to borrow at the discount window conditional on the information available in the morning and $\phi^m (r^p + \Delta d_s < \chi d]$ is the probability of having to borrow at the discount window when constrained by LST. Thus, the liquidity premium on reserves $r^k - r^m$ depends on the tightness of the reserve requirement and the probability of hitting the LST constraint.

### 4.2 Benchmark Without LST Regulation

We first examine a benchmark case with the reserve requirement as the only binding constraint. This case corresponds to the setting studied in Poole (1968) extended to include a bank-to-shadow-bank repo market. We study how the repo market and the fed funds market interact with each other. In this benchmark case, banks act as unconstrained arbitrageurs between these two market segments and ensure that the two rates remain equal at all times.
We first derive the demand for repo financing. Shadow banks fund themselves in the repo market unless the repo rate is above the daylight overdraft cost. That is,

$$\varphi^* = \begin{cases} 
  b - n & \text{if } r_p^* < \lambda, \\
  (0, b - n) & \text{if } r_p^* = \lambda, \\
  0 & \text{if } r_p^* > \lambda.
\end{cases}$$

When the repo rate $r_p^*$ is below the daylight overdraft cost $\lambda$, shadow banks will finance their entire portfolio with repos. When the repo rate is above $\lambda$, they prefer to pay the daylight overdraft cost on their entire portfolio, and their repo demand falls to zero. In the edge case where the repo rate is precisely equal to $\lambda$, shadow banks are indifferent between repo funding and penalty on unauthorized overdrafts. Proposition 1 characterizes the main properties of the unconstrained equilibrium for money market rates.

**Proposition 1.** In an economy in which LST is never binding, the repo rate is always equal to the fed funds rate, and both of these rates are bounded by the interest on reserves below and the discount window rate above:

$$r^m \leq r^f = r^p \leq r^m + r^w.$$ 

This proposition unfolds from the first order conditions for reserves and repo holdings:

$$r^m + r^w \phi^m = r^f = r^p, \tag{6}$$

where $\phi^m \equiv \mathbb{P}_* [m + \Delta d_* - \Delta p_* < \chi d]$ is the probability of having to borrow at the discount window conditional on available information in the afternoon. According to equation (6), banks’ first order conditions translate into a no-arbitrage condition between repos and fed funds rates. As the probability $\phi^m$ is between zero and one, the two rates are contained within the boundaries of the interest rate on reserve $r^m$ and the discount window rate $r^m + r^w$. When unconstrained, banks supply any quantity necessary for money market rates to adjust to the interest rate on reserves plus a liquidity premium on reserves accounting for the marginal benefit of lowering discount window risk. This outcome corresponds to the traditional result of Poole (1968) for the fed funds market—here extended to the repo market.

[Figure 11 about here]

Figure 11 plots the simultaneous equilibrium in the two markets. The upper left panel
corresponds to the traditional corridor diagram and shows the fed funds rate $r_f^r$ as a function of the quantity of reserves. The downward blue line shows the demand schedule for reserves as a negative function of the fed funds rate. When the fed funds rate is high, banks want to lend more in fed funds (and repo) markets and therefore have less precautionary demand for reserves. This demand schedule is bounded above by the discount window rate as this rate is the worst rate possible when facing a liquidity shock. At the bottom, it is bounded by the interest on reserves: the value of holding reserves without liquidity benefits. The vertical red line represents the supply of reserves as provided by the central bank. The larger the supply of reserves, the lower the liquidity premium on these and the closer the fed funds rate $r_f^r$ is to the interest on reserves $r^m$.

The upper right panel displays the simultaneous equilibrium in the repo market. The repo supply schedule in red is an increasing function of the repo rate $r_p^r$—higher repo rates provide incentives for banks to lend larger amounts in repos at the cost of resorting to the discount window more often. The blue line represents the excess repo demand from shadow banks: the inelastic shadow bank demand to fund an overdraft position minus the supply by the household sector. When rates reach $\lambda$, shadow banks prefer to pay the daily overdraft cost rather than borrow in repo markets. This outside option, therefore, caps shadow banks’ willingness to pay for repo.

The two lower panels (b) and (c) illustrate how the equilibrium adjusts to a negative shock to households’ preference for repos. As households want to hold less repo, shadow banks’ excess demand for repos is larger—resulting in the blue line moving rightward in the right panel. As a consequence, banks want to lend more in repo markets to benefit from higher rates. Doing so increases banks’ liquidity risk and, hence, precautionary motives for holding reserves. The demand for reserves increases in the left panel as well. Panel (c) shows that, to keep the fed funds rate on target, the central bank needs to increase the supply of reserves to banks. Thus, banks can lend more in the repo market without increasing their liquidity risk.

In this benchmark economy, banks always act as arbitrageurs between money market segments. In doing so, they play the essential role of transporting the liquidity from the market for reserves into the broader repo market. A key assumption that makes these arbitrage always possible—however large the repo demand—is that banks can go to a negative reserves balance within the day by drawing on their central bank overdrafts.
4.3 Intraday Liquidity Shortage

In this section, we characterize an economy that is subject to the LST constraint. We first show that when the constraint is binding, the repo rate disconnects from the fed funds rate and jumps above the opportunity cost of reserves. We find that this economy qualitatively behaves similarly to an economy in which banks cannot run intraday overdrafts at the central bank. We then show that the probability that the system hits the constraint and generates a repo spike depends both on monetary policy and fiscal policy, and has consequences for T-bond yields.

Money Markets This first part explores the implications of the LST constraint for money markets. LST alters money market rate dynamics by preventing banks from intermediating liquidity from reserves markets to repo markets, resulting in spikes in repo rates. When LST binds for traditional banks, quantities of repo supplied are constrained and traditional banks cannot take advantage of high repo rates. In this situation, the market is rationed, shadow banks are short of funds, and the repo rate jumps above the discount window rate up to the overnight credit cost $\lambda$.

Proposition 2. When LST is binding, that is, $\phi^p = 1$,

- the repo rate is above the discount window rate: $r^p > r^m + r^w$;
- there are no transactions in the fed funds market: $f_* = 0$.

Proposition 2 describes the effect of a binding LST constraint on money market rates. When LST binds for traditional banks, quantities of repo supplied are constrained and traditional banks cannot take advantage of high repo rates. Banks would like to lend more in repo but cannot because they don’t have enough reserves to settle the corresponding transactions. Because the liquidity risk of shadow banks is not bounded by the discount window rate $r^m + r^w$—to which they don’t have access—but rather by their overnight credit cost $\lambda$, the repo rate can jump above the discount window rate. Finally, the repo rate disconnects from the fed funds rate. As no bank is able to lend reserves, no transaction occurs in the fed funds market and the fed funds rate is never observed above the discount window rate.

Proposition 3. An economy in which LST is sometimes binding, that is, $\phi^p > 0$,

- the repo rate is sometimes above the discount window rate: $\mathbb{P}[r^p > r^m + r^w] > 0$;
is qualitatively equivalent to an economy without intraday overdrafts at the central bank—that is, in which \( LST \) is replaced by
\[
 m - \max\{0, p_*\} - \max\{0, f_*\} \geq 0.
\]

Proposition 3 states that when \( LST \) is sometimes binding (depending on shock realizations and agents’ portfolio choices), the economy is characterized by jumps in the repo rate. When the \( LST \) constraint binds, the repo rate spikes up to \( \lambda \). These dynamics are qualitatively identical to an alternative setting in which banks do not have access to intraday overdrafts at the central bank. In such an economy, banks would stop lending after having exhausted all reserves in their balance sheets. Both economy are characterized by banks facing a hard constraint on the amount they can lend in repo to shadow banks.

![Figure 12 about here](image)

Figure 12 illustrates this result. The upper panels represent an economy without the \( LST \) constraint. The left panel (a) illustrates a possible equilibrium for the repo market when the supply of reserves available is larger than the quantity of repo that is used to lend in the repo market. In this example, the repo demand meets repo supply to determine a repo rate inside the space between the interest on reserves and the discount window rate. The right panel (b) displays an example of equilibrium with a larger repo demand. In this depiction, the quantity of reserves available is not large enough for banks to be able meet the repo demand. Hence, banks end up running a negative intraday reserve balances—an overdraft—at the central bank. These temporary intraday loans of reserves from the central bank allow banks to fully meet the repo demand. As banks increase their lending in repo markets, the probability of having to borrow at the discount window at the end of the day rise exactly up to the point where fed funds and repo rates are equalized.

Lower panel (c) display an example economy in which banks are subject to the \( LST \) constraint and repo demand is low. In this case, as in panel (a), there are enough reserves available relative to demand for the market to clear with repo rates inside the corridor. One nonetheless notices that the shape of the repo supply schedule has changed to feature a kink when the \( LST \) constraint is binding Panel (d) plots an economy with the \( LST \) constraint and a large repo demand. Despite a larger total amount of reserves than in the upper panels (a) and (b), there are not enough reserves available to meet the repo demand because a large portion is used and \( locked \) in to meet \( LST \). In contrast to panel (c), banks cannot extend their repo lending beyond the reserves they have available through an overdraft because doing so would require them to first fully deplete their reserve account, which is not possible under \( LST \). In that sense, \( LST \) is responsible for removing the elasticity provided by intraday
overdrafts at the Fed. LST therefore creates an inelastic kink in the repo supply at the locus where all available reserves have already been used. When the repo demand exceeds the quantity of available reserves, scarcity arises in the repo market and repo rates spike up to shadow banks’ overnight credit charge $\lambda$.

**Fiscal Policy**  This second section explores the implication of fiscal policy on money markets in an economy with LST. Larger T-bond stock and issuance is found to increase the probability of seeing a surge in repo rates through distinct and cumulative channels. To simplify the exposition, we consider the two polar cases where the cash raised by the Treasury is (i) entirely kept within its account at the Fed (i.e., $\Delta B = \Delta G$) and (ii) entirely distributed to households, resulting in a corresponding increase their future tax liabilities (i.e., $\Delta B = \Delta T$). Panel (b) and (c) in Figure 13 provides a sectoral balance sheet illustration of these two scenarios relative to panel a baseline represented in panel (a).

**Proposition 4.** In an economy in which LST is sometimes binding ($\phi^p > 0$), the short-term impact of an increase in the quantity of T-bonds $B$ that is entirely absorbed by an increase in Treasury account ($\partial T/\partial B = 0$) results in an increase of the probability of a repo spike:

$$\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T)}{\partial B} = \frac{1}{(1 - \alpha) d \partial B} \frac{\partial \tilde{o}}{\partial B} - \frac{1}{(1 - \alpha) d \partial B} \frac{\partial \tilde{m}}{\partial B} > 0.$$ 

This increase in the repo spike probability operates through two distinctive channels:

- (i) a larger quantity of T-bonds outstanding increases the demand for shadow bank repo financing (first term is positive);
- (ii) a larger treasury account decreases the supply of reserves available to banks (second term is negative).

**Proposition 5.** In an economy in which LST is sometimes binding ($\phi^p > 0$), the short-term impact of an increase in the quantity of T-bonds $B$ that keeps the size of the treasury account $G$ constant ($\partial T/\partial B = 1$) results in an increase of the probability of a repo spike:

$$\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T(B))}{\partial B} = \frac{1}{(1 - \alpha) d \partial B} \frac{\partial \tilde{o}}{\partial B} - \frac{o - m}{(1 - \alpha) d^2 \partial T} \frac{\partial \tilde{d}}{\partial T} > 0.$$
This increase in the repo spike probability operates through two distinctive channels:

- (i) a larger quantity of T-bonds outstanding increases the demand for shadow bank repo financing (first term is positive);
- (ii) a larger spot issuance of T-bonds increases the settlement needs for reserves (second term is negative).

Combining Propositions 4 and 5, there are three channels through which the issuance of new T-bonds influences the probability of a repo spike. First, an increase in the supply of T-bonds generates a larger need for excess repo financing from shadow banks. This feature arises in the model as a consequence of all T-bonds having to be held by shadow banks, whereas households want to invest part of their tax liabilities in deposits.\(^{14}\) With a large excess demand for repo, the economy moves closer to hitting the LST constraint, and the probability of a spike increases. Second, by increasing the size of the treasury account on a given date, T-bond issuance removes from the volume of reserves available to banks to settle their repo lending. Third, by increasing predicted reserve outflows for traditional banks, new T-bond issuance also increases the quantity of reserves needed under LST. The two latter channels shift the inelastic kink in the repo supply to the left and increase the chance of a repo spike happening.

**Monetary Policy** We then explore how monetary policy affects money market conditions. We find that a contractionary monetary policy reducing the size of the central bank balance sheet increases the risk of a repo spike in two ways. Such a scenario is represented in panel (d) of Figure 13 relative to the baseline in panel (a).

Proposition 6. *In an economy in which LST is sometimes binding \((\phi^p > 0)\), the short-term impact of an increase in the central bank portfolio results in a decrease of the probability of a repo spike:*

\[
\frac{\partial \tilde{\phi}^p(n, \bar{n}^h, M, B, T(B))}{\partial M} = \frac{1}{(1 - \alpha) d \partial M} \frac{\partial \tilde{o}}{\partial \tilde{\phi}^{h}} - \frac{1}{(1 - \alpha) d \tilde{M}} < 0. 
\]

*This decrease in the repo spike probability operates through two distinctive channels:*

\(^{14}\)Note that this result would hold with a less restrictive preferred habitat assumption as long as households’ allocation between deposits and repos does not exactly correspond to how T-bonds are distributed between traditional and shadow banks.
• (i) a lower quantity of T-bonds have to be absorbed by shadow banks (first term is negative);

• (ii) a larger quantity of reserves allows traditional banks to lend more in the money markets (second term is positive).

By reducing the size of its balance sheet, the central bank effectively acts on both the demand and the supply of repo. On the asset side, the T-bonds that the central bank sells have to be absorbed into the shadow banking sector, which increases the demand for repo financing. On the liability side, the central bank removes from the pool of reserves available to banks. The two adjustments have a positive cumulative impact on the probability of a repo spike.

Treasury Yields Last, we characterize in Proposition 7 how T-bond spreads are affected by an increase in the probability of a repo spike.

**Proposition 7.** In an economy in which the LST is sometimes binding \((\phi^p > 0)\), the short-term impact of an increase in the probability of a repo spike results in an increase in T-bond spreads:

\[
\frac{\partial (\tilde{r}^b(n, \tilde{n}^h, M, B, T) - \tilde{r}^m(n, \tilde{n}^h, M, B, T))}{\partial \phi^p(n, \tilde{n}^h, M, B, T)} > 0.
\]

The intuition behind this result is that when the repo market is more likely to be disrupted the afternoon, shadow banks bear increased repo liquidity risk and require a larger return to hold T-bonds to reflect this risk.

**Long-Term Dynamics** We then analyze the dynamics of repo rates when allowing bankers to adjust their allocation of equity between traditional and shadow banks flexibly and interpret this exercise as the long-term dynamics with the intuition that equity buy-backs and issuances are complex and costly operations. Proposition 8 reveals that when bankers’ equity is fully flexible, the previous short-term result is entirely reversed.

**Proposition 8.** The long-term impact of an increase in the quantity of T-bonds \(B\) results in a decrease of the probability of a repo spike:

\[
\frac{\partial \phi^p(\tilde{n}^h, M, B, T)}{\partial B} < 0 \quad \text{if } \phi^m(\tilde{n}^h, M, B, T) > 0 \text{ and } 0 \text{ otherwise.}
\]
As shown above, a short-term increase in T-bonds increases the quantity of repo demanded by shadow banks while decreasing the amount of reserves available for traditional banks. Absent discount window risk $\phi^m$, bankers will rebalance their equity from traditional banks to shadow banks to reflect the greater financing needs from the second institution, and restore the previous equilibrium. However, in the general case, the decrease in the amount of reserves available to traditional banks also increases traditional banks’ discount window risk. Consequently, it becomes optimal for bankers to decrease further the leverage of the shadow banking sector, which alleviates the pressure on repo markets beyond the initial equilibrium. This mechanism illustrates the non-trivial interaction between the multiple functions of reserves and regulations and indicates potential adaptation capabilities from the economy to the existence of spikes in a longer-term horizon.

5 Empirical Analysis

In this section, we present empirical evidence confronting the main predictions of the model to the data. These predictions are that money market spreads tend to be larger on days with high settlement volumes and when the supply of reserves available to banks is low relative to Treasuries. This observation complements the results of Copeland, Duffie, and Yang (2021), documenting a strong relationship between late payment timing—an indicator of intraday liquidity scarcity—and money market spreads.

5.1 Data Description

As our primary measure of money market spreads, we use the Treasury General Collateral Rate over the interest paid on excess reserves ($TGCR - IOER$). The former is collected directly from the Depository Trust & Clearing Corporation website and tracks the average daily interest rate on the most-traded GCF repo contract for U.S. Treasuries (DTCC GCF Repo Index), while the latter is publicly available from the online FRED database. As additional dependent variables, we also examine the MBS general collateral rate ($MBSGCF$) spread (also from DTCC), the Secured Overnight Financing Rate ($SOFR$, from the Federal Reserve Bank of New York), and the JPY-USD FX swap implied dollar funding rate from Correa et al. (2020), all in excess of the IOER.

To test our model’s predictions, we collect multiple variables as implied drivers of the repo spreads and money market spikes by the model. First, we consider the daily change
in central bank reserves available to banks (Reserves). We use the Total Reserve Balances Maintained at the Fed, collected from FRED, and released from the Board of Governors’ H.3 Tables. As the variable is available only on a Wednesday-level basis, we construct the daily series by subtracting the cumulative change in the Treasury General Account (from the Daily Treasury Statement from the Bureau of Fiscal Services) from each Wednesday observation to fill the missing observations. To measure days with high settlement volumes and intraday timing stress we use a set of calendar and event dummies, as used in the literature.\footnote{see Judson and Klee (2009), Carpenter and Demiralp (2008), Ashcraft et al. (2011), Furfine (2001), McAndrews and Kroeger (2016) and Bech et al. (2012). These earlier studies used calendar fixed effects to proxy for high expected daily payment activity and volatility in the federal funds rate to identify banks’ demand for reserves and study the liquidity channel on the federal funds rate and repo markets.} We include quarter-end fixed effects to measure days of high expected payment flow and to capture the post-regulation window-dressing spikes as documented in Correa et al. (2020).

In addition, we include dummies for days corresponding to corporate tax payment deadlines (TaxDeadlineDays\(_t\)). Major tax deadlines occur regularly in the middle of the month and are therefore exogenously expected to drive repo market activity higher. If any of these days fall outside of intra-week days or on business holidays (according to the SIFMA calendar), then the first business day after is used. For example, 09/15/2019 was a Sunday. Therefore the tax deadline dummy is equal to 1 on 09/16/2019. We also include a separate dummy for the day following tax deadlines (TaxDeadlineDays\(_{t+1}\)) to capture potentially lagged reactions\footnote{Such a case can arise due to bank transfers typically settling with some time gap. For example, during the September 2019 event and the Covid crisis, our spread measures record the largest spikes on the day after the trading day closest to the corporate tax deadline, respectively 09/17/2019 and 03/17/2020.}.

Furthermore, we measure daily US Treasury issuance by separately including new T-Bill issues (T-Bills Issuance), Treasury coupons (Coupons Issuance) (comprising notes and bonds), and total Treasury redemptions (Tot. Treas. Redemptions). The series is collected from the Daily Treasury Statement website. Finally, we consider the role of the relative supply of public Treasuries and include the total marketable Treasury debt outstanding held by the public (Tot. Treasury Outstanding), collected from the Monthly Treasury Statement (MTS). The analysis and final specifications are estimated over the post-regulation period from 2015-12-15 until 2020-09-09. The end period is chosen as the Board of Governors
discontinued the statistical release of our variable for the Reserve Balances since that date. Table 1 reports summary statistics for our variables over the specified period.

5.2 Discussion

Table 2 reports the estimated results from OLS regressions of the money market spreads on the independent variables. All specifications and estimates are correcting for HAC using a Newey-West covariance matrix estimator with a three-month lag. The first five columns use the TGCF-IOR spread as the dependent variable.

Across all columns, we observe that an increase in reserve balances at the Fed is associated with a lower spread. The effect is statistically significant at the 1% level, and the coefficient is stable across all specifications. In the second and third columns, we add, respectively, the quarter-end calendar effect and the set of variables related to higher settlement and payment activity. Both quarter-end and the corporate tax deadline dummies are associated with larger repo spreads, although the coefficients are not statistically significant due to higher standard errors. As mentioned, corporations and money market funds pull out cash from banks in the run-up to and at corporate tax deadlines to make the payments to the Treasury, decreasing the supply of reserves in the system. This channel implies a correlation between tax deadlines and a change in Reserves that we already include in our regression, as the increase in the TGA balance sterilizes reserves from the pool available to banks, which could account for a lack of statistical significance.

In Column (3), we also include the three Treasury issuance variables. We find a positive and statistically significant effect from both Treasury Bills and Coupons issuance and an opposite (but not significant) effect from Total Treasury redemptions. While our regressions do not allow us to establish a causal effect, the sign of coefficients are in line with the economic mechanism implied by the model. From the supply side, higher Treasury issuance increases repo rates through the conversion of banks’ reserves into TGA balances at the Fed as well as banks’ hoarding of liquidity (lower willingness to lend in the repo market) in order to meet larger settlement volumes and comply with the intra-day liquidity regulation. From the demand side, primary dealers require larger repo financing as they need to purchase the new Treasury securities at auctions. Notably, we observe that the upward pressure estimated from coupons issuance is significantly larger than from T-Bills issuance and remains statistically significant across all specifications. This result is consistent with
the analysis of Vandeweyer (2019), who argues that T-bills are direct substitute assets for repo as these are held directly by money funds. In contrast, chunky coupon supply days tend to cause episodic spikes in overnight general collateral repo volumes. Nevertheless, given the higher volatility of T-Bills issuance (partly driven by our inclusion of irregular CMBs in our measure), the overall economic magnitude of the effect from both types of security issuance is comparable.\footnote{For example, given the summary statistics presented in Table 1, in our most complete specification (Column (4)), our estimates suggest that a one-standard-deviation increase in coupons (bills) issuance is associated with 3.08 (3.43) basis points higher repo spread, or approximately 1.8% higher.}

Finally, we test our hypothesis that the relevant variable is the relative availability of reserves to Treasuries available to the public. As such, we explicitly measure this balance as the ratio of our Reserves and Tot. Treasury Outstanding variables and we create a dummy Low Balance which is equal to one when the relative supply of reserves to Treasury outstanding is in the bottom 5th percentile.\footnote{To increase the precision of this threshold, we estimate the percentile from our full sample of data observations spanning from January 2010 until September 2020. The results are robust when using a larger percentile threshold or when estimating the distribution statistics from the reduced post-regulatory sample.} Hence, in Column (4), we add to our set of explanatory variables the Low Balance dummy, both separately and interacted with the Reserves variable, and comment two interesting findings in line with our predictions. First, we find that on days in which the amount of reserves available to banks relative to the supply of Treasuries available to the public is in the bottom 5th percentile of the distribution, the TGCF-IOER spread is higher by approximately 370 basis points, and the effect is statistically significant at the 1% level, with the other coefficients broadly unchanged. The goodness of fit of the model, as captured by the adjusted $R^2$ from this specification, increases by 7%, and our estimate suggests it can explain the magnitude of the large spikes observed during the money market dislocation events. We complement the regression results with graphical results in Figure 7, which illustrates a scatter plot of the TGCF spread against the ratio of Reserves to total Treasury outstanding. As we explicitly highlight in the figure, the three major repo spike events, in September 2019, March 2020, and December 2018, all occurred at times when the amount of reserves available to banks was the lowest relative to the Total supply of debt.

To further motivate our model’s prediction that intraday liquidity regulation contributed to the dislocations, we perform a placebo analysis by re-estimating this last specification but over the pre-regulatory period, using the sample spanning from January 2010 until December 2015, and present the regression results in Column (5). The coefficients on the Treasury issuance and tax deadline dummies remain positive (and statistically significant for Coupon
Issuance and \textit{Tax Deadline Day}_{t+1}), which confirms that market participants already expect these factors to lead to higher repo spread. On the other hand, the coefficient on the Low Balance dummy now drops significantly from 372.1 to 66.81. This element supports the hypothesis that, over this period, the relative supply of reserves to Treasuries could not account for the observed magnitude of the recent money market dislocations and that rather the consolidation of the post-crisis regulations in the second sample (Column 4) can be the interacting trigger that surprised market participants with the realized size of the repo spikes. We point out that our analysis does not aim at claiming a causal relationship but provides supporting evidence for our model’s predictions. In line with Copeland, Duffie, and Yang (2021), we conclude that “reserves were not so ample after all”: banks were likely constrained through regulations, despite the still large nominal amount of reserve balances compared to the pre-crisis period, by the larger amount of Treasuries in the system in need for repo financing.

Finally, from specification (4), we also find a negative and statistically significant coefficient on the interaction term between the Low Balance dummy and Total Reserves. We interpret this result as corroborating evidence for Proposition 5. When the amount of reserves relative to Treasuries is low in the current post-regulatory regime, as measured by the interaction term, an increase in reserves available to banks is associated with a lower rise in repo spread, ceteris paribus.

6 Conclusion

In this paper, we propose a framework to examine how \textit{intraday} liquidity interacts with \textit{overnight} liquidity to generate spikes in money market rates. Our analysis finds that imposing regulation requiring banks to pre-fund intraday outflows with reserves results in a hard constraint on banks’ ability to supply repo to shadow banks. In other words, a shortfall in intraday liquidity affecting banks creates a shortage of overnight liquidity for shadow banks. Because the latter institutions do not have access to the lender-of-last-resort function of central banks, a slight change in repo demand may generate large spikes in repo rates. Our analysis further yields that, in an economy in which Treasury debt is largely held by shadow banks leveraged through repo, an increase in the balance between Treasury outstanding and reserves available makes the probability of a repo spike more likely and Treasury spreads to surge to compensate shadow banks for increased liquidity risk.

These findings are instructive in the context of two important policy debates on central
bank liquidity provision. First, since the generalization of RTGS systems that started in the 80s, most central banks adopted generous daylight overdraft policies. As RTGS systems require banks to pre-fund their gross outflows of funds, daylight overdrafts were indeed deemed an essential tool to accommodate the ensuing expansion in demand for intraday liquidity. The 2008 crisis altered this situation. On the one hand, as a side product of QE, the quantity of reserves increased enough to accommodate most needs for intraday liquidity so that bank overdrafts fell to near-zero levels. On the other hand, the crisis renewed regulatory concerns on the financial stability consequences of the accumulation of large credit-financed outflows by banks. These concerns eventually turned into new regulations requiring banks to pre-fund daylight outflows with reserves with the (possibly unintended) consequence of making the use of Fed overdrafts for arbitrage purposes impossible and turning the payment system from a “credit system” to a “token system” (Pozsar, 2019). As the Fed reversed the expansion of its balance sheet and removed the quantity of reserves-token available, the banking system started to hit limits on its ability to settle repo transactions on days characterized by large intraday flows. The middle of September and March when corporations are required to pay taxes while bank reserves are made scarcer by inflows to the TGA are examples of such vulnerable periods.

This papers’ analysis suggests two directions for policymakers aiming at minimizing these disruptions: stick to a token system but ensure that the quantity of reserves available is sufficient or revert back to a credit system. The first is, de facto, the situation in 2021 following the new rounds of QE brought about by the Covid pandemic. The second requires institutional changes to relax intraday regulation, such as including discount window liquidity in intraday liquidity stress-tests computations (Quarles, 2020) or introducing a daylight borrowing facility. These two arrangements would likely remove obstacles to bank intraday liquidity provision but not necessarily fully solve the challenge of providing overnight liquidity to shadow banks. As observed in 2008 and on quarter-ends thereafter, other frictions may still prevent banks to provide liquidity to shadow banks. Thereby, our analysis also points to a permanent Fed repo facility with a wide range of counter-parties as an important tool for money market rate stabilization.
References


Copeland, Adam M., Duffie, James Darrell, and Yang, Yilin (David). Reserves were not so ample after all. Staff Report No. 974, FRB of New York, July 2021.


Appendices

A Omitted Derivations and Proofs

State Variables and Recursive Formulation Below, we characterize the economy as a recursive Markov equilibrium. Note our notation for the Markov economy: $x$, instead of $x_t$, denotes morning variables, while $x_*$, instead of $x_t^*$, denotes afternoon variables. We define the set of state variables as $X = (\tilde{n}^b, M, B, T)$ and $X_* = (X, \alpha_*)$.

Bankers We can write the problem of bankers in recursive form as:

$$V^b(n^b, X) = \max_{c^b \leq n, n^b \geq 0, n^b \geq 0} \mathbb{E} \left[ c^b + \beta V^b(n^{b'}, X') \right]$$

subject to the morning balance sheet constraint:

$$n + n^b = n^b - c^b,$$

where returns on their portfolio are such that:

$$n^b = n' + n'.$$

Thus, the return on investing in traditional and shadow banks must be equal:

$$\frac{n'}{n} = \frac{n'}{n}.$$

Traditional Banks We can write the problem of traditional banks as:

$$\max_{k \geq 0, m \geq 0, o \geq 0, d \geq 0} \mathbb{E} \left[ \max_{p*, f*} \mathbb{E}_r \{ n' \} \right]$$

subject to the morning balance sheet constraint:

$$q^k + m + o = n + d,$$

where returns on their portfolio are such that:

$$n' = n + kr^k + m_*r^m + or^o + o_*\lambda + p_*r^p + f_*r^f - d_*r^d - (\chi^m d - m_*) r^w \mathbb{1} \{ m_* < \chi^m d \}.$$

The LST constraint can be written as:

$$\mathbb{P}(m - \max\{0, p_*\} - \max\{0, f_*\} \geq \chi^m d + \Delta^o d_*) = \zeta.$$
With further algebra, we get
\[ m - \max\{0, p_*\} - \max\{0, f_*\} \geq \Phi^d d, \]
where the size of the reserve buffer is defined as:
\[ \Phi^d \equiv \chi^m + \sigma (1 - \zeta) + \mu^o, \]
given the distribution of the outflow deposit shock \( \varepsilon^o \). This constraint can be split into two constraints depending on whether \( p_* \) is positive:
\[ \lambda^m : \quad m \geq \Phi^d d, \]
\[ \lambda^p : \quad m - p_* \geq \Phi^d d. \]
The LST constraint is satisfied if and only if both of these constraints are satisfied. The lambdas denote the corresponding Lagrangian multipliers. We set \( f_* = 0 \) in the \( \lambda^p \) constraint. This is without loss of generality. Indeed, as we show in the proof of Proposition 2, if \( \lambda^p > 0 \) then \( f_* = 0 \).

The first-order condition with respect to consumption \( c^b \) can be written as:
\[ 1 = \beta \mathbb{E}[V^b(n'; X')(1 + r^k)]. \]
Together with the envelope condition:
\[ V^b(n; X) = \beta \mathbb{E}[V^b(n'; X')(1 + r^k)], \]
we get:
\[ 1 = \beta (1 + r^k). \]
(9)
The first-order condition with respect to reserves \( m \) is given by:
\[ r^m + \phi^m r^w + \lambda^\ell + \mathbb{E}[\lambda^p] = r^k, \]
(10)
where
\[ \phi^m = \mathbb{P}[m + \Delta d_* - p_* - f_* + o < \chi^m d]. \]
The first-order condition with respect to deposits \( d \) is given by:
\[ r^d - r^k + \mu^d (r^d - r^m) = (\mu^m - \phi^m \chi^m) r^w - \lambda^p \Phi^p - \mathbb{E}[\lambda^p] \Phi^p, \]
(11)
where
\[ \mu^m = \mathbb{E} [\Delta d_* / d \I \{m + \Delta d_* - p_* - f_* + o < \chi^m d}] . \]
The first-order condition with respect to repo transactions $p_*$ is given by:

$$r^p_+ = r^m + \phi^m r^w + \lambda^p_*, \quad (12)$$

where

$$\phi^m_* = \mathbb{P}_*[m + \Delta d_* - p_* - f_* + o < \chi^m d].$$

The first-order condition with respect to fed fund transactions $f_*$ is given by:

$$r^f_* = r^p_*. \quad (13)$$

Last, the first-order condition with respect to daily overdrafts $o$ is given by:

$$r^o + \phi^o \lambda + (1 - \phi^o)(r^m + \phi^m r^w) = r^k. \quad (14)$$

**Shadow Banks** We can write the problem of shadow banks as:

$$\max_{b \geq 0, o \geq 0} \mathbb{E}\left[ \max_{s} \left\{ n' \right\} \right] \quad (15)$$

subject to the morning balance sheet constraint:

$$b = n + o, \quad (16)$$

where returns on their portfolio are such that:

$$n' = n + b r^b - o r^o - \Delta \mathbb{2}_* r^p_* - \lambda \max\{0, o - \Delta \mathbb{2}_*\}.$$

Daily overdrafts are chosen in order to satisfy the balance sheet constraint. If the repo rate is below the overnight credit charge $\lambda$, shadow bankers choose to fund the entirety of their daily overdrafts with repo. When the repo rate reaches $\lambda$, they are indifferent. That is,

$$\Delta \mathbb{2}_* = \begin{cases} \frac{o}{[0, o]} & \text{if } r^p_* < \lambda, \\ \frac{\lambda o}{[0, o]} & \text{if } r^p_* = \lambda. \end{cases} \quad (17)$$

Implicitly, we omit the possibility for $p_* < 0$ and $r^p_* > \lambda$ as it will never be an equilibrium outcome. Thus, we can rewrite the law of motion of wealth as:

$$n' = (n - \xi)(1 + r^o + r^p_*) + b(r^b - r^o - r^p_*).$$

The first-order condition with respect to consumption $r^b$ can be written as:

$$1 = \beta \mathbb{E}[V^b(n'; X')(1 + r^o + r^p_*)]$$
Together with the envelope condition:
\[ V^b(n; X) = \beta \mathbb{E}[V^b(n'; X')(1 + r^o + r^p)] , \]
we get:
\[ 1 = \beta \mathbb{E}[1 + r^o + r^p] . \] (18)
Lastly, the first-order condition with respect to bonds \( b \) is given by:
\[ r^b = \mathbb{E}[r^o + r^p] . \] (19)

**Households** We can write the problem of households in recursive form as:
\[ V^h(n^h; X) = \max_{c^h \leq n^h, d^h \geq 0} \mathbb{E} \left\{ \max_{p^h \geq 0} \left\{ U^h(c^h, d^h; p^h, \alpha^h) + \beta V^h(n^{h'}; X') \right\} \right\} \] (20)
subject to the morning balance sheet constraint:
\[ d^h + t^h = n^h + t^h - c^h \] (21)
where returns on their portfolio are such that:
\[ n^{h'} = n^h - c^h + d^h r^d + p^h r^p - t^h \tau . \]
The Leontief aggregator implies that the relative demand of deposits and repo is such that:
\[ (1 - \alpha^h)d^h = \alpha^h p^h . \]
Using the law of motion for deposits and repos, the utility function simplifies to:
\[ U^h = c^h + \varphi \left( \frac{(d^h)^{1-\gamma}}{1-\gamma} \right) . \]
Deposits are chosen in order to satisfy the balance sheet constraint. The first-order condition for \( c^h \) is given by:
\[ 1 - \varphi(d^h)^{-\gamma} = \beta \mathbb{E}[V^h(n^{h'}; X')(1 + r^d)] . \]
Together with the envelope condition:
\[ V^h(n^h) = \varphi(d^h)^{-\gamma} + \beta \mathbb{E}[V^h(n^{h'}; X')(1 + r^d)] , \]
we get:
\[ 1 - \varphi(d^h)^{-\gamma} = \beta(1 + r^d) . \] (22)
For the tax rebates to be fairly priced and consistent with aggregate quantities, the following pricing equation needs to be satisfied:

\[(1/\beta - 1)L = (1/\beta - 1)T - \tau T.\]

**Definition 2 (Markov Equilibrium).** A Markov equilibrium in \(X = (n^h, M, B, T)\) and \(X_* = (X, \alpha^h)\) is a set of functions \(g = g(X)\) and \(g_* = g(X_*)\) for (i) prices \(\{q, r^k, r^m, r^o, r^b, r^d, r^p, r^f\}\); (ii) individual controls for bankers \(\{c^b, n, \mu\}\); (iii) individual controls for traditional banks \(\{k, m, o, d, p, f, \sigma\}\); (iv) shadow banks \(\{h, \beta, \beta_*\}\); (v) households \(\{c^h, d^h, p^h, f^h\}\); and (vi) value of liquidity services \(\{l^h, L^h\}\) such that:

1. Agents solve their respective problems defined in equations (7), (8), (15), and (20).
2. The balance sheet constraint of the central bank and the treasury are satisfied.
3. Morning markets clear:
   - (a) capital: \(\int_0^1 k(i) di = 1,\)
   - (b) deposits: \(\int_0^1 d^h(h) dh = \int_0^1 d(i) di,\)
   - (c) T-bonds: \(\int_0^1 b(j) dj + B = B,\)
   - (d) reserves: \(\int_0^1 m(i) di + G = M,\)
   - (e) overdrafts: \(\int_0^1 o(i) di = \int_0^1 o(j) dj,\)
   - (f) output: \(\int_0^1 c^h(h) dh + \int_0^1 c^b(b) db = a,\)

4. Afternoon markets clear:
   - (g) repo: \(\int_0^1 p_*(i) di + \int_0^1 p^*_h(h) dh = \int_0^1 \phi_*(j) dj,\)
   - (h) fed funds: \(\int_0^1 f_*(i) di = 0,\)

5. The laws of motion for the state variables \(n\) and \(\mu\) are consistent with equilibrium functions and demographics,

6. The aggregation of liquidity services and tax liabilities are consistent:
   - \(\int_0^1 l^h(h) dh = L,\)
   - \(\int_0^1 t^h(h) dh = T.\)

**Equilibrium Characterization**

**Lemma 1.** We get that \(q = \frac{a}{1/\beta - 1}\) and \(\phi^p < 1\) for all equilibria.
Proof. First, since $1 = \beta(1+r^k)$ and $1+r^k = \mathbb{E}\left[\frac{a+q}{q}\right]$ in all periods, we have that $q = \frac{a}{1/\beta-1}$.

If $\phi^p = 1$, the first-order condition with respect to reserves $m$ in equation (10) together with the first-order condition for repo transactions $p_*$ in equation (12) implies that $r^k = \lambda$, which is a contradiction since $\lambda > 1/\beta - 1$.

As mentioned in the main text, we restrict the analysis to the set of equilibria such that the quantity of reserves available jointly with LST are never constraining the issuance of deposits by traditional banks in the morning with condition C5. That is, we study only the state space where $\lambda^m = 0$. When $\lambda^m > 0$, the equilibrium is a limit case in which LST is so restrictive that the central bank loses the control of the interest on reserves as he interest on reserves is not pinned down by the quantity of reserves but by the tightness of LST: $r^m + \phi^m r^w < r^k$.

Thus, from the shadow bankers’ first-order conditions for consumption and bonds in equations (18) and (19), we have:

$$r^b = \mathbb{E}[r^o + r^p_*] = 1/\beta - 1.$$  

Using the first-order condition of traditional bankers for reserves $m$ in equation (10), we get:

$$r^m + \phi^m r^w + \mathbb{E}[\lambda^p_*] = r^k.$$  

Using the first-order condition of traditional bankers with respect to repo transactions $p_*$ in equation (12) yields:

$$r^p_* = r^m + \phi^m r^w + \lambda^p_*.$$  

When $\lambda^p_* > 0$, LST is binding, $r^p_*=\lambda$, and $\lambda^p_* = \lambda - r^m - \phi^m r^w$. When $\lambda^p_* = 0$, $r^p_* = r^m + \phi^m r^w$. Thus,

$$\mathbb{E}[r^p_*] = \phi^p \lambda + (1-\phi^p)(r^m + \phi^m r^w) = r^k,$$

and

$$(1-\phi^p)(r^m + \phi^m r^w) + \phi^p \lambda = r^k.$$  

Thus, if $\phi^p < 1$,

$$r^m = \frac{1/\beta - 1 - \phi^p \lambda}{1-\phi^p} - \phi^m r^w.$$  

From the first-order condition of traditional bankers with respect to daily overdrafts $o$ in
equation (14), we get:

\[ r^\circ = r^k - \phi^p \lambda - (1 - \phi^p)(r^m + \phi^m r^w) = 0. \]

Thus, at equilibrium, the following is indeed satisfied:

\[ r^h = r^k. \]

Combining the first-order condition of households and traditional bankers with respect to deposits \( d^h \) and \( d \) in equations (22) and (11) yields:

\[
1 - \frac{\varphi(d^h)^{-\gamma}}{\beta} - 1 = \frac{r^k + \mu^d r^m + (\mu^m - \phi^m \chi^m) r^w - \phi^p (\lambda - r^m - \phi^m r^w) \Phi^d}{1 + \mu^d}.
\] (23)

This equation pins down the equilibrium aggregate quantity of deposits. In the following paragraph, we can characterize \( \phi^m, \phi^*_m, \mu^m, \) and \( \phi^p \).

Firstly, note that from the accounting identities for deposits flows and overdrafts, we have that:

\[ \mu^d d = \mu^*_d d - \mu^*_o d = -p^h_* \quad \text{and} \quad p_* = \Delta o_* = p_* + p^h_. \]

This means that in equilibrium, the end-of-the-day stock of reserves is simply given by:

\[ m_* = m + \Delta d_* + \Delta o_* - p_* = m + d\varepsilon_*^i - d\varepsilon_*^o. \]

The probabilities of having to borrow at the discount window \( \phi^m \) and \( \phi^*_m \) are given by:

\[ \phi^m = \phi^*_m = \max \left\{ 0, \min \left\{ 1, \frac{\chi^m - m}{2\sigma} \right\} \right\}, \]

and the expected size of the reserve gap:

\[ \mu^m = \mathbb{E} [\Delta d_*/d \mathbbm{1} \{m + \Delta d_* - p_* + o < \chi^m d\}] = \phi^m \mu^d + \left( \max \left\{ -\sigma, \min \left\{ \sigma, \frac{\chi^m - m}{4\sigma} \right\} \right\} \right)^2 - \sigma^2. \]

Finally, the traditional bankers are constrained by \( \Phi^d d \) when:

\[ m - p_* \leq \Phi^d d. \]

That is, when the supply of repo by the households is too low:

\[ p^h_* \leq o + \Phi^d d - m. \]
Given the distributional assumption on $\alpha_*$, the probability of a spike is then given by:

$$
\phi^p = \mathbb{P} \left[ p^h \leq o + \Phi^d d - m \right] = \max \left\{ 0, \min \left\{ 1, \frac{o + \Phi^d d - m}{(1 - \alpha)d} \right\} \right\},
$$

and $\mu^o = 0.5 - \alpha/2$ and $\mu^d = -\mu^o$.

Using the morning balance sheet constraints of shadow bankers and households in equations (16) and (21), we can characterize $d$ and $o$ in terms of $(\tilde{n}^h, n)$:

$$
d = T + \tilde{n}^h,
$$

$$
o = B - (q - n - \tilde{n}^h).
$$

In the following paragraphs, we characterize the equilibrium in the state space $(\tilde{n}^h, M, B, T)$. To do so, we look at the value of $n$ that solves the equilibrium conditions. We define the implicit function $D(n, \tilde{n}^h, M, B, T) = 0$ to find the equilibrium $n$ given $(\tilde{n}^h, M, B, T)$.

Thus, from equation (23), we can define the object $D$:

$$
D(n, \tilde{n}^h, M, B, T) \equiv \frac{1 - \varphi^d}{\beta} - \frac{r^k + (\alpha/2 - 0.5)r^m + (\mu^m - \phi^m \chi^m)r^w - \phi^p(\lambda - r^m - \phi^m r^w)\Phi^d}{0.5 + \alpha/2} - 1.
$$

The necessary and sufficient conditions for $(n, \tilde{n}^h, M, B, T)$ to be an equilibrium are given by:

$$
n \geq 0, \quad \tilde{n}^h \geq 0, \quad q - n - \tilde{n}^h \geq 0, \quad q - \tilde{n}^h - n \leq B - M, \quad \tilde{n}^h < (M - B + T)/\Phi^d - T, \quad D(n, \tilde{n}^h, M, B, T) = 0.
$$

C4 arises from the restriction that $q$ cannot be negative. C5 was set previously such that $\lambda^m = 0$. The set of conditions C1 to C5 represent a set with a polygon as perimeter in the space $(n, \tilde{n}^h)$. Given $(M, B, T)$, define $\mathcal{C}$ the set of pairs $(\tilde{n}^h, n)$ that satisfy C1 to C5 and $\mathcal{E}$ the set of equilibrium pairs $(\tilde{n}^h, n)$ that satisfy C1 to C6. Lemma 2 provides the necessary and sufficient conditions for the existence of the set of equilibrium pairs $\mathcal{E}$.

**Lemma 2.** Given $\tilde{n}^h \in \mathcal{C}$, if and only if $D(\tilde{n}^h, \max\{0, q + M - B - \tilde{n}^h\}, M, B, T) > 0 \geq D(\tilde{n}^h, q - \tilde{n}^h, M, B, T)$, there exists one unique $n$ such that the pair $(\tilde{n}^h, n)$ is an equilibrium with $\phi^p > 0$.

Also, given $\tilde{n}^h \in \mathcal{C}$, if and only if $D(\tilde{n}^h, \max\{0, q + M - B - \tilde{n}^h\}, M, B, T) = 0$, then for all $n \leq \bar{n}(\tilde{n}^h) \in \mathcal{C}$, the pair $(\tilde{n}^h, n)$ is an equilibrium with $\phi^p = 0$.
Proof. First, we derive the following partial derivative:

\[
\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} = \frac{1 - \alpha - \phi^p 2\Phi^d}{1 + \alpha} \frac{\partial r^m}{\partial n} + \frac{(\lambda - r^m - \phi^m r^w) 2\Phi^d}{1 + \alpha} \frac{\partial \phi^p}{\partial n},
\]

where

\[
\frac{\partial r^m}{\partial n} = \frac{1}{\beta} \frac{1}{(1 - \phi^p)^2} \leq 0, \quad \text{and} \quad \frac{\partial \phi^p}{\partial n} = \frac{1}{(1 - \alpha)d} > 0 \quad \text{if} \quad \phi^p > 0 \quad \text{and} \quad 0 \quad \text{otherwise}.
\]

With further algebra we get:

\[
\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} = \frac{1}{\beta} \frac{1}{1 - \lambda - \phi^p} \frac{1 - \alpha - 2\Phi^d}{(1 - \phi^p)^2} \frac{1}{(1 - \alpha)^2d} \geq 0,
\]

since \(1/\beta - 1 - \lambda < 0\), \(1 - \alpha - 2\Phi^d < 0\), and \(\alpha < 1\) by definition. Define \(\pi(\tilde{n}^h)\) such that \(\phi^p(n, \tilde{n}^h, M, B, T) = 0\) for all \(n \leq \pi\) and \(\phi^p(\tilde{n}^h, n) > 0\) for all \(n > \pi\). That is,

\[
\pi(\tilde{n}^h) = 2(M - B) + (1 - \Phi^d)T + q - (1 + \Phi^d)\tilde{n}^h.
\]

Thus, if \(n \leq \pi\), then \(\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} = 0\) and if \(n > \pi\), then \(\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} > 0\). □

**Proof of Proposition 1** If \(\phi^p = 0\), from equations (10), (13), (12) we get:

\[
r^m + \phi^m r^w = r^f_* = r^p_* = 1/\beta - 1.
\]

Thus, \(r^m \leq r^f_* \leq r^m + r^w\) as \(\phi^m \in [0, 1]\). □

**Proof of Proposition 2** First, we show that if LST is binding then \(f_* = 0\) for all traditional banks. As \(\lambda^m = 0\), the constraint is given by:

\[
m - \max\{0, p_*\} - \max\{0, f_*\} \geq \Phi^d d.
\]

Assume that it is binding and that \(f_* > 0\) for a traditional bank. From the market clearing condition, it must mean that \(f_* < 0\) for another traditional bank. Because \(m\) and \(d\) are identical across banks for the first-order conditions (10) and (11) to hold, it means that the constraint is not binding for the bank with \(f_* < 0\) and the first-order condition for repo (10) cannot hold. Thus, it must be that if LST is binding, then \(f_* = 0\).

Second, if \(\phi^p_* = 1\), from equations (17) and for the repo market to clear with limited supply, \(r^f_*\) must be equal to \(\lambda\) such that shadow bankers are indifferent between paying the overnight credit charge and paying the repo rate. Furthermore,

\[
r^m + r^w = \frac{1/\beta - 1 - \phi^p \lambda}{1 - \phi^p} + (1 - \phi^m)r^w < 1/\beta - 1 + (1 - \phi^m)r^w < 1/\beta - 1 + r^w < \lambda.
\]

49
Thus, $r_p^r = \lambda > r^m + r^w$. □

**Proof of Proposition 3** If $\phi^p > 0$, the repo rate $r_p^r$ has a probability $1 - \phi^p$ to be equal to $r^m + r^w$, which is a continuously decreasing function of $\phi^p$, and a probability $\phi^p$ to be equal to $\lambda$. If intraday overdrafts at the central bank are not permitted, it means that the stock of reserves has to stay positive at all times during the day. That is,

$$m - \max\{0, p_*\} - \max\{0, f_*\} \geq 0.$$  

That constraint is equivalent to LST when $\lambda^m = 0$ and $\Phi^d = 0$. □

With this characterization of $E$, we can derive comparative statics with respect to monetary and fiscal policies. To do so, we first study short-term impact of a change in policy then long-term impact. We define the short-term impact as the partial derivative of an equilibrium Markov policy, such as the probability of a spike $\phi^p(n, M, B, T)$, with respect to the policy decision set $(M, B, T)$, but keeping $(n, \tilde{n})$ constant. Thus, we define the function $\tilde{\phi}^p(n, M, B, T) = \phi^p(n, M, B, T)$, and take the partial derivative of that function with respect to the policy variables.

The long-term impact is then the partial derivative without that constraint.

**Proof of Proposition 4** Given equation (24), we get that if $\phi^p > 0$:

$$\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T)}{\partial B} = \frac{1}{(1 - \alpha)d} \frac{\partial \tilde{o}(n, \tilde{n}^h, M, B, T)}{\partial B} - \frac{1}{(1 - \alpha)d} \frac{\partial \tilde{m}(n, \tilde{n}^h, M, B, T)}{\partial B} = \frac{2}{(1 - \alpha)d} > 0.$$  

The first term represents the change in the demand for shadow bank repo financing $o$ while the second term represents the change in the supply of reserves available to traditional banks $m$:

$$\tilde{o}(n, \tilde{n}^h, M, B, T) = B - M - n, \quad \tilde{m}(n, \tilde{n}^h, M, B, T) = M - B + T.$$  

□

**Proof of Proposition 5** Instead of keeping $T$ constant, assume that $T$ adjusts such that $G$ stays constant—that is:

$$\frac{\partial T(B)}{\partial B} = 1.$$  

50
Then, we get that if $\phi^p > 0$:

$$\frac{d\tilde{\varphi}(n, \bar{n}^h, M, B, T(B))}{dB} = \frac{1}{(1 - \alpha)d} \frac{\partial \tilde{o}(n, \bar{n}^h, M, B, T)}{\partial B} - \frac{o - m}{(1 - \alpha)d^2} \frac{\partial \tilde{d}(n, \bar{n}^h, M, B, T)}{\partial T}. $$

Note that this time the supply of reserves available to traditional banks $m$ is not affected. The second term represents the change in the quantity of deposits which directly impacts the settlement needs for reserves to satisfy the LST constraint: $d(n, \bar{n}^h, M, B, T(B)) = T + n^h$. Thus,

$$\frac{\partial \tilde{\varphi}(n, \bar{n}^h, M, B, T(B))}{\partial B} = \frac{d + m - o}{(1 - \alpha)d^2} > 0,$$

which is strictly positive since $o < m$.

Proof of Proposition 6  Given equation (24), we get that if $\phi^p > 0$:

$$\frac{\partial \tilde{\varphi}(n, \bar{n}^h, M, B, T)}{\partial M} = \frac{1}{(1 - \alpha)d} \frac{\partial \tilde{o}(n, \bar{n}^h, M, B, T)}{\partial M} - \frac{1}{(1 - \alpha)d} \frac{\partial \tilde{m}(n, \bar{n}^h, M, B, T)}{\partial M}.$$

The first term represents the change in the quantity of T-bonds that have to be absorbed by shadow banks:

$$\frac{\partial \tilde{o}(n, \bar{n}^h, M, B, T)}{\partial M} = -1.$$

The second term represents the change in the quantity of reserves available to traditional banks to lend in repo markets:

$$\frac{\partial \tilde{m}(n, \bar{n}^h, M, B, T)}{\partial M} = 1.$$

Summing over the two terms gives:

$$\frac{\partial \tilde{\varphi}(n, \bar{n}^h, M, B, T)}{\partial M} = -\frac{2}{(1 - \alpha)d} < 0,$$

which is strictly negative.

Proof of Proposition 7  We can solve for the equilibrium spread between the yield on treasuries and interest on reserves as:

$$r^b - r^m = 1/\beta - 1 - \frac{1/\beta - 1 - \phi^p \lambda}{1 - \phi^p} + \phi^m r^w.$$
We can solve for the short-term change in the Markov equilibrium policy \( \phi^p(\tilde{n}^h, M, B, T) \) and \( \phi^m(\tilde{n}^h, M, B, T) \) following a change in \( M, B, \) or \( T \):

\[
\begin{align*}
\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T)}{\partial M} &= -\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T)}{\partial B} = 2 \frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial T} = -\frac{2}{(1 - \alpha)d}, \\
\frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial M} &= -\frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial B} = \frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial T} = -\frac{1}{2\sigma d}.
\end{align*}
\]

Thus, we have that, following a change of either \( M, B, \) or \( T, \)

\[
\frac{\partial \phi^m(\tilde{n}^h, M, B, T)}{\partial X} = \frac{1}{2\sigma} - \frac{1}{\alpha}
\]

where \( X = M, B, \) or \( T. \) Finally, we can solve the following:

\[
\frac{\partial (r^b(\tilde{n}^h, M, B, T) - r^m(\tilde{n}^h, M, B, T))}{\partial M} = \frac{1}{\beta} - \frac{1}{\sigma} - \frac{1}{\phi^p} + \frac{r^w}{2\sigma}  > 0.
\]

If \( \phi^m = 0 \) or \( \phi^m = 1, \) the second term is equal to zero.

\[
\square
\]

**Proof of Proposition 8** Given equation (24), we get that if \( \phi^p > 0: \)

\[
\frac{\partial \phi^p(\tilde{n}^h, M, B, T)}{\partial B} = \frac{1}{(1 - \alpha)d} \frac{\partial o}{\partial B} - \frac{1}{(1 - \alpha)d} \frac{\partial m}{\partial B}.
\]

The first term represents the change in the demand for shadow bank repo financing:

\[
\frac{\partial o(\tilde{n}^h, M, B, T)}{\partial B} = \frac{\partial o(n, \tilde{n}^h, M, B, T)}{\partial B} + \frac{\partial o(n, \tilde{n}^h, M, B, T)}{\partial n} \frac{\partial n}{\partial B} = 1 + \frac{\partial n}{\partial B}.
\]

Furthermore, using the implicit function theorem, we get: \( \frac{\partial n}{\partial B} = -\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} \frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial \tilde{n}^h}. \)

We can also derive the following partial derivatives:

\[
\begin{align*}
\frac{\partial \tilde{\phi}^p(n, \tilde{n}^h, M, B, T)}{\partial B} &= \frac{2}{(1 - \alpha)d}, \\
\frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial B} &= \frac{\alpha/2 - 0.5}{d} + \frac{1}{d \sigma} \frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial B}, \\
\frac{\partial r^m(n, \tilde{n}^h, M, B, T)}{\partial B} &= \frac{1}{\beta} - \frac{1}{\sigma} + \frac{1}{\phi^p} - \frac{r^w}{2\sigma} \frac{\partial \tilde{\phi}^m(n, \tilde{n}^h, M, B, T)}{\partial B}.
\end{align*}
\]

52
So, we get that, if $0 < \phi^m < 1$,

\[
\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial B} = \frac{1 - \alpha - 2\phi^p \Phi^d}{1 + \alpha} \frac{\partial \tilde{r}^m}{\partial B} + \frac{2r^w}{1 + \alpha} \frac{\partial \tilde{r}^m}{\partial B} + \frac{2\Phi^d(\lambda - r^m - \phi^m r^w)}{1 + \alpha} \frac{\partial \phi^p}{\partial B}
\]

\[
+ \frac{2(\chi^m - \phi^p \Phi^d)w}{1 + \alpha} \frac{\partial \tilde{\phi}^m}{\partial B} = \frac{2\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} + \frac{r^w m}{1 + \alpha \sigma^2}.
\]

(We already derived $\frac{\partial D(n, \tilde{n}^h, M, B, T)}{\partial n} > 0$ above.) Thus, we get:

\[
\frac{\partial \phi^p}{\partial B} = -1 - \frac{r^w}{1 + \alpha \sigma^2} \frac{m}{(1 - \phi^p)^2} \frac{1/\beta - 1 - \lambda 1 - \alpha - 2\Phi^d}{(1 - \alpha^2)d^2},
\]

and adding the second term reflecting the reduction in reserves available to banks, we get:

\[
\frac{\partial \phi^p}{\partial B} = -\frac{mr^w}{\sigma^2} \frac{1/\beta - 1 - \lambda 1 - \alpha - 2\Phi^d}{(1 - \phi^p)^2} \frac{(1 - \alpha^2)d^2}{d^2}.
\]

Since $1/\beta - 1 - \lambda < 0$ and $1 - \alpha - 2\Phi^d < 0$ by definition, $\frac{\partial \phi^p(n^h, M, B, T)}{\partial B} < 0$.

Otherwise, if $\phi^m = 0$ or $\phi^m = 1$, $\frac{\partial \phi^p(n^h, M, B, T)}{\partial B} = 0$.  

□
B Figures

Figure 1: Repo Rates Spiking. The figure displays the time series for the DTCC GCF Repo Index tracking the average daily interest rate paid for the most-traded GCF Repo contracts for US Treasury General Collateral along with the lower and upper bounds for the Fed’s fed fund rate target range.
Source: Depository Trust & Clearing Corporation and Federal Reserve Economic Data.

Figure 2: Relationship Between Treasury GCR-IOER Spreads and Banks’ Repo lending. The figure provides a scatter plot representation of the relationship between week-to-week differences in the Treasury GCR-IOER spread, and aggregate banks reverse repo (lending) positions along with an OLS regression line computed excluding the four outliers. The sample period is 2010/01/01 to 2019/09/30.
Source: Depository Trust & Clearing Corporation
Figure 3: Evolution of the Liability Side of the Fed’s Balance Sheet. The figure displays the large increase in Fed’s reserves following QE waves, followed by a reduction driven by the tapering of Fed’s portfolio and the growth of other liability items.

Source: Federal Reserve Economic Data

Figure 4: Evolution of Total Outstanding and Fed Holdings of Treasuries. The figure displays the acceleration in the growth the supply of Treasury securities available to the public after the Trump election and the start of the tapering of the Fed balance sheet. The series was smoothed using an expanding window mean. The sample is between January 2013 and October 2019.

Source: Federal Reserve Economic Data and CRSP Treasury Data
Figure 5: Evolution of Domestic Banks and Non-banks Treasury Holdings. The figure shows the increase in US bank and non-bank portfolio of Treasury securities, with an acceleration in 2019. Non-bank holdings include hedge funds and security brokers-dealers. The sample is between January 2013 and October 2019.

*Source: Federal Reserve’s Z1 Financial Accounts*

Figure 6: Evolution of Non-bank Treasury Holdings and Bank Repo. The figure shows a sharp increase in both non-bank holdings of Treasury securities and repo lending from banks. Non-bank holdings include hedge funds and security brokers-dealers. The sample is between January 2013 and October 2019.

*Source: Federal Reserve’s Z1 Financial Accounts and H8 Banks Balance Sheet Reports*
Figure 7: Reserves Balance to Public Debt Outstanding Ratio and TGCF-IOR Spreads. The figure displays the scatter plot and OLS fitted line between these two series with significant repo spike dates highlighted. Source: Federal Reserve and Treasury Direct
Figure 11: No LST Benchmark This figure presents a qualitative depiction of an equilibrium in fed funds and repo markets under the assumption that the deposit shock is normally distributed and there is no LST constraint. The blue lines represent the demand for reserves and repo (in the left and right panels respectively). The red lines represent the supply of reserves and repo (in the left and right panels respectively).
Figure 12: Repo Markets with and without LST
The figure presents graphical representations of repo markets under various scenarios along with the amount of reserves lent in repo markets relative to excess reserves. The two upper panels display an economy that is not subject to the LST constraint. The two lower panels display an economy with relatively more excess reserves but are subject to the LST constraint. The right panels represent the two economies with a high repo demand.
Figure 13: Balance Sheet Reaction to Fiscal and Monetary Policy. This figure proposes a balance sheet representation of sectoral positions after changes in fiscal and monetary policy. Panel (a) represents the economy before any change as a baseline. Panel (b) represents the economy after the issuance of additional T-bonds when the Treasury hoards the proceed in its account with the Central Bank. Panel (c) presents the economy after the issuance of additional T-bonds when the Treasury redeems the proceed to households. Panel (d) represents the economy after the Central Bank has sold T-bonds. Shaded areas highlight changes relative to the benchmark.
<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>p5</th>
<th>p50</th>
<th>p95</th>
<th>max</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves ($bln)</td>
<td>2,101.23</td>
<td>408.32</td>
<td>1,338.12</td>
<td>1,503.82</td>
<td>2,150.09</td>
<td>2,867.42</td>
<td>3,252.80</td>
<td>1,185</td>
</tr>
<tr>
<td>Tot. Treasury Outstanding ($bln)</td>
<td>14771.13</td>
<td>1,633.66</td>
<td>12783.28</td>
<td>12972.87</td>
<td>14176.65</td>
<td>18766.69</td>
<td>19712.63</td>
<td>1,185</td>
</tr>
<tr>
<td>T-Bills Issuance ($bln)</td>
<td>35.26</td>
<td>58.92</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>156.01</td>
<td>272.86</td>
<td>1,185</td>
</tr>
<tr>
<td>Coupons Issuance ($bln)</td>
<td>10.60</td>
<td>32.73</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>104.37</td>
<td>218.27</td>
<td>1,185</td>
</tr>
<tr>
<td>Tot. Treas. Redemptions ($bln)</td>
<td>39.98</td>
<td>58.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>155.00</td>
<td>258.75</td>
<td>1,185</td>
</tr>
<tr>
<td>T-Bills Redemptions ($bln)</td>
<td>32.25</td>
<td>53.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>141.99</td>
<td>201.77</td>
<td>1,185</td>
</tr>
<tr>
<td>TGCF-IOER (bp)</td>
<td>0.57</td>
<td>18.60</td>
<td>-30.20</td>
<td>-19.70</td>
<td>1.50</td>
<td>16.90</td>
<td>390.70</td>
<td>1,185</td>
</tr>
<tr>
<td>MBS GCF-IOER (bp)</td>
<td>2.51</td>
<td>19.65</td>
<td>-26.70</td>
<td>-18.10</td>
<td>2.90</td>
<td>20.20</td>
<td>459.90</td>
<td>1,185</td>
</tr>
<tr>
<td>SOFR GCF-IOER (bp)</td>
<td>-6.81</td>
<td>14.83</td>
<td>-29.00</td>
<td>-24.00</td>
<td>-6.00</td>
<td>8.00</td>
<td>315.00</td>
<td>1,185</td>
</tr>
<tr>
<td>FX JPY-USD-IOER (bp)</td>
<td>26.55</td>
<td>88.05</td>
<td>-86.44</td>
<td>-15.93</td>
<td>9.34</td>
<td>95.42</td>
<td>1,221.87</td>
<td>1,030</td>
</tr>
<tr>
<td>Quarter End</td>
<td>0.02</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1,185</td>
</tr>
<tr>
<td>TaxDeadlineDays_t</td>
<td>0.02</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1,185</td>
</tr>
<tr>
<td>Low Balance</td>
<td>0.11</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1,185</td>
</tr>
</tbody>
</table>

**Table 1: Summary Statistics** The sample period is 2015-12-15 to 2020-09-09.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves ($bln)</td>
<td>-0.0153**</td>
<td>-0.0166**</td>
<td>-0.0118**</td>
<td>-0.00338</td>
<td>-0.0124**</td>
<td>-0.0113**</td>
<td>-0.0269**</td>
<td></td>
</tr>
<tr>
<td>Quarter End</td>
<td>29.95</td>
<td>23.89</td>
<td>22.71</td>
<td>7.968</td>
<td>296.9</td>
<td>7.968</td>
<td>296.9</td>
<td></td>
</tr>
<tr>
<td>T-Bills Issuance ($bln)</td>
<td>0.0809**</td>
<td>0.0582*</td>
<td>0.0227</td>
<td>0.0514</td>
<td>0.0617*</td>
<td>0.0722</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coupons Issuance ($bln)</td>
<td>0.114**</td>
<td>0.0940**</td>
<td>0.0526**</td>
<td>0.0834**</td>
<td>0.0763**</td>
<td>0.0671</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tot. Treas. Redemptions ($bln)</td>
<td>-0.0520</td>
<td>-0.0294</td>
<td>-0.0124</td>
<td>-0.0233</td>
<td>-0.0242</td>
<td>0.0126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TaxDeadlineDays_t</td>
<td>4.703</td>
<td>4.276</td>
<td>1.393</td>
<td>3.692</td>
<td>1.518</td>
<td>18.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Balance=1</td>
<td>372.1***</td>
<td>66.81***</td>
<td>460.3**</td>
<td>288.2**</td>
<td>236.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>32.73</td>
<td>31.87</td>
<td>31.24</td>
<td>21.59</td>
<td>13.60</td>
<td>79.58**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,185</td>
<td>1,185</td>
<td>1,185</td>
<td>1,185</td>
<td>1,185</td>
<td>1,185</td>
<td>1,185</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Regression Models of Money Market Spreads The dependent variables are the Treasury GCF, MBS GCF, SOFR and JPY-USD FX implied dollar rate over the IOER. Newey-West adjusted standard errors are reported in parenthesis. The sample period is 2015-12-15 to 2020-09-09.